Choice of the Empirical Definition of Zero in the Translog Multiproduct Cost Functional Form

By

Yapo G. N'Guessan[a], Allen Featherstone[b], Hanas A. Cader[c]

[a] Ph D Candidate (Contact Author), Department of Agricultural Economics, Kansas State University, Manhattan, KS 66502, Phone: 785 532 4438, Fax: 785 532 6925, e-mail: nguessan@agecon.ksu.edu [b] Professor, Department of Agricultural Economics, Kansas State University, Manhattan, KS 66502, Phone: 785 532 4441, Fax: 785 532 6925, e-mail: afeather@agecon.ksu.edu [c] Ph D Candidate, Department of Agricultural Economics, Kansas State University, Manhattan, KS 66502, Phone: 785 532 4438, Fax: 785 532 6925, e-mail: acmhanas@agecon.ksu.edu

Selected Paper prepared for presentation at the Southern Agricultural Economics Association Annual Meetings, Orlando, Florida, February 5-8, 2006

Choice of the Empirical Definition of Zero in the Translog Multiproduct Cost Functional Form

Abstract

This study examines the impacts of empirical definition of zero output values on price elasticities, economy of scope, and scale using the Translog cost function. A system of cost and factor share equations with regularity conditions imposed is estimated. Results show that the choice of default values affects policy recommendations.

Keyword words: curvature, translog function, zero output value

Introduction

The Translog function, first introduced by Christensen, Jorgenson and Lau in 1973 is one of the most widely used functional forms in empirical analysis in the modeling of the indirect cost and profit functions. The Translog multiproduct cost functional form is attractive because it places no priori restrictions on the substitution possibilities among factors of production and allows for the computation of scale economies (Christensen and Greene). Nevertheless, the classical translog functional form is not without flaws (Berger, Hunter and Timme, Caves, Christensen and Tretheway). One of the flaws is related to the modeling of zero output values. To remedy to this problem, different alternatives have been suggested. One alternative is to “clear the data” by removing from the data set observations that have zero output values. But this method can lead to bias estimates if the number of zero value observations is relatively high. In addition, information on the cost structure of specialized firms is eliminated. As Caves, Christensen and Tretheway pointed out, the use of the full sample give better stable estimates. Furthermore, from economic standpoint, calculating economies of scope or product specific scale economies are more robustly estimated with zero observations.

When the zero output value observations are kept without modification, a generalized translog multiproduct cost model was been suggested. This method also called the Box-Cox transformation replaces the original output Y by $(Y^{\lambda}-1)/\lambda$ (Parker; Caves, Christensen and Tretheway; Caves, Christensen and Swanson). The other alternative commonly used is to replace those output data points by some small positive values (Akridge and Hertel; Cowing and Holtmann; Schroeder). A common choice is to use 10% of the mean value of the variables to replace their respective zero data points.

But, replacing the zero value by arbitrarily chosen small numbers may introduce bias in the resulting parameter estimates. Clearly the implication of this ad hoc choice on the conclusions from the analysis is an empirical issue that needs to be addressed. Do policy recommendations change given the choice of the default values used? This question has not had much attention in the economic literature.

Another important flaw of the translog form is that like most of other flexible functional forms derived from duality theory, the model is unfortunately associated with important violations of economic theory (Terrell). The translog in particular is not globally regular. Many violations of monotonicity and curvature are reported when using this functional form, resulting in positive own price demand elasticities. Attempts to globally impose regularity conditions using the classical Cholesky decomposition often lead to a significance loss in its flexibility (Westbrook and Buckley). A successful alternative approach to impose curvature without altering the flexibility of the functional form appeals for a Bayesian approach that involves the use of the Markov Chain Monte Carlo integration procedure using the Metropolis-Hasting algorithm (Chib and Greenberg, Griffiths, O'Donnell and Cruz). This method allows the imposition of the regularity condition at each of the data points.

In this article, we investigate the impact of the empirical definition of zero using the Translog model on the compensated input elasticities, the economies of scope and the economies of scale that are key economic measures usually derived in empirical studies. We estimate a system of cost and factor share equations in which curvature conditions are imposed using the Markov Chain Monte Carlo integration procedure with the Metropolis-Hasting algorithm. The elasticities and the scope and scale measures are computed at the means. The output data set contains zero output values that are replaced by some various arbitrarily chosen values. For the purpose of this analysis, we use zero-output values of 20%, 15%, 10%, 5%, 2.5%, and 1% of the mean values. For each value, the cost function and share equations are estimated and the elasticities and scope and scale economies measures are derived. This research provides evidence of the sensitivity of key economic measures to the default values set to replace the zero output data points when using the Translog model and illustrates the impact on policy recommendation derived from the empirical analysis.

