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Land, Wealth, and Taxation

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Abstract

We examine the role of land in wealth dynamics, and its consequences on efficiency and inequality by focusing on the interplay among agents' bidding for location, mortgage market imperfections, and inheritance. We develop a model in which altruistic agents leave to their heirs a financial bequest and their housing wealth. The borrowing constraint generates a housing return premium and spatial wealth sorting, which translate into persistent inequality. Since altruism and the borrowing constraint distort land price formation, we discuss different corrective tax schedules. Land taxation cannot be disconnected from inheritance taxation, and must be levied on the inheriting generation.

Keywords: mortgage market imperfections, spatial sorting, wealth distribution, wealth taxation, efficiency.

JEL Classification: D31, E21, H21, R14.

Terre, richesse et fiscalité

Résumé

Nous analysons le rôle de la terre dans la dynamique de la richesse et ses conséquences sur l'efficacité et l'inégalité en nous concentrant sur l'interaction entre les enchères des agents pour la localisation, les imperfections du marché du crédit immobilier et l'héritage. Nous développons un modèle dans lequel les agents altruistes lèguent à leurs héritiers un héritage financier et leur patrimoine immobilier. La contrainte de crédit génère un rendement du logement plus élevé pour les plus riches et une ségrégation spatiale de la richesse, qui se traduisent par une inégalité persistante. L'altruisme et la contrainte de crédit impliquant une distorsion dans la formation du prix du foncier, nous discutons différents systèmes d'imposition. L'impôt foncier ne peut être déconnecté de l'impôt sur les successions et doit être prélevé sur la génération bénéficiant de l'héritage.

Mots-clés: imperfections du marché du crédit immobilier, ségrégation spatiale, inégalité, fiscalité de la richesse, efficacité.

Classification JEL: D31, E21, H21, R14.

> Land [...] is not the only monopoly, but it is by far the greatest of monopolies – it is a perpetual monopoly, and it is the mother of all other forms of monopoly.
>
> *Winston Churchill, speech made to the House of Commons on May 4, 1909.*

1. Introduction

Land is a key driver of wealth dynamics that has crucial joint implications for efficiency and inequality. On the one hand, land amounts to a large and increasing share of housing value, which itself represents a large part of inherited wealth (Piketty and Zucman, 2014, Knoll *et al.*, 2017, Kuhn *et al.*, 2020).[1] Borrowing constraints in mortgage markets amplify its impact on wealth dynamics, as modest households facing limited access to credit are prevented from acquiring housing in the most attractive sites[2] and leaving substantial housing wealth to their descendants. Although households' wealth, housing/land values, and intergenerational wealth transmission are tightly intertwined, little attention has been paid to the impact of residential choices on wealth dynamics. On the other hand, land in attractive locations is scarce, and generates economic rents distorting resource allocation.[3] Since land is a durable good that is not subject to depreciation, rents are transmitted between generations and can affect the dynamic efficiency of the economy. However, the macroeconomic literature does not regard land as a locational space deriving its value from its location. How do wealth dynamics interact with households' bidding for location? Does land rent taxation lead to an efficient allocation of resources when residential choices and wealth transmission are interrelated? These are the questions we aim to answer in this paper.

For our purpose, we develop a model of a dynamic economy in which housing wealth reflects residential choices and the access to mortgage credit is imperfect. We consider an infinite sequence of non-overlapping generations connected *via* intergenerational transfers. One-period-lived agents are heterogeneous in inherited wealth, composed of a financial bequest and the proceeds from the sale of their parents' house. To capture the spatial dimension of housing wealth, we consider a system of cities in which places of residence are defined by their attributes that vary spatially (*e.g.*, accessibility to jobs, amenities,

[1] For example, among the developed countries studied by Knoll *et al.* (2017), the share of land in total housing value ranges from 37% in Germany to 77% in Japan. Housing wealth amounts to 50% of households' portfolios on average in OECD countries (Causa *et al.*, 2019). In France, in 2018, 37.7% of inherited wealth and 48.6 % of wealth received as a donation is represented by real estate (Insee, 2018).

[2] Land rent varies widely across and within urban areas (see Albouy *et al.*, 2018, for the US; Combes *et al.*, 2019, for France; Gaigné *et al.*, 2022, for the Netherlands). For example, in New York, the land value in the center (Empire State Building) has 22-fold the land value in the periphery (10 miles from the center) (Albouy *et al.*, 2018).

[3] Urban land value captures the benefits accruing to landowners from external developments (*e.g.*, the presence of public infrastructure such as transportation networks and amenities) and not from their efforts.

facilities), and city governments set a local tax to finance public expenditures. Given that parents are altruistic and transmit their home to their heirs, their residential choice can be interpreted as an investment in a location. The future sale price corresponds to the investment yield, while the current costs are the price of housing, which is endogeneous, and mobility costs, which exogenously vary across locations. Returns to location/housing are determined by land and mortgage markets. The presence of a borrowing constraint in the form of a downpayment requirement can prevent agents with low inherited wealth from purchasing a home in their most desired location.[4] The borrowing constraint is endogenous because it is specified on inherited wealth and the equilibrium house price, which varies across space.

We first characterize the residential equilibrium that arises at each date depending on the shape of the wealth distribution. Starting with the case of homogeneous agents, we propose a novel mechanism that leads to *symmetry-breaking* in the sense of Matsuyama (2000, 2006) that has not previously been considered in the literature. The population polarizes into rich agents, who enjoy a higher utility that is location dependent and leave a higher inheritance to their offspring, and poor agents, who obtain a lower homogeneous utility and leave a lower inheritance to their descendants. Thus, wealth inequality occurs endogenously. This is a consequence of the borrowing constraint, which limits agents' access to credit in attractive locations and imposes a cap on the equilibrium rent. Hence, constrained agents pay a lower price than what they would pay without any borrowing constraint, and can increase their consumption and inheritance to their children. We define this extra amount of lifetime wealth arising from the presence of borrowing constraints as "constraint rent".

We then turn to a heterogeneous population setting, and show that the borrowing constraint generates *spatial wealth sorting*: The wealthier the agent, the closer she resides to the most attractive city locations. The key mechanism is that wealthy agents are less likely to be constrained and limited in paying to live in a location. The borrowing constraint thus creates heterogeneity in agents' ability to pay, with wealthy agents outbidding poorer agents in the most attractive locations. We also show that for a given wealth distribution, the city space is partitioned into distinct areas where residents hold the same status: constrained or unconstrained. There are two types of winners capturing the constraint rent: (i) the constrained agents who pay the price capped by the borrowing constraint, and (ii) the wealthiest agents who are not credit constrained but face less competition to buy land/housing in attractive sites. This highlights that *the housing market generates*

[4] As purchasing a home in a given location involves large expenditures amounting to many years of income for most households, access to mortgage credit becomes an important determinant of homeownership. However, frictions such as agency costs prevent agents from pledging the full value of their property for repayment. The tightness of borrowing constraints increased in the aftermath of the financial crisis, as credit institutions tightened mortgage credit availability (Acolin *et al.*, 2016).

implicit transfers between the various agent types.

Furthermore, we characterize the long-run steady-state wealth distributions. Either the wealth of all agents converges to a common wealth level because no agent is constrained and the rate of return to financial assets equals the rate of return to location assets, or the population is polarized into dynasties with different wealth levels. Wealthy dynasties are located in the most attractive sites. The borrowing constraint implies that they benefit from a rate of return to location that is greater than the rate of return to financial assets, allowing them to transfer more wealth to their heirs. By contrast, poor dynasties are excluded from living in the most attractive locations, do not benefit from this return premium, and reside in poverty trap areas. Hence, spatial sorting translates into persistent inequality, emphasizing the importance of space in understanding long-run wealth inequality. When we introduce heterogeneity in human capital investment among agents, the steady state features wealth sorting *as well as* skill sorting, that is, wealthy dynasties living in the most attractive sites are also more educated than the rest of the population and earn a higher wage. As a corollary, our analysis suggests that land use restrictions magnify long-run wealth inequalities.

After characterizing the competitive intertemporal equilibrium, we then consider the social optimum and its decentralization. In our setting with scale economies, mobile altruistic agents and "constraint rents" that are transferred between generations, we show that the decentralization of the social optimum can be achieved by using two tax instruments. First, when agents are altruistic and credit markets are perfect, a tax on inherited land wealth paid by *donees* is efficient and sufficient to finance public expenditures. This is a dynamic version of the *Henry George Theorem* (HGT), formalized by (Arnott and Stiglitz, 1979) in a static context without intergenerational transfers.[5] Second, when the borrowing constraint is binding, an additional tax is required. Indeed, the borrowing constraint causes the market price of land to deviate downwards from its shadow price. A specific tax on the donor's lifetime wealth equal to her "constraint rent" must be implemented. However, when policymakers have no information about agents' willingness-to-pay to live in a neighborhood, they cannot implement this individual-specific tax on "constraint rents". Under these circumstances, a taxation schedule involving downpayment subsidies, land value tax, and a lump-sum tax is welfare improving. Therefore, our framework bears strong implications for tax design. A *donee*-based tax must be levied on the inherited land wealth to cover public expenditures and achieve the optimal solution involving no long-run inequality.

[5]The HGT posits that a confiscatory tax on land rents is sufficient to finance urban public expenditures and to reach the optimum social. Henry George noted in his famous 19th century book *Progress and Poverty* that land taxation is efficient as taxing pure economic rents does not create a distortion.

Finally, we consider the case in which bequests finance productive capital. Indeed, a large capital stock can be attributable to financial assets transferred between generations (Dynan *et al.*, 2002). Since land is a durable good, its price impacts the structure of the wealth left to children (*e.g.*, housing assets *vs.* financial assets) and, in turn, capital accumulation. We show that without any borrowing constraint, the presence of altruistic agents bidding for location implies high land rents that reduce savings for bequest motives, and cause underinvestment in productive capital. Our analysis confirms that the existence of a land market can lead to a shift in portfolio allocation away from productive capital and toward land (as in Drazen and Eckstein (1988) and Deaton and Laroque (2001) who consider land as a production factor). By contrast, if some residents are unable to invest in their desired level of housing due to borrowing constraints, then the downward pressure on capital investment is dampened. We also show that a tax on the land value received by donees yields a better allocation of resources between productive capital and housing capital, and decreases the *wealth-to-income* ratio.

Literature review. We first contribute to the macroeconomics literature on the mechanisms that generate persistent wealth inequalities. This literature shows that when credit markets are imperfect, households with little initial wealth face limited investment opportunities, and their children remain poor (Banerjee and Newman, 1993, Galor and Zeira, 1993, Mookherjee and Ray, 2002, 2003, 2010, Matsuyama, 2000, 2006). We show that spatial sorting generates persistent wealth inequality. Our model can be viewed as similar to these occupational choice models because locations are vertically differentiated like occupations, and access to the most attractive locations/high-skilled occupations is costly and may be prevented by borrowing constraints. We depart from these works because households' wealth reflects residential choices involving a trade-off between locational attributes and house prices. In our model, residential location costs are endogenously determined as households compete for locations and each land plot is assigned to the highest bidder.[6] In addition, the possibility to borrow in one location depends on household's wealth and the house price arising in each period. Benhabib *et al.* (2011) also emphasize that heterogeneity in returns to wealth explains the unequal wealth distribution (see also Benhabib *et al.*, 2019; Garbinti *et al.*, 2020; Fagereng *et al.*, 2020). Our analysis demonstrates that the wealth distribution dynamics and the location return premium are intertwined.

Second, our paper is at the intersection of the literature on land taxation (Mieszkowski and Zodrow, 1989) and the literature on inheritance taxation (Scheuer and Slemrod, 2021),

[6] In models of wealth transmission and occupational choice, occupation/training costs vary *exogenously*. In our model, *endogenous* location costs imply that, in equilibrium, (i) wealthy agents have access to returns to investments in locations that are higher than the return to financial investments, and (ii) they leave higher financial bequests than poorer agents (differently from Mookherjee and Ray, 2010, where the financial bequests of wealthy agents equal zero).

which have been disconnected. Since the return to land can be viewed as an economic rent, it is not surprising that the interest in land taxation to finance public expenditures has resurged recently among economists (Stiglitz, 2015, Bonnet *et al.*, 2021). There is a long history of arguments in favor of land taxation. Since the return to land is an economic rent, and land supply is not responsive to price, it can be taxed without significantly distorting economic behavior. In particular, according to the HGT, a confiscatory tax on land rents is the only tax needed to finance urban public expenditures (Arnott and Stiglitz, 1979). In our dynamic setting, the optimal tax scheme involves a tax on land assets transferred between generations. Departing from the famous *zero* tax results provided by Chamley (1986) and Judd (1985), recent inheritance taxation models find that some positive tax or subsidy can be socially optimal (Farhi and Werning, 2007, Piketty and Saez, 2013). Uncertainty (implying accidental bequests) and/or at least two sources of inequality (parents differ in their taste for bequests and their productivity/wage) are prerequisites for deriving a positive optimal inheritance tax rate. In our framework, even though there is no mortality risk and one-period-lived agents share both the same preferences and ability and exert the same effort, inheritance taxation must be positive because inherited wealth includes land/location rents.

Third, to the best of our knowledge, our residential choice model is the first to incorporate both a borrowing constraint and wealth transfers between generations. While Bilal and Rossi-Hansberg (2021) consider a lifecycle model in which the residential choice of individuals is modeled as an asset investment decision, they do not consider transfers between generations, and their borrowing constraint is exogenous. Our scope is also different because we consider wealth inequality dynamics and land taxation issues. Furthermore, our model highlights the borrowing constraint as a new source of spatial sorting, while the literature focuses on productivity and amenity spillovers (Eeckhout *et al.*, 2014, Diamond, 2016), as well as nonhomothetic preferences (Handbury, 2021, Gaigné *et al.*, 2022). Our dynamic framework with credit market imperfections derives a modified Alonso-Muth condition: Agents' bid rents depend on the future value of housing and their ability to pay for a place of residence, so that the rent gradient also reflects the shape of the wealth distribution and the stringency of the borrowing constraint.

Finally, our paper contributes to the literature that studies the effect of credit market imperfections on the housing market (Stein, 1995, Ortalo-Magné and Rady, 2006). These models include an initial downpayment to obtain a mortgage that constrains low-income households (especially young households) from becoming homeowners. The housing market then reacts to any change in the credit market through the change in the homeownership rate of low-income households. We depart from these models in two ways: First, in our model with a continuous space, the borrowing constraint is *location dependent*; Second, the wealth distribution results from the residential equilibrium.

The remainder of the paper is organized as follows. Section 2 introduces the main model. In Section 3, we determine the spatial sorting arising at each date. We analyze the long-run wealth distributions and taxation policy when wage and interest rate are exogenous in Section 4, and endogenous in Section 5. Concluding remarks and a discussion about the empirical implications and some possible extensions of our model follow.

2. The model

In this section, we present a model of wealth transmission in a spatial context in a relatively parsimonious form. Admittedly, some assumptions would be debatable from an empirical perspective, but they permit to derive simple expressions to identify the main drivers of wealth dynamics associated with housing and land markets. In Section 5 and Appendix C, we consider extensions that imply cumbersome expressions and do not alter our main results.

2.1. Time, space, and preferences

Time is discrete and extends to infinity. Each agent lives one period and has a unique offspring at the end of her life. Dynasties are formed by each infinite parent-child sequence. At the beginning of each period, every agent receives a bequest $y_t \in [\underline{y}_t, \overline{y}_t]$ from her parent, with $0 < \underline{y}_t \leqslant \overline{y}_t < \infty$. We denote by $F_t(y)$ the share of agents with wealth below y_t at the beginning of period t. The initial wealth is the only source of *ex ante* heterogeneity across agents. To isolate the role of land markets, we ignore the role of labor markets by assuming that agents are endowed with the same ability and skills, and supply one unit of labor (the case where the skill level differs across agents is considered in Section 5.1).

We consider a system of cities where total population is constant, while the population of each city L_t is endogenous (the number of cities is also endogenous). In each period, the existence of a city requires an amount of fixed costs G, which include costs of public facilities (*e.g.*, transport infrastructure). Each city levies taxes on inhabitants to finance public expenditures G. Locations within each city are heterogeneous and vertically differentiated, that is, locations are more or less attractive places to live. The heterogeneity dimension stems from the disutility from commuting and/or monetary costs (including opportunity costs of time) associated with distance to jobs or amenities attributes and service facilities.[7] For the sake of simplicity, we consider a linear and monocentric city defined over the one-dimensional space $\mathbb{R}_+$, where locations differ with respect to access

[7] Combes *et al.* (2019) find that French households devote 13.5% of their expenditure to transportation. The opportunity cost of the time spent in commuting represents three to six weeks of work for a typical New Yorker, and, on average, four weeks of work for a resident of Greater Paris (Proost and Thisse, 2019). Moreover, individuals perceive commuting as one of their most stressful and unpleasant activities (Kahneman *et al.*, 2004).

to the central business district (hereafter CBD), which hosts all jobs, located at the origin $x = 0$, and where agents earn the same wage w_t (there is no difference in ability or effort across agents who supply their unit of labor inelastically to producers). Commuting costs, given by $\kappa(x)$, increase with distance x between agents' residential location and the CBD. Our results remain valid if the model is extended to a map formed by roads and railway junctions modeled by means of a topological network, with locations characterized by distance to various job centers, facilities, and exogenous amenities. In such a case, there must exist a location-quality index that subsumes into a single scalar the different accessibility costs (Gaigné *et al.*, 2022).[8]

Agents have an altruistic concern for their children and have the same utility function u, which is increasing in consumption of a nondurable good c_t, housing services s_t generated by the housing investment decision made in the current period, and the inheritance left y_{t+1}. The nondurable good market is perfectly competitive, and there are no transportation costs, so that the price of the nondurable good does not vary across cities and is normalized to one. While we analyze the short-run equilibrium, we do not need to specify the altruism utility component. The equilibrium characteristics are the same whether we assume that parents have *warm glow* preferences (the *joy of giving* motive), that is $u\left(c_t, s_t, y_{t+1}\right)$, or preferences *à la* Barro and derive utility from the offspring's utility, for example $U_t = u(c_t, s_t) + \delta U_{t+1}$, where δ is the discount factor between generations.

The wealth left to the offspring is assumed to be the sum of some voluntary parental bequest b_t and the future housing value net of transaction costs $\theta_{t+1}(x)$ owned and occupied by the parent in the previous period:

$$y_{t+1}(x) = (1 + r_{t+1})b_t + \theta_{t+1}(x), \tag{1}$$

where voluntary bequests are assumed to earn a rate of return r_{t+1}. Hence, voluntary bequests can be regarded as financial assets transmitted to the next generation. Unlike the literature on inheritance, the wealth left to the offspring comprises not only financial assets, but also owner-occupied housing. This assumption is in line with empirical studies showing that a large share of household wealth consists of housing wealth (Causa *et al.*, 2019).[9] Given that housing wealth is location dependent, a key feature of our model is that inherited wealth depends on parents' residential choice.

[8]In the Supplementary Appendix, we assume that location affects agents' utility through the consumption of amenities available at the place of residence or commuting costs generating utility loss.

[9]For example, the main residence of households living in the Eurozone countries accounts for approximately 50% of their assets.

2.2. Housing market and mortgage market imperfections

Housing investment h_t generates a flow of housing services s_t *via* a standard linear technology, $s_t = h_t$, that transforms the housing investment into housing services in the same period. Houses are modeled as discrete-size durable goods (housing investment is assumed to be lumpy and indivisible). The housing supply is fixed in each location $h_t(x)$. For simplicity, the amount of housing available at each location x is $h_t = 1$, so that households consume only one unit of housing, regardless of their residential location (in turn, $s_t = 1$). We thus set $u(c_t, 1, y_{t+1}) \equiv u(c_t, y_{t+1})$. In Appendix C.2, we relax the assumption of fixed lot size and assume, in accordance with the macroeconomics literature on housing (Piazzesi and Schneider, 2016), that houses come in different sizes restricted by a set that consists of a finite number of housing sizes at the city level.[10]

Housing is transmitted across generations using the services of intermediaries, named real estate companies. At each date t, real estate companies buy the housing unit available at location x at price $\theta_t(x) \geqslant R_A$ from the previous owners living at $t-1$, where R_A is the opportunity cost of land, and sell it to agents born at t at price $p_t(x)$.[11] We initially assume that the housing unit cannot be rented and that all agents purchase a house. We relax this assumption in Appendix C.1. We also assume no depreciation of the housing stock over time and no cost of adjusting housing services from the previous period to the current period (this extension is developed in Section 5.2). The profit of the intermediary associated with location x is $\boldsymbol{v}_t(x) = p_t(x) - \theta_t(x)$. We assume free entry, which implies $\boldsymbol{v}_t(x) = 0$ and $\theta_t(x) = p_t(x)$.

Within each period, we consider the following sequence of events to account for inherited wealth as the key variable determining access to credit. First, housing markets open at the beginning of each period. Agents endowed with y_t choose their residential location x and pay $p_t(x)$, the price of housing. Second, they work in the CBD, earn w_t, incur the commuting cost $\kappa(x)$, consume the composite good $c_t(x)$, leave their bequest b_t, and pay the lump-sum tax τ_t.

If $p_t(x) > y_t$, households need to borrow. There are many different microfoundations for credit market imperfections, based, for example, on moral hazard and adverse selection. Asymmetric information between borrowers and lenders implies a maximum amount of credit that an agent can borrow (as in Matsuyama, 2000, 2006), and/or that the interest rate is higher for borrowers than for lenders (as in Galor and Zeira, 1993). We assume

[10] Urban economic theory considers the extreme case where housing size can adjust *freely* to new conditions, making the analysis much more cumbersome without affecting the nature of our results. This case is discussed in the Supplementary Appendix.

