On the Choice of Functional Forms in the Measurement of Scale and Scope Economies: Generalized Box-Cox and Composite Cost Functions

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Abstract

This paper estimates and compares generalized Box-Cox and composite cost functions to identify scale and scope economies. The robustness of the outcomes to different functional specifications was examined. Increasing returns to scale was common for product-specific and the overall measures. Generalized Leontief and composite forms yielded more robust elasticity, scale and scope measures. The generalized Box-Cox model was selected as the appropriate functional form as all the special cases were rejected.

1. **Introduction**

The search for appropriate functional forms in production agriculture has continued to dominate the economics literature. Various functional forms have been used to more accurately explain or predict producer behavior. The parameter estimates of interest can, however, be very sensitive to the types of functional forms used because of the different restrictions and assumptions that are basically imposed by the different forms. Some of the time, researchers have failed to test the sensitivity of such estimates to different forms and under different assumptions. In some cases, general comparisons of estimates are made to other related studies without considering the differing kernels and motivations for such studies. The foregoing has implications for the recommendation and the utilization of the economic measures.

Berndt and Khaled (1979) proposed a generalized Box-Cox cost function, with and without technological change, that takes on special or limiting cases of generalized Leontief (GL), generalized square-root quadratic (GSRQ), and translog (TLOG) cost functions. Its one-output case, incorporating technological change, was applied by these authors to the manufacturing sector of the U.S. economy. An extension of their generalized form into the multiple-output case was undertaken by Bruno De Burger (1992) to estimate cost structure and productivity growth in Belgian railroad operations.

In the context of the parametric estimation of scale and scope economies, indirect cost functions, which are based on duality theory, have generally been estimated. The generalized translog function has been commonly used, while the composite form is becoming increasingly preferred by many analysts. A two-input (labor and capital) two-output (life insurance and superannuation) version of the generalized translog form was adopted by Khaled, Adams and Pickford (2001) to estimate a variety of measures of economies of scale and scope using 135 pooled observations for the non-bank life insurance operations in New Zealand. They found that product-specific returns to scale were increasing for the small business but approximately constant for the larger ones, and that there were substantial economies of scale in the superannuation output for firms of all sizes. There were diseconomies of scope in the small- and medium-sized firms. The large firms, however, did not experience either economies or diseconomies of scope.

Neither functional form – the generalized Box-Cox nor the composite - has been commonly applied in production agriculture. Aside from its use in a study involving the estimation of elasticities of substitution for U.S agricultural production (Chalfant, 1984), the generalized Box-Cox has not been used in farm analysis. The composite counterpart, which is relatively new, has not been estimated for production agriculture. Its advantages over a number of the common functional forms are mentioned in section 3 of this paper.

This paper analyzes the agricultural sector of Kansas economy via economies of scale and scope and attempts to determine the robustness of these measures under alternative functional forms.

The paper is organized as follows. The multiple-output Box-Cox cost function and its economic properties are reviewed in section 2. In section 3, the composite form of cost function and its properties are discussed. These two forms are estimated using Kansas data in section 4. After the empirical results are discussed, the paper ends with concluding comments.

2. Generalized Box-Cox Cost Function

The generalized Box-Cox cost specification, including its special and limiting cases of GL, GSRQ and TLOG (Berndt and Khaled 1979, Bruno De Burger 1992) is presented below.

$$C = \left[1 + \lambda\, G(P)\right]^{\frac{1}{\lambda}} \left[\prod_{k=1}^{K} Z_k^{\beta_k (Z,P)}\right] \tag{1}$$

where
$$G(P) \equiv \alpha_0 + \sum_{i=1}^{N} \alpha_i P_i(\lambda) + \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} P_i(\lambda) P_j(\lambda) \tag{2}$$

$$\beta_k(Z,P) \equiv \beta_k + \sum_{l=1}^{K} \frac{\theta_{lk}}{2} \ln Z_l + \sum_{i=1}^{N} \phi_{ki} \ln P_i \tag{3}$$

$$P_i(\lambda) \equiv \frac{(P_i^{\frac{\lambda}{2}} - 1)}{(\frac{\lambda}{2})} \tag{4}$$

The vector P consists of input prices, while the vector Z consists of the outputs. The symmetry assumption implies that $\gamma_{ij} = \gamma_{ji}$ and $\theta_{lk} = \theta_{kl}$. Linear homogeneity in prices, according to Berndt and Khaled (1979), occurs if and only if:

$$(a)\ \sum_{i=1}^{N}\alpha_i = 1+\lambda\alpha_0,\quad (b)\ \sum_{j=1}^{N}\gamma_{ij} = \frac{\lambda}{2}\alpha_i\ \ \forall i,\quad (c)\ \sum_{i=1}^{N}\phi_{ki} = 0\ \ \forall k. \tag{5}$$

Imposing the homogeneity restrictions (5) on the GBC cost function (1) results in:

$$C = \left[\frac{2}{\lambda}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\,P_i^{\frac{\lambda}{2}}P_j^{\frac{\lambda}{2}}\right]^{\frac{1}{\lambda}}\left[\prod_{k=1}^{K}Z_k^{\beta_k(P,Z)}\right]. \tag{6}$$

