Agrekon

Agricultural Economics Research, Policy and Practice in Southern Africa

ISSN: 0303-1853 (Print) 2078-0400 (Online) Journal homepage: www.tandfonline.com/journals/ragr20

Risk efficiency of optimal water allocation within a single- and multi-stage decision-making framework

Primrose Madende & Bennie Grové

AGREKON
2020, VOL. 59, NO. 1, 78–92
https://doi.org/10.1080/03031853.2019.1636668

Risk efficiency of optimal water allocation within a single- and multi-stage decision-making framework

Primrose Madende and Bennie Grové

Department of Agricultural Economics, University of the Free State, Bloemfontein, South Africa

ABSTRACT

The South African government has put legislation in place to exercise better control over irrigation water usage. Thus, proper planning of irrigation areas and scheduling of irrigation events in order to stay within water quotas has become more important. Currently, the available methodologies to assist irrigation farmers overlook the complexities and interrelated relationships between crop-area planning and the multi-stage nature of irrigation-scheduling decisions within a stochastic dynamic environment. This paper contributes to knowledge through the development of a bio-economic model that uses evolutionary algorithms to optimise water use, taking cognisance of the complex interrelationships between crop-area planning, the multistage decision-making nature of irrigation-scheduling decisions, and the stochastic dynamic environment under conditions of limited water supply. The results show that gross margin variability is reduced and the expected outcomes are improved due to improved irrigation-scheduling decisions made sequentially in multiple stages. Multi-stage decisions tend to make the impact of risk aversion less profound because taking account of unfolding weather information is risk reducing. Ignoring the risk-reducing impact of sequential decision-making will over-estimate the cost of water restrictions. Caution is hence necessary when formulating agricultural water-allocation policies based on crop water optimisation models that overlook the complex nature of irrigation decisions.

ARTICLE HISTORY
Received 5 September 2018
Accepted 19 June 2019

KEYWORDS
Irrigation; sequential decision-making; limited water supply; bio-economic model; evolutionary algorithms; optimisation; risk

1. Introduction

The agricultural sector is the largest consumer of water in developing countries, with South Africa (SA) being no exception. The recurring droughts in the region, coupled by erratic rainfall patterns, have aggravated the dire crisis of a scarcity of water resources in SA, specifically water for agricultural purposes (Department of Water and Sanitation [DWS] 2018). The dwindling water resources hence render the improvement of irrigation water management decisions greater priority in an effort to alleviate the extent of the problem. Furthermore, the newly gazetted legislation requires irrigation farmers to measure and report their water use on a regular basis (DWS 2017). Most irrigation schemes measure water use through indirect measurements based on planned irrigation area and crop water-use factors. The mere fact that direct measurements will prevail in the future emphasises the importance of proper planning of irrigation areas and scheduling of irrigation events in order to stay within water quotas. Crop water-use computer models play an important role in assisting irrigation farmers with irrigation planning (Van Heerden et al. 2009). The effectiveness of irrigation

CONTACT Primrose Madende madendep@ufs.ac.za Department of Agricultural Economics, University of the Free State, PO Box 339, Bloemfontein 9300, South Africa

planning, however, is vastly dependent on the ability of such models to represent the complex nature of irrigation decisions.

Within irrigation water management systems, decision-making complexities are aggravated by the dynamic nature of irrigation water, combined with day-to-day, seasonal and inter-annual climatic variabilities (Kusunose and Mahmood 2016). The interaction between different decisions made at different times during the growing season, such as crop, area and water-scheduling decisions, hence complicates the proper planning of irrigation water use. Many farm management decisions are thus formulated within a multi-stage decision-making process characterised by a sequence of decisions made to meet farmer's objectives. Accordingly, decisions on irrigation water allocation are considered complicated decisions that are made sequentially in multiple stages, taking into consideration the stock nature of field water supply dynamically throughout the growing season (Dai and Li 2013; Robert et al. 2016, 2017). By implication, the amount of irrigation water applied in one period affects the availability of extractable water by crops in the next time period, since water can be stored in the soil. The time periods that divide the decision-making process are referred to as stages and represent the moments when decisions must be made (Robert et al. 2016). The sequential nature of irrigation decisions is thus independent of the scale of production. The farmer will decide when and how to irrigate in one stage, given the influence the decision made in the previous stage had on soil–water status.

Crop-area decisions made at the beginning of the growing season, when the climatic conditions of the entire growing season are still unknown to the decision-maker, are essential for agricultural water management as they facilitate irrigation strategy decisions, given the amount of water available (Karrou and Oweis 2012; Zeng et al. 2010). Within the South African context, the Water Administration System (WAS) is used efficiently to facilitate water supply management within canal water-distribution systems to ensure that all users along a canal are supplied with sufficient water (Benadé 2011). For a given water-availability scenario, the area decisions determine the irrigation strategy that will be followed, given the amount of water that could be applied on a per hectare basis as guided by the area decision. In contrast, irrigation water-scheduling decisions are made sequentially throughout the growing season as the uncertain weather conditions unfold, given the crop-area decision already made. The sequential decisions made by irrigation farmers facilitate the adjustment of irrigation water schedules for each consecutive stage, depending on the currently prevailing weather conditions. Thus, the decision-maker is able to manage production risk by taking cognisance of new information from unfolding weather states. Modelling farmers' decision-making processes by including adaptations when representing farmers' practices is thus considered an important challenge for the agricultural research community (Robert et al. 2016).

Substantial progress has been made internationally with the development and application of techniques that acknowledge the multi-stage, dynamic and uncertain nature of irrigation decisions to solve irrigation water-allocation problems (Bryant, Mjelde, and Lacewell 1993; Chen et al. 2017; Li and Huang 2011; Parsinejad et al. 2012; Robert et al. 2017). Nonetheless, modelling adaptation to uncertainty as a representation of farmers' practices and decision-making processes has been addressed on different temporal and spatial scales due to the curse of dimensionality associated with dynamic programming. As a result, limited decision stages were included in the models, resulting in models failing to explicitly incorporate the sequential nature of irrigation water-allocation decisions. The association of dynamic programming with the curse of dimensionality results in integrating the timing scope of farmers' adaptive behaviour within a solution procedure difficult to overcome when designing farming systems. A recursive stochastic solution procedure that solves problems through forward recursion was explored in this research, hence providing an alternative solution procedure to solve complex dynamic problems without the limitation of the curse of dimensionality (Blanco and Flichman 2002).