The model and Estimation Method

The model is the multiproduct translog variable cost functional form that is expressed as follows:

$$\begin{aligned}\ln C = \alpha_0 + \sum_{i=1}^{m}\alpha_i \ln w_i + \sum_{i=m+1}^{n}\alpha_{iY} Y_i + .5\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\gamma_{ij}(\ln w_i \ln w_j) + \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ijY}(\ln Y_i \ln Y_j)\right) \\ + \sum_{i=1}^{m}\sum_{j=1}^{n}\gamma_{ijY} \ln w_i \ln Y_j\end{aligned} \quad (1)$$

where C represents total cost, w_i represents the price of input i, and Y_i quantity of output i and the alphas and gammas (α and γ) are parameter estimates. In our case, there are 8 inputs (m) and 2 outputs (n).

For a well behaved cost function, economic theory requires that the cost function should be symmetric and homogeneous of degree 1 in input prices. So the following restrictions are imposed on parameters:

$$\gamma_{ij} = \gamma_{ji}, \ \sum_{i=1}^{m}\alpha_i = 1, \ \sum_{i=1}^{m}\gamma_{ij} = \sum_{j=1}^{m}\gamma_{ji} = \sum_{j=1}^{m}\gamma_{iY} = 0 \quad (2)$$

The system of factor share equations is derived using Shephard's Lemma:

$$S_i = \alpha_i + \sum_{i=1}^{m-1} \sum_{j=1}^{n} \gamma_{ij} \ln w_{ij} + \sum_{i=1}^{m-1} \sum_{j=1}^{n} \gamma_{ijY} \ln Y_j \tag{3}$$

where S_i is the cost share of input i. The cost function will be estimated jointly with the system of share equations to have more efficient results. One of the share equations is dropped to avoid singularity given that only 7 out of the 8 share equations are linearly independent (Christensen and Greene). Monotonicity and curvature are imposed numerically using the Metropolis-Hasting algorithm, a Markov Chain Monte Carlo method to draw a posterior density function of the parameter estimates and derive the moment of their respective marginal density (Griffiths, O'Donnell and Cruz). The candidate values of the parameter estimates are used to evaluate the regularity conditions at the mean of the data at each iteration. Monotonicity is satisfied if each of the estimated factor shares is positive.

On the input side, concavity will be satisfied if the Hessian matrix of second order derivative is negative semi-definite. We need non-positive eigenvalues to assure that the Hessian matrix is negative semi-definite. The ij^{th} elements of the Hessian matrix are derived as follows: The non-diagonal element of the Hessian matrix is for

$$i \neq j,\ h_{ij} = \left(\frac{\gamma_{ij}}{S_i} + S_j \right) * \left(\frac{S_i * C}{w_i w_j} \right) \tag{4}$$

and the diagonal element of the Hessian matrix is given by

$$h_{ii} = \left(\frac{\gamma_{ii}}{S_i} + S_i - 1 \right) * \left(\frac{S_i * C}{(w_i)^2} \right) \tag{5}$$

Curvature is also imposed on the output side. Curvature is satisfied on the output side if the Hessian matrix of second order derivatives is positive semi-definite which requires non-negative eigenvalues. For the cross terms, the Hessian terms are:

$$hh_{ij} = \left(\alpha_{ijY} + \alpha_{iY} * \alpha_{jY}\right) * \left(\frac{C}{Y_i * Y_j}\right) \tag{6}$$

and the Hessian non-cross terms $hh_{ii} = \left(\alpha_{iiY} - \alpha_{iY} + (\alpha_{iY})^2\right) * \left(\frac{C}{(Y_i)^2}\right)$ (7)

In this article, our model imposes monotonicity, curvature in input side and curvature in output side at the mean of the data set containing 4,780 observations. The initial starting values that satisfy the stated regularity conditions above are chosen arbitrarily. From these starting values, the algorithm generates candidates that are evaluated against the three conditions. Monotonicity is satisfied if the evaluated predicted factor share is positive. Concavity is satisfied in the input side if the maximum eigenvalue of the estimated Hessian matrix (h_{ii}) is negative. On the output side, concavity is satisfied if the minimum eigenvalue of the estimated Hessian matrix (hh_{ii}) is positive. To be accepted, each candidate needs to satisfy all three conditions. Whenever a condition is violated, a new set of candidate is randomly generated and evaluated. The burn in period and sample size used are 120,000 and 280,000 respectively, set sufficiently large to allow the model to converge. We set the tuning parameter h used to manipulate the acceptance rate to be equal to 0.00001 which provided an acceptance rate of 48%. Using the estimated cost function that satisfies the regularity conditions, we derive the elasticities and the economies of scope and scale estimates at the mean of the data. We run the model for each of the six zero-output values. The starting values are adjusted for each zero-output definition.