[11] At $t = 0$, we could suppose that there are some original agents who are given some property titles and sell their property to the real estate companies.

that agents who borrow to purchase the housing unit may face a borrowing constraint. Following the literature on the housing market and credit rationing (Rosenthal *et al.*, 1991, Stein, 1995, Ortalo-Magné and Rady, 2006), agents can borrow up to a certain fraction λ of the house value, with $0 \leqslant \lambda < 1$, meaning that they are able to borrow if

$$\lambda p_t(x) \geqslant p_t(x) - y_t. \tag{2}$$

This downpayment requirement implies that agents must be endowed with at least a level of wealth equal to $(1-\lambda)\, p_t$ to be able to purchase the house at x. Hence, if agents' inherited wealth is higher than the house price, $y_t \geqslant p_t(x)$, then they do not need to borrow. Otherwise, agents can borrow without facing a borrowing constraint when $p_t(x) > y_t \geqslant (1-\lambda)p_t(x)$, or they can borrow up to the limit $\lambda p_t(x)$ when $y_t < (1-\lambda)p_t(x)$. The key feature of our borrowing constraint is that it is *location specific*, as it depends on the price of housing, which varies with location x. Note that we assume, without loss of generality, that lenders' interest rate and borrowers' interest rate are normalized to 0.[12] Thus, the household budget constraint is written as

$$w_t + y_t - p_t(x) = c_t + b_t + \kappa(x) + \tau_t, \tag{3}$$

where τ_t is a (lump-sum) income tax on each inhabitant to finance public expenditures. The budget constraint of the city implies that $\tau_t = G/L_t$. In Section 4, we discuss the appropriate design of a tax system.

2.3. Bequest and bid rent

At a given location and depending on whether agents are borrowing-constrained or unconstrained, agents maximize $u(c_t, y_{t+1})$ with respect to c_t and b_t, considering transmitted wealth (1), under budget constraint (3). This yields the following first-order condition:

$$u_c(c_t, y_{t+1}) = (1 + r_{t+1})u_y(c_t, y_{t+1}), \tag{4}$$

with $u_c \equiv \partial u/\partial c_t$ and $u_y \equiv \partial u/\partial y_{t+1}$.[13]

Note that the house price setting in our framework differs from the current approach

[12] We could set the lender's interest rate to r_t without modifying our results. In addition, as shown in the Supplementary Appendix, the fact that borrowers pay a higher interest rate than the lenders does not play a role in wealth inequality (but has an impact on optimal tax policies). A more realistic modeling of credit rationing on housing demand would require us to add a second constraint: an upper limit on the share of current income that owner-occupiers can spend on housing (Rosenthal *et al.*, 1991). We set aside this income constraint because we focus on the wealth dynamics, and therefore we need only constraint (2) for our purpose. Moreover, we assume that agents have the same income.

[13] To avoid burdensome expressions, the altruism motive is assumed to be sufficiently strong to have $b_t > 0$ in equilibrium.

used in quantitative models with housing. Indeed, our objective is to determine which agent occupies a particular location, and land at a particular location does not correspond to a single commodity whose price is obtained by the interplay between a large number of sellers and buyers (Fujita and Thisse, 2002). Indeed, land at a location is not a homogeneous good, but rather a continuously differentiated good. Therefore, it appears to be convenient to determine the land use equilibrium by using the bid-rent function in the tradition of economic urban theory. Hence, agents bid for available housing units, and real estate companies sell housing units to the highest bidder. We define the bid-rent as the *maximum* price per unit of housing that an agent endowed with wealth y_t would be willing to pay to live in a location where she enjoys the utility level $u[c_t(x), y_{t+1}(x)]$. The bid rent depends on the extent of competition from other bidders and the imperfections of the credit market.

Let us start with location choices when the borrowing constraint is slack. We denote by Ψ_t the maximum bid rent that solves the equilibrium condition $u'[c_t(x), y_{t+1}(x)] = 0$ (a prime denotes d/dx) or, equivalently, $c_t'(x) = -y_{t+1}'(x)u_y(c_t, y_{t+1})/u_c(c_t, y_{t+1})$. Using (1), (3), and (4), we can rewrite the equilibrium condition:[14]

$$\Psi_t'(x) + \kappa'(x) = \frac{p_{t+1}'(x)}{1 + r_{t+1}}. \tag{5}$$

Integrating (5), we can express the maximum rent an agent y_t can pay for residing at location x without binding the borrowing constraint as follows:

$$\Psi_t(x, K_t) = K_t - \Upsilon_t(x) \quad \text{with} \quad \Upsilon_t(x) \equiv \kappa(x) - \mu_{t+1}p_{t+1}(x), \tag{6}$$

where $\mu_{t+1} \equiv 1/(1 + r_{t+1}) < 1$ and K_t stands for the constant of integration, which is independent of x and will be determined by the pattern of residential choices and borrowing capacities arising in equilibrium. Assuming $p_{t+1}'(x) \leqslant 0$ (which will be shown later), it follows that $\Upsilon_t'(x) > 0$ and $\Psi_t(x, K_t)$ is a continuous and decreasing function of x. We obtain the standard trade-off between the price of land and commuting costs: Moving further from the city center, the bid rent decreases to compensate agents for higher commuting costs. In our case, there is a second trade-off: By moving away from the CBD, the bid rent decreases to compensate the lower discounted value of housing wealth (as $\mu_{t+1}p_{t+1}(x)$ decreases with x).

A key feature of our setup is that the rent endogenously determines whether each agent needs to borrow, possibly facing the borrowing constraint. Given that the bid rent is

[14] The equation $c_t' = -y_{t+1}'u_y/u_c$ is similar to the "mobility Euler equation" of Bilal and Rossi-Hansberg (2021), although we consider one-period-lived agents who optimize their inheritance by choosing how many financial assets and how much housing wealth to transmit.

decreasing with distance, it turns out that a y_t agent is more likely to borrow at more attractive locations where prices are high. For any y_t agent, we can define the threshold location $\widehat{x}_t\left(y_t, K_t\right) \in [0, L_t]$ such that the borrowing constraint (2) is binding

$$(1-\lambda)\Psi_t\left(\widehat{x}_t, K_t\right) = y_t. \tag{7}$$

For any location $0 \leqslant x \leqslant \widehat{x}_t\left(y_t, K_t\right)$, the agent endowed with an initial wealth y_t is borrowing constrained. Hence, for any location $0 \leqslant x \leqslant \widehat{x}_t\left(y_t, K_t\right)$, agent y_t is no longer able to make a trade-off between commuting costs and rent and obtain the same return rate for both the financial and housing assets (Equation (5) is no longer satisfied). In other words, when the borrowing constraint is binding, the marginal utility of moving closer to the CBD is higher than the land rent gradient, which is nil.

From the implicit function theorem, given K_t, $\mathrm{d}\widehat{x}_t/\mathrm{d}y_t < 0$ and $\mathrm{d}\widehat{x}_t/\mathrm{d}\lambda < 0$. The wealthier the agent/the smaller the loan-to-value ratio, the smaller is the set of locations where the borrowing constraint binds. Note that the borrowing constraint is never binding for sufficiently wealthy households if $(1-\lambda)\Psi_t\left(0, K_t\right) < y_t$, since Ψ_t decreases with x.

For all locations $x \geqslant \widehat{x}_t\left(y_t, K_t\right)$, a y_t agent is not borrowing constrained, and her maximum bid rent is $\Psi_t\left(x, K_t\right)$ to reside at x. We thus define the bid-rent function with a borrowing constraint denoted by ψ for any agent y_t as follows:

$$\psi(x, y_t, K_t) = \begin{cases} \frac{y_t}{1-\lambda} & \text{for } x \in \left[0, \widehat{x}_t\left(y_t, K_t\right)\right], \\ \Psi_t\left(x, K_t\right) = K_t - \Upsilon_t(x) & \text{for } x \in \left[\widehat{x}_t\left(y_t, K_t\right), L_t\right]. \end{cases} \tag{8}$$

Inserting (8) and (1) in (3), we obtain $c_t(x, y_t) + y_{t+1}(x, y_t)/(1 + r_{t+1}) = \mathcal{W}_t(x, y_t)$, where $\mathcal{W}_t(x, y_t)$ corresponds to the "lifetime" wealth of agent y_t equal to

$$\mathcal{W}_t(x, y_t) = \begin{cases} w_t + y_t - \Upsilon_t(x) - \frac{y_t}{1-\lambda} - \tau_t & \text{for } x \in \left[0, \widehat{x}_t\left(y_t, K_t\right)\right], \\ w_t + y_t - K_t - \tau_t & \text{for } x \in \left[\widehat{x}_t\left(y_t, K_t\right), L_t\right]. \end{cases} \tag{9}$$

The bid-rent function (8) stresses that the capacity of any agent to pay to reside at location x depends on whether she needs to borrow. Mortgage market imperfections impact agents' ability to pay. For locations x where the borrowing constraint does not bind, that is $x \geqslant \max\left\{0, \widehat{x}_t(y_t, K_t)\right\}$, the agent obtains the same lifetime wealth whatever her place of residence and whether she is a borrower or a saver. This is a direct consequence of the bid rent fully compensating for commuting costs and the future sale price. By contrast, for locations where the agent borrows up to the borrowing limit, the rent $y_t/\left(1-\lambda\right)$ does not capitalize commuting costs and the future sale price. Hence, the closer she resides to the CBD, the higher her lifetime wealth. *Any agent strictly prefers to live at locations*

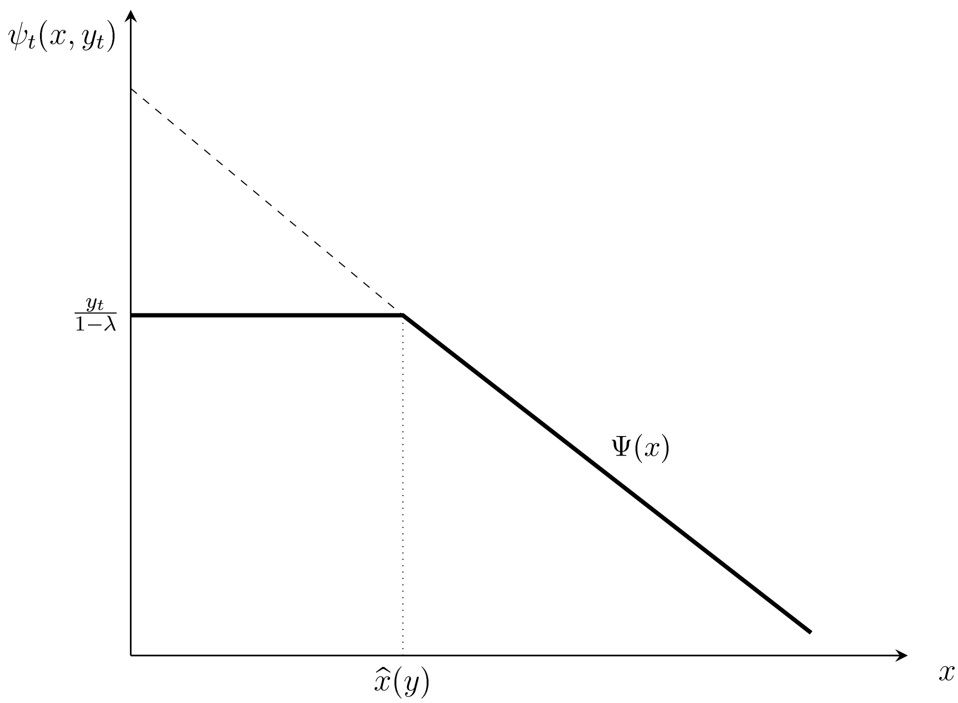

Figure 1: Bid rent with linear commuting costs for a y_t agent

closer to the CBD where she borrows up to the limit. This relies on the very nature of the borrowing constraint, which imposes a cap on agents' bid rent and implies that the marginal utility of moving marginally closer to the CBD exceeds the house price gradient.

Land is allocated to the highest bidder, so that the equilibrium price function is given by

$$p_t(x) = \max \left\{ \max_{y_t \in [\underline{y}_t, \overline{y}_t]} \psi(x, y_t, K_t), R_A \right\}, \tag{10}$$

where $p_t(x)$ is a continuous function. Given that the price received by real estate companies must exceed the opportunity cost of land ($\theta_t(x) \geqslant R_A$), we must have $p_t(x) \geqslant R_A$ for any $x \in [0, L]$.

By contrast, when there are no mortgage market imperfections, the bid-rent curve does not depend on individual wealth, and the slope of the bid-rent curve does not vary across agents. The house price in equilibrium is given by $\max\{\Psi_t(x, \mathcal{K}_t), R_A\}$ with $\Psi_t(x, \mathcal{K}_t) = \mathcal{K}_t - \Upsilon_t(x)$, where $\mathcal{K}_t \equiv (1 - \mu_{t+1})R_A + \kappa(L_t)$. Indeed, as $\Psi_t(x, \mathcal{K}_t)$ decreases with distance, $\mathcal{K}_t$ is such that $\Psi_t(L_t, \mathcal{K}_t) = R_A$.

Given that agents are altruistic, the residential choice may be viewed as an asset investment decision. Agents bear current urban costs (housing price and mobility costs), and the investment yields the future housing value. As agents must reside at one location, the average return to residential location x is then

$$\varrho_{t+1}(x) = \frac{p_{t+1}(x) - R_A}{[p_t(x) + \kappa(x)] - [R_A + \kappa(L)]}. \tag{11}$$

When the borrowing constraint is not binding, $p_t(x) = \Psi_t(x, \mathcal{K}_t)$, and it is straightforward to check that the return to location equals the financial return, that is, $\varrho_{t+1}(x) = 1/\mu_{t+1} = 1 + r_{t+1}$. Indeed, agents care about the total wealth left to their children (y_{t+1}), and choose their residential location so that they are indifferent between more financial bequest and a higher future price of housing. However, when some agents face the hurdle of the borrowing constraint, the location asset yields a higher return, that is, $\varrho_{t+1}(x) > 1 + r_{t+1}$ when $p_t(x) = y_t/(1-\lambda)$ (because $y_t/(1-\lambda) < \Psi_t(x, \mathcal{K}_t)$).[15] Such a result emerges because the housing price is lower than the maximum price an agent would be willing to pay.

3. Spatial sorting and wealth

In this section, we characterize the urban configuration that arises at any date t given a wealth distribution $F_t(y)$ in any city. Without any borrowing constraint, the maximum bid rent is identical across individuals. In equilibrium, $p_t(x) = \Psi_t(x, \mathcal{K}_t)$ for all $x \leqslant L_t$, and there is no spatial sorting. There exists a continuum of residential equilibria because agents are indifferent among all city locations. In the following paragraphs, we show that when we introduce the borrowing constraint, the competition for land differs, leading to spatial sorting.

Symmetry-breaking. Let us first provide the intuition for the mechanisms at work by assuming that at date t all agents own the same wealth y_t^0. As in Matsuyama (2000, 2006), credit market imperfections can give rise to *symmetry-breaking*, leading an initially homogeneous population to endogenously split into different wealth classes. Our novel feature is that *symmetry-breaking* relies on the interplay between credit market imperfections and agents' location choices. Such a configuration occurs if the wealth level y_t^0 is such that the borrowing constraint binds for an interior location $\widehat{x}_t^0$, that is, $(1-\lambda)\,\Psi_t(\widehat{x}_t^0) = \widehat{y}_t^0$, with $\widehat{x}_t^0 \in [0, L_t]$. Hence, if the city is characterized by a perfectly equal distribution of wealth, then the agents residing at $x \in [0, \widehat{x}_t^0]$ obtain a higher utility level. Indeed, their lifetime wealth is $\mathcal{W}_t(x) = w_t - \Upsilon_t(x) - \lambda y_t^0/(1-\lambda) - \tau_t$, with $\mathcal{W}_t'(x) < 0$, while the rest of the population living at any $x \in [\widehat{x}_t^0, L_t]$ achieves the same level of lifetime wealth, given by $\mathcal{W}_t^0 = w_t + y_t^0 - \mathcal{K}_t - \tau_t < \mathcal{W}_t(x)$. There is credit rationing because some agents cannot borrow up to their borrowing limit to live close to the center, and are relegated to areas where they leave a lower inheritance. Residents in area $[0, \widehat{x}_t^0]$ enjoy a higher lifetime

[15] See Appendices B.2 and B.4 for computations at the steady state.

wealth and greater utility because the price they pay is capped at level $y_t^0/(1-\lambda)$.[16] The mortgage market imperfection generates an economic rent that we call the "constraint rent". In the subsequent period, the society polarizes into wealthy agents, who receive a high inheritance from their parents residing in area $[0, \widehat{x}_t^0]$, and poor agents, whose parents lived in area $[\widehat{x}_t^0, L_t]$. Thus, *wealth inequality occurs endogenously.* By contrast, without any credit constraint, market equilibrium requires that any agent has the same utility and leaves the same bequest, regardless of where she lives. No wealth inequality would arise under these conditions.

Spatial sorting with heterogeneous agents. Assume now that the initial wealth differs across agents. To grasp the intuition for the heterogeneous case, we consider a wealth distribution with σ wealthy agents endowed with $\overline{y}_t$, while the rest of the agents $L-\sigma$ are poor and receive initial wealth $\underline{y}_t < \overline{y}_t$. This case is presented in Figure 2. Wealthy agents are not borrowing-constrained whatever the location x, while poor agents would be constrained in the city center, that is, in locations $x < \widehat{x}_t \equiv \widehat{x}_t(\underline{y}_t) \in (0, L_t)$. Hence, $\overline{y}_t$ agents can outbid $\underline{y}_t$ agents at $x \in [0, \widehat{x}_t)$ to be better off. The borrowing constraint generates spatial sorting of heterogeneous agents. If $\sigma < \widehat{x}_t$, then all the wealthy agents live in the most attractive sites. However, under this configuration, they do not pay $\Psi_t(x, \mathcal{K}_t)$, as poor agents cannot outbid residents at $x \in [0, \sigma]$ to be better off. The bid rent of each $\overline{y}_t$ agent is $\Psi_t\left(x, \overline{K}_t\right) = \overline{K}_t - \Upsilon_t(x)$, where $\overline{K}_t$ is such that $\overline{K}_t - \Upsilon_t(\sigma) = \underline{y}_t/(1-\lambda)$, with $\Psi_t\left(x, \overline{K}_t\right) < \Psi_t(x, \mathcal{K}_t)$. In other words, the presence of the borrowing constraints makes competition for land less intense, including for wealthy agents. The constraint rent captured by a wealthy agent increases with a lower level of wealth owned by the poorest agents and with a lower mass of wealthy agents since they would locate closer to the CBD. A fraction of poor agents lives in $x \in [\sigma, \widehat{x}_t)$ and pays a price $\underline{y}_t/(1-\lambda)$ lower than $\Psi_t(x, \mathcal{K}_t)$. The remaining poor agents occupy the least attractive sites in $[\widehat{x}_t, L_t]$ and pay $\Psi_t(x, \mathcal{K}_t)$. Therefore, the house price and agents' lifetime wealth depend on the initial wealth distribution.

Stemming on the bid-rent formation process, *the housing market generates implicit transfers between the various types of agents.* There are two types of winners: (i) the wealthiest agents who are not constrained and pay a house price that is lower than the price they would pay without the borrowing constraint, therefore capturing the rent represented by area A in Figure 2, and (ii) the lucky poor agents who are borrowing-constrained and capture the rent represented by area B in Figure 2. The resulting agents' lifetime wealth reflects *both their ranking in the wealth distribution* and *luck*.

[16] The choice of rationing rule that would be implemented to split identical agents into different categories is beyond the scope of the paper. The rationing rule would inevitably be *ad hoc* but would not affect the occurrence of symmetry-breaking.

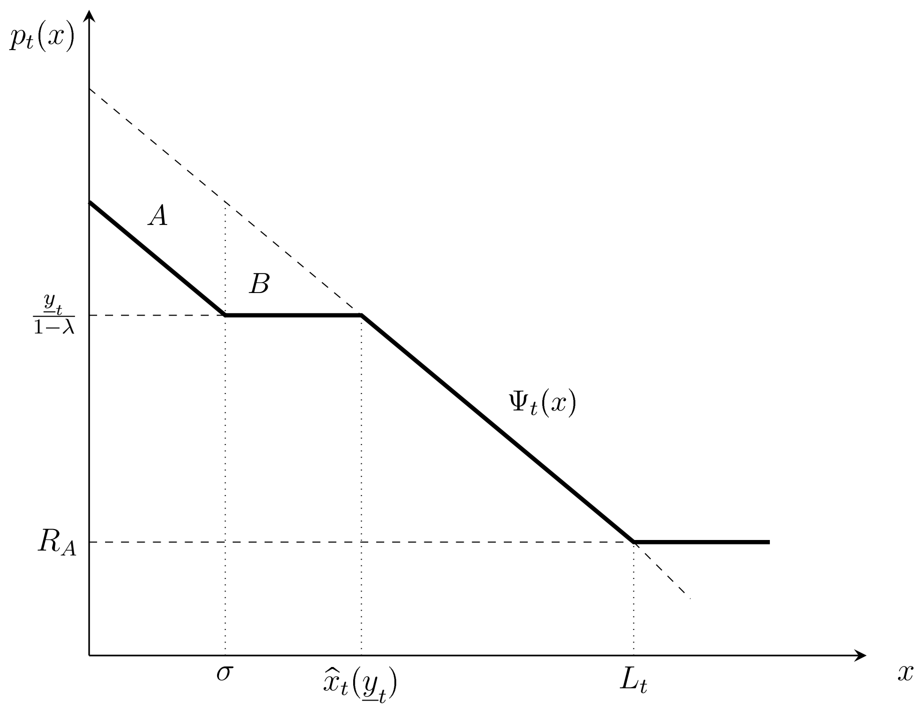

Figure 2: Equilibrium with two wealth classes and linear commuting costs

More generally,

Proposition 1
The borrowing constraint leads to spatial wealth sorting. The wealth mapping $y_t^\star(x)$ from the location set to the wealth set is such that $1 - F_t(y) = x/L_t$. Consequently, $y_t^\star(x)$ decreases with x as $y_t^{\star\prime}(x) = -1/[L_t f_t(y)]$.