Considerable research efforts have also been made in South Africa on crop water-use management for both limited and full water quota supply conditions (Botes 1990; Botes, Bosch, and Oosthuizen 1996; Gakpo et al. 2005; Grové and Oosthuizen 2010; Haile 2017; Haile et al. 2014; Venter, Grové, and Van der Stoep 2017). Research efforts have explored the concept of modelling the interdependency of water applications between irrigation applications in different time periods. The crop water-use models, however, focused primarily on the depth of irrigation amounts without explicitly considering the timing of irrigation events. Hence, only four research efforts have accounted for the dynamic, intertemporal nature of irrigation decisions through mathematical programming and simulation optimisation (Botes 1990; Grové and Oosthuizen 2010; Haile 2017; Venter, Grové, and Van der Stoep 2017). Dynamic models with complex inter-actions are difficult to solve with linear programming, hence the application of simulation and non-linear programming models. Recent research by Haile (2017) successfully developed an integrated bio-economic simulation optimisation model that is able to solve complex stochastic water-allocation optimisation problems under salinity, while also considering production risk. Seven states of nature were included in the simulation model. However, water was assumed to be a state-general input. Thus, the optimal irrigation strategy was determined such that it would maximise utility irrespective of the state of nature that unfolds. Therefore, no adaptive decision-making was included in the model. Overall, none of the research efforts considers the sequential nature of irrigation decisions and the fact that irrigation farmers could manage production risk through sequential decision-making. In all cases, the crop-area decision and the irrigation-scheduling decisions are made within a single stage. The interaction between crop, area planted and water availability on the ability to supply enough irrigation water on a per hectare basis to produce a non-stressed crop, was disregarded.

Considering the studies identified in this review, the association of dynamic programming with the curse of dimensionality limits the applicability of the identified solution procedures to complex dynamic problems when area-planted and irrigation-scheduling decisions need to be considered in multiple stages when allocating limited water supplies. None of the researchers considered recursive solution techniques that enable one to solve complex water-allocation models that represent the actual manner in which irrigators make irrigation water-allocation decisions in reality. The question, hence, is not whether irrigators should adopt a sequential decision-making framework or not. Rather, the problem is that the methodologies that are currently applied to model irrigation water-allocation decisions do not explicitly represent the aforementioned decision-making processes. Currently, no unified framework exists within a South African context to model the interaction between water availability, irrigation area and irrigation-scheduling decisions as multi-stage sequential decisions. The currently available crop water-use optimisation solution techniques lack complexity, resulting in dynamics of irrigation water use and the associated production risk being only approximated, if not overlooked. Consequently, decision support under limited water supply conditions is hampered. A new line of solution techniques that utilises evolutionary algorithms to optimise complex simulation models offers an alternative technique to solve complex multi-stage irrigation decisions. Evolutionary algorithms are capable of solving complex dynamic models without the complexity of the model rendering the solution infeasible.

Therefore, the main objective of this research is to formulate a multi-stage decision-making framework and compare the results obtained when modelling irrigation water-allocation decisions under such a framework with those obtained within a single-stage decision-making framework under a full water quota and a restricted water quota. Comparing the results of the two decision-making frameworks for the two alternative water quotas will allow for the determination of the impact of modelling irrigation water-allocation decisions within a multi-stage sequential manner on:

- Total gross margin risk and irrigation management decisions.
- The monetary value that will result from the improved modelling of irrigation water-allocation decisions for risk-averse decision-makers.
- The monetary cost of valuing the impact of restricted water use resulting from ignoring the improved modelling of irrigation water-allocation decisions for risk-averse decision-makers.

The paper proceeds with a description of procedures that were followed to develop and solve the bio-economic risk simulation model within both a single-stage and a multi-stage decision-making framework, and the results thereof.

2. Procedures and data

Achieving the main objective of this research required some form of agricultural water-use optimisation. The optimisation procedure used in this research deviates from the normal mathematical programming approaches typically used in SA as it uses the evolutionary algorithms embedded in Excel® to optimise water use. Evolutionary algorithms use random realisations of the decision variables as a basis to evolve to a better solution. Thus, the "optimal" solution is not achieved when optimality conditions are satisfied, and therefore only near-optimal solutions are possible. Applying the evolutionary algorithm to optimise agricultural water requires the development of a bio-economic model that is able to simulate the economic consequences resulting from changes to the key decision variables that need to be optimised. The methods and procedures adopted in this research are mainly based on the research done by Venter, Grové, and Van der Stoep (2017) on the sustainable management of electricity and water use in irrigation farming systems. This research was initiated, managed and funded by the Water Research Commission of South Africa (WRC).

2.1 Bio-economic risk simulation model

A bio-economic model was developed to simulate the impact of different irrigation schedules on crop yield while considering centre pivot technology. The pumping rate of the irrigation system provides a direct link between the water application and the pumping hours necessary to calculate electricity costs. Firstly, to determine the impact of the timing and amount of irrigation on crop yield, the Stewart multiplicative yield response function was used to calculate the resulting yield of maize and wheat for a given irrigation schedule in each state of nature. The Stewart multiplicative formula represents a simple heuristic multiplicative form of a crop water production function model that represents the complex functional relationship between crop yield and consumptive water use estimated by the actual evapotranspiration (ETa) (Stewart et al. 1977). The complexity of the relationship between crop yield and ETa stems from the fact that the independent effects of crop water stress in different periods (weekly, monthly or crop-growth stages) of the growing season differ (Doorenbos and Kassam 1979; Jensen 1968; Rao, Sarma, and Chander 1988). The multiplicative crop water production function hence suggests that crop water deficits in different crop-growth stages may reduce the resulting crop yield in a multiplicative manner. The function is presented by the following equation (De Jager 1994):

$$Y_{c,s} = ym_{c,s} \times \pi_{g=1}^{4}\left(1 - ky_{c,g}\left(1 - \left(\frac{\sum_{t\epsilon g} \mathrm{ETa}_{c,i,s}}{\sum_{t\epsilon g} \mathrm{ETm}_{c,i,s}}\right)\right)\right) \tag{1}$$

where $Y_{c,s}$ is the actual yield for crop c in state of nature s (t/ha); $ym_{c,s}$ represents the maximum (potential) yield for crop c in state of nature s (t/ha); $ky_{c,g}$ is the yield response factor for crop c in growth stage g; $\mathrm{ETa}_{c,i,s}$ is the daily actual crop evapotranspiration for crop c on day i in state of nature s (mm); and $\mathrm{ETm}_{c,i,s}$ represents the daily maximum crop evapotranspiration for crop c on day i in state of nature s (mm).

