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**Accuracy of Implied Volatility Approximations Using
“Nearest-to-the-Money” Option Premiums.**

by

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Abstract

Implied volatility is a useful bit of information for futures and options hedgers and speculators. However, extraction of implied volatility from Black-Scholes (BS) option pricing model requires a numeric search. Since 1988, there have been numerous simplifying modifications to the BS formula proposed and presented in the applied economics and finance literature to allow approximation of implied volatility directly. This study identifies and tests the accuracy of these approximation methods using call only and put-call average elicitation of an implied volatility estimate. Results show that accuracy varies by method and whether call only or put-call average approaches are applied.

Introduction

Futures markets are important sources of information for forming price expectations. If futures markets are efficient, then the prevailing futures price represents today's best estimate of expected price (Fama, 1970). With an estimate of the implied variance, which can be gained from the options market, price distributions could be developed. Price forecasts based on these distributions should be a useful management tool because they are derived from the pooled probabilistic assessments of all traders in the market.

The Black-Scholes model is widely used for estimating volatility of futures prices as implied by the option premiums.

$$(1.1) \quad C = SN(d_1) - (Xe^{-r\tau}N(d_2)),$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{\tau},$$

where C is the call premium, S is the current underlying futures price, X is the discounted exercise price, r is the continuously compounded risk free interest rate, τ is the time until the expiration of the option, σ is the implied volatility for the underlying futures price and N is the cumulative normal density function. The accuracy, assumptions and characteristics of the implied volatility estimates from the Black-Scholes model are well established in the existing

literature. The drawback of the Black-Scholes formula for estimating implied volatility is a lack of a closed form solution, which includes computational difficulty. Within a Black-Scholes model the implied volatility is determined through an iterative process that equates the market-observed option premium to the imbedded variables which are underlying futures price, strike price, time to expiration and interest rates. This task can be performed using statistical or mathematic software packages but cannot be easily done in a spreadsheet or a simple calculator. This feature of the Black-Scholes formula instigated a number of studies that proceed in two general directions: one theoretical, that explores mathematical properties of this implied volatility measure and provides alternative approaches (e.g. Dupuire, 1994, Koekebakker and Lien, 2004), and the other more pragmatic that concentrates on providing acceptable approximations to the Black-Scholes formula that can be easily calculated in a spreadsheet.

This study concentrates on the second line of literature and evaluates and compares the accuracy of several implied volatility approximation formulae relative to the benchmark of the Black-Scholes implied volatility estimates using “nearest-to-the-money” option premiums. While numerous approximations of the Black-Scholes formula have been proposed, little is known about their relative accuracy. Previous studies (Chambers and Nawalkha, 2001; Li, 2005) limit their analysis to just a few models and use hypothetical options to demonstrate the benefits of their proposed method to the alternative methods. The analysis of accuracy in these studies is usually very limited. Therefore, the purpose of this study is to provide a comprehensive analysis of accuracy of all available approximations of Black-Scholes implied volatility. Differently from the previous studies, which used hypothetical options, this analysis uses actual daily settlement prices for December futures and options contracts for corn and cotton from December 1990 to October 2005. Because some approximation formulae are only valid for at-the-money contracts,

nearest-to-the-money options were selected for computations. The use of empirical data for two commodities allows examination of the impact of deviations of futures and strike prices and time to maturity on the accuracy of approximations. The use of a large data set and four alternative accuracy measures ensures the robustness of statistical tests.

The results of this analysis will provide evidence regarding relative accuracy of alternative approximations of the Black-Scholes implied volatility. The results will also demonstrate the accuracy levels of these methods for corn and soybean options and reveal how these levels change with time to maturity and moneyness. Finally, this study will suggest specific recommendations regarding the use of particular approximation methods.

Approximations of the Black-Scholes Implied Volatility

Several earlier studies took an approach of developing an approximation to the Black-Scholes formula for a simplified case when a futures price is exactly equal to a discounted strike price. Brenner and Subrahmanyam (1988) proposed a following formula:

$$(1.2) \quad \sigma \approx \sqrt{\frac{2\pi}{\tau} \frac{C}{S}},$$

where the variables are as defined above. Feinstein (1988) independently derived an essentially identical formula. The accuracy of formula (1.2) depends on the assumption that a futures price is equal to a discounted exercise price. Following Brenner and Subrahmanyam, an at-the-money option is defined as one with futures price equal to discounted strike price, $S = Xe^{-r\tau}$. For convenience, the discounted strike price is denoted by K (i.e. $K = Xe^{-r\tau}$) in the remainder of this paper.

Curtis and Carriker (1988) developed a closed form solution for Black-Scholes volatility for at-the-money options ($S = K = Xe^{-r\tau}$). Since in this case $d_1 = \sigma\sqrt{\tau}/2$, $d_2 = -\sigma\sqrt{\tau}/2$, the Black-Scholes formula becomes

$$(1.3) \quad C = S \left[N(\sigma\sqrt{\tau}/2) - N(-\sigma\sqrt{\tau}/2) \right] = S(2N(\sigma\sqrt{\tau}/2) - 1).$$

Which can be solved for σ .

$$(1.4) \quad \sigma = (2/\sqrt{\tau})\varphi((C+S)/2S),$$

Where $\varphi = N^{-1}$. It can be shown that formula (1.2) is an approximation of formula (1.4) if one makes a Taylor development of order one for φ about $1/2$ (Chargoy-Corona and Ibarra-Valdez, 2006).

