Testing Separability in a Generalized Ordinary Differential Demand System: The Case of Nigerian Demand for Meat

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Abstract

This study investigates consumer demand for meat products in Nigeria and tests if any of the meat products are separable from one another. Estimating a generalized nested demand system, the Rotterdam model was selected using an adjusted likelihood ratio test. Results indicate that mutton and pork are more elastic than beef and poultry. Only pork was found to be separable from other meat types.

1. Introduction

To date there has been very few empirical studies focusing on demand analysis for Nigeria. The majority of studies have focused on production issues because of limited data availability for consumers and other constraints. An exhaustive search on consumer demand for meat in Nigeria using various national and international archives (including that of Agricola) has not been fruitful. Pursuing an analysis of consumer demand for meat types in Nigeria will provide initial insight into consumer responsiveness to price changes and allow more precise welfare calculations that are necessary for policy analysis.

The study's objectives are: (i) to investigate the appropriate demand model in the analysis of the consumer demand for meat using the Generalized Ordinary Differential Demand System; (ii) to test the separability structure between the meat products in order to gain a better understanding of Nigerian consumer choices in protein (meat) demand; and (iii) to suggest the policy as well as the disciplinary implications of the findings.

The paper is organized as follows. First, some background information on Nigeria is provided. Then, Section 2 presents the separability concepts and the review of related studies. The discussion of the data and the estimation procedure are given in

Section 3. Section 4 contains results and discussion. Section 5 includes the summary and the conclusion.

1.1 Background Information

Domestic consumption of Nigerian meat products: beef, mutton, goat meat, pork and poultry have been met largely by domestic production over the years. Available information indicates that mutton and goat meat have never been imported into Nigeria (table 1). Sheep and goat have been raised almost by every household especially in the rural communities, where agriculture is still the major occupation and their meat are mainly consumed in local restaurants and rural markets. The values shown in the beef column show that domestic beef production has not been adequate to meat its demand, although substantial part of these values could be said to represent import of veal included in beef information and which were normally consumed by foreigners mainly from the west. The northern normads are the predominant rearers of cattle and aside from the fact that they need the female calves for local milk production, the male calves are usually kept either for an indication of wealth or reared for future sales. Slaughtering calves for market is thus never a usual nor an acceptable practice for the normads. Beef consumption, on the whole, has been most erratic of all meat type (figure 1). The fact that about half of Nigerian population are Muslims has had an adverse effect on production, importation and consumption of pork. The average of 1.7 percent and 8.8 percent of pork import in the later years might have been due to reduction in local production that necessitates its importation or foreigners demand for import.

Poultry has been increasingly popular among Nigerians over the years and local production has been immense due to huge government's investment in day-old chicks production. Its importation was however noted during the late 70s till early 80s. This coincided with the time of meat shortage following the great draught of early 70s.

Although, there has been generally high dependence on import in order to meet national food requirement, there has not been major import of meat products in Nigeria. Their demand has, however, been growing due to the nutritional need of the ever increasing population, which the World Bank (1996) estimated to be growing at 2.9 percent per annum. Importation, generally, has reduced in real term. This might be due to the declining income (figure 2), which presently stood below US$400 per capita. The decline could have a detrimental effect on household food consumption in general since Nigerian households devote about 75 % of their total expenditure to food (figure 3).

2. Conceptual Framework and Literature Review

2.1 Concept of Separability

The foundation work on separability was laid by Leontief (1947). Three concepts of separability in demand analysis have been summarized by Goldman and Uzawa (1964). These are based on the earlier works of Leontief (1947), Sono (1961), Strotz (1957, 1959), Gorma (1959), Frisch (1959), Houthakker (1960) and Pearce (1961). In the sense of Blackorby, Primont, and Rusell (1978) as well as Goldman and Uzawa (1964), these three concepts have been found to be extremely useful and effective for economic analysis of the structure of consumers' preference because of its wide ranging implications for the existence of consistent aggregates and the decentralization of optimization decisions.

