# A 'Differential' Differential Demand System: An Application to US Meat Demand *

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**Abstract**

I present a 'differential' (threshold) Rotterdam demand model. I argue that a standard model, which holds parametric relationships constant across all sizes of shocks to prices and expenditures, may produce misleading elasticity estimates. This arises because consumers may react less or even not at all to small shocks to prices and expenditures. However, when shocks are large enough to exceed any adjustment or transactions costs, consumers may react more strongly, thereby producing more elastic responses. I estimate the demand for meats in the US over the 1980-2019 period. I consider four meat products—beef, pork, chicken, and turkey—and assume that meats are weakly separable from other demand decisions. I confirm that reactions to larger shocks will result in more elastic price and expenditure adjustments. A test of the difference in the regimes delineated by the size of shocks to prices and expenditures is statistically significant.

Key Words: Rotterdam Model, Threshold Model, Meat Demand, Regime Switching Model

*JEL* Codes: Q11, Q13

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# A 'Differential' Differential Demand System: An Application to US Meat Demand

Conventional approaches to the study of US meat demand have typically assumed a specific functional form for a demand system, such as the Rotterdam or AIDS models, and pursued standard maximum likelihood estimation procedures. A rather large body of research has examined the demand for meat in developed and developing economies. Recent studies include those by Roosen, Staudigel, and Rahbauer (2022), Aborisade, Carpio, and Boonsaeng (2024), and Malla, Klein, and Presseau (2020).[1] These studies considered various specific issues including separability, translation, scaling, and structural change. Understanding the demand for meats and food commodities in general remains an important issue for comprehending market relationships. Food accounted for about 15% of total personal consumption expenditures in the fourth quarter of 2024. Red meat and poultry accounted for 16.2% of total food expenditures.[2]

Demand analysis, by its very nature, relates changes in consumption to changes in prices and incomes or expenditures. Depending on the functional form, demand specifications typically assume a smooth continuum of responses such that adjustments to small price changes are characterized by the same parametric relationships as those adjustments to large price changes. It is, however, possible that consumers react to small changes in prices and expenditures differently from large changes. Changes in purchasing and consumption patterns generally require some adjustment costs. Such costs could result in moderated reactions to small shocks to prices and expenditures—shocks not large enough to overcome the costs of adjustment. The analysis reported in this paper seeks to address this issue.

We adopt a Rotterdam model of meat demand. We allow the parameters of the Rotterdam model to differ according to the magnitude of shocks to prices and expenditures. The

[1] Meat demand was the focus of many studies during the 1980s and 1990s. See, for example, Chavas (1983), and Moschini and Meilke (1984).

[2] Consumption statistics taken from the Bureau of Labor Statistics detailed personal consumption expenditures.

magnitude of shocks is estimated using residuals from a median regression of log changes in meat expenditures on log changes in the prices of meat commodities. Four meats are considered—beef, pork, chicken, and turkey.[3] Our Rotterdam model is applied to quarterly price and consumption data.

# The Rotterdam Model

The Rotterdam demand model was introduced by Barten (1964) and Theil (1965) and is distinguished from other system models of demand by its differential form.[4] The Rotterdam model represents a decomposition of demand relationships in log differential form, where quantities, prices, and expenditures are written as the expenditure-share weighted change from one period to another in the logarithmic transformation of prices and quantities.[5] An equation of the system can be written as

$$\bar{w}_{it} dq_{it} = \beta_i d\bar{X}_t + \sum_{j=1}^{k} \gamma_{ij} dp_{jt} + \nu_{it}, \qquad (1)$$

where $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{it-1})$ is the average budget share for good $i$ between periods $t$ and $t-1$, $d\bar{X}_t = \sum_{i=1}^{k} \bar{w}_{it} dq_{it}$ (which is equivalent to the log change in real expenditures), $\nu_{it}$ is a random disturbance, and $\beta_i$ and $\gamma_{ij}$ are parameters to be estimated. To be consistent with theory, the model should satisfy the following parametric restrictions $\sum_i^k \beta_i = 1$, $\sum_i^k \gamma_{ij} = 0$ and $\gamma_{ij} = \gamma_{ji}$. In cases where consumption patterns may be subject to a trend, an intercept term ($\alpha_i$ in our case) can be added to capture such trends and its parameters must also sum to zero to ensure adding up.