The following studies concentrated on developing approximate formulas which would be valid when stock prices deviate from discounted strike prices. Corrado and Miller (1996) proposed a following quadratic formula:

$$(1.5) \quad \sigma \approx \sqrt{\frac{2\pi}{\tau}} \frac{1}{S+K} \left[C - \frac{S-K}{2} + \sqrt{\left(C - \frac{S-K}{2} \right)^2 - \frac{(S-K)^2}{\pi}} \right].$$

This model may be considered an extension of Brenner and Subrahmanyam's approach as it uses a higher order Taylor expansion to achieve a formula that would be valid for a wider range of strike prices. As pointed out by Chambers and Nawalkha (2001), a potential problem with Corrado and Miller's model (1.5) is that it includes a square root term that in some cases does not have a real solution. "Specifically, for short term options that are very substantially away from the money the formula requires the square root of a negative value" (Chambers and Nawalkha, 2001, p. 95).

Bharadia et al. (1996) derived a highly simplified volatility approximation as:

$$(1.6) \quad \sigma \approx \sqrt{\frac{2\pi}{\tau} \frac{C - (S - K)/2}{S - (S - K)/2}}.$$

Chance (1996) extended Brenner and Subrahmanyam formula (1.2) for in-or out-of-the-money calls. Chambers and Nawalkha (2001) developed a simplified extension of the Chance model. The approach taken in these two papers uses the first and second derivatives of the call price with respect to volatility. However, these approaches need a reasonable estimate of volatility to serve as a starting point to the approximation.

More recently, Li (2005) developed an explicit formula which is valid for the wide band of option moneyness and time to maturity. His formula has several *formulations* depending on the moneyness of the option. If the moneyness is defined as

$$(1.7) \quad \eta = \frac{K}{S},$$

$\eta=1$ represents at the money, $\eta>1$ represents out-of-the money and $\eta<1$ represents in-the-money options. Li's formula for deep in- or out-of-the-money calls is:

$$(1.8) \quad \sigma \approx \frac{\tilde{\alpha} + \sqrt{\tilde{\alpha}^2 - \frac{4(\eta-1)^2}{1+\eta}}}{2\sqrt{\tau}},$$

where $\tilde{\alpha} = \frac{\sqrt{2\pi}}{1+\eta} \left[\frac{2C}{S} + \eta - 1 \right]$. Note that this formula reduces to formula (1.2) when $\eta=1$ and is

only slightly different from formula (1.5) in the last term under the square root (for details see Lee, 2005, p. 617). Li's formula for nearly at-the-money calls is

$$(1.9) \quad \sigma \approx \frac{2\sqrt{2}}{\sqrt{\tau}} \tilde{z} - \frac{1}{\sqrt{\tau}} \sqrt{8\tilde{z}^2 - \frac{6\tilde{\alpha}}{\sqrt{2}\tilde{z}}},$$

where $\tilde{z} = \cos \left[\frac{1}{3} \cos^{-1} \left(\frac{3\tilde{\alpha}}{32} \right) \right]$ and $\tilde{\alpha}$ is as defined previously. The combination of formulas

(1.8) and (1.9) depends on the following variable:

$$(1.10) \quad \rho = \frac{|\eta - 1|}{(C/S)^2} = \frac{|K - S|S}{C^2}.$$

When $\rho \leq 1.4$, formula (1.9) should be used, if $\rho > 1.4$, formula (1.8) should be used.

Finally, Chargoy-Corona and Ibarra-Valdez develop an approximate formula for implied volatility by means of an asymptotic representation of the Black-Scholes formula. In their study, they start with the same approach as Curtis and Carriker (1988) and extend formula (1.4) to cases where futures prices are different from exercise prices ($S \neq Xe^{-r\tau}$). If moneyness is defined as $\alpha = \log(S / Xe^{r\tau}) = \log(S / X) + r\tau$, their approximate formula for the Black-Scholes model is:

$$(1.11) \quad C = Se^{-\alpha} [2N(\theta/2) - 1]$$

If $\theta = \tilde{\sigma}\sqrt{\tau}$, an approximate formula for volatility can be derived by using $\varphi = N^{-1}$:

$$(1.12) \quad \tilde{\sigma} = \left(2/\sqrt{\tau} \right) \varphi \left(\frac{Ce^{r\tau} + X}{2X} \right).$$

Note that when $\alpha=0$ (at-the-money) formula (1.12) reduces to formula (1.4). Additionally, Chargoy-Corona and Ibarra-Valdez is the only study that provides an analytical formula for bounds of the errors of their approximation method (for details, see Chargoy-Corona and Ibarra-Valdez, 2006, p. 686).

Data

This study evaluates the accuracy of alternative volatility estimates using daily settlement prices for December options contracts for corn and cotton from December 1990 to October 2005.

The futures and options data were provided by INFOTECH. Due to data availability cotton series use 10 months of data from January to October for each December contract, and corn series use 11 month of data from previous December to October for each December contract. This approach provided 3662 observations for corn and 3118 observations for cotton. Because some approximate formulae are only valid for at-the-money contracts, nearest-to-the-money premiums and strike prices were selected for computations. Cotton options are traded in 1 cent intervals and corn options are traded in 10 cent (5 cents for the two closest to expiration) intervals. These deviations will reflect the impact of moneyness on the accuracy of approximations. The use of 10 or 11 month of data for each December contract will demonstrate the impact of time to maturity on the accuracy of approximations. The use of 15 years of data will ensure the robustness of statistical results.

