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# A Comparative Analysis of Water Allocation Institutions and Economic Efficiency

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## Abstract

Water demand is estimated to soon surpass average supplies in many arid climates, which has implications for the institutional efficiency of water administration and governance. This article evaluates the economic efficiency of different water allocation mechanisms and how market efficiency is impacted by water shortages, return flows, and heterogeneity in the initial allocation of water rights. I develop theoretical water allocation models for a hypothetical river basin, and simulations of surface water supply conditions are used to evaluate changes in economic value under different basin compositions and institutional frameworks. Prior appropriation, the predominant institution in the western United States, is compared to alternative, share-based water allocation regimes. I find that due to return flows—the positive externalities associated with water use—the initial allocation of water rights under an appropriative doctrine can distort the total availability of the resource. When senior users appropriate water downstream, the amount of water available for use can be substantially limited, particularly during times of shortage. Subsequently, market-based solutions to institutional inefficiencies arising from appropriative water rights do not always generate more value than share-based mechanisms. Comparatively, share-based mechanisms can allow the users of a river system to internalize more return flows by ensuring more upstream diversions in some instances.

**Keywords:** prior appropriation, water sharing, water trading, water allocation, optimization

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# 1 Introduction

The western United States (US), one of the fastest-growing regions in the country, encompasses the driest areas of North America. New and competing interests for water continue to emerge, yet surface water shortages are expected to worsen due to increasing frequency and severity of droughts induced by climate change (Gutzler and Robbins, 2011). The extent to which increasing water scarcity affects western river basins will depend on the ability of human systems to adapt. Although technological innovations have led to marked improvements in water use efficiency, institutional changes in the governance of western water are occurring at a slower pace (Brewer et al., 2006; Libecap, 2011; Leonard et al., 2019). Achieving outcomes close to a social optimum through technological improvement alone is unlikely if the mechanism governing water use inherently introduces inefficiencies. The present research aims to compare the efficiency of different water allocation institutions and evaluate the implications of return flows, heterogeneous users, and user locations (i.e., upstream or downstream) on allocative performance and market efficiency. I focus on comparing prior appropriation and alternative, share-based mechanisms—all with and without water markets. While it is well established in the economics literature that a system of prior appropriation creates inefficiencies due to asymmetry in risk across water users, less attention has been given to how the initial allocation of water rights can exacerbate or alleviate these inefficiencies. Although perfectly competitive markets can reallocate resources to an efficient outcome, the initial distribution of property rights has implications for how much reallocation must occur within the market. This is particularly relevant for contemporary water trading, which are typically associated with high transaction costs. Additionally, I find that due to return flows—the positive externalities associated with water use—the initial location of prior appropriative water rights along a stream can affect the total amount of water available for use in the onset.

The riparian water rights doctrine, rooted in English common law, is prevalent in the humid, eastern US. It grants landowners abutting water bodies the right to use adjacent water

“reasonably,” without significantly diminishing its supply for downstream users. However, in the arid regions of the western US, where the prevalence of irrigated agriculture requires large diversions and sometimes low return flows, a riparian doctrine is insufficient for ensuring the right of downstream users to divert water. Consequently, the western US has adopted a doctrine of appropriative water rights (Jones and Cech, 2009). This system awards water rights based on the actual physical diversion and beneficial use of water, adhering to a “first in time, first in right” principle. The adoption of beneficial consumptive use as the criterion for water rights aims to encourage investment in water diversion infrastructure and limit excessive speculation on unused waters. There is also evidence that appropriative water rights incentivized *collective* investment in western irrigation infrastructure due to decreased risk for senior users (Libecap, 2007; Leonard and Libecap, 2019).

Under a doctrine of prior appropriation, users must obtain a water right to divert water from any natural system. Once a water right is administered, it is placed within a hierarchy of priority. When basin water supplies are insufficient to fulfill all allotments, older (senior) water rights have priority, potentially leaving newer (junior) rights with nothing. This system introduces uneven risk across water users, making prior appropriation widely considered inefficient in the absence of a competitive water rights market (Burness and Quirk, 1979, 1980; Huffaker et al., 2000). In their seminal work, Burness and Quirk (1979, 1980) analyze the efficiency of prior appropriation with simplifying assumptions, focusing on the long-run equilibrium of expected diversions and profits when firms with identical, separable profit functions share a waterway. They acknowledge that an efficient use of resources under an appropriative framework would require a competitive leasing market contingent on significant investment in the construction of diversion facilities to capture overall water capacity. However, over 40 years later, many western basins are now over-appropriated, and several assumptions in Burness and Quirk (1979, 1980) may not align with contemporary conditions. Firstly, although modern water markets do exist, they are typically constrained to localized trading with high transaction costs encompassing legal fees, negotiation costs,

and the expenses associated with transferring rights ([Brewer et al., 2006](#); [Libecap, 2011](#); [Leonard et al., 2019](#)). This renders current water allocations relatively inflexible, which can hinder the transfer of water to more beneficial uses as new and potentially more valuable uses emerge ([Brewer et al., 2008](#)). Consequently, the initial allocation of water rights becomes an important determinant of water use efficiency, as high transaction costs may prevent the market from correcting inefficiencies in allocation. Secondly, the authors do not address scenarios where average water availability is insufficient to satisfy all administered water rights. This situation is particularly pertinent in regions experiencing water scarcity, where the hierarchy of water rights can result in junior users being deprived of water despite holding legal entitlements. Not only does this prevent users who potentially have a high marginal benefit of water from generating value, there are implications for return flows when curtailed users are located upstream. Lastly, the authors focus on the case in which value is only generated from consumptive use. Many water uses generate some value non-consumptively, such as hydroelectric power generation, recreational activities, ecosystem services, and many household uses. Under a strict prior appropriative regime, it is possible that no water may be diverted for such uses during times of shortage even if the consumptive use is relatively low.

One alternative institution to prior appropriation are water sharing mechanisms, which aim to redistribute risk. For example, some farmers have formed cooperative organizations called “irrigation districts,” pooling the priority of their water rights and redistributing risk evenly across shares. Water is then allocated among members according to the number of shares they own. This system ensures nonzero water allocations each year for each shareholder, which creates more certainty for users to plan for the future. [Ji and Cobourn \(2018\)](#) find that irrigation districts allocate larger portions of land to drought-sensitive, high-value crops compared to areas with similar overall priority without a share-based system, resulting in a 6% average increase in profit. Water sharing plans have also been adopted at larger scales in Australia and Chile ([Grafton et al., 2011](#); [Tingey-Holyoak, 2014](#)).

Although shared-based mechanisms can remedy the uneven risk introduced by prior appropriation, efficiency ultimately depends on the benefits generated by individual water users (Bennett et al., 2000). Return flows also require consideration, as water diverted is not a perfectly rivalrous good. Most water uses necessitate diverting more water than needed for consumption, with return flows ranging from 5-90% depending on the use (Burness and Quirk, 1980). In the South Platte Basin of Colorado, annual surface water diversions double the average natural inflow (CWCB, 2015). Efficiency, therefore, depends not only on the heterogeneous value generation across users but also on their physical location within a river basin. For instance, if an upstream user with high return flows diverts one unit of water, a large portion of that unit can continue to generate value as it reenters the stream for the next user. The allocative efficiency of priority based mechanisms can further change depending on whether the upstream user is junior or senior and high value or low value. Given the complexities arising from return flows and user heterogeneity, no single water allocation institution is likely to be preferable across all western basins. The most efficient institution for a given basin will depend on its specific composition. Figure 1 highlights the heterogeneity of watersheds in the western US, displaying HUC8 classifications from Duan et al. (2018) and the location of relatively big cities. Large population centers are located within both headwater (upstream) and terminus (downstream) HUC8 basins and across states with varying water uses and needs. For example, California’s water demands are remarkably diverse. Large urban areas, such as Los Angeles and San Francisco along the state’s central and southern coast, rely heavily on water for domestic use, industry, and maintaining green spaces in cities. Agriculture, a significant sector in California’s economy, demands large quantities of water for irrigation. The state’s water sources are equally varied, encompassing snowmelt from the Sierra Nevada mountains in northern California and major river systems like the Colorado River and the Sacramento-San Joaquin River Delta.

In general, the current body of economic research offers important perspectives on optimizing water resource use under institutional constraints, particularly emphasizing the role

of market mechanisms. The Coase Theorem (Coase, 1960), a cornerstone in the field of environmental economics, posits that if property rights are clearly defined and transaction costs are negligible, the allocation of resources will be efficient regardless of the initial distribution of these rights. This theorem stresses the potential for private negotiations to address externalities and achieve optimal resource allocation. However, the application of Coasean bargaining to water rights governed by a doctrine of prior appropriation is not clear cut. When high priority water rights are located downstream, the ability of upstream junior users to divert water and generate return flows can be stifled, potentially distorting the total amount of water that can be reallocated through a market. This challenges the assumption that initial allocations do not affect efficiency, particularly under conditions where the physical characteristics of the resource impose unique constraints.

In contrast to fixed-allocation mechanisms like prior appropriation, share-based mechanisms, where water rights are allocated as proportional shares of the available supply rather than as fixed quantities, offer a different paradigm. These mechanisms provide two possible improvements to prior appropriation. First, they always ensure some amount of upstream diversions, which can generate more return flows in some cases. Second, water sharing plans inherently reduce the asymmetry in risk across users, potentially leading to an initial allocation that reflects more efficient use from the outset. Share-based systems may therefore reduce the subsequent reliance on markets to achieve social optimality. The key insight here is that the initial allocation of property rights not only potentially affects the availability of the resource due to the positive externalities of return flows, but also determines the degree of market participation necessary to reach an economically efficient use of resources. In scenarios where the initial allocation generates a more efficient distribution, the market's role in adjusting allocations to meet changing demands and values is less critical.