Several of the special or limiting cases can be seen in model (6). For instance, when $\lambda =$ 2, the result is the generalized square-root quadratic (GSRQ) form:

$$C = \left[\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\,P_iP_j\right]^{\frac{1}{2}}\left[\prod_{k=1}^{K}Z_k^{\beta_k(P,Z)}\right]. \tag{7}$$

When $\lambda = 1$, a GL function results:

$$C = \left[2\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\sqrt{P_iP_j}\right]\left[\prod_{k=1}^{K}Z_k^{\beta_k(P,Z)}\right]. \tag{8}$$

To obtain the TLOG case, the limit of equation (9), derived by rewriting the GBC (1) as $\lambda \to 0$ yields the TLOG function (10).

$$G(P) = \frac{\left[C\Big/\left(\sum_k Z_k^{\beta_k(Z,P)}\right)\right]^{\lambda} - 1}{\lambda} \tag{9}$$

$$\ln C = \alpha_0 + \sum_{i=1}^{N}\alpha_i \ln P_i + \sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{ij}\ln P_i \ln P_j + \sum_{k=1}^{K}\beta_k \ln Z_k$$

$$+\sum_{k=1}^{K}\sum_{l=1}^{K}\frac{\theta_{lk}}{2}\ln Z_k + \sum_{k=1}^{K}\sum_{i=1}^{N}\phi_{ki}\ln P_i \ln Z_k\ . \tag{10}$$

The TLOG, however, involves an additional parameter restriction on (5), because as $\lambda \to 0$, the expression (a) becomes:

$$\lim_{\lambda\to 0}\ \sum_{i=1}^{N}\alpha_i = 1.$$

Corresponding to the GBC (6), the following factor demand system can be derived (De Burger) (1992):

$$X_i = \left[\frac{2}{\lambda} \sum_{j=1}^{N} \gamma_{ij} \left(\frac{P_j}{P_i} \right)^{\frac{\lambda}{2}} \right] \left[\prod_{k=1}^{K} Z_k^{\lambda\, \beta_k(Z,P)} \right] \left(\frac{C}{P_i} \right)^{1-\lambda} + \sum_{k=1}^{K} \frac{C}{P_i} \left(\phi_{ki} \ \ln Z_k \right). \qquad (11)$$

In addition, the Allen partial elasticities of substitution (σ_{ij}) are specified as:

$$\sigma_{ij} = 1 - \lambda + \gamma_{ij} \frac{(P_i P_j)^{\frac{\lambda}{2}}}{s_i s_j} M + \lambda \frac{F_j(Z)}{s_j} + \lambda \left[1 - \frac{F_j(Z)}{s_j} \right] \frac{F_i(Z)}{s_i}, \quad i \neq j, \qquad (12)$$

and

$$\sigma_{ii} = 1 - \lambda + \gamma_{ii} \frac{p_i^{\lambda}}{s_i^2} M + \lambda \frac{F_i(Z)}{s_i} + \lambda \left[1 - \frac{F_i(Z)}{s_i} \right] \frac{F_i(Z)}{s_i} + \frac{\lambda}{2} \left[1 - \frac{F_i(Z)}{s_i} \right] \frac{1}{s_i} - \frac{1}{s_i}, \qquad (13)$$

where $M = \left[\frac{2}{\lambda} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \, P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}} \right]^{-1},$ (14)

$$F_i(Z) = \sum_{k=1}^{K} \theta_{ki} \ln Z_k \ , \qquad i = 1, \ldots\ldots, N \qquad (15)$$

$$s_i = \frac{P_i X_i}{C}, \qquad i = 1, \ldots\ldots\ldots, N \qquad (16)$$

The input price elasticities are then derived from the Allen Partial elasticities:

$$\eta_{ij} = s_j \sigma_{ij} \, , \qquad i, j = 1, \ldots\ldots\ldots, N \qquad (17)$$

Finally, the cost elasticities with respect to the outputs are given as:

$$\varepsilon_k = \frac{\partial \ln C}{\partial \ln Z_k} = \beta_k + \sum_{l=1}^{K} \theta_{lk} \ln Z_l + \sum_{i=1}^{N} \phi_{ki} \ln P_i \, . \qquad (18)$$

Expressions 17 and 18 are used to verify if the regularity conditions implied by the economic theory are satisfied is this analysis.

In a multiple-output estimation, scale economies can result from two sources: scope economies and/or product-specific economies (Featherstone and Moss, 1994).

