Opportunities of Using Random Sets to Model Uncertainties in Agricultural Field Boundaries Observed from Remote Sensing Images

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Authors' contributions

This work was carried out in collaboration between all authors. Authors AS and AAA designed the study, wrote the protocol and supervised the work. Author AGE carried out all analyses. Author AS managed the analyses of the study. Author AGE wrote the first draft of the manuscript and managed the literature searches. Author AS edited and finished the manuscript. All authors read and approved the final manuscript.

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ABSTRACT

Random sets are common spatial statistical concepts that allow quantifying uncertainty in spatial objects. For objects extracted from remote sensing images, quantification of the uncertainty is important, as many objects are relatively small with respect to the pixel size and are sometimes poorly defined. Remote Sensing (RS) data are important in land cover identification, classification and estimation. The aim of this paper is to address problems associated with the presence of edges between objects. Such edges occur on images in different shapes, for example as borders

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between agricultural parcels. The study was applied on an NDVI map of a Landsat 5 TM image. Field boundaries are normally irregular and often transitional. Modeling agricultural fields as spatial objects helps to identify the extensional uncertainties and therefore to characterize inaccuracy in parcel size estimation. The study was carried out in the Sharifabad region in Iran. The Douglas Paucker algorithm was used to establish a single boundary that separates different parcels of agricultural fields. The results of the study indicate that Gaussian thresholding of image segmentation generated random sets for six agricultural fields. Quantification of extensional uncertainty presented two parcels with a larger extensional uncertainty than the other four parcels. A question we addressed in this study was identification of the boundaries between two adjacent parcels. An overall accuracy of 91% shows that random sets were effective for modeling the extensional uncertainty of the agricultural fields and for the delineation of the agricultural field boundaries. We conclude that the geometric model used to delineate the agricultural field boundaries is able to properly handle irregular shape boundaries.

1. INTRODUCTION

Random sets are defined as a relation between a set and a measure space. This is an extension of random variables that relate a number with a measure space. Agricultural parcels were extracted from remote sensing images. Their boundaries are defined as level sets for different probability values applying Gaussian thresholding.

Modeling uncertainties in spatial studies is focusing increasingly on an object based approach. A major reason is that object oriented classification [1,2] is becoming important and is a common approach [3] nowadays for spatial studies. In addition, methods that were used in the past were based primarily on a fuzzy approach and hence were lacking a solid probabilistic frame [4]. The third reason is that spatial data quality is an important field of science that has been developing rapidly during the last decades [5] and has resulted so far mainly within a remote sensing context in pixel based approaches, whereas an object based approach is missing. Here, the use of modern spatial statistical methods becomes more and more convenient.

Spatial Data Quality (SDQ) is defined as the precision and accuracy of spatial data in relation to its fitness for use. From this definition, there is a clear link between the basic element that is identified (being either the pixel or the object) to the intended use of the information. Since long [6, 7, 8] it has been realized that spatial variation is present, and that the point data have a spatial dependence. As the scale of variation is usually relatively large as compared to the scale of the objects of interest to be identified from an image,

the spatial variation translated into spatial dependence cannot be ignored. It depends upon the user, however, to decide upon the relevance within the application domain.

Spatial data are data that are either measured directly or data that are observed from a distance. Examples of point data are data on crops, vegetation, water and health. Examples of objects are agricultural parcels, water bodies or buildings. The decision maker (the user) ranges in these examples from the farmer through the land manager towards the water manager. They have to make a decision often based upon uncertain information, where the uncertainty as such may be difficult to get and understand.

Spatial data quality becomes a major issue when precise information has to be collected and processed. In such cases, spatial statistics can be of a major assistance. There are several issues of SDQ where spatial statistics has shown ways for resolving problems. For point-like data, positional accuracy has been addressed at length, as for remote sensing images in the particular case of spatial resolution, and likewise attribute accuracy. The same applies to temporal quality with the recent advent of spatio-temporal statistics [9].

