Nonparametric Estimation of Multiproduct and Product-Specific Economies of Scale

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Selected Paper, SAEA Annual Meetings
February 14-18, 2004, Tulsa, OK

Keywords: nonparametric estimation, scale economies, scope economies, multiproduct, product-specific

Introduction

In recent years, numerous studies have utilized nonparametric methods to analyze efficiency in various industries (for example see Banker and Maindiratta, Jaforullah and Whiteman, Chavas and Cox). In such studies, several types of efficiency are generally estimated to determine if a firm is producing on the production or cost frontier, whether the firm is optimally allocating inputs, or if the firm is operating at the most efficient size. Additionally, Chavas and Aliber developed a nonparametric method to measure scope economies. The nonparametric approach, in both contexts, has several desirable attributes. The most notable is that it is not necessary to restrict the technology to a specific functional form. Furthermore, the approach can be easily modified to deal with multiple products and multiple inputs. These qualities have been cited as reasons to opt for nonparametric estimation in lieu of a traditional econometric approach.

Much of the existing applied duality work has concentrated on parametric estimation of multiproduct economies of scale and product-specific economies of scale (for example see Aigner, Lovell, and Schmidt). There has been less effort to compare results from the nonparametric method with the parametric method and determine if the approaches yield comparable results. In fact, a method for estimating product-specific economies of scale using the nonparametric methods does not appear in the literature to our knowledge. Empirically, nonparametric estimation is desirable since the mathematical programs that estimate efficiency measures are linear. This avoids many of the solver difficulties encountered when complex functions are specified and empirically estimated for multiple inputs and outputs. This benefit of the nonparametric approach, however, raises a seldom-addressed concern.

The linear models used to estimate efficiency measures approximate a production or cost frontier. These frontiers are composed of multiple linear segments as opposed to the smooth curve that is assumed econometrically. Representing the cost or production function in such a way results in a reliable approximation with one exception. If the firm in question is producing at a point such that they are located on a "kink" (where two linear segments of different slope join) in the frontier then the marginal cost estimation needed for estimation of economies of scale and product-specific economies of scale are not unique. This is because the function is not continuous at that point and, therefore, no partial derivative (which would define marginal cost) exists. In addition to the formal specification and presentation of nonparametric estimation of multiproduct and product specific economies of scale, this study will offer an approach to recognize when this situation occurs.

This study has three major objectives. The first of these is the specification and presentation of the mathematical programming models necessary for nonparametrically estimating product-specific economies and multiproduct economies of scale. The next objective is the application of these models to a sample dataset. This will include identifying the aforementioned points where marginal costs are not unique. The data will also be used to parametrically calculate multiproduct scale and product-specific scale measures assuming a quadratic cost function. This will provide a comparison of the two methods.

Data and Methods

The data were collected from 106 Kansas farms in 1998. These farms are enrolled in the Kansas Farm Management Association Program. The data set contains two outputs (crops and livestock) and seven input measures (machinery, seed, fertilizer and pesticides, feed, energy, family and hired labor, and land and structures). Price indices of inputs and outputs were obtained from the U.S. Department of Agriculture's *Kansas Farm Facts* and *Agricultural Outlook*. Output quantities were obtained by dividing accrual revenue from farming by the corresponding prices. Descriptive statistics of the data are presented in Table 1. This cross section of farm output gives a snapshot of the productivity of Kansas crop and livestock farms. If it is assumed that all farms face the same technology, that is the same production function, then the scale and scope efficiency of these farms can be compared.

Calculation of Cost Measures

The first step toward calculating scale efficiency measures is the determination of the minimum costs of production. This study will build on the methodology used by Fare, Grosskopf, and Lovell to calculate other efficiency (i.e., technical, allocative) measures. Minimum cost of production for the ith are calculated beginning with the following model.

$$(1)\ \text{Min} \sum_{n} w_{i,n} x_i^* = C_{i,\text{all}}$$

s.t.

$$(2)\ \sum_{k} x_{k,n} z_k \leq x_i^* \ \forall n$$

$$(3)\ \sum_{k} y_{k,p} z_k - y_{i,p} \leq 0 \ \forall p$$

$$(4)\ \sum_{k} z_k = 1$$

In this formulation, there are k firms (k=106), n inputs and p outputs. The decision variable is x_i^*. The costs of the inputs for the ith firm are represented by $w_{i,n}$. The objective function value ($C_{i,all}$) is the minimum attainable cost to produce all outputs at the observed level of firm i.

