Nonpoint Source Pollution, Incomplete Information and Learning: An Entropy Approach

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Abstract

Kullback's Cross Entropy, a methodology for modeling incomplete information and learning, is applied to nonpoint source pollution management. By definition, incomplete information on the linkages between nonpoint source and load exists. We have explicitly model monitoring and learning, to focus attention on the manager's budget tradeoff between monitoring and abatement.

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1. Introduction

Recently, policy makers have turned to the problem of nonpoint source (NPS) pollution in an effort to further improve water quality. In the past, most of the implemented control measures have focused on the easier problem of reducing point source pollution (Congdon et al.; Helfand; Larson et al.). However, the majority of the nation's waters are polluted by NPS (USEPA).

In this paper, we provide a methodology for empirically modeling NPS control in the presence of incomplete information and learning. By definition, incomplete information on the linkage between source and load exists for NPS, and thus creates uncertainty on the efficient abatement level. Over time, the manager can reduce uncertainty on the linkages by learning from the collected information. Ultimately, the manager decides if reducing uncertainty warrants monitoring, and if so, at what frequency.

Two empirical applications motivate this research: Redwood National Park, Orick, CA and the Hoopa Valley Indian Reservation, Hoopa, CA. In both cases, logging activity and related road networks have altered the "natural" rate of sediment loading. If perfect information exists on the linkages, the managers can allocate all of their limited budgets to abatement control. With incomplete information, the management of sediment loading requires an explicit or implicit allocation of resources between information collection and abatement control.

The information contained in the collected data is fundamental to the analysis. For instance, if all collected data contains information, then monitoring expenditures are justified if the marginal value of monitoring equals or exceeds the monitoring costs. If however, there is no information in the collected data the marginal value of monitoring is equal to zero. Furthermore, we may expect that the manager will learn about the linkages between source and load over time and thus further reduce the uncertainty and monitoring expenditures.

In this analysis, the imperfect information is defined to be one of incomplete information. Pollution control problems have typically been cast as non-cooperative, asymmetric information games because the private polluter has better information on the cost of abatement than the water quality manager (Russell et al.; Garvie and Keeler; Harford; Harford and Harrington; and Romstad and Bergland). However, when polluting activities have ceased or logging related activities are publicly controlled, as is the case in Redwood National Park and the Hoopa Valley Indian Reservation, respectively, asymmetric information does not exist. The managers and the polluters are not fully informed on each sources' contribution to the pollution load but each knows the cost of abatement.

By explicitly modeling monitoring and learning into a pollution control model, the analysis allows us to focus on the tradeoffs between information content and abatement productivity. In the analysis, Kullback's Cross Entropy (CE) formalism, a method for modeling information and learning when data is ill-posed, is applied to sediment load management. Ill-posed statistical problems arise when the number of parameters to be estimated exceeds the data points available. Given the limited data from ambient pollutant measures in a river and the large number of possible combinations of sources and pollution loads, the estimation of relationships among NPS pollution problems is often ill-posed.

In the following section, a brief literature review is provided. Section 3. describes the NPS management model with incomplete information and learning. Section 4. provides a discussion on the entropy approach and it's application to NPS modeling. Section 5 concludes with some final remarks.

2. Literature Review

Economic analysis of the pollution problem has been slow to consider the problem of managing or enforcing water quality standards. The economic literature on controlling point and NPS pollution has mostly analyzed the economic efficiency of alternative policy instruments such as command and control verse pigouvian taxes or tax/refund incentives (see, for example, Griffin and Bromley; Shortle and Dunn; Segerson; Helfand; Malik et al.; and Larson et al.). Past empirical work on pollution control is limited. LaPointe and Rilstone (1996) and Magat and Viscusi (1990) analyze the impact that inspections have on emissions from pulp and paper mills. Gray and Deily consider manager behavior in a similar empirical analysis of point source pollution control.

After a thorough review of the literature, it appears that the explicit modeling of information collection and learning in NPS analyses has been neglected. Cabe and Herriges provide a theoretical model of NPS with Bayesian learning in a social welfare framework. They do not explicitly constrain the manager to work within a limited budget. Kolstad (1996a) describes learning as either, active, purchased or passive depending upon how the information is collected. In his work, Kolstad (1996b) examines passive learning in the case of climate change. In our case of NPS, we analyze active learning where the manager actively collects information for the purpose of reducing the uncertainty on the sediment loading parameters.