This study focuses on agricultural fields. Those are generally well identifiable remote sensing objects, at current images well identifiable as regular or irregular polygons. At various resolutions there is a clear variation visible within the fields. In addition, the resolution of the images can be such that fields are covered with a small number of pixels, and that hence a large uncertainty as concerns the boundaries of the field is present. In that sense, agricultural fields

could serve as uncertain objects, and current theory can be applied to those.

The overall objective of this application was to use random sets for modeling the extensional uncertainty of the agricultural fields in space. As different random set models for neighboring parcels may partly overlap, modeling the boundary between random sets was identified as the objective of study. The novelty of the current study, however, is to emphasize the borders between the fields that are all modeled as random sets. It is this identification of borders and their uncertainty that has triggered our interest.

2. METHODS

2.1 Study Area

The study area is located close to Sharifabad in the center of the Ghazvin province, NW Iran. The NDVI pixels of the selected study area are shown in Fig. 4. Table 1, shows the statistical information for the six parcels selected for further investigation within the study area using the existing parcel boundaries. We chose these parcels because the parcels are adjacent and neighboring fields have spectrally similar.

The procedure to identify the parcels was running as follows. From the LANDSAT image, after geometric correction, an NDVI image of the study area was derived. A segmentation was carried out by setting minimum and maximum thresholds on the NDVI image. This resulted in six parcels in the study area (Fig. 4) that were close to each. For each parcel the covering function (Equation 2.2) was determined.

2.2 From Pixels to Objects

Spatial information on buildings, vegetation and land use was extracted from remotely sensed imagery. Traditionally, remote sensing and earth observations data were analyzed using pixel oriented approaches. Out of the pixels segments were created and classified. There is now more attention to an object based approach than in the past. These objects, however, can be uncertain. Uncertainty mainly comes from imprecise boundaries, uncertain class definitions, spectral overlap between objects, spatial variation at the earth surface, mixture of classes in the sampling grid along the object boundary, atmospheric distortions, whereas other SDQ components play a role as well.

In order to address these problems, random sets have been adopted recently [10]. Random sets were developed in the context of stochastic geometry [11,12]. They serve as a generalization of probability, possibility, interval analysis and evident theory [13] that have all been used in the past (Fig. 1). They are more flexible, with applications including image processing [14], particle statistics in material science [15], vegetation patches [16], glacier debris [17] and traffic islands [18].

2.3 Random Variables

As random sets are not so common in spatial studies, we now introduce them starting with recalling random variables. Random variables are defined as follows. We assign to each outcome ω of an experiment (or of an event) a number $X(\omega)$. This establishes a relationship between the elements ω of Ω and numbers $X(\omega)$. Such a function is called a random variable. A random variable represents a process of assigning to every outcome of an experiment a number $X(\omega)$. Thus a random variable is a function with domain given by the set of experimental outcomes and range contained in a set of numbers (Fig. 2).

A probability space is a triple $(\Omega, \sigma_\Omega, Pr_\Omega)$. The symbols denote the following: Ω is the set of all possible outcomes for which a probability is to be defined, σ_Ω is a σ-algebra on Ω that defines the events and Pr_Ω is the probability function that applies to the events (Fig. 3). All probabilities are defined to produce real numbers. That means that the range of the probability function Pr_Ω is a measure space. Both for continuous and for discrete probabilities this holds. Formally, we let $(\Omega, \sigma_\Omega, Pr_\Omega)$ be a probability space and (Ξ, σ_Ξ) be a measure space. A random variable $X(\omega)$ is a measurable function from the probability space Ω to the measure space Ξ.

The concept of a random variable easily expands towards random sets. Heuristically, we replace the numbers by sets. A random set Γ is defined as a random variable from the sample space Ω to U, where U is a set of subsets of Ξ, i.e. $U \subseteq P(\Xi)$.