The elasticities ε_{ij} are computed according to the following:

$$\varepsilon_{ij} = \frac{\gamma_{ij} + S_i^2 - S_i}{S_i} \text{ for } i = j \text{ and } \varepsilon_{ij} = \frac{\gamma_{ij} + S_i * S_j}{S_i} \text{ for } i \neq j \tag{8}$$

The Economies of scope $SC(Y)$ are

$$SC(Y) = \frac{\sum_{i=1}^{n} C(Y_i) - C(Y)}{C(Y)} \tag{9}$$

where $C(Y_i)$ is the cost of producing output i and $C(Y)$ is total cost. The multiproduct scale economies are $S_N(Y) = \frac{\sum_{i=1}^{n} C(Y)}{\sum_{i=1}^{n} Y_i * \frac{\partial C(Y)}{\partial Y_i}}$ (10)

where the marginal cost of producing Y_i is

$$MC(Y_i) = \frac{\partial C(Y)}{\partial Y_i} = \frac{C(Y)}{Y_i} * \left[\alpha_{iY} + \alpha_{iiY} \ln Y_i + \sum_{i=1}^{m-1} \sum_{j=1}^{n} \gamma_{ijY} \ln w_i \right] \tag{11}$$

The product specific scale economy is $PSE_i = \frac{C(Y) - C(Y_j)}{MC(Y_i) * Y_i}$ (12)

Description of the data

The data used in this analysis is obtained from the Kansas Farm Management Association from 239 farms enrolled in their program from 1984 to 2003 giving a total of 4780 observations over 20 years. The data contains 2 outputs: crops (Y_1) and livestock (Y_2) and 8 inputs: seed, fertilizer, chemicals, feed, fuel, labor, land and machinery. Total cost(C) is also reported. Input prices were obtained from the Agricultural Outlook or Agricultural Prices, USDA. Summary statistics of the variables used in this estimation are reported in Table 1.

The original data has 833 zero output data points for livestock (Y_2). No zero value is reported for the second output crop (Y_1). The logarithm of zero is not defined, so a transformation of the original data is necessary. If the original output quantity Y is less than x% of the mean of Y, we set Y equal to x% of the mean of Y. Cost and share equations are estimated using Gauss 5.0.

Results and Discussion

Parameter Estimates

The parameter estimates and their respective standard errors are summarized in table 2. At 5% level of significance, 62% of the parameter estimates are statistically significant when we take the estimation with 20% of the mean. The percentage is 64%, 56%, 56%, 65% and 58% for the output default value equals to 15%, 10%, 5%, 2.5% and 1% of the mean respectively.

Eigenvalues

The Hessian matrix of second order derivative is negative-semidefinite attesting that the estimated cost function is concave at the mean. Furthermore one of the eigenvalues is exactly equal to zero for all the cases, indicating that the Hessian matrix is singular (Griffiths, O'Donnell and Cruz).

Elasticities

Compensated input elasticities are reported in Table 3. All the row sums for the elasticities equal to zero, due to the imposition of homogeneity. The own price elasticities are all negative which is consistent with the imposition of the restrictions required by economic theory. The magnitude and the sign of the elasticities change depending on the zero-value definition used. Generally, we can conclude that the demand for factors are inelastic with own price and cross price elasticities being less than 1. Only chemicals, at 20%, 15% and 2.5% and feed at 10% exhibit an elastic own price elasticity. The own price elasticities for feed at 15%, 10% and 2.5% and fertilizer at 15% are close to unity.

Another noticeable feature is that complementarity and substitution relationships between inputs are influenced by the choice of the zero-value definition. When using 20% and 5% of the mean, fertilizer and seed are net substitutes (negative cross price elasticity), but they are net complements (positive cross price elasticity) when we use 15%, 10%, 2.5% and 1% of the mean. The same occurs for the pair feed and fertilizer and for the pair fuel and labor, although for different range of default values.

Scope and scale measures

Short run economies of scope (SC), overall scale economies (SN) and product specific scale economies (PSE) are reported in Table 4. A positive scope economy represents the variable cost savings that are attributable to the joint production of livestock and crops. From the results, economies of scope exist for all the zero values used although the magnitudes are not the same. We observe more cost saving when we define zero as 20% of the mean (SC at 0.14) and less cost saving with 5% (SC at 0.08). The scale economies measures the change in the total cost to a proportionate change of the quantity of the two outputs. The results show increasing returns to scale for multiproduct firms ($SN \geq 1$) for all the zero values used, though the magnitude increases from 1.25 to 2.11 as we change the definition of zero from 20% of the mean to 1% of the mean. The Product specific scale economies measure the impact of increasing one output while keeping the other outputs constant. The PSE reveals also an increasing return to scale for both livestock and crops at all zero values except for crops when using 20% of the mean where the PSE is less than 1 (0.997). Substantial differences in the magnitude to these values occur depending on the definition of zero-output.

Conclusion

In this article, we have evaluated the impact of the empirical definition of zero on the compensated input elasticities and the economies of scope and scale, key economic measures usually derived in economic analysis using the Translog cost functional form. Because the logarithm of zero is not defined, researchers usually use some arbitrarily chosen small values to replace the zero output data points. We replaced the zero output data points by 20%, 15%, 10%, 5%, 2.5%, and 1% of the output mean values respectively. For each value, we estimated the system of cost and share equations after the imposition of the regularity conditions (homogeneity, monotonicity and curvature in input side and output side). Curvature conditions are imposed using a Markov Chain Monte Carlo method. The elasticities and scope and scale economies measures are then computed.