The initial allocation of property rights and its impact on market efficiency is a theme not unique to water rights. In the context of land demarcation, (Libecap and Lueck, 2011) show that different property allocation regimes have enduring impacts on land values, transaction

costs, and the structure of land markets. They find that the rectangular system, by providing standardized, secure property rights, enhances economic outcomes compared to the more flexible metes and bounds system. In the context of sulfur dioxide cap-and-trade systems, (Henry III et al., 2011) demonstrate that the initial allocation of pollution permits can lead to increased social costs by shifting emissions to areas of higher marginal damage. Such examples in the broader economics literature highlight the principle that the initial distribution of property rights can influence the efficiency of resource allocation, even in the presence of well-functioning markets.

The present research builds upon the current literature by exploring the nuanced aspects of water allocation that remain relatively unexplored, examining how the initial allocation of water rights and physical characteristics of a river basin can either aggravate or mitigate potential inefficiencies. I develop a series of optimization models that emulate different institutions that exist throughout the world and use Monte Carlo simulations of water supply to examine their sensitivity to randomly generated physical characteristics of a river basin. I find that system-wide value generation under prior appropriation is particularly sensitive to the location of priority along a stream and the utilization of return flows during times of shortage, even in the presence of a perfectly competitive leasing market. For simulations in which there is no water shortage, markets unsurprisingly and unambiguously generate the most value. Ideal conditions for priority-based mechanisms include relatively senior upstream water users that either generate high value or high return flows. However, as the number of water users in the model increases, these ideal conditions become less likely, and share-based mechanisms can generate more value than a market under certain conditions. By examining the physical factors that impact institutional efficiency, this study provides valuable insights for policymakers and stakeholders in the development and implementation of effective and efficient water allocation strategies.

The remainder of the article is organized as follows. First, I present a series of modeling frameworks that emulate different water allocation mechanisms for a hypothetical river basin.

I begin by outlining a simple two-user case to emphasize the impact of basin composition on economic efficiency and extend the models to accommodate  $n$ -users for generalizability. Next, I employ Monte Carlo simulations to evaluate efficiency outcomes of each mechanism under various basin compositions and random water supply conditions. I outline the conditions that have the greatest impact on market efficiency, and regression models are applied to the simulation results to characterize basin compositions that bode well for specific water allocation mechanisms. Lastly, I discuss potential policy considerations and implications.

## 2 Theoretical Models of Water Allocation Institutions

### 2.1 Two Users

#### 2.1.1 Social Planner

Before examining real-world water allocation institutions, consider the social planner (SP), whose objective is to maximize the total value of production within a river basin by allocating water  $w_i$  to two users with value functions  $v_i(w_i)$ , where  $i =$  upstream ( $u$ ) or downstream ( $d$ ). Available water in a given year is a random draw  $\tilde{w} \in \mathbf{W}$ , where  $\mathbf{W}$  is the distribution of possible water supply conditions. Some proportion of water  $\theta \in (0, 1)$  diverted to the upstream user returns to the stream and becomes available for the downstream user as a return flow. The SP's deterministic maximization problem with two water users in a given year is

$$\begin{aligned} \max_{w_u, w_d \geq 0} \quad & v_u(w_u) + v_d(w_d) \\ \text{s.t.} \quad & w_u \leq \tilde{w} \\ & w_d \leq \tilde{w} - (1 - \theta)w_u. \end{aligned} \tag{1}$$

If the two users are homogeneous, i.e.,  $v_u = v_d = v$ , it can be easily shown that  $w_u^* > w_d^*$  and  $\partial v / \partial w_d^* > \partial v / \partial w_u^*$  at the optimum. Due to return flows, more water is diverted to the upstream user, resulting in a lower marginal benefit of water compared to the downstream

user.<sup>1</sup> When  $v_u \neq v_d$ , optimal allocations will depend on the relative value functions.

### 2.1.2 Prior Appropriation

Now consider the same basin layout, but water allocation is determined by PA. Each user owns a water right  $R_i^m$  that determines a maximum amount of water allowable for diversion, where  $m = \text{senior } (s) \text{ or junior } (j)$ . Under PA,  $R_i^s$  must be met before the junior user can divert any water. In practice, any water use by a junior user that prevents any senior user from receiving their full allotment is prohibited. Water allocation is therefore no longer determined by a maximization problem, but by the location of the junior and senior user, their respective water rights, and the total water supply. There are two possible cases for a given level of water supply: (i.) the senior user is upstream or (ii.) the senior user is downstream.

- i. When the upstream user is senior and the downstream user is junior, the resulting water allocations are:

$$\begin{aligned}
 w_u^s &= \begin{cases} \tilde{w} & \text{if } \tilde{w} < R_u^s \\ R_u^s & \text{if } \tilde{w} \geq R_u^s \end{cases} \\
 w_d^j &= \begin{cases} \theta \tilde{w} & \text{if } \tilde{w} \leq R_u^s \\ \tilde{w} - (1 - \theta)R_u^s & \text{if } R_u^s < \tilde{w} < (1 - \theta)R_u^s + R_d^j \\ R_d^j & \text{if } \tilde{w} \geq (1 - \theta)R_u^s + R_d^j. \end{cases} \tag{2}
 \end{aligned}$$

- ii. When the upstream user is junior and the downstream user is senior, the resulting

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<sup>1</sup>The role of return flows in determining upstream versus downstream diversions is further demonstrated in the [Appendix](#).

water allocations are:

$$\begin{aligned}
 w_d^s &= \begin{cases} \tilde{w} & \text{if } \tilde{w} < R_d^s \\ R_d^s & \text{if } \tilde{w} \geq R_d^s \end{cases} \\
 w_u^j &= \begin{cases} 0 & \text{if } \tilde{w} \leq R_d^s \\ \frac{\tilde{w} - R_d^s}{1 - \theta} & \text{if } R_d^s < \tilde{w} < (1 - \theta)R_u^j + R_d^s \\ R_u^j & \text{if } \tilde{w} \geq (1 - \theta)R_u^j + R_d^s. \end{cases} \tag{3}
 \end{aligned}$$

In each case,  $(1 - \theta)R_u^m + R_d^m$  characterizes the total amount of water necessary to fulfill the maximum allowable diversion for both users. For the senior user, location is irrelevant and  $w_u^s = w_d^s$  in any water supply scenario. However, the junior user faces different outcomes in times of scarcity, i.e.,  $\tilde{w} < (1 - \theta)R_u^m + R_d^m$ , depending on location. If downstream, the junior user will always receive at least the return flows from the upstream user,  $\theta\tilde{w}$ . If upstream, the junior user faces the possibility of receiving nothing if the basin water supply is insufficient to fulfill even the senior right, i.e.,  $\tilde{w} < R_d^s$ . Although simple, the two user case clearly conveys how risk is disproportionately assumed by the junior user. [Table 1](#) provides a stylized example of how water allocations differ for the junior user under different stream location and supply scenarios. If the users have the same value functions in both cases, case (ii.) unambiguously results in lower basin-wide value during shortages due to the upstream, junior user receiving less water.

One criticism of the doctrine of prior appropriation is the economic inefficiency introduced through an uneven distribution of risk. During times of shortage, some users who could generate significant value per unit of water receive nothing, while others with higher priority rights may not put all of the resource to its most productive use. This uneven risk can lead to a suboptimal allocation of resources. Alternatively, share-based institutions can partly remedy this inefficiency by ensuring that all users receive a non-zero allocation of water, even during times of scarcity. Under a PS system, water is allocated proportionally according to

shares until all endowments are satisfied in a descending order from highest to lowest number of shares. If a basin were to transition from PA to PS, shares may be determined by users' previous priorities or endowments or both. Share-based institutions aim to redistribute risk more equitably among water users, hopefully promoting a more efficient use of resources.

### 2.1.3 Proportional Sharing

Before addressing PS, here I characterize how water rights may have formed along a stream. This intermediate step has the benefit of ensuring return flows are accounted for when developing a stylized PS system. The senior user first appropriates some portion  $\alpha_i^m \in (0, 1)$  of average water supply from a completely unappropriated basin, and then the junior user appropriates some portion of the remaining available water. The characterization again depends on the location of each user, where  $\bar{w}$  is the mean of  $\mathbf{W}$ .

- i. The upstream user is senior, the downstream user is junior, and the resulting water rights are:

$$\begin{aligned} R_u^s &= \alpha_u^s \bar{w} \\ R_d^j &= \alpha_d^j \bar{w} [1 - (1 - \theta) \alpha_u^s]. \end{aligned} \tag{4}$$

- ii. The upstream user is junior, the downstream user is senior, and the resulting water rights are:

$$\begin{aligned} R_d^s &= \alpha_d^s \bar{w} \\ R_u^j &= \alpha_u^j \bar{w} \left( \frac{1 - \alpha_d^s}{1 - \theta} \right). \end{aligned} \tag{5}$$

Again consider the same basin layout as in the SP and PA examples, but water allocation is now determined by PS. Assume this institution has replaced PA and the existing water users trade in their water rights for a share

$$\delta_i^m = \frac{R_i^m}{\sum_i R_i^m} \in (0, 1). \tag{6}$$

A more realistic formulation of shares that is driven by seniority is introduced in the n-user

framework, however this simple version is sufficient for examining how allocations change under a share-based mechanism. Under PS, if  $\tilde{w}$  is not sufficient to fulfill the sum of all previous water rights, the shortage is distributed equally according to shares. Again, there are two cases depending on the location of the users.