Economies of scope measures the cost advantage associated with producing several output simultaneously. For two outputs, a measure of scope economies (SCP), according to Baumol, Panzar and Willig (1982), is:

$$SCP = \frac{\left[C(y_1,0) + C(0,y_2) - C(y_1,y_2)\right]}{\left[C(y_1,y_2)\right]} \tag{19}$$

where $C(y_1,0)$ is the cost of production if good y_2 is not produced. $C(0,y_2)$ is the cost of production if good y_1 is not produced, while $C(y_1,y_2)$ is the cost of producing y_1 and y_2 together. Economies of scope exists (does not exist) if SCP is greater (less) than zero. Product-specific economies of scale (SCL_i) measures the short-run impact of expanding the production of a single product while input prices, fixed inputs and other output levels remain constant. The SCL_i are measured as:

$$SCL_i = \frac{AIC_i}{MC_i},\ i = 1,2 \tag{20}$$

where (MC_i) represents the marginal cost of output *i* and the average incremental cost (AIC_i) are depicted by:

$$(a)\ AIC_1 = \frac{\left[C(y_1,y_2) - C(0,y_2)\right]}{y_1}, \quad (b)\ AIC_2 = \frac{\left[C(y_1,y_2) - C(y_1,0)\right]}{y_2} \tag{21}$$

An overall measure of the returns to scale for an individual firm, also referred to as scale economies (*SCL*), can result from the combination of both economies of scope and product-specific economies. For a two-output firm, this is stated as:

$$SCL = \frac{\left(\theta_1 SCL_1 + \theta_2 SCL_2\right)}{\left(1 - SCP\right)} \tag{22}$$

where $\theta_1 + \theta_2 = 1,\ \ and\ \ \theta_1 = \frac{y_1 MC_1}{\sum_j y_j MC_j}$, j = 1, 2. (23)

The existence of economies of scale is therefore sensitive to the relative magnitudes and nature of SCL_i and the SCP. For instance, if economies of scope equal zero, economies of scale will exist if one of the output exhibits increasing returns (IRS) to scale while the other output has constant returns to scale (CRS). Different outcomes are expected if scope economies exist.

3. The Composite Cost Function

The composite form of cost function was proposed by Pulley and Braunstein and was applied to economies of scope in the banking industry. It is a flexible multiproduct function, in the sense of Diwert (1974), which combines the log-quadratic input price structure of the translog model with a quadratic structure for multiple outputs as well as satisfies the properties of linear homogeneity in input prices, non *a priori* imposition of separability between ouputs and inputs, and has the ability to model cost behavior in the range of zero outputs, a limitation of translog forms. The non-imposition of separability in the composite model gives it an added advantage over other multiplicatively separable forms such as the quadratic, the translog and the CES-Quadratic functions. The motivation to develop the composite function was aroused by an earlier suggestion of a quadratic output structure in the measurement of scale and scope economies as well as subadditivity by Baumol, Panzar and Willig.

Very few empirical studies (Pulley and Braunstein, Khaled) that have been known to have applied the composite model in the estimation of multiple-output technologies have been in the banking and insurance sectors. None of such use has however been found in agricultural fields. The composite model is given by:

$$C = \left[\alpha_0 + \sum_i \alpha_i q_i + \frac{1}{2} \sum_i \sum_j \alpha_{ij} q_i q_j + \sum_i \sum_k \delta_{ik} q_i \ln r_k \right] \exp[f(r)] + \varepsilon \qquad (24)$$

where C is the cost, ε is an error term, $q_{i,j}$ refers to the outputs and r_k are the input prices. The exponential in 24 is stated as:

$$f(r) = \beta_0 + \sum_k \beta_k \ln r_k + \frac{1}{2} \sum_k \sum_l \beta_{kl} \ln r_k \ln r_l + \sum_i \sum_k \mu_{ik} q_i \ln r_k \qquad (25)$$

Equation (25) is a translog function of the input prices (r_k). The symmetry condition requires that $\beta_{kl} = \beta_{lk}$, $\alpha_{ij} = \alpha_{ji}$, while linear homogeneity of cost in input prices requires that $\sum_k \beta_k = 1,\ \sum_l \beta_{kl} = 0,\ and\ \sum_k \delta_{ik} = 0$. It was suggested that β_0 be chosen arbitrarily since it is not defined (Pulley and Humphrey). The price and output interaction terms both in the output and the input structures of the composite model are included in order to avoid the imposition of separability. Equation 24 can be written in its logarithmic form, the only change being that the multiplicative exponential part involving the price structure now becomes additive. The logarithmic version may however provide better description of some data sets because of the combination of the outputs quadratic structure with that of the log-quadratic for input prices as described above.

In order to gain better understanding on the choice of empirical specification in their analysis, the proponents of the composite model opted for the adoption of the "transform-both-sides" procedure of Carroll and Ruppert (1984, 1987, 1988). This procedure suggests a Box-Cox transformation of both sides of equation 24 to enable its simultaneous estimation with the logarithmic counterpart. Incorporating the transformation on both sides of the composite cost function results in its hybrid representation presented as follows:

$$C^{(\phi)} = \left\{ \left[\alpha_0 + \sum_i \alpha_i q_i^{'} + \frac{1}{2}\sum_i \sum_j \alpha_{ij} q_i^{'} q_j^{'} + \sum_i \sum_k \delta_{ih} q_i^{'} \ln r_h \right] \exp[f(r)] \right\}^{(\phi)} + \varepsilon \qquad (26)$$

where the superscript (ø) refers to the Box-Cox transformation and q'=q-1. This implies that equation 24 and its logarithmic version are special cases of equation 26 when ø equals 1 and 0, respectively[1]. Applying Shephard's Lemma to equation 26 results in the share equations shown as:

$$C^{(\phi)} = \left[\alpha_0 + \sum \alpha_i q_i^{'} + \frac{1}{2}\sum \sum \alpha_{ij} q_i^{'} q_j^{'} + \sum \sum \delta_{ih} q_i^{'} \ln r_h \right]^{-1} * \left(\sum \delta_{is} q_i^{'} \right) + \beta_s + \sum \beta_{sl} \ln r_l + \sum \mu_{is} q_i^{'} \qquad (27)$$