The effect of crop water stress on yields can be evaluated through the quantification of the relative evapotranspiration deficit (1 − (ETa/ETm)), as shown in the Stewart multiplicative yield response function. Estimating crop evapotranspiration (ET) is hence necessary to determine the relationship between soil moisture stress and crop yield. ET is useful for estimating crop water requirements, as the actual amount of water lost through evapotranspiration represents the amount of water required by the crop to compensate for water deficits through rainfall or irrigation. Subsequently, the scheduling of irrigation events to avoid crop water stress is facilitated. ETa is determined by measuring various components of the soil–water balance through the computation of a daily water budget (Allen et al. 1998). The actual evapotranspiration rate of the crop in each state of nature for a given day was calculated according to the following equation:

$$\mathrm{ETa}_{c,i,s} = \min\Big|^{\mathrm{ETm}_{c,i,s}}_{\mathrm{ETm}_{c,i,s}}\left(\frac{\mathrm{RWC}_{c,i,s}}{\mathrm{TAW}_{c,i,s} - \mathrm{RAW}_{c,i,s}}\right) \tag{2}$$

where $\mathrm{ETm}_{c,i,s}$ is the maximum crop evapotranspiration for crop c on day i under standard condition for state in nature s (mm); $\mathrm{RWC}_{c,i,s}$ is the actual root-water content for crop c on day i in state of nature s (mm); $\mathrm{TAW}_{c,i,s}$ is the total available water for crop c on day i in state of nature s (mm); and $\mathrm{RAW}_{c,i,s}$ is the readily available water for crop c on day i in state of nature s (mm).

ETa retains the minimum value generated between the ETm value and the ETa value under water-stress conditions when the root-zone depletion exceeds the RAW. After the root-zone depletion exceeds RAW, ETa is limited to less than the potential or maximum values, as it begins to decrease in proportion to the amount of water remaining in the root zone. The minimum function hence indicates that ETa cannot exceed the potential or maximum evapotranspiration of a given crop generated under standard conditions. Water deficits in soils, and the resulting water stress on plants, thus influence crop evapotranspiration and, subsequently, crop yield (Kallitsari, Georgiou, and Babajimopoulos 2011). The reduction in ETa below ETm due to soil–water deficits consequently affects the resulting crop yield. Determining the yield-moisture stress relationship hence facilitates the effective scheduling of timing and the amount of irrigation water. Under non-standard conditions, one should hence determine the RWC level where crops do not experience water stress to successfully simulate the timing and amount of irrigation events. The RWC is determined in the daily water budget using the following equation:

$$\mathrm{RWC}_{c,i,s} = \mathrm{RWC}_{c,i-1,s} - \mathrm{ETa}_{c,i,s} + \mathrm{R}_{c,i,s} + \mathrm{IR}_{c,i,s} + \mathrm{DP}_{c,i,s} \tag{3}$$

where $\mathrm{RWC}_{c,i-1,s}$ represents root-water content in the root zone at the end of the previous day, $i-1$ for crop c in state of nature s (mm); $\mathrm{R}_{c,i,s}$ is the rainfall received on day i for crop c for state of nature s, taking surface runoff into account (mm); $\mathrm{IR}_{c,i,s}$ represents the net irrigation depth on day i that infiltrates the soil for crop c in state of nature s (mm); and $\mathrm{DP}_{c,i,s}$ is the water draining below the root zone by deep percolation on day i for crop c in state of nature s (mm).

The RWC accounts for all the water fluxes within the root zone on a daily basis. The actual capacity of the soil–water cannot exceed the root-zone water-holding capacity (RWCAP). Otherwise, the soil drains the water from below the roots by deep percolation (DP). For a detailed description of the necessary equations to compute DP through a daily soil–water budget, the reader is referred to Grové and Oosthuizen (2010).

Given the irrigation amount determined in the daily water budget and the resulting crop yield for each state of nature, the number of hours required daily to pump the irrigation water is determined, taking the design properties of the irrigation system into consideration. The simulation model assumes that irrigation events could occur every consecutive day. Thus, the model implicitly assumes that the irrigation hours could be spread over a two-day period to make better use of the time-differentiated electricity tariff structure of Ruraflex. The crop yields and the pumping hours thus provide the link in the economics module to quantify the economic implications of different irrigation schedules. Thus, any adjustments in the water budget will alter the gross margin, depending on the response of crop yields and total pumping hours. The following equation was used to calculate the distribution of gross margins:

$$\mathrm{GM}_{c,s} = \sum_{c,s} \mathrm{p}_c \times \mathrm{Y}_{c,s} \times \mathrm{A}_c - \sum_{c,s} \mathrm{A}_c \times \mathrm{YDC}_{c,s} - \sum_{c,s} \mathrm{A}_c \times \mathrm{ADC}_c - \sum_{c,s} \mathrm{kWPH}_{c,s} \times \mathrm{IDC}_{c,s} \tag{4}$$

where p_c is the contracted price for crop c (R = South African rand); $\mathrm{Y}_{c,s}$ is the actual yield for crop c in state of nature s (ton/ha); A_c is the area under production of crop c (ha); $YDC_{c,s}$ is the total yield-dependent costs for crop c in state of nature s (R); ADC_c is the total area-dependent costs for crop c (R); $PH_{c,s}$ represents the required pumping hours to irrigate crop c in state of nature s (hours); and $IDC_{c,s}$ is the total irrigation-dependent costs for crop c for state of nature s (R/h).

The crop yield estimations determined in the water budget influence the gross margin through the production income. Yield-dependent costs entail all production costs that change as the yield produced changes, while costs that change subject to a change in the size of the area under production are referred to as area-dependent costs. $IDC_{c,s}$ is a function of total electricity costs, total labour costs, total water costs and total repair and maintenance costs incurred during the production of a crop for a specific irrigation system with a specific kilowatt. The total pumping hours are accounted for within the total electricity costs, which influence the irrigation-dependent costs. For a detailed description of the equations used to calculate area and yield costs, the reader is referred to Venter, Grové, and Van der Stoep (2017).

2.2 Model setup

Secondary economic, agronomic and irrigation-dependent data was used to set up the bio-economic model. All economic data was standardised to conform to the 2016/2017 production season. The research was conducted in Douglas, a town situated close to the convergence of the Vaal and Orange Rivers in the Northern Cape province of SA. Douglas provides a typical location of an irrigation farm where farmers source irrigation water from the Vaal River and the Orange River.

2.2.1 Agronomic data

Agronomic input data includes weather-related, soil, water-allocation, root-growth and yield response factors data. The agronomic data was used for water budget calculations in the model. The economic impact of irrigation water-allocation decisions within single and multi-stage decision-making frameworks was determined for an intra-sessional crop production of maize and wheat. Weather- or climate-related input parameters, such as the reference evapotranspiration (ETo), crop coefficient (Kc) and rainfall, were obtained from SAPWAT3 (South African procedure for estimating irrigation water requirements) (Van Heerden, pers. comm.). Weather data extracted from the V13D weather station was used in SAPWAT3 to estimate the daily ETo, Kc and rainfall for each crop for a growing period of 120 and 148 days for maize and wheat respectively over a period of 49 years. ETm was calculated as a function of the ETo and Kc for each state of nature. A soil with a water-holding capacity (WHC) and a depth of 130 mm/m and 1.2 m respectively was used in the model. The yield response factors (Ky factors) and the length of growth stages (Ky days) proposed by Doorenbos and Kassam (1979) were used in the model.