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a) Rough sets, defined by an inner and an outer polygon.

b) Fuzzy sets as membership values expressed as gray levels to a set of three elements {Dark,Gray,Bright}

c) Fractals as regular patterns to model uncertain boundaries

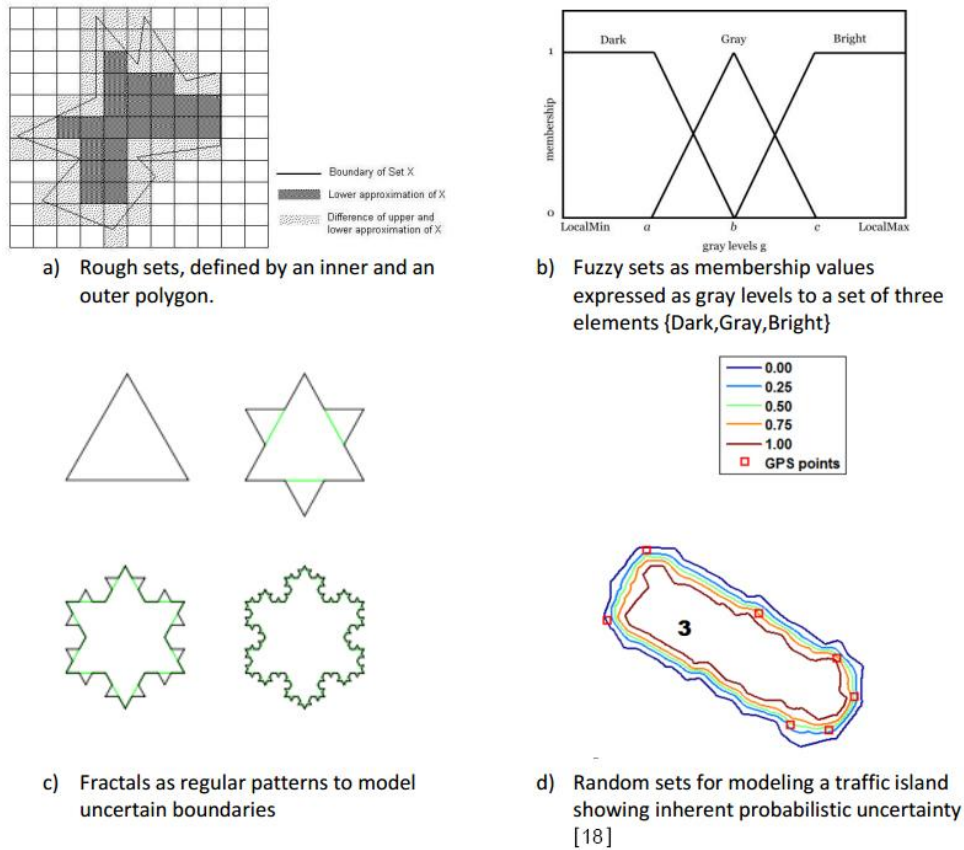d) Random sets for modeling a traffic island showing inherent probabilistic uncertainty [18]

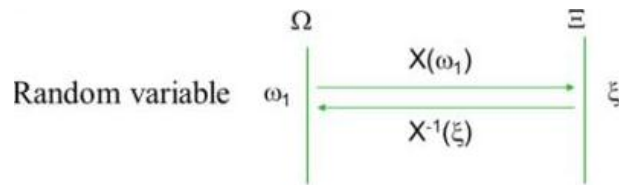**Fig. 1. Four different ways of modeling uncertainty of spatial objects**

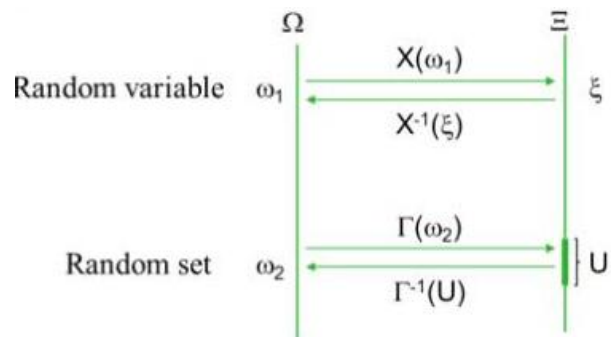**Fig. 2. The definition of a random variable from the space of outcomes Ω to a variable ξ in the measure space Ξ**

Fig. 3. The definition of a random set from the space of outcomes Ω to a set U in the measure space Ξ