The appropriate point of entry for the price-output interaction terms needs to be investigated. Pulley and Braunstein found that incorporating the terms through the output structure by deleting μ_{ik} from the models resulted into best fit in their study of the banking industry.

For the composite form, Khaled gives the product-specific scale economies (SCL_i), the scope economies (SCP) of Pulley and Humphrey and the overall scale economies (SCL) as equations 28, 29 and 30, respectively:

$$SCL_i = 1 - \frac{0.5\alpha_{ii} q_i}{\alpha_i + \sum_j \alpha_{ij} q_j + \sum_k \delta_{ik} \ln r_k} \qquad (28)$$

[1] This approach of Box-Cox-Composite (BCC) procedure could be estimated, but equation 24 is used in this paper.

$$SCP = \frac{(n-1)\alpha_0 - 0.5\sum_{i\neq}\sum_{j}\alpha_{ij}q_i q_j}{\alpha_0 + \sum_i \alpha_i q_i + 0.5\sum_i\sum_j \alpha_{ij}q_i q_j + \sum_i\sum_k \delta_{ik}q_i \ln r_k} \quad (29)$$

$$SCL = \frac{\alpha_0 - 0.5\sum_i\sum_j \alpha_{ij}q_i q_j}{\sum_i \alpha_i q_i + \sum_i\sum_j \alpha_{ij}q_i q_j + \sum_i\sum_k \delta_{ik}q_i \ln r_k} \quad (30)$$

The magnitude of SCL_i depends on α_{ii}. SCL_i will be less than one if α_{ii} is greater than zero, which signifies increasing returns to scale and greater than one if α_{ii} is less than zero, a case of decreasing returns to scale. In both cases, the cost function must satisfy output regularity in form of positive marginal costs. If, however, α_{ii} equals zero, returns to scale are constant at all output levels, i.e. product-specific economies of scale neither arise owing to economies of scope ($\alpha_{ij} < 0$) nor due to cost complementarities.

Considering equation 29, the existence of economies of scope is determined either by α_0, which represents a spread of fixed cost over a variety of outputs or α_{ij}, which is the cross output interactions. If the non-specific fixed cost component α_0 is close to zero and α_{ij} are all of the same signs, economies of scope either exist or not at all sizes of outputs. The measure in equation 30 is in line with Baumol, Panzar, and Willig in that economies of scale depends on both product-specific economies of scale and economies of scope. Whether there is increasing or decreasing returns to scale will depend however on the signs on the individual components of α_{ij} in relation to α_o at different output levels.

4. Data and Estimation Procedure

The data used in this paper comprise of the prices of eight inputs (seed, fertilizer, pesticides, feed, energy, labor, land and machinery) as well as the gross output quantities for crop and livestock production. The observations span across 106 farms over 26 years, amounting to 2756 observations[2]. The normalized versions of both functional forms (GBC and Composite) using machinery were estimated[3]. This might seem reasonable since they are only needed in this context in the evaluation of curvature property, which was not imposed in this paper.

Systems of cost and factor demand or share equations including the error terms were estimated using a Marquardt nonlinear estimation procedure in SAS. The system of nonlinear equations can be estimated using a seemingly unrelated regression (SUR) method. However, this will be inappropriate in this context since it may generate inconsistent estimates because of the endogeneity of the observed cost, which appears on the right hand side of the factor demand equations in the Box-Cox specifications. As an alternative, iterative 3-stage least squares (IT3SLS), which guarantees a level of consistency is therefore used in this paper. This requires inclusion of instruments defined as lagged prices, cost and outputs up to a 3rd-lag.

The coefficient 'lambda' in the Box-Cox model, in conjunction with other parameters, was estimated parametrically and hypothesis testing was done, using Wald, test. The parametric estimation of lambda was difficult until a range (bound) of $0.3 \leq \lambda \leq 5$ was imposed in the SAS proc model. Numbers outside this were problematic, especially those very close to zero or involving negative signs. This is understandable

[2] Zero output quantities were substituted with a 10 percent of the mean to avoid missing values as well as estimation problems when the natural logarithm was taken.
[3] The share equation for machinery input was not recovered.

since equations 3 and 4 will only be finite for lambda values that are greater than zero. Estimation involving lambda equals zero and lambda equals one (i.e. special or limiting cases of TLOG and GL cost functions, respectively) therefore proved to be complex. To manage this problem, a grid search[4] was done by estimating lambda at values close to one (GL case). The estimated lambda values that do not increase the objective values[5] away from 1 was adopted in place of the exact values that were problematic. Trials with different values yielded 1.13 for lambda equals 1(GL). This value was adopted in computing other economic estimates of interest since it is not expected to perform poorly. Another alternative, in the sense of Bruno De Burger, would be to specify and estimate the translog and the generalized Leontief functions separately without incorporating them within the generalized Box-Cox specification. A separate function was therefore specified and estimated for the translog case. In both cases of the translog and the composite cost function, the same estimation procedures (Marquardt and IT3SLS) as obtained for generalized Box-Cox were used. This is to enable consistent comparison of the parameters that are obtained from them.