3. Management Model

Recently, Amacher and Malik introduced the pollution control problem as a cooperative game in light of the bargaining potential that exists and can be observed in actual management strategies. The manager reduces asymmetric information by bargaining technology changes for less compliance. In our cooperative management model, the manager does not rely on bargaining but rather on strategic monitoring to reduce the incomplete information.

In the model, the manager has imperfect knowledge of the pollution each source contributes to the ambient water quality, and must rely on estimated sediment loading parameters $q = (q_1, q_2, q_3, \ldots\ldots, q_I)$, where q_i is the sediment loading from source i. To reduce the uncertainty on the sediment loading parameters, the manager takes samples in addition to an initial downstream sample. These samples are taken during high storm events with frequency, α. After the rain season has ended, the manager chooses the abatement effort for each source , $X = (x_1, x_2, x_3, \ldots\ldots, x_I)$, in order to maximize abatement.

We now introduce an information processing rule $I(q(\alpha))$. In deriving curvature properties for the information process it is assumed that every monitoring sample has some information content. The curvature properties are explicitly derived below.

First we consider

$$\frac{dI}{d\alpha} = \left(\frac{\partial I(q(\alpha))}{\partial q}\right)\left(\frac{\partial q(\alpha)}{\partial \alpha}\right) > 0\,.$$

Because I is an efficient information processing rule, when monitoring increases and the expected sediment loading parameter decreases we have gained information. Similarly, if increased monitoring raises the expected contribution from a site we have again increased the information. Thus, the signs on the partial derivatives have the same sign.

To derive the sign on the second derivative on the information rule,

$$\frac{d^2 I}{d\alpha^2} = \left(\frac{\partial^2 I(q(\alpha))}{\partial q(\alpha)^2}\right)\left(\frac{\partial q(\alpha)}{\partial \alpha}\right)^2 + \left(\frac{\partial I(q(\alpha))}{\partial q(\alpha)}\right)\left(\frac{\partial^2 q(\alpha)}{\partial \alpha^2}\right) < 0\,,$$

we choose an information processing rule that is also a shrinkage rule. This implies that

$$\frac{d^2 I}{dq^2} < 0, \frac{d^2 q}{d\alpha^2} < 0,\ \text{if}\ \frac{dq}{d\alpha} > 0\ \text{and}\quad \frac{d^2 q}{d\alpha^2} > 0\,,\ \text{if}\ \frac{dq}{d\alpha} < 0\,.$$

So, given a proper shrinkage rule, which cross entropy satisfies, the first right hand side term is always negative. Now, recall $\frac{dI}{dq}$ and $\frac{dq}{d\alpha}$ have the same sign. Since $\frac{d^2 q}{d\alpha^2}$ and $\frac{dq}{d\alpha}$ have opposite signs, the second right hand side term is also negative and thus the second derivative on information with respect to monitoring is always negative.

With monitoring, the manager can improve her information on each source. Under uncertainty, the manager chooses α and X to maximize expected abatement based on the information she has on each source's contribution. The objective function for the management model defines an expected abatement production function where the productivity depends on the chosen level of abatement and the expected level each source contributes to the ambient water quality. The ability to collect information and implement abatement technology is constrained by the management budget. The abatement production function $f(X; I(q(\alpha)))$ is twice differentiable with the following curvature: $f_I>0, f_x>0, f_{xx}<0, f_{xI}>0$. The manager's objective function is:

$$\text{Max } \Omega(X, \alpha) = f_e(X, I(q(\alpha))) \tag{1}$$

s.t.

$$\alpha m + cX' = B \tag{2}$$

where, m is the per unit monitoring cost, $c = (c_1, c_2, c_3, \ldots\ldots, c_I)$ is the per unit cost of abatement effort at each source, B is the annual budget and λ is the Lagrangian multiplier on (2).