In the sense of Goldman and Uzawa (1964), denoting a set of all n finite commodities by N, where $N = \{1,\ldots\ldots,n\}$, and which can be partitioned into a class of mutually exclusive and exhaustive subsets, $\{N_1,\ldots,N_s\}$; namely,

$$N = N_1 \cup \ldots \cup N_s, \quad N_s \cap N_t \text{ is empty for } s \neq t.$$

A commodity bundle, $x = (x_1,\ldots, x_n)$, may be correspondingly partitioned into $(x^{(1)},\ldots,x^{(s)})$, where, for each s, the subvector $x^{(s)}$ is composed of x_i, $i \in N_s$. Given that $\{N_1,\ldots,N_s\}$ is a partition or subset of the set N, let $u(x)$ be a utility function that states the necessary and the sufficient conditions of a preference relation. Let $u_i(x)$ and $u_j(x)$ be partial derivatives of $u(x)$ with respect to x_i and x_j, respectively, there are several concepts of separability that can be stated as follows.

A utility function u(x) is *weakly separable with respect to the partition* $\{N_1,\ldots,N_s\}$ if the marginal rate of substitution $u_i(x)/u_j(x)$ between two commodities i and j from subsets N_s does not depend upon the quantities of commodities outside of N_s, namely,

$$\frac{\partial\left[u_i(x) \;/\; u_j(x)\right]}{\partial x_k} = 0, \qquad \text{for all } i, j \in N_s \text{ and } k \notin N_s \tag{2.1}$$

A utility function u(x) is *strongly separable with respect to the partition* $\{N_1,\ldots,N_s\}$ if the marginal rate of substitution $u_i(x)/u_j(x)$ between two commodities i and j from different subsets N_s and N_t, respectively, does not depend upon the quantities of commodities outside of N_s and N_t; namely,

$$\frac{\partial\left[u_i(x)\ /\ u_j(x)\right]}{\partial x_k} = 0, \qquad \text{for all } i \in N_s,\ j \in N_t, \text{ and } k \notin N_s \cup N_t\ (s \neq t) \qquad (2.2)$$

Goldman and Uzawa (1964) defined *Pearce separability* as the third concept, which demands that the marginal rate of substitution $u_i(x)/u_j(x)$ between two commodities i and j from the same subset N_s be independent upon the quantities of all other commodities, that is,

$$\frac{\partial\left[u_i(x)\ /\ u_j(x)\right]}{\partial x_k} = 0, \qquad \text{for all } i, j \in N_s \text{ and } k \neq i, j\ .$$

(3)

Another concept of separability that has been used in the literature is *additive separability*. This is the case in which each good is its own group and its partial derivative does not depend upon any other goods.

All the separability concepts stated above in terms of utility functions can be characterized by the Slutsky terms of the corresponding demand function.

2.2 Literature Review

Previous studies have investigated separability using alternative approaches. These are the demand system approach (ordinary and inverse form) as well as differential demand system approach. Moschini, Moro, and Green (1994) tested for the structure of preferences in three different demand systems, based on the work of Balckorby, Davidson, and Schworm (1991). Eales and Wessells (1999) argued that the easiest demand system within which such test can be conducted is the Rotterdam model, the system used in Capps *et al* (1994), who examined the demand for meat products in the

Pacific Rim region. The Rotterdam model does not assume a specific functional form and different versions were developed by Barten (1964) and Theil (1965). One unique feature in a Rotterdam system is that the restrictions used to test for separability (either asymmetric or symmetric) depend only upon coefficients and not on any variable and the results of such test are global. If the Rotterdam system is not consistent with a particular data set, then such tests are unavailable (Eales and Wessells, 1999).

According to Eales, Durham, and Wessells (1997), "a number of studies have examined the plausibility of theoretically consistent inverse demand systems, e.g., Barten and Bettendorf (1989), Moschini and Vissa (1993), Eales and Unnevehr (1993, 1994), and Brown, Lee and Seale (1995). Barten and Bettendorf develop differential inverse demands for application to monthly demand for fish in Belgium. Specifically, they develop inverses of the Rotterdam, Central Bureau of Statistics (CBS; Keller and van Driel (1985), Laitinen and Theil (1979)), and differential Almost Ideal Demand System (AIDS) models as well as the interpretations of the coefficients. Just as the ordinary CBS demand model is a hybrid of the ordinary Rotterdam and AIDS models, the inverse CBS (originally developed by Laitinen and Theil) is a combination of inverse AIDS scale effects. The National Bureau of Research demand model (NBR; Neves (1987)) is also a hybrid. It has an inverse analog, the inverse NBR, which combines inverse AIDS quantity effects with an inverse Rotterdam scale effect".