The choice of the arbitrarily chosen zero-output value matters and can affect policy recommendations. The own price elasticities can be elastic or inelastic depending on the default value chosen. The relations between inputs are also sensitive to this choice. Two inputs that are complements according to the analysis when using 10% of the mean to replace the zero output value become substitutes when using 5% instead. Although, we concluded that economies of scope and increasing return to scale exist, the differences in the magnitude of these estimates are quite large and varied across the six definitions used.

Table 1. Descriptive Statistics

Variables	Mean	Standard Deviation	Minimum Value	Maximum Value
Price of seed	111.2	16.911	93.000	154.000
Price of fertilizer	106.0	11.469	86.000	125.000
Price of chemical	106.9	13.406	87.000	122.000
Price of feed	105.2	10.549	83.000	129.000
Price of fuel	98.0	17.196	76.000	140.000
Price of labor	111.9	24.563	77.000	157.000
Price of land	33.3	2.167	28.600	36.000
Price of machinery	113.1	22.649	83.000	150.000
Cost share of seed	0.055	0.045	0.000	0.550
Cost share of fertilizer	0.091	0.053	0.000	0.359
Cost share of chemical	0.053	0.043	0.000	0.322
Cost share of feed	0.110	0.136	0.000	0.877
Cost share of fuel	0.072	0.031	0.000	0.319
Cost share of labor	0.038	0.047	0.000	0.743
Cost share of land	0.342	0.136	0.009	0.815
Cost share of machinery	0.239	0.078	0.032	0.688
Total cost	177990	138920	17384	1380100
Crops quantity	1096	1005.4	5.446	11316
Livestock quantity	845.6	1332.4	0.000	18484

Table 2. Parameter Estimates

	Estimated Parameters with their Standard Deviations					
Variables	20%	15%	10%	5%	2.5%	1%
Intercept	10.6350**	10.6360**	10.6579**	10.6415**	10.6336**	10.6377**
	(0.0022)	*(0.0026)*	*(0.0066)*	*(0.0020)*	*(0.0026)*	*(0.0018)*
Seed	0.0274**	0.0276**	0.0284**	0.0272**	0.0277**	0.0280**
	(0.0003)	*(0.0002)*	*(0.0004)*	*(0.0005)*	*(0.0002)*	*(0.0002)*
Fertilizer	0.2351**	0.2353**	0.2350**	0.2352**	0.2350**	0.2348**
	(0.0002)	*(0.0003)*	*(0.0003)*	*(0.0003)*	*(0.0001)*	*(0.0002)*
Chemical	0.02758*	0.0288**	0.0280**	0.0281**	0.0282**	0.0265**
	(0.0002)	*(0.0003)*	*(0.0001)*	*(0.0003)*	*(0.0003)*	*(0.0005)*
Feed	0.0320**	0.0334**	0.0355**	0.0332**	0.0356**	0.0351**
	(0.0004)	*(0.0004)*	*(0.0009)*	*(0.0010)*	*(0.0012)*	*(0.0006)*
Fuel	0.0237**	0.0229**	0.0233**	0.0234**	0.0232**	0.0223**
	(0.0002)	*(0.0002)*	*(0.0003)*	*(0.0002)*	*(0.0002)*	*(0.0002)*
Labor	0.0366**	0.0352**	0.0357**	0.0363**	0.0355**	0.0355**
	(0.0003)	*(0.0005)*	*(0.0002)*	*(0.0002)*	*(0.0004)*	*(0.0003)*
Land	0.3100**	0.3121**	0.3082**	0.3104**	0.3126**	0.3123**
	(0.0007)	*(0.0008)*	*(0.0013)*	*(0.0007)*	*(0.0010)*	*(0.0006)*
Crops	-0.0383**	-0.0489**	-0.0370**	-0.0413**	-0.0458**	-0.0452**
	(0.0027)	*(0.0016)*	*(0.0032)*	*(0.0023)*	*(0.0021)*	*(0.0020)*
Livest.	-0.0985**	-0.0915**	-0.1002**	-0.0952**	-0.0999**	-0.0983**
	(0.0019)	*(0.0011)*	*(0.0016)*	*(0.0008)*	*(0.0017)*	*(0.0014)*
Seed/Seed	0.0201**	0.0137**	0.0064	0.0175**	0.0187**	0.0298**
	(0.0032)	*(0.0062)*	*(0.0130)*	*(0.0066)*	*(0.0050)*	*(0.0062)*
Seed/Fert.	-0.0159**	-0.0024	-0.0089	-0.0177**	-0.0076*	0.0031
	(0.0040)	*(0.0059)*	*(0.0054)*	*(0.0038)*	*(0.0042)*	*(0.0102)*
Seed/Chem.	0.0033	-0.0097**	-0.0211**	-0.0164**	-0.0101**	-0.0021
	(0.0045)	*(0.0032)*	*(0.0048)*	*(0.0047)*	*(0.0051)*	*(0.0046)*
Seed/Feed	-0.0116**	-0.0128**	-0.0028	0.0035	0.0050**	-0.0061
	(0.0045)	*(0.0053)*	*(0.0031)*	*(0.0035)*	*(0.0025)*	*(0.0067)*
Seed/fuel	0.0006	0.0000	-0.0049**	0.0017	-0.0001	-0.0018
	(0.0026)	*(0.0011)*	*(0.0020)*	*(0.0017)*	*(0.0020)*	*(0.0014)*
Seed/Labor	0.0029	0.0051	*0.0104***	-0.0008	-0.0152**	0.0025
	(0.0081)	*(0.0091)*	*(0.0050)*	*(0.0055)*	*(0.0041)*	*(0.0037)*
Seed/Land	0.0016	-0.0085*	-0.0011	-0.0047	-0.0140**	-0.0243**
	(0.0050)	*(0.0048)*	*(0.0051)*	*(0.0052)*	*(0.0061)*	*(0.0067)*