The first order conditions are:

$$\left(\frac{\partial f_e(X, I(q(\alpha)))}{\partial I}\right)\left(\frac{\partial I(q(\alpha))}{\partial q}\right)\left(\frac{\partial q(\alpha)}{\partial \alpha}\right) = \lambda m \tag{3}$$

and

$$\frac{\partial f_e(X, I(q(\alpha)))}{\partial X} = \lambda c' \tag{4}$$

From (3) we see that the manager optimally allocates resources such that the marginal benefit from monitoring equals the marginal cost of monitoring. From (4) we see that resources are allocated optimally when the marginal abatement productivity equals the marginal abatement cost. Equating (3) and (4) shows that at the optimum, the manager will recursively choose α and X such that the ratio of marginal benefit and cost is equal across abatement and monitoring efforts.

Decomposing equation (3) we see that there are three components. The first component is the reduction in uncertainty given the change in information on sediment loading parameters. The second and third components shows information changes with a change in the monitoring frequency. These latter components have been difficult to obtain empirically. The following section unravels this ill-posed problem.

4 The Ill-Posed Problem and the Entropy Approach

Although there is data to analyze the NPS pollution problem, it is insufficient for conventional estimation. Aggregate NPS data yields an estimation problem that is generally ill-posed. Given the limited data from ambient pollutant measures in a river and the large number of possible combinations of sources and pollution loads, the estimation of relationships among NPS pollution problems is often ill-posed.

Shannon and Jaynes established the maximum entropy (ME) approach as a logical basis for making inferences from ill-posed problems. Kullback showed how the cross entropy (CE) approach incorporates prior information to reconstruct posterior distributions consistent with ill-posed data. ME-CE is a recent innovation in applied economic analyses, and has yet to be used in the estimation of NPS pollution parameters, particularly, the value of information.

ME-CE provide several benefits over traditional econometric techniques when faced with ill-posed problems. First, estimating parameters with limited data requires restrictive assumptions. With ME-CE, no underlying functional form is necessary and a non-parametric distribution around the parameters is derived from the available data. What ME-CE does is reconstruct the probability distribution most likely to have produced the observed data. Second, formal modeling of Bayesian learning by integrating over joint density functions is often very complicated and time consuming. Fortunately, CE is consistent with Bayes Rule and thus is an efficient information processing rule (Golan et al. 1996). Given that CE is easier to empirically model and has desirable consistency properties, it is employed in the reconstruction of the unknown sediment loading parameters.

As mentioned, entropy has been applied to a few economic problems. For example, Golan et al. (1993) use limited incomplete multisectoral economic data to recover expenditure, trade and income flows. Paris and Howitt apply a ME approach to ill-posed production data. Their study considers the typical data set of multiproduct/multi-input production where the available data consists of total input use. In their analysis they reconstruct input cost shares for each of the output products.

To illustrate the ill-posed problem, suppose the ambient water quality samples are taken which measure turbidity, an indicator of suspended sediment resulting from soil erosion within the watershed. Further, suppose that there are seven identified pollution sources. When the manager sets up the monitoring sites, she equips each site to take multiple samples to measure ambient levels during high storm events. For each storm, assume the manager has optimally decided that each monitoring site collects three samples at fixed intervals. Figure 1 illustrates the matrix of sediment loading parameters.

Figure 1: Matrix of Sediment Loading Parameters.

	S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**Total Loading**
Turbidity1	q_1+e_{11}	q_2+e_{12}	q_3+e_{13}	q_4+e_{14}	q_5+e_{15}	q_6+e_{16}	q_1+e_{17}	Q_1
Turbidity2	q_1+e_{21}	q_2+e_{22}	q_3+e_{23}	q_4+e_{24}	q_5+e_{25}	q_6+e_{26}	q_1+e_{27}	Q_2
Turbidity3	q_1+e_{31}	q_2+e_{32}	q_3+e_{33}	q_4+e_{34}	q_5+e_{35}	q_6+e_{36}	q_1+e_{37}	Q_3

This problem is ill-posed. The task we face is to recover the 7 sediment loading parameters from each ambient sample. The relationship is written as:

$$Q = Sq' + e$$

where we observe $S = (S_1, S_2, S_3, \ldots\ldots, S_I)$ the potential pollution sources, and $Q = (Q_1, Q_2, Q_3)'$ the ambient water quality level for each sample, with $e = (e_1, e_2, e_3)'$, the measurement error where $e_j = \sum_{i=1}^{7} e_{j,i}, \forall j, j = 1,2,3.$

The more structure we can place on the matrix, the smaller the space in which the expected parameters are present. First we know that the horizontal sum of each row must equal the ambient level, *Q* with noise, *e*. Furthermore, since we have three samples to analyze we can infer that the sediment loading parameter for each source and each sample will not change given the short time between sample collection. In order to incorporate learning and derive posterior distributions on the sediment loading parameters we turn to the cross entropy (CE) approach.

