A Distance Function Approach to Multifactor Productivity Measurement in U.S. Agriculture

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Abstract

A new procedure is developed to derive estimates of productivity. Distance function values are calculated between observed netputs and a reference technology constructed by augmenting observed netputs for quality changes. MFP growth rates average around 2% over the postwar period. Discrepancies occur between the distance function estimates and traditional measures.

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There is a continuing interest in both measuring and explaining productivity changes in agriculture. Two traditional approaches to the measurement of productivity are based on econometric methods or on index number approaches to quantify changes in aggregate output not explained by corresponding changes in aggregate inputs. Unfortunately, both of these approaches may misspecify the underlying structure of the production technology.

Restrictions arising from model specification and time series considerations can result in erroneous estimates of technical change using econometric approaches (Lambert and Shonkwiler, 1995). Although the adoption of flexible functional forms has improved the approximation properties of functional specifications, disparities may arise between various estimates of productivity depending upon specification (Guilkey, Lovell, and Sickles, 1983).

Index numbers may also impose structure on the underlying technology. For example, the Törnquist index presumes a translog technology, competitive behavior, Hicks neutral technical change, input-output separability, and constant returns to scale (Antle and Capalbo, 1988). Biases may be introduced into the technical change measure when the assumptions underlying the aggregator functions are violated (Capalbo, 1988).

Nonparametric approaches to measuring productivity have recently been developed to avoid these structural restrictions. One approach in the nonparametric measurement has been developed byChavas and Cox. The Chavas and Cox procedures derive estimates of netput technological augments consistent with Varian's weak axiom of profit maximization. The resulting augmentation estimates are then used to construct measures of multifactor productivity. Unfortunately, these MFP estimates themselves are dependent upon Hicks neutral technical change, a condition often violated when deriving the augmentation parameters.

More recently, distance function approaches have been applied to measuring MFP. The purpose of this paper is to present a new measure of technical change based on distance function measurement from a reference technology constructed to rationalize a time series of production observations. The reference technology is consistent by construction with WAPM, or the weak axiom of profit maximization. Varian (1984) has shown that if a data set satisfies WAPM, then there exists a closed, convex, negative monotonic production set that rationalizes the data. Although the exact nature of this production set cannot be determined, upper and lower bounds can be found on the region containing the true production set. Distance function measurements can then be calculated between these bounds and the observed netput bundles. These values then serve as bounds on the multifactor productivity of the observed bundles from the reference technology production possibilities set.

Identifying Changes in Technology

Consider the production possibilities set S available at time *t*:

[1] $$S^t = \{ (\mathbf{x}, \mathbf{y}) : \mathbf{x} \text{ can produce } \mathbf{y} \}$$

where $\mathbf{x} \in \Re^n$ is a vector of inputs and $\mathbf{y} \in \Re^m$ is a corresponding vector of outputs. S^t is conditional upon the technology available at time *t*. Consider the production possibilities set available at another time period *s*:

$$[2] \qquad S^s = \{ (\mathbf{x}, \mathbf{y}) : \mathbf{x} \text{ can produce } \mathbf{y} \}$$

Definition: Technical change has occurred between two periods *t* and *s* if $S^t \subset S^s$, but $S^s \not\subset S^t$.

Procedures to identify technical change obviously require representations of the production possibilities sets for two or more periods. Although an exact characterization of the production possibilities set is problematic, the existence of the set is confirmed if observed production relationships are consistent with WAPM (Varian, 1984). If we have a series of observations $(\mathbf{x}^i, \mathbf{y}^i)$, $i = 1,..., T$, consistency with WAPM requires $\mathbf{p}^i\mathbf{y}^i\text{-}\mathbf{r}^i\mathbf{x}^i \geq \mathbf{p}^i\mathbf{y}^j\text{-}\mathbf{r}^i\mathbf{x}^j$, for all i , j = 1, ..., T. In other words, decision makers made the profit maximizing choices of $(\mathbf{x}, \mathbf{y})$ for each set of prices they faced. If WAPM is satisfied, then there exists a closed, convex, negative monotonic production set that rationalizes the data.