Analysis of Accuracy

While numerous approximations of the Black-Scholes formula have been proposed, little is known about their relative accuracy. Chambers and Nawalkha (2001) provide analysis of accuracy for Chance's (1996) model, their modification of Chance's model, Corrado-Miller's (1996) model, and Bharadia et al.'s (1996) model by evaluating mean absolute errors of the estimates generated by these models relative to the Black-Scholes implied volatilities. The errors were calculated for a hypothetical call option for various levels of strike prices and times to maturity. The authors found that their modification of Chance's model produced the smallest mean absolute error, followed by Chance's model, Corrado-Miller's model, and Bharadia et al.'s model. Li compared the accuracy of his formula (1.9) to Brenner and Subrahmanyam's formula (1.2) and the combination of formulas (1.9) and (1.8) to Corrado-Miller's formula (1.5) and

found that the average errors from using his formulas was consistently lower than those of alternative methods. Note that the use of average error for such analysis may be misleading as positive and negative errors will cancel out thus producing a measure of bias, not accuracy.

In the present study we evaluate the accuracy of alternative approximate formulas relative to Black-Scholes estimates using four different measures of accuracy to ensure robustness of results. Two measures are defined using percent errors, $p_t = [(A_t - B_t) / B_t] \times 100$, and two using absolute errors $e_t = A_t - B_t$, where B is the Black-Scholes implied volatility, A is the volatility approximation, and t denotes the date. The following accuracy measures were computed for each commodity and each approximation method:

$$(1.13) \text{ mean absolute percent error, } MAPE = \frac{1}{T} \sum_{t=1}^T |p_t|,$$

$$(1.14) \text{ root-mean-squared percent error, } RMSPE = \sqrt{\frac{1}{T} \sum_{t=1}^T p_t^2},$$

$$(1.15) \text{ mean absolute error, } MAE = \frac{1}{T} \sum_{t=1}^T |e_t|,$$

$$(1.16) \text{ and root-mean-squared error, } RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T e_t^2}.$$

This analysis allows evaluation of the relative accuracy of the alternative formulas.

Furthermore, the following descriptive statistics were calculated for p_t and these statistics were used for comparison of the five approximations:

$$(1.17) \text{ mean of } p_t, \quad M = (\sum p_t / T)$$

$$(1.18) \text{ 95\% upper prediction limit for } p_t, \quad U = M + t_{(\alpha/2, n-1)} * S / \sqrt{(1 + 1/n)}$$

$$(1.19) \text{ 95\% lower prediction limit for } p_t, \quad L = M - t_{(\alpha/2, n-1)} * S / \sqrt{(1 + 1/n)}$$

$$(1.20) \text{ 50}^{\text{th}} \text{ percentile of the } p_t \text{ values (median)}$$

(1.21) 2.5th percentile of the p_t values

(1.22) 97.5th percentile of the p_t values

The prediction limits indicate the range of possible future p_t values and were of great interest for comparing the five approximations, but the limits depend on the assumption of normality for the p_t value distributions. The percentiles were included to basically ensure that the comparisons of the approximations based on the prediction limits were not influenced by a highly skewed (non-normal) distribution of p_t values. The comparisons of the five approximations using the means and medians were equivalent, and the comparisons using the prediction limits and the 2.5th and 97.5th percentiles were equivalent; and therefore the tables and graphs used for the comparisons only include the means and prediction intervals based on means.

Three additional statistical methods were used in the comparisons of the approximations. First, a hypothesis test (t-test and p-value) was used to determine if the true mean p_t was not equal to zero for the approximations (test of bias).

$$(1.23) \quad t = \frac{\mu - 0}{s / \sqrt{n}} \sim T\left(0, \frac{n-1}{n-3}\right),$$

where n is the number of observations, μ is the mean and s is the standard deviation of p_t .

Second, changes in the p_t values were explored due to time to maturity. Therefore the comparisons of the means and prediction limits of the approximations were conducted for each month. Third, a pattern in the five p_t values was explored due to differences between strike and futures prices.

An additional issue that is addressed in this paper is that of averaging of implied volatilities from both puts and calls to help reduce measurement error. While this approach has been suggested in the literature (e.g., Jorion, 1995) and is widely used, it may not always yield

desirable results. Using the put and call averages may sometimes increase the variability volatility estimates depending on the variance of the puts, the variance of the calls, and the covariance of the puts and calls.

$$(1.24) \quad Var_{\sigma_{AV}^2} = \frac{1}{4}(\sigma_c^2 + \sigma_p^2) + \frac{1}{2}cov(\sigma_c^2, \sigma_p^2)$$

If the variance of volatility using puts is for whatever reason much greater than the variance of volatility using calls and/or the covariance between two measures is positive and substantial, the use of the average may increase the variability of the volatility estimates. Therefore, the analysis was conducted for implied volatilities and approximations using only call premiums and for averages using both put and call premiums.