The distribution of random set Γ is defined as:

$$\Pr_\Gamma(B) = \Pr_\Omega(\Gamma^{-1}(B)) = \Pr_\Omega\{\omega \,|\, \Gamma(\omega) \in B\}, \forall_{B \in \sigma_U} \quad (2.1)$$

In the special case that *B* is a singleton and $\xi \in \Xi$, we are back at the random variables, i.e. the random set Γ becomes a random variable. The distribution of the random set Γ in this special case is called the one point coverage function or covering function:

$$\Pr_\Gamma(\{\xi\}) = \Pr_\Omega\{\omega \,|\, \xi \in \Gamma(\omega)\}, \forall_{\xi \in \Xi} \quad (2.2)$$

To be able to make inference on random sets, use is commonly made of a covering function. An estimator of the covering function of the random set Γ is defined as:

$$\Pr_\Gamma(x) = \frac{1}{n} \sum_{i=1}^{n} I_{O_i}(x), x \in R^2, O_i \in U \quad (2.3)$$

where $I_{O_i}(x)$ is the indicator function of O_i, i.e. $I_{O_i}(x) = 1$ if $x \in O_i$ and $I_{O_i}(x) = 0$ if $x \notin O_i$ and O_i are the realizations of Γ. With the distribution collected, the next step was to define and use its moments and parameters. The set

$$\Gamma_p = \left\{ x \in R^2, 0 \le p \le 1 : \Pr_\Gamma(x) \ge p \right\} \quad (2.4)$$

is called a *p*-level set, with as special cases: the *median set* defined the 0.5-level set, the *support* is the 0-level set, i.e. the possible part of Γ and the core is the 1-level set, i.e. the certain part of Γ. The core, median and support sets of random regions have been obtained from the p-level set.

The core set equals:

$$\Gamma_c = \{R^2 \in H : Pr_\Gamma(x) = 1\} \quad (2.5)$$

the support set equals to:

$$\Gamma_s = \{R^2 : H : Pr_\Gamma(x) > 0\} \quad (2.6)$$

and the median set equals:

$$\Gamma_{median} = \{R^2 : H : Pr_\Gamma(x) \ge 0.5\} \quad (2.7)$$

The mean of the random set has been defined in several ways. It is common to use the Vorob'ev expectation defined as

$$\Gamma_m = \left\{ x \in R^2, 0 \le p_m \le 1 : \Pr_\Gamma(x) \ge p_m \right\} \quad (2.8)$$

where p_m is such that Γ_m has the mean area $E(A(\Gamma))$ of the random set:

$$E(A(\Gamma_m)) = E(A(\Gamma)) = \int_{R^2} \Pr_\Gamma(x) dx \quad (2.9)$$

Using the mean, we can now continue to also define the set-theoretic variance of a random set as:

$$\Gamma_{var}(x) = E\left(I_{O_i}(x) - \Pr_\Gamma(x)\right)^2 \quad (2.10)$$

and the coefficient of variation (CV) is defined as

$$CV = \frac{\int_{\Gamma_s} \sqrt{\Gamma_{var}(x)} dx}{\int_{\Gamma_s} \Pr_\Gamma(x) dx} \quad (2.11)$$

These definitions are all used to model identified objects. They describe the size of the area, the likeliness that a point belongs to an object and the variation within the objects. In the past, interest focused on the identification of objects themselves, as stand-alone objects within otherwise stable environment. The current study extends upon this, by addressing the possibility that two (or more) random sets touch each other and hence show a boundary that it is uncertain.

3. RESULTS

3.1 Application: Delineation of Agricultural Field Boundaries

The problem that we address in this application concerns the identification of agricultural field boundaries where the crops of neighboring fields have spectral similarity.

The NDVI pixels of the selected study area are shown in Fig. 4. Table 1, shows the statistical information for the six parcels selected for further investigation within the study area using the existing parcel boundaries. We chose these parcels because the parcels are adjacent and neighboring fields have spectrally similar crops. On the one hand, existing parcel boundaries were determined using visual interpretation. In addition, we used random sets to model the uncertain parcel boundaries, where uncertainty is provided by i) the spectral similarity of the crops and ii) the relatively coarse resolution of the image as compared to the sizes of the agricultural crops.