As far as Generalized demand models are concerned, Barten (1993) developed a synthetic differential demand model, which nests such differential demand systems as Rotterdam (Barten 1964, Theil 1965), the differential AIDS, and the two hybrid demand systems: CBS and NBR. Lee, Brown, and Seale (1994) applied this system to examine

Taiwanese demands. This should, however, not be confused with the generalized inverse demand system developed by Brown, Lee and Seale (1995). An alternative parameterization to the Barten's synthesis above, known as Generalized Ordinary Differential Demand System (GODDS), was developed later by Eales, Durham and Wessells (1997) in a study of Japanese demand for fish. This system nests the Rotterdam demand system and the differential form of the AIDS as well as the two hybrid models: NBR and CBS models.

3. Data and Analytical Procedures

3.1 Data Collection and Derivation

Time-series annual data (1961 – 1999) on consumption of meat products as well as their nominal retail prices were collected from the Food Balance Sheet section of the Food and Agriculture Organization (FAO) as well as the Nigeria's Federal Office of Statistics, Lagos. The consumption data per 1000 metric tonnes are presented in aggregate form and these were converted to per capita level per kilogram, using the population data also available in FAO as well as the metric conversion. The total expenditure were derived by adding up individual expenditure, which is a product of the observed quantity that is consumed and its corresponding nominal price, while expenditure share of each of the meat products was derived from a division of the individual expenditure by the total expenditure.

3.2 Demand Modeling

The Generalized Ordinary Differential Demand System (GODDS) discussed in sub-section 2.2 is used in this study. The content (equations 3.1 to 3.10 below) as well as method of testing for separability are defined Eales and Wessells (1999) and others as follows:

$$dw_i = (\beta_i + \theta_1 \overline{w}_i) d\ln(Q) + \sum_{k=1}^{N} \left(\gamma_{ik} + \theta_2 \overline{w}_i (\delta_{ik} - \overline{w}_k)\right) d\ln(p_k), \tag{3.1}$$

where:

$$\left.\begin{array}{ll} d\ln(Q) = \sum_j \overline{w}_j d\ln(q_j) & (\textit{Divisia volume index}), \\ w_i = p_i q_i / x \ \textit{and} \ \overline{w}_i = 0.5\left(w_i + lag(w_i)\right) & (\textit{budget shares}), \\ \beta_i & (\text{exp}\,\textit{enditure coefficients}), \\ \gamma_{ik} & (\textit{price coefficients}), \\ \theta_1 \ \textit{and} \ \theta_2 & (\textit{nesting parameters}), \\ \delta_{ik} & (\textit{Kronec}\,\text{ker's}\,\textit{delta}). \end{array}\right\} \tag{3.2}$$

The outcome of different restrictions on the nesting parameters is as shown in table 2.

The coefficient $\hat{\gamma}_{ij}$ will be Almost Ideal price effects if $\hat{\theta}_2 = 0 = \hat{\theta}_1$ and Rotterdam price effects if $\hat{\theta}_2 = 1$, $\hat{\theta}_1 = -1$. For other values of $\hat{\theta}_2$, the price effect is neither that of the Almost Ideal nor the Rotterdam systems. Further discussion of the relationship between the models can be found in Neves'(1994). In the analysis below, the functional forms nested in GODDS were tested with homogeneity and symmetric restrictions in place to investigate their consistency with the data in order determine their appropriateness for further analysis.

Important economic effects can be derived using the appropriate functional form. The compensated, cross-price elasticities for the GODDS model are:

$$e^*_{ik} = \frac{\gamma_{ik}}{w_i} + (\theta_2 - 1)(\delta_{ik} - w_k), \tag{3.3}$$

while expenditure elasticities are:

$$e_i = \frac{\beta_i}{w_i} + \theta_1 + 1 \tag{3.4}$$

The restrictions implied by asymmetric separability for the GODDS model are of the form (for i, j ∈ A, which is asymmetrically separable from k ∈ B):

$$\gamma_{ik} = \frac{\left(\beta_i + w_i(\theta_1 + 1)\right)}{\left(\beta_j + w_j(\theta_1 + 1)\right)}\left(\gamma_{jk} - w_j w_k(\theta_2 - 1)\right) + w_i w_k(\theta_2 - 1) \qquad (3.5)$$