Proof. From (8), for any agent y_t we have

$$\psi'(x, y_t, K_t) = \begin{cases} 0 & \text{for } x \in [0, \widehat{x}_t(y_t, K_t)[\,, \\ \Psi_t'(x, K_t) < 0 & \text{for } x \in]\widehat{x}_t(y_t, K_t), L_t] \end{cases}$$

considering that ψ is not differentiable at $\widehat{x}_t(y_t, K_t)$.

First, suppose that $\widehat{x}_t(y_t, K_t) \in [0, L_t]$ for some y_t. Since $\widehat{x}_t(y_t, K_t)$ decreases with y_t, we can define the threshold $\widehat{y}_t(K_t) \in [\underline{y}_t, \overline{y}_t]$ such that, at a given location x, we have $x = \widehat{x}(\widehat{y}_t, K_t)$. Considering a bid-rent function $\psi(., y_t, K_t)$, we take a point on that curve $(\widehat{x}_t(\widehat{y}_t, K_t), \psi(\widehat{x}_t(\widehat{y}_t, K_t), y_t, K_t))$. By holding the value of ψ constant, we can then rank the bid-rent slopes according to the wealth level for any $x = \widehat{x}_t(\widehat{y}_t, K_t) \in (0, L_t)$:

$$\psi'(x, y_t, K_t)|_{x=\widehat{x}(\widehat{y}_t, K_t), \psi=\text{const}} = \begin{cases} \Psi_t'(x, K_t) & \text{for } y_t > \widehat{y}_t, \\ 0 & \text{for } y_t < \widehat{y}_t. \end{cases}$$

where, from (6), $\Psi_t'(x, K_t) = -\Upsilon_t'(x) < 0$. Steeper bid rents imply locations closer to the CBD, as agents with steeper bid rents bid away the agents with flatter bid rents. In other words, agents endowed with an initial wealth $y_t > \widehat{y}_t$ can outbid agents poorer than $\widehat{y}_t$ in more attractive areas, *i.e.*, in the area $[0, \widehat{x}(\widehat{y}_t, K_t)[$.

If $L_t > \widehat{x}(\overline{y}_t, K_t) > 0$, then for any $x \in [0, \widehat{x}(\overline{y}_t, K_t)]$, bid-rent slopes are nil for any agent. However, as the bid rent $y_t/(1-\lambda)$ is strictly increasing with the agent's wealth and any constrained borrower strictly prefers to live close to the city center, there is perfect sorting in the area $[0, \widehat{x}(\overline{y}_t, K_t)]$.

If $0 < \widehat{x}(\underline{y}_t, K_t) < L_t$, then for any $x \in [\widehat{x}(\underline{y}_t, K_t), L_t]$, this implies that there is no $\widehat{y}_t \in [\underline{y}_t, \overline{y}_t]$ such that $x = \widehat{x}(\widehat{y}_t, K_t)$ for $x \in [\widehat{x}(\underline{y}_t, K_t), L_t]$. No agent y_t is constrained by the borrowing constraint at any $x \in [\widehat{x}(\underline{y}_t, K_t), L_t]$. There is no sorting in this area, as for locations $x \in [\widehat{x}(\underline{y}_t, K_t), L_t]$ the bid-rent slope is the same whatever the agent y_t.

Second, assume that there is no $\widehat{x}_t(y_t, K_t) \in [0, L_t]$ for any y_t. This arises under two cases:

(i) The wealth distribution is such that $(1-\lambda)\Psi_t(x, K_t) > \overline{y}_t$ for any $x \in [0, L_t]$, meaning that there does not exist $\widehat{x}_t(y_t, K_t) \in [0, L_t]$ for any y_t. This implies that all agents are borrowing constrained (all agents bid $y_t/(1-\lambda)$ for land, and there is perfect sorting in the area $[0, L_t]$).

(ii) The wealth distribution is such that $(1-\lambda)\Psi_t(0, K_t) < \underline{y}_t$, meaning that there does not exist $\widehat{x}_t(y_t, K_t) \in [0, L_t]$ for any y_t. This implies that, for no agent and no location, the borrowing constraint is binding. One could not rank bid-rent slopes because all agents would be able to pay Ψ_t.

■

Proposition 1 stresses the role of the borrowing constraint in generating segregation. Since wealthy agents are less likely to borrow and to confront the obstacle of the borrowing constraint, the bid-rent slope is increasing with agents' wealth. It turns out that wealthy households are able to outbid poorer households. This leads to spatial wealth sorting, that is, the wealthier the residents, the better the place where they live. In other words, the borrowing constraint acts as a barrier preventing less-favored agents to accede to the most attractive locations.[17]

Spatial wealth sorting occurs as soon as the wealth distribution and the bid rent are such that the poorest agents are credit constrained at the CBD, that is, $(1-\lambda)\Psi_t(0, \mathcal{K}_t) > \underline{y}_t$. Otherwise, any agent pays rent $\Psi_t(x, \mathcal{K}_t)$, and the urban configuration resembles the urban equilibrium of the standard monocentric city model (Fujita and Thisse, 2002). Furthermore, we assume that the poorest agents are not credit constrained at the city fringe ($\underline{y}_t > (1-\lambda)\Psi_t(L_t, \mathcal{K}_t)$); otherwise, they would be expelled from the city.

[17] In a different setting of endogenous formation of jurisdictions, Bénabou (1996) also emphasizes that credit market imperfections are sufficient to cause social segregation.

Rent gradient and the intergenerational transmission of wealth. We are also able to characterize the rent gradient in the following proposition:

Proposition 2
At any residential equilibrium, the rent function is nonincreasing with distance x. Its gradient in a given area depends on whether residents are borrowing constrained.

The Alonso-Muth condition is satisfied for all locations where agents are not borrowing constrained because the rent decreases with distance to compensate for higher commuting costs and the lower future sale price. This is not the case for locations where the borrowing constraint is binding because the rent gradient is equal to $y_t^{\star\prime}(x) / (1-\lambda) < 0$. The rent capitalizes neither commuting costs nor the future selling price, and its gradient depends solely on the wealth distribution and the intensity of the borrowing constraint λ. The greater the wealth gap between two adjacent locations, captured by a steep mapping $y_t^\star(x)$, the steeper is the rent function. Note that the rent function is flat in locations where agents are constrained and have the same wealth level.

Next, we turn to the intergenerational transmission of wealth. Given the lifetime wealth (9), we have for any $y_t^\star(x)$ we have

$$\mathcal{W}_t'(x, y_t^\star(x)) = \begin{cases} -\Upsilon_t'(x) + (y_t^{\star\prime}(x) - \psi_y(x, y_t^\star) y_t^{\star\prime}(x)) & \text{if borrowing constrained,} \\ y_t^{\star\prime}(x) & \text{otherwise.} \end{cases}$$

From Proposition 1, we know that $y_t^{\star\prime}(x) < 0$. Moreover, $\psi_y(x_t, y_t^\star) = 0$ in the urban equilibrium. Hence, for both constrained and unconstrained residents, we deduce that $\mathcal{W}_t'(x, y_t^\star(x)) < 0$. Since y_{t+1} is an increasing function in $\mathcal{W}_t$, this allows us to formulate the following statement:

Proposition 3
In any residential equilibrium, the more attractive the parents' location, the higher is the inheritance received by their offspring.

Propositions 1 and 3 together imply that the spatial sorting and wealth ranking are maintained along the transition path. The farther the parents live from the CBD, the less their offspring inherits. The place where an agent lives thus translates into the position of her offspring in the wealth distribution. As a consequence, the lower the inherited wealth, the farther away the offspring will reside. It turns out that each generation lives at the same distance to the CBD even if their forebears live in a different city. Of course, this relies on our assumption that there is no idiosyncratic shock to wealth, preventing any dynasty from experimenting social mobility or any change in its place of residence.

The social structure and size of cities. Propositions 1, 2, and 3 hold regardless of constrained agents' locations. In the Supplementary Appendix, we characterize the spatial organization of cities as a set of areas that are subsets of space that host agents who have the same status, constrained or not. The size, number, and location of the areas depend on the shape of the wealth distribution. In this subsection, we discuss the urban configuration when commuting costs $\kappa(x)$ are linearly increasing with distance and inherited wealth y_t follows a bounded Pareto distribution. Figure 3 illustrates this case while the demonstration is reported in Appendix A. When perfect sorting occurs, $y_t^\star(x)$ is decreasing and *convex* since the cumulative distribution function of a Pareto variable is an increasing and concave function. There are also three distinct areas (formally, two *cutoff locations*). When there is at least one area hosting credit-constrained agents, inhabitants in areas closer to the city center pay lower house prices regardless of whether they are constrained. According to the distribution of wealth, there is scope for luck. Some middle-wealth agents are lucky because they reside in attractive, but not the most attractive, places, while poorer agents cannot outbid the lucky agents. Hence, lifetime wealth differentials in this model arise from inherited wealth differentials, magnified by the assignment of agents across locations.

The social structures of all cities are identical when agents are free to move across cities. Given the budget constraint of a city, $G/L_t = \tau_t$, the existence of fixed costs provides an incentive for city formation. However, a rise in city population implies higher house prices due to land competition. A spatial equilibrium arises when no agent has an incentive to migrate to another city. The agents endowed with the same initial wealth must achieve

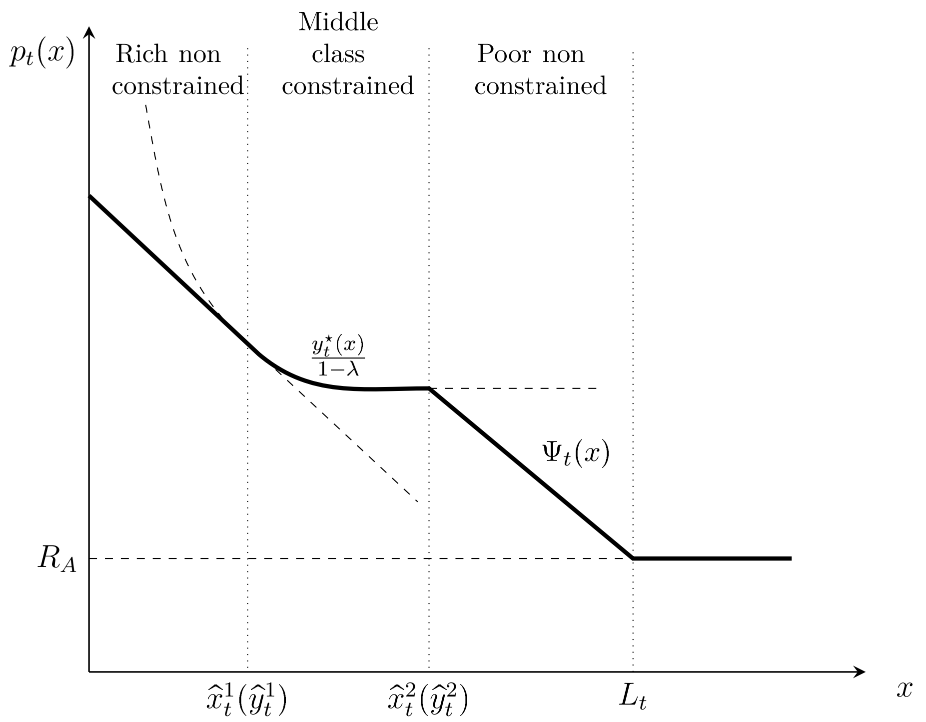

Figure 3: Equilibrium with Pareto distribution and linear commuting costs

the same level of utility across cities in each period $V_t^\star(y_t)$. When the borrowing constraint applies, it is straightforward to check that there is spatial splitting of identical agents in equilibrium. Agents who have the same initial wealth are split across cities and live in locations with the same commuting cost, so that they reach the same level of utility. The wealthy agents are attracted to cities hosting agents endowed with a lower initial wealth who are credit constrained. They also have an incentive to disperse across cities to avoid fierce competition for land. The share of agents endowed with y_t living in a city is $1/N$, where N is the number of cities. The social composition of all cities is therefore identical. The city size L_t is such that the poorest agents obtain the same utility level across cities, *i.e.*, $w_t + \underline{y}_t - \mathcal{K}_t - G/L_t$ does not vary across cities.

4. Long-run wealth and taxation with fixed wages and interest rate

We now analyze the joint dynamics of the urban equilibrium and the wealth distribution. We first consider the long-run wealth distribution under *laissez-faire*, when agents are assumed to have perfect foresight of future house prices and the wage and the interest rate are held fixed. We then determine whether market mechanisms lead to efficient outcomes and tax policies allow a better allocation of resources. Our framework captures two sources of inefficiency: (i) mortgage market imperfections, and (ii) head taxation with mobile agents and scale economies in the production of local public goods.[18]

4.1. Long-run wealth distribution under laissez-faire

To obtain sharp predictions on the long-run equilibrium, we assume Cobb-Douglas preferences $u\left(c_t, y_{t+1}\right) = c_t^{1-\alpha} y_{t+1}^{\alpha}$. As a result, agents' transmitted wealth and consumption are constant fractions of their lifetime wealth. Homotheticity is not essential. We could use preferences leading wealthy agents to leave as inheritance a larger fraction of their wealth than poorer agents. In addition, the nondurable good consumption is assumed to be produced under constant returns and perfect competition, using labor only (we consider productive capital below). Similarly, urban local markets are perfectly competitive, so that the urban wage is given by $w_t = \varphi$, where φ measures the marginal product of a labor unit, which is the same for all periods. The interest rate is also assumed to be constant ($r_t = r$ and, in turn, $\mu_t = \mu$ and $\mathcal{K}_t = \mathcal{K}$). Since in each period cities are symmetric at equilibrium, and the total size of population is constant, we have $L_t = L$.

The wealth dynamics differ between constrained agents and unconstrained agents and are

[18] Some readers might wonder why head taxation is inefficient in our context. Inefficiency arises because the intercity mobility of agents implies a fiscal externality as the per capita cost of local public good G/L_t varies with the intercity distribution of agents (see Mieszkowski and Zodrow, 1989).

given by

$$y_{t+1} = \beta \begin{cases} \widetilde{w} + y_t - \kappa[x^\star(y_t)] - \frac{y_t}{1-\lambda} + \mu p_{t+1}[x^\star(y_t)] & \text{if borrowing-constrained} \\ \widetilde{w} + y_t - K_t & \text{otherwise,} \end{cases} \tag{12}$$

where $x^\star(y_t)$ is the inverse of the wealth mapping, $\beta \equiv \alpha(1+r)$, and $\widetilde{w} \equiv \varphi - G/L$ is the wage net of the lump-sum tax.

The steady state is associated with the wealth limit distribution $F_\infty(y)$ and the limit house price $p_\infty(x)$. Note that given (6), the rent dynamics are forward-looking and can have many solutions. As a first step, we abstract from any dynamics with housing bubbles, allowing us to obtain a tractable solution of the bid rent Ψ_t. We discuss the consequences of housing bubbles in Appendix C.3.

Full convergence. We first discuss the case where individual wealth converges to the same steady state. This case arises only when *all* agents are not borrowing constrained. Then, the agent living at location x pays the rent given by $\Psi_t(x, \mathcal{K}) = \mathcal{K} - \kappa(x) + \mu\Psi_{t+1}(x, \mathcal{K})$. Hence, we have $\Psi_\infty(x, \mathcal{K}) = [\mathcal{K} - \kappa(x)]/(1-\mu)$ (see Appendix B.1 for details). Given (12), agents' steady-state wealth is given by the fixed point of $y_{t+1} = \beta(\widetilde{w} + y_t - \mathcal{K})$, *i.e.*

$$y_\infty = \frac{\beta(\widetilde{w} - \mathcal{K})}{1-\beta} \equiv y_\infty^n. \tag{13}$$

The sufficient and necessary conditions for the existence of a steady state in which there is no wealth inequality are $\beta < 1$ and $y_\infty^n \geqslant (1-\lambda)\max_{x\in[0,L]} \Psi(x, \mathcal{K})$. Using (13) and $\max_{x\in[0,L]} \Psi(x, \mathcal{K}) = \Psi(0, \mathcal{K}) = \mathcal{K}/(1-\mu)$, the latter inequality is equivalent to

$$R_A \leqslant \underline{R}_A(\lambda) \equiv \frac{1}{1-\mu}\left[\frac{\widetilde{w}}{1+\rho(\lambda)} - \kappa(L)\right] \quad \text{with} \quad \rho(\lambda) \equiv \frac{(1-\beta)(1-\lambda)}{\beta(1-\mu)}. \tag{14}$$

As long as the inequality in (14) is satisfied, a steady state exists in which all agents maintain the same level of wealth, although they do not pay the same rent. Land abundance (low R_A) and low commuting costs make the borrowing constraint less stringent. The concavity of the utility function and the absence of heterogeneity in productivity imply that wealth inequality declines over time, and that each dynasty will own the same steady-state wealth.

Persistent wealth inequality. We now consider steady states with an unequal distribution of wealth, *i.e.* some dynasties are borrowing constrained. With $\min_{x\in[0,L]} \Psi(x, \mathcal{K}) = (\mathcal{K} - \kappa(L))/(1-\mu)$, the existence of persistent inequality requires $(1-\lambda)(\mathcal{K}-\kappa(L))/(1-\mu) \leqslant$

$y^n_\infty < (1-\lambda)\Psi_\infty(0,\mathcal{K})$ or, equivalently, $\underline{R}_A(\lambda) < R_A < \overline{R}_A(\lambda)$ with

$$\overline{R}_A(\lambda) \equiv \frac{1}{1-\mu}\left[\frac{\widetilde{w}-\kappa(L)}{1+\rho(\lambda)}\right] \tag{15}$$

and $\overline{R}_A > \underline{R}_A$. In such a steady state, the borrowing constraint leads to permanent spatial sorting. The poorest dynasties live at the city fringe, $x \in [\widehat{x}^*_\infty, L]$, and are unconstrained. Accordingly, their wealth must converge to y^n_∞, with $\widehat{x}^\star_\infty$, such that $y^n_\infty = (1-\lambda)\Psi(\widehat{x}^\star_\infty,\mathcal{K})$ or, equivalently,

$$\kappa(\widehat{x}^\star_\infty) = \mathcal{K} - \frac{\widetilde{w}-\mathcal{K}}{\rho(\lambda)}. \tag{16}$$

The constrained agents are sorted by increasing wealth as the distance to the CBD decreases from $x = \widehat{x}^*_\infty$. Wealth dynamics converge to the following steady state:

$$y_\infty(x) = \frac{\beta[\widetilde{w}-\kappa(x)]}{(1-\beta)[1+1/\rho(\lambda)]} \equiv y^c_\infty(x) \tag{17}$$

with $y^c_\infty(\widehat{x}^*_\infty) = y^n_\infty$ and $y^c_\infty(x) > y^n_\infty$ when $x \in [0,\widehat{x}^*_\infty]$. In Appendix B.2, we show that the wealth of credit-constrained dynasties living at x converges to y^c_∞, provided that the following convergence condition holds:

$$\lambda < \frac{1-\beta\mu}{1+\beta} \equiv \overline{\lambda} \tag{18}$$

It follows that the rent gradient is smaller for locations occupied by constrained agents since they pay $y_\infty(x)/(1-\lambda)$, and that $|\mathrm{d}\Psi(x,\mathcal{K})/\mathrm{d}x| > |\mathrm{d}y_\infty(x)/\mathrm{d}x|/(1-\lambda)$ regardless of $\kappa(x)$, μ, $\lambda < 1$, and $\beta < 1$.

Note that given our assumptions on wealth dynamics, a class of wealthy unconstrained agents living in the most attractive places cannot emerge in the long run (see Appendix B.3). The emergence of wealthy agents who are not credit constrained is a transitory configuration. In the long run, their wealth converges to the wealth of the wealthiest constrained agents. In Section 5.1, we show that the introduction of an endogenous occupational choice leads to a steady state with both unconstrained wealthy (skilled) agents and middle wealth (unskilled) agents who are credit constrained.