Weather data was available to define 49 states of nature. However, only 12 states of nature were included in the simulation model to quantify the impact of weather risk on irrigation management decision-making to reduce the dimensionality of the simulation model. The 49 possible weather states were reduced to 12 representative states using cluster analysis (CA). The Ward's hierarchical cluster method proposed by Ward (1963), which generates clusters by minimising the within-group sum of squares, was applied. In each cluster, the state of nature with the minimum sum of squared differences was chosen as a representative state for that cluster. For a detailed description of the CA procedure, the reader is referred to Madende (2017).

2.2.2 Economic data

Some of the economic data utilised in the model was obtained from the cost guide published by the Griekwaland-Wes Korporatief (GWK) (2016). Input data such as the crop price, area and target yield for both maize and wheat was extracted from the enterprise budgets of the cost guide. The maximum potential yield for each crop in each state of nature was calculated using the yield index method. The cost-reduction method developed by Grové (1997) forms the basis of the yield-dependent cost calculations. A scaling factor was used to calculate the actual yield-dependent costs according to a method proposed by Venter, Grové, and Van der Stoep (2017).

2.2.3 Irrigation-dependent input data

The electricity tariffs, water tariff, labour wage rate, irrigation system design, and repair and maintenance data were important inputs in the model for the calculation of irrigation-dependent costs. The Ruraflex tariffs obtained from Eskom (2016/17) were used to calculate the electricity costs given the total pumping hours and the kilowatt usage. The Ruraflex tariff option chosen is based on the transmission zone, ranging between 300 and 600 km, a voltage of less than 500 V, and a monthly utilised capacity ranging between 100 and 500 kVA. A minimum wage determined by the Department of Labour ([DOL] 2016), of R13.37 per hour, was used in the model, accounting for 0.58 labour hours for every 24 h. A repair and maintenance tariff of R0.413217 was used, expressed per 1000 h pumped and based on a method proposed by Meiring (1989).

2.3 Solution procedure

The objective of the model was to optimise the certainty equivalent (CE) of the distribution of gross margins associated with changes in irrigation area and irrigation schedules. The negative exponential utility function was used to calculate CE, assuming constant risk aversion (CARA) (Babcock, Choi, and Feinerman 1993).

The absolute risk-aversion coefficient (r_a) was calculated according to the relationship between $r_a(x)$ and a standardised measure of risk aversion ($r_s(x^s)$). The minimum and maximum levels of $r_s(x^s)$ according to the plausible range for $r_s(x^s)$ of between 0 and 2.5, as determined by Grové and Oosthuizen (2010), were used to calculate the risk-aversion coefficient. The plausible range was adopted, given that most research has reported that the majority of farmers within the South African context are risk averse in nature, rather than risk seeking (Ferrer 1999).

A genetic algorithm (GA) was employed in Visual Basic for Applications (VBA) in Excel® to facilitate the recursive optimisation of the model according to the methodology developed by Blanco and Flichman (2002). Two macros were programmed in Excel® VBA, with one solving the risk model within a single-stage decision-making framework and the other solving within a multi-stage decision framework for both a full water quota and a restricted water quota supply scenario for a risk-neutral and risk-averse decision-maker. The water budget calculations were replicated for each state of nature to simulate the impact of changes in the irrigation schedule on the key output variables and to determine an irrigation schedule that would maximise utility, irrespective of the state of nature that unfolds. Figure 1 depicts the single- and multi-stage decision-making.

Within a single-stage decision-making framework, the assumption is that decisions on both the area irrigated and the irrigation schedule are made for the whole season at the beginning of the season, when the weather for the rest of the season is unknown. As a result, a single optimisation will determine the optimal area and irrigation schedule that will maximise the CE, irrespective of the state of nature occurring, as indicated by the single optimisation in Figure 1.

Within a multi-stage decision-making framework, the area decision is made in the first stage and the irrigation-scheduling decisions are made sequentially as more information becomes available on a weekly basis, as illustrated in Figure 1. The first stage of decision-making within a multi-stage decision-making framework is the same as the first stage within a single-stage decision-making framework where an optimal area and irrigation schedule is generated irrespective of state of nature unfolding. Thereafter, irrigation decisions are made sequentially in multiple stages, taking into cognisance additional weather and soil–water information (ETm and rainfall) as it unfolds.

Excel® Solver implements a genetic algorithm (GA) technique to achieve a near-optimal solution. During the second decision-making stage, denoted by I_2 in Figure 1, the irrigation decision

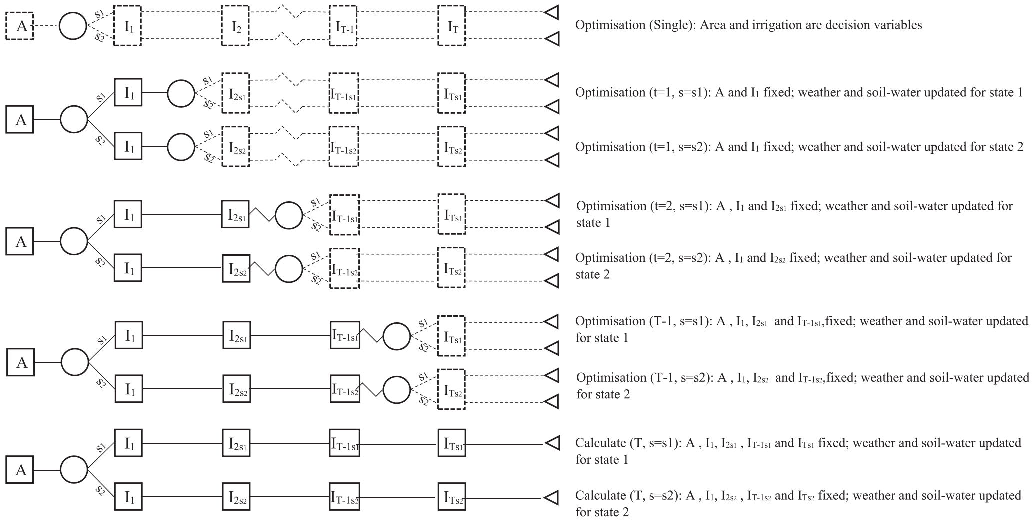

Figure 1. Schematic representation of decision-making within a single-stage and multi-stage decision-making framework, where ▭ represents fixed decisions, ◯ represents possible events to unfold, ⬚ represents optimised decisions, ◁ represents the outcome, **A** represents the area, **I** represents the irrigation decisions, **T** represents the decision stage and **S** represents the possible state of nature to unfold.

determined during the first week (I_1) is fixed and the consecutive irrigation decisions are optimised after updating the ETm and rainfall data of all the other states with that of the state assumed to occur. For instance, if state of nature s_1 is assumed to occur, all states of nature are updated with the ETm and rainfall data of s_1, with the irrigation decision I_1 fixed before optimising decisions of the current week (I_{2s_1}), as shown in Figure 1. The resulting optimised irrigation decisions for that week are hence the same in each state, regardless of s_1 occurring. If s_2 is assumed to occur, all states of nature are updated with the ETm and rainfall data of s_2, with the irrigation decision I_1 fixed before optimising decisions of the current week (I_{2s_2}), as shown in Figure 1. The recursive updating procedure occurs on a weekly basis after each optimisation as commanded by the multi-stage updating macro. It is important to note that the optimised irrigation decision from the previous stage is fixed for each consecutive optimisation. Twelve different optimisations were hence carried out, with each optimisation assuming that one of the twelve states would occur.