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Table 1. The means (Mean) and standard deviations (SD) of the NDVI pixel values for the six different parcels

Parcel	Mean	SD
1	0.4	0.05
2	0.04	0.005
3	0.13	0.01
4	0.38	0.05
5	0.036	0.002
6	0.34	0.01

Some parcels are quite similar to each other in terms of their NDVI values, like parcels 1, 4 and 6, which have relatively high NDVI values, indicating presence of crop on the land. Parcels 2 and 5 have similar, but low NDVI values indicating absence of crop, whereas parcel 3 is in between and may contain remnants of previous crops, weeds, or a freshly planted crop.

On the basis of the coverage functions, six random sets were generated, using 60 iterations for each of the parcels. This resulted into 60 coverage functions. In doing so, we found the support set (Equation 2.6), the median set (Equation 2.7) and the core set (Equation 2.5) of the six parcels as shown in Fig. 5.

Fig. 4. Left: Location of the study area in the Ghazvin province in Iran in a snapshot of Google Earth from the area. Middle: The six parcels with the existing parcel boundaries. Right: The NDVI map of the LANDSAT image for the study area

Contours of:

⎯⎯ Core set (Γ_c)   ⎯⎯ Median ($\Gamma_{0.5}$)   ⎯⎯ Support set (Γ_s)
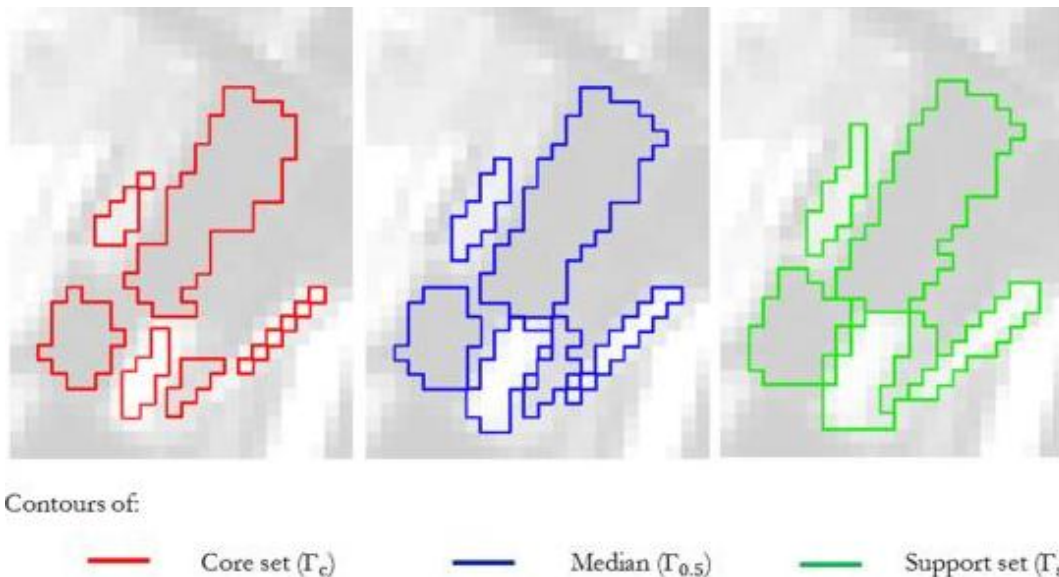
Fig. 5. The core set (left), median set (middle) and support set (right) of the six parcels. The core sets with coverage function equal to 1 (Equation 2.5) show clearly different objects, whereas the support indicating a coverage function above 0 (Equation 2.6) shows overlapping fields. The median set with coverage function equal to 0.5 (Equation 2.7) is in between

Clearly, the six different core sets (Equation 2.5) are all well separated, indicating that the parts of the field that were covered with vegetation were clearly different from each other. However, median sets (Equation 2.7) already partly overlap, which indicates that the presence of relatively large pixels and edge effects result in transition zones. That is even more apparent when considering the support sets (Equation 2.6), where we notice that overlap between the different parcels could in principle be substantial. When considering the mean sets of the random sets (Equation 2.8), a similar picture emerges (Fig. 6).