Existence of a true production set does not provide information on the curvature of the production surface. However, lower and upper bounds can be placed on the true production set S. Employing Varian's terminology, the tightest inner bound of the true production set can be defined:

$$[3] \qquad SI = \overline{\text{com}} \{ (\mathbf{x}^i, \mathbf{y}^i) \quad \text{for all } i = 1, ..., T \},$$

where $com^{-}\{\}$ is the negative monotonic hull of S.

The tightest outer bound on the true production possibilities set is defined (Varian, 1984):

[4] $SO = \{ (\mathbf{x}, \mathbf{y}) : \mathbf{p}^i\mathbf{y} - \mathbf{r}^i\mathbf{x} \leq \mathbf{p}^i\mathbf{y}^i - \mathbf{r}^i\mathbf{x}^i \quad \text{for all } i = 1, ..., T\},$

Varian (1984) proves that SO rationalizes the data and envelopes all production sets S that rationalize the data.

Assume that a true production possibilities set S is known to rationalize a set of netput vectors. If a previously unobserved netput is not within the bounds, the new vector ($\mathbf{y}$, $\mathbf{x}$) might represent either inefficient production (i.e., ($\mathbf{y}$, $\mathbf{x}$) is strictly contained in SI) or infeasible production bundles given the technology underlying S (($\mathbf{y}$, $\mathbf{x}$) $\notin$ SO). In this manner, the distance of a candidate netput vector from set S might represent changes in MFP resulting from outward shifts in the production possibilities surface.

The technology underlying production set S can be completely described by the output distance function (Färe and Primont, 1994):

[5] $$D_o^t(\mathbf{x}, \mathbf{y}) = \inf\{\theta : (\mathbf{x}, \mathbf{y}/\theta) \in S^t\}$$

$$= (\sup\{\theta : (\mathbf{x}, \theta\mathbf{y}) \in S^t\})^{-1} .$$

The output distance function is the reciprocal of the maximum proportional expansion in output $\mathbf{y}$ given $\mathbf{x}$. Values of $D_o^t(\cdot)$ less than 1 will lie within the boundary of S^t, implying that production associated with this observation is inefficient. Output could be increased to **y**/θ given inputs **x**. Production on the frontier is efficient, resulting in $D_o^t(\cdot)$ equaling 1. Values of $D_o^t(\cdot)$ greater than 1 cannot be produced given S^t. θ would indicate the minimal shrinkage of **y** to be on the boundary of S^t. In this case, the distance function value represents an increase in MFP between the periods, or how much additional output is feasible in period *s* than in period *t* for a given factor complement.

Most, if not all, studies employing a distance function approach to productivity measurement define changes in MFP relative to the inner bound representation of technology constructed from panel or cross-sectional data (Färe et al., 1994; Bureau et al., 1995). We turn next to a procedure to estimate productivity changes based upon a distance function approach when panel data are not available.

Creation of a Reference Technology

Creation of a reference technology S using time series data is problematic since only one observation per period is available. However, production sets can be compared to a common reference technology. Although this technology can be arbitrary (Malmquist, 1953), we propose constructing a reference technology consistent with the minimal behavioral assumption of profit maximization.

It is often found in empirical work that WAPM is violated when comparing earlier to later time periods. Technical change has occurred that render what would be profit maximizing choices using year *s* technology unachievable in year *t*. The quality of the netputs has somehow been affected by technical change. The augmentation hypothesis asserts that observed netputs actually can be decomposed to a quantity value consistent with other observed levels of the netput and a netput augment representing the state of technology (Chavas and Cox, 1990).