The Rotterdam model has been widely applied and has the advantage of being locally flexible in that there are no a priori restrictions on the elasticity estimates implied by the

[3] Note that lamb and mutton are minor meat commodities in the US and were not included in our study due to a lack of consumption data.

[4] See Mountain (1988) for a discussion of the approximation properties of the Rotterdam model.

[5] This derivation of the Rotterdam model is known as the absolute price version. An alternate relative price version is also sometimes used. See Barten (1964) and Theil (1965) for a discussion of the alternate versions of the Rotterdam model.

model. The model is also distinguished from other flexible functional forms due to its derivation through differential changes to quantities and prices.[6] This is in contrast to other approaches that assume a specific flexible functional form for the expenditure or indirect utility function at the individual consumer level. The Rotterdam model has an alternative interpretation as representing aggregate preferences rather than imposing a representative consumer example, which is then subject to aggregation. This is because no specific representation of *individual* preferences is asserted.

Standard demand system models assume a continuity of responses to price and expenditure changes. This continuity is reflected in the parameters that make up the demand system and that are assumed to be constants. In reality, consumers may respond to small price and expenditure shocks to a lesser degree than is the case for responses to large shocks. This difference in responses may, in part, reflect the costs associated with adjusting purchases in response to such price and expenditure shocks. Some consumers may not respond to small shocks at all since a degree of transactions and adjustment costs is typically associated with such adjustments. Standard demand models with constant parameters are unable to capture such differences and may therefore bias the resulting elasticity estimates. Of course, any model of aggregate demand conditions is made up of the multitude of adjustments that individual consumers make in response to changes in prices and expenditures.

# Empirical Application and Results

We consider the demand for four meats, which are the primary protein sources for most US consumers. We assume that the four meats are weakly separable from other consumption decisions.[7] We utilize USDA data on meat prices and consumption (disappearance) per capita.

[6] Regime switching could also be accommodated in other demand models, including the AIDS model of Deaton and Muellbauer (1980) and the EASI model of Lewbel and Pendakur (2009). We selected the Rotterdam model for its flexibility properties, its differential form, and its ease of application.

[7] Weak separability allows us to consider the demand for meats in exclusion to demands for other goods. This imposes specific restrictions on the cross price elasticities for goods within the group (meats) and goods not included in the group (all other goods).

The data are observed quarterly and span 1980.1 through 2019.4. We deliberately exclude later data due to the effects of the COVID pandemic, which triggered significant departures from typical food demand conditions. We allow for a single threshold that separates two regimes, one corresponding to small price and expenditure changes less than the threshold and another for larger changes.

The trigger, or forcing variable, whose value corresponds to shifts between the alternate regimes, is the residual from an $L1$ (median) regression of changes in total meat expenditures on changes in logarithmic prices:

$$d\bar{X}_t = \theta_0 + \sum_{i=1}^{k} \theta_i dp_{it} + \epsilon_t, \tag{2}$$

where $|\epsilon_t|$ is the forcing variable that triggers shifts across the two regimes. Larger than typical shocks to prices or total meat expenditures will result in larger values of $|\epsilon_t|$. We include intercept terms and quarterly dummy variables to capture seasonality.

Our use of a discrete threshold model could be taken to imply that price or expenditure shocks that are too small to elicit a response by consumers should imply elasticities of zero (no response) for small changes. However, our data represent aggregations across a multitude of consumers, each making their own consumption decisions. To the extent that consumers' responses to shocks are heterogeneous, we simply anticipate smaller elasticity values for smaller price and expenditure changes rather than no response at all.

Our estimation approach involves a grid search across all values of $|\epsilon_t|$, with each regime's parameters being estimated by maximum likelihood. We select the supremum value of a standard Chow test for structural changes across the regimes. The $sup(F-test)$ is illustrated in Figure 1. The figure illustrates values of the Chow test statistic across different break points between regimes. The optimal break point is that which maximizes the value of the Chow test statistic. We trim the 25 lowest and highest values from the search in order to have sufficient data to estimate the Rotterdam system.[8]

[8] As the figure suggests, the results were not sensitive to the amount of trimming.