We summarize these results in the following proposition, and in Figure 4, which depicts the ranges of parameter values in the plane (λ, R_A) associated to a given long-run wealth distribution:

Proposition 4

If and only if $\beta < 1$ and *convergence condition (18) hold, the long-run city is characterized by one of the following steady-state distributions:*

(i) Full convergence: If $R_A \leqslant \underline{R}_A$*, all agents' wealth converges to the same steady state* y^n_∞*.*

(ii) Persistent inequality: If $\underline{R}_A \leqslant R_A < \overline{R}_A$*, a fraction* $L - \widehat{x}^\star_\infty$ *of agents ends up with the long-run wealth* y^n_∞ *given by (13), while a fraction* $\widehat{x}^\star_\infty$ *of constrained agents obtains long-run wealth that is location dependent,* $y^c_\infty(x)$*, and* $\widehat{x}^\star_\infty$ *is the unique interior solution of* $\Psi_\infty(\widehat{x}^\star_\infty, \mathcal{K})(1-\lambda) = y^n_\infty$*.*

(iii) Maximal persistent inequality: If $\overline{R}_A \leqslant R_A$*, all agents end up with the long-run wealth* $y^c_\infty(x)$ *given by (17).*

Proposition 4 highlights the role played by credit market imperfections, and the place of residence on persistent wealth inequality. Item (i) states that when all agents are wealthy enough, they can afford to locate anywhere, the borrowing constraint not being binding. As the rent exactly offsets commuting costs and the future house sale price, wealth dynamics are not location dependent. The return to location $\varrho_\infty(x)$ (see (11)) equals the rate of return of financial assets, *i.e.* $1+r$. Even if poor dynasties start with lower amounts of wealth, their wealth converges to the same steady state. Therefore, the long-run wealth distribution is degenerate. Item (ii) provides a characterization of the long-run equilibrium with persistent inequality, that is, the poorest agents are unconstrained at the city limit and at the CBD ($\underline{R}_A < R_A < \overline{R}_A$). Wealthier agents who live at the CBD are borrowing-constrained, and benefit from a rate of return to their location asset $\varrho_\infty[y^c_\infty(x)]$ that is now greater than $1+r$ (the proof is reported in Appendix B.2). This translates into persistent inequality between constrained and unconstrained dynasties. *Persistent inequality thus reflects heterogeneity across locations.*

From Proposition 1, the long-run wealth distribution must be such that $1 - F_\infty[y_\infty(\widehat{x}^\star_\infty)] = \widehat{x}^\star_\infty/L$. The borrowing constraint is not binding for residents living farther away from $\widehat{x}^\star_\infty$, and they converge to $y^n_\infty \leqslant y^c_\infty(x)$. Note that if the condition $R_A < \overline{R}_A$ does not hold, all agents would be limited in their capacity to borrow and pay the rent $y_t/(1-\lambda)$. There would be perfect sorting along $[0, L]$. This case leads to the most unequal wealth distribution, with $y_\infty(x)$ given by (17) for all $x \in [0, L]$. Note further that, although the initial urban configuration is characterized by several threshold locations $\widehat{x}^\star_0$, at most one threshold location $\widehat{x}^\star_\infty$ arises in the long run. Wealth convergence conditions prevent multiple threshold locations in the long run. Indeed, if there were two thresholds $\widehat{x}^{1\star}_\infty < \widehat{x}^{2\star}_\infty$, we would have $\Psi_\infty(\widehat{x}^{1\star}_\infty)(1-\lambda) = y^c_\infty(\widehat{x}^{1\star}_\infty) = y^n_\infty$ and $\Psi_\infty(\widehat{x}^{2\star}_\infty)(1-\lambda) = y^c_\infty(\widehat{x}^{2\star}_\infty) = y^n_\infty$, which is impossible because the land rent is strictly decreasing with distance.

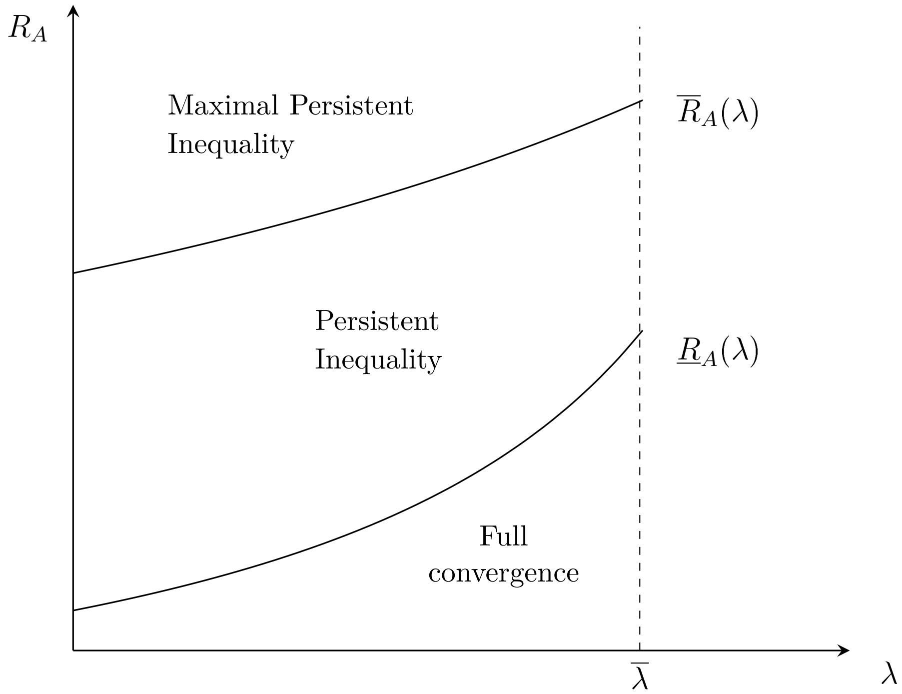

Figure 4: Wealth inequality in the (λ,R_A)-space

Furthermore, Proposition 4 states that steady states are not path dependent. This property is reminiscent of endogenous inequality models (Matsuyama, 2000) and stems from the endogeneity of the borrowing constraint. Many studies on the role of imperfect capital markets have stressed that initial conditions matter in predicting the long-run equilibrium because individuals have to be wealthier than an exogenous wealth level to have access to credit (Galor and Zeira, 1993, Banerjee and Newman, 1993). In our model, the agent's position in the wealth distribution determines her capacity to borrow.

As depicted in Figure 4, for a high level of credit market imperfections (low λ), or when land is scarce (high R_A), persistent inequality can arise because the borrowing constraint becomes tighter. Regarding the effect of the downpayment requirement, we have that $\mathrm{d}y^c_\infty/\mathrm{d}\lambda < 0$. A tighter downpayment requirement (lower λ) leads to an increase in the constrained agents' wealth. This is because the constrained agents' bid rent $y_t/(1-\lambda)$ decreases. Expenditures for housing decrease, and constrained agents end up wealthier, thus allowing for higher bequests. Hence, *in the long run, a tighter borrowing constraint increases the long-run wealth of the wealthiest agents.*[19]

[19]It follows that, even though returns to wealth and income do not differ among agents in the long run, the wealth distribution may not inherit the properties of the income distribution (see Benhabib and Bisin, 2018, which emphasize the role of earnings distribution and heterogeneous returns on wealth in wealth inequality).

4.2. Achieving a better allocation of resources with land taxation

Efficient allocation of resources. Consider an economy-wide *policymaker* whose objective is to maximize the discounted sum of per capita utilities of current and future generations. The policymaker acquires the land needed for each city from landowners (the agents) for the amount $\mathcal{R}_t \geqslant R_A$, and chooses the population size L and the taxation scheme in each city, as well as agents' location, the consumption of the composite good c_t and the bequest b_t. For a straight comparison between decentralized and centralized allocations of resources, we adopt the approach in which the policymaker considers the bequest received by the next generation and wages to be determined by competitive markets, so that $b_{t+1} = (1+r)b_t$ and $w_t = \varphi$.[20] In addition, to focus on the own effects of credit and land markets, we assume that households are homogeneous (although households differ in their initial wealth, they end up homogeneous in the first-best configuration).

As all cities are identical, it is sufficient to focus on the representative city. Formally, the policy maker maximizes $\sum_{t=0}^{\infty} \delta^t u(c_t, y_{t+1})$ with $\delta \leqslant 1$ being the social discount factor and $y_{t+1} = (1+r)b_t + \mathcal{R}_t$, where $\mathcal{R}_t$ is the value of the land property right. Hence, children receive bequests and a land property right. The per capita resource constraint of the economy is:

$$c_t = \varphi + y_t - b_t - \mathcal{R}_t - \Omega(L) \quad \text{with} \quad \Omega(L) \equiv G/L + \mathcal{T}(L)/L, \tag{19}$$

where $\mathcal{T}(L) = \int_0^L \kappa(x)\mathrm{d}x$ is the total transport cost. The value of land, bequests, and city size are endogenously determined by the planner. The optimality condition associated with bequests implies:

$$u_c(c_t, y_{t+1}) = (1+r)u_y(c_t, y_{t+1}) + \delta(1+r)u_c(c_{t+1}, y_{t+2}). \tag{20}$$

A higher voluntary bequest reduces the consumption of donors, generating a utility loss, while it raises not only the current welfare of donors (joy-of-giving effect), but also the welfare of the next generation as the consumption of donees increases. Using (20), we have $u_c(c^o_\infty, y^o_\infty) = (1+r)u_y(c^o_\infty, y^o_\infty)/[1-\delta(1+r)]$ at the steady state. Therefore, given a Cobb-Douglas utility function, $c^o_\infty = (1-\alpha)(1-\delta/\mu)y^o_\infty/\beta$, the dynamics of inherited wealth converge to the following steady state:

$$y^o_\infty = \frac{\beta[\varphi - \Omega(L) - (1-\mu)\mathcal{R}_\infty]}{(1-\beta)(1-\tilde{\delta})} \quad \text{with} \quad \tilde{\delta} \equiv \frac{\delta}{\mu}\frac{1-\beta\mu}{1-\beta} < 1, \tag{21}$$

[20] Note that the social objective could be the discounted sum of generational utilities, each purged of its altruistic component. There is a debate about whether the social objective should consider the altruism component of individuals' preferences (see Michel and Pestieau, 2004 and Farhi and Werning, 2007). In our context, intergenerational altruism does not make the social objective time-inconsistent.

where we have used (19).

We can now determine the long-run value of the land property right $\mathcal{R}_\infty^o$ and city size L^o that maximize steady-state social welfare. Because y_∞^o decreases with $\mathcal{R}_\infty$, while c_∞^o increases with y_∞^o, we have $\partial u(c_\infty^o, y_\infty^o)/\partial\mathcal{R}_\infty < 0$. Hence, the optimal value of the land property right is equal to the opportunity cost of land ($\mathcal{R}_\infty^o = R_A$). The effect of city size on welfare is ambiguous. Indeed, an increase in population size has two effects: The per capita cost of the public good G/L decreases (economies of scale), while the per capita commuting cost $\mathcal{T}(L)/L$ increases since $\mathcal{T}(L)$ is strictly increasing and convex in L (diseconomies of scale).[21] If the population size is chosen to maximize the utility level of the city's residents, then $G = (\mathrm{d}\mathcal{T}/\mathrm{d}L)L - \mathcal{T}(L)$ with $\mathrm{d}\mathcal{T}/\mathrm{d}L = \kappa(L)$. Hence, regardless of the functional form of the utility function, the utility is maximized when $L = L^o$, where L^o is implicitly given by

$$G = \kappa(L^o)L^o - \mathcal{T}(L^o). \tag{22}$$

It can be checked that the population size increases with the fixed requirement in public expenditures and decreases with the distance elasticity of the commuting cost.[22]

We define the shadow rent $S_t(x)$ on land at a distance x from the CBD as the resource saving from having an additional unit of land in this location. Moving an individual from the city limit to the additional unit of land at x would result in the discounted sum of resource saving $\kappa(L) - \kappa(x)$. The shadow rent on land at the city limit equals the shadow rent on land in nonresidential use, which equals R_A. Hence, $S_t(x) = R_A + \sum_{\varsigma=t}^{\infty}\delta^{\varsigma-t}[\kappa(L) - \kappa(x)]$. As a result, when the city size is optimal, the present value of aggregate differential (shadow) land rents is

$$\mathrm{ASLR}_t = \sum_{\varsigma=t}^{\infty}\delta^{\varsigma-t}\int_0^{L^o}\kappa(L^o) - \kappa(x)\mathrm{d}x = [\kappa(L^o)L^o - \mathcal{T}(L^o)]\sum_{\varsigma=t}^{\infty}\delta^{\varsigma-t} = \frac{G}{1-\delta}.$$

When $\delta = 0$, we fall back to the HGT in a static environment, so that, at the optimal population, public expenditures equal the *instantaneous* value of aggregate differential (shadow) land rents ($G = \mathrm{ASLR}$). Our dynamic framework implies a variant of the HGT. The present value of aggregate differential (shadow) land values equals the present value of public expenditures (given by $\sum_{\varsigma=t}^{\infty}\delta^{\varsigma-t}G = G/(1-\delta)$).

We are now equipped to determine whether the steady-state equilibrium of the decentralized economy coincides with the planner's optimal solution, and to discuss tax instruments that allow for the decentralization of the social optimum. The debate on which tax base

[21] The aggregate commuting cost is convex, regardless of the structure of the commuting cost as long as the commuting cost incurred by an agent increases with the distance to jobs. For example, if $\kappa(x) = \kappa x^\epsilon$ with $\kappa > 0$ and $\epsilon > 0$, then $\mathcal{T}(L) = \kappa L^{1+\epsilon}/(1+\epsilon)$, so that $\mathcal{T}(L)$ is convex with city size.

[22] For example, if $\kappa(x) = \kappa x^\epsilon$, then the optimal size of cities is $L^o = [(1 + 1/\epsilon)G/\kappa]^{1/(1+\epsilon)}$.

to target for raising revenues to finance public expenditure has resurged recently among economists (Schwerhoff *et al.*, 2020). Land taxation has received considerable attention because the return to land can be viewed as an economic rent. Indeed, urban land value primarily captures benefits that do not come from the efforts of landowners (*e.g.*, the presence of public facilities, accessibility of jobs, and amenities). In addition, the price elasticity of land supply is very low in the short run, and land cannot be moved. Land thus represents a tax base on which a tax can be levied to finance public expenditures without significantly distorting economic behavior. Developing a general, but static, framework, Arnott and Stiglitz (1979) show that in a city of optimal population size, differential land rents (*i.e.*, the aggregate urban land rents less the opportunity cost of land in nonurban use) equal public expenditure (the HGT), so that taxing differential land rents is sufficient and optimal. In what follows, we show that altruism and the credit constraint imply a distortion in land price formation, and preclude the decentralization of Pareto optimal allocations even though the population size of the decentralized city is L^o. The HGT needs to be adjusted in our distorted economy.

Taxation with no credit constraint. We first assume that the mortgage market is perfect. Given the specification of our utility function (impure altruism), the decentralized city implies that the ratio y_∞/c_∞ is lower than that of the planner. Indeed, agents do not take fully into account the infinite stream of their descendants' utilities. To attain an efficient allocation of resources between consumption and intergenerational transfer, the city government has to implement a standard Pigouvian tax on consumption equal to $1/(\mu/\delta - 1) \equiv \tau^c_\infty$, where tax revenues are recycled in a lump-sum transfer scheme.[23] This is a well-known feature of models with impure altruism (Michel and Pestieau, 2004). By contrast, in the presence of pure altruism (parents derive utility from the offspring's utility), the ratio of inherited wealth to consumption in the *laissez-faire* equilibrium is efficient.

Furthermore, cities must satisfy their budget constraint that requires the present value of public expenditures, $G/(1-\delta)$, to be equal to the present value of tax revenue levied by the city government. Under pure or impure altruism, the present value of the steady-state differential land price when the credit constraint is not binding is $\sum_{\varsigma=t}^{\infty} \delta^{\varsigma-t}[\theta_t(x) - R_A] = [\kappa(L^o) - \kappa(x)]/[(1-\mu)(1-\delta)]$, which is higher than the present value of differential shadow price $S_t(x) - R_A$ because of altruistic preferences (captured through parameter μ in (6)). With intergenerational altruism, a single 100% differential land rent tax would be sufficient to finance public expenditures. The presence of altruistic agents makes the differential land rent higher to finance public expenditures. When parents have an altruistic concern

[23] Equivalently, the government could implement a Pigouvian subsidy on voluntary bequests equal to $\delta(1+r)$ financed by a lump-sum tax.

for their children, the socially optimal decentralization is possible with a tax paid by *donees* on *inherited land assets* so that the inheritance received by donees is $y_{t+1} = (1+r)b_t + \theta_{t+1}(x) - \tau^d_{t+1}(x)$, with $\tau^d_{t+1}(x) = \theta_{t+1}(x) - R_A$ leading to $y_{t+1} = (1+r)b_t + R_A$. Under this tax regime, bid rents at period t equal $R_A + \kappa(L) - \kappa(x) \equiv \Psi^d(x)$, which now corresponds to the instantaneous value of the shadow land price. Such a result emerges because donors' bid rents no longer depend on μ, since the wealth left to offspring does not vary with the future value of land ($\partial y_{t+1}/\partial x = 0$ when $y_{t+1} = (1+r)b_t + R_A$). Note that this tax would be inefficient if it were paid by the *donors*, as their bid rent would depend on the future value of land, and their allocation of resources between consumption and wealth left to donees would be distorted. Hence, public expenditures can be financed by a confiscatory tax on land assets (up to its opportunity cost) received by the descendants equal to $\tau^d_t(x)$, and y^o_∞ is given by

$$y^o_\infty = \frac{\beta(\varphi - \mathcal{K})}{(1-\beta)(1-\widetilde{\delta})} \tag{23}$$

where we have used (22) and $\mathcal{R}^o_\infty = R_A$. Therefore, *the optimal land taxation cannot be disconnected from inheritance taxation.*

Proposition 5
When the mortgage market is perfect and the city population size is optimal, a single 100% tax on differential land rents received by donees is efficient and sufficient to finance public expenditures.

It is worth stressing that a residential property tax, instead of a tax on land, could be implemented to finance public expenditures. However, if a property tax is more efficient than a head tax, this taxation scheme is not optimal (see Appendix B.4). Indeed, in our dynastic framework, a property tax corresponds to a tax on land asset owned by donors. Their bid rent still depends on the future value of land that distorts the allocation of their resources between consumption and wealth left to heirs. With intergenerational transfers and altruistic parents, we show that the tax on *differential land rent must be paid by donees* to achieve a better resource allocation.

Taxation under credit constraint. Let us now consider imperfect mortgage markets, while still keeping the Pigouvian tax on consumption $\tau^c_\infty = 1/(\mu/\delta - 1)$ to get the ratio y_∞/c_∞ optimal and the tax on land assets left to offspring $\tau^d_\infty(x) = \theta_\infty(x) - R_A$, so that $y_{t+1} = (1+r)b_t + R_A$ and bid rents at period t equal the instantaneous value of the shadow land price $\Psi^d(x)$. As credit constraints cause the market price of land at the most attractive locations to be lower than the shadow prices of land $S_t(x)$, Proposition 5 is no longer valid, and the optimal tax schedule must be amended to obtain further fiscal resources to fund public expenditures. Indeed, under these circumstances, the present value of aggregate differential land rents denoted by ALR^d_∞ equals

$$\text{ALR}^d_\infty = \frac{1}{1-\delta}\int_0^{\widehat{x}^\star_\infty}\left[\frac{y^c_\infty(x)}{1-\lambda} - R_A\right]\mathrm{d}x + \frac{1}{1-\delta}\int_{\widehat{x}^*_\infty}^{L}\left[\Psi^d(x) - R_A\right]\mathrm{d}x, \tag{24}$$

where $\widehat{x}^*_\infty$ is defined so that $\Psi^d(\widehat{x}^\star_\infty) = y^c_\infty(\widehat{x}^\star_\infty)/(1-\lambda)$, and $y^c_\infty(x)$ is the wealth of credit-constrained agents when $y_{t+1} = (1+r)b_t + R_A$. Using $G/(1-\delta) = \text{ASLR}_\infty$ when $L = L^o$, we obtain $\text{ALR}^d_\infty = [G - \Theta(\widehat{x}^*_\infty)]/(1-\delta) < \text{ASLR}_\infty$ with

$$\Theta(\widehat{x}^*_\infty) \equiv \int_0^{\widehat{x}^*_\infty} \tau^\lambda_\infty(x)\mathrm{d}x \quad \text{where} \quad \tau^\lambda_\infty(x) \equiv \Psi^d_\infty(x) - \frac{y^c_\infty(x)}{1-\lambda}, \tag{25}$$

where $\tau^\lambda_\infty(x)$ corresponds to the "constraint rent", so that $G/(1-\delta) = \text{ALR}^d_\infty + \Theta(\widehat{x}^*_\infty)/(1-\delta)$. As a result, a single 100% land rent tax is not sufficient to finance public expenditures when the borrowing constraint is binding because the wealthiest agents capture a fraction of aggregate shadow land rents ($\Theta(\widehat{x}^\star_\infty)$), which is allocated to consumption c_t and the inheritance left y_{t+1}. To reach the first best, the policymaker could increase λ up to the point at which no household would be constrained. In this case, the rent function capitalizes any urban features, and taxing land asset value would be sufficient to finance public expenditures. Nevertheless, the borrowing constraint is implemented to manage financial market failures and is not a policy instrument to regulate land markets. In addition, the policymaker could also implement a tax on *donors*' lifetime wealth equal to the "constraint rent" $\tau^\lambda_\infty(x)$ for any $x \in [0, \widehat{x}^*_\infty]$, which varies with the wealth of agents residing in the more attractive sites. Combined with a tax on differential land rents received by donees, this tax schedule is optimal as the wealth of all agents converge to y^o_∞ and the present value of public expenditures equals the aggregate differential land rents ALR^d_∞, plus the present value of additional tax on lifetime wealth $\tau^\lambda_\infty(x)$. However, the tax on the "constraint rent" $\tau^\lambda_\infty(x)$ relies on the feasibility of individual-specific lump-sum taxes, and requires the government to know households' willingness to pay for location, which they might not be willing to reveal.