Beneath each coefficient estimate, we report the standard deviation in parenthesis
****Denotes significant at the 5% level.**
***Denotes significant at the 10% level.**

	Estimated Parameters with their Standard Deviations					
Variables	20%	15%	10%	5%	2.50%	1%
Fert./Fert.	0.0045	-0.0153	0.0199**	-0.0099**	0.0009	0.0114**
	(0.0094)	*(0.0113)*	*(0.0077)*	*(0.0045)*	*(0.0118)*	*(0.0031)*
Fert./Chem.	0.0082*	0.0159**	0.0172**	-0.0268**	0.0141**	-0.0045
	(0.0048)	*(0.0037)*	*(0.0057)*	*(0.0030)*	*(0.0040)*	*(0.0075)*
Fert./Feed	-0.0054*	0.0051	-0.0029	-0.0082*	0.0172**	0.0056*
	(0.0028)	*(0.0046)*	*(0.0027)*	*(0.0048)*	*(0.0058)*	*(0.0034)*
Fert./Fuel	-0.0044**	-0.0116**	-0.0032*	0.0024	-0.0019	0.0048**
	(0.0010)	*(0.0019)*	*(0.0017)*	*(0.0023)*	*(0.0029)*	*(0.0016)*
Fert./Labor	0.0047	-0.0008	-0.0060	0.0100*	-0.0113	0.0087
	(0.0055)	*(0.0106)*	*(0.0036)*	*(0.0052)*	*(0.0091)*	*(0.0065)*
Fert./Land	0.0171**	0.0012	0.0080	0.0072**	0.0118**	-0.0068
	(0.0040)	*(0.0034)*	*(0.0052)*	*(0.0030)*	*(0.0033)*	*(0.0048)*
Chem./Chem.	-0.0078	-0.0256	0.0095*	0.0120**	-0.0111*	0.0331**
	(0.0085)	*(0.0210)*	*(0.0056)*	*(0.0040)*	*(0.0065)*	*(0.0038)*
Chem./Feed	-0.0048**	-0.0056**	0.0034	-0.0050	-0.0015	0.0003
	(0.0022)	*(0.0014)*	*(0.0027)*	*(0.0045)*	*(0.0019)*	*(0.0019)*
Chem./Fuel	-0.0051**	-0.0010	-0.0043**	0.0021*	-0.0059**	-0.0026
	(0.0019)	*(0.0016)*	*(0.0016*	*(0.0011)*	*(0.0029)*	*(0.0017)*
Chem./Labor	-0.0198**	-0.0059	-0.0210**	0.0106	-0.0196**	-0.0065
	(0.0069)	*(0.0047)*	*(0.0080)*	*(0.008)4*	*(0.0082)*	*(0.0045)*
Chem./Land	-0.0003	-0.0074	-0.0082**	-0.0115**	-0.0005	-0.0200**
	(0.0078)	*(0.0087)*	*(0.0040)*	*(0.0041)*	*(0.0049)*	*(0.0041)*
Feed/Feed	0.0008	-0.0039	-0.0063	-0.0062	-0.0074	0.0200**
	(0.0057)	*(0.0069)*	*(0.0050)*	*(0.0066)*	*(0.0086)*	*(0.0069)*
Feed/Fuel	-0.0083**	0.0001	-0.0017	-0.0008	0.0044**	0.0020
	(0.0020)	*(0.0023)*	*(0.0022)*	*(0.0024)*	*(0.0015)*	*(0.0022)*
Feed/Labor	-0.0056**	0.0033	0.0011	-0.0057	-0.0024	-0.0130**
	(0.0014)	*(0.0037)*	*(0.0025)*	*(0.0060)*	*(0.0053)*	*(0.0037)*
Feed/Land	-0.0195	-0.0055	0.0085	-0.0287*	-0.0041	-0.0108
	(0.0161)	*(0.0067)*	*(0.0053)*	*(0.0153)*	*(0.0055)*	*(0.0090)*
Fuel/Fuel	0.0088**	0.0177**	0.0121**	0.0150**	0.0051	0.0124**
	(0.0019)	*(0.0009)*	*(0.0020)*	*(0.0007)*	*(0.0043)*	*(0.0010)*
Fuel/Labor	-0.0036*	-0.0149**	-0.0011	-0.0031*	0.0015	-0.0115**
	(0.0021)	*(0.0049)*	*(0.0018)*	*(0.0022)*	*(0.0024)*	*(0.0034)*
Fuel/Land	0.0012	-0.0012	0.0010	-0.0037	-0.0034	-0.0114**
	(0.0013)	*(0.0030)*	*(0.0026)*	*(0.0021)*	*(0.0025)*	*(0.0016)*
Labor/Labor	-0.0102	-0.0219**	0.0165	-0.0110	0.0137**	-0.0218**
	(0.0169)	*(0.0101)*	*(0.0122)*	*(0.0293)*	*(0.00560*	*(0.0093)*