Consider a series of observed production bundles ($\mathbf{x}^i$, $\mathbf{y}^i$), $i = 1, ..., T$. Call the vectors of effective quantities, ($\mathbf{x*}^i$, $\mathbf{y*}^i$), where $\mathbf{y*} = y(\mathbf{y}, \mathbf{a})$, and $\mathbf{x*} = x(\mathbf{x}, \mathbf{b})$. The functions y and x are one-to-one correspondences between each input and output. Augmentation factors are sought that would result in $\mathbf{p}^i\mathbf{y*}^i\textbf{-}\mathbf{r}^i\mathbf{x*}^i \geq \mathbf{p}^i\mathbf{y*}^j\textbf{-}\mathbf{r}^i\mathbf{x*}^j$, i , j = 1,...,T. Values of **a** and **b** that satisfy the inequalities for all i and j indicate the existence of a stable production technology over the entire period,

[6] $$S^* = \{ (\mathbf{x*}^i, \mathbf{y*}^i) : \mathbf{x*}^i \text{ can produce } \mathbf{y*}^i \text{ for all } i = 1, ..., T\}$$

The proof of [6] follows directly from Varian's theorem 3 (1984, page 584).

The relationship between S* and observed netput vectors ($\mathbf{x}^t$, $\mathbf{y}^t$), t = 1,..., T, can indicate the extent to which factor-product relationships have changed from the reference technology. A measure of productivity change for period t can result by finding the distance function from ($\mathbf{x}^t$, $\mathbf{y}^t$) to the reference technology:

[7] $$m\left(\mathbf{x}^{\mathbf{t}}, \mathbf{x*}, \mathbf{y}^{\mathbf{t}}, \mathbf{y*}\right) = \mathrm{D}_{\mathrm{o}}^{*}\left(\mathbf{x}^{\mathbf{t}}, \mathbf{y}^{\mathbf{t}}\right) = \left(\sup\left(\theta : \left(\mathbf{x}^{\mathbf{t}}, \theta\, \mathbf{y}^{\mathbf{t}}\right) \in \mathrm{S}^*\right)\right)^{-1}$$

The observed netput bundles are evaluated with respect to the reference technology S*. For a given observation, the productivity index will be less than, equal to, or greater than 1. If, for example, the reference technology approximates actual production in the middle of the sample period, distance function values will be less than (greater than) 1 when comparing earlier (later) observed production bundles with the reference technology. The index will thus trend upwards in the event of nonregressive technical change.

The distance of a particular observation ($\mathbf{x}^t$, $\mathbf{y}^t$) from the inner bound SI* can be found by solving of the following linear programming problem (Färe et al., 1994):

[8] $$\left(D_o^{SI*}\left(\mathbf{x^t}, \mathbf{y^t}\right)\right)^{-1} = \text{Max} \quad \theta$$

$$\text{subject to} \qquad \theta\ \mathbf{y^t} \le \sum_{i=1}^{T} \lambda_i\ \mathbf{y}*^i$$

$$\sum_{i=1}^{T} \lambda_i\ \mathbf{x}*^i \le \mathbf{x^t}$$

$$\lambda_i\ , \theta \ge 0$$

The output distance function with respect to the outer bound will measure the minimum proportional change in output necessary for situating a candidate observation, ($\mathbf{x}^t$, $\mathbf{y}^t$) on the surface of SO*. This deflation factor ϕ can be found in a similar fashion to the distance function calculations under SI:

[9] $$\left(D_o^{SO*}\left(\mathbf{x^t}, \mathbf{y^t}\right)\right)^{-1} = \text{Max} \quad \phi$$

subject to $\mathbf{p}^i (\phi\ \mathbf{y}^t) - \mathbf{r}^i\mathbf{x}^t \le \mathbf{p}^i\mathbf{y*}^i\mathbf{-r}^i\mathbf{x*}^i$ for all i = 1, ..., T

Construction of a WAPM consistent set of effective inputs and outputs requires finding solution values of $\mathbf{a}$ and $\mathbf{b}$ to $\mathbf{p^t}$ [$y(\mathbf{y}^t, \mathbf{a}^t) - y(\mathbf{y}^s, \mathbf{a}^s)$] $\mathbf{- r}^t$ [$\text{x}(\mathbf{x}^t, \mathbf{b}^t) - \text{x}(\mathbf{x}^s, \mathbf{b}^s)$] ≥ 0 for all t, s = 1, ..., T, $t \neq s$. We use the scaling hypothesis in augmenting observed netputs so that $y(\mathbf{y}^t, \mathbf{a}^t) = \mathbf{y}^t \cdot \mathbf{a}^t$. The objective function minimizes the sum of absolute deviations between the observed and effective netputs, or $\min \sum_{t=1}^{T} \left(\sum_{i=1}^{m} |a_{it}| + \sum_{i=1}^{n} |b_{it}| \right)$, subject to the effective netputs satisfying WAPM.