The CE approach minimizes the entropy between a prior estimate and the reconstructed probability. If the cross entropy measure is greater than zero we have gained information on the prior and thus learning has occurred. In the presence of repeated samples, cross entropy acts as a shrinkage rule so that the reconstructed probability approaches the true probability as the sample size approaches infinity (Golan et al., 1996).

Reparameterizing the problem for a generalized cross entropy (GCE) optimization model, discrete random variables are constructed with prior weights, *z* and *V*, over k finite supports that reflect non-sample information about q and *e* (Golan et al., 1996), which yields:

$$Q_j = \sum_{i=1}^{7} S_i \sum_{k=1}^{K} z_{i,k} \beta_{i,k} + \sum_{k=1}^{K} V_{j,k} w_{j,k} \quad \forall j, j=1,2,3$$

where, $q_i = \sum_{k=1}^{K} z_{i,k} \beta_{i,k}$ and $e_j = \sum_{k=1}^{K} V_{j,k} w_{j,k}$.

Now, suppose β_o is the prior on the source probabilities. The GCE program for each sample is written as:

$$\underset{\beta,w}{Min}\left(\sum_{i=1}^{7} \sum_{k=1}^{K} \beta_{i,k} \ln\left(\beta_{i,k} \big/ \beta_{o,i} \right) + \sum_{k=1}^{K} w_k \ln(w_k) \right)$$

$s.t.$

$$Q = \sum_{i=1}^{7} S_i \sum_{k=1}^{K} z_{i,k} \beta_{i,k} + \sum_{k=1}^{K} V_k w_k \quad \forall j, j=1,2,3 \tag{5}$$

$$\sum_{i=1}^{7} q_i = Q_j \quad \forall j, j=1,2,3 \tag{6}$$

To reconstruct learning in the management model, we let the manager choose the number of samples to be taken during each storm event. The cross entropy problem now relies on the previous sample estimates on q as the priors.

With these priors, the manager will optimize:

$$\sum_{j=1}^{J} \underset{\beta,w}{Min} \left(\sum_{i=1}^{7} \sum_{k=1}^{K} \beta_{j,i,k} \ln\left(\beta_{j,i,k} \big/ \beta_{j-1,i,k} \right) + \sum_{k=1}^{K} w_{j,k} \ln(w_{j,k}) \right)$$

$s.t$

(5)-(6).

With this model then we can empirically model the learning that occurs from repeated samples. These results can now be incorporated into the management model defined in (1)-(4).

5. Concluding Remarks

The model presented in this paper provides an empirical method for deriving the value of information and learning for NPS pollution control. Given this theoretical and methodological basis the research can be empiricized by collecting monitoring and abatement data from currently engaged management programs and apply ME-CE approach in order to estimate the value of information and learning. Currently, data is being collected from Redwood National Park and the Hoopa Valley Indian Reservation, where logging and related practices have degraded water quality by increasing the sediment loading in the respective watersheds. Both of these cases are unique in that the information problem is one of incomplete information only, since logging has ceased in the Redwood case and in the Hoopa case the logging activities and water quality management are coordinated through the Tribal government.

This research shows how empirical measures of the value of information for NPS situations can be estimated. We also intend to emphasize the often neglected cost of monitoring in designing efficient management programs. An obvious extension of this research will address asymmetric information in NPS pollution problems.

References

Amacher, G.S., and A.S. Malik. "Bargaining in Environmental Regulation and the Ideal Regulator." *Journal of Environmental Economics and Management* 30(1996):233-253.

Cabe, R. and J.A. Herriges. "The Regulation of Non-Point Source Pollution Under Imperfect Information and Asymmetric Information." *Journal of Environmental Economics and Management* 22(1992):134-146.

Congdon, C.H., T.F. Young and B.E. Gray. "Economic Incentives and Nonpoint Source Pollution: A Case Study of California's Grassland Region." *Hastings West-Northwest Journal of Environmental Law and Policy* 2(Spring 1995):185-247.