Clearly, parcels 3 and 4 have a large probability of overlapping, whereas for example parcel 6 is well separated from the other parcels. As the next stage we considered the boundaries between the different parcels, and we focused in the analysis on the boundary between parcels 1 and 3.
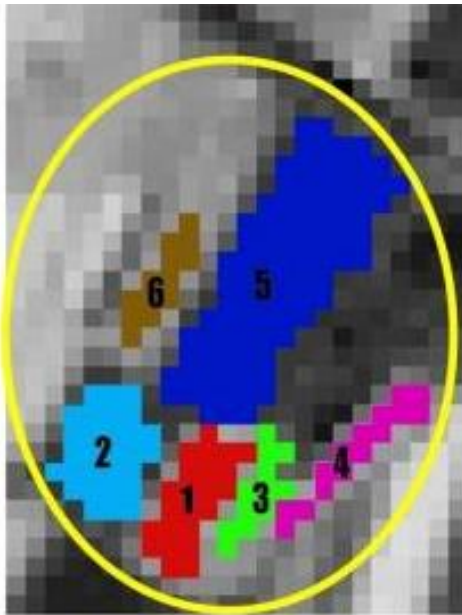

Fig. 6. Mean sets (Equations 2.8 and 2.9) of the six different parcels

Figs. 6 and 7 illustrate the typical pixel-based representation of a Landsat image segmentation resulting in pixelated edges instead of the required smooth and straight GIS object boundaries that correspond to field conditions. The boundaries of the actual agricultural fields are approximated by the pixel edges and in this representation lead to a blocky pixelated

structure. To improve the representation of the boundaries we analyzed the pixel numbers of boundaries with the related covering function values for each pixel. Fig. 7 illustrates the boundary between parcels 1 and 3 with its related pixel numbers.
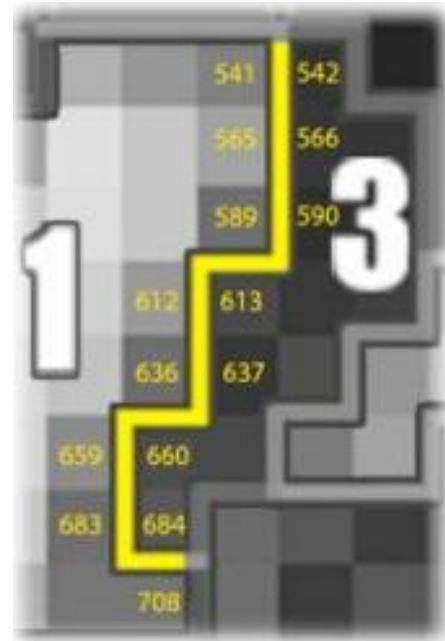

Fig. 7. The boundary between parcels 1 and 3 with related pixel numbers and the typical pixel-based pixilated representation of agricultural field boundaries