The $sup(Chow)$ test has been discussed by Hansen (2000). Because the test statistic is a supremum value selected from all possible values in the data, its distribution is nonstandard. To obtain critical and p-values, we simulate the statistic under the null of no difference in regimes. We randomize the dependent variables in the demand system and repeat the analysis 500 times to derive the appropriate p-value associated with the test statistic. The test statistic and associated p-value are reported along with parameter estimates for the Rotterdam system in Table 1. The statistic rejects the null hypothesis that parameter estimates are equal across regimes. This likewise implies that the elasticity estimates are statistically significantly different across regimes.[9]

Estimates for the standard Rotterdam model along with the two-regime threshold Rotterdam model are presented in Table 1. Regime I corresponds to price and expenditure shocks (represented by $|\epsilon_t|$) that are beneath the threshold (small shocks) while Regime II corresponds to larger shocks. Table 2 contains compensated (Hicksian), uncompensated, and expenditure elasticities for each of the Rotterdam systems. As expected, elasticities corresponding to the regime with greater price and expenditure shocks are considerably larger in magnitude in most cases. For example, the uncompensated beef price elasticity rises from -1.598 to -1.732 when price and expenditure shocks are greater. The uncompensated own price elasticity for chicken rises from -0.563 to -0.619. The uncompensated own price elasticity for pork is quite similar across the regimes, with values of -0.338 and -0.367. The uncompensated own price elasticity for turkey rises significantly in magnitude from -0.689 to -1.076. In many cases, the results suggest a greater degree of substitutability. For example, the cross price elasticity of the price of beef in the chicken equation rises from 0.827 to 1.141. Beef is found to have a large expenditure elasticity relative to the other meats, suggesting that beef is more of a 'luxury' meat. Turkey also has a large expenditure elasticity which rises from 1.128 to 1.810 across the regimes. In contrast, expenditure elasticities for

[9] The Hicksian elasticity estimates in a Rotterdam model are given by the simple ratio of the parameters for prices to the budget share, which is held fixed within a regime in order to evaluate elasticities.

pork and chicken are negative, indicating that, within the meat complex, these are inferior commodities.

The compensated (Hicksian) price elasticities suggest that the meats are all Hicks-Allen substitutes for one-another. This degree of substitutability rises in nearly every case when once considers larger shocks to prices and expenditures, indicating a greater degree of substitutability when price and expenditure shocks are large. As would be expected, the standard Rotterdam model yields elasticity estimates that generally fall between the two alternative regimes. Considerable differences between the alternative regimes and the standard model exist, suggesting the potential for inferential biases in the standard model.

# Concluding Remarks

We present a 'differential' (threshold) Rotterdam demand model. We argue that a standard model, which holds parametric relationships constant across all sizes of shocks to prices and expenditures, may produce misleading elasticity estimates. This arises because consumers may react less or even not at all to small shocks to prices and expenditures. However, when shocks are large enough to exceed any adjustment or transactions costs, consumers may react more strongly, thereby producing more elastic responses. We estimate the demand for meats in the US over the 1980-2019 period. We consider four meat products—beef, pork, chicken, and turkey—and assume that meats are weakly separable from other demand decisions. We confirm our suspicions that reactions to larger shocks will result in more elastic price and expenditure adjustments. A nonstandard test of the difference in the regimes delineated by the size of shocks to prices and expenditures finds that the differences across different regimes are statistically significant.