The next step is to calculate the incremental cost of producing each output. The model is solved omitting one output constraint. The resulting objective function value is the cost of producing all outputs except the one omitted. For the sake of simplicity, consider the two-output example application presented in this paper. If the livestock constraint is omitted the objective function value is the cost of producing crops alone ($C_{i,c}$). Likewise, if crops constraint is omitted the result is the cost of producing livestock ($C_{i,l}$). This approach, however, need not be limited to two outputs.

Incremental cost of producing output n is then calculated by subtracting from C_{all} the value of producing all outputs except output n. Again, for simplicity consider the case of the crop and livestock farm. Incremental costs of the outputs (l and c) for firm i are calculated as follows.

(5) $IC_{i,l} = C_{i,all} - C_{i,c}$

(6) $IC_{i,c} = C_{i,all} - C_{i,l}$

This measure is the cost to firm i resulting from the production of a given product. It is necessary to now calculate the average (or per unit) adjusted incremental cost ($AAIC_{i,p}$).

(7) $$AAIC_{i,p} = IC_{i,p} \Big/ \left(y_{i,p} - \sum_k z_k^p y_{k,p} \right)$$

$AAIC_{i,p}$ is "adjusted" in the denominator by the predicted output level. This is due to the fact that even when a constraint for an output is removed the model may predict that some of the output is produced. Therefore, incremental cost measures are not truly for the amount of observed output, but rather a smaller amount. Neglecting to perform this adjustment would result in understating per-unit incremental costs. In addition to $AAIC_{i,p}$ the marginal cost of producing one more unit of output is known. This marginal cost for each output ($MC_{i,p}$) is simply the shadow price on the relevant output constraint (Equation 3).

Multiproduct and Product-specific Economies of Scale

To arrive at a measure of product-specific economies of scale, $AAIC_{i,p}$ and $MC_{i,p}$ must be compared. Specifically the ratio of the former to the latter is evaluated.

$$(8)\quad PSE_{i,p} = \frac{AAIC_{i,p}}{MC_{i,p}}$$

$PSE_{i,p}$ is interpreted as a typical scale measure. That is, if $PSE_{i,p} = 0.5$, doubling (i.e., increasing by 100%) the production of output p would increase production costs of p by only 50%. A multiproduct economies of scale measure (MSE_i) can also be calculated.

$$(9)\quad MSE_i = \frac{C_{all}}{\sum_p MC_{i,p}\, y_{i,p}}$$

The interpretation MSE_i of is same as that of $PSE_{i,p}$ except in this case an equal increase or decrease of all outputs and their production cost is considered.

The scale measures indicate whether a firm might benefit from expanding its enterprises. Assuming a competitive market, increasing production will result in revenue being increased by a proportionate amount. A scale measure of less than one indicates that the firm's increase in costs from the expansion will be less than the increase in revenue, proportionally. Therefore, a scale measure of less than one indicates a firm should consider expanding. If the measure is a product-specific one the interpretation applies only to the relative output and if it is a multiproduct measure the interpretation applies to all outputs.

Economies of Scope

Since minimum costs for producing each input both separately and jointly have been obtained, a measure for economies of scope (SC_i) is easily attained.

$$(10)\quad SC_i = \frac{\left(C_{i,c} + C_{i,l} - C_{i,all}\right)}{C_{i,all}}$$

If SC_i is greater than zero then economies of scope exist for firm i. In other words, it is less expensive to producer all outputs jointly than individually. The cost benefit of joint production is represented by the reciprocal of 1+ SC_i. For example, if $SC_i = 0.5$ then producing all outputs jointly can be done at two-thirds the cost of doing so separately.

Application to Kansas Farms

The methodology presented in the previous section was used to calculate scale and scope measures using the dataset of a set of Kansas farms producing crops and livestock described in that section. All 106 farms were used to specify the programming model shown in Equations 1 through 4. The model was solved including both crops and livestock and then once including each output individually. At this point, concerns about the methodology outlined at the beginning of this paper were addressed.

Any farm i for which $z_k = 1$ where i=k was deemed to have non-unique marginal cost measures. Such a result indicates that the most efficient method of production is the farm's own method. That means the farm is already on the cost frontier. Also, since this cost frontier is made up of linear segments, an optimal solution on the frontier must be at a corner or "kink" in the frontier where two segments join. Seven farms met these criteria.