We now consider an approach under which the agents' willingness to pay for land is a private information, so that the policymaker has no information about the "constraint rent", while she knows the inherited wealth y_t and the equilibrium land prices ($p_t(x)$ and R_A). To ease the comparison with the decentralized economy presented in Section 4.1, we abstract from the Pigouvian tax on consumption τ^c_∞. Hence, we restrict the tax instruments to a tax on donees' inherited land assets $\tau^d_\infty(x) = \theta_\infty(x) - R_A$ as above, and a downpayment subsidy $\zeta(y_t, p_t)$ financed by a lump-sum tax which is not specific to individuals τ^s_∞. Precisely, the downpayment subsidy $\zeta(y_t, p_t)$ is such that, for any y_t

household and any land price level p_t, the borrowing constraint is binding. Formally,

$$\zeta(y_\infty, p_\infty) = \max\left\{p_\infty(1-\lambda) - y_\infty, 0\right\}, \tag{26}$$

where we focus on the long-run steady-state equilibrium of the economy. It turns out that no household is credit constrained and bid rents equal the instantaneous value of the shadow land price $\Psi^d(x)$. Hence, applying a 100% tax on the differential land rents received by donees suffices to finance public expenditures G.

The problem for the government is then to finance the downpayment subsidy, that is $\tau^s_\infty L^o = \int_0^{\widehat{x}^\star_\infty} \zeta(y_\infty, p_\infty)\mathrm{d}x$, where $\widehat{x}^*_\infty$ is such that $\zeta(y_\infty, p_\infty(\widehat{x}^\star_\infty)) = 0$. Using (26), the government budget constraint can be rewritten as:

$$\tau^s_\infty = \frac{1-\lambda}{L^o}\int_0^{\widehat{x}^\star_\infty}\left[\Psi^d(x) - \frac{y^s_\infty}{1-\lambda}\right]\mathrm{d}x < \frac{(1-\lambda)G}{L^o}, \tag{27}$$

where y^s_∞ is the wealth level of agents under this tax regime. Since no agent is borrowing-constrained under this taxation schedule, individual wealth converges to the same steady-state level implicitly given by:[24]

$$y^s_\infty = \frac{\beta\left(\varphi - \mathcal{K} - \tau^s_\infty\right)}{1-\beta}. \tag{28}$$

Using (13), (27), and (28), it is easy to check that $y^s_\infty > y^n_\infty$. This restricted tax schedule is welfare improving regarding to a lump-sum tax funding the public services as in Section 4.1, but does not achieve the steady state wealth level when the mortgage market is perfect.

To summarize,

Proposition 6

When the mortgage market is imperfect and the city population size is optimal, a tax schedule with a 100% inherited land wealth tax and a downpayment subsidy financed by a lump-sum tax is welfare improving and reduces wealth inequality.

Notice that if pursuing equality of opportunity is an important goal for policy, our model provides an argument for taxing wealth transfers between generations to reduce the advantage that some agents have from being born in a wealthy family. Borrowing constraints imply inequality in inherited wealth resulting from differences in opportunity. According to most theories of justice, it is unfair that two agents with the same behavior and characteristics (φ in our case) enjoy unequal welfare levels because one individual received a large inheritance while the other did not (see Fleurbaey, 2008).

[24]Under this configuration, the agents' steady-state wealth is given by the fixed point of $y_\infty = \beta[\varphi + y_\infty + \mu R_A - (R_A + \kappa(L^o)) - \tau^s_\infty]$.

5. Long-run wealth and taxation with endogenous prices of labor and capital

Thus far, we have assumed that agents are identical in their innate productivity, so that they earn the same wage. They may differ, however, in their investment in human capital due to educational investment constraints and differences in inherited wealth. Moreover, we have assumed that production occurs according to a linear technology and there is no capital. In this section, we discuss the implications of introducing (i) a difference in agents' investment in human capital due to educational investment constraints, and (ii) the production of consumption good and housing good depending on the stock of capital and endogenous labor and capital prices.

5.1. Skill premium and mortgage market

Assume that at the beginning of life, agents have to decide whether to invest in their own education before entering the labor and housing markets. Education increases their human capital, yielding more efficiency units of labor. If agents do not invest in education, they acquire one efficiency unit of labor (unskilled workers). Given that the wage rate per unit of efficient labor is φ, their labor income is $w = \varphi$ (as in Section 5.1). Alternatively, agents may invest in education at fixed cost ϕ. In this case, they supply $e > 1$ efficiency units of labor (skilled workers), and their labor income reaches $\varphi e > w$. Therefore, agents invest in education if both conditions $y_t > \phi$ and $\phi < \varphi(e-1) \equiv \phi^e$ are met.[25]

First, assume that there are no imperfections in the mortgage market. Under these circumstances, the bid-rent curves are given by $\Psi_t(x, \mathcal{K})$, which does not depend on wage and wealth, so there is no spatial wealth sorting. The dynamics for each dynasty's wealth in this economy are, therefore, given by the transition rule $y_{t+1} = \beta(\widetilde{w} + y_t - \mathcal{K})$, if $y_t < \phi$, and $y_{t+1} = \beta(\widetilde{w} + \phi^e + y_t - \phi - \mathcal{K})$, if $y_t > \phi$. Unsurprisingly, there are two equilibria, and long-run outcomes depend on the initial condition. The long-run wealth of agents with initial wealth lower than ϕ will converge to y^n_∞ given by (13), while the long-run wealth of those with greater inherited wealth will converge to $y^n_\infty + \beta(\phi^e - \phi)/(1-\beta)$. In this context, long-term wealth inequality is driven by educational investment constraints rather than the mortgage market, and depends on the initial condition. If all agents have the same initial wealth, there is no symmetry-breaking, and all agents become either skilled workers or unskilled workers. In addition, a single 100% land rent tax is sufficient to finance public expenditures.

Assume now that some unskilled workers face credit constraints when attempting to obtain a mortgage. We seek to build a steady state where there is spatial wealth sorting: cities host unconstrained skilled agents, unskilled agents who are credit constrained, and

[25] For simplicity and without loss of generality, agents with wealth $y_t \leqslant \phi$ cannot borrow to cover the costs of education.

unskilled agents who are unconstrained. In this case, the steady-state wealth of unskilled workers who are credit constrained is $y^c_\infty(x)$ (see (17)), and the steady-state wealth of unskilled workers who are unconstrained is y^n_∞ (see (13)). The residential area of the latter class is still $[\widehat{x}^\star_\infty, L]$, where $\widehat{x}^\star_\infty$ is given by (16). As $y^c_\infty(x)$ increases when the distance to the city center declines, a class of skilled workers emerges if the distance cutoff $\widehat{x}^e_\infty$, given by $y^c_\infty(\widehat{x}^e_\infty) = \phi$, belongs to $(0, \widehat{x}^\star_\infty)$. Hence, $\widehat{x}^e_\infty$ corresponds to the right endpoint of the residential area formed by agents investing in education who reside in the most attractive sites $[0, \widehat{x}^e_\infty]$. In Appendix B.5, we show that some workers invest in education and reside in the most attractive site $(0 < \widehat{x}^e_\infty < \widehat{x}^\star_\infty)$ when $\lambda < \lambda^e(\phi)$, and their wealth converges to y^e_∞, with

$$\lambda^e(\phi) \equiv 1 - \frac{\beta(1-\mu)}{(1-\beta)[\Lambda(\phi)-1]} \quad \text{and} \quad y^e_\infty \equiv \frac{\beta(\phi^e - \phi)}{1-\beta} + \phi, \tag{29}$$

where $\Lambda(\phi) \equiv \widetilde{w}\beta/[\phi(1-\beta)] > 1$.[26] Furthermore, at the steady state, the emergence of two types of unskilled agents (credit constrained and unconstrained) requires $(1-\lambda)R_A < y^n_\infty < (1-\lambda)\Psi_\infty(\widehat{x}^e_\infty, \mathcal{K})$ or, equivalently,

$$\underline{R}^e_A(\phi) \leqslant R_A \leqslant \overline{R}_A(\lambda) \quad \text{with} \quad \underline{R}^e_A(\phi) \equiv \frac{\widetilde{w} - \kappa(L)}{1-\mu} - \frac{\phi}{1-\mu}\frac{1-\beta}{\beta}, \tag{30}$$

and $\overline{R}_A$ is given by (15). In the presence of skilled workers, the unskilled workers who are unconstrained reside in remote areas $(\widehat{x}^\star_\infty, L)$ while the unskilled workers who are credit constrained live in $[\widehat{x}^e_\infty, \widehat{x}^\star_\infty]$ when $\underline{R}^e_A(\phi) \leqslant R_A \leqslant \overline{R}_A(\lambda)$. We summarize our findings in Figure 5 and in the following proposition:

Proposition 7

Assuming $\beta < 1$, $\lambda < \overline{\lambda}$, and $\phi < \phi^e$, the long run city is characterized by one of the following steady-state distributions:

(i) If $\underline{R}^e_A < R_A < \overline{R}_A$ and $\lambda < \min\{\lambda^e, \overline{\lambda}\}$, then three types of agents emerge in equilibrium: high-wealth agents (skilled workers), middle-wealth agents (credit-constrained unskilled workers), and low-wealth agents (unconstrained unskilled workers).

(ii) If $\overline{R}_A < R_A$ and $\lambda < \min\{\lambda^e, \overline{\lambda}\}$, then two types of agents emerge in equilibrium: skilled workers and credit-constrained unskilled workers.

(iii) If $R_A \leqslant \min\{\underline{R}^e_A(\phi), \underline{R}_A(\lambda)\}$ and $\lambda < \min\{\lambda^e(\phi), \overline{\lambda}\}$, then the long-run wealth inequality depends on the initial wealth distribution.

(iv) If $\underline{R}_A < R_A$ and $\lambda^e < \lambda < \overline{\lambda}$, then all agents are unskilled, and persistent inequality occurs.

(v) If $R_A < \underline{R}_A$ and $\lambda^e < \lambda < \overline{\lambda}$, then full convergence occurs.

[26] We do not study the case where a fraction of skilled workers are credit constrained.

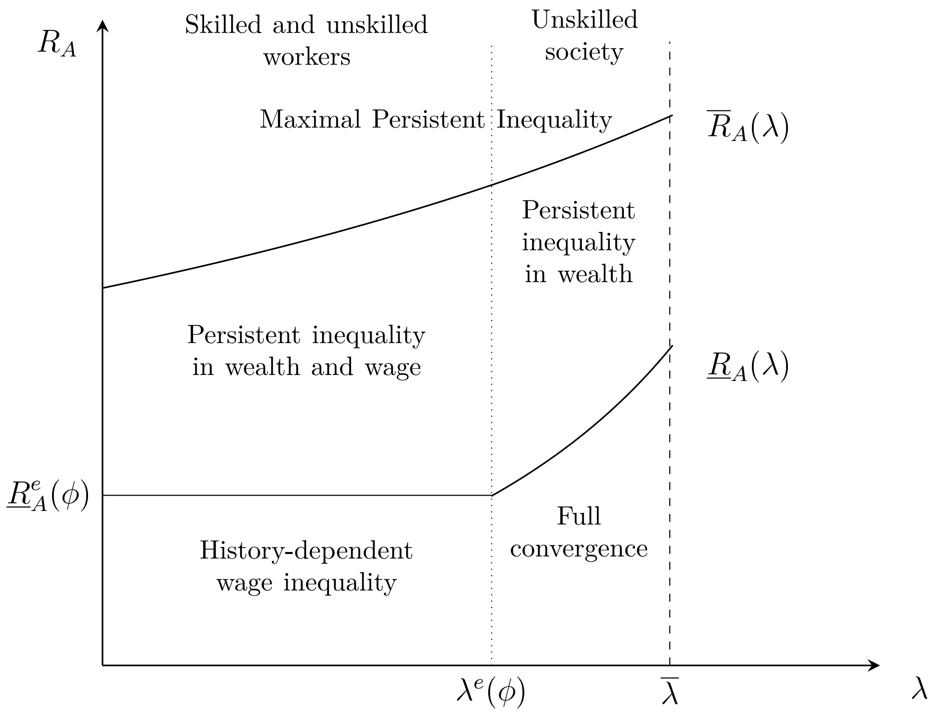

Figure 5: Wealth and wage inequalities in the (λ,R_A)-space

Therefore, mortgage market imperfections have an impact on occupational choices and spatial wealth sorting, as well as spatial skill sorting. The equilibrium labor income relies on the interplay between the inherited wealth distribution and access to mortgages. As in Sections 3 and 4, the credit constraint forces the partition of agents within the city according to the distribution of wealth y_t, which also depends on occupational choices. Agents living in an area close to the city center are wealthier than agents residing in areas farther away. There is perfect skill sorting and perfect wealth sorting. The wealthiest agents (skilled workers) live in the most attractive locations, while middle-class agents (lucky unskilled workers) are credit-constrained and can live in better places than the other unskilled workers. Skilled agents also benefit from the presence of constrained agents because they face less competition in the housing market and pay a lower housing price (see Appendix B.5). Hence, the emergence of wealthy classes who are not credit-constrained is not a transitory configuration, and their wealth converges to y^e_∞. Mortgage market imperfections magnify the wage inequality and make wealth more unequally distributed than labor income. Notice also that the long-run return to location x for skilled agents is higher than the return to financial asset because of the presence of credit-constrained unskilled workers (see Appendix B.5).

In our context, where occupational choices are affected by the distribution of wealth and mortgage market imperfections, taxation policy needs to be adjusted to reach the optimal outcome. Indeed, the presence of fixed costs associated with investment in education may generate an under-investment in human capital. The per capita production is now

$\varphi(e\sigma + 1 - \sigma)$, where σ is the share of skilled workers in the economy. Furthermore, some resources must now be allocated to the funding of per capita education expenditures equal to $\phi\sigma$. It is straightforward to verify that, when $\phi < \phi^e$, the optimal outcome is that all agents become skilled workers. Hence, Proposition 7 shows that the segmentation of the labor force between skilled and unskilled workers when mortgage market is imperfect differs from the efficient allocation. As a result, in the presence of mortgage market imperfections, wealth distribution has a long-lasting effect on aggregate income. To achieve the optimal solution, a 100% land rent received by donees and a tax on lifetime wealth of skilled workers and "lucky" unskilled workers (which amounts to the "constraint rent"), is required to finance non-education public expenditures, leading the wealth distribution to mirror the labor income distribution. To finance the education of all workers, a lump-sum tax ϕ can be optimally implemented. In this case, all agents become skilled workers, and their wealth converges to $y^n_\infty + \beta(\phi^e - \phi)/(1-\beta)$ (unskilled workers are better off while no agent is worse off).

5.2. Productive capital vs. housing capital

We now discuss the case in which the production of goods requires also capital, while the wage and interest rate are endogenous. The capital stock is assumed to adjust to the total savings in the economy at each period, which corresponds to the bequests of all parents, given by $L\int_{\underline{y}_t}^{\overline{y}_t} b_t(y)f(y_t)\mathrm{d}y_t$ (see also Chapter 9 in Acemoglu, 2009). Agents lend their savings b_t as capital to consumption good producers and real estate companies at period t, and their children receive the return at time $t+1$.

In each period, a representative firm located in each city produces the consumption good and uses two inputs, capital and labor. We assume a standard neoclassical production function that satisfies the Inada conditions, given by $g(k_t)$, where $g(.)$ is per capita production and k_t is the per capita capital stock used in the consumption good industry. As usual, the wage rate is $w_t = g(k_t) - \widetilde{r}_t k_t \equiv w(k_t)$, where $\widetilde{r}_t$ is the gross rate of return on capital ($\widetilde{r}_t \equiv 1 + r_t$). As the marginal productivity of capital equals its price in equilibrium, we have $\widetilde{r}_t = \partial g(.)/\partial k_t \equiv g_k(k_t)$.

In addition, at the beginning of each period, real estate companies must undertake maintenance worth $1/\varphi_h$ units of capital to produce one unit of housing. Without maintenance, the home is uninhabitable. As long as essential maintenance is performed, housing service is constant. The profit of the intermediary associated with location x is now $v_t(x) = p_t(x) - \widetilde{r}_t/\varphi_h - \theta_t(x)$. Free entry implies $\theta_t(x) = p_t(x) - \widetilde{r}_t/\varphi_h$ and $p_t(L) = R_A + \widetilde{r}_t/\varphi_h \equiv p_{A,t}$ at the city border. As usual, urban land value accounts for the value of the accessibility to jobs through $p_t(x)$, and depends on the cost of capital and the housing production technology through the value of φ_h. In accordance with empirical evidence, the relative share of land in the property price $\theta_t(x)/p_t(x) = 1 - \widetilde{r}_t/(\varphi_h p_t(x))$ decreases with distance to jobs and increases with housing industry productivity (φ_h), as $\widetilde{r}_t$ does not vary across space.

The per capita total capital stock at time $t+1$ is given by $k_{t+1} + 1/\varphi_h = i_t$, where i_t is the per capita investment at period t and capital is assumed to fully depreciate after use. The total investment is equal to the sum of individual savings/bequests at period t, so that $k_{t+1} + 1/\varphi_h = b_t$, where the per capita saving is given by $b_t = \int_{\underline{y}_t}^{\overline{y}_t} b_t(y) f(y_t) \mathrm{d}y_t$.

Efficient allocation of resources between productive capital and consumption. For a direct comparison of the model with physical capital and the model developed above, we still assume that the bequest received by the next generation and wages are determined by competitive markets, where we now have $b_t = \widetilde{r}_t b_{t-1}$ and $w_t = w(k_t)$. Note also that the optimal value of the land property right is still equal to the opportunity cost of land R_A for the same reasons described above. Hence, $y_{t+1} = \widetilde{r}_{t+1} b_t + R_A$, and the per capita resource constraint is now given by:

$$c_t = g(k_t) - b_t - \Omega_t(L_t) \quad \text{with} \quad \Omega_t(L_t) \equiv G/L_t + \mathcal{T}(L_t)/L_t, \tag{31}$$

as $w(k_t) + y_t - \widetilde{r}_t/\varphi_h - R_A = w(k_t) + \widetilde{r}_t b_{t-1} - \widetilde{r}_t/\varphi_h = w(k_t) + \widetilde{r}_t k_t = g(k_t)$. Agents are workers and capital holders. The city government sets the bequest b_t that maximizes $\sum_{t=0}^{\infty} \delta^t u(c_t, y_{t+1})$ using $\widetilde{r}_{t+1} = g_k(k_{t+1})$ and $k_{t+1} = b_t - 1/\varphi_h$, which implies that the optimal levels of consumption c_t^o and wealth left to heirs y_{t+1}^o are such that:

$$u_c(c_t^o, y_{t+1}^o) = u_y(c_t^o, y_{t+1}^o)(1-\eta_t)\widetilde{r}_{t+1} + \delta g_k(k_{t+1}) u_c(c_{t+1}^o, y_{t+2}^o), \tag{32}$$

where $\eta_t \equiv -\frac{\partial g_k(b_t)}{\partial b_t}\frac{b_t}{g_k(b_t)} > 0$ is the elasticity of the interest rate to a change in the productive capital stock (note that $g_{kk}(k_{t+1}) < 0$). The difference between (20) and (32) arises because the city government internalizes the impact of bequest b_t on future prices of capital (captured through η_t).

The population size that maximizes the utility level of the city's residents, L_t^o, is implicitly given by:

$$G = \kappa(L_t^o) L_t^o - \mathcal{T}(L_t^o) + g_k(k_t) k_t L_t^o. \tag{33}$$

An increase in population size has now an impact on per capita production, which is negative because of diseconomies scale in the production of the consumption good. As the marginal productivity of labor is decreasing, an additional agent in a city has a negative impact on the revenue received by each incumbent $g(k_t)$. As a result, each city must charge a fee equal to the "crowding" cost generated by an additional resident and imposed on all users ($g_k(k_t)k_t$).[27] Hence, when the city size is optimal, public expenditure equals the aggregate differential land rent, plus the optimal fee collected from all agents. It is straightforward to check that the optimal population size adjusts negatively with the stock of capital ($\partial L_t^o/\partial k_t < 0$) as $g_{kk}k_t + g_k(k_t) > 0$. In addition, (33) yields $g(k_t) - \Omega_t(L_t^o) = w_t(k_t) - \kappa(L_t^o)$, which is increasing with k_t.

We confine the analysis to steady states. Given the Cobb-Douglas utility function and (32), the optimal consumption c_∞^o and inherited wealth y_∞^o at steady state are such that $c_\infty^o = \widetilde{\alpha}(b_\infty)y_\infty^o/\widetilde{r}_\infty^o$, and given (31) and $y_\infty^o/\widetilde{r}_\infty^o = b_\infty + R_A/g_k(b_\infty^o)$, the capital stock at the steady state is implicitly given by

$$b_\infty^o = \frac{\mathcal{W}_\infty^o}{1+\widetilde{\alpha}(b_\infty^o)} - \frac{R_A}{g_k(b_\infty^o)}, \tag{34}$$

with

$$\widetilde{\alpha}(b_\infty^o) \equiv \frac{1-\alpha}{\alpha}\frac{1-\delta g_k(b_\infty^o)}{1-\eta(b_\infty^o)} > 0 \quad \text{and} \quad \mathcal{W}_\infty^o \equiv w(k_\infty^o) - \kappa(L_\infty^o) + \frac{R_A}{g_k(b_\infty^o)}.$$

We assume that $b_\infty^o > 1/\varphi_h$. The level of lifetime wealth at the steady state $\mathcal{W}_\infty^o$ increases with b_∞^o as $g_{kk}(b_\infty^o) < 0$ and $w(k_\infty^o) - \kappa(L_\infty^o)$ increases with b_∞^o. The share of lifetime wealth allocated to saving $b_\infty^o/\mathcal{W}_\infty^o$ is not constant because the price of capital is endogenous and preferences are not homogenous of degree 1 in c and b when $R_A > 0$. It is easy to check that a higher opportunity cost of land reduces capital accumulation. We do not discuss here the existence and stability of the steady states. Rather, we determine whether an optimal stationary allocation, which is feasible in the long run, can be decentralized when the city size is set to L_t^o.