5. Empirical Results and Discussion

The estimated results from the four different cases of the generalized Box-Cox cost function (GBC, GL, GSRQ, and TLOG) as well as those from the composite cost function are reported in tables 1, 2, 3 and 4.

Table 1 presents the parameter estimates for generalized Box-Cox cost functions. Of the 47 coefficients, 40 and 31 were significantly different from zero at the 5% level

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[4] Thanks to Dr. James Chalfant of University of California, Davis and Dr. Bruno De Burger of University of Antwerp, Antwerp, Belgium for their suggestions in this regard.

[5] Since this is a cost minimization exercise.

for the GBC and the GL, respectively. For the TLOG, 38 coefficients were significant. In the remaining special case, GSRQ, much less statistical significance was observed. The use of the Wald (table 3) test statistic indicates that the special cases are rejected at the 5% level.

The composite cost function results, depicted in table 3, indicate that 41 out of 55 coefficients are significantly different from zero at the 5% level. Estimated elasticity measures (both of substitution and price) are presented in tables 5 to 8. The marginal costs are in table 9, while the cost elasticities measures with respect to the output of crop and livestock are depicted in table 10. All elasticity estimates were calculated at the mean of the price and output variables. These estimates seem reasonable, but only that some of the own-price elasticities are not consistent with the theory. There is therefore a need for the curvature imposition, although was not carried out as mentioned earlier.

The marginal costs are all positive except for the GBC function (table 9). The negative values for the GBC result in the negative values of the cost elasticities. Table 11 contains the estimates for the multi-product (overall) economies of scale as well as the product-specific counterpart. The product-specific scale measures are positive unless that for livestock under the TLOG and the GBC. There are mixed outcomes for these measures, indicating all three types of returns to scale – increasing returns to scale (IRS) if measures are greater than 1, decreasing returns to scale (DRS) if lower than 1, and constant returns to scale (CRS) if close to 1. IRS seems to dominate for the product-specific economies. In the case of the overall scale economies, both DRS and IRS dominate.

The economies of scope measures are presented in table 12. The GSRQ produces a negative scope measure, therefore indicating that diseconomies of scope exist. They are, however, positive under other forms. This shows that economies of scope exist meaning that splintering production into single crop or livestock operation would not be cost efficient. Both scope and product-specific measures have therefore contributed in varying degree to cost efficiency of farms as recorded in the results tables.

Overall, using the t-test, the GL and the composite form appear to have performed better than the other functional specifications. This is because they result in many significant coefficients as well as more robust economies measures. The TLOG measures of scale and scope might have been affected by the method used via the incremental, marginal and total costs calculated using the simple calculus. When the Wald test is administered, GBC with a lambda value of 0.3712 stands out to be the appropriate functional form. This lambda value could be approximated to zero, which makes the TLOG also a candidate for appropriate form although it is rejected with the other special cases of GL and GSRQ.

6. Summary and Conclusion

Five functional forms, four of which are special cases in Box-Cox specification and the composite type were used in this paper. By making series of estimations with different bounds, lambda was parametrically estimated. All the special cases for the Box-Cox (GL, TLOG, GSRQ) were rejected when tested against the GBC at the 5% chi-square level. Nevertheless, the GL and the composite forms give the more robust measures. The results also show the IRS is common for product-specific economies, while DRS and IRS prevails for the overall counterpart. It could be summarized that major contribution to cost efficiency has comes from joint production of crop and livestock.

The results could be improved in some ways. The first and the most important suggestion for further research is that of curvature imposition since the results need to be consistent with the theory. The elasticity estimates and other economic parameters derived from the functional forms will only be reliable if the regularity properties are satisfied. Alternative specification of GL is needed since it was approximated by using grid search in the neighborhood of one. The use of a full information maximum likelihood (FIML) method is recommended. This enables easy computation of the generalized R-square, which is appropriate to test the goodness of fit of the different forms specifications.

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Table 1: Estimated Generalized Box-Cox (GBC) Cost Function.