Sixteen farms had a marginal cost of zero for producing crops. In other words, the shadow price on Equation 3 where p=crops was zero. In general these farms were very large in terms of land but had few crops planted and seemed to use substantially more of certain inputs relative to other farms. The unused resources and inefficient use of inputs make expanding the crop production appear to be simply a matter of better management, which bears no cost in this framework. This is problematic, in that, a marginal cost of zero will not allow for most measures to be calculated. Another issue was that twenty-six farms produced only crops in 1998, which makes the scope measure almost meaningless. There was some overlap among these three problem categories. In total, 44 farms fell into at least one category. These 44 farms are not included in the results reported in this study. However, the resulting calculations for these farms are available upon request from the authors. After dropping these 44, there were 62 farms with all of the quantifiable measures outlined in the previous section.

Since there are such a large number of farms the scale and scope results will be presented as averages of groups of farms. These groups are formed by ranking farms in ascending order according to the economic measure of interest and dividing the farms into 10 roughly equal groups. The full set of results can be obtained from the authors. The results of the measure of product-specific economies of scale for livestock (LSE)[1] are shown in Table 2. Note that excepting for the first and last groups, standard deviations are very low, which indicates that we do no lose much qualitative value in presenting the results in this manner. Measures of land, crops produced, and livestock produced are also presented to give an indication of the average size of farms in each group.[2]

The majority of livestock producers in this sample could actually expand in a cost-efficient manner. A cursory evaluation of the results reveals that farms with an average livestock output of 1000 to 1500 would actually be the most efficient in expansion, while those with an output of over 1600 will experience constant returns to scale in terms of cost. It is also

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[1] Notice that this is the equivalent of $PSE_{i,p}$ (where p=livestock), as presented in the methodology section. It is simply a notational convenience to drop output subscript and since results are presented for groups of firms, the firm subscript is not useful. The same convention will be used for crop-specific economies of scale (CSE) and multiproduct economies of scale (MSE).

[2] While gross revenue might be a more appropriate measure of farm size the data used here are limited to farm characteristics (land, livestock output, crop output). These should, however, be highly correlated with gross revenue and, therefore, serve as a reasonable proxy for farm size.

interesting that these large livestock producers with an LSE of 1.00 have very few crops. This is a possible indication that these groups have specialized in livestock production and have become very efficient at it, reaching (or nearly reaching) their maximum efficient size.

The picture for expanding crop enterprises is very similar. These results are presented in Table 3. Over 60% of farms would apparently benefit from planting more crops. The distribution of farm size is also similar. There are large farms (in terms of average crop production and land) at the low end of the crop-specific economies of scale (CSE) ranking and at the high ranking with smaller farms in between. It is also apparent that groups with a CSE of 1.0 generally have a low output of livestock. It seems that once again, those farms specializing have reached their limit for efficient expansion while some of the more diverse operations could benefit from expanding.

Table 4 offers the multiproduct economies of scale (MSE) results. About 70% of the farms would not benefit by simultaneously expanding both livestock and crop enterprises. That is, costs would increase more, proportionally, than revenue. It is also interesting to note that, in general, it is bigger farms that would benefit from expansion. For example, the group with the lowest MSE also has the third largest amount of land and the second largest production of crops and livestock of all groups. The group with the highest MSE, however, ranks last or next to last in all three size measures. It appears that the larger operations have the advantage in across-the-board expansion. This result is contrary to findings of Chavas and Aliber in their nonparametric analysis of Wisconsin crop and livestock farms.

The last measure to be considered is a measure of economies of scope (SC). These are presented in Table 5. All farms are better off by producing crops and livestock as opposed to producing each separately. However, it seems that the benefit is greater for smaller farms. This is somewhat intuitive. A farm that produces a lot of livestock and crops is obviously putting a lot of effort into each enterprise. Over time there will be specializations in each that do not complement the other. This will result in the two operations being less related. On the other hand, smaller farms might still practice procedures such as raising all their own feed and so on. For these producers, the enterprises are very closely related and to separate them would increase costs dramatically.