These restrictions are imposed at the mean shares unless the resulting model is Rotterdam. Consequently, the test of separability in GODDS are local (unless the restrictions that result in the Rotterdam model are imposed. To test for separability between the meat products, numbers can be given to them so that 1 – 4 is for beef and veal; mutton and goat meat; pork as well as poultry, respectively. The nonredundant restrictions that are necessary and sufficient for the asymmetric separability are thus (e.g., separability of beef/veal from mutton/goat, pork, and poultry):

$$\frac{\sigma_{12}}{\sigma_{14}} = \frac{e_2}{e_4}, \quad \frac{\sigma_{13}}{\sigma_{14}} = \frac{e_3}{e_4} \qquad (3.6)$$

where σ_{ij} is the Allen-Uzawa elasticity of substitution given as:

$$\sigma_{ik} = \frac{\gamma_{ik} + (\theta_2 - 1)w_i(\delta_{ik} - w_k)}{w_i w_k}. \qquad (3.7)$$

Using the above framework, there are two steps of analysis. The first one involves the model that was restricted by homogeneity and symmetry using iterative Seemingly Unrelated Regression (ITSUR), while the second estimation incorporates the restrictions implied by the separability of meat products, using the appropriate model in a nonlinear framework. All restrictions in equations (3.5 – 3.8) are tested by comparing estimates from the above two estimations using a likelihood-ratio test.

4. Analysis and Results

4.1 Test of Models and Separability

The nested model (GODDS) is restricted for homogeneity and symmetry and estimated with a system of equations by ITSUR in SHAZAM. Conducting an adjusted likelihood ratio test yielded the Rotterdam as the appropriate model (Table 3). Given the restrictions of separability are easily done and the results of its tests are global in Rotterdam, further analysis was therefore done using the Rotterdam model.

The results for the asymmetric separability tests are presented in Table 4. A detailed description of the method for imposing separability in the Rotterdam model can be found in the articles of Moschini, Moro and Green (1994) and Capps *et al.* (1994). The results indicate that only pork is statistically separable from other meat types at the 5% level. The result is not surprising since as pointed out in section 1, the composition of the Nigerian population has a tremendous effect on pork consumption.

4.2 Elasticities of Demand

Table 5 reports the compensated elasticity estimates from the Rotterdam model. Curvature was imposed using Cholesky decomposition for consistency with the economic theory. This is because initial results violated curvature conditions. All of the own-price elasticities are inelastic, which indicates that if an own-price increases by 1%, consumption decreases by less than 1% in all meat type. Moreover, beef and poultry are less responsive to own-prices than are mutton or pork. In regards to cross-price effects, the meat products are more often substitutes than complements. Pork and poultry are indicated to be compliments, while almost all others are substitutes.

The expenditure elasticity estimates, are also provided within Table 5. Beef, pork, and poultry have positive expenditure elasticities and are thus normal goods. Importantly, as income increases, consumers tend to purchase more beef than other meat products. This is consistent with expectations as beef is relatively more expensive than other meat in Nigeria and is consumed by those with higher income. Mutton and goat meat appear to be the inferior good, since their combined consumption is negatively responsive to income change. This is understandable since Nigerians spend about 70% of their disposable income on food generally and there is tendency for consumption of meat products to shift from the low patronized products to the expensive ones as income increases.

5. Summary and Conclusion

The study estimated the demand for meat products in Nigeria. To do this, it is appropriate to identify the model providing the most robust estimates that could be adopted for policy actions. The GODDS that nests four differential models was used and tested, yielding the selection of the Rotterdam model, which was not only used to impose and test for separability but also to estimate the price elasticities of consumer meat demand.

As would be expected, pork is the only meat type that is separable from others and this should be noted when modeling demand for meat in Nigeria. The own price elasticities are all inelastic, indicating that an increase in own prices by 1 percent will not change the demand for meat products much.

The areas for further research are as follows: (i) the incorporation of additional meat substitutes such as fish, demographic variables as well as food safety variables (if any) that can have impact on meat demand, (ii) analyzing the demand for meat with more data than used here could generate better estimates, and (iii) investigating further the effect of separability restrictions on the elasticity estimates and other economic parameters.

References

Barten, A. P. “Consumer Allocation Models: Choice of Functional Forms.” *Emprical Econ.* 18 (1993): 129-58.

Blackorby, C., R. Davidson, and W. Schworm. “Implicit Separability: Characterization and Implications for Consumer Demands.” *J. Econ. Theory* 55 (Dec. 1991): 364 – 99.

Capps, O., Jr., R. Tsai, R. Kirby, and G.W Williams. “A Comparison of Demands for Meat Products in the Pacific Rim Region.” *J. Agr. And Resour.* Econ. 19,1 (1994): 210-24.