The initial solutions generated from the initial optimisations were poor approximations of the global optimal, hence the optimisation is repeated for as long as the subsequent solution is greater than the previous solution. The optimal solution is achieved when the CE generated in the subsequent optimisation is equal to that achieved in the previous solution. In addition, the model was also solved with a different mutation rate to ensure that the solutions that were generated were the best possible solutions.

3. Results

The results for a combined inter-seasonal production of maize and wheat are presented in three sections. The first section presents the results of the gross margin variations and the responses of a decision-maker within a single-stage decision-making framework (SSDF) and a multi-stage decision-making framework (MSDF). Cumulative distribution functions (CDF) are used to quantify the level of gross margin risk faced by a risk-neutral (RN) and a risk-averse (RA) decision-maker within the two alternative decision-making frameworks for both a full water quota (FQ) and a restricted water quota (RQ) scenario. The optimised gross margins (GMs) for each of the 12 states of nature under each water-supply scenario are used to graph the CDF. The second section presents the results of the estimated value or benefit of considering a multi-stage decision-making framework within the two alternative water-supply scenarios and risk preferences of the decision-maker, as indicated by the optimised certainty equivalents. The final section presents the results of the cost of a water restriction.

3.1 Gross margin variability

3.1.1 Single-stage decision-making framework

Figure 2 illustrates the resulting gross margin variations within a single-stage decision-making framework and how these variations alter under a full water quota and a restricted water quota water-supply scenario, taking the risk preferences of the decision-maker into cognisance. The resulting variations of the GMs within an SSDF are attributed to the response of the decision-maker with regard to areas planted, irrigation water use and the resulting crop yields under the two alternative water-supply scenarios and risk preferences. A single average best irrigation schedule is applied over all 12 states of nature. As highlighted in Figure 2, a similar distribution of GMs results under both water-supply scenarios and risk preferences, with a notably lower tail. Irrigation decisions made within an SSDF under an RQ scenario resulted in a similar full area production of 30.1 ha for maize for both an RN and an RA decision-maker, as generated within an FQ scenario. However, wheat production is reduced to 20 and 20.7 ha for a risk-neutral and risk-averse decision-maker respectively, given the restricted irrigation water under an RQ scenario resulting in lower GMs. Hence, a shift of the CDFs to the left is noted within an RQ scenario within both risk preferences. The extreme level

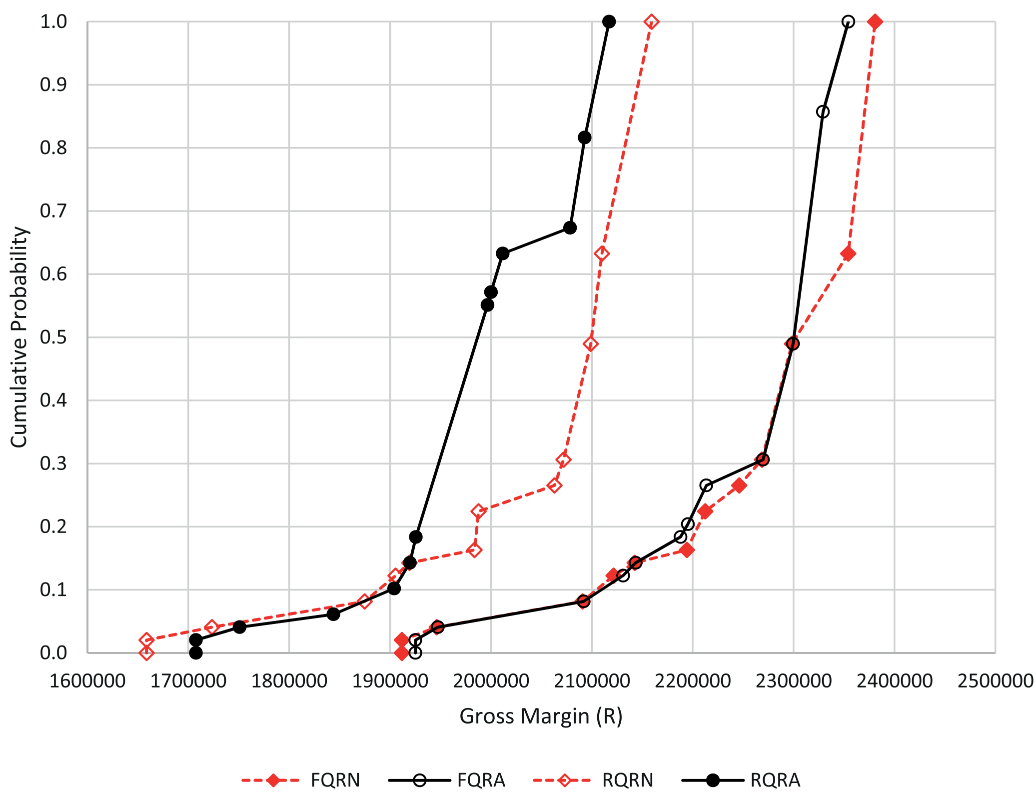

Figure 2. Gross margin variability for a risk-neutral (RN) and risk-averse (RA) decision-maker within a single-stage decision-making framework for a full water quota (FQ) and a restricted water quota (RQ) scenario.

of risk aversion considered in the study resulted in a reduction in average crop yields and irrigation water use when an RA decision-maker was considered within both water-supply scenarios, regardless of water being considered a risk-reducing input.

The positive impact on GMs resulting from lower irrigation-dependent costs was greater than the negative impact owing to lower crop yields for states of nature within the lower tail of the CDFs. A distribution of higher minimum and lower maximum GMs is thus generated for both water-supply scenarios when RA is considered, reflecting the substantial emphasis placed on an improved lower tail of the CDF under RA. Nevertheless, almost maximum, if not maximum potential yields were achieved in each state of nature for both crops for an RN and an RA decision-maker, regardless of the reduced water use under risk aversion.

3.1.2 Multi-stage decision-making framework

The resulting gross margin variations within a multi-stage decision-making framework, and how these variations alter under an FQ and an RQ water-supply scenario taking risk preferences into consideration, are depicted in Figure 3. As aforementioned, the resulting variations in the GMs are attributed to the response of the decision-maker with regard to area planted, irrigation water use and the resulting crop yields under the two alternative water-supply scenarios and risk preferences. A state-specific optimal irrigation schedule was applied in each state of nature.