Fig. 8 describes the pixel numbers in horizontal axis and the values of the covering function along the vertical axis of the graph. Apparently, values of the covering function for the related pixels of the boundary in the parcel 1 differ from those of parcel 3. In other words, from the graph it is apparent that for determining the boundaries of the agricultural fields as real world objects, the typical pixel-based and pixelated representation derived from the pixel edges is problematic. As such, methods that are able to simplify and smooth the boundaries have to be used. Similar analyses could be done for the boundaries between other fields. To do so, we used the Douglas Paucker algorithm that generates a single boundary between different parcels. After the application of this algorithm the following boundaries were identified (Fig. 9). After resolving the small polygons issue, the final NDVI map with the agricultural field boundaries (parcels 1-6) overlaid was obtained (Fig. 10).
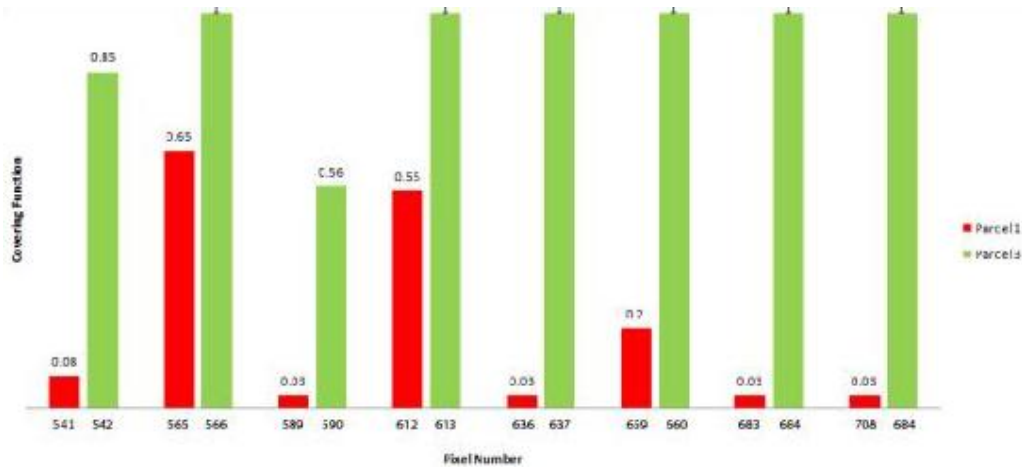
Fig. 8. Covering function values for the pixels of the boundary between parcels 1 and 3
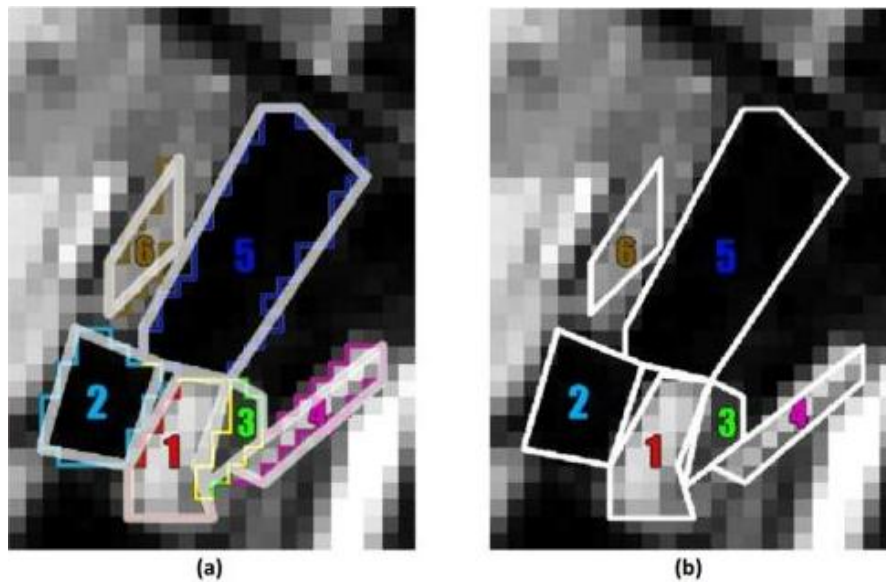

Fig. 9. Implementation of the Douglas Paucker algorithm (a) Before and (b) After applying the algorithm

4. DISCUSSION

This paper shows a step forward in relation to previous studies, in the sense that it describes innovative way of identifying boundaries between random sets. Identification of sets as such has been done before, but the novelty lies in the interaction between sets, in the sense of the covariance between them.

Random sets have a logical place in modeling spatial uncertainty. The use of random sets allows us to better understand the spatial uncertainty of objects identified from remote sensing images. We have seen examples in the past on vegetation patches, glaciers and road objects, whereas the current study also shows that the approach works fine for agricultural parcels. Here uncertainty is caused by the following: small size of agricultural fields/parcels compared with the size of a pixel (Landsat TM of 30x30meters), i.e. every parcel is covered with at most some 10 pixels. In our agricultural study area, about 40percent of the pixels of the Landsat TM image fall in field edges. This results in an uncertainty in the boundary characterization. Moreover, parcels in this province of Iran tend to be separated by a