Results

Four measures of accuracy of the Black-Scholes Implied Volatility approximations calculated using call premiums are presented in Table 1. The percentage measures are self-explanatory, while the absolute measures should be interpreted relative to the average Black-Scholes volatility of 23.2% for corn and 20.3% for cotton. All four measures provided consistent results which suggest that Corrado and Miller's (1996) formula is the most accurate as it yields the smallest errors. For example, *MAPE* of this formula was only 1.16% for corn and 0.8% for cotton. Bharadia et al's (1996) and Li's (2005) formulas produced similar results that were only slightly worse than Corrado and Miller's approximations. Curtis and Carriker's (1988) and Brenner and Subrahmanyam's (1988) formulas also produced similar results, but were a lot less accurate than the first three approximations. For example, *MAPEs* of Curtis and Carriker's and Brenner and Subrahmanyam's approximations were about 8 times greater than *MAPEs* of Corrado and Miller's approximations and about 6 times greater than *MAPEs* of Bharadia et al's

and Li's approximations in corn and about 5.5 times greater than alternative approximations in cotton. This finding is likely due to the fact that Corrado and Miller's, Bharadia et al's and Li's (2005) formulas are valid for a wide range of options, while Curtis and Carriker's and Brenner and Subrahmanyam's formulas are designed for at-the-money options. Since corn options are traded in greater increments than cotton options (10 vs. 1 cent), relative sizes of errors are likely caused by deviations of futures from strike prices, which would be greater in corn than in cotton.

Table 1 shows that on average for implied volatility approximations calculated using call premiums, Corrado and Miller's formula provides the most accurate approximation of the Black-Scholes volatility. Table 2 looks into distributional characteristics of percent errors of the various approximations using call premiums.¹ The first three columns of table 2 provide a test of bias. The results demonstrate that all approximations of implied volatility for corn futures were biased at the 0.05 level of significance. The means of p_t for all approximations in corn were significantly greater than zero, meaning that these approximations tended to overestimate volatility. The performance of the implied volatility approximation methods in cotton was better with the null hypothesis of $M = (\sum p_t / T) = 0$ not rejected for Curtis and Carriker's, Brenner and Subrahmanyam's, and Bharadia et al.'s methods. Corrado and Miller's method tended to underestimate implied volatility while Li's method tended to overestimate it.

The next two columns show that these approximations also occasionally yielded very large errors. For example, the maximum percent error of the Curtis and Carriker's method was 72% for corn and 44.5% for cotton. On the other hand, Corrado and Miller's method produced the maximum error of 22% for corn and 7% for cotton. The last three columns show the 95% confidence intervals for percent errors of each approximation. These results indicate that 95% of errors from Corrado and Miller's model were less than 6% from the Black-Scholes estimates in

corn and less than 3% in cotton. On the other hand, Curtis and Carriker's method and Brenner and Subrahmanyam's method were far less accurate, with 95% confidence interval ranges of 52% for corn and 24% for cotton. Overall, the descriptive statistics in table 2 support the analysis of accuracy and rankings of alternative approximations from Table 1.

Further analysis of approximations of Black-Scholes implied volatilities includes investigation of the impacts of deviations of futures from strike prices and time to maturity on accuracy. For this analysis observations were sorted based on absolute percent difference between futures and strike and than 95% confidence intervals were computed. As shown in figure 1, two basic patterns were detected. Corrado and Miller's, Bharadia et al's and Li's methods do not appear sensitive to deviations of futures prices from strike. The accuracy of Brenner and Subrahmanyam's and Curtis and Carriker's methods decreases after the absolute percent difference between strike and futures goes above 0.10%. If the absolute percent difference between strike and futures is 0.10% or less, the confidence intervals for all approximations are essentially the same. Figure 2 explores the impact of time to maturity on 95% confidence intervals of percent errors of implied volatility approximations. This figure provides for several interesting conclusions. The accuracy of all approximations of implied volatility using December call premiums deteriorates after August, i.e. about 3 month prior to expiration. Again, two methods appear less accurate (Brenner and Subrahmanyam's and Curtis and Carriker's) with confidence intervals going from -53% to +59% in October. Corrado and Miller's approximations remain in -11% to +16% bounds in October. Li's and Bharadia's methods show a tendency to overestimate volatility in October with mean percent errors of 5% and positive bounds of 19% versus negative bounds of -9%.

Finally, the analysis was conducted by constructing approximations using averages from implied volatilities derived from calls and implied volatilities derived from puts as a way to minimize measurement error suggested by Jorion (1995). First, there was no significant change in Black-Scholes implied volatility measures with corn mean implied volatility using calls only equaling 23.204% using calls only and equaling 23.213% using call and put averages. Similarly, cotton mean implied volatility using calls only equaled 20.311% using calls only and equaled 20.309% using call and put averages. However, the results of the accuracy tests of the volatility approximations presented in table 3 are remarkably different. If the results presented in table 3 are compared to results presented in table one, it becomes apparent that averaging was very beneficial for Brenner and Subrahmanyam's and Curtis and Carriker's methods and not beneficial for the other three methods. For example, MAPE for Curtis and Carriker's method reduces from 9.40% for implied volatilities using calls only to 1.76% for implied volatilities using averages from puts and calls. At the same time, MAPE for Corrado and Miller's method deteriorated from 1.16% to 5.10% by using average implied volatility from puts and calls. These results are consistent between all statistical tests and for both commodities.