Decentralization and taxation with productive capital. Under a free land market, the budget constraint of an agent living at x is $c_t = w(k_t) - G/L_t^o + y_t - b_t - \kappa(x) - p_t(x)$, while the wealth left to offspring remains $y_{t+1} = \widetilde{r}_{t+1}b_t + \theta_{t+1}(x)$. Assuming that mortgage market is perfect, the bid rent is now $p_t(x) = \mathcal{K}_t - \kappa(x) + \theta_{t+1}(x)/\widetilde{r}_{t+1}$ for any x, so that $\theta_t(x) = \mathcal{K}_t - \kappa(x) + \theta_{t+1}(x)/\widetilde{r}_{t+1} - \widetilde{r}_t/\varphi_h$ with $\mathcal{K}_t = \kappa(L) + r_{t+1}R_A/\widetilde{r}_{t+1} + \widetilde{r}_t/\varphi_h$ (since $\theta_t(L) = R_A$). Bequests and consumption depend on the sequence of all current and future prices $\{w_\varsigma, \widetilde{r}_{\varsigma+1}, \theta_{\varsigma+1}\}_{\varsigma=t}^{\varsigma}$.

The equilibrium levels of consumption c_∞ and wealth left to heirs y_∞ at the steady state are such that $c_\infty/y_\infty = [(1-\alpha)/\alpha]/g_k(b_\infty)$ when agents are endowed with the Cobb-Douglas utility function. Because capital stock is the outcome of decisions made by myriad agents

[27] In this context, a city can be viewed as a "production club", very much as a city is considered as a "consumption club" with a congestible public good. The optimal fee here depends only on the mass of residents living in a city, and not their residential location within the city (anonymous crowding).

belonging to the previous generations, we face a potential pecuniary externality because the actions of agents living at period t affect the welfare of the next generation through the future prices of capital $\widetilde{r}_{t+1}$, which depend on b_t. Since $\partial\widetilde{r}_{t+1}/\partial b_t < 0$, the presence of altruistic preferences generates a negative pecuniary externality through the wealth left to offspring y_{t+1}. As in overlapping generation models, dynamic inefficiency arises when the interest rate is endogenously determined (see Acemoglu, 2009). Hence, free markets involve misallocation of resources between bequest and consumption. Again, to reach the efficient allocation of resources between consumption and intergenerational transfer, the city government has to implement a standard Pigouvian tax on consumption, which is now given by $\tau^c_\infty = [\delta g_k(b_\infty) - \eta(b_\infty)]/[1 - \delta g_k(b_\infty)]$.

In Appendix B.6, we show that if the Pigouvian tax on consumption is implemented such that the ratio $c_t/(y_{t+1}/\widetilde{r}_{t+1})$ in equilibrium is equal to $c^o_\infty/(y^o_\infty/\widetilde{r}^o_\infty) = \widetilde{\alpha}(b^o_\infty)$, then the per capita capital stock in period $t+1$, given by $\frac{1}{L^o_t}\int_0^{L^o_t} b_t(x)\mathrm{d}x \equiv \bar{b}_t$,

$$\bar{b}_t = \frac{1}{1+\widetilde{\alpha}(b^o_\infty)}\left[w(k_t) - \kappa(L^o_t) + \frac{R_A}{\widetilde{r}_{t+1}}\right] - \frac{R_A}{\widetilde{r}_{t+1}} - \frac{\widetilde{\alpha}(b^o_\infty)}{1+\widetilde{\alpha}(b^o_\infty)}\frac{1}{L^o_t}\int_0^{L^o_t}\frac{\theta_{t+1}(x) - R_A}{\widetilde{r}_{t+1}}\mathrm{d}x. \quad (35)$$

Clearly, if $\theta_{t+1}(x) = R_A$ at the steady state, then $\bar{b}_\infty = b^o_\infty$ as $r_{t+1} = g_k(b_t)$. However, under land competition with altruistic agents, we have $\theta_{t+1}(x) > R_A$, implying $\bar{b}_\infty < b^o_\infty$. In other words, starting from the optimal stock of capital, the *marginal* impact of land competition with altruistic agents implies a lower stock of capital. The presence of the land asset in inherited wealth yielding a transfer of rents $(\theta_{t+1}(x) - R_A)$ between generations implies an *underaccumulation* of productive capital. As in the context without productive capital, the optimum decentralization is possible with a tax paid by donees on inherited land assets so that $y_{t+1} = \widetilde{r}_{t+1}b_t + R_A$ and capital stock increases. It is worth stressing that a single 100% tax on differential land rent received by donees decreases the *wealth-income* ratio, given by $y_t/g(k_t)$, as y_t decreases while $g(k_t)$ increases.

Proposition 8

A single 100% tax on differential land rent received by donees allows a better allocation of resources between housing and productive investment, and decreases the wealth-income *ratio.*

If a fraction of the population becomes credit constrained, then the voluntary bequests left by credit-constrained parents increase. Hence, the credit constraint causes upward pressure on the accumulation of capital. The positive effect of the credit constraint offsets, or may reverse, the negative effects of land rents on the accumulation of productive capital. The role of credit limits has been extensively discussed in the literature on macroeconomic fluctuations since Kiyotaki and Moore (1997). Empirical evidence shows a positive causal effect of house prices on household spending (*e.g.* Andersen and Leth-Petersen, 2020) and on corporate investment through the collateral value (*e.g.*, Chaney *et al.*, 2012). As real estate assets constitute a significant share of the pledgeable assets owned by firms, higher real estate prices are expected to relax the borrowing constraint of firms and, in turn, increase their productive investments (collateral channel). We identify a new channel. By making land competition across households less fierce, the credit constraint favors lower house prices and, in turn, yields more savings and investment opportunities (cost channel).

6. Concluding remarks

This paper presents a model of the transmission of wealth from parents to children. The key feature is that inherited wealth depends on parents' residential choices, which are themselves impacted by a borrowing constraint that limits agents' access to credit. Given that the borrowing constraint ties agents' bid-rent function to their wealth, wealthy agents can outbid the rest of the population in the most attractive locations, leading to *spatial wealth sorting*. We characterize the residential equilibrium for any given wealth distribution, showing that the borrowing constraint strongly impacts the pattern of price formation and wealth dynamics. We also show that in any equilibrium, bequests are location-dependent: Agents living closer to attractive locations leave a higher bequest than agents living farther away, and spatial sorting translates into persistent inequality. Our model also provides an argument for taxation on land assets paid by donees that improves efficiency and reduces wealth inequality.

Our main results are robust to various extensions of the model. First, when considering occupational choices, skilled and unskilled agents are sorted across space, leading to long-run wealth inequality. Second, when the economy accumulates physical capital, taxation on differential land rent alleviates under-accumulation of productive capital due to agents' willingness to transmit land assets. Third, when real estate companies can rent housing units instead of selling them to constrained agents, spatial sorting and symmetry-breaking can occur in areas where real estate companies still find it profitable to sell to constrained agents. Fourth, when housing demand is endogenous, the borrowing constraint makes land competition less fierce, and generates wealth inequalities. Finally, a housing bubble makes housing more expensive, increasing the need to borrow, and relaxes the constraints as the pledgeable value of investment increases. We claim that the spatial sorting result and its long-run effect on wealth inequality hold.

The predictions of our framework can be tested in various directions. First, our model suggests that any shock affecting housing wealth in a specific location can have long-lasting implications for future generations. Such shocks impact the inheritance left to subsequent generations, their location choices, and the economic rent they can extract from living in desirable areas. Hence, spatial inequality amplifies wealth inequality. A fruitful line of inquiry for empirical research is to assess the long-run implications of exogenous local shocks impacting the parents' wealth on the descendants' wealth (*e.g.*, urban disasters such as hurricanes, great fires, earthquakes, or unanticipated place-based policies). Second, our model bears some empirical implications for the estimation of the urban rent gradient. The empirical literature finds that transportation costs, city size, and income are key determinants of the price of housing (*e.g.*, Combes *et al.*, 2019). Our model shows that the level of wealth should be considered when estimating housing/land prices because of borrowing constraints.

Finally, our theoretical setup could be expanded in various directions. For example, our paper does not address the important issue of endogenous local public goods and segregation. Land and housing values reflect the quality of local public goods, which depends on the socioeconomic composition of the neighborhood. A thorough theoretical analysis of the impact of segregation on housing values and wealth dynamics would be a fruitful line of research. In addition, land use and building regulations have substantial impacts on land and housing values, and thus on the demand for mortgage credit. Our approach could be extended to shed light on how the interaction between land use, building regulations, and mortgage imperfections drives spatial inequality and wealth dynamics.

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth.* Princeton University Press.

Acolin, A., Bricker, J., Calem, P., and Wachter, S. (2016). Borrowing Constraints and Homeownership. *American Economic Review*, 106(5): 625–629.

Albouy, D., Ehrlich, G., and Shin, M. (2018). Metropolitan Land Values. *The Review of Economics and Statistics*, 100(3): 454–466.

Andersen, H. Y. and Leth-Petersen, S. (2020). Housing wealth or collateral: How home value shocks drive home equity extraction and spending. *Journal of the European Economic Association*, 19(1): 403–440.

Arnott, R. J. and Stiglitz, J. E. (1979). Aggregate Land Rents, Expenditure on Public Goods, and Optimal City Size. *The Quarterly Journal of Economics*, 93(4): 471–500.

Banerjee, A. V. and Newman, A. F. (1993). Occupational Choice and the Process of Development. *Journal of Political Economy*, 101(2): 274–298.

Bénabou, R. (1996). Equity and Efficiency in Human Capital Investment: The Local Connection. *The Review of Economic Studies*, 63(2): 237–264.

Benhabib, J. and Bisin, A. (2018). Skewed Wealth Distributions: Theory and Empirics. *Journal of Economic Literature*, 56(4): 1261–1291.

Benhabib, J., Bisin, A., and Luo, M. (2019). Wealth Distribution and Social Mobility in the US: A Quantitative Approach. *American Economic Review*, 109(5): 1623–1647.

Benhabib, J., Bisin, A., and Zhu, S. (2011). The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents. *Econometrica*, 79(1): 123–157.

Bilal, A. and Rossi-Hansberg, E. (2021). Location as an Asset. *Econometrica*, 89(5): 2459–2495.

Bonnet, O., Chapelle, G., Trannoy, A., and Wasmer, E. (2021). Land is back, it should be taxed, it can be taxed. *European Economic Review*, 134: 103696.

Causa, O., Woloszko, N., and Leite, D. (2019). Housing, wealth accumulation and wealth distribution: Evidence and stylized facts. Technical report, OCDE, Paris.

Chamley, C. (1986). Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica*, 54(3): 607–622.

Chaney, T., Sraer, D., and Thesmar, D. (2012). The Collateral Channel: How Real Estate Shocks Affect Corporate Investment. *American Economic Review*, 102(6): 2381–2409.

Combes, P.-P., Duranton, G., and Gobillon, L. (2019). The Costs of Agglomeration: House and Land Prices in French Cities. *The Review of Economic Studies*, 86(4): 1556–1589.

Deaton, A. and Laroque, G. (2001). Housing, Land Prices, and Growth. *Journal of Economic Growth*, 6(2): 87–105.

Diamond, R. (2016). The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000. *American Economic Review*, 106(3): 479–524.

Drazen, A. and Eckstein, Z. (1988). On the Organization of Rural Markets and the Process of Economic Development. *American Economic Review*, 78(3): 431–443.

Dynan, K. E., Skinner, J., and Zeldes, S. P. (2002). The Importance of Bequests and Life-Cycle Saving in Capital Accumulation: A New Answer. *American Economic Review*, 92(2): 274–278.

Eeckhout, J., Pinheiro, R., and Schmidheiny, K. (2014). Spatial Sorting. *Journal of Political Economy*, 122(3): 554–620.

Fagereng, A., Guiso, L., Malacrino, D., and Pistaferri, L. (2020). Heterogeneity and Persistence in Returns to Wealth. *Econometrica*, 88(1): 115–170.

Farhi, E. and Werning, I. (2007). Inequality and Social Discounting. *Journal of Political Economy*, 115(3): 365–402.

Fleurbaey, M. (2008). *Fairness, Responsibility, and Welfare*. OUP Oxford.

Fujita, M. and Thisse, J.-F. (2002). *Economics of Agglomeration: Cities, Industrial Location, and Regional Growth*. Cambridge University Press.

Gaigné, C., Koster, H. R. A., Moizeau, F., and Thisse, J.-F. (2022). Who lives where in the city? Amenities, commuting and income sorting. *Journal of Urban Economics*, 128: 103394.

Galor, O. and Zeira, J. (1993). Income Distribution and Macroeconomics. *The Review of Economic Studies*, 60(1): 35–52.

Garbinti, B., Goupille-Lebret, J., and Piketty, T. (2020). Accounting for wealth-inequality dynamics: Methods, estimates, and simulations for france. *Journal of the European Economic Association*, 19(1): 620–663.

Handbury, J. (2021). Are Poor Cities Cheap for Everyone? Non-Homotheticity and the Cost of Living Across U.S. Cities. *Econometrica*, 89(6): 2679–2715.

Henderson, J. V. and Ioannides, Y. M. (1983). A Model of Housing Tenure Choice. *The American Economic Review*, 73(1): 98–113.

Insee (2018). Présentation statistique - Enquête Histoire de vie et patrimoine 2017-2018.

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, 28(1): 59–83.

Kahneman, D., Krueger, A. B., Schkade, D. A., Schwarz, N., and Stone, A. A. (2004). A Survey Method for Characterizing Daily Life Experience: The Day Reconstruction Method. *Science*, 306(5702): 1776–1780.

Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2): 211–248.

Knoll, K., Schularick, M., and Steger, T. (2017). No Price Like Home: Global House Prices, 1870-2012. *American Economic Review*, 107(2): 331–353.

Kuhn, M., Schularick, M., and Steins, U. I. (2020). Income and Wealth Inequality in America, 1949–2016. *Journal of Political Economy*, 128(9): 3469–3519.

Matsuyama, K. (2000). Endogenous Inequality. *The Review of Economic Studies*, 67(4): 743–759.

Matsuyama, K. (2006). The 2005 Lawrence R. Klein Lecture: Emergent Class Structure. *International Economic Review*, 47(2): 327–360.

Michel, P. and Pestieau, P. (2004). Fiscal Policy in an Overlapping Generations Model with Bequest-as-Consumption. *Journal of Public Economic Theory*, 6(3): 397–407.

Mieszkowski, P. and Zodrow, G. R. (1989). Taxation and The Tiebout Model: The Differential Effects of Head Taxes, Taxes on Land Rents, and Property Taxes. *Journal of Economic Literature*, 27(3): 1098–1146.

Mookherjee, D. and Ray, D. (2002). Is Equality Stable? *American Economic Review*, 92(2): 253–259.

Mookherjee, D. and Ray, D. (2003). Persistent Inequality. *The Review of Economic Studies*, 70(2): 369–393.

Mookherjee, D. and Ray, D. (2010). Inequality and Markets: Some Implications of Occupational Diversity. *American Economic Journal: Microeconomics*, 2(4): 38–76.

Ortalo-Magné, F. and Rady, S. (2006). Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints. *The Review of Economic Studies*, 73(2): 459–485.

Piazzesi, M. and Schneider, M. (2016). Housing and Macroeconomics. In *Handbook of Macroeconomics*, Taylor, J. B. and Uhlig, H. (eds.), volume 2, *Elsevier*, : 1547–1640.

Piketty, T. and Saez, E. (2013). Optimal Labor Income Taxation. In *Handbook of Public Economics*, Auerbach, A. J., Chetty, R., Feldstein, M., and Saez, E. (eds.), volume 5, *Elsevier*, : 391–474.

Piketty, T. and Zucman, G. (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *The Quarterly Journal of Economics*, 129(3): 1255–1310.

Proost, S. and Thisse, J.-F. (2019). What Can Be Learned from Spatial Economics? *Journal of Economic Literature*, 57(3): 575–643.

Rosenthal, S. S., Duca, J. V., and Gabriel, S. A. (1991). Credit rationing and the demand for owner-occupied housing. *Journal of Urban Economics*, 30(1): 48–63.

Scheuer, F. and Slemrod, J. (2021). Taxing Our Wealth. *Journal of Economic Perspectives*, 35(1): 207–230.

Schwerhoff, G., Edenhofer, O., and Fleurbaey, M. (2020). Taxation of Economic Rents. *Journal of Economic Surveys*, 34(2): 398–423.

Stein, J. C. (1995). Prices and Trading Volume in the Housing Market: A Model with Down-Payment Effects. *The Quarterly Journal of Economics*, 110(2): 379–406.

Stiglitz, J. E. (2015). New Theoretical Perspectives on the Distribution of Income and Wealth among Individuals: Part IV: Land and Credit. NBER Working papers n. 21192.

Appendix A The social structure of cities under Pareto wealth distribution and linear commuting cost

We illustrate the model by developing an example in which commuting costs are linear and increasing with distance and wealth y_t is Pareto distributed, truncated to the support $[\underline{y}_t, \overline{y}_t]$ with shape parameters $\omega > 0$: $F_t(y) = [1-(y/\underline{y}_t)^{-\omega}]/\phi_t$ with $\phi_t \equiv 1-(\overline{y}_t/\underline{y}_t)^{-\omega} \in (0,1]$. A low value of ω means that the wealth distribution is close to uniform among agents, whereas that distribution gets more and more skewed towards low-wealth agents for larger values of ω. In addition, ϕ_t increases with $\overline{y}_t/\underline{y}_t$. In addition, when perfect sorting occurs, we can deduce that $y_t^\star(x)$ is decreasing and *convex* since the cumulative distribution function of a Pareto variable is an increasing and concave function. From Proposition 1, assortative matching requires that the x% of the wealthiest agents be matched with the x% of the least distant locations. Formally, the matching function $y_t(x)$ can be retrieved from the following condition:

$$\int_{y_t}^{\overline{y}_t} f_t(y)\mathrm{d}y = \frac{1}{L}\int_0^x \mathrm{d}x$$

The proportion of the population with wealth greater than or equal to y_t being $1-F_t(y_t)$, a simple calculation then shows that the equilibrium wealth mapping is such that

$$y_t^\star(x) = \underline{y}_t\left[\frac{1}{1-\phi_t+(\phi_t/L)x}\right]^{1/\omega}. \tag{A.1}$$

Consider the case where the city structure is characterized by three areas, each one hosting agents with a particular status, constrained or non-constrained. The poorest individuals who are assumed to not face any borrowing constraint live at the least attractive places, *i.e.* the city fringe $[\widehat{x}_t^2, L]$ and pay $\Psi_t(x, \mathcal{K})$ where the threshold location $\widehat{x}_t^2$ is such that $y_t^\star(\widehat{x}_t^2) = (1-\lambda)\Psi_t(\widehat{x}_t^2, \mathcal{K})$. The richest agents can afford to live in the most attractive locations $[0, \widehat{x}_t^1]$ without being borrowing contrained while the intermediate-wealth agents, the only agents who are borrowing constrained, reside in the area $[\widehat{x}_t^1, \widehat{x}_t^2]$ and pay $y_t^\star(x)/(1-\lambda)$. The house price paid by the wealthiest agents is given by the bid rent $\Psi_t^1(x, K_t^1) = K_t^1 - \Upsilon_t(x)$ where K_t^1 is obtained from $K_t^1 - \Upsilon_t(\widehat{x}_t^1) = y_t^\star(\widehat{x}_t^1)/(1-\lambda)$ and the location cutoff $\widehat{x}_t^1$ is such that the marginal agent endowed with wealth $\widehat{y}_t^1 \equiv y_t^\star(\widehat{x}_t^1)$ cannot outbid credit-constrained agents, *i.e.*

$$V[\Psi_t^1(x, K_t^1), \widehat{y}_t^1] \geqslant V[y_t^\star(x)/(1-\lambda), \widehat{y}_t^1]$$

for any $x \in [\widehat{x}_t^1, \widehat{x}_t^2]$ where $V(.)$ is the indirect utility. This condition holds if $y_t^{\star\prime}(x) = -(1-\lambda)\Upsilon_t'(x)$ evaluated at $\widehat{x}_t^1$ and $y_t^\star(x)$ is strictly convex. Stated differently, *the gain in housing expenditures by marginally moving further from the CBD and paying a lower rent*

$y_t^\star(x)/(1-\lambda)$ *must be lower than additional costs associated with distance in equilibrium.* Given Pareto wealth distribution, $y_t^{\star\prime}(x) = -(1-\lambda)\Upsilon_t'(x)$ becomes

$$\frac{\phi_t(\widehat{y}_t^1)^{1+\omega}}{\omega L \underline{y}_t^\omega} = (1-\lambda)\Upsilon_t'(\widehat{x}_t^1).$$

Since $\Upsilon(x)$ increases linearly with distance while $y_t^\star(x)$ is decreasing and convex, there is a single solution (see Figure 3). As a consequence, $\Psi_t^1(x, K_t^1)$ is the tangent to the curve $y_t^\star(x)/(1-\lambda)$ at $\widehat{x}_t^1$. Moreover, it is below the curve $y_t^\star(x)/(1-\lambda)$ and parallel to $\Psi_t(x, \mathcal{K})$ (see Figure 3). Using the equilibrium condition for which the utility level of the individual living in $\widehat{x}_t^1$ is such that $V(\Psi_t^1, \widehat{y}_t^1) = V(\widehat{y}_t^1/(1-\lambda), \widehat{y}_t^1)$, we have $\Psi_t^1(\widehat{x}_t^1, K_t^1) = \widehat{y}_t^1/(1-\lambda)$ and $\Psi_t^1(x, K_t^1) < \Psi_t(x, \mathcal{K})$ so that $K_t^1 < \mathcal{K}$. (see Figure 3). Further, for any $x \in [0, \widehat{x}_t^1]$, we have $p_t^\star(x) = \Psi_t^1(x, K_t^1) < y_t/(1-\lambda)$ for all y_t agents with $y_t \in [0, \widehat{y}_t^1]$, implying that they are not borrowing-constrained. *Even though the wealthiest agents are not credit-constrained, they gain from credit market imperfections* as their housing expenditures is reduced by the amount $\mathcal{K} - K_t^1$. Note that if $\widehat{x}_t^1$ does not exist, the city structure would be characterized by two areas: the wealthiest agents with $y_t \geqslant \widehat{y}_t^2$ reside at $x \in [0, \widehat{x}_t^2]$ and are borrowing constrained and the poorest agents live in $x \in [\widehat{x}_t^2, L]$ and are not constrained.