Eales, J., C. Durham, and C. Wessells. “Generalized Models of Japanese Demand for Fish.” *Amer. J. Agr. Econ.* 79 (November 1997): 1153-63.

Frisch, R. “A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors.” *Econometrica,* Vol. 27 (1959): 177-96.

Goldman, S. M. and H. Uzawa. “A Note on Separability in Demand Analysis.” *Econometrica,* Vol 32, Issue 3 (Jul.,1964): 387-398.

Gorman, W. M. “Separable Utility and Aggregation.” *Econometrica,* Vol. 27 (1959): 469-81.

Hauthakker, H. S. “Additive Preferences.” *Econometrica,* Vol. 28 (1960): 244-57

Lee, J. Y., M.G. Brown, and J. L. Seale, Jr. “Model Choice in Consumer Analysis: Taiwan, 1970-1989.” *Amer. J. Agr. Econ.* 76 (August 1994): 504-12.

Leontief, W. W. “A Note on the Interrelation of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives.” *Bulletin of the Amer. Math. Society,* Vol. 53 (1947): 343-50.

Leontief, W. W. “Introduction to the Theory of the Internal Structure of Functional Relationships.” *Econometrica,* Vol. 15 (1947): 361-73.

Moschini, G., D. Moro, and R. D. Green. “Maintaining and Testing Separability in Demand Systems”. *Amer. J. Agr. Econ.* 76 (February 1994): 61-73.

Neves, P. D. “A Class of Differential Demand Systems.” *Econ. Letters* 44 (1994): 83-86.

Pearce, I. F. “An Exact Method of Consumer Demand Analysis.” *Econometrica,* Vol. 29 (1961): 499-516.

Sono, M. “The Effect of Price Changes on the Demand and Supply of Separable Goods.” *International Econ. Review,* Vol. 2 (1961): 239-71.

Strotz, R. H. “The Empirical Implication of Utility Tree.” *Econometrica,* Vol. 25 (1957): 269-80.

Strotz, R. H. “The Utility Tree-A Correction and Further Appraisal.” *Econometrica,* Vol. 27 (1959): 482-88.

World Bank Development Reports, 1980 – 96 Issues. *Oxford Univ. Press.*

Table 1: Import Quantity Percentage Share of Nigerian Domestic Consumption (5-year averages)[1]

Year Span	Percentage Share			
	Beef[a]	Mutton[b]	Pork	Poultry
1961-1965	1.2	0	0	0
1966-1970	0	0	0	0
1971-1975	1.0	0	0	0
1976-1980	6.9	0	0	12.1
1981-1985	3.9	0	0	6.4
1986-1990	0.4	0	0	0
1991-1995	0	0	1.7	0
1996-1999[c]	0.3	0	8.8	0

Source: Computed from FAO Food Balance Sheet, 1961 – 1999.
a = including Veal; b = including goat meat; c = only 4-year average. 1 indicates that per capita values are used to derive the percent share.

Table 2. Restrictions on the Generalized Models Which Yield Alternative Functional Forms

Model	Restrictions	
	2_1	2_2
AIDS	0	0
Rotterdam	-1	1
CBS	0	1
NBR	-1	0

Source: Testing Separability of Japanese Demand for Meat and Fish Within Differential Demand Systems, Eales and Wessells (1999), J. of Agric. & Res. Econs., p117.
Note: Notation follows that of equations (3.1) and (3.2) in the text.

Table 3: Tests of Models Nested within GODDS

	Log-Likelihood	Adj LR Statistic
GODDS	298.64	
AIDS	294.31	7.40
CBS	297.74	1.54
NBR	294.63	6.85
Rotterdam	298.62	0.03

