# Threshold Models in Theory and Practice

by

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**Abstract**

Threshold models have gained much recent attention in applied economics for modeling nonlinear behavior. The appeal for these models is in part due to the observable pattern that many economic variables follow, such as asymmetric adjustment towards equilibrium. Recent developments in model specification derive error-correction models as a specific type of threshold models. This paper summarizes the developments in threshold modeling over the past two decades and reviews a sample of empirical works in agricultural economics. Guidance is provided for obtaining software programs.

### Introduction

The majority of the empirical econometric modeling work in agricultural economics assumes that relationships are linear. Economic theory plays a passive role on this issue, and thus most applied research finds it convenient to assume linearity. Recently, arguments have been presented, based on regularities observed in economic and financial data, that nonlinear specifications may be a more realistic representation of data generation processes. In finance, for instance, stock returns tend to be more correlated when there is low volatility than when volatility is high. A similar behavior has been observed in exchange rate mechanisms where the exchange rate may be constrained to lie within a pre-defined target zone (Franses and Dijk, 2000). To accommodate this kind of dynamic behavior using time series data, regime-switching models (RSM) have been introduced (Priestley, 1980 & 1988; Granger and Terasvirta, 1993). One particular model that begins to regularly appear in the agricultural economics literature is the threshold autoregressive (TAR) model introduced by Tong, Tong and Lim (1980), and extensively discussed in Tong (1990). The TAR model assumes that the regime is determined by a variable $q_t$ relative to a *threshold value* (. The empirical existence of a threshold seems plausible in various economic settings. In agricultural marketing, for instance, there is abundant literature on price transmission reporting that there exist asymmetries in price adjustments at various levels of the food marketing system. One claim is that output prices tend to respond faster to input price decreases than to increases, and that this asymmetric response to cost shocks is substantial and enduring in producer and consumer goods markets. A similar argument emerges in the literature on spatial market integration, where it is argued that transaction costs cause threshold effects that play a role in the mechanism that leads to equilibrium in spatially separated markets. The recent developments in threshold modeling also seem to provide unique ways to accommodate nonstationary and cointegration properties of economic data. Standard cointegration models (error-correction models) assume linearity and symmetric adjustment (Engle & Granger, 1987). Threshold models provide a general model specification; in fact, when adjustment is symmetric in the error-correction term, TAR reduces to the standard error correction model (Balke and Fomby, 1997; Enders and Granger, 1998; Enders and Siklos, 2001; and Hansen, 1997 & 2002).

The main objective of this paper is to provide a concise review of literature on estimation and inference with nonlinear econometric time series models with emphasis on the usefulness of threshold models to agricultural economists. This objective is accomplished by organizing the paper in three parts: a) presenting a concise review of literature on modeling nonlinear behavior in economics with an emphasis on theory and concepts, b) summarizing applied work relevant to agricultural economists, and c) providing guidance for the implementation of these models in empirical work, along with web links to software.

## Review of Literature

### *Econometric Theory*

Most econometric-time series models found in empirical applications are linear. The class of RSM that can be estimated depends on whether the regime-determining variable is observable or not. In the first class of models, regimes determined by observable variables, a regime that occurs at time $t$ can be determined by an observable variable $q_t$. Perhaps the most popular model in this class is the Threshold Autoregressive (TAR) model introduced by Tong, Tong and Lim (1980), and extensively discussed in Tong (1990). As pointed out in the introduction, the TAR model assumes that the regime is determined by a variable $q_t$ relative to a *threshold value* (. If $q_t$ is equal to the dependent variable, say $y_t$, in an autoregressive regression, the model is referred to as *Self-Exciting TAR* (SETAR) model. We confine this theory section to the TAR type of models.[1] The theory section below draws heavily from Franses and van Dijk (2000).