- i. The upstream user was senior, the downstream user was junior, and the resulting shares are:

$$\begin{aligned}\delta_u^s &= \frac{\alpha_u^s}{\alpha_u^s + \alpha_d^j [1 - (1 - \theta)\alpha_u^s]} \\ \delta_d^j &= \frac{\alpha_d^j}{\alpha_u^s + \alpha_d^j [1 - (1 - \theta)\alpha_u^s]}.\end{aligned}\tag{7}$$

- ii. The upstream user was junior, the downstream user was senior, and the resulting shares are:

$$\begin{aligned}\delta_d^s &= \frac{\alpha_d^s}{\alpha_d^s + \alpha_u^j \left(\frac{1 - \alpha_d^s}{1 - \theta}\right)} \\ \delta_u^j &= \frac{\alpha_u^j}{\alpha_d^s + \alpha_u^j \left(\frac{1 - \alpha_d^s}{1 - \theta}\right)}.\end{aligned}\tag{8}$$

In each case, the junior user is now guaranteed a nonzero allocation of water in a given period.

## 2.2 N Users

In this section I outline four institutional mechanisms for allocating water relative to the social planner. The first is a strict doctrine of prior appropriation, where no water can be traded among users. The second and third mechanism also do not have markets, and water is allocated equally or proportionally according to shares. The final mechanism emulates a leasing market, in which water allocations from any of the prior institutions can be traded.

### 2.2.1 Social Planner

The two-user case provides helpful intuition for how stream position and seniority can affect the overall value of water within a basin. The SP problem can be easily extended to accom-

modate an  $n$  number of users, where  $i$  indexes stream position from furthest upstream (1) to furthest downstream ( $n$ ):

$$\begin{aligned}
 & \max_{w_1, \dots, w_n \geq 0} \sum_{i=1}^n v_i(w_i) \\
 & \text{s.t.} \quad \left. \begin{aligned}
 & w_1 \leq \tilde{w} \\
 & w_2 \leq \tilde{w} - (1 - \theta_1)w_1 \\
 & \vdots \\
 & w_n \leq \tilde{w} - \sum_{i=1}^{n-1} (1 - \theta_i)w_i
 \end{aligned} \right\} \text{Physical constraints.}
 \end{aligned} \tag{9}$$

There are now  $n$  number of constraints, where each imposes the maximum amount of water that can be physically delivered to the  $i^{th}$  user. Again, the SP problem conveys the upper bound on the total value that can be generated from the available water in a given period. [Figure 2](#) provides a visual representation of the hypothetical river basin outlined in equation (9) with  $n = 3$  users. Notably, return flows generated from user  $i$  are available to all downstream users.<sup>2</sup>

### 2.2.2 Prior Appropriation

With  $n$  users, it is not feasible to discretely characterize every possible stream position and priority hierarchy as in the two-user case. Instead, the institution of prior appropriation can be emulated with a constrained linear programming problem. The following maximization

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<sup>2</sup>All else equal, the Social Planner will always divert more water to upstream users, as demonstrated in the [Appendix](#).

problem will hereinafter be referred to as the PA allocation function,

$$\begin{aligned}
& \max_{w_1^m, \dots, w_n^m \geq 0} && \sum_{i=1}^n \beta_i^m w_i^m \\
& \text{s.t.} && \left. \begin{aligned}
& w_1^m \leq \tilde{w} \\
& w_2^m \leq \tilde{w} - (1 - \theta_1)w_1^m \\
& \vdots \\
& w_n^m \leq \tilde{w} - \sum_{i=1}^{n-1} (1 - \theta_i)w_i^m
\end{aligned} \right\} \text{Physical constraints} \\
& && \left. \begin{aligned}
& w_i^m \leq R_i^m \quad \forall i \in \{1, \dots, n\}
\end{aligned} \right\} \text{Institutional constraints}
\end{aligned} \tag{10}$$

where  $m$  indexes priority from 1 to  $M$  with 1 being the highest priority user, and  $\beta_i^m$  and  $R_i^m$  correspond to a priority coefficient and maximum allowable quantity for user  $i$ , respectively. For the highest priority user,  $\beta_i^m$  must be sufficiently large enough such that the PA allocation function always gives the next unit of water to the user with  $m = 1$  until  $w_i^m = R_i^m$ , and so on and so forth down the priority ranking. The  $n$  institutional constraints are imposed to ensure no user is allocated more than  $R$  and the physical constraints are identical to the SP problem, resulting in a total of  $2n$  constraints.

For the PA allocation function,  $i$  still indexes stream position, but it does not reflect the hierarchy of priority. It is not necessarily the case that  $\beta_1^m$  corresponds to the highest or lowest priority user. In the following water supply simulations section, both  $i$  and  $m$  are randomly assigned to each user to analyze how efficiency changes under different hierarchies and stream positions. The solution to the PA allocation function yields a vector of water delivered to each user. Once the allocations are determined, they can be plugged into the function  $\sum_{i=1}^n v_i(\hat{w}_i^m)$  to determine the total value generated under prior appropriation.

### 2.2.3 Proportional Sharing

Suppose users under a prior appropriation system elect to pool their risk and assign shares according to priority levels. Under a proportional sharing (PS) system, water is allocated

proportionally according to each user's number of shares until all endowments are satisfied in a descending order from highest to lowest number of shares. This ensures non-zero allocations but still maintains some manner of hierarchy. Let  $\gamma_i^m$  represent a user's shares and  $\delta_i^m$  their proportional allocation:

$$\gamma_i^m = \sum_{j=m}^n \frac{1}{j} \quad (11)$$

$$\delta_i^m = \frac{\gamma_i^m}{\sum_{m=1}^n \gamma_i^m}. \quad (12)$$

Different formulations of  $\gamma_i^m$  can be used to tighten or loosen the level of hierarchy.<sup>3</sup> Essentially, all users are allocated water at a rate proportional to  $\delta_i^m$ .

The PS mechanism can be modeled with an iterative linear optimization approach. Again, let  $R_i^m$  be the water right that determines the maximum amount of water allowable for user  $i$  with priority  $m$ . PS can be emulated as follows:

1. Calculate the minimum amount of water necessary,  $\dot{w}_1$ , to satisfy the maximum allowable quantity for the highest priority user ( $R_i^1$ ) while abiding by the physical and institutional constraints below.

$$\begin{array}{ll} \min_{\dot{w}} & \dot{w} \\ \text{s.t.} & \left. \begin{array}{l} w_1^m \leq \dot{w} \\ w_2^m \leq \dot{w} - (1 - \theta_1)w_1^m \\ \vdots \\ w_n^m \leq \dot{w} - \sum_{i=1}^{n-1} (1 - \theta_i)w_i^m \end{array} \right\} \text{Physical constraints} \\ & \left. \begin{array}{l} w_i^1 = R_i^1 \\ w_i^m = \delta_i^m (\sum_{j=1}^n w_j) \end{array} \right\} \text{Institutional constraints} \end{array} \quad (13)$$

<sup>3</sup>For  $n = 4$ ,  $\delta_i^1 = 0.520833$ ,  $\delta_i^2 = 0.276833$ ,  $\delta_i^3 = 0.145833$ , and  $\delta_i^4 = 0.0625$  using equation (12).

2. If  $\tilde{w} \leq \dot{w}_1$ , use the following maximization problem to determine allocations.

$$\begin{aligned}
& \max_{w_1^m, \dots, w_n^m \geq 0} \sum_{i=1}^n w_i^m \\
& \text{s.t.} \quad \left. \begin{aligned}
& w_1^m \leq \tilde{w} \\
& w_2^m \leq \tilde{w} - (1 - \theta_1)w_1^m \\
& \vdots \\
& w_n^m \leq \tilde{w} - \sum_{i=1}^{n-1} (1 - \theta_i)w_i^m
\end{aligned} \right\} \text{Physical constraints} \\
& \left. \begin{aligned}
& w_i^m \leq R_i^m \\
& w_i^m = \delta_i^m (\sum_{j=1}^n w_j)
\end{aligned} \right\} \text{Institutional constraints}
\end{aligned} \tag{14}$$

If  $\tilde{w} > \dot{w}_1$ , store the allocations from the minimization problem in step 1 in vector  $\hat{\mathbf{w}}_1 = (\hat{w}_1^m, \hat{w}_2^m, \dots, \hat{w}_n^m)$  and proceed to step 3.

3. Subtract  $\hat{\mathbf{w}}_1$  from the vector of maximum allowable quantities  $\mathbf{R}$  to get the remaining amount of water each user requires to satisfy their water right,  $\hat{\mathbf{R}}_1 = (R_1^m - \hat{w}_1^m, R_1^2 - \hat{w}_1^2, \dots, R_n^m - \hat{w}_n^m)$ . Subtract  $\dot{w}_1$  from  $\tilde{w}$  to get the remaining amount of water in the stream,  $\tilde{w}_1$ . Set  $\delta_i^1 = 0$  since  $R_i^1 - \hat{w}_i^1 = 0$ , and generate new shares and proportions using equation (14) for  $i - 1$  users. Calculate the minimum amount of water necessary,  $\dot{w}_2$ , to satisfy the maximum allowable quantity for the second highest priority user ( $R_i^2 - \hat{w}_i^2$ ), who now holds the largest share, using the same minimization problem from step 1 with the updated parameters.
4. If  $\tilde{w}_1 \leq \dot{w}_2$ , use the maximization problem from step 2 with updated parameters to determine second stage allocations and add them to  $\hat{\mathbf{w}}_1$  to get final allocations. If  $\tilde{w}_1 > \dot{w}_2$ , repeat step 3 with updated parameters.
5. Repeat the process until there is no water remaining or all water rights are satisfied.