Chi-square critical value at 5% with 2 df = 7.38

Table 4. Separability Test Results

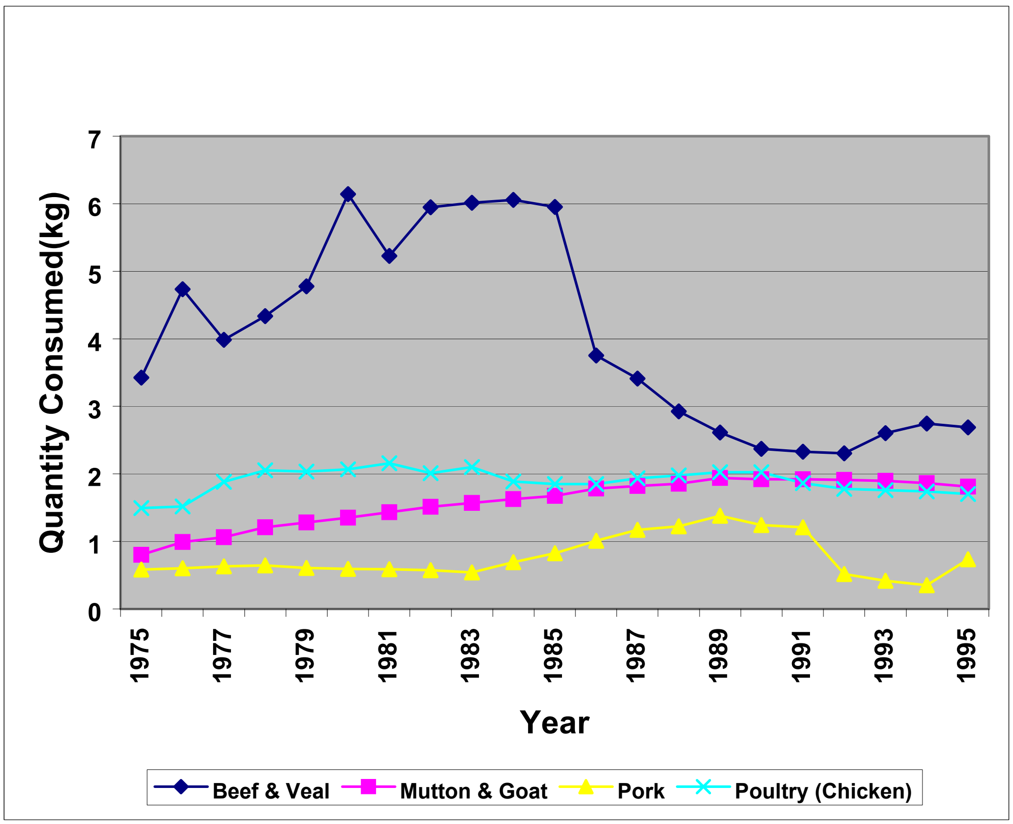
Separability Structure	# Restrictions	Log-Likelihood	Adj LR Statistic
Rotterdam Unrestricted[a]		303.83	
Beef vs. rest of meat	2	298.56	4.50
Mutton vs. rest of meat	2	298.62	4.45
Pork vs. rest of meat	2	250.91	45.23

Chi-square critical value at 5% with 2 df = 7.38

Table 5. Compensated Price Elasticity Estimates

	Beef [a]	Mutton[b]	Pork	Poultry	Income
Beef [a]	-0.0550	-0.0198	0.0718	0.0030	1.7953
Mutton[b]	0.3827	-0.8073	0.6775	0.2529	-0.0022
Pork	0.1340	0.5719	-0.8270	-0.1211	0.2082
Poultry	-0.0617	0.0779	-0.0442	-0.0280	0.0829

[a] beef includes veal and [b] mutton includes goat.



<u>Source:</u> Drawn from the data in FAO Food Balance Sheet, 1975-95

Fig 1. Per Capita Meat Consumption in Nigeria (1975 – 1995)