Table 4 shows the descriptive statistics and the 95% confidence intervals for percent errors of volatility approximations constructed using averages from put and call premiums. Consistent with table 2, which shows similar statistics for approximations using call premiums only, all approximations of implied volatility in corn show a significant positive bias. In cotton, the null hypothesis of no bias was not rejected at 0.05 level for Brenner and Subrahmanyam's and Corrado and Miller's method, while the other approximations tended to overestimate Black-Scholes implied volatility. The remainder of table 4 appears almost like a mirror image of table 2 in the sense that methods that performed well when constructed using calls only, performed

poorly when constructed using averages from calls and puts. These results are consistent with the analysis of accuracy presented in table 3.

Summary and Conclusions

While numerous approximations of the Black-Scholes formula have been proposed, little is known about their relative accuracy. Previous studies (Chambers and Nawalkha, 2001; Li, 2005) limit their analysis to just a few models and use hypothetical options to demonstrate the benefits of their proposed method to the alternative methods. The analysis of accuracy in these studies is usually very limited. Therefore, the purpose of this study was to provide a comprehensive analysis of accuracy of all available approximations of Black-Scholes implied volatility. Differently from the previous studies, which used hypothetical options, this analysis uses actual daily settlement prices for December futures and options contracts for corn and cotton from December 1990 to October 2005. Because some approximate formulas are only valid for at-the-money contracts, nearest-to-the-money options were selected for computations. The use of empirical data for two commodities allows examination of the impact of deviations of futures and strike prices and time to maturity on the accuracy of approximations. The use of a large data set and four alternative accuracy measures ensures the robustness of statistical tests.

The results of the empirical analysis for Black-Scholes Implied Volatility approximations calculated using call premiums suggest that Corrado and Miller's (1996) formula is the most accurate as it yields the smallest errors. Bharadia et al's (1996) and Li's (2005) formulas produced similar results that were only slightly worse than Corrado and Miller's approximations. Curtis and Carriker's (1988) and Brenner and Subrahmanyam's (1988) formulas also produced similar results, but were a lot less accurate than the first three approximations. This finding is

likely due to the fact that Corrado and Miller's, Bharadia et al's and Li's (2005) formulas are valid for a wide range of options, while Curtis and Carriker's and Brenner and Subrahmanyam's formulas are designed for at-the-money options. Since corn options are traded in greater increments than cotton options (10 vs. 1 cent), relative sizes of errors were about two times greater in corn than in cotton. Further analysis revealed the all approximations were biased toward overestimating in corn and all but Curtis and Carriker's, Brenner and Subrahmanyam's, and Bharadia et al's methods were biased in cotton.

Results of the analysis conducted for approximations using averages from implied volatilities derived from calls and puts were remarkably different. Averaging was very beneficial for Brenner and Subrahmanyam's and Curtis and Carriker's methods and not beneficial for the other three methods. Brenner and Subrahmanyam's and Curtis and Carriker's methods become the most accurate methods followed by Corrado and Miller's, Bharadia et al.'s and Li's method. Similarly to the first set of results, all approximations of implied volatility in corn show a significant positive bias. In cotton, the null hypothesis of no bias was not rejected for Brenner and Subrahmanyam's and Corrado and Miller's method, while the other approximations tended to overestimate Black-Scholes implied volatility.

Additional analysis was conducted to reveal the patterns in accuracy relative to moneyness and time to maturity. Corrado and Miller's, Bharadia et al's and Li's methods do not appear sensitive to deviations of futures prices from strike. The accuracy of Brenner and Subrahmanyam's and Curtis and Carriker's methods decreases after the absolute percent difference between strike and futures goes above 0.10%. If the absolute percent difference between strike and futures is 0.10% or less, the confidence intervals for all approximations are essentially the same. With respect to time to maturity, the accuracy of all approximations of

implied volatility for December contracts deteriorates after August, i.e. about 3 month prior to expiration. Again, two methods appear less accurate (Brenner and Subrahmanyam's and Curtis and Carriker's) while Corrado and Miller's approximations remain in relatively tight bounds (-11% to +16%) in October. Li's and Bharadia's methods show a tendency to significantly overestimate volatility in October.

Overall, these results suggest that Corrado and Miller's method yields relatively more accurate results when only call premiums are used. The accuracy of this method is closely followed by Bharadia et al's and Li's method. The users of Curtis and Carriker's and Brenner and Subrahmanyam's methods will get significantly more accurate results by averaging implied volatilities from calls and puts. The opposite is also true for the other methods. When the increment at which the futures are traded is fairly large (as in corn) all methods tend to overestimate implied volatility. With smaller trading increments (like in cotton) these biases are either small or non-existent.

Finally, it is important to recognize that while general accuracy patterns should not be sample-specific, some of these conclusions reflect the characteristics of the empirical sample used and may not be valid for other data sets. It is also important to keep in mind that the methods reviewed in this paper are the approximations of the Black-Scholes formula, hence all the problems associated with the Black-Scholes estimates (such as the infamous volatility smile) are likely to be found in these approximations as well.

References.