It is also important to notice the relationship between all these measures. One way to get that relationship is to measure correlation. Correlations between all the farm size measures and all the economic measures (and all possible combinations of each group) are shown in Table 6. Qualitatively, these correlations are consistent with the discussion offered in this section. That is, all scale measures (with the exception of LSE to Corn) are negatively related to the size measures. The relationship of LSE to all size measures is weaker that those of CSE and MSE. It is very interesting that SC has a positive correlation with LSE, CSE, and MSE. This, combined with the fact that SC is negatively related to all size measures, would indicate that farms that would likely benefit more (relative to other farms in this study) by expansion are realizing less benefit from producing their outputs jointly. This is consistent with the reasoning that larger, specialized farms will benefit from expanding but do not benefit a great deal from producing crops and livestock jointly. This result is also consistent with the relationship between SC and farm size reported in the Chavas and Aliber study referenced earlier.

Conclusions

The nonparametric approach to measuring economies of scale and scope can readily be expanded to firms producing more than one output. This framework is relatively easy to solve empirically and does not impose functional restrictions on the cost measures. The application of that approach in this study reveals many things about Kansas crop and livestock farms.

In general, larger farms will find it easier to expand operations. That is, they experience decreasing returns to scale in regards to cost. It is also true that larger farms realize less benefit from jointly producing crops and livestock instead of doing so separately. In other words, the larger producers do not enjoy economies of scope to the degree that smaller producers do. While these results are logically sound, it is wise to introduce some caution in broadly applying them. For example, in this approach we must implicitly assume that all units of inputs are equally productive. For inputs like land or machinery, it is obvious that for some farms this will be an incorrect assumption. That is likely why there is so much variability in the size measures within groups as the results were presented.

Addressing the issue of non-unique marginal costs should bolster confidence in the results from nonparametric analysis. This eliminates a relevant problem that often goes unmentioned in this body of literature. By doing just that and by offering an approach to quantifying product-specific economies of scale, the methodology presented herein offers a useful, readily applicable extension to the existing nonparametric efficiency analysis tools.

References

Aigner, D.J., C.A.K. Lovell, and P. Schmidt. "Formulation and estimation of Stochastic Frontier Production Function Models." *Journal of Econometrics*, 6(1977): 21-37.

Banker, Rajiv D. and Ajay Maindiratta. "Nonparametric Analysis of Technical and Allocative Efficiencies in Production." *Econometrica*, 56,69(November 1988): 1315-1332.

Chavas, Jean-Paul and T.L. Cox. "A Nonparametric Analysis of Agricultural Technology." *American Journal of Agricultural Economics*, 70(1988): 303-310.

Chavas, Jean-Paul and M. Aliber. "An Analysis of Economic Efficiency in Agriculture: A Nonparametric Approach." Journal of Agricultural and Resource Economics, 18(1993): 1-16.

Fare, R., S. Groskopf, and C.A.K Lovell. *The Measurement of Efficiency of Production*, Boston: Kluwer-Nijhoff, 1985.

Jaforullah, Mohammad and John Whiteman. "Scale Efficiency in the New Zealand Dairy Industry: A Non-parametric Approach." *The Australian Journal of Agricultural and Resource Economics*, 43(4): 523-541.

Table 1. Summary Statistics of Farm Production Levels and Prices

	Mean	St. Dev.	C.V.	Min	Max	Price*
Outputs						
Livestock	504.87	951.11	188.39%	0.00	7022.13	154.00
Crops	1011.02	882.82	87.32%	67.85	4696.54	130.00
Inputs						
Seed	69.67	77.83	111.51%	0.00	374.98	194.00
Fertilizer	122.28	102.44	83.78%	10.58	482.45	155.00
Chemicals	98.64	115.36	117.17%	0.00	600.36	173.00
Feed	272.03	631.15	232.01%	0.00	3621.38	130.00
Fuel	63.25	53.75	84.98%	1.24	260.73	189.00
Labor	53.19	97.12	182.61%	0.00	600.59	253.00
Land	1725.94	1221.90	70.80%	240.00	8393.00	35.50
Machinery	179.43	147.60	82.26%	10.85	785.27	271.00

*Prices are constant since the data are taken from a single year.