The iterative approach to PS allocation provides the vector of water delivered to each user. This vector is the sum of all water allocated through each iteration, culminating in the the maximization problem in step 2 when there is not enough water left to satisfy the next

highest priority user. In essence, the PS allocation process always diverts the next available unit of water in proportion to each user's number of shares while abiding by the physical constraints of the basin.

#### 2.2.4 Equal Sharing

A simple, and naive, share-based mechanism for water allocation is one where all users are required to receive the same amount of water. An institution where water is equally shared (ES) across all water users can also be imitated with a linear programming problem. The following is the ES allocation function:

$$\begin{aligned}
 & \max_{w_1, \dots, w_n \geq 0} \quad \sum_{i=1}^n v_i(w_i) \\
 & \text{s.t.} \quad \left. \begin{aligned}
 & w_1 \leq \tilde{w} \\
 & w_2 \leq \tilde{w} - (1 - \theta_1)w_1 \\
 & \vdots \\
 & w_n \leq \tilde{w} - \sum_{i=1}^{n-1} (1 - \theta_i)w_i
 \end{aligned} \right\} \text{Physical constraints} \\
 & \quad \quad \quad \left. \begin{aligned}
 & w_i \leq R_i \quad \forall i \in \{1, \dots, n\} \\
 & w_i = w_j \quad \forall i \neq j
 \end{aligned} \right\} \text{Institutional constraints}
 \end{aligned} \tag{15}$$

where additional institutional constraints ensure all users receive the same amount of water.

#### 2.2.5 Leasing Markets without Transaction Costs

The mechanisms above each allocate water according to a set of institutional constraints that do not consider the value generated by individual users and therefore do not attempt to equate marginal value across users. A leasing market can remedy this introduced inefficiency, which allows the trading of water among users. Users with lower-valued uses can sell water to those who can generate higher value from the water, leading a more efficient reallocation of resources. The following model emulates a leasing market in which users are allowed to

trade water *after* allocations are determined from the PA, PS, or ES allocation function. Annual water supplies are often estimated from the amount of winter snow pack, so the initial allocations can be interpreted as expected allocations for the given period.

To proxy the outcome of a perfectly competitive market with no transaction costs, the model is set up as a Social Planner's problem with additional institutional constraints. For the leasing market, let  $\hat{w}_i$  be the initial allocation for user  $i$  who can choose to buy or sell an amount  $t_i$ :

$$\begin{aligned}
& \max_{t_1, \dots, t_n \geq 0} \sum_{i=1}^n v_i(\hat{w}_i + t_i) \\
& \text{s.t.} \quad \left. \begin{aligned}
& \hat{w}_1 + t_1 \leq \tilde{w} \\
& \hat{w}_2 + t_2 \leq \tilde{w} - (1 - \theta_1)(\hat{w}_1 + t_1) \\
& \vdots \\
& \hat{w}_n + t_n \leq \tilde{w} - \sum_{i=1}^{n-1} (1 - \theta_i)(\hat{w}_i + t_i)
\end{aligned} \right\} \text{Physical constraints} \\
& \left. \begin{aligned}
& \hat{w}_i + t_i \geq 0 \quad \forall i \in \{1, \dots, n\} \\
& \sum_{i=1}^n t_i = 0
\end{aligned} \right\} \begin{array}{l} \text{Non-negative allocations} \\ \text{Market clearing condition.} \end{array}
\end{aligned} \tag{16}$$

If  $t_i > 0$ , user  $i$  can be considered a net buyer and if  $t_i < 0$ , a net seller. The first set of constraints ensure that user  $i$  can only buy water up to an amount such that  $\hat{w}_i + t_i$  is less than or equal to the total amount of water in the river at their point of diversion. The second set of constraints ensures that no user can sell more water than they were initially allocated. The last constraint is a market clearing condition that ensures that sum of all water bought is equal to the sum of all water sold. The solution to the water market optimization problem,  $\hat{\mathbf{t}}$ , can then be added to the initial vector of allocations  $\hat{\mathbf{w}}$  to get the final allocations.

## 3 Results

### 3.1 Water Supply Simulations

In this section I employ a Monte Carlo simulation to randomly generate water supply conditions to compare allocation and value outcomes across the institutional mechanisms outlined above. For the two-user case, I compare four simple basin compositions to highlight the sensitivity of prior appropriation to the location of priority and value along a waterway. For the  $n$ -user case, basin compositions are randomly generated. Users are randomly assigned a priority number, stream position, return flow parameter, and value function to assess how robust water allocation mechanisms are to varying institutional and physical constraints.

#### 3.1.1 Two Users

To compare the efficiency of each institution, I allow users to be heterogeneous in terms of their value functions and use a Monte Carlo simulation to randomly sample 5,000 draws of  $\tilde{w}$  from a log-normal distribution. Four cases are compared across the three institutions (SP, PA, PS): (i.) the senior user is upstream and high value, (ii.) the senior user is downstream and high value, (iii.) the senior user is upstream and low value, and (iv.) the senior user is downstream and low value. In each scenario, the junior user assumes the opposite characteristics. The outcomes of PA and PS would be identical when there is enough water to fulfill the endowments of both users, i.e.,  $\tilde{w} \geq (1 - \theta)R_u^m + R_d^m$ , so the following results examine only the iterations where there was a water *shortage*. The models are parameterized such that:  $v_{high}(w) = 20w^{\frac{1}{2}}$ ,  $v_{low}(w) = 10w^{\frac{1}{2}}$ ,  $\theta = 0.25$ ,  $\alpha_i^s = 0.6$ ,  $\alpha_i^j = 0.75$ , and  $\bar{w} = 500$ .

The results of the simulations are presented in [Figure 2](#), which displays distributions of total value generated for each institution across all random water supply draws. Distributions with significant area on the right side of each panel indicate a larger proportion of simulations where high value is generated. In each scenario, the SP's distribution characterizes the upper

bound on efficiency. In the scenario where the senior user is upstream and high value, PA generates more basin-wide value from water than PS on average, as evidenced by the higher peak on the right hand side of the panel. Under PA, water is first allocated to the higher value user upstream, and return flows are then available for the less valuable use downstream. Moving from PA to PS reallocates some water away from a high value use. In the scenarios where the senior user is low value, PS unambiguously generates more value than PA. When the senior user is downstream and low value (bottom right panel), there is potential for PA to generate very little relative value during extreme shortages, as shown in the peak in the left tail. An ambiguous finding results from when the senior user is downstream and high value. For minor shortages, PA may be preferred to PS, but during extreme shortages where the junior user receives no water, the senior uses all available water to a point of low marginal benefit. In general, [Figure 2](#) conveys how PA is more sensitive to basin composition than PS in terms of risk. Moving from a desirable to less desirable composition causes the distribution to flatten, increasing the risk of low-value outcomes in the left tail.

### 3.1.2 N Users

Similar to the two-user simulations, the allocative efficiency of the following mechanisms with and without a subsequent leasing market are compared using a Monte Carlo simulation: 1. prior appropriation (PA), 2. proportional sharing (PS), and 3. equal sharing (ES). For the leasing market scenarios (PAM, PSM, ESM), transaction costs are set to zero to capture a perfectly efficient market. Water supplies are randomly sampled from a log-normal distribution and the maximum amount any user can divert is 500 units of water. For each water supply draw, users along the stream are randomly assigned a stream position, a priority (and a subsequent number of shares), a value function, and a return flow parameter. The value functions are all non-negative, monotonically increasing, and display diminishing marginal returns. Detailed information on all parameters of the hypothetical river basin are outlined in [Table 2](#). The optimization problems for each of the mechanisms are then solved

for that particular composition of users. This process is repeated for a system with three to 36 users, with 15,000 water supply draws for each case.<sup>4</sup> Table 3 displays an example of a simulation with  $n = 10$  users and a randomly sampled water supply. In this example, the optimization models are each parameterized according to their relevant values in this table, and the outcomes are compared for  $\tilde{w} = 2, 252$ .

For each institution and simulation, I also compute the absolute difference between the water allocations without the market (PA, PS, ES) and with the market (PAM, PSM, ESM), to generate a percentage that provides a total measure of how much water had to be shifted under each mechanism to reach the efficient (market) outcome. A higher percentage indicates a greater shift in allocations due to the introduction of the market, and, all else equal, plausibly higher participation and transaction costs required to reach that optimal outcome. The calculation for PA would be:

$$\% \text{ Reallocation}_{\text{PA}} = \frac{\sum_{i=1}^n |w_i^{\text{PA}} - w_i^{\text{PAM}}|}{2 \times \sum_{i=1}^n w_i^{\text{PA}}} \times 100 \quad (17)$$

where  $w_i^{\text{PA}}$  and  $w_i^{\text{PAM}}$  are the water allocations to the  $i^{\text{th}}$  user under PA without and with the perfectly competitive leasing market, respectively. The division by 2 is included to account for the fact that reallocating water from one user to another is counted in both the reduction for one and the increase for the other, thus doubling the total apparent reallocation when summed absolutely.