A similar distribution of expected GMs is noted for both an FQ and an RQ scenario within an MSDF, as depicted in Figure 3. Similar to the results generated within an SSDF, a shift of the CDFs to the left results under risk aversion. It is important to note that the areas under production

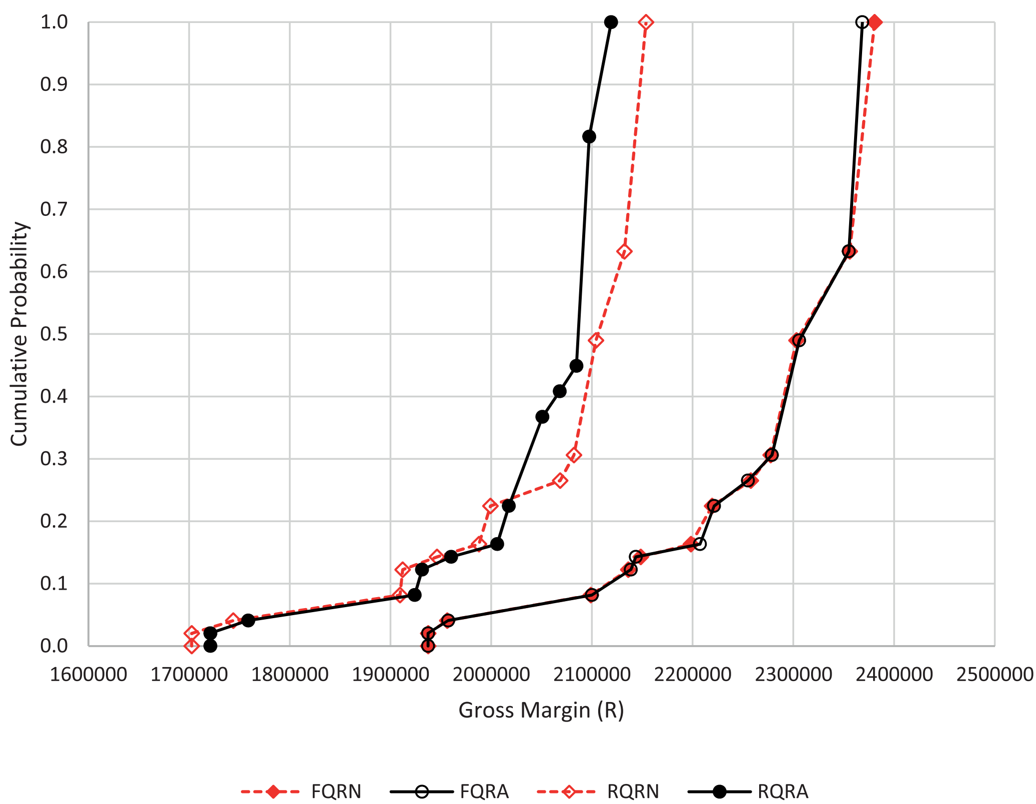

Figure 3. Gross margin variability for a risk-neutral (RN) and risk-averse (RA) decision-maker within a multi-stage decision-making framework (MD) for a full water quota (FQ) and a restricted water quota (RQ) scenario.

for both maize and wheat generated within an SSDF correspond to those generated within an MSDF, as the first stage of the MSDF is the same as a single stage within an SSDF. However, a state-specific irrigation schedule is generated for each state of nature within MSDF. Consequently, irrigation-dependent costs and the successive yields have a significant impact on the resulting GM for each state of nature. As expected under risk aversion, a distribution of slightly higher minimum GMs and lower maximum GMs is generated for a risk-averse decision-maker. A significant reduction in irrigation water applied for each state of nature resulted under risk aversion. A noteworthy trend of relatively higher GMs resulting from reduced irrigation water use, coupled with slight variations in crop yields in each state between the two risk preferences and water-supply scenarios under risk aversion, was hence deduced for states of nature within the lower tail of the CDFs. In contrast, the negative effect of reduced GMs due to lower yields was greater than the positive impact of reduced irrigation-dependent costs due to reduced water use for states of nature within the upper tail of the CDFs.

The positive and negative impacts of the decision-maker's responses on the gross margins for the states of nature within the lower and upper tails of the CDFs respectively when risk aversion is taken into account, are relatively trivial in comparison to those under an SSDF. The slight shift of the lower and upper tails of the CDFs when an RA decision-maker is considered within an MSDF in comparison to that noted within an SSDF elucidates the abovementioned. Thus, the impact of risk aversion indicated by the shifts in the CDF to left is more visible within an SSDF, implying a less significant impact of risk aversion within an MSDF. By implication, decisions made within an MSDF already account for risk.

3.2 The value of a multi-stage decision-making framework

The value of switching to a multi-stage decision-making framework is estimated by comparing the certainty equivalents (CE) of a single-stage decision-making framework to that of a multi-stage decision-making framework for both a risk-neutral and a risk-averse decision-maker under a full and a restricted water quota, as presented in Table 1. As indicated in Table 1, under an FQ the value of an MSDF is R4 261 and R11 149 for a risk-neutral and a risk-averse decision-maker respectively. The value is determined by subtracting the CE generated within an SSDF from that generated within an MSDF. The value represents the minimum sure amount that has to be paid to a decision-maker to justify a switch from making irrigation decisions within an SSDF to within an MSDF. The gain realised within an MSDF under an FQ is attributed to improved irrigation water management from taking additional water budget information into account as sequential irrigation decisions are made over the course of the production season. With the improved risk management within an MSDF as highlighted in Section 3.1.2, the certain minimum amount that both a risk-neutral and a risk-averse decision-maker can receive increases within an MSDF.

The resulting value of an MSDF for the extreme level of risk aversion considered is R6 888 (R11 149 – R4 261) more than that generated under risk neutrality under an FQ. The noteworthy increase in the value of an MSDF is due to the risk-reducing nature of the decision-making framework, hence more favourable for a risk-averse decision-maker. The rand value of switching from an SSDF to an MSDF is R7 019 and R14 413 for a risk-neutral and a risk-averse decision-maker

Table 1. The value of a multi-stage decision framework for a risk-neutral and risk-averse decision-maker under full and restricted water quota scenarios, 2016.

	Full water quota		Restricted water quota	
	Risk neutral	Risk averse	Risk neutral	Risk averse
Single-stage framework certainty equivalent (R)	2,291,835	2,077,144	2,073,516	1,864,631
Multi-stage framework certainty equivalent (R)	2,296,097	2,088,293	2,080,535	1,879,044
Value (R)	4261	11,149	7019	14,413

respectively under a restricted quota water-supply scenario. In other words, the minimum amount of money that a risk-neutral and a risk-averse decision-maker has to receive to consider an MSDF is R7 019 and R14 413 respectively. Nonetheless, irrigation decisions made within an MSDF have a greater value if water supply is restricted compared to a full quota scenario for both a risk-neutral and a risk-averse decision-maker, as highlighted in Table 1. In addition, the value of an MSDF for a risk-averse decision-maker under an RQ is R7 394 (R14 413 – R7 019) greater than that generated under risk neutrality, and is also greater than the difference between the value under risk neutrality and risk aversion of R6 887, as noted under an FQ. Generally, efficient and effective irrigation management is more beneficial under restricted water conditions, as the responses of the decision-maker will have an impact on the resulting GMs, hence the greater value of an MSDF under a restricted water quota.