Table 2. Farm Characteristics and Livestock-Specific Economies of Scale (LSE)

Range	LSE		Land		Livestock		Crops	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
1-6	0.53	0.06	1861.67	653.13	1233.05	644.26	417.39	448.22
7-12	0.63	0.02	2316.83	990.16	1056.91	323.97	644.07	587.19
13-18	0.68	0.02	2568.83	1045.49	1555.59	980.03	957.68	555.39
19-24	0.73	0.01	2185.50	1517.47	1127.71	583.19	860.26	660.05
25-30	0.76	0.01	1874.17	1082.53	1140.12	878.95	1087.96	918.41
31-36	0.80	0.02	1999.67	1392.02	976.17	1281.40	633.57	729.22
37-42	0.86	0.03	1977.50	1984.78	443.72	210.38	794.84	965.81
43-48	0.95	0.04	2158.50	3103.02	406.18	200.71	837.03	1785.93
49-54	1.00	0.00	2214.33	685.03	1602.48	895.04	50.56	31.05
55-62	1.00	0.00	1768.50	658.46	1882.85	788.78	134.86	74.08

Note: Range is the number of the farms (ranked in ascending according to the LSE measure) upon which descriptive were based. Crops and Livestock refer to the observed output of crop and livestock for the farms.

Table 3. Farm Characteristics and Crop-Specific Economies of Scale (CSE)

Range	CSE		Land		Livestock		Crops	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
1-6	0.66	0.10	3583	898	2754	492	1469	506
7-12	0.90	0.02	1520	745	1027	768	102	72
13-18	0.92	0.00	1304	389	796	411	242	154
19-24	0.94	0.00	1798	1033	1438	1063	260	150
25-30	0.95	0.00	2419	942	1160	550	269	122
31-36	0.96	0.01	1655	301	1074	271	204	81
37-42	0.99	0.01	1781	981	1467	1052	19	17
43-48	1.00	0.00	1575	1106	906	533	1255	987
49-54	1.00	0.00	2941	2803	891	418	1649	1477
55-62	1.00	0.00	2205	1701	404	97	744	659

Note: Range is the number of the farms (ranked in ascending according to the CSE measure) upon which descriptive were based. Crops and Livestock refer to the observed output of crop and livestock for the farms.

Table 4. Farm Characteristics and Multiproduct Economies of Scale (MSE)

Range	MSE		Land		Livestock		Crops	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
1-6	0.72	0.09	3005	1142	2453	1127	1454	519
7-12	0.91	0.02	3124	1497	820	358	1442	259
13-18	0.94	0.02	3494	2730	1336	473	2191	1191
19-24	1.01	0.00	1932	688	2525	530	138	88
25-30	1.02	0.00	1995	698	1363	285	74	49
31-36	1.04	0.01	2107	1286	901	339	240	224
37-42	1.05	0.00	1638	829	977	100	325	99
43-48	1.07	0.01	1834	333	804	79	228	31
49-54	1.10	0.02	1273	368	444	122	244	155
55-62	1.22	0.07	837	510	323	52	95	63

Note: Range is the number of the farms (ranked in ascending according to the MSE measure) upon which descriptive were based. Crops and Livestock refer to the observed output of crop and livestock for the farms.

Table 5. Farm Characteristics and Economies of Scope (ES)

Range	SC		Land		Livestock		Crops	
	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
1-6	0.08	0.03	4502	2434	2268	1344	1946	1454
7-12	0.15	0.01	2145	943	1559	1146	930	919
13-18	0.17	0.01	2492	600	2081	551	762	827
19-24	0.21	0.01	2200	817	1204	343	949	737
25-30	0.25	0.02	2726	1367	1451	346	827	549
31-36	0.30	0.01	1853	810	1020	123	338	126
37-42	0.34	0.01	1542	408	801	265	267	114
43-48	0.38	0.02	1586	459	716	172	219	81
49-54	0.47	0.04	1481	739	537	109	136	91
55-62	0.73	0.11	742	367	311	45	67	44

Note: Range is the number of the farms (ranked in ascending according to the SC measure) upon which descriptive were based. Crops and Livestock refer to the observed output of crop and livestock for the farms.

Table 6. Correlation of Economies of Scale and Scope Measures and Farm Characteristics

	Land	Crops	Livestock	LSE	CSE	MSE	SC
Land	1.00						
Crops	0.29*	1.00					
Livestock	0.70*	0.18	1.00				
LSE	-0.06	0.05	-0.19	1.00			
CSE	-0.24	-0.61*	-0.18	0.18	1.00		
MSE	-0.52*	-0.65*	-0.60*	0.23	0.64*	1.00	
SC	-0.57*	-0.66*	-0.52*	0.06	0.26*	0.77*	1.00

* represents statistical significance at the 0.05 level