Figure 4 displays the average value generated (left panel) and average water reallocated (right panel) across all simulations by institution and number of users. The left panel of the figure presents a clear trend: in the absence of a leasing market, share-based mechanisms outperform PA in terms of average total value created, with the disparity widening as the user base expands. This suggests that share-based mechanisms have the capacity to allocate water more efficiently in the absence of an active market. With a leasing market in place, as

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<sup>4</sup>Due to the complex and nonlinear optimization problems, the solver fails to find a solution for at least one institution in roughly 9.1% of the draws. When a solution cannot be found, that draw is omitted.

indicated by the overlapping red, yellow, and orange curves, total average value is effectively equivalent across institutions, which aligns with the principle that the efficiency of resource allocation should not depend on the initial allocation of resources. Nonetheless, the right panel provides a nuanced perspective on this narrative. Despite achieving similar levels of value creation, the proportion of water that necessitates reallocation under the PA market (PAM) system is substantially higher—almost double—compared to the equal sharing market (ESM) system. The need for significant reallocation in the PAM system to attain efficiency relative to the ESM system highlights an often-overlooked dimension of Coasean bargaining: the initial distribution of rights has substantial implications for the volume of resources that must be reallocated to achieve efficiency. This suggests that more efficient initial allocations, as evidenced in the share-based systems, could reduce the need for market reallocations, thereby minimizing transaction costs and potential barriers to achieving an optimal distribution of resources.

Due to return flows, as demonstrated in the [Appendix](#), the Social Planner allocates more water upstream to balance the marginal benefits of the last units of water diverted to each user. However, as shown in Section 2.1.2 and [Table 1](#), a senior user downstream of a junior user can result in less overall diversions. So, for each randomly generated basin composition, I calculate a Pearson correlation coefficient  $\rho$  to examine the strength and direction of the relationship between stream position and user priority under PA. For example, consider a case with  $n = 3$  users where the priorities assigned (in order of stream position) are 1, 2, 3. Here, the stream position correlates perfectly with the user priorities, yielding a correlation coefficient of  $\rho = 1$ . In contrast, if the user priorities are assigned as 3, 2, 1, the stream position correlates perfectly negatively with the user priorities, resulting in a correlation coefficient of  $\rho = -1$ . In essence, a positive correlation would suggest that users' priorities are aligned with the natural flow of the stream, i.e., upstream users have higher priority. Conversely, a negative correlation implies that high-priority rights are downstream.

[Figure 5](#) displays the relationship between stream-priority location and the total amount

of water diverted to all users. This relationship is calculated simply by regressing  $\rho$  on the percentage of total water that is diverted in excess of the natural inflow  $\tilde{w}$  across all simulations. For the ES mechanism, stream-priority location has no impact on the amount of water diverted, because the institutional constraint in equation (15) simply forces everyone to divert the same amount of water. The PA mechanism displays the steepest slope, indicating that when downstream users have senior rights, there is a constraint on the ability of upstream users to divert water, which could lead to decreased overall utilization of the resource. However, a higher correlation mimics the strategy of the Social Planner to divert more water upstream, so there are then some instances when PA with an active market is the most efficient mechanism. The magnitude of the slope is reduced for the PS mechanism, as it can be interpreted as a middle ground institution between ES and PA. In summary, the findings presented in [Figure 4](#) and [Figure 5](#) convey that the distribution of user priorities across the stream not only determines the level of market transactions required to achieve social optimality, but it also has a direct impact on the total volume of water that can be harnessed by the system.

[Figure 4](#) demonstrates that each of the market mechanisms generates the same amount of total value on average, but this is not the case for all stream compositions. As shown in [Figure 5](#), the total amount of water diverted is impacted by the location of users. As such, [Figure 6](#) shows the distribution of total value generated (relative to the Social Planner) across all simulations that fall into four categories: the combinations of high priority downstream ( $\rho \leq -0.5$ ) and high priority upstream ( $\rho \geq 0.5$ ), and high and low shortage. The high and low shortage scenarios are defined as the top and bottom 25<sup>th</sup> percentile of shortage per user, which is calculated as the minimum amount of water required to satisfy all user endowments minus the natural inflow  $\tilde{w}$  and divided by the number of users. In [Figure 6](#), the top four panels correspond to the mechanisms without a market, and it is evident that PA is particularly sensitive to both the location of users and the level of shortage. The peaks of each distribution move the left as high priority users move downstream, indicating

greater efficiency losses, and the distributions flatten as the magnitude of shortage increases. The share-based mechanisms display less sensitivity in these dimensions. The bottom four panels of [Figure 6](#) correspond to the mechanisms with a market, and unsurprisingly all distributions display a rightward shift relative to no market. However, PAM still displays a sensitivity to the four scenarios. Given that these markets have zero transaction costs and the same value functions across users, the only explanation for differences in efficiency is that different amounts of water are being traded (despite the same natural inflow). This further demonstrates that the total volume of water harnessed by the system can be impacted by the location of priority along the stream and potentially further exacerbated during times of extreme shortage. In fact, ES *without* a water market still generates more value than PAM in 17.5% of simulations. Of these simulations, the average stream position-priority correlation coefficient is -0.2 and the average shortage per user is 180 units, which corresponds to the 27<sup>th</sup> percentile across all simulations.

## 4 Conclusion and Policy Discussion

This research examines the impact of water allocation institutions on economic efficiency by comparing prior appropriation and alternative, share-based mechanisms. As water scarcity intensifies due to climate change, this study underscores the need for adaptive institutional changes beyond technological improvements. I demonstrate how the initial allocations of water rights, influenced by return flows and the location of users, affect both market efficiency and the total availability of water. Prior appropriation, the predominant water allocation mechanism in the western US, is particularly sensitive to basin composition. Additionally, share-based water allocation mechanisms can provide more equitable risk distribution and efficiency compared to prior appropriation in many instances, even in the absence of a water market. The findings also suggest that market mechanisms, while important, are not a panacea; institutional configurations must be tailored to basin-specific characteristics and

the physical realities of water resources.

This analysis shortcomings that must best addressed. First, the parameterization of the theoretical models in this analysis is somewhat arbitrary. The purpose of these models is solely to highlight the costs and benefits of different institutional constraints, not to adequately mimic reality. To apply the findings of this article to an existing river basin, it would be necessary to generate realistic value functions and return flow dynamics, which can be challenging. Secondly, the models presented in this research do not account for decreases in water *quality*. I claim that, in some instances, water sharing mechanisms generate more value due to a higher capacity to capitalize on return flows. However, return flows tend to be associated with decreases in water quality, which can impact downstream value generation. Lastly, to consider the feasibility of policy action, moving away from a property rights system like PA can be exceptionally difficult.

Despite some limitations, the results from this paper have important policy implications. First, it is a poignant reminder for policymakers that the design of water allocation systems must account for the geophysical context to ensure both the efficient allocation and the maximal utilization of water resources. The composition of a river basin and the initial allocation of property rights have implications for both the efficiency of markets and the total volume of water that can be used. The presence of return flows and the location of users need to be considered when designing rules for governance.

For future work, the modeling framework presented in this study could be modified to further investigate the availability and timing of return flows, which could also impact institutional efficiency. In reality, the return flows from one user may not be available for all downstream users. There may also be uncertainty about *when* that water reenters the system. Additional constraints could also be included to emulate instream flow water rights. States such Arizona and Colorado have incorporated the use of water rights to ensure certain sections of rivers maintain a minimum flow for ecological purposes. These minimum flow requirements would generate ecological benefits, though there may be trade offs for the

location and seniority of instream flow water rights.

## 4.1 The Colorado River

The findings presented in this paper hold significant importance in the context of ongoing negotiations among Western states for reducing withdrawals of Colorado River water. The US Bureau of Reclamation has issued warnings of imposing more stringent regulations on water release from reservoirs if a total demand reduction of 4 million acre-feet is not achieved (Flavelle, 2023a). California, Arizona and Nevada came to a temporary agreement to reduce water use, however the federal government will pay \$1.2 billion to various stakeholders to achieve that reduction (Flavelle, 2023b). As climate change exacerbates water scarcity in the western United States, there is an urgent need to develop and implement efficient water allocation institutions that can adapt to evolving environmental and socioeconomic conditions.

The Colorado River is a crucial water source for seven states in the US and two states in Mexico, supporting over 40 million people and extensive agricultural activities. Given the river's importance, the ongoing negotiations between states aim to address the increasing demand for its water resources and balance the competing interests of various stakeholders. This research can inform these negotiations by providing insights into the relative efficiency of different water allocation mechanisms under different basin compositions and water supply conditions. Understanding the strengths and weaknesses of existing mechanisms, such as prior appropriation and share-based systems, can help policymakers identify potential improvements in water governance that foster more equitable and sustainable water use.

Moreover, the US Bureau of Reclamation's warning highlights the need for effective demand management strategies to avoid severe restrictions on water releases from reservoirs, which could have significant consequences for water-dependent communities and ecosystems. The results from this analysis can guide decision-makers in identifying water allocation mechanisms that promote more efficient water use. By examining the impacts of heterogeneity in

value generation and user location on allocative efficiency, this study can also inform targeted policy interventions that address the specific needs and constraints of different user groups within a river basin. If water markets are the preferred intervention, this article provides insights into how much participation is necessary to outperform other mechanisms. Ultimately, this research has the potential to contribute to more sustainable water management practices in the western United States and other arid climates. By identifying the most efficient water allocation mechanisms under various basin compositions, it can support the development of policies that promote more equitable and efficient water use in the face of growing demands and climate change-induced water scarcity.