	GBC		GL		GSRQ	
	Parameter	Std Error	Parameter	Std Error	Parameter	Std Error
γ11	4.1592*	1.2671	0.0139*	0.0064	0.0053	0.0047
γ12	-0.6405	0.9123	-0.0035	0.0028	-0.0024	0.0024
γ13	0.4277	0.8734	-0.0050	0.0031	-0.0028	0.0031
γ14	-2.9627*	0.8385	-0.0033	0.0021	0.0025	0.0021
γ15	0.8685	0.6003	0.0007	0.0014	0.0013	0.0014
γ16	-4.6448*	1.2093	-0.0057	0.0033	-0.0032	0.0029
γ17	6.6477*	2.3126	0.0145	0.0115	-0.0008	0.0000
γ22	11.1730*	2.2042	0.0056	0.0035	-0.0085	0.0063
γ23	0.4355	0.9122	0.0019	0.0026	0.0039	0.0033
γ24	2.2904*	0.8627	0.0045	0.0023	0.0206	0.0147
γ25	-1.4965*	0.5625	-0.0013	0.0013	-0.0034	0.0026
γ26	4.3792*	1.3054	-0.0138*	0.0056	-0.0066	0.0051
γ27	-24.0088*	5.3881	0.0661*	0.0252	-0.0036	0.0000
γ33	-2.4540*	1.0258	-0.0039	0.0030	-0.0092	0.0072
γ34	-4.0270*	1.0102	-0.0019	0.0020	0.0038	0.0031
γ35	0.1015	0.5300	-0.0047*	0.0022	-0.0053	0.0039
γ36	2.0104*	0.9156	0.0208*	0.0084	0.0099	0.0072
γ37	8.2845*	2.5365	-0.0375*	0.0183	-0.0003	0.0000
γ44	-1.5413	1.0694	0.0045	0.0024	-0.0394	0.0284
γ45	3.2787*	0.8014	0.0044*	0.0019	0.0122	0.0086
γ46	-0.9379	0.8649	0.0014	0.0023	0.0063	0.0048
γ47	10.6039*	3.3329	-0.0731*	0.0292	-0.0059	0.0000
γ55	9.6914*	1.9752	0.0066*	0.0029	0.0070	0.0050
γ56	2.5550*	0.8274	-0.0166*	0.0062	-0.0110	0.0078
γ57	-26.2777*	5.9200	0.0523*	0.0189	-0.0007	0.0000
γ66	7.2711*	1.7305	-0.0081*	0.0039	0.0080	0.0057
γ67	-11.3067*	2.9660	0.0331*	0.0134	-0.0034	0.0000
γ77	15.0992*	4.2821	0.4845*	0.2027	1.6100	1.1925
βc	-1.2023*	0.0400	0.7650*	0.0636	1.1341*	0.0830
βl	-0.6424*	0.0476	0.9910*	0.0533	2.0962*	0.0412
θcc	0.1940*	0.0076	0.1035*	0.0068	0.1439*	0.0095
θlc	0.1461*	0.0043	-0.2041*	0.0067	-0.2530*	0.0065
θll	0.0071*	0.0030	0.1501*	0.0032	0.0241*	0.0045
ϕc1	-0.0050*	0.0013	0.0143*	0.0009	0.0001*	0.0000
ϕc2	0.0395*	0.0012	-0.0013	0.0012	0.0001*	0.0000
ϕc3	-0.0152*	0.0016	0.0238*	0.0009	0.0001*	0.0000

$\phi c4$	-0.0965*	0.0035	-0.0270*	0.0029	-0.0005*	0.0000
$\phi c5$	0.0556*	0.0019	-0.0024*	0.0007	0.0001*	0.0000
$\phi c6$	-0.0263*	0.0020	0.0281*	0.0016	0.0000*	0.0000
$\phi c7$	0.2590*	0.0088	-0.1130*	0.0037	0.0001*	0.0000
$\phi l1$	-0.0079*	0.0009	-0.0102*	0.0005	-0.0001*	0.0000
$\phi l2$	-0.0142*	0.0007	-0.0011	0.0008	-0.0001*	0.0000
$\phi l3$	-0.0032*	0.0011	-0.0118*	0.0005	0.0000*	0.0000
$\phi l4$	0.0949*	0.0021	0.0769*	0.0019	0.0006*	0.0000
$\phi l5$	-0.0088*	0.0016	0.0042*	0.0005	0.0000*	0.0000
$\phi l6$	0.0099*	0.0013	-0.0054*	0.0010	0.0000*	0.0000
$\phi l7$	-0.0445*	0.0077	0.0560*	0.0031	-0.0004*	0.0000
λ	0.3712	-	1.13	-	2.0	-

Std Error=Standard error; GBC=Generalized Box-Cox;
GL=Generalized Leontief; GSRQ=Generalized Square-Root Quadratic.
* indicates significant at the 5 percent level