An Application to U.S. Agriculture

We use Fisher price and quantity indices recently compiled by Ball et al. (1996) for U.S. agriculture from 1948 through 1994. We used a single measure of output and 13 input categories (hired labor, self-employed labor, durables, real capital, inventories, energy, chemicals, feed, seed, purchased livestock, services, onfarm consumption, and miscellaneous inputs). Preliminary examination of the data found 1,091 violations of WAPM in the 2,162 interyear comparisons. The violations were consistent with technological change affecting the production set. 91.9 percent of the backwards comparisons ($s < t$) were consistent with WAPM. However, 92.8 percent of the forward comparisons ($t < s$) violated WAPM. Adjustments for technical change would thus appear to be warranted.

Netput augments are presented in figures 1 and 2. By minimizing the sum of absolute errors, augments are centered around a presumed median of zero. Therefore, earlier period output augments tend to be positive, indicating that observed outputs are strictly contained within the reference technology S*. For example, 1948 observed output would need to be increased by a factor of 1.42 to be on the frontier of S*. The augments estimated in this study are similar to those derived in Cox and Chavas (1990). An assertion of Hicks neutrality is not supportable. Input augments affect real capital, and, to lesser extent, self-employed labor, durable equipment, feed, and onfarm consumption.

The presence of biases in technical change undermines the MFP measures based on the Törnquist index and the nonparametric measure developed in Cox and Chavas (1990). The distance function approach proposed here is not restricted to the Hicks neutrality

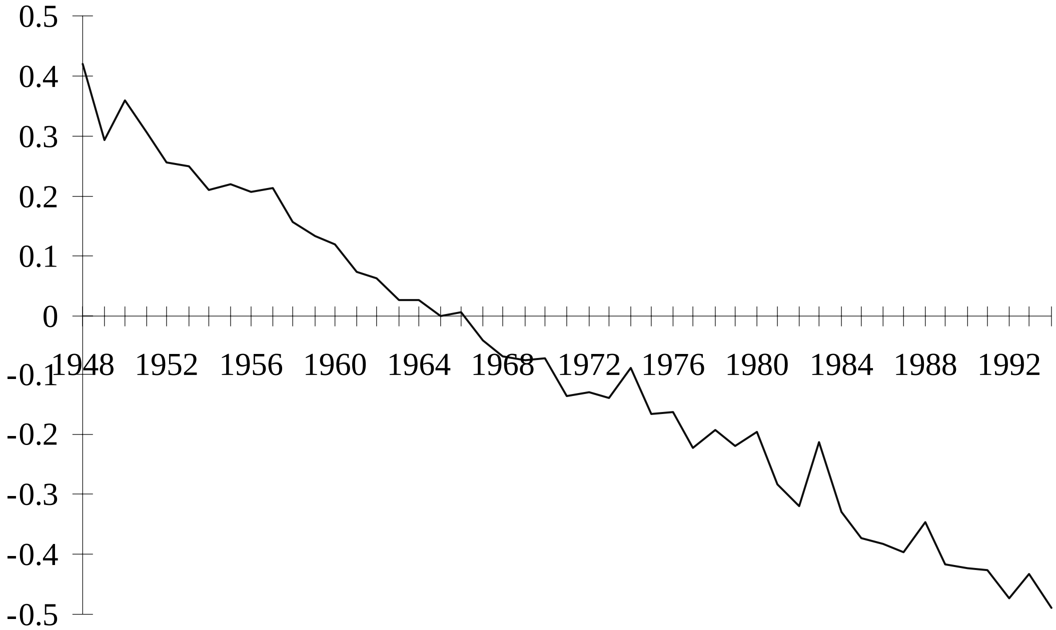

Figure 1. Output augments for aggregate farm production.