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## Tables

Table 1: Water allocation outcomes under different water supply scenarios;  $R_i^s = 250$ ,  $R_i^j = 200$ ,  $\theta = 0.25$

$\tilde{w}$	(i.)			(ii.)		
	$w_u^s$	$w_d^j$	Total	$w_d^s$	$w_u^j$	Total
100	100	<b>25</b>	<b>125</b>	100	<b>0</b>	<b>100</b>
300	250	<b>112.5</b>	<b>362.5</b>	250	<b>66.7</b>	<b>316.7</b>
500	250	200	450	250	200	450

Table 2: Detailed parameters for water supply simulations

Parameter	Sample Set	Replacement
Water Supply ( $\tilde{w}$ )	Log-normal distribution	With
User Endowments ( $R$ )	500	N/A
Number of Users ( $n$ )	3 to 36 in increments of 1	N/A
Stream Position ( $i$ )	1 to $n$	Without
Priority Number ( $m$ )	1 to $n$	Without
Priority Coefficient ( $\beta$ )	$20^{-m}$	N/A
Return Flow Coefficient ( $\theta$ )	0.1, 0.2, 0.3, ..., 0.9	With
Value Coefficient ( $v$ )	Normal distribution	With
Value Function	$v \cdot w^{\frac{1}{2}}$	N/A

Table 3: Parameters for a sample simulation with ten users and  $\tilde{w} = 2,252$

<b>Parameter</b>	<b>User 1</b>	<b>User 2</b>	<b>User 3</b>	<b>User 4</b>	<b>User 5</b>	<b>User 6</b>	<b>User 7</b>	<b>User 8</b>	<b>User 9</b>	<b>User 10</b>
Stream position ( $i$ )	1	2	3	4	5	6	7	8	9	10
Priority no. ( $m$ )	5	6	4	10	7	8	2	1	3	9
Share coef. ( $\delta$ )	0.085	0.065	0.11	0.01	0.048	0.034	0.193	0.293	0.143	0.021
Value coef. ( $v$ )	68	738	435	225	950	454	121	592	321	534
Return flow coef. ( $\theta$ )	0.4	0.1	0.8	0.6	0.7	0.8	0.8	0.3	0.7	0.3
Endowment ( $R$ )	500	500	500	500	500	500	500	500	500	500

# Figures

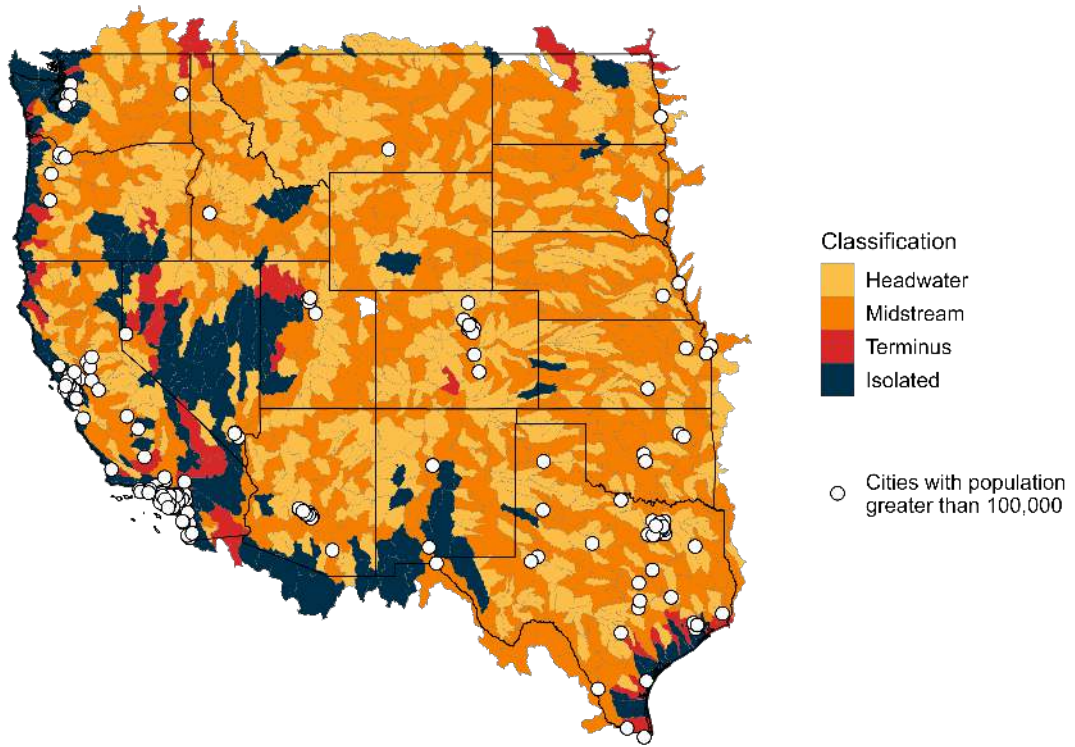


Figure 1: Spatial heterogeneity in hydraulic connectivity of western HUC8 basins per [Duan et al. \(2018\)](#)

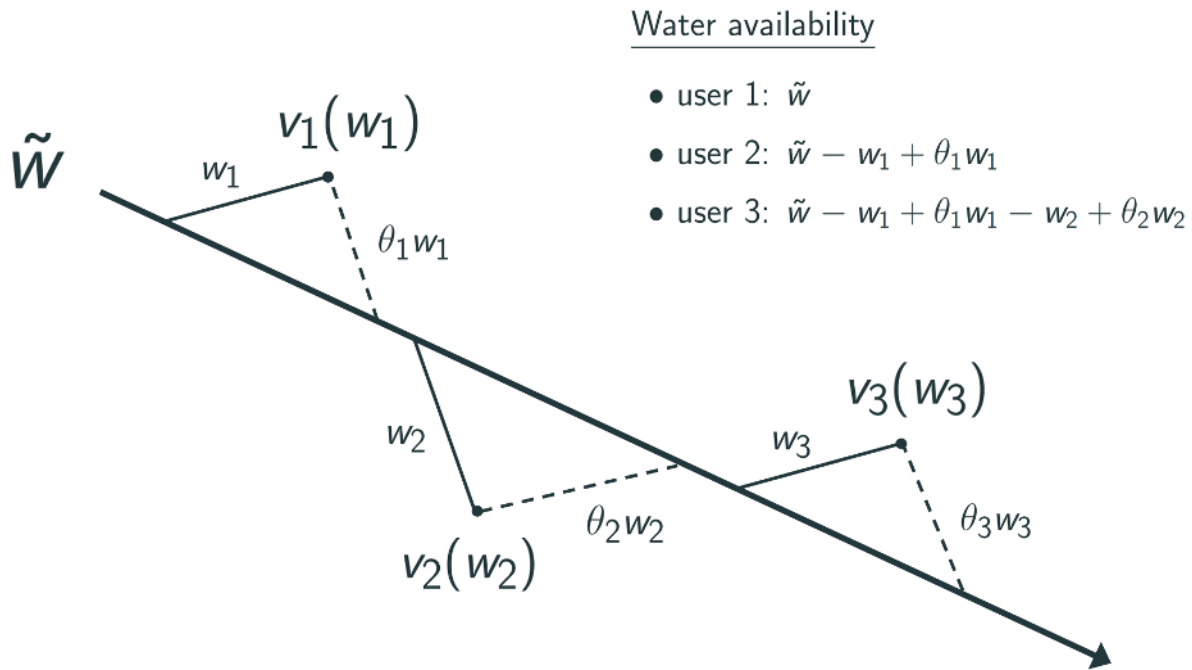


Figure 2: Visual representation of a hypothetical waterway with three users in which return flows from an upstream user are available to all downstream users

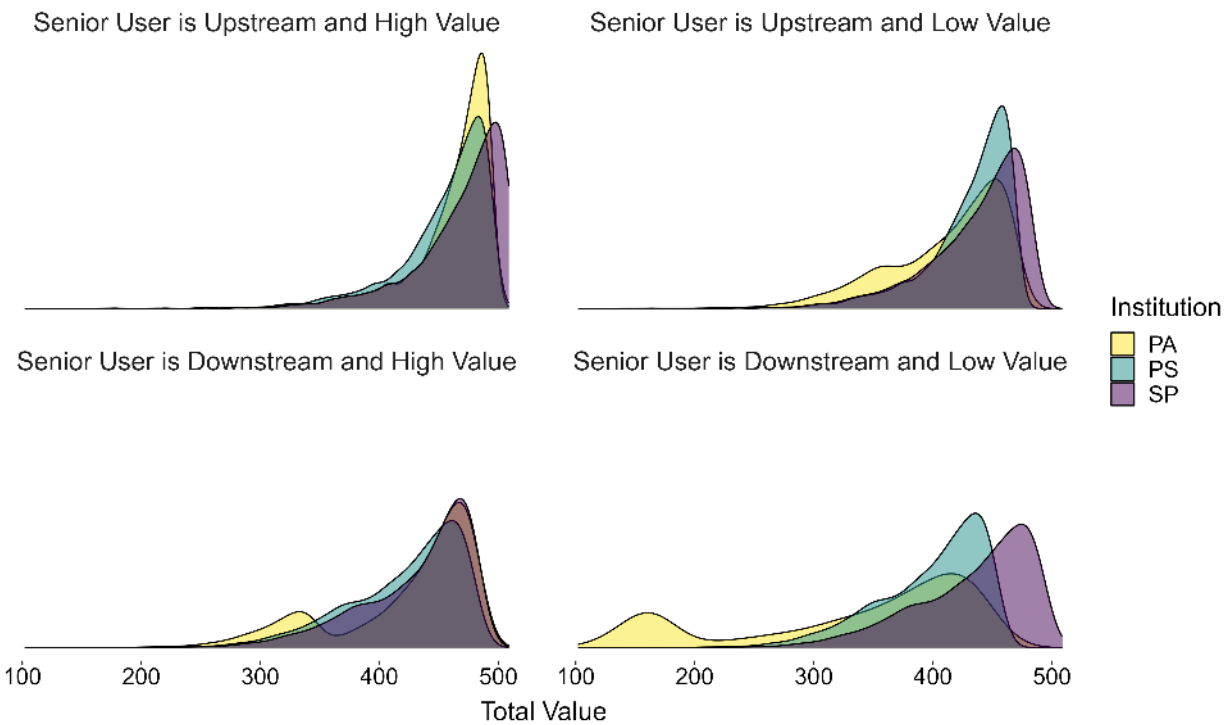


Figure 3: Monte Carlo generated total value distributions under different water allocation mechanisms with two water users