Appendix B Long-run land price and wealth

B.1 Rent dynamics and long-run wealth for non-constrained dynasties

We define a non-constrained dynasty a sequence of generations living at the city fringe such that the active agent and all her descendants are never constrained. Consider that these agents live at locations x where the rent is $\Psi_t(x, \mathcal{K})$. Let us consider the dynamics of land price $p_t(x) = \Psi_t(x, \mathcal{K})$ expressed as follows

$$p_t(x) = \mathcal{K} - \kappa(x) + \mu p_{t+1}(x), \tag{B.2}$$

where $\mathcal{K} \equiv \kappa(L) + (1-\mu)R_A$. We consider that there are some locations x such that these dynamics hold at any date t. In particular, if at some date t, $\underline{y}_t > R_A(1-\lambda)$, and if the sequence $\{\underline{y}_\varsigma\}_{\varsigma=t}^{\infty}$ is monotonously increasing, then these rent dynamics prevail at the city fringe. In this case, the rent can thus be solved by iterating forward the system. Hence,

$$p_t(x) = \lim_{\varsigma \longrightarrow \infty} \mu^{\varsigma+1} p_{t+1+\varsigma}(x) + \lim_{\varsigma \longrightarrow \infty} \sum_{j=0}^{\varsigma} \mu^j \left[\mathcal{K} - \kappa(x)\right].$$

In order to find a solution, we abstract from the presence of any housing bubble and we impose a transversality condition, that is, $\lim_{t\rightarrow\infty} p_t(x) < \infty$. As $\mu < 1$, we obtain

$$p_\infty(x) = \frac{\mathcal{K} - \kappa(x)}{1-\mu}. \tag{B.3}$$

Given (12), if $\beta < 1$, then the wealth of non-constrained dynasties living at the city fringe converges to

$$y_\infty = y_\infty^n \equiv \frac{\beta(\widetilde{w} - \mathcal{K})}{1-\beta}. \tag{B.4}$$

At the steady state, these dynasties reside at $x \in [\widehat{x}_\infty, L]$, where $\widehat{x}_\infty$ is given by $(1-\lambda)\, p_\infty(\widehat{x}_\infty) = y_\infty^n$. Note that if $\widehat{x}_\infty$ exists, it is necessarily unique. By rearranging terms, we finally obtain

$$\kappa(\widehat{x}_\infty) = \mathcal{K}\left(1 + \frac{1}{\rho}\right) - \frac{\widetilde{w}}{\rho} \quad \text{with} \quad \rho \equiv \frac{(1-\beta)(1-\lambda)}{\beta(1-\mu)}. \tag{B.5}$$

B.2 Long run wealth and rent of constrained dynasties

Consider now constrained dynasties. They pay the rent $p_t^\star(x) = y_t^\star(x)/(1-\lambda)$ at each t. Thus, the dynamics of the equilibrium rent $p_t^\star(x)$ follows the dynamics of $y_t^\star(x)$. From (12), constrained agents follow the dynamics:

$$y_{t+1}(x) = \beta \left\{ \widetilde{w} - \kappa(x) + \left(y_t^\star(x) - \frac{y_t^\star(x)}{1-\lambda} \right) + \mu \frac{y_{t+1}^\star(x)}{1-\lambda} \right\}.$$

As we assume that bequest is positive, we have $w - \kappa(x) + y_t^\star(x) - \frac{y_t^\star(x)}{1-\lambda} > 0$. Hence, in order for the above equation to hold for $y_{t+1} > 0$ a necessary condition is that

$$1 - \frac{\beta\mu}{1-\lambda} > 0 \Leftrightarrow 1 - \beta\mu > \lambda. \tag{B.6}$$

Rearranging terms leads to

$$y_{t+1}(x) = \frac{\beta(1-\lambda)}{1-\lambda-\beta\mu} \left[\widetilde{w} - \kappa(x) - \frac{\lambda y_t(x_t)}{1-\lambda} \right].$$

If

$$1 > \frac{\beta\lambda}{1-\lambda-\beta\mu} \Leftrightarrow \lambda < \frac{1-\beta\mu}{1+\beta} \tag{B.7}$$

then the wealth converges to $y_\infty^c(x)$ with

$$y_\infty^c(x) \equiv \frac{\beta\left[\widetilde{w} - \kappa(x)\right]}{(1-\beta)(1+1/\rho)} > 0. \tag{B.8}$$

These households live in the area $[0, \widehat{x}_\infty]$, with $\widehat{x}_\infty$ defined by $y_\infty(\widehat{x}_\infty) = (1-\lambda)\Psi_\infty(\widehat{x}_\infty, \widehat{K}_\infty)$ and it can be checked that it leads to (B.5).

The long-run return to location x for constrained agents is

$$\varrho_\infty^c(x) = \frac{\frac{y_\infty^c(x)}{1-\lambda} - R_A}{\frac{y_\infty^c(x)}{1-\lambda} + \kappa(x) - [R_A + \kappa(L)]} \tag{B.9}$$

where $\varrho_\infty^c(x) > 1 + r$ is equivalent to

$$\frac{(1-\mu)R_A + \kappa(L) - \kappa(x)}{1-\mu} > \frac{y_\infty^c(x)}{1-\lambda}. \tag{B.10}$$

Because the left-hand side of this inequality is equal to $\Psi_\infty(x, \mathcal{K})$, which is higher than $y_\infty^c(x)/(1-\lambda)$, $\varrho_\infty^c(x)$ is higher than $1 + r$ when agents are credit constrained.

B.3 Long-run urban configurations

Claim *In the long run, (i) non-constrained agents always live further away than constrained agents and (ii) there can be at most one threshold location* $\widehat{x}_\infty$*.*

(i) By contradiction, assume a long run urban equilibrium with non-constrained agents living in $[0, \widehat{x}_\infty]$ and constrained-agents living in $[\widehat{x}_\infty, L]$. The non-constrained agents would pay the rent $\Psi_\infty(x, \widehat{K}_\infty)$ with $\widehat{K}_\infty$ such that $y_\infty(\widehat{x}_\infty) = \Psi_\infty(\widehat{x}_\infty, \widehat{K}_\infty)(1-\lambda)$ and, given the dynamics (12), they would all end up with the same long-run wealth level $y^n_\infty = y_\infty(\widehat{x}_\infty) = \Psi_\infty(\widehat{x}_\infty, \widehat{K}_\infty)(1-\lambda)$. By assumption, for any $x \in [0, \widehat{x}_\infty[$, $y^n_\infty > \Psi_\infty(x, \widehat{K}_\infty)(1-\lambda)$ which is a contradiction as Ψ_∞ is stricly decreasing with x.

(ii) By contradiction, assume, w.l.o.g., a long run urban equilibrium with 3 threshold-locations denoted by $\widehat{x}^1_\infty$, $\widehat{x}^2_\infty$, $\widehat{x}^3_\infty$ such that there are constrained residents in $[0, \widehat{x}^1_\infty]$ and $[\widehat{x}^2_\infty, \widehat{x}^3_\infty]$ and non-constrained agents in between. Applying the same logic as in item (i) there cannot be constrained agents further away from the CBD than non-constrained agents. According to this claim, the long run spatial sorting is that constrained agents live close to the CBD and non-constrained further away.

B.4 Steady-state wealth and property tax

We consider a standard uniform tax on residential property τ^p instead of lump-sum tax when the city size is given by $L = L^o$ and there is no borrowing constraint. Therefore, the budget constraint becomes

$$w_t + y_t = c_t + b_t + p_t(x)(1 + \tau^p) + \kappa(x) \tag{B.11}$$

with $b_t = y_{t+1}/(1 + r) - \mu p_{t+1}$ while the balanced-budget constraint for each city is now given by

$$G = \tau^p \int_0^{L^o} p_t(x)\mathrm{d}x. \tag{B.12}$$

This tax does not distort land market as the land supply is fixed and housing size is fixed. It is easy to check that the equilibrium land price is then given by $p_t(x) = R_A + [\kappa(L^o) - \kappa(x)](1 - \mu + \tau^p)$. A higher property tax is capitalized into lower land prices. The capitalization of the property tax into land prices offers support for the idea that property taxes can efficiently finance public expenditures. Therefore, (B.12) can be rewritten as follows

$$G = \tau^p \frac{1 - \mu + \tau^p}{1 - \mu + \tau^p} \int_0^{L^o} p_t(x) - R_A \mathrm{d}x + \tau^p L^o R_A \tag{B.13}$$

As $(1-\mu+\tau^p)\int_0^L [p_t(x) - R_A]\mathrm{d}x = \int_0^L [\kappa(L) - \kappa(x)]\mathrm{d}x = \text{ASLR} = G$ when $L = L^o$, the equilibrium property tax is implicitly given by

$$\tau^p R_A = \frac{1-\mu}{1-\mu+\tau^p}\frac{G}{L^o} \tag{B.14}$$

Hence, the left wealth is $y_{t+1} = \beta[w_t + y_t - (1-\mu+\tau^p)R_A - \kappa(L^o)]$ so that the steady-state wealth is

$$\begin{aligned} y_\infty &= \frac{\beta}{1-\beta}\left[\varphi - \frac{G}{L^o} - \mathcal{K} + \left(\frac{G}{L^o} - \tau^p R_A\right)\right] \\ &= \frac{\beta}{1-\beta}\left[\varphi - \frac{G}{L^o} - \mathcal{K} + \frac{\tau^p}{1-\mu+\tau^p}\frac{G}{L^o}\right], \end{aligned} \tag{B.15}$$

which is higher than y^n_∞.

In addition, if we consider the Pigouvian tax on consumption τ^c, the steady-state wealth becomes

$$y_\infty = \frac{\beta}{(1-\beta)(1-\widetilde{\delta})}\left[\varphi - \mathcal{K} - \frac{1-\mu}{1-\mu+\tau^p}\frac{G}{L^o}\right], \tag{B.16}$$

which is lower than than y^o_∞.

B.5 Steady-state wealth and wage inequality

The steady state wealth of unskilled workers who are credit constrained is $y^c_\infty(x)$ (see (17)) and the steady state wealth of unskilled workers who are unconstrained is y^n_∞ (see (13)). The residential area of the latter class is still $[\widehat{x}^\star_\infty, L]$, where $\widehat{x}^\star_\infty$ is given by (16). The residential area formed by agents investing in education is $[0, \widehat{x}^e_\infty]$. We determine $\widehat{x}^e_\infty$ such that $y^c_\infty(\widehat{x}^e_\infty) = \phi$ yielding

$$\kappa(\widehat{x}^e_\infty) = \phi\frac{1-\beta}{\beta}\left[\Lambda(\phi) - 1 - \frac{1}{\rho(\lambda)}\right] \quad \text{where} \quad \Lambda(\phi) \equiv \frac{\widetilde{w}}{\phi}\frac{\beta}{1-\beta}. \tag{B.17}$$

In addition, if a class of skilled workers exists, their bid rent is $\Psi^e_\infty(x, K^e_\infty)$, where K^e_∞ is the integration constant of the bid rent of skilled workers. As $\Psi^e_\infty(\widehat{x}^e_\infty, K^e_\infty) = y^c_\infty(\widehat{x}^e_\infty)/(1-\lambda)$ must hold in equilibrium, $K^e_\infty = [\Lambda(\phi) - 1]\phi(1-\beta)/\beta$, with $K^e_\infty - \kappa(\widehat{x}^e_\infty) > 0$ and $K^e_\infty < \mathcal{K}$ (so that $\Psi^e_\infty(x, K^e_\infty) < \Psi_\infty(x, \mathcal{K})$). Since the lifetime wealth of skilled workers is $we - \phi - K^e_\infty - G/L$, their wealth converges to

$$y^e_\infty = \frac{\beta(\phi^e - \phi)}{1-\beta} + \phi. \tag{B.18}$$

Therefore, some workers invest in education if $y^e_\infty > \phi$ and $\widehat{x}^e_\infty \in (0, \widehat{x}^\star_\infty)$. The former condition holds as long as $\phi < \phi^e$. The condition $\widehat{x}^e_\infty > 0$ occurs when $\lambda < \lambda^e \equiv 1 - \frac{\beta(1-\mu)}{(1-\beta)[\Lambda(\phi)-1]}$ which decreases with ϕ (notice that $\Lambda(\phi) > 1$ as $K^e_\infty > \kappa(\widehat{x}^e_\infty) > 0$). We have $\widehat{x}^e_\infty < \widehat{x}^\star_\infty$ if and only if $\kappa(\widehat{x}^e_\infty) < \kappa(\widehat{x}^\star_\infty)$. Using (16) and (B.17), the latter condition is equivalent to $y^n_\infty < \phi$ which always hold when skilled workers and unskilled workers emerge at the steady state. Furthermore, at the steady state, the emergence of two types of unskilled agents (credit constrained and unconstrained) requires $(1-\lambda)R_A < y^n_\infty < (1-\lambda)\Psi_\infty(\widehat{x}^e_\infty, \mathcal{K})$ or, equivalently,

$$\underline{R}^e_A(\phi) \leqslant R_A \leqslant \overline{R}_A(\lambda) \quad \text{with} \quad \underline{R}^e_A(\phi) \equiv \frac{\widetilde{w} - \kappa(L)}{1-\mu} - \frac{\phi}{1-\mu}\frac{1-\beta}{\beta} \tag{B.19}$$

and $\overline{R}_A$ is given by (15). $\overline{R}_A(\lambda) > \underline{R}^e_A(\phi) > \underline{R}_A(\lambda)$ as long as $\widehat{x}^e \in (0, L)$ and with $\underline{R}^e_A(\lambda^e) = \underline{R}_A(\lambda^e)$.[28] In the presence of skilled workers, the unskilled workers who are unconstrained reside in the remote areas $(\widehat{x}^\star_\infty, L)$ while the unskilled workers who are credit constrained live in $[\widehat{x}^e_\infty, \widehat{x}^\star_\infty]$ when $\underline{R}^e_A(\phi) \leqslant R_A \leqslant \overline{R}_A(\lambda)$.

The long-run return to location x for skilled agents is

$$\varrho^e_\infty(x) = \frac{\Psi^e_\infty(x, K^e_\infty) - R_A}{\Psi^e_\infty(x, K^e_\infty) + \kappa(x) - [R_A + \kappa(L)]} \tag{B.20}$$

where $\varrho^e_\infty(x) > 1 + r$ is equivalent to

$$\frac{(1-\mu)R_A + \kappa(L) - \kappa(x)}{1-\mu} > \Psi^e_\infty(x, K^e_\infty). \tag{B.21}$$

Given that the left-hand side of this inequality is equal to $\Psi_\infty(x, \mathcal{K})$, which is higher than $\Psi^e_\infty(x, K^e_\infty)$ $(\mathcal{K} > K^e_\infty)$, $\varrho^c_\infty(x)$ is higher than $1 + r$ when agents are skilled workers.

B.6 Steady-state capital accumulation

If the agent is located at x, the wealth left to her offspring is $y_{t+1} = (1 + r_{t+1})b_t + \theta_{t+1}(x)$. Assuming that credit market is perfect, her bid-rent is now $p_t(x) = \mathcal{K}_t - \kappa(x) + \theta_{t+1}(x)/\widetilde{r}_{t+1}$. As $p_t(x) = \theta_t(x) + \widetilde{r}_{t+1}/\varphi_h$ from the zero-profit condition, we obtain

$$\theta_t(x) = \mathcal{K}_t - \widetilde{r}_{t+1}/\varphi_h - \kappa(x) + \theta_{t+1}(x)/\widetilde{r}_{t+1} \tag{B.22}$$

with $\mathcal{K}_t = \kappa(L^o_t) + r_{t+1}R_A/\widetilde{r}_{t+1} + \widetilde{r}_{t+1}/\varphi$ (so that $\theta_t(L^o_t) = R_A$). Plugging the bid rent into the budget constraint of an agent living at x and using $y_{t+1}(x) = (1 + r_{t+1})b_t + \theta_{t+1}(x)$ lead to $c_t + y_{t+1}/\widetilde{r}_{t+1} = w(k_t) + y_t - G/L^o_t - \mathcal{K}_t$. If the Pigouvian tax on consumption

[28] It is straightforward to check that $\underline{R}^e_A = \underline{R}_A(\lambda) + \frac{1}{1-\mu}\frac{\rho}{1+\rho}\kappa(\widehat{x}^e)$ and $\overline{R}_A(\lambda) = \underline{R}_A(\lambda) + \frac{1}{1-\mu}\frac{\rho}{1+\rho}\kappa(L)$, so that $\overline{R}_A(\lambda) > \underline{R}^e_A > \underline{R}_A(\lambda)$.

denoted by τ_t^c is implemented so that the ratio $c_t/(y_{t+1}/\tilde{r}_{t+1})$ at the equilibrium is equal to $c_\infty^o/(y_\infty^o/\tilde{r}_\infty^o) = \tilde{\alpha}(b_\infty^o)$. In this context, utility-maximizing bequest is given by

$$b_t(x) = \frac{1}{1+\tilde{\alpha}(b_\infty^o)}\left[w(k_t) + y_t - \frac{G}{L_t^o} - \mathcal{K}_t\right] - \frac{\theta_{t+1}(x)}{\tilde{r}_{t+1}}.$$

The capital stock per capita available for production of consumption and housing goods in period t is $k_t + 1/\varphi_h = \frac{1}{L_t^o}\int_0^{L_t^o} b_{t-1}(x)\mathrm{d}x \equiv \bar{b}_{t-1}$. As $y_t(x) = \tilde{r}_t b_{t-1}(x) + \theta_t(x)$ and $g(k_t) = w_t(k_t) + \tilde{r}_t k_t$, the capital stock per capita in period $t+1$ is

$$\begin{aligned}\bar{b}_t &= \frac{1}{L_t^o}\int_0^{L_t^o} b_t(x)\mathrm{d}x \\ &= \frac{1}{1+\tilde{\alpha}(b_\infty^o)}\left[g(k_t) + \frac{\tilde{r}_{t+1}}{\varphi_h} - \frac{G}{L_t^o} - \mathcal{K}_t + \frac{1}{L_t^o}\int_0^{L_t^o}\theta_t(x)\mathrm{d}x\right] - \frac{1}{L_t^o}\int_0^{L_t^o}\frac{\theta_{t+1}(x)}{\tilde{r}_{t+1}}\mathrm{d}x\end{aligned}$$

Using (B.22) and $G/L_t^o = \kappa(L_t^o) - \mathcal{T}(L_t^o)/L_t^o + g_k(k_t)k_t$, we get

$$\begin{aligned}\bar{b}_t &= \frac{1}{1+\tilde{\alpha}(b_\infty^o)}\left[w(k_t) - \kappa(L_t^o) + \frac{1}{L_t^o}\int_0^{L_t^o}\frac{\theta_{t+1}(x)}{\tilde{r}_{t+1}}\mathrm{d}x\right] - \frac{1}{L_t^o}\int_0^{L_t^o}\frac{\theta_{t+1}(x)}{\tilde{r}_{t+1}}\mathrm{d}x \\ &= \frac{1}{1+\tilde{\alpha}(b_\infty^o)}\left[w(k_t) - \kappa(L_t^o) + \frac{R_A}{\tilde{r}_{t+1}}\right] - \frac{R_A}{\tilde{r}_{t+1}} - \frac{\tilde{\alpha}(b_\infty^o)}{1+\tilde{\alpha}(b_\infty^o)}\frac{1}{L_t^o}\int_0^{L_t^o}\frac{\theta_{t+1}(x) - R_A}{\tilde{r}_{t+1}}\mathrm{d}x\end{aligned} \tag{B.23}$$

Clearly, if $\theta_{t+1}(x) = R_A$ at the steady state, then $\bar{b}_\infty = b_\infty^o$. However, under land competition with altruistic agents, we have $\theta_{t+1}(x) > R_A$ implying $\bar{b}_\infty < b_\infty^o$.

Appendix C Richer models of residential location and wealth transmission

Our urban model predicts that without a borrowing limit, the housing market mechanism intrinsically promotes the convergence of the wealth of different agents. Credit market imperfections modify the distribution of surplus across agents because the borrowing constraint can cap the house price paid by wealthy or lucky agents. We explore three extensions to check the robustness of our results. First, we allow real estate companies to rent out the housing units and examine how the borrowing constraint impacts agents' tenure choice. Second, the agents are assumed to be free to choose the size of housing units they want to consume. Finally, we study how housing bubbles impact the borrowing constraint and the urban equilibrium. We show that our findings are not specific to the model considered thus far.