- Brenner, M. and M. G. Subrahmanyam. "A Simple Formula to Compute the Implied Standard Deviation." *Financial Analysts Journal* 5 (1988): 80-83.
- Bharadia, M.A., N. Christofides, and G.R. Salkin. "Computing the Black-Scholes Implied Volatility." *Advances in Futures and Options Research* 8(1996): 15-29.
- Chambers, D.R. and S.K. Nawalkha. "An Improved Approach to Computing Implied Volatility." *The Financial Review* 38 (2001): 89-100.
- Chance, D.M. "A Generalized Simple Formula to Compute the Implied Volatility." *The Financial Review* 31(1996): 859-867.
- Chargoy-Corona, J. and C. Ibarra-Valdez. "A Note on Black-Scholes Implied Volatility." *Physica A* 370 (2006): 681-688.
- Corrado C. J. and T. W. Miller. "A Note on a Simple, Accurate Formula to Compute Implied Standard Deviations." *Journal of Banking and Finance* 20 (1996): 595-603.
- Curtis, C.E., and G.L. Carriker. "Estimating Implied Volatility Directly from "Nearest-to-the-Money" Commodity Option Premiums. Working Paper 081588 (1988). Department of Agricultural Economics, Clemson University.
- Dupuire, B. "Pricing with a Smile." *Risk* 7(1994):18-20.
- Fama, E. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance* 25 (1970):383-417.
- Feinstein, S. "A Source of Unbiased Implied Volatility." Working paper 88-9, Federal Reserve Bank of Atlanta, December 1988.
- Jorion, P. "Predicting volatility in the foreign exchange market." *Journal of Finance* 50 (1995): 507-528.
- Koekebakker, S. and G. Lien. "Volatility and Price Jumps in Agricultural Futures Prices – Evidence from Wheat Options." *American Journal of Agricultural Economics* 86(4) (2004): 1018-1031.
- Li, S. "A New Formula for Computing Implied Volatility." *Applied Mathematics and Computation* 170 (2005): 611-625.

Table 1. Accuracy and Size of Errors of Black-Scholes Implied Volatility Approximations Calculated Using Call Premiums.

Approximation	<i>MAPE</i>		<i>MAE</i>		<i>RMSPE</i>		<i>RMSE</i>	
	(%)	Rank	(%)	Rank	(%)	Rank	(%)	Rank
Panel A: Corn, N=3662								
Curtis and Carriker (1988)	9.40	5	2.06	5	13.07	5	2.73	5
Brenner and Subrahmanyam (1988)	9.38	4	2.05	4	13.05	4	2.72	4
Bharadia et al. (1996)	1.58	2	0.35	2	3.11	2	0.66	2
Corrado and Miller (1996)	1.16	1	0.27	1	2.66	1	0.59	1
Li (2005)	1.66	3	0.37	3	3.14	3	0.67	3
Panel B: Cotton, N=3118								
Curtis and Carriker (1988)	4.48	3	0.88	3	5.96	4	1.15	3
Brenner and Subrahmanyam (1988)	4.48	3	0.88	3	5.95	3	1.15	3
Bharadia et al. (1996)	0.84	2	0.18	2	1.36	2	0.30	2
Corrado and Miller (1996)	0.80	1	0.17	1	1.30	1	0.29	1
Li (2005)	0.84	2	0.18	2	1.36	2	0.30	2

Note: *MAPE* is mean absolute percent error; *MAE* is mean absolute error; *RMSPE* is root-mean-squared percent error; *RMSE* is root-mean-squared error. Mean Black-Scholes volatility for corn was 23.2% and for cotton was 20.3%.

Table 2. Descriptive Statistics and 95% Confidence Intervals for Percent Errors of Volatility Approximations Using Call Premiums.

Approximation	Mean	<i>t</i> -statistic	Probability	Minimum	Maximum	95% Confidence Interval		
						Lower	Upper	Range
Panel A: Corn, N=3662				-----%-----				
Curtis and Carriker (1988)	0.87	4.04	0.00	-48.15	72.24	-25.23	26.97	52.20
Brenner and Subrahmanyam (1988)	0.75	3.48	0.00	-48.15	72.20	-25.32	26.82	52.14
Bharadia et al. (1996)	1.19	25.11	0.00	-11.54	25.97	-4.56	6.95	11.51
Corrado and Miller (1996)	0.64	14.90	0.00	-13.32	22.31	-4.54	5.81	10.35
Li (2005)	1.31	27.87	0.00	-11.52	25.98	-4.39	7.02	11.41
Panel B: Cotton, N=3118								
Curtis and Carriker (1988)	-0.14	-1.29	0.20	-28.86	44.46	-12.06	11.78	23.84
Brenner and Subrahmanyam (1988)	-0.21	-2.02	0.04	-28.86	44.46	-12.12	11.69	23.81
Bharadia et al. (1996)	0.00	0.07	0.95	-9.08	8.76	-2.72	2.72	5.44
Corrado and Miller (1996)	-0.11	-4.86	0.00	-9.12	7.18	-2.70	2.48	5.18
Li (2005)	0.08	3.24	0.00	-9.07	8.76	-2.63	2.79	5.42

Table 3. Accuracy and Size of Errors of Black-Scholes Implied Volatility Approximations Calculated Using Averages from Call and Put Premiums.