*Stationary SETAR Models*

The SETAR model is a convenient way to specify a TAR model because $q_t$ is defined simply as the dependent variable ($y_t$). In this case, the process can be formally written as

$$y_t = \begin{cases} \phi_{0,1} + \phi_{1,1} y_{t-1} + \phi_{2,1} y_{t-2} + \ldots + \phi_{p_1,1} y_{t-p_1} + \varepsilon_t \text{ if } y_{t-1} \leq c \\ \phi_{0,2} + \phi_{1,2} y_{t-1} + \phi_{2,2} y_{t-2} + \ldots + \phi_{p_2,1} y_{t-p_2} + \varepsilon_t \text{ if } y_{t-1} > c \end{cases}$$

Note that in general the threshold variable $q_t$ has a delay coefficient *d* that in the SETAR model is assumed to equal 1, that is, $q_t = y_{t-d} = y_{t-1}$. There are several approaches that may be used to determine the order of the AR lags, $p_1$ and $p_2$. Sin and White (1996) find that statistical selection criteria (such as the AIC or BIC) will choose the lag order with probability one asymptotically. These procedures, however, require the estimation of a SETAR model for all combinations of $p_1$ and $p_2$. Formulas for both criteria may be found in Franses and van Dijk for the two-regime SETAR. Franses and Dijk point out that little is known about the conditions under which the SETAR model generates time series that are stationary. Such conditions have been derived for the simple SETAR(1) model. A recent treatment of testing for unit roots in SETAR models can be found in Caner and Hansen (2001), and Enders and Granger (1998).

The two-regime model identified above can be extended to multiple regimes, and it is possible that the multiple regimes can be determined by a single variable, by multiple variables, or by linear combinations of variables (cointegration case). An extended discussion on possible extensions of this model is found in the section below on nonstationariy.

It is interesting to highlight that the estimation of SETAR models requires the application of least squares procedures only, more specifically, sequential conditional least squares. For the two-regime SETAR model, the steps can be outlined as follows:

[1] The second class of RSM models assumes that the regime is determined by an unobservable variable, say $s_t$, the most popular of which is the Markov-Switching (MSW) model (Hamilton, 1989) which has not found wide applicability in economics and finance.

- Step 1: Set $p_1=p_2=p$ for simplicity and estimate the AR coefficients conditional on the value of the threshold (c).
- Step 2: Calculate conditional residuals $\hat{\varepsilon}_t(c)$ and estimated variances $\hat{\sigma}(c)$ from the coefficients in step 1.
- Step 3: Obtain least squares estimates of c by minimizing the residual variance $\hat{\sigma}^2(c)$ over all possible values of the threshold coefficient c, that is,

$$\hat{c} = \operatorname{argmin} \hat{\sigma}^2(c).$$

This minimization requires a direct search over the ordered values of $y_t$. Hansen (1997) provides an approach free of nuisance parameters that are involved in estimation. Also note that in more complex models, the direct search may be over values of c and d, and thus, the estimate of the variance in Step 3 would be conditional on values of c and d (e.g., Hansen and van Dijk).

- Step 4: Obtain final estimates of $\hat{\phi}$ and $\hat{\sigma}^2$ from results in Step 3.

The estimation of the threshold values in Step 3 requires that each regime contains enough observations for reliable estimation of the AR coefficients. About fifteen percent (15%) of the observations on each regime seems to work well.

*TAR Models with Nonstationary and Cointegration TAR models*

Empirical works with time series data in agricultural economics often report that variables tend to be nonstationary with one-unit root. When unit roots exists and there is cointegration, error correction (ECM) modeling is the appropriate methodology to use to capture dynamics. ECMs are a general specification that, with appropriate restrictions, reduces to other simpler models (e.g., Lutkepohl; Enders; Hamilton). For instance, if there are unit roots but no cointegration, an ECM reduces to a VAR in differences. If the time series are stationary in levels, the ECM further reduces to a VAR. Because of these simplifications, and others that similarly apply to threshold models, this section will explain a general specification of TAR models called a “momentum” TAR (M-TAR) with asymmetric adjustment.

Using Engle-Granger (1987) two-step approach to error-correction modeling, the M-TAR approach first estimates an OLS regression of the long-run equilibrium between retail (RP) and farm (FP) prices:

(1) $$RP_t = c + b * FP_t + u_t \ .$$

In the classical cointegration analysis, OLS would be used to estimate $\rho$ in the following equation:

(2) $$\Delta u_t = \rho u_{t-1} + \varepsilon_t$$

where $\varepsilon_t$ is a white noise process. If the residuals in (1) are stationary with mean zero then cointegration is found. When there is asymmetric adjustment, Enders and Granger (1998) propose to use the M-TAR framework, which is represented by