3.3 The cost of a water restriction

To determine the cost of a water restriction, the certainty equivalents generated within the two alternative decision-making frameworks under a full quota and a restricted quota water-supply scenario are compared for a risk-neutral and a risk-averse decision-maker. The results of the cost of a water restriction are presented in Table 2. Given that the certain amount that a risk-neutral decision-maker receives reduces significantly when a water restriction is enforced, the cost of a water restriction is represented by the difference between the CE generated under a full water quota and that under a restricted water quota for each decision-making framework. Table 2 indicates the cost of a water restriction for an SSDF of R218 319, which is greater than that of R215 561 faced within a multi-stage decision-making framework for a risk-neutral decision-maker. By implication, the cost of a water restriction is over-estimated within an SSDF. The true quantification of risk is hence imperative if the true cost of a water restriction is to be determined.

Likewise, a greater cost of a water restriction of R212 513, results within an SSDF in comparison to the R209 249 generated within an MSDF under risk aversion, as depicted in Table 2. An over-estimation of the cost of a water restriction is hence also noted for a risk-averse decision-maker. Nonetheless, the costs of a water restriction when risk aversion is considered are lower than under risk neutrality for both an SSDF and an MSDF. The lower costs can be attributed to the fact that risk-averse decision-makers already make conservative decisions, hence a water restriction will have a relatively lesser impact on such a decision-maker.

The cost differences between the SSDF and the MSDF are small because the weather states included in the model were assumed to be constant between both water quotas. Consequently, the impact of water restrictions might be under-estimated, since the weather states were not correlated with the prevailing conditions associated with water restrictions.

4. Conclusion

Higher gross margins resulted when irrigation decisions were modelled within a multi-stage decision-making framework in comparison to that under a single-stage decision-making framework. The overall increase in gross margins for each state of nature within a multi-stage decision-making

Table 2. Certainty equivalents for each water-supply scenario for the two alternative decision-making frameworks for a risk-neutral and a risk-averse decision-maker, 2016.

	Risk neutral			Risk averse		
	Full quota	Restricted quota	Cost (R)	Full quota	Restricted quota	Cost (R)
Single-stage framework certainty equivalent (R)	2,291,835	2,073,516	218,319	2,077,144	1,864,631	212,513
Multi-stage framework certainty equivalent (R)	2,296,097	2,080,535	215,561	2,088,293	1,879,044	209,249

framework is attributed to the lower irrigation-dependent costs incurred, given the reduced per-state irrigation water use. The updating of additional water budget information as it becomes available with irrigation decisions made sequentially reflects the true risk that a decision-maker faces, and hence the higher expected gross margins resulting from improved irrigation water scheduling.

Risk aversion does not significantly affect the resulting area and yields under a full water quota. However, irrigation water use is significantly reduced in an effort to increase the minimum outcomes. Although water is considered a risk-reducing input, the extreme level of risk aversion used resulted in a reduction in irrigation water applied in an effort to improve the minimum expected values as irrigation-dependent costs decreased. Under a restricted water quota, significant differences are noted between yields generated for a risk-neutral and a risk-averse decision-maker, hence implying the noteworthy impact of risk aversion on the responses of a decision-maker when water supply is restricted. The interaction between crop, area planted and water availability facilitated during multi-stage decision-making is hence more significant when water supply is restricted.

In the light of the noted responses of decision-makers and the corresponding variations in gross margins, the conclusion is that modelling irrigation decisions within a multi-stage decision-making framework is worthwhile considering its resulting value when the certainty equivalents of the two alternative decision-making frameworks are compared. Also, the utility weighted risk premium of a multi-stage decision-making framework is more significant under restricted water supply, hence it is imperative to consider such a framework given the worsening problem of water scarcity. Modelling of irrigation decisions within a multi-stage decision-making framework also provides a true reflection of the value of the certainty equivalents and can significantly guide irrigation-scheduling decisions.

The results of the costs of a water restriction confirm the notion under which the aim of this study was constructed, namely that modelling irrigation decisions within a single-stage decision-making framework, considering area and irrigation-scheduling decisions as one decision, will lead to an over-estimation of the cost of a water restriction.

In this case, the cost of a water restriction is over-estimated within a single-stage decision-making framework, which might result in imprecise modelling of irrigation decision tools, especially under restricted water scenarios taking risk preferences into account. Caution is hence necessary when formulating agricultural water-allocation policies based on crop water optimisation models that ignore the multi-stage decision-making framework within which irrigation decisions are made. Ignoring modelling irrigation decisions as sequential dynamic decisions results in over-estimating the impact of any given policy on water-use management. It is therefore critical to analyse farm-level profitability within a framework that better represents farmers' decision-making to provide policy decision-makers with improved decision-making tools.

The characterisation of weather risk was done using historical data. The results could be further improved by integrating the multi-stage decision-making framework MSDF with weather forecasts that take cognisance of the prevailing weather conditions when the actual decisions are made.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Water Research Commission (WRC) of South Africa [grant no. K5/2279], 'The optimisation of electricity and water use for sustainable management of irrigation farming systems'. The views expressed by the authors do not necessarily reflect those of the WRC.

References

Allen, R.G., L. Pereira, D. Raes, and M. Smith. 1998. Crop evapotranspiration: Guidelines for computing crop water requirements. FAO Irrigation and Drainage Paper No 56, FAO, Rome, Italy.

Babcock, B.A., E.K. Choi, and E. Feinerman. 1993. Risk and probability premiums for CARA utility functions. *Journal of Agricultural and Resource Economics* 18: 17–24.

Benadé, N. 2011. Development of training material and providing training for implementation of the water use efficiency accounting report of the water administration system. WRC Report No KV283/11, Water Research Commission, Pretoria, South Africa.

Blanco, M., and G. Flichman. 2002. Dynamic optimisation problems: different resolution methods regarding agricultural and natural resource economics. Proceedings of the international conference on risk and uncertainty in environmental economics, March, in Wageningen University, Wageningen, The Netherlands.

Botes, J.H.F. 1990. An economic evaluation of wheat irrigation scheduling strategies using stochastic dominance (Afrikaans). MSc thesis, University of the Free State, Bloemfontein, South Africa.

Botes, J.H.F., D.J. Bosch, and L.K. Oosthuizen. 1996. A simulation and optimisation approach for evaluating irrigation information. *Agricultural Systems* 51: 165–83.