Beneath each coefficient estimate, we report the standard deviation in parenthesis
****Denotes significant at the 5% level.**
***Denotes significant at the 10% level.**

	Estimated Parameters with their Standard Deviations					
Variables	20%	15%	10%	5%	2.50%	1%
Labor/Land	-0.0157**	0.0104**	-0.0234**	0.0215**	0.0241**	0.0125*
	(0.0044)	*(0.0037)*	*(0.0090)*	*(0.0050)*	*(0.0074)*	*(0.0069)*
Land/Land	0.0809**	0.0969**	0.1031**	0.0927**	0.0878**	0.0852**
	(0.0127)	*(0.0107)*	*(0.0071)*	*(0.0113)*	*(0.0114)*	*(0.0092)*
Crops/Crops	0.0635**	0.0564**	0.0595**	0.0559**	0.0512**	0.0540**
	(0.0029)	*(0.0069)*	*(0.0039)*	*(0.0041)*	*(0.0042)*	*(0.0050)*
Crops/Livest.	0.0225**	0.0175**	0.0116**	0.0102**	0.0065**	0.0036**
	(0.0020)	*(0.0015)*	*(0.0010)*	*(0.0017)*	*(0.0011)*	*(0.0015)*
Livest./Livest.	0.0267**	0.0317**	0.0298**	0.0273**	0.0282**	0.0286**
	(0.0023)	*(0.0016)*	*(0.0030)*	*(0.0042)*	*(0.0024)*	*(0.0029)*
Seed/Crops	-0.0009**	0.0011**	-0.0020**	-0.0009**	-0.0010**	-0.0004**
	(0.0001)	*(0.0003)*	*(0.0007)*	*(0.0002)*	*(0.0004)*	*(0.0002)*
Seed/Livest.	0.0159*	0.0264**	0.0138*	0.0127**	0.0131**	0.0108
	(0.0087)	*(0.0077)*	*(0.0080)*	*(0.0064)*	*(0.0065)*	*(0.0072)*
Fert./Crops	-0.0181**	-0.0207**	-0.0200**	-0.0173**	-0.0184**	-0.0184**
	(0.0004)	*(0.0007)*	*(0.0003)*	*(0.0002)*	*(0.0002)*	*(0.0004)*
Fert./Livest.	-0.0003	0.0005	0.0001	0.0001	0.0005**	-0.0007**
	(0.0003)	*(0.0003)*	*(0.0002)*	*(0.0002)*	*(0.0002)*	*(0.0003)*
Chem./Crops	0.0000	-0.0006**	0.0004	-0.0010*	-0.0009**	-0.0005*
	(0.0003)	*(0.0003)*	*(0.0002)*	*(0.0006)*	*(0.0002)*	*(0.0003)*
Chem./Livest.	0.0009**	0.0003**	0.0006**	0.0001	-0.0005*	0.0005*
	(0.0001)	*(0.0001)*	*(0.0002)*	*(0.0002)*	*(0.0003)*	*(0.0003)*
Feed/crops	-0.0024**	-0.0018**	0.0008	-0.0006*	0.0022**	0.0032**
	(0.0009)	*(0.0005)*	*(0.0010)*	*(0.0003)*	*(0.0007)*	*(0.0005)*
Feed/Livest.	0.0007*	0.0013**	0.0018**	-0.0004	-0.0003	-0.0006*
	(0.0004)	*(0.0006)*	*(0.0008)*	*(0.0003)*	*(0.0002)*	*(0.0003)*
Fuel/Crops	-0.0001	0.0009**	0.0002	-0.0002	0.0008**	-0.0001
	(0.0003)	*(0.0002)*	*(0.0002)*	*(0.0002)*	*(0.0003)*	*(0.0001)*
Fuel/Livest.	-0.0003**	-0.0003**	-0.0002**	-0.0004**	0.0002	0.0000
	(0.0001)	*(0.0001)*	*(0.0001)*	*(0.0001)*	*(0.0001)*	*(0.0001)*
Labor/crops	0.0216**	0.0193**	0.0207**	0.0207**	0.0199**	0.0187**
	(0.0004)	*(0.0002)*	*(0.0003)*	*(0.0002)*	*(0.0003)*	*(0.0005)*
Labor/Livest.	-0.0004	-0.0005*	0.0001	-0.0007**	0.0001	-0.0004*
	(0.0003)	*(0.0003)*	*(0.0002)*	*(0.0003)*	*(0.0003)*	*(0.0002)*
Land/Crops	-0.0007	0.0031**	-0.0002	-0.0003	-0.0009	0.0012
	(0.0007)	*(0.0009)*	*(0.0008)*	*(0.0006)*	*(0.0010)*	*(0.0011)*
Land/Livest.	0.0016**	-0.0004**	0.0026**	0.0065**	0.0071**	0.0018**
	(0.0005)	*(0.0009)*	*(0.0010)*	*(0.0014)*	*(0.0013)*	*(0.0004)*