Approximation	<i>MAPE</i>		<i>MAE</i>		<i>RMSPE</i>		<i>RMSE</i>	
	(%)	Rank	(%)	Rank	(%)	Rank	(%)	Rank
Panel A: Corn, N=3662								
Curtis and Carriker (1988)	1.76	2	0.39	2	3.21	2	0.69	2
Brenner and Subrahmanyam (1988)	1.72	1	0.38	1	3.18	1	0.68	1
Bharadia et al. (1996)	9.96	4	2.19	4	13.72	4	2.89	4
Corrado and Miller (1996)	5.10	3	1.13	3	7.14	3	1.52	3
Li (2005)	10.00	5	2.20	5	13.75	5	2.89	4
Panel B: Cotton, N=3118								
Curtis and Carriker (1988)	0.87	2	0.18	2	1.37	1	0.30	1
Brenner and Subrahmanyam (1988)	0.86	1	0.18	1	1.37	1	0.30	1
Bharadia et al. (1996)	4.61	3	0.91	3	6.12	2	1.19	2
Corrado and Miller (1996)	4.63	5	0.91	3	6.14	3	1.19	2
Li (2005)	4.62	4	0.91	3	6.12	2	1.19	2

Note: *MAPE* is mean absolute percent error; *MAE* is mean absolute error; *RMSPE* is root-mean-squared percent error; *RMSE* is root-mean-squared error. Mean Black-Scholes volatility for corn was 23.2% and for cotton was 20.3%.

Table 4. Descriptive Statistics and 95% Confidence Intervals for Percent Errors of Volatility Approximations Using Averages from Call and Put Premiums.

Approximation	Mean	<i>t</i> -statistic	Probability	Minimum	Maximum	95% Confidence Interval		
						Lower	Upper	Range
Panel A: Corn, N=3662								
				-----%-----				
Curtis and Carriker (1988)	1.34	27.74	0.00	-12.08	26.97	-4.50	7.17	11.67
Brenner and Subrahmanyam (1988)	1.21	25.00	0.00	-12.10	26.95	-4.67	7.10	11.76
Bharadia et al. (1996)	1.58	7.03	0.00	-48.79	75.22	-25.67	28.84	54.51
Corrado and Miller (1996)	0.81	6.91	0.00	-32.08	43.62	-13.39	15.01	28.39
Li (2005)	1.71	7.59	0.00	-48.79	75.27	-25.57	28.99	54.57
Panel B: Cotton, N=3118								
Curtis and Carriker (1988)	0.09	3.67	0.00	-9.00	8.53	-2.66	2.84	5.50
Brenner and Subrahmanyam (1988)	0.01	0.52	0.61	-9.01	8.53	-2.74	2.77	5.51
Bharadia et al. (1996)	0.21	1.93	0.05	-27.31	41.44	-12.05	12.48	24.53
Corrado and Miller (1996)	0.10	0.93	0.35	-35.63	38.53	-12.18	12.38	24.56
Li (2005)	0.29	2.64	0.01	-27.31	41.45	-11.99	12.57	24.55