Table 1: Rotterdam Model Parameter Estimates

| Parameter | Standard Model Estimate | Standard Model Std Error | Regime I Estimate | Regime I Std Error | Regime II Estimate | Regime II Std Error |
|---|---|---|---|---|---|---|
| $\alpha_b$ | 0.2756 | 0.0171* | 0.2502 | 0.0185* | 0.3094 | 0.0362* |
| $\beta_b$ | 1.1624 | 0.0141* | 1.1555 | 0.0178* | 1.1803 | 0.0281* |
| $\gamma_{bb}$ | −0.2278 | 0.0076* | −0.2064 | 0.0075* | −0.2661 | 0.0164* |
| $\gamma_{bp}$ | 0.0755 | 0.0037* | 0.0728 | 0.0043* | 0.0905 | 0.0071* |
| $\gamma_{bc}$ | 0.1419 | 0.0069* | 0.1288 | 0.0071* | 0.1611 | 0.0141* |
| $\alpha_p$ | −0.1053 | 0.0079* | −0.1026 | 0.0100* | −0.1322 | 0.0148* |
| $\beta_p$ | −0.0699 | 0.0065* | −0.0675 | 0.0097* | −0.0556 | 0.0116* |
| $\gamma_{pp}$ | −0.1059 | 0.0068* | −0.1069 | 0.0085* | −0.1120 | 0.0112* |
| $\gamma_{pc}$ | 0.0152 | 0.0066* | 0.0225 | 0.0081* | 0.0087 | 0.0110 |
| $\alpha_c$ | −0.0722 | 0.0158* | −0.0641 | 0.0180* | −0.0674 | 0.0315* |
| $\beta_c$ | −0.1466 | 0.0130* | −0.1330 | 0.0174* | −0.1855 | 0.0246* |
| $\gamma_{cc}$ | −0.1646 | 0.0092* | −0.1604 | 0.0106* | −0.1766 | 0.0166* |
| $Q1_p$ | −0.0367 | 0.0027* | −0.0451 | 0.0058* | −0.0374 | 0.0052* |
| $Q2_p$ | −0.0307 | 0.0026* | −0.0298 | 0.0028* | −0.0388 | 0.0066* |
| $Q3_p$ | −0.0270 | 0.0026* | −0.0250 | 0.0026* | −0.0371 | 0.0072* |
| $Q1_c$ | −0.0112 | 0.0055* | 0.0215 | 0.0104* | −0.0303 | 0.0112* |
| $Q2_c$ | 0.0488 | 0.0053* | 0.0495 | 0.0050* | 0.0528 | 0.0140* |
| $Q3_c$ | 0.0398 | 0.0052* | 0.0409 | 0.0048* | 0.0519 | 0.0154* |
| $Q1_b$ | 0.0557 | 0.0060* | 0.0310 | 0.0107* | 0.0727 | 0.0128* |
| $Q2_b$ | −0.0092 | 0.0057* | −0.0113 | 0.0052* | −0.0040 | 0.0160 |
| $Q3_b$ | −0.0051 | 0.0056 | −0.0086 | 0.0050 | −0.0053 | 0.0176 |
| $sup\ (Chow)\ Test$ | 0.800 | [0.046][a] | | | | |
| $R-Square_b$ | 0.9887 | | 0.9869 | | 0.9917 | |
| $R-Square_p$ | 0.8674 | | 0.8658 | | 0.8833 | |
| $R-Square_c$ | 0.8472 | | 0.8802 | | 0.8366 | |

[a]P-Value for sup(Chow) test.