Beneath each coefficient estimate, we report the standard deviation in parenthesis
****Denotes significant at the 5% level.**
***Denotes significant at the 10% level.**

Table 3. Compensated Input Elasticities

Zero output values replaced by 20% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.71486	-0.0383	0.060608	-0.04629	0.024545	0.227668	0.23253	0.254095
Fertilizer	-0.05688	-0.86189	0.130422	-0.01816	-0.0312	0.259775	0.417804	0.160129
Chemicals	0.223015	0.323159	-1.18851	-0.09388	-0.12546	-0.36403	0.210615	1.015086
Feed	-0.1331	-0.03517	-0.07336	-0.93791	-0.16577	0.078146	-0.21707	1.484222
Fuel	0.160151	-0.13706	-0.22247	-0.37616	-0.53401	0.019564	0.281989	0.807992
Labor	0.142394	0.109408	-0.06188	0.016998	0.001875	-0.84534	0.143224	0.493315
Land	0.135444	0.163874	0.03334	-0.04397	0.025174	0.133384	-0.41222	-0.03503
Machinery	0.124028	0.052632	0.134655	0.251962	0.060447	0.384998	-0.02935	-0.97937

Zero output values replaced by 15% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.71405	0.083317	-0.00694	-0.02044	0.026997	0.178977	0.177622	0.274521
Fertilizer	0.200226	-1.06959	0.20569	0.091362	-0.09695	0.147719	0.228685	0.292855
Chemicals	-0.0432	0.533082	-1.67221	-0.1186	-0.00155	-0.00674	0.012066	1.29714
Feed	-0.12617	0.234624	-0.11752	-1.07058	0.030891	0.247327	0.065216	0.736209
Fuel	0.226916	-0.33909	-0.00209	0.042071	-0.31389	-0.4001	0.171499	0.614676
Labor	0.257583	0.088464	-0.00156	0.057676	-0.06851	-0.98324	0.281901	0.367678
Land	0.18566	0.099465	0.002025	0.011045	0.021327	0.204738	-0.33501	-0.18925
Machinery	0.295404	0.131131	0.224108	0.128365	0.078692	0.27491	-0.19483	-0.93778

Zero output values replaced by 10% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.8309	0.005041	-0.14585	0.018463	-0.02218	0.303429	0.191046	0.480953
Fertilizer	0.006588	-0.68034	0.246921	0.010039	-0.01611	0.13988	0.295221	-0.0022
Chemicals	-0.36537	0.473321	-0.74116	0.119464	-0.07579	-0.26397	0.017317	0.836181
Feed	0.046936	0.019528	0.12123	-1.10148	-0.01728	0.235077	0.395779	0.300209
Fuel	-0.11422	-0.06347	-0.15579	-0.03501	-0.41551	0.157946	0.246153	0.379895
Labor	0.160272	0.056537	-0.05566	0.048844	0.016201	-0.71137	0.089916	0.39526
Land	0.105437	0.124674	0.003815	0.085923	0.026381	0.093949	-0.28621	-0.15397
Machinery	0.188589	-0.00066	0.130885	0.046306	0.028927	0.293425	-0.10939	-0.57808

Zero output values replaced by 5% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.73237	-0.0476	-0.10863	0.092651	0.036822	0.143729	0.202031	0.41337
Fertilizer	-0.05014	-0.98338	-0.21226	-0.01452	0.043997	0.243514	0.309876	0.662899
Chemicals	-0.34411	-0.63834	-0.631	-0.07632	0.079517	0.444814	-0.07611	1.241542
Feed	0.170513	-0.02536	-0.04434	-1.03918	0.009303	0.057879	-0.22167	1.092854
Fuel	0.192379	0.218242	0.131154	0.02641	-0.28965	0.007228	0.074579	-0.36034
Labor	0.10855	0.174609	0.106055	0.023752	0.001045	-0.92232	0.386091	0.122221
Land	0.094486	0.137592	-0.01124	-0.05633	0.006676	0.239085	-0.37552	-0.03475
Machinery	0.176903	0.26934	0.167735	0.254128	-0.02952	0.069256	-0.0318	-0.87605