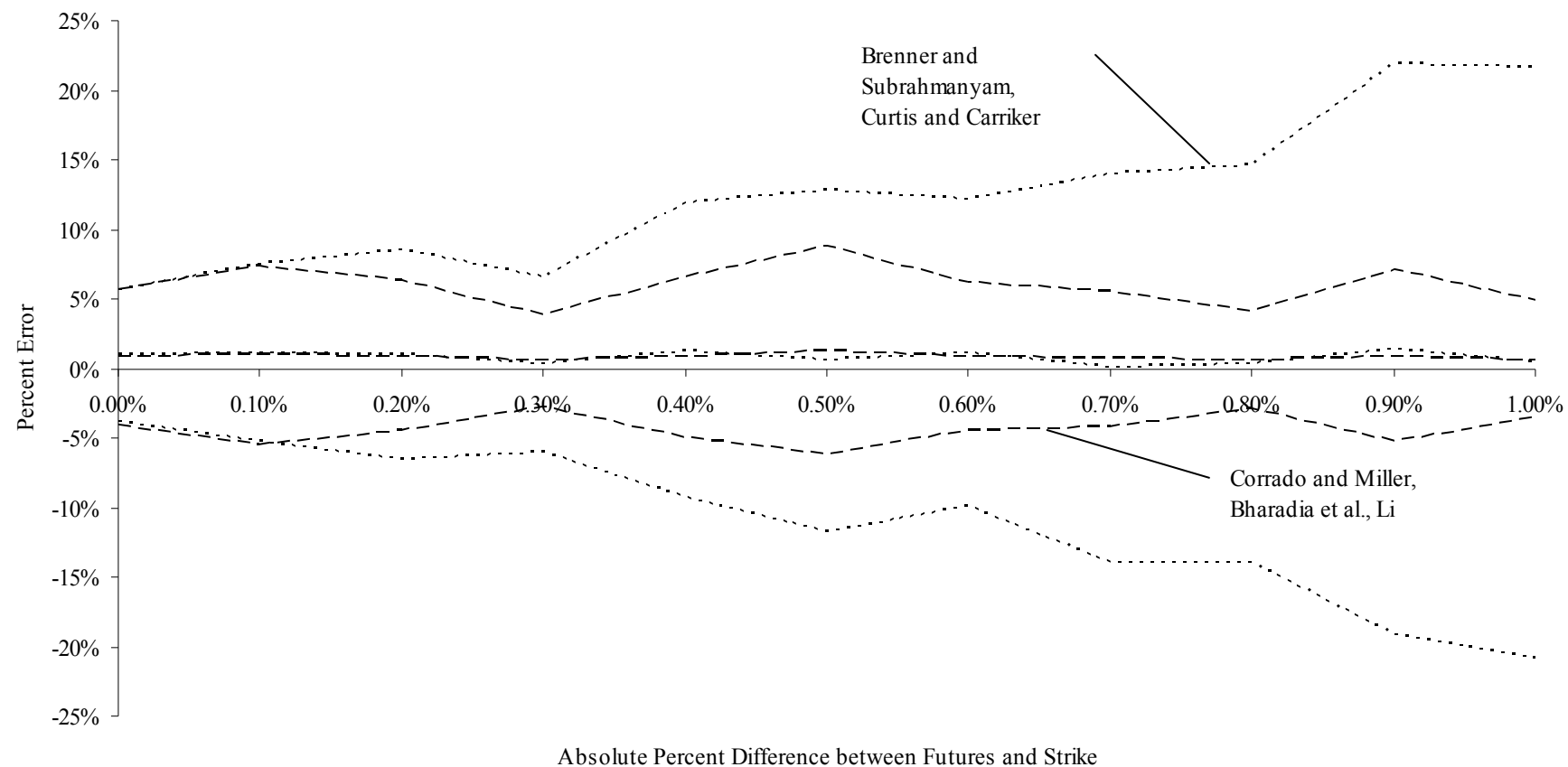


Figure 1. Impact of Deviations of Futures from Strike on 95% Confidence Intervals of Percent Errors of Implied Volatility Approximations Using Corn Call Premiums

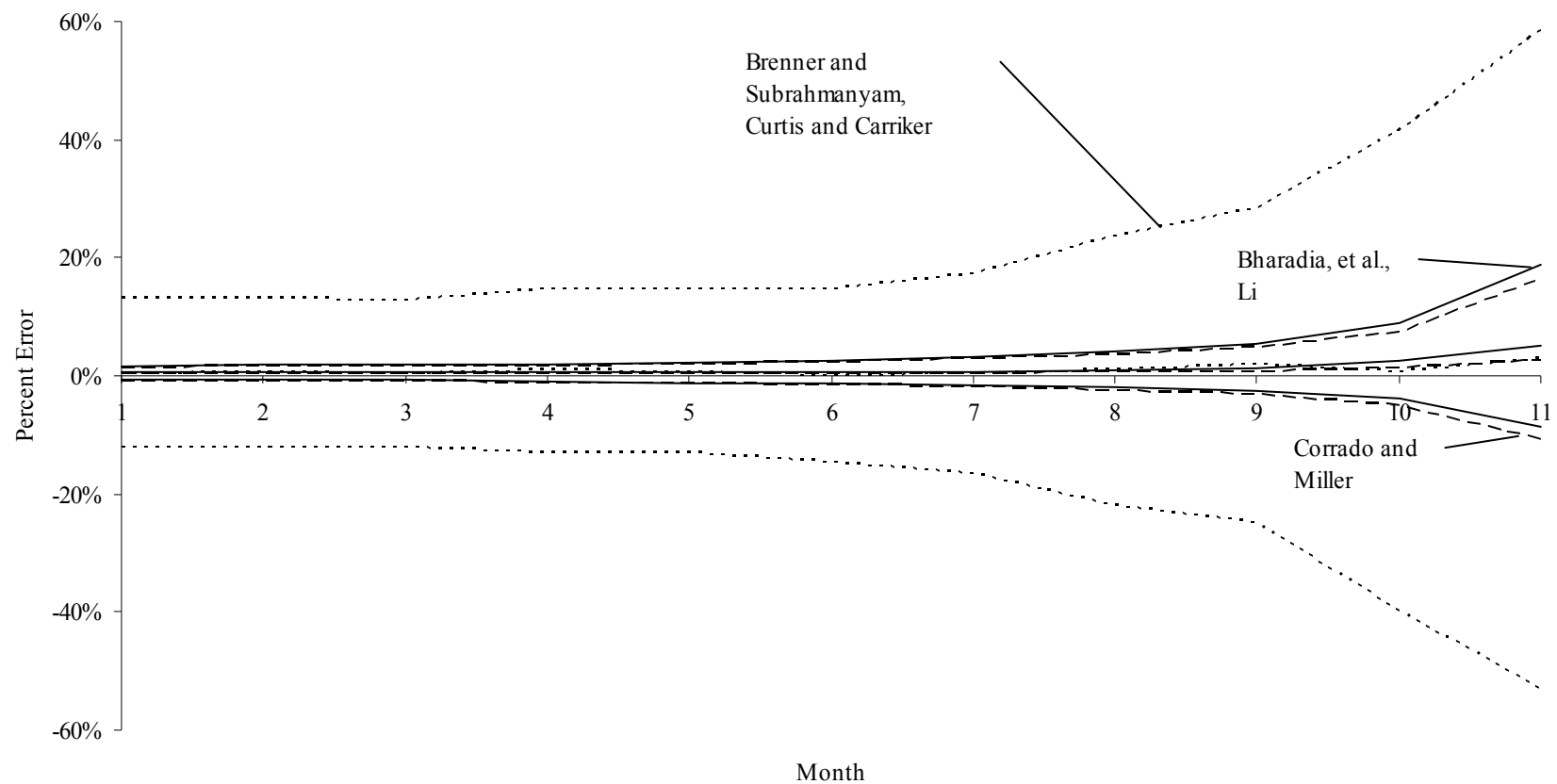


Figure 2. Impact of Time to Maturity on 95% Confidence Intervals of Percent Errors of Implied Volatility Approximations Using Corn Call Premiums

¹ The same analysis was conducted using absolute errors and yielded consistent results. Details available from authors upon request.