Table 2: Rotterdam Elasticity Estimates

| | | | | | |
|---|---|---|---|---|---|
| **Standard Rotterdam Model** | | | | | |
| | **Uncompensated Price Elasticities of Demand** | | | | |
| | **Beef** | **Pork** | **Chicken** | **Turkey** | **Expenditures** |
| Beef | −1.645 | −0.490 | −0.257 | −0.070 | 2.462 |
| Pork | 0.411 | −0.332 | 0.118 | 0.068 | −0.265 |
| Chicken | 0.932 | 0.238 | −0.580 | 0.057 | −0.647 |
| Turkey | −0.404 | 0.026 | −0.128 | −0.937 | 1.444 |
| | **Compensated Price Elasticities of Demand** | | | | |
| Beef | −0.482 | 0.160 | 0.301 | 0.022 | |
| Pork | 0.286 | −0.402 | 0.058 | 0.058 | |
| Chicken | 0.627 | 0.067 | −0.727 | 0.033 | |
| Turkey | 0.278 | 0.407 | 0.199 | −0.883 | |
| **Regime I: Small Market Shocks** | | | | | |
| | **Uncompensated Price Elasticities of Demand** | | | | |
| Beef | −1.598 | −0.496 | −0.295 | −0.088 | 2.478 |
| Pork | 0.396 | −0.338 | 0.144 | 0.054 | −0.256 |
| Chicken | 0.827 | 0.249 | −0.563 | 0.063 | −0.577 |
| Turkey | −0.405 | −0.005 | −0.030 | −0.689 | 1.128 |
| | **Compensated Price Elasticities of Demand** | | | | |
| Beef | −0.443 | 0.156 | 0.276 | 0.010 | |
| Pork | 0.276 | −0.406 | 0.085 | 0.044 | |
| Chicken | 0.559 | 0.097 | −0.696 | 0.040 | |
| Turkey | 0.121 | 0.292 | 0.231 | −0.644 | |
| **Regime II: Large Market Shocks** | | | | | |
| | **Uncompensated Price Elasticities of Demand** | | | | |
| Beef | −1.732 | −0.461 | −0.203 | −0.052 | 2.449 |
| Pork | 0.443 | −0.367 | 0.079 | 0.056 | −0.210 |
| Chicken | 1.141 | 0.264 | −0.619 | 0.059 | −0.845 |
| Turkey | −0.440 | −0.098 | −0.197 | −1.076 | 1.810 |
| | **Compensated Price Elasticities of Demand** | | | | |
| Beef | −0.552 | 0.188 | 0.334 | 0.030 | |
| Pork | 0.341 | −0.423 | 0.033 | 0.048 | |
| Chicken | 0.734 | 0.040 | −0.804 | 0.031 | |
| Turkey | 0.432 | 0.382 | 0.201 | −1.015 | |

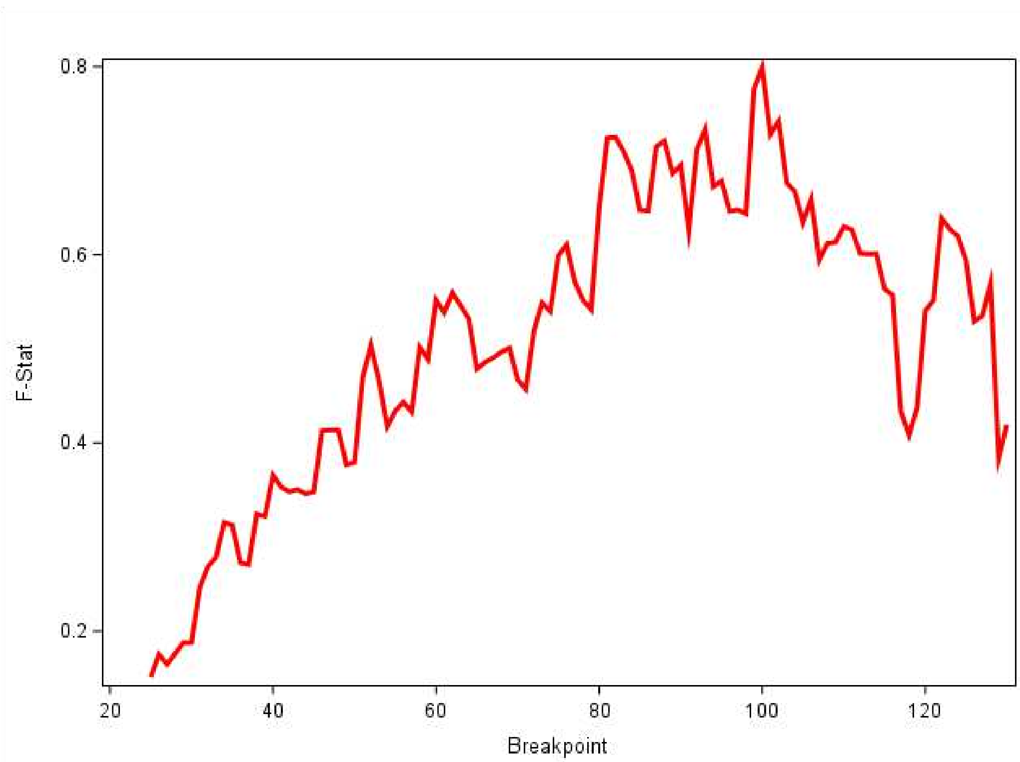


Figure 1: F-Statistic and Forcing Variable (L1 Residual) Breakpoints

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