Zero output values replaced by 2.5% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.72535	0.03481	-0.05923	0.092375	0.032518	0.027161	0.138923	0.458792
Fertilizer	0.04635	-0.89569	0.169331	0.233282	0.013253	0.028955	0.373735	0.030788
Chemicals	-0.36367	0.780806	-1.51998	-0.01911	-0.25129	-0.80547	0.22498	1.95373
Feed	0.221955	0.420971	-0.00748	-1.08862	0.116779	0.102413	0.170812	0.063171
Fuel	0.122954	0.037635	-0.15476	0.183767	-0.81259	0.192571	0.14743	0.282988
Labor	0.023294	0.018651	-0.11252	0.036556	0.043681	-0.75955	0.412492	0.337387
Land	0.070483	0.142408	0.018591	0.036068	0.019783	0.244012	-0.39835	-0.133
Machinery	0.210762	0.010622	0.146181	0.012078	0.034382	0.180713	-0.12042	-0.47431

Zero output values replaced by 1% of the mean

	Prices							
Zij	Seed	Fertilizer	Chemicals	Feed	Fuel	Labor	Land	Machinery
Seed	-0.63836	0.133097	0.032717	0.017126	0.019387	0.170126	0.042148	0.22376
Fertilizer	0.155778	-0.78685	0.007921	0.117683	0.07838	0.230931	0.170988	0.025173
Chemicals	0.083663	0.017306	-0.28245	0.070763	-0.01833	0.020361	-0.1699	0.278578
Feed	0.033091	0.194284	0.05347	-0.62921	0.064491	-0.04724	0.06885	0.26226
Fuel	0.072974	0.252078	-0.02698	0.125633	-0.59932	-0.18958	-0.10458	0.469779
Labor	0.143024	0.165875	0.006694	-0.02055	-0.04234	-0.99373	0.316333	0.424696
Land	0.022862	0.079243	-0.03604	0.019327	-0.01507	0.2041	-0.40163	0.127208
Machinery	0.122227	0.011748	0.059508	0.07414	0.068173	0.275947	0.128104	-0.73985

Table 4. Economies of Scope and Scale

	20%	15%	10%	5%	2.50%	1%
Economies of scope	0.1454	0.1246	0.1244	0.0805	0.1366	0.1305
Scale economies	1.2509	1.4126	1.5695	1.7459	2.0629	2.1087
Product specific scale economies Crops	0.9980	1.1676	1.2927	1.5114	1.7372	1.8423
Product specific scale economies Livestock	1.2361	1.3659	1.5831	1.8638	1.9040	1.8074

References

Akridge, J.T., and T.W. Hertel. “Multiproduct Cost Relationships for Retail Fertilizer Plants.” *American Journal of Agricultural Economics* 68(1986):928-938.

Berger, A.N., W.C. Hunter, and S.G. Timme. “The Efficiency of Financial Institutions: A Review and Preview of Research Past, Present, and Future.” *Journal of Banking and Finance* 17(1993):221-249.

Caves, D.W., L.R. Christensen, and J.A. Swanson. “Productivity in U.S. Railroads, 1951-1974.” *The Bell Journal of Economics* 11(1980):166-181.

Caves, D.W., L.R. Christensen, and M.W. Tretheway. “Flexible Cost Functions for Multiproduct Firms” *The Review of Economics and Statistics* 62(1980):477-481.

Chib, S., and E. Greenberg. “Understanding the Metroplolis-Hasting Algorithm.” *The American Statistician* 49(1995):327-335.

Christensen, R.L., and W.H. Greene. “Economies of Scale in U.S. Electric Power Generation.” *The Journal of Political Economy* 84(1976):655-676.

Christensen, R.L., D.W. Jorgenson, and L.J. Lau. “Transcendental Logarithmic Production Frontiers.” *The Review of Economics and Statistics* 55(1973):28-45.

Cowing, T.G., and A.G. Holtmann. “Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data.” *Southern Economic Journal* 49(1983):637-653.

Griffiths, W.E., C.J. O’Donnell, and A.T. Cruz. “Imposing Regularity Conditions on a System of Cost and Factor Share Equations.” *The Australian Journal of Agricultural and Resource Economics* 44(2000):107-127.

Parker, E. “The Accuracy of Generalized Cost Function Estimation: A Monte Carlo Approach.” *Southern Economic Journal* 60(1994): 907-926.

Schroeder, T.C. “Economies of Scale and Scope for Agricultural Supply and Marketing Cooperatives.” *Review of Agricultural Economics* 14(1992):93-103.

Terrell D. “Incorporating Monotonicity and Concavity Conditions in Flexible Functional Forms.” *Journal of Applied Econometrics* 11(1996):179-194.

Westbrook, M.D., and P.A. Buckley. “Flexible Functional Forms and Regularity: Assessing the Competitive Relationship between Truck and Rail Transportation.” *The Review of Economics and Statistics* 72(1990):623-630.