Bryant, K.J., J.W. Mjelde, and R.D. Lacewell. 1993. An intra-seasonal dynamic optimisation model to allocate irrigation water between crops. *American Journal of Agricultural Economics* 75, no. 4: 1021–9.

Chen, S., D. Shao, W. Gu, B. Xu, H. Li, and L. Fang. 2017. An interval multi-stage water allocation model for crop different growth stages under inputs uncertainty. *Agricultural Water Management* 186: 86–97.

Dai, Z.Y., and Y.P. Li. 2013. A multistage irrigation water allocation model for agricultural land use planning under uncertainty. *Agricultural Water Management* 129: 69–79.

De Jager, J.M. 1994. Accuracy of vegetation evaporation ration formulae for estimating final wheat yield. *Water SA* 20, no. 4: 307–14.

Department of Labour (DOL). 2016. *Minimum wage for farm worker sector with effect from 1 March 2016*. Pretoria: Department of Labour.

Department of Water and Sanitation (DWS). 2017. *National water and sanitation master plan: Ready for the future and ahead of the curve, draft 2.6*. Pretoria: Department of Water and Sanitation.

Department of Water and Sanitation (DWS). 2018. *National water and sanitation master plan, volume 1: Ready for the future and ahead of the curve*. Pretoria: Department of Water and Sanitation.

Doorenbos, J., and A.H.I. Kassam. 1979. *Yield response to water*. Rome: Food and Agriculture Organization of the United Nations.

Eskom. 2016/17. *Tariffs and charges booklet*. http://www.eskom.co.za/content/2015_16DraftTariffBookver3.pdf (accessed November 29, 2016).

Ferrer, S.R.D. 1999. Risk preferences and soil conservation decisions of South African commercial sugarcane farmer. PhD thesis, University of Natal, Pietermaritzburg, South Africa.

Gakpo, E.F.Y., J. Tsephe, F. Nwonwu, and M.F. Viljoen. 2005. Application of stochastic dynamic programming (SDP) for the optimal allocation of irrigation water under capacity sharing arrangements. *Agrekon* 44, no. 4: 436–51.

Griekwaland-Wes Korporatief (GWK). 2016. *Input costs guide*. Douglas: GWK.

Grové, B. 1997. Modellering van die ekonomiese effekte van wisselvallige waterbeskikbaarheid vir besproeiingsboere in die Wintertongebied met inagneming van minimum binnestroomvloeivoorsiening. MSc dissertation, University of the Free State, Bloemfontein, South Africa.

Grové, B., and L.K. Oosthuizen. 2010. Stochastic efficiency analysis of deficit irrigation with standard risk aversion. *Agricultural Water Management* 97, no. 6: 792–800.

Haile, B.O. 2017. An economic analysis of salinity management with evolutionary algorithms in Vaal Harts. PhD thesis, University of the Free State, Bloemfontein, South Africa.

Haile, B.O., B. Grové, J. Barnard, and L.D. van Rensburg. 2014. Exploring genetic algorithms for improved salinity management using SWAMP. Proceedings of the 6th South African National Commission on irrigation and drainage symposium, November 18–20, in Glenburn Lodge.

Jensen, M.E. 1968. Water consumption by agricultural plants. In *Water deficits and plant growth, vol. 2*, ed. T.T. Kozlowski, 1–22. New York: Academic Press.

Kallitsari, C., P. Georgiou, and C. Babajimopoulos. 2011. Evaluation of crop water production functions under limited soil water availability with SWBACROS model. In *Proceedings of the European Federation for Information Technology in Agriculture, Food and the Environment World Congress on computers in agriculture*, ed. E. Gelb and K. Charvát, 585–96, July 11–14, in Prague, Czech Republic.

Karrou, M., and T. Oweis. 2012. Water and land productivities of wheat and food legumes with deficit supplemental irrigation in a Mediterranean environment. *Agricultural Water Management* 107: 94–103.

Kusunose, Y., and R. Mahmood. 2016. Imperfect forecasts and decision making in agriculture. *Agricultural Systems* 146: 103–10.

Li, Y.P., and G.H. Huang. 2011. Planning agricultural water resources system associated with fuzzy and random features. *Journal of the American Water Resources Association* 47, no. 4: 841–60.

Madende, P. 2017. Risk efficiency of optimal water allocation within a single and multi-stage decision-making framework. MSc dissertation, University of the Free State, Bloemfontein, South Africa.

Meiring, J.A. 1989. 'n Ekonomiese evaluering van alternatiewe spilpuntbeleggingstrategieë in die Suid-Vrystaat substree met inagneming van risiko. MSc dissertation, Department of Agricultural Economics, University of the Orange Free State, Bloemfontein, South Africa.

Parsinejad, M., A.B. Yazdi, S. Arahhinejad, A.P. Nejadhashemi, and M.S. Tabrizi. 2012. Optimal water allocation in irrigation networks based on real-time climatic data. *Agricultural Water Management* 117: 1–8.

Rao, N.H., P.B.S. Sarma, and S. Chander. 1988. A simple dated water production function for use in irrigated agriculture. *Agricultural Water Management* 13: 25–32.

Robert, M., J. Dury, A. Thomas, O. Theron, M. Sekhar, S. Badiger, L. Ruiz, and J. Bergez. 2016. CMFDM: A methodology to guide the design of a conceptual model of farmers' decision-making processes. *Agricultural Systems* 148: 86–94.

Robert, M., A. Thomas, M. Sekhar, S. Badiger, L. Ruiz, H. Raynal, and J. Bergez. 2017. Adaptive and dynamic decision-making processes: A conceptual model of production systems on indian farms. *Agricultural Systems* 157: 279–91.

Stewart, J.I., R.E. Danielson, R.J. Hanks, E.B. Jackson, R.M. Hagan, W.O. Pruitt, W.T. Franklin, and J.P. Riley. 1977. *Optimizing crop production through control of water and salinity levels in soil*. Logan: Utah Water Research Laboratory.

Van Heerden, P.S., C.T. Crosby, B. Grové, N. Benade, R.E. Schulze, and M.H. Tewolde. 2009. Integrating and updating of SAPWAT and PLANWAT to create a powerful and user-friendly irrigation-planning tool. WRC Report No. TT391/08, Water Research Commission, Pretoria, South Africa.

Venter, M., B. Grové, and I. Van der Stoep. 2017. The optimisation of electricity and water use for sustainable management of irrigation farming systems: Main report. WRC Report No. TT717/17, Water Research Commission, Pretoria, South Africa.

Ward, J.H. 1963. Hierarchical grouping to optimise an objective function. *Journal of the American Statistical Association* 58: 236–44.

Zeng, X.T., S.Z. Kang, F.S. Li, L. Zhang, and P. Guo. 2010. Fuzzy multi-objective linear programming applying to crop area planning. *Agricultural Water Management* 8: 134–42.