(3) $$\Delta u_t = \begin{cases} \rho_1 u_{t-1} + \varepsilon_t \text{ if } \Delta \mathrm{u}_{\mathrm{t-1}} \ \geq \ 0 \\ \rho_2 u_{t-1} + \varepsilon_t \text{ if } \Delta \mathrm{u}_{\mathrm{t-1}} \ < \ 0 \end{cases}$$

If the above sequence is stationary, the least squares estimates of $\rho_1$ and $\rho_2$ have an asymptotic multivariate normal distribution. The process is formally specified as:

(4) $$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t)\rho_2 u_{t-1} + \varepsilon_t$$

where $\mathbf{I_t}$ is referred to as the Heaviside indicator function such that

(5) $$I_t = \begin{cases} 1 \text{ if } u_{t-1} \geq 0 \\ 0 \text{ if } u_{t-1} < 0 \end{cases}$$

where 0 represents a critical threshold value. Equations (4) and (5) are referred to as M-TAR. Note that when the values of $\rho_1$ and $\rho_1$ are the same, then M-TAR reduces to equation (2), the traditional symmetric ECM specification; therefore, a symmetric ECM is a special case of the M-TAR model. Thus, an asymmetric ECM specification is needed to capture M-TAR properties, referred to as M-$TAR_{ECM}$, which for retail prices may be written as

(6) $$\Delta RP_t = \rho_{1,1} I_t u_{t-1} + \rho_{2,1}(1 - I_t)u_{t-1} + \sum lagged(\Delta RP_t, \Delta FP_t) + v_t \ .$$

This M-$TAR_{ECM}$ has properties consistent with the asymmetric momentum in producer-retail price movements. For example, if $|\rho_2| < |\rho_1|$, this model exhibits little decay for negative changes in $\Delta u_{t-1}$ but substantial decay for positive changes, a property consistent with observed asymmetries in retail and farm prices. An application to pork prices in the Swiss market is found in Abdulai (2002). Much remains to be known about univariate and multivariate properties of this model. The specification in equation (6), however, provides a useful first approach to econometric modeling of asymmetric price behavior consistent with non-stationary time series properties of commodity prices and with the argument that there is more momentum in price changes in one direction than another. Recent developments in threshold modeling (e.g., Balke and Fomby (1997); Tsay (1998); and Hansen (2002)) are shedding light on better procedures for modeling multivariate adjustment mechanisms, a subject of considerable ongoing research.

## Applications in Agricultural Economics

A concern of students of agricultural commodity markets is with how well the price signal gets transmitted across the different marketing levels. The study of price transmission is guided by market theory and econometric measurement of expected relationships. The relative frequencies of use as evidenced by citations in published works determine which bodies of market theory and econometric models are deemed seminal. In the study of asymmetric price adjustments, the identification of these seminal works establishes a context for the threshold models identified in this paper.

Chronological ordering of published works suggests that Tweeten and Quance led initial inquiries in the estimation of irreversible supply functions by developing a technique for segmentation of a price series into its price increasing segment and its price decreasing segment. Wolffram in 1971 claimed that the procedure was (1) applicable to any number of economic applications, not just supply irreversibility and (2) inferior to one he had earlier developed because it was “mathematically incorrect both for (a) quantification of irreversible supply reactions to increasing and decreasing prices and (b) differentiating the partial influence of an independent variable during certain periods of investigation” (Wolffram : 356-357). In 1977, Houck improved upon the Wolffram price segmentation procedure that became seminal in the study of price transmission in agricultural commodity markets. Houck’s procedure implicitly assumed that the properties of the price series being used as data input into the procedure included linearity and stationarity. In cases where those properties were not the reality, the output of Houck’s procedures potentially carried incorrect implications for inference about market symmetry.

Between the appearance of the procedures by Wolffram and Houck, Gardner outlined a static model of a multilevel marketing system. That economic model became seminal in price transmission studies. Gardner created a system of simultaneous demand and supply equations to model a food system with primary demand at retail and derived demand at the farm and with primary supply at the farm and derived supply at retail. System includes intermediate marketing services. Equations allow for the construction of elasticities and for the derivation of farm-retail price spreads. Elasticities and spreads allow for the investigation of economic phenomena such as (1) changes in primary (retail) demand due to changes in population (a demand shifter); (2) farm (primary) product supply due to changes in weather ( a supply shifter); and (3) a change in the intermediate input market (via taxes). Gardner notes that there are no simple markup pricing rules that can capture the relationships between the farm and retail price because these prices move together in different ways depending upon whether the changes in the determining variables events are associated with a shift in retail demand, farm supply or the supply of marketing inputs. (Gardner: 406). Reference to the co-movement of farm and retail prices finds expression in the term "cointegration" in the discussion of the properties of the price series used in the nonlinear models.