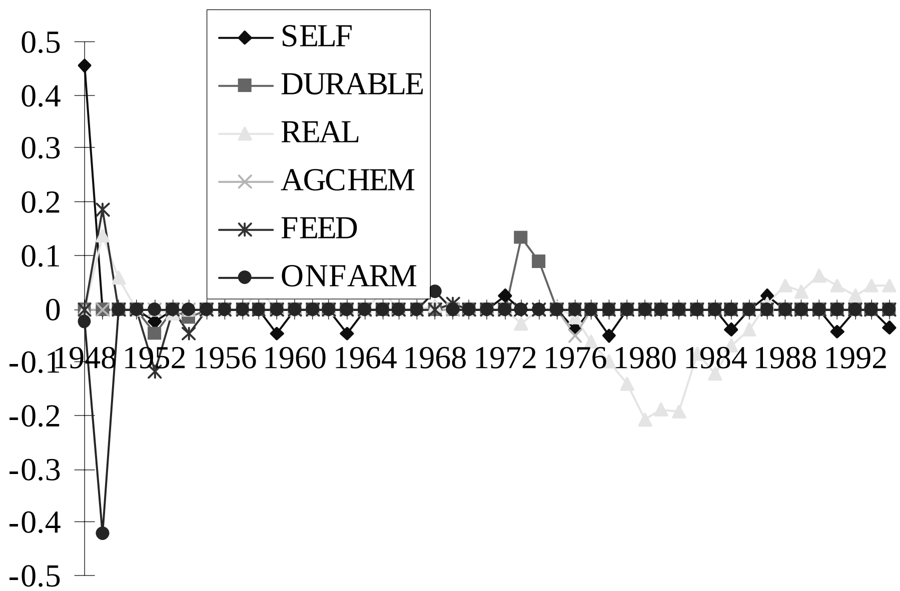

Figure 2. Input augments for self-employed labor, durable equipment, real capital, agricultural chemicals, feed, and onfarm consumption.

assertion, and thus may provide a less restrictive measure of technical change not predicated upon Hicks neutrality or constant returns to scale.

Comparisons of Three Approaches to Estimating Multifactor Productivity

Three measures of multifactor productivity changes were estimated: an econometric model, the Törnquist index, and the distance function measures developed in this paper. The econometric approach utilized Capalbo's model 2 (Capalbo, 1988, page 165). A single output translog cost function was estimated simultaneously with 5 of the 6 share equations corresponding to six inputs: labor, durable equipment, real capital, inventories, materials, and onfarm consumption. This input aggregation resulted from explicitly testing for input separability using Varian's nonparametric tests (Varian, 1984).

We followed Capalbo's imposition of symmetry and linear homogeneity in input prices. This model does allow for input bias in technical change.

The Törnquist index was calculated using the standard formulation,

$$[10] \quad \ln\left(\frac{TFP_t}{TFP_{t-1}}\right) = \ln\left(\frac{y_t}{y_{t-1}}\right) - 0.5\sum_i \left(S_{it} + S_{i,t-1}\right)\ln\left(\frac{x_{it}}{x_{i,t-1}}\right)$$

Distance function values from the reference technology set S* were derived by solving the linear programming problems [8] and [9]. MFP estimates resulting from all three approaches are illustrated in figure 3.

Except for the first two years, the inner and outer bound distance function measures track one another very closely. Growth in MFP has been fairly consistent over the period. The Törnquist measure deviates from the two DF measures in most years