Table 2. Estimated Translog(TLOG) Cost Function

	Parameter	Std Error		Parameter	Std Error		Parameter	Std Error
$\alpha 0$	-5.9747*	1.3069	$\gamma 26$	-0.0008	0.0120	βl	0.6997*	0.1890
$\alpha 1$	0.0000	0.0286	$\gamma 27$	-0.0885*	0.0168	θcc	0.1134*	0.0138
$\alpha 2$	-0.0525	0.0323	$\gamma 33$	0.0017	0.0048	θlc	0.4238*	0.0341
$\alpha 3$	-0.0123	0.0252	$\gamma 34$	-0.0091	0.0067	θll	0.1212*	0.0056
$\alpha 4$	-0.4719*	0.0294	$\gamma 35$	-0.0204*	0.0053	$\phi c1$	0.0141*	0.0007
$\alpha 5$	0.0912*	0.0203	$\gamma 36$	0.0042	0.0102	$\phi c2$	0.0128*	0.0010
$\alpha 6$	0.1290*	0.0345	$\gamma 37$	0.0133	0.0132	$\phi c3$	0.0161*	0.0007
$\alpha 7$	1.3165*	0.0585	$\gamma 44$	0.0455*	0.0057	$\phi c4$	-0.0178*	0.0020
$\gamma 11$	0.0339*	0.0073	$\gamma 45$	0.0204*	0.0045	$\phi c5$	0.0000	0.0006
$\gamma 12$	-0.0138	0.0095	$\gamma 46$	0.0318*	0.0075	$\phi c6$	0.0142*	0.0010
$\gamma 13$	-0.0143	0.0083	$\gamma 47$	-0.1902*	0.0136	$\phi c7$	-0.0395*	0.0020
$\gamma 14$	-0.0038	0.0073	$\gamma 55$	0.0233*	0.0025	$\phi l1$	-0.0082*	0.0005
$\gamma 15$	-0.0119	0.0059	$\gamma 56$	-0.0518*	0.0065	$\phi l2$	-0.0135*	0.0007
$\gamma 16$	-0.0640*	0.0120	$\gamma 57$	-0.0047	0.0107	$\phi l3$	-0.0040*	0.0005
$\gamma 17$	-0.0032	0.0151	$\gamma 66$	-0.0109*	0.0087	$\phi l4$	0.0684*	0.0014
$\gamma 22$	0.0447*	0.0064	$\gamma 67$	0.1015*	0.0181	$\phi l5$	-0.0048*	0.0004
$\gamma 23$	-0.0013	0.0083	$\gamma 77$	0.1604*	0.0152	$\phi l6$	0.0040*	0.0007
$\gamma 24$	0.0173*	0.0087	βc	0.0212*	0.2443	$\phi l7$	-0.0419*	0.0014
$\gamma 25$	-0.0092	0.0057						

* indicates significance at 5 percent level.

Table 3. Test Results on GBC

	λ Value	Wald Statistic
TLOG	0	1709.10
GL	1	4905.70
GSRQ	2	32914.00
GBC	0.37	-

Note: GBC cannot be rejected. All others forms are rejected.

Table 4: Estimated Composite Cost Function

(a) Output Portion:

	Parameter	Std Error		Parameter	Std Error
α0	30.0957	20.1372	γc5	-0.0007	0.0024
α1	0.3771*	0.0836	γc6	0.0497*	0.0127
α2	0.4688*	0.1040	γc7	-0.2106*	0.0456
α11	-0.0001*	0.0000	γl1	-0.0020	0.0046
α12	-0.0004*	0.0001	γl2	-0.0283*	0.0074
α22	0.0000*	0.0000	γl3	0.0277*	0.0070
γc1	0.0504*	0.0111	γl4	0.3149*	0.0673
γc2	0.0290*	0.0070	γl5	-0.0159*	0.0043
γc3	0.0631*	0.0138	γl6	0.0648*	0.0160
γc4	-0.0268*	0.0092	γl7	-0.3168*	0.0680

(b)Translog Portion (i.e., of input prices):

	Parameter	Std Error		Parameter	Std Error
β1	-0.0383	0.1084	β26	0.0715*	0.0121
β2	0.7108*	0.0753	β27	-0.0556*	0.0027
β3	-0.4366*	0.0781	β33	0.0346*	0.0117
β4	-0.1353*	0.0404	β34	-0.0277*	0.0082
β5	0.1532*	0.0522	β35	-0.0101	0.0052
β6	0.9976*	0.0968	β36	-0.0145	0.0124
β7	-0.2514*	0.1132	β37	0.0077*	0.0026
β11	0.0782*	0.0144	β44	0.0645*	0.0141
β12	-0.0072	0.0090	β45	-0.0025	0.0046
β13	-0.0017	0.0095	β46	-0.0332*	0.0110
β14	-0.0211*	0.0080	β47	-0.0181*	0.0055
β15	0.0028	0.0055	β55	0.0681*	0.0044
β16	-0.0502*	0.0137	β56	-0.0198*	0.0075
β17	-0.0077*	0.0023	β57	-0.0406*	0.0017
β22	0.0994*	0.0113	β66	0.1801*	0.0161
β23	-0.0024	0.0092	β67	0.0046	0.0048
β24	-0.0145	0.0087	β77	-0.2896*	0.0064
β25	0.0051	0.0048			

* shows significace at the 5 percent level

Table 5: Elasticities of Substitution between Inputs (GBC)

	Seed	Fert	Pest	Feed	Energy	Labor	Land
Seed	14.4241	-0.3476	0.5930	-6.8131	7.1878	-33.9020	10.1578
Fert	-0.3476	6.5508	3.3887	3.8005	-2.6964	16.4790	-9.5408
Pest	0.5930	3.3887	-29.9173	-10.4393	5.1341	11.5289	13.5612
Feed	-6.8131	3.8005	-10.4393	-6.0375	7.0608	-3.0022	6.0877
Energy	7.1878	-2.6964	5.1341	7.0608	4.2563	15.6117	-16.7315
Labor	-33.9020	16.4790	11.5289	-3.0022	15.6117	41.8664	-3.9009
Land	10.1578	-9.5408	13.5612	6.0877	-16.7315	-3.9009	-9.5188

Fert=Fertilizer; Pest=Pesticides; GBC=Generalized Box-Cox.