relatively large boundary, which is covered by grass, mud roads and water bodies. Such a mixture of almost linear features leads to a large uncertainty in parcel identification. In addition, there are the almost common issues like Landsat-TM sensor's spatial response and atmospheric distortion that lead to uncertain boundaries. This is of interest for resolving the mixed boundary pixels, particularly in the case of small objects (compared with the size of a pixel).
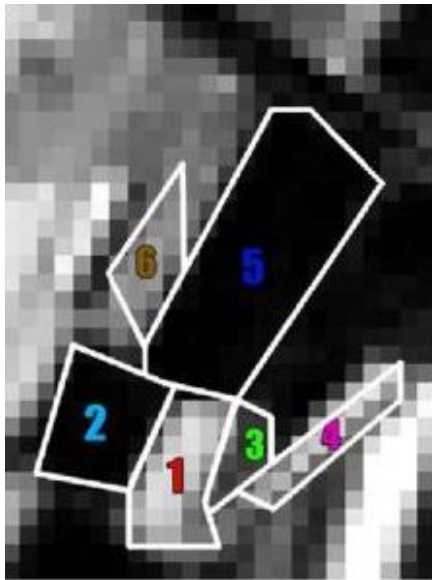

Fig. 10. NDVI map with the agricultural field boundaries (parcels 1-6) overlaid, after resolving the small polygons issue shown in Fig. 9b

Still work has to be done on to identify a single boundary, including the uncertainty. The algorithm that we applied is a standard algorithm, and it is not so sure whether it is the best approach in the situation as described. We may, in particular whether a boundary might not be better described as a line object, similar as was done with geological lineaments [19]. We also have the intention to use GIS/object-based delineation, representation and modeling, particularly for the manmade agricultural objects with a common boundary as developed as a topology-based classification by Janssen et al. [20], Abkar et al. [21] and Abkar et al. [22]. The focus will be to generate smoother agricultural field boundaries with common boundaries between random regions, by incorporating the analysis at the object level instead of pixel level to better understand the spatial uncertainty of agricultural field boundary characterization.

Another issue will be to consider the impact of a sensor Point-Spread Function (PSF) on the segmentation results and for the case of median sets (Equation 2.7) that object are partly overlapping each other, following Abkar et al. [22], Townshend et al. [23] and Huang et al. [24].

In Fig. 11, a synthetic image composed of two agricultural fields allocated to different crops to illustrate the need for estimating the sensor PSF to estimate the object's edge parameters is shown.
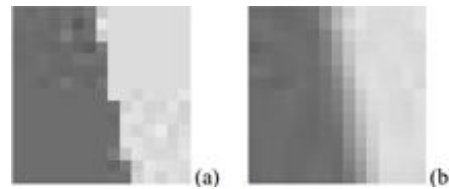

Fig. 11. True land cover map with two crops, dark grey = Wheat (Class A), light grey = Barley (Class B), Grey represents a mixture of classes at the boundary of two crop fields. (a) Map when the effect of the sensor PSF is not considered. (b) True land cover map using sensor PSF with width 3. For more information See Abkar (Section 4.4.2) [25]

For future research it is recommended to test the method for separating the agricultural fields covered with vegetation from the fields that are not fully covered with vegetation, e.g. containing freshly planted or recently harvested crops. Separating the fields that are partially covered with vegetation for example at the beginning of the growth stage from other parcels should be tested as well. If a canopy is too sparse, the background signal, e.g. the soil, can change NDVI significantly [26], thus affecting the results. Here, other vegetation indices could be more advantageous.

5. CONCLUSIONS

The paper describes the use of random sets to model uncertainties in spatial studies. The application problem addressed in this paper is the delineation of uncertain agricultural field boundaries from a Landsat TM image from Iran. For us, it was most important to provide a reliable estimate of the boundaries between areas planted with specific crops. Analysis of the image in this agricultural area was a challenge, because of the spectral confusion of crop types and mixed boundary pixels. Agricultural field boundaries have been delineated using the basic parameters

of random sets, i.e. the mean, covering function, level sets and variance with an overall accuracy of 91%.

We conclude that the geometric model used to delineate the agricultural field boundaries is efficient in handling non-rectangular shapes as provided by irregular shape boundaries. Hence, the algorithm can create irregularly shaped segments. This makes the approach generally applicable to a wide range of similar cases.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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