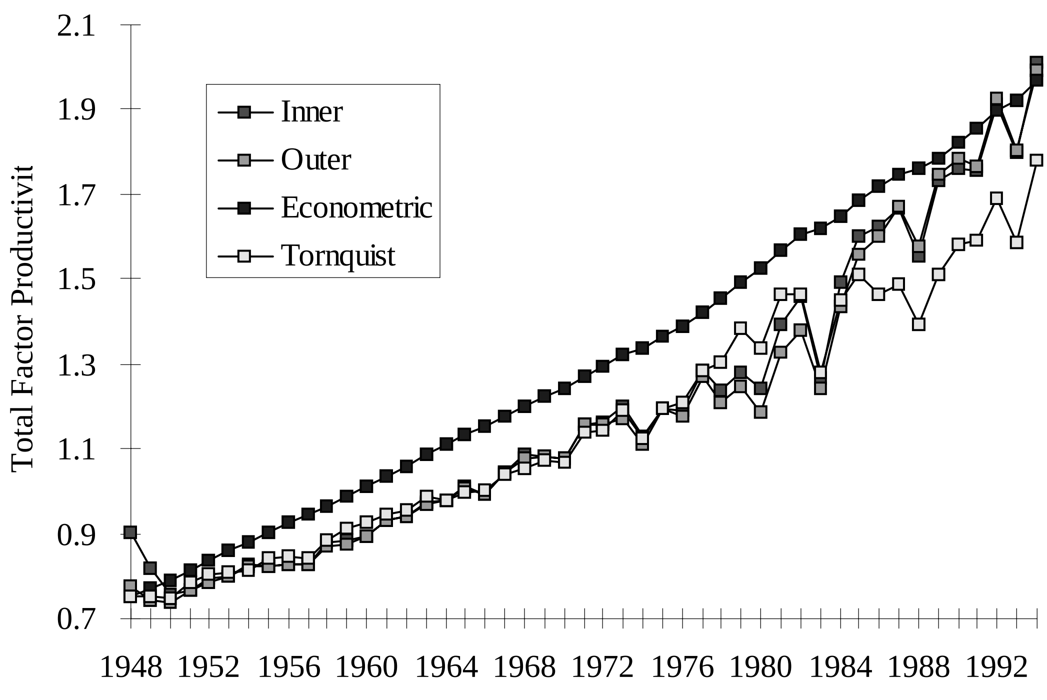

Figure 3. Comparisons of total factor productivity.

from about 1977 onwards. This deviation may result from theTörnquist measure assumption that technical change is Hicks neutral. Figure 2 illustrates that bias in real capital occurs in every year from 1976 onwards. These input augments indicate Hicks neutrality is not a valid assertion, and an MFP measure dependent upon this assertion may yield biased estimates.

The rather simple deterministic trend underlying the econometric model results in relatively stable estimates of annual technical change. It is interesting, however, that the econometric estimate intersects the DF measures at the beginning and the end of the period. This results in the similarities among all three approaches in terms of rates of annual change in MFP (Table 1).

Table 1. Annual MFP Growth Rates Measures for U.S. Agriculture, 1948-94

	Inner DF	Outer DF	Törnquist	Econometric	Ball et al.
1948-94	1.920	2.200	2.001	2.115	1.997
1949-54	-1.298	1.045	1.342	2.663	1.343
1955-64	1.708	1.744	1.878	2.337	1.886
1965-74	1.548	1.333	1.443	1.899	1.430
1975-84	3.165	2.903	2.820	2.111	2.809
1985-94	3.190	3.512	2.257	1.786	2.255

Few studies other then the present and Ball et al. (1996) have estimated MFP past the mid-80's. The earlier studies find annual MFP growth in the range of 1.3-1.8% from the late 1940s through the early-1980s (Capalbo, 1988a). These values are fairly close to the estimates derived using the distance function approach (Table 1). Trends underlying the Ball et al. data indicate a greater rate of average MFP growth from 1975 on, regardless of procedures used to generate the estimates. Further research into the recently observed increases in MFP are certainly warranted. The adoption of precision agriculture techniques, other computer-assisted decision and farming technologies, and the effects of farm policies such as CRP may be potential contributors to the observed trends.

Conclusions

This paper has presented a procedure to estimate multifactor productivity based on distance function measurements from a reference technology constructed from a time series. The procedure imposes no restriction upon technology other than the reference technology satisfies the weak axiom of profit maximization. The estimates of MFP are seen to closely track other measures, such as the Törnquist. Discrepancies in levels appear to occur, however, in those years in which the assumptions of the Törnquist are violated.

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