Gardner's contribution was to provide for a theoretical model of price relationships between different market levels (farm and retail) and to use prices between those two levels to create the marketing margin as the difference between the price at retail and the price at the farm. Gardner modeled a static conceptualization of the market. Heinen's contribution was to render Gardner's model dynamic in the determination of prices and quantities at the farm, retail and intermediate services market levels.

Heinen noted that Gardner's framework did not handle inventories (carryovers) between time periods and introduced disequilibrium by noting that time is required for markets to clear. He contended that it is in the treatment of inventory policy with price adjustment equations following the excess demand approach that a market model incorporating disequilibrium is closed. His model also allowed for testing the hypothesis of markup pricing at the retail level using Granger-Sims causality tests but pointed out that results could be sensitive to filtering.

Heinen performed asymmetry test (retailers treat increases in wholesale prices differently than they do decreases) for 22 commodities using Houck's seminal procedure. A finding of market asymmetry was declared for 12 of the 22 commodity markets. The t-value had the correct sign and exceeded two standard errors for only 5 of the 22 markets. Of the 22 commodities, two had very short shelf life attributes (lettuce, fresh tomatoes); 11 had intermediate shelf life attributes (bread, potatoes, apples, oranges, milk, eggs, beef, pork, chicken, butter, and fresh orange juice); and 9 had long shelf life attributes (sugar, soft drinks, canned tomatoes, rice, vegetable shortening, margarine, salad oil, chocolate bars, and frozen french fries).

Raw milk provides a unique case study of price transmission relationships because of product characteristics and institutional pricing influences. Kinnucan and Forker note that raw milk can be converted into fluid, soft manufactured and hard manufactured dairy products. The costs of those conversions are not only a component of the marketing margin but the relative quantities involved in those conversions influence the farm price series under the provisions of federal milk market order regulations. The farm-retail price transmission process in the dairy sector was assessed asymmetric.

Work by Carmen focused on the relationships between farm and retail fluid milk prices in California. The sale of milk in California differs from the sale of milk under federal milk market orders in that the solids content of all fluid milk sold in California must be maintained at specified levels. Consumer desirability for low fat fluid milk means the substitution of nonfat solids for fat solids. The marketing margin necessarily reflects the costs of maintaining that solids standard. Carman found no statistical difference in the total amount that retail prices increase or decrease in response to a one-dollar producer price increase or decrease. Although Carmen found no evidence of asymmetry in California's milk markets, he reported that retailers take a month longer to fully respond to a farm price decrease then to a farm price increase, and that this delay benefits retailers at the expense of consumers.

Asymmetry of prices is an emotional issue with respect to milk and other agricultural commodities. It calls into question market theory and heighten concerns about market failure.The findings in the two studies were derived using Houck's segmentation model. Over a decade after the first study was published, Cramon-Taubadel noted that the problem with a test of symmetry based on a model that assumed linearity in the price series was flawed.

Price symmetry may be an underlying assumption of markets that tend toward equilibrium where equilibrium means that prices serve as signals that bring about resource allocations consistent with effectiveness and efficiency. It may very well be that the market is not an equilibrium phenomenon, but a process phenomenon. The validity of the bedrock equilibrium principle of markets merits investigation. In this age of information technology, where more nonprice information now coordinates resources, threshold models may become more prominent in the analysis of markets.

The emergence of non-linear models expands the number of engines of analysis that can be employed in the analysis of price asymmetry. In addition to non-linearity, there is the phenomenon of dynamics. The appropriate engine of analysis depends upon the properties of the price series being analyzed. These properties are either linear or non-linear, static or dynamic, stationary or non-stationary, and cointegrated or not cointegrated. These properties reflect the uniquenesses associated with the physical, economic and institutional characteristics of the products being marketed. Technology and social changes can alter these characteristics so the challenge is to employ models that can capture those changes so that the risk of making incorrect inferences about market performance due to model output not truly reflective of model input characteristics can be minimized.