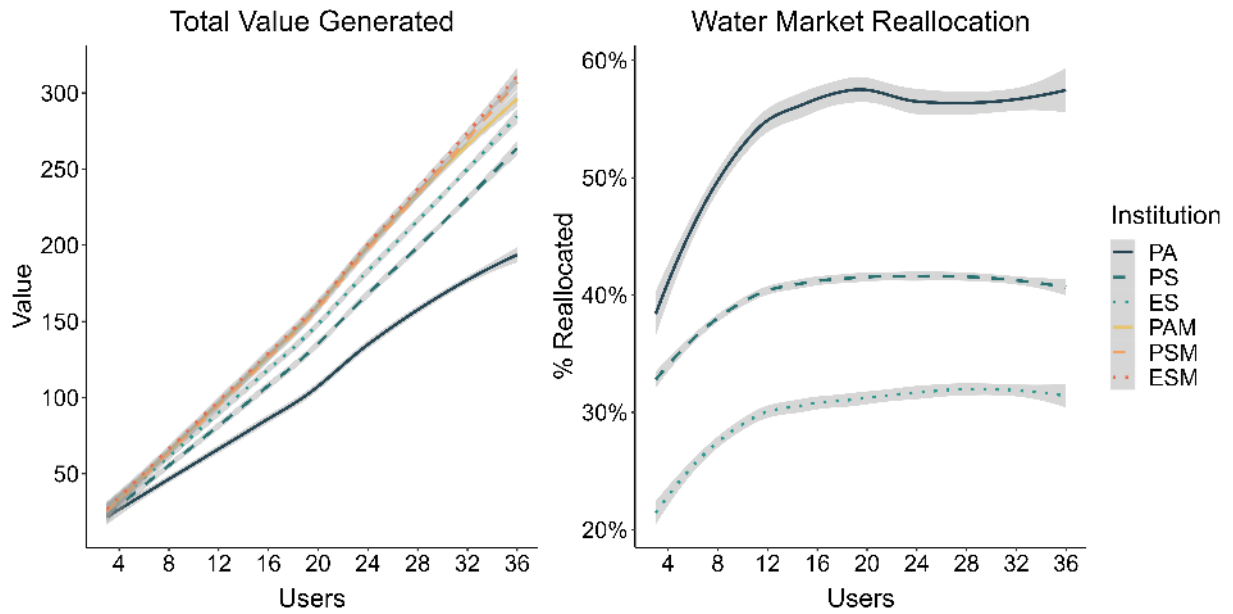


Figure 4: Average total value generated and percentage of reallocated water by institution and number of water users

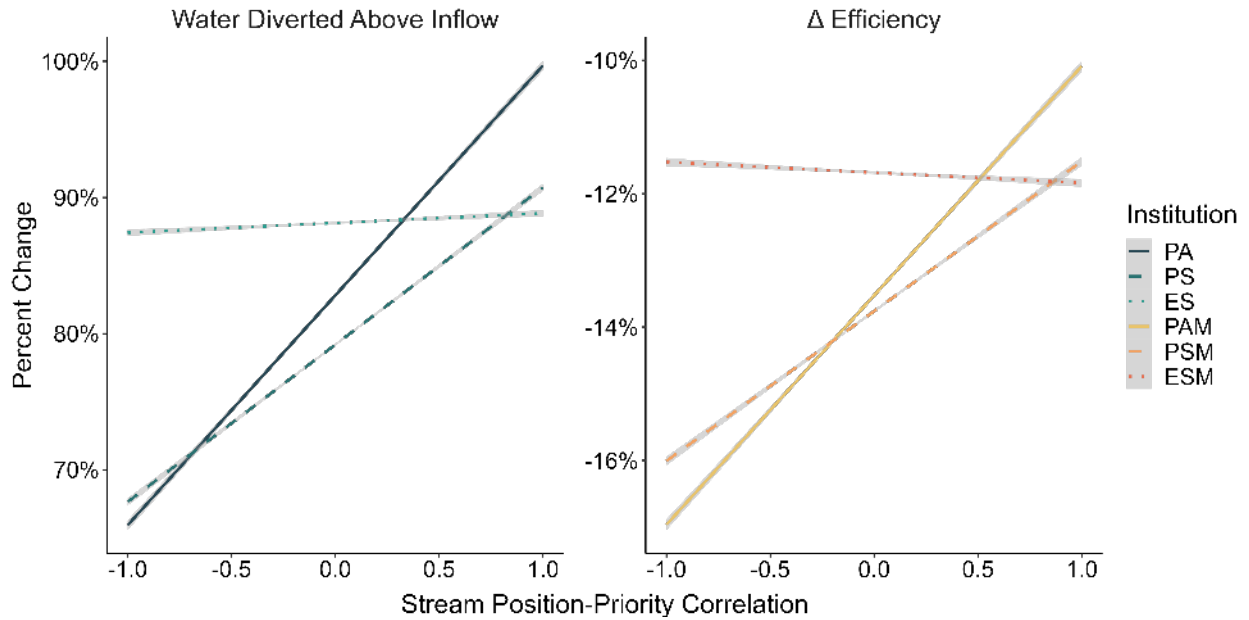


Figure 5: How priority location impacts the total amount of water available for trade

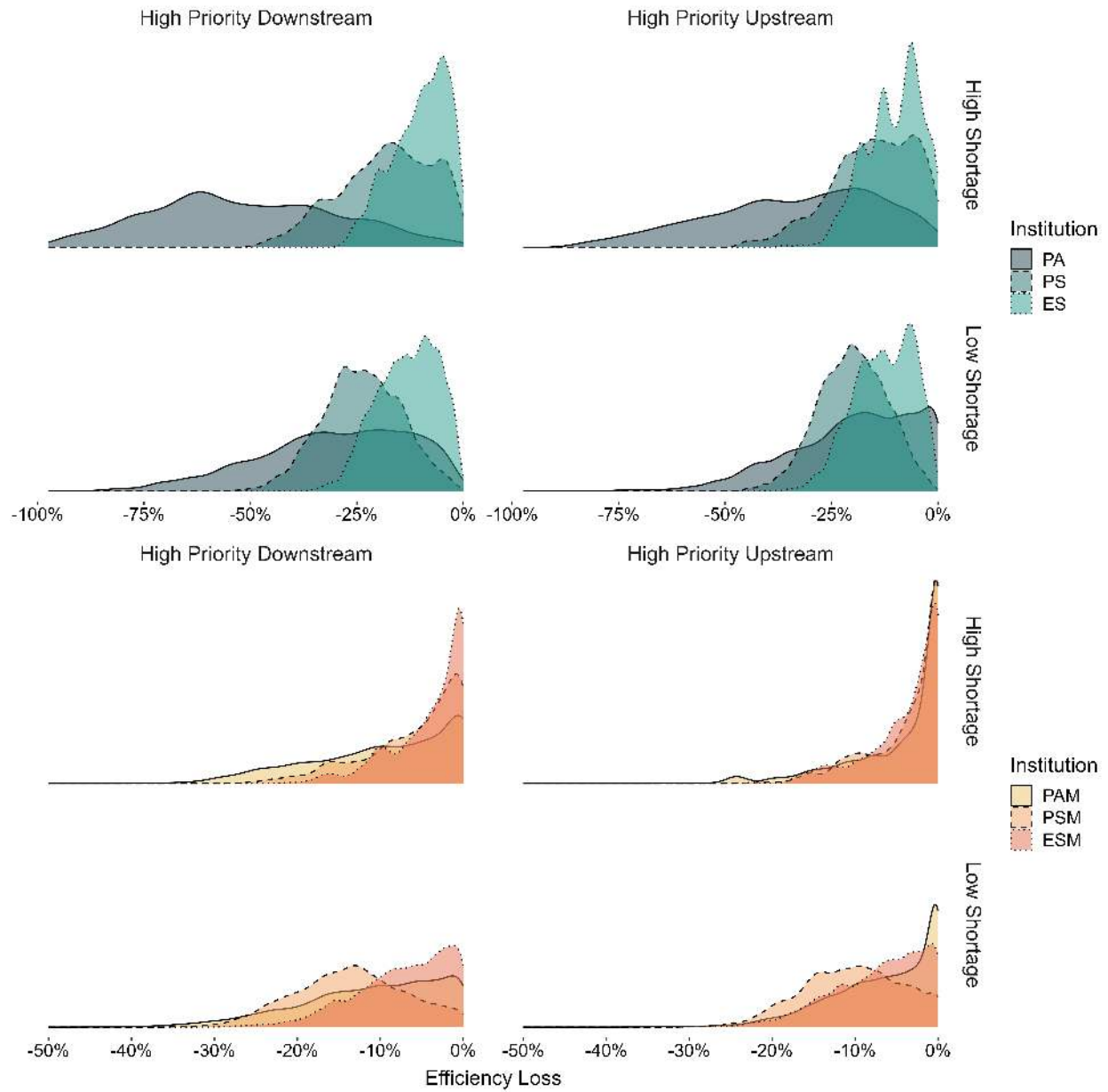


Figure 6: Distributions of total value losses relative to the Social Planner by institution, priority composition, and water shortage scenarios

## 5 Appendix

### 5.1 The Role of Return Flows in Water Allocation

Consider  $n$  users along a river, with a uniform return rate  $\theta$  for water used by each user. Each user  $i$  has a concave, differentiable value function  $v(w_i)$ , representing the total value generated from water diverted  $w_i$ . The objective of the Social Planner is to maximize the total value from water allocations subject to the total available water  $\tilde{w}$ . The simplified problem is formulated as:

$$\max_{w_1, \dots, w_n \geq 0} \sum_{i=1}^n v(w_i) \quad \text{s.t.} \quad w_1 + (1 - \theta)w_2 + \dots + (1 - \theta)^{n-1}w_n \leq \tilde{w} \quad (18)$$

The Lagrangian for this optimization problem is:

$$\mathcal{L} = \sum_{i=1}^n v(w_i) - \lambda \left( \sum_{i=1}^n (1 - \theta)^{i-1} w_i - \tilde{w} \right), \quad (19)$$

where  $\lambda$  is the Lagrange multiplier for the water availability constraint. Differentiating  $\mathcal{L}$  with respect to  $w_i$  yields the first-order conditions:

$$\frac{dv}{dw_i} = \lambda(1 - \theta)^{i-1}. \quad (20)$$

Since  $(1 - \theta) \in (0, 1)$  and  $i$  indexes stream position from upstream to downstream, this condition shows that the marginal value of water allocation decreases as we move downstream, adjusting for the return flow effect. Thus, the allocation  $w_i$  must decrease for increasing  $i$  to balance marginal value across users, leading to  $w_1 > w_2 > \dots > w_n$ . This pattern indicates that, due to return flows, more water is allocated upstream *ceteris paribus*.