Gray, W.B., and M.E. Deily. "Complinace and Enforcement: Air pollution Regulation in the U.S. Steel Industry." *Journal of Environmental Economics and Management* 31(1996):96-111.

Garvie, D., and A. Keeler. "Incomplete Enforcement with Endogenous Regulatory Choice." *Journal of Public Economics* 55(1994):141-62.

Golan, A., Judge, G. and D. Miller. *Maximum Entropy Econometrics: Robust Estimation with Limited Data*. John Wiley and Sons, 1996.

Golan, A., Judge, G., and S. Robinson. "Recovering Information from Incomplete or Partial Multisectoral Economic Data." Department of Agricultural and Resource Economics, University of California, Berkeley, September 1993.

Griffin, R.C. and D.W. Bromley. "Agricultural Runoff as a Nonpoint Externality: A Theoretical Development." *American Journal of Agricultural Economics* 64 (1982):547-52.

Harford, J.D. "Measurement Error and State-Dependent Pollution Control Enforcement." *Journal of Environmental Economics and Management* 23(1991):67-81.

Harford, J.D., and W. Harrington. "A Reconsideration of Enforcement Leverage when Penalties are Restricted." *Journal of Public Economics* 45(1991):391-95.

Helfand, G.E. 1994. "Alternative pollution Standards for Regulating Nonpoint Source Pollution." *Journal of Environmental Management* 45(1994):231-241.

Jaynes, E.T. "Information Theory and Statistical Mechanics." *Physics Review* 106(1957):620-30.

Kolstad, C.D. “Learning, Irreversibilities and Climate.” paper prepared for Seventh Annual Conference of the European Association of Environmental and Resource Economists. Lisbon, Portugal. June 27-29, 1996,a.

Kolstad, C.D. “Learning and Stock Effects in Environmental Regulation: The Case of Greenhouse Gas Emissions.” *Journal of Enviromental Economics and Management* 31(1996,b):1-18.

Kullback, J. *Information Theory and Statistics*. New York: John Wiley & Sons, 1959.

LaPointe, B. and P. Rilstone 1996. “Environmental Inspections and Emissions of Pulp and Paper Industry in Quebec.” *Journal of Environmental Economics and Management* 31(1996):19-36.

Larson, D.M., Helfand, G.E. and B.W. House, “Second-Best Tax Polices to Reduce Nonpoint Source Pollution.” *American Journal of Agricultural Economics* 78(November 1996): 1108-1117.

Magat, W.A. and W.K. Viscusi. “Effectiveness of the EPA’s Regulatory Enforcement: The Case of Industrial Effluent Standards.” *Journal of Law and Economics* 33(1990):331-60.

Malik, A.S., Larson, B.A. and M. Ribaudo. *Agricultural Nonpoint Source Pollution and Economic Incentive Policies: Issues in Reauthorization of the Clean Water Act,* , Resources and Technology Division, Economic Research Service, US Department of Agriculture, Staff Report No. AGES 9229, US Government Press, Washington DC. 1992.

Paris, Q., and R.E. Howitt. “The Analysis of Ill-Posed Problems in Production Economics.” presented paper at the annual meeting of the American Agricultural Economics Association, San Antonio, Texas, July 28-31, 1996.

Romstad, E., and O. Bergland. “Inducing Individual Firm Compliance to Emission Quotas When Abatement Costs are Private Knowledge.” Department of Economics and Social Sciences, The Agricultural University of Norway, 1993.

Russell, C.S., Harrington, W., and W.J. Vaughan. *Enforcing Pollution Control Laws*. Washington, DC: Resources for the Future, 1986.

Segerson, K. “Uncertainty and Incentives for Nonpoint Source Pollution Control.” *Journal of Environmental Economics and Management* 15(1988):87-98.

Shannon, C.E. “A Mathematical Theory of Communication.” *Bell System Technical Journal* 27(1948):379-423.

Shortle, J.S. and J.W. Dunn. "The Relative Efficiency of Agricultural Source Water Pollution Control Policies." *American Journal of Agricultural Economics* 68(1986):668-77.

United States Environmental Protection Agency. *Section 319 National Monitoring Program: An Overview, March 1995,* http://www.epa.gov/OWOW/NPS/Section 319/overview.html.