In the late 1990's, work with nonlinear models had advanced so that many assumptions of previous econometrics models could be relaxed. New procedures were introduced to estimate nonlinear, nonstationary and cointegrated models. Threshold models belong to a class of nonlinear models and their attributes of linking a variable with a regime of other variables constitutes a potential contribution to the assessment of price transmission in agricultural commodity markets. Two applications in agricultural economics that use the TAR approach presented in this paper are those of Abdulai and Goodwin.

**Software**

Various papers found in the literature offer programs developed by the authors which can be obtained by contacting them directly. One example is the work by Enders and Falk (1998) which studies out-of-sample prediction of threshold autoregressive models. This program requires some knowledge of programming in RATS but is well written and easy to follow.

A rich source of programs and papers, including unpublished and ongoing research, is that of Hansen at the University of Wisconsin, Madison, and these can be accessed at: http://www.ssc.wisc.edu/~bhansen/progs/progs.htm which are organized by date of publication and by subject for published and unpublished papers. This is probably the most comprehensive site on the subject and contains work completed over the past decade. Threshold models are listed by subject and cover (as of December 2002) the following papers by Hansen:

- "Inference when a nuisance parameter is not identified under the null hypothesis."

Econometrica, (1996).

- "Inference in TAR models." Studies in Nonlinear Dynamics and Econometrics, (1997).
- "Threshold effects in non-dynamic panels: Estimation, testing and inference." Journal of Econometrics, (1999).
- "Testing for Linearity." Journal of Economic Surveys, (1999).
- "Sample splitting and threshold estimation." Econometrica, (2000).
- "Threshold Autoregression with a Unit Root." Econometrica (2001), with Mehmet Caner.
- "How responsive are private transfers to income? Evidence from a laissez-faire economy." (11/2002) with Donald Cox and Emmanuel Jimenez, unpublished working paper.
- "Testing for two-regime threshold cointegration in vector error correction models," with Byeongseon Seo, Journal of Econometrics (2002).
- "Instrumental Variable Estimation of a Threshold Model", with Mehmet Caner, unpublished working paper (2001).

A user would need considerable experience in GAUSS programming to be able to understand the programs. Running the programs is somewhat less problematic but modifying them requires a bit of effort. It is an excellent source for students of threshold models. The programs are well structured, which makes the estimation and testing of theory easier to interpret.

Another useful source for code and other related routines can be found in Franses and van Dijk on "Non-linear time series models in empirical finance;" GAUSS code for many of the programs can be downloaded from www.few.eur.nl/few/people/franses. This is an extensive source of code and covers TAR, SETAR, STAR, ARCH and GARCH type models.

## Summary and Future Pespectives

This paper is about threshold autoregressive models with specific reference to those that are capable of dealing with economic series that are nonlinear, nonstationary and cointegrated. It is about expanding the catalogue of econometric models so that measurement can better serve economic theory and improve policy prescriptions.

Threshold models, as a specific class of nonlinear models, were introduced as tools of analysis which had the potential to minimize the introduction of biases into the analysis of prices that may result from assuming linearity and stationarity. The seminal tool developed by Houck for segmenting an independent price variable into its increasing and decreasing segments constitutes a linear model. Its employment with data series that are nonlinear increases the probability that the contributions of the underlying economic theory will be diminished and / or the inferences based on model findings will be flawed. In addition to identifying the threshold model as the engine of analysis for markets with non-linear price series, the appropriateness of different types of threshold models for varying combinations of price series properties was outlined.

The economic models advance by Gardner and Heinen assumed market equilibrium. That assumption underlies the criteria for assessing the presence or absence of market symmetry with its implications for the acceptance of economic theory and its prescription for policy.

The note by Wolffram that the price segmentation procedure has many applications in economics continues to hold true. Threshold models have the potential of working with price series with properties identified by Heinen and earlier writers of nonlinearity, nonstationarit y and not cointegrated.

Cramon-Taubadel recently wrote about asymmetric price transmission as a fact or artefact and argues that, because of structural breaks in the data, the methods of analysis used could lead to conclusions of asymmetry which may, in fact, be more artifact than fact. Although threshold models offer a more natural way to model asymmetric adjustment with nonstationary price series, much work is needed on developing models consistent with the structure and dynamics of agricultural commodity markets.

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