Table 6: Elasticities of Substitution between Inputs (GL)

	Seed	Fert	Pest	Feed	Energy	Labor	Land
Seed	23.5662	-4.2057	-13.5464	-1.7151	1.9450	-15.9314	2.7116
Fert	-4.2057	-0.3791	4.7877	4.1395	-1.3562	-16.9336	3.6745
Pest	-13.5464	4.7877	-45.7285	-1.7317	-7.5895	67.5496	-3.8361
Feed	-1.7151	4.1395	-1.7317	-9.0150	4.6619	0.3685	-0.9864
Energy	1.9450	-1.3562	-7.5895	4.6619	2.5218	-32.1512	5.0965
Labor	-15.9314	-16.9336	67.5496	0.3685	-32.1512	-79.1947	13.5098
land	2.7116	3.6745	-3.8361	-0.9864	5.0965	13.5098	1.4668

Fert=Fertilizer; Pest=Pesticides; GL=Generalized Leontief.

Table 7: Elasticities of Substitution between Inputs (GSRQ)

	Seed	Fert	Pest	Feed	Energy	Labor	Land
Seed	-304.637	64.190	160.286	-46.845	-56.142	208.528	0.368
Fert	64.190	106.643	-106.479	-176.258	66.674	204.766	1.862
Pest	160.286	-106.479	537.296	-70.5134	226.145	-665.743	-0.513
Feed	-46.845	-176.258	-70.513	223.999	-162.306	-132.234	2.206
Energy	-56.142	66.674	226.145	-162.306	-216.870	534.746	-0.105
Labor	208.528	204.766	-665.743	-132.234	534.746	-598.858	5.326
Land	0.368	1.862	-0.513	2.206	-0.105	5.326	-1.748

Fert=Fertilizer; Pest=Pesticides; GSRQ=Generalized Square Root Quadratic.

Table 8: Price Elasticity of Demand for Seed

	Seed	Fert	Pest	Feed	Energy	Labor	Land
GBC	0.7080	0.6520	-1.3274	-0.8600	0.3374	1.9563	-2.6446
TLOG	0.0491	0.0996	0.0444	0.1433	0.0792	0.0467	0.2776
GL	1.1567	-0.0377	-2.0289	-1.2842	0.1999	-3.7005	0.4075
GSRQ	-14.9520	10.6141	23.8388	31.9079	-17.1933	-27.9823	-0.4856
Comp	0.0511	0.0981	0.0491	0.1994	0.0770	0.0622	0.2617

Fert=Fertilizer; Pest=Pesticides. GBC=Generalized Box-Cox; TLOG=Traslog; GL=Generalized Leontief; GSRQ=Generalized Square Root Quadratic; and Comp=Composite.

Table 9: Estimated Marginal Cost

	Crop	Livestock
GBC	-0.7618	-0.1493
TLOG	1.4613	1.1746
GL	0.8529	0.9853
GSRQ	1.0195	2.0713
Composite	0.4041	0.6067

GBC=Generalized Box-Cox; TLOG=Traslog; GL=Generalized Leontief; GSRQ=Generalized Square Root Quadratic; and Comp=Composite.

Table 10: Cost Elasticities Measures

	Crop	Livestock
GBC	-0.6480	-0.0986
TLOG	1.2432	0.7760
GL	0.7256	0.6509
GSRQ	0.8673	1.3684
Composite	0.3438	0.4008

GBC=Generalized Box-Cox; TLOG=Traslog; GL=Generalized Leontief; GSRQ=Generalized Square Root Quadratic; and Comp=Composite.

Table 11: Scale Economies

	Multi-product	Product Specific	
		Crop	Livestock
GBC	-0.6842	2.3068	-4.5694
TLOG	0.3007	0.1011	-1.863
GL	1.0240	0.9911	0.5643
GSRQ	0.4424	0.5570	0.9133
Composite	1.4279	1.3371	1.2254

GBC=Generalized Box-Cox; TLOG=Traslog; GL=Generalized Leontief; GSRQ=Generalized Square Root Quadratic; and Comp=Composite.

Table 12: Scope Economies

GBC	3.0441
TLOG	3.1737
GL	0.2293
GSRQ	-0.4696
Composite	0.1057

GBC=Generalized Box-Cox; TLOG=Traslog; GL=Generalized Leontief; GSRQ=Generalized Square Root Quadratic; and Comp=Composite.