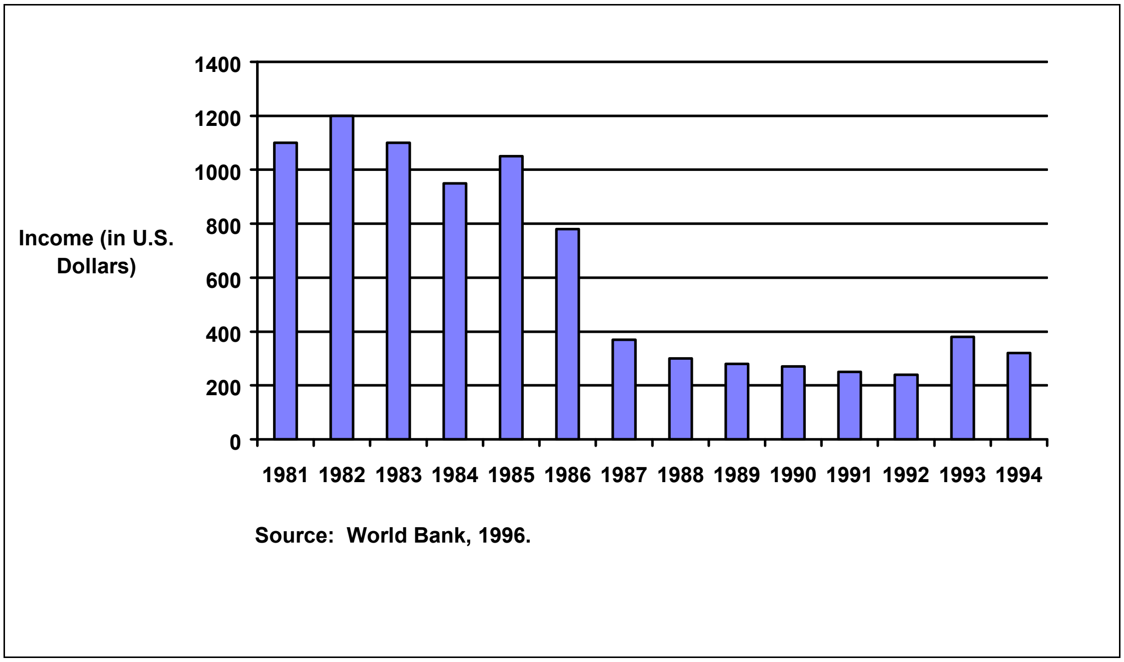


Fig 2: Per Capita Income in Nigeria, 1980 – 1994.

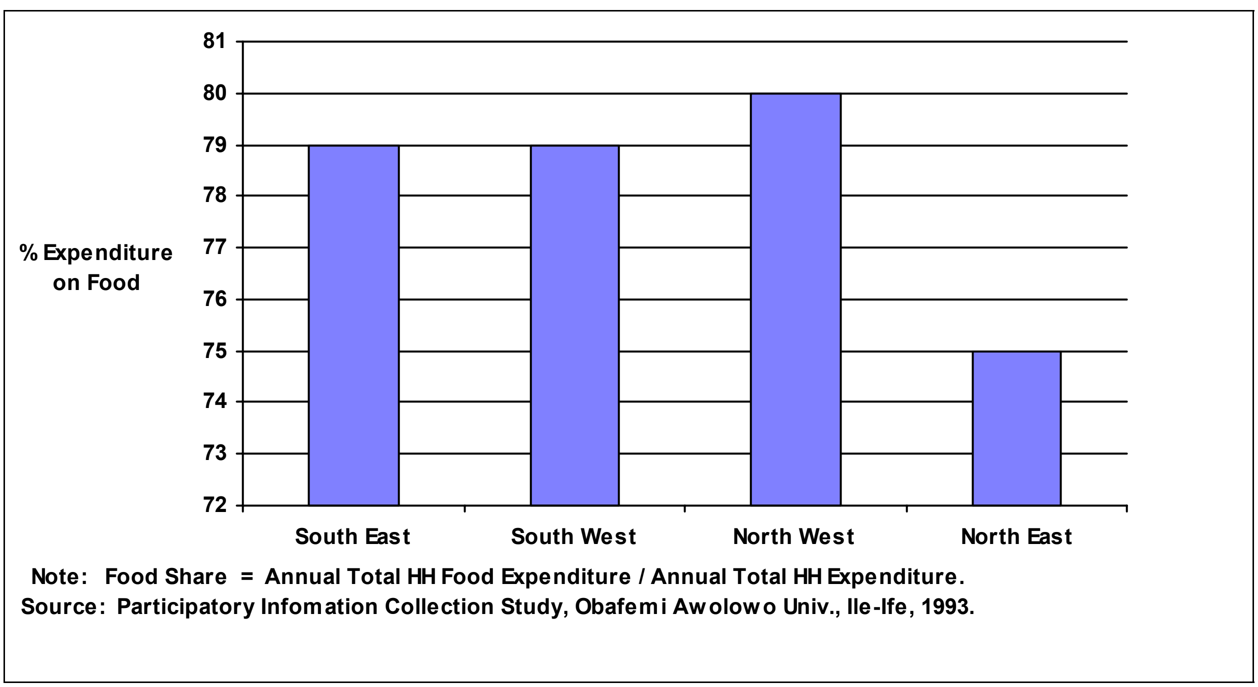


Fig. 3: Percentage of Household Expenditure on Food (Food Share) by Region: 1993