C.1 Tenure choice

When the borrowing constraint exerts downward pressure on house sale prices, real estate companies could decide to rent their housing units instead of selling them to increase their profits. In this case, wealthy agents could bid to rent housing units located in attractive locations at the price of not leaving any housing wealth to their offspring. Hence, the effect of the rental housing market on wealth inequality is unclear. In what follows, we extend our framework considering the endogenous supply of houses both for renting and available for sale.

The real estate industry. Real estate companies now play a crucial role in the pattern of residential choices emerging in equilibrium. At the beginning of each period, a real estate company owning the house located at x has two options: it can sell the house at price $p_t^H(x)$, and the value of the firm is denoted by $\boldsymbol{v}^H(x)$ and expressed as follows

$$\boldsymbol{v}_t^H(x) = p_t^H(x) - \theta_t(x), \tag{C.24}$$

where $p_t^H(x) \geqslant p_A$; alternatively, it can rent the house out and sell it to the next generation of real estate companies at the end of the period. In this case, the associated value of the firm is

$$\boldsymbol{v}_t^T(x) = p_t^T(x) - \Gamma p_t^T(x) - \theta_t(x) + \mu p_{t+1}(x), \tag{C.25}$$

where $\Gamma > 0$ is the cost of adjusting housing services in the case of renting to take into account tenants' behavior that generates additional maintenance costs (Henderson and Ioannides, 1983). We also assume that $\boldsymbol{v}_t^T(L)$ is positive, that is, $p_t^T(L) \geqslant (1-\mu)R_A/(1-\Gamma)$. Each firm chooses the best option so that its profit is given by $\boldsymbol{v}(x) = \max\{\boldsymbol{v}^H(x), \boldsymbol{v}^T(x)\}$.

Agents. Agents decide whether to rent or to purchase the house they occupy. We make the following assumptions that are not restrictive. First, there is free access to the rental housing market, which amounts to there being no borrowing requirement for tenants. Second, all agents have the same initial wealth y_t^0. Third, home ownership does not generate a positive non pecuniary benefit ("pride of ownership"). Considering a heterogeneous wealth distribution would not change the main results at the expense of a more cumbersome presentation of the urban equilibrium. We can thus write the budget constraint

$$c_t^z + \frac{y_{t+1}(x)}{1+r} = \mathcal{W}_t^z(x) \equiv \widetilde{w} - \kappa(x) + y_t^0 - p_t^z(x) + \mathbf{1}_z \mu p_{t+1}(x) \tag{C.26}$$

where $z = H$ if the agent is a homeowner, with $\mathbf{1}_H = 1$, and $z = T$ if the agent is a tenant, with $\mathbf{1}_T = 0$. Unlike homeowners, tenants do not leave any housing wealth to their offspring.

By maximizing the utility function with respect to c_t and y_{t+1} under (C.26), we obtain $y_{t+1} = y_{t+1}\left[\mathcal{W}_t^z(x)\right]$ and $V[\mathcal{W}_t^z(x)]$. Then, for any agent $z = T, H$, the bid rent $\Psi_t^z\left(x, K_t^z\right)$ is derived from the equilibrium condition, $\mathrm{d}V\left[\mathcal{W}_t^z(x)\right]/\mathrm{d}x = 0$. Hence,

$$\Psi_t^z(x, K_t^z) = K_t^z - \kappa(x) + \mathbf{1}_z \mu p_{t+1}(x), \text{ for } z = T, H, \tag{C.27}$$

where the constants K_t^H and K_t^T are obtained from the urban configuration arising in equilibrium. Note that only homeowners' bid rent capitalizes the future house sale price.

Tenants are never constrained by assumption, so that they can pay $\Psi_t^T(x, K_t^T)$ at any location x. Homeowners can only pay up to $y_t^0/(1-\lambda)$ when the borrowing constraint is binding. The bid-rent function for homeowners is then

$$\psi^H(x, y_t^0, K_t) = \begin{cases} \frac{y_t^0}{1-\lambda}, \text{ when } x \in [0, \widehat{x}_t^0] \\ \Psi_t^H\left(x, K_t^H\right), \text{ when } x \in [\widehat{x}_t^0, L], \end{cases} \tag{C.28}$$

where $\widehat{x}_t^0$ is such that $y_t^0 = \Psi_t^H(\widehat{x}_t^0, K_t^H)(1-\lambda)$. Given that agents own the same wealth y_t^0, unconstrained agents and tenants must enjoy the same utility level in equilibrium. Otherwise, some agents would find it profitable to change their location and tenure status by outbidding some already settled residents. Hence, in equilibrium, we have $\mathcal{W}_t^H = \mathcal{W}_t^T$ where $\mathcal{W}_t^H = \widetilde{w} + y_t^0 - K_t^H$ and $\mathcal{W}_t^T = \widetilde{w} + y_t^0 - K_t^T$ for any $x \geqslant \widehat{x}_t^0$, yielding $K_t^H = K_t^T = K_t$ or, equivalently,

$$\Psi_t^T(x, K_t) = \Psi_t^H\left(x, K_t\right) - \mu p_{t+1}(x) \leqslant \Psi_t^H\left(x, K_t\right).$$

The urban equilibrium. We build an equilibrium with the additional requirement that firms choose their best option between renting and selling given the agents' best location choice. A real estate company sells the house if and only if $\Delta(x) \equiv \boldsymbol{v}_t^H\left[\psi^H(x, y_t^0)\right] - \boldsymbol{v}_t^T\left[\Psi_t^T(x, K_t)\right] > 0$ with

$$\Delta(x) = \psi^H(x, y_t^0) - (1-\Gamma)\Psi_t^T(x, K_t) - \mu p_{t+1}(x) \tag{C.29}$$

where we have plugged (C.27) and (C.28) into (C.24) and (C.25). Three comments are in order. First, given the bid rents and the borrowing constraint, real estate companies are more likely to rent houses located in the most attractive places, *i.e.*, where the borrowing constraint is binding and imposes a limit on the price ($x \in [0, \widehat{x}_t^0]$). Second, as the lifetime wealth of homeowners and tenants is identical when $x \in [\widehat{x}_t^0, L]$, with $\mathcal{W}_t^T(x) = \mathcal{W}_t^H(x) = \mathcal{W}_t^T$, there is no wealth inequality across agents living in the less attractive places, whatever the status of agents (homeowner or tenant). However, the equilibrium spatial structure of the city has an impact on the *level* of wealth. Indeed, $K_t = \mathcal{K}$ if the agents living at the city fringe are homeowners or $K_t = \kappa(L) + (1-\mu)R_A/(1-\Gamma) > \mathcal{K}$ if the agents living at the city fringe are tenants. Since wealth inequalities across agents living in the less attractive locations ($x \in [\widehat{x}_t^0, L]$) do not emerge, we do not need to analyze the conditions under which real estate companies prefer to either sell or to rent for locations $x \in [\widehat{x}_t^0, L]$. Third, regardless of the status of agents in the less attractive sites ($x \in [\widehat{x}_t^0, L]$), the size of the area in which agents are credit-constrained decreases when credit frictions become less severe ($\partial\widehat{x}_t^0/\partial\lambda < 0$). Indeed, $y_t^0 = \Psi_t^H(\widehat{x}_t^0, \mathcal{K})(1-\lambda)$ implies $\mathcal{K} - y_t^0/(1-\lambda) = \kappa(\widehat{x}_t^0) - \mu p_{t+1}(\widehat{x}_t^0)$.

When $0 < \widehat{x}_t^0 < L$, wealth inequality can emerge. Indeed, we have $\psi^H = y_t^0/(1-\lambda)$ and $\Delta'(x) = (1-\Gamma)\kappa'(x) - \mu p'_{t+1}(x) > 0$ when $x \in [0, \widehat{x}_t^0]$. If $\Delta(0) > 0$ (which is akin to when the borrowing constraint is not very severe, *i.e.*, λ is high), we return to our framework developed in Section 3. There is no area hosting tenants, and the lifetime wealth of homeowners located in the most attractive sites is higher than the wealth of agents residing in the rest of the city. If $\Delta(0) < 0$ (the borrowing constraint is more stringent, λ becomes low), there exists a unique $\overline{x}_t \in [0, \widehat{x}_t^0]$ such that the firm located at $\overline{x}_t$ is indifferent between selling or renting, that is, $\Delta(\widehat{x}_t^0) = \Gamma\Psi_t^T(\widehat{x}_t^0, K_t^T) > 0$. Under this configuration, real estate companies at any location $x \leqslant \overline{x}_t$ rent their properties out, while those at any location $x > \overline{x}_t$ sell houses at $\psi^H = y_t^0/(1-\lambda)$. With respect to the preceding case without any rental market, the number of constrained homeowners is reduced. Real estate companies renting their property in $[0, \overline{x}_t]$ extract more revenues from the tenants. In the area $[\overline{x}_t, \widehat{x}_t^0]$, there are constrained homeowners who pay the rent $y_t^0/(1-\lambda)$. Because $\Delta(\overline{x}_t) = 0$, we have $\partial\overline{x}_t/\partial\lambda < 0$. A tighter borrowing constraint (lower λ) favors the emergence of an area hosting tenants in the best places ($\overline{x}_t$ increases). As long as $\widehat{x}_t^0 > \overline{x}_t$, there is still *symmetry-breaking* because constrained homeowners

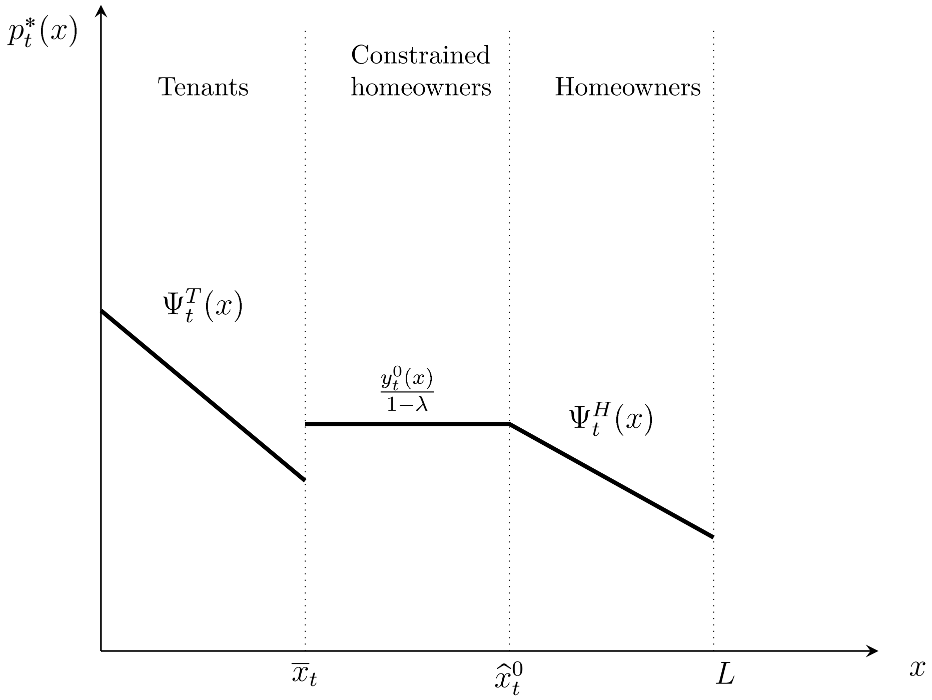

Figure C.1: Urban equilibrium with tenure choice

leave a higher bequest than tenants and unconstrained homeowners. However, the lucky homeowners no longer reside in the most attractive locations but are settled at $\overline{x}_t$. Hence, the rental market reduces overall inequality since it decreases the number of constrained homeowners, and it shrinks the utility gap between constrained and unconstrained agents. Using $\mathcal{W}_t^T = \mathcal{W}_t^H(\widehat{x}_t^0)$, the lifetime wealth wedge between the wealthiest agent and the unconstrained agent is $\mathcal{W}_t^H(\overline{x}_t) - \mathcal{W}_t^T = \kappa(\widehat{x}_t^0) - \kappa(\overline{x}_t) + \mu\left[p_{t+1}(\overline{x}_t) - p_{t+1}(\widehat{x}_t^0)\right]$.[29]

The introduction of borrowing constraints makes a fraction of agents wealthier. However, a stricter borrowing constraint implies that real estate companies may prefer to rent the houses located in better places because agents are prepared to become tenants and pay a higher rent to live in these better sites. Note further that a housing policy that would cap the rent of tenants would shrink the area where it is profitable for real estate companies to supply houses for renting (formally, it could reduce Δ).

[29]as $\mathcal{W}_t^H(\overline{x}_t) - \mathcal{W}_t^T = -\frac{y_t^0}{1-\lambda} - \kappa(\overline{x}_t) + \mu p_{t+1}(\overline{x}_t) + \mathcal{K}$.

C.2 Heterogeneous housing size

We now consider the case where the supply of housing units varies across locations. Housing units are available in discrete and fixed sizes $h_t \in \{\underline{h}, ..., \overline{h}\}$ where $\underline{h}$, and $\overline{h}$ represent the minimum and maximum sizes, respectively. The case in which housing units are instead a continuum of sizes is reported in the Supplementary Appendix. In accordance with empirical evidence, we assume that housing size increases with distance to the CBD. For example, $h_t(x) = \underline{h}$ for $x \in (0, \ell]$ and $h_t(x) = \overline{h}$ for $x \in (\ell, L]$. The utility function is $u(c_t, s_t, y_{t+1})$ with $s_t = h_t$, while the bequest becomes $y_{t+1} = (1+r)b_t + p_{t+1}(x)h_t(x)$. Agents face the budget constraint $c_t + b_t + p_t h_t = w_t - \kappa(x) + y_t - \tau_t$. Note that p_t is the price *per unit of housing*. Maximizing $u(.)$ under the budget constraint implies $u_c(c_t, s_t, y_{t+1})p_t = u_s(c_t, s_t, y_{t+1})$ with $u_s \equiv \partial u / \partial s_t$. The maximum bid rent per unit of housing $\Psi_t(x, y_t)$ is such that $u'[c_t(x), s_t(x), y_{t+1}(x)] = 0$ or, equivalently,

$$\Psi_t^{'}(x, h) = \frac{-\kappa'(x)}{h_t(x)} + \mu p_{t+1}^{'}(x). \tag{C.30}$$

Contrary to the case with exogenous lot size, the slope of the bid rent depends on housing size. This is a modified version of the Alonso-Muth condition. Agents living far from the CBD are compensated for their long and costly commutes by enjoying larger housing. As a result, the maximum bid rent per unit of housing is

$$\Psi_t(x, h) = K_t - \frac{\kappa(x)}{h_t(x)} + \mu p_{t+1}(x), \tag{C.31}$$

where $K_t = \mathcal{K}/\overline{h}$ if no agent is credit constrained.

The borrowing constraint is now given by $\lambda p_t(x)h_t(x) \geqslant p_t(x)h_t(x) - y_t$, where $p_t(x)h_t(x)$ represents housing expenditures such that agents can borrow if and only if

$$p_t(x) \leqslant \frac{y_t}{1-\lambda}\frac{1}{h_t(x)} \equiv \widehat{p}_t(h). \tag{C.32}$$

When there are two sizes of housing, there are now at most two areas ($(0, \widehat{x}_t)$ and $(\ell, \widehat{x}_t^{\ell})$) in which the borrowing constraint is binding for each agent (see Figure C.2). The threshold locations $\widehat{x}_t \in (0, \ell)$ and $\widehat{x}_t^{\ell} \in (\ell, L)$ are such that $\Psi_t(\widehat{x}_t^{\ell}, \underline{h}) = \widehat{p}_t(\underline{h})$ and $\Psi_t(\widehat{x}_t^{\ell}, \overline{h}) = \widehat{p}_t(\overline{h})$, respectively. We can generalize this result to any number of housing size classes. The same mechanisms presented in the previous sections are at work. In different places, the equilibrium house prices paid by the credit-constrained agents and the wealthiest agents is lower than the maximum price that an agent would be willing to pay. As in the previous sections, credit constraints can give rise to symmetry-breaking and lead to spatial wealth sorting, which can translate into persistent inequality. A tax on both land rents and lifetime wealth must also be implemented to achieve a better allocation of resources.

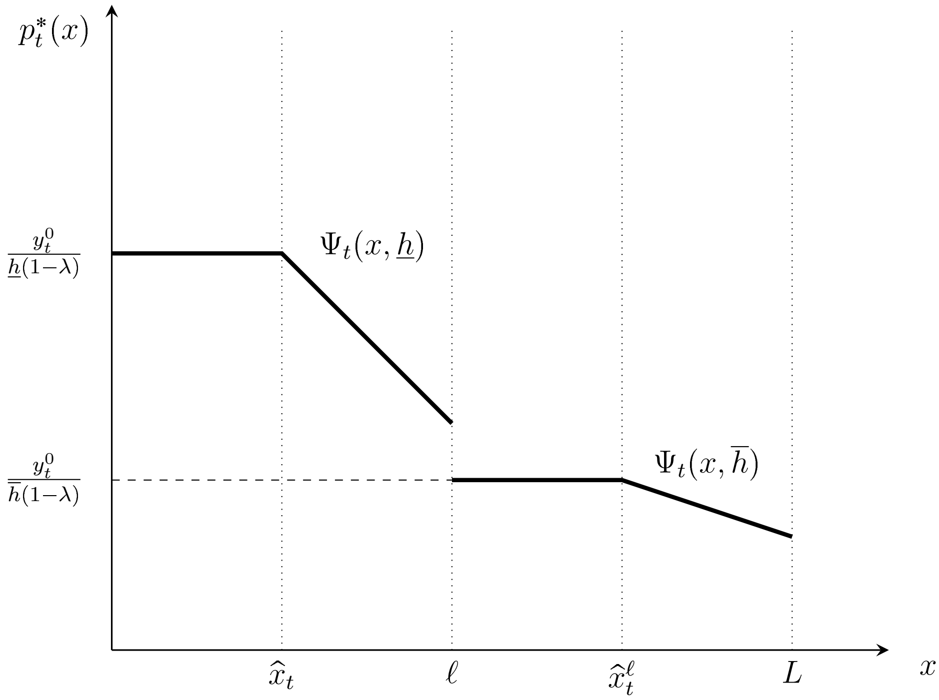

Figure C.2: Urban equilibrium with two housing sizes and no wealth heterogeneity.

C.3 Bubbly dynamics

Thus far, we have studied the rent dynamics by assuming away housing bubbles. If we do no more impose the transversality condition, a wider class of solutions may exist. We now explore rent dynamics with bubbles where house prices are now expressed as $\widetilde{p}_t(x) = p_t(x) + a_t$ with $p_t(x)$ being the fundamental solution of Equation (B.2) and a_t being a bubble. Under this configuration, we have

$$p_t(x) + a_t = \mu \left[p_{t+1}(x) + \mathbb{E}_t(a_{t+1})\right] + \mathcal{K} - \kappa(x). \tag{C.33}$$

where $\mathbb{E}_t$ stands for the expectation operator. Any a_t such that $a_t = \mu\mathbb{E}_t(a_{t+1})$ is a solution of Equation (B.2). As $\mu < 1$, a_t explodes in expected value. All agents expect the sale price to increase in the future. The bubble a_t does not modify the wealth dynamics of unconstrained agents because the rent internalizes the future sale price and thus the bubble. However, the bubble has some key consequences for the borrowing constraint. As it is recognized that rapidly rising house prices increase pressure to relax borrowing constraints (Acolin *et al.*, 2016), we integrate into the downpayment requirement the expectation of the future increase in house prices as follows: $\widetilde{p}_t(x) - y_t < \lambda\widetilde{p}_{t+1}(x)$, which is equivalent to

$$p_t(x) - \lambda p_{t+1}(x) - (\lambda/\mu - 1)a_t < y_t$$

with p_t given by (B.2) and where we have used (C.33) and $a_t = \mu \mathbb{E}_t(a_{t+1})$. The dynamics of $\widehat{x}_t^\star$ depend on the trend followed by the bubble a_t. The bubble generates two opposite effects on the borrowing constraint. On the one hand, expectations of an increase in the pledgeable sale price make the borrowing constraint less stringent. On the other hand, rising sale prices will increase the need to borrow and tighten the borrowing constraint. The overall effect depends on the credit market imperfections and on the type of bubble.

Assume that the bubble follows a deterministic increasing trend, given by $a_t = \mu^{-t} a_0$ with a_0 being an arbitrary initial condition. In this case, if credit market frictions are weak (resp., strong), *i.e.*, $\lambda/\mu > 1$ (resp.,$<$) the borrowing constraint becomes less tight (resp., tighter). In the long run, the whole population ends up satisfying the borrowing constraint (resp., being borrowing-constrained) holding the same long-run wealth level, (resp., remaining unequal in the long run).

Assume now that the bubble follows stochastic dynamics given by

$$a_{t+1} = \begin{cases} \frac{1}{\mu\pi} a_t + \xi_{t+1} \text{ with probability } \pi, \\ \xi_{t+1} \text{ with probability } 1 - \pi. \end{cases}$$

with $\mathbb{E}_t \left[\xi_{t+1}\right] = 0$. It amounts to assuming that if the bubble bursts with some probability $1 - \pi$ or continues to grow with probability π, the dynamics of the city would change. Consider $\lambda/\mu > 1$, as in the deterministic case: As long as the bubble inflates, the borrowing constraint would become less stringent, and credit would be easier. After a bursting of the bubble, the borrowing constraint can suddenly be binding for a part of the population, thereby generating wealth inequality.

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