## 5.2 Regression Exercises

In the above analysis, institutional performance is compared across hundreds of thousands of simulations under randomly generated basin compositions and different supply scenarios. While the mechanisms that consistently generate a large amount of value relative to others show robustness to varying basin characteristics, it is perhaps still not entirely clear how the institutional and physical constraints may mitigate or exacerbate inefficiencies. As a final exercise to try and characterize why certain mechanisms perform better or worse under different circumstances, I employ a series of regressions to explain why a mechanism generates the most value for a given simulation. Specifically, I run separate linear regression models for each water allocation mechanism:

$$\frac{y_k^j - y_k^{SP}}{y_k^{SP}} = \alpha^j + \beta_1^j X_k^j + \beta_2^j C_k + \varepsilon_k^j, \quad j = 1, \dots, J, \quad (21)$$

where  $\frac{y_k^j - y_k^{SP}}{y_k^{SP}}$  is the proportional decrease in value generated for mechanism  $j$  relative to the social planner in simulation iteration  $k$ . Matrix  $X_k^j$  contains variables characterizing institution-specific constraints, such as the correlation between priority and stream position and the correlation between priority and value. Matrix  $C_k$  contains information characterizing the physical characteristics of the basin such as the number of users, the level of shortage, and the correlation between stream position and return flows. The goal of this exercise is to identify which mechanisms are most sensitive to institutional constraints and basin compositions. Regression output from equation (18) is available in [Table A1](#).

## 5.3 Other Considerations

In addition to system-wide value, there are other outcomes from allocating water that warrant attention. For a 36 user system, [Figure A1](#) displays distributions for total value, average value per unit of diverted water, total diverted water, and basin outflow for each institution.

Table A1: Cross-sectional regression results assessing the impacts of basin composition and water shortage on institutional efficiency relative to the Social Planner

Model:	PA (1)	PAM (2)	PS (3)	PSM (4)	ES (5)	ESM (6)
Constant	-0.034*** (0.0008)	-0.078*** (0.0005)	-0.141*** (0.0004)	-0.095*** (0.0004)	-0.086*** (0.0003)	-0.037*** (0.0003)
# Users	-0.017*** ( $7.6 \times 10^{-5}$ )	-0.005*** ( $5.02 \times 10^{-5}$ )	-0.009*** ( $4.15 \times 10^{-5}$ )	-0.005*** ( $3.96 \times 10^{-5}$ )	-0.010*** ( $3.46 \times 10^{-5}$ )	-0.008*** ( $3.27 \times 10^{-5}$ )
# Users Sq.	0.0003*** ( $1.94 \times 10^{-6}$ )	$5.35 \times 10^{-5}$ *** ( $1.28 \times 10^{-6}$ )	0.0001*** ( $1.06 \times 10^{-6}$ )	$7.94 \times 10^{-5}$ *** ( $1.01 \times 10^{-6}$ )	0.0002*** ( $8.82 \times 10^{-7}$ )	0.0001*** ( $8.32 \times 10^{-7}$ )
Shortage	-0.001*** ( $6.86 \times 10^{-6}$ )	0.0003*** ( $4.53 \times 10^{-6}$ )	-0.0004*** ( $3.75 \times 10^{-6}$ )	0.0001*** ( $3.58 \times 10^{-6}$ )	0.0002*** ( $3.13 \times 10^{-6}$ )	0.0002*** ( $2.95 \times 10^{-6}$ )
Shortage Sq.	$-9.13 \times 10^{-7}$ *** ( $2.53 \times 10^{-8}$ )	$-8.67 \times 10^{-7}$ *** ( $1.67 \times 10^{-8}$ )	$1.22 \times 10^{-6}$ *** ( $1.38 \times 10^{-8}$ )	$2.34 \times 10^{-7}$ *** ( $1.32 \times 10^{-8}$ )	$-4.02 \times 10^{-7}$ *** ( $1.15 \times 10^{-8}$ )	$-3.71 \times 10^{-7}$ *** ( $1.09 \times 10^{-8}$ )
Prio. SP Corr.	0.080*** (0.0005)	0.035*** (0.0003)	0.025*** (0.0003)	0.023*** (0.0003)	-0.0005** (0.0002)	-0.0006*** (0.0002)
Prio. Value Corr.	0.148*** (0.0005)	0.004*** (0.0003)	0.106*** (0.0003)	0.002*** (0.0003)	0.0004 (0.0002)	0.0002 (0.0002)
RF SP Corr.	-0.044*** (0.0005)	-0.033*** (0.0003)	-0.023*** (0.0003)	-0.025*** (0.0003)	-0.032*** (0.0002)	-0.033*** (0.0002)
<i>Fit statistics</i>						
Observations	467,773	467,773	467,773	467,773	467,773	467,773
R <sup>2</sup>	0.59827	0.18693	0.38230	0.21444	0.40189	0.37203

*IID standard-errors in parentheses*  
*Signif. Codes: \*\*\*: 0.01, \*\*: 0.05, \*: 0.1*

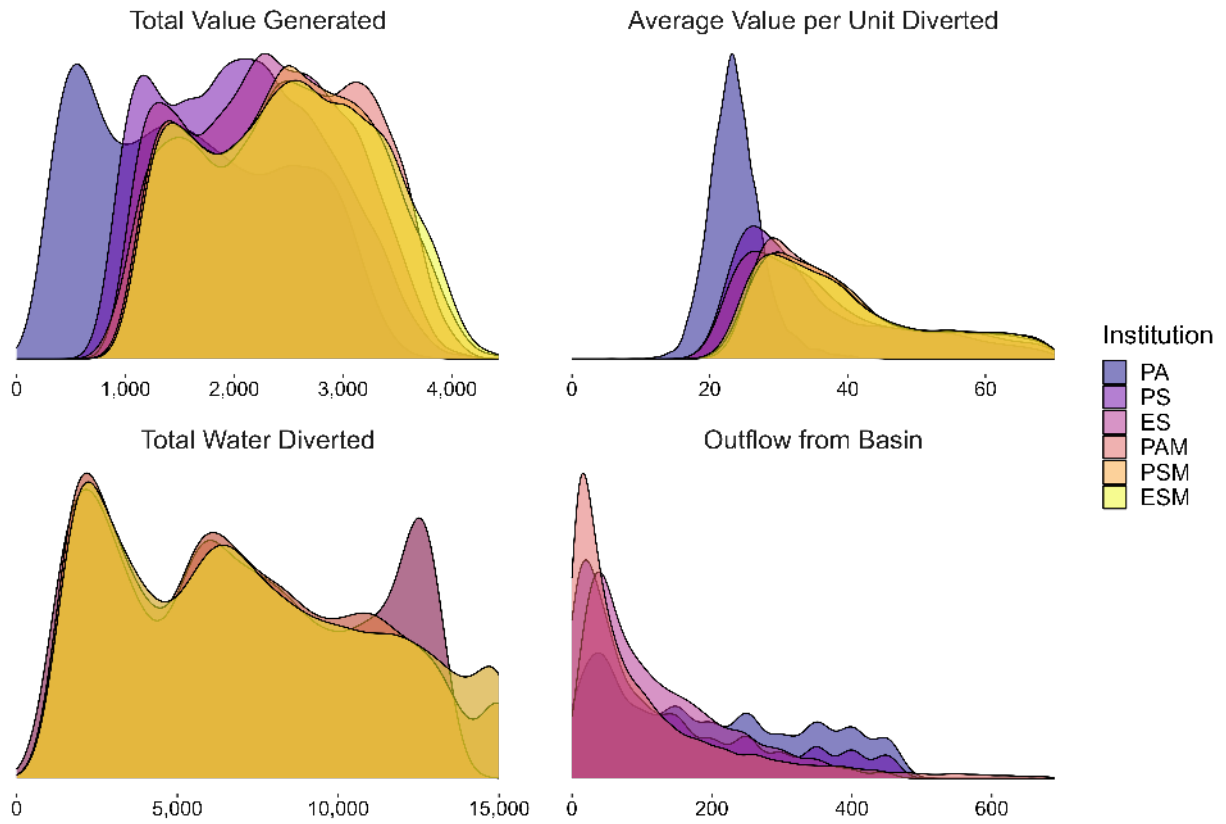


Figure A1: Monte Carlo generated total value, average value, total water, and basin outflow distributions with 36 water users