Option Pricing Revisited:

The Role of Price Volatility and Dynamics

Jean-Paul Chavas, University of Wisconsin-Madison, USA. Email: jchavas@wisc.edu.

Jian Li, Huazhong Agricultural University, China. Email: jli@mail.hzau.edu.cn.

Linjie Wang, Huazhong Agricultural University, China. Email: lwang829@wisc.edu.

Correspondence to J. Li.

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Abstract: The analysis of option pricing in derivative markets has commonly relied on the Black-Scholes model. This paper presents a conceptual and empirical analysis of option pricing with a focus on the validity of key assumptions embedded in the Black-Scholes model. Going beyond questioning the lognormality assumption, we investigate the role played by two assumptions made about the nature of price dynamics: quantile-specific departures from a unit root process, and the role of quantile-specific drift. Our analysis relies on a Quantile Autoregression (QAR) model that provides a flexible representation of the price distribution and its dynamics. Applied to the soybean futures market, we examine the validity of assumptions made in the Black-Scholes model along with their implications for option pricing. We document that price dynamics involve different responses in the tails of the distribution: overreaction and local instability in the upper tail, and underreaction in the lower tail. Investigating the implications of our QAR analysis for option pricing, we find that failing to capture local instability in the upper tail is more serious than failing to capture "fat tails" in the price distribution. We also find that the most serious problem with the Black-Scholes model arises in its representation of price dynamics in the lower tail.

Keywords: option pricing, futures price distribution, volatility, dynamics, quantile, soybean

JEL: G12, C31, Q11

1. Introduction

The last few decades have seen rapid growth in derivative securities which have offered new opportunities to manage risk in agricultural, energy, metal, and other commodity markets around the world (Dionne, 2013). Options have a long history in finance. They are now available on a wide array of financial instruments (Hull, 2022). The analysis of option pricing benefited from the seminal contribution made by Black and Scholes in 1973, leading to the derivation of an explicit formula for the pricing of options (Black and Scholes, 1973; Black, 1976; Merton, 1990). The availability of a simple formula representing option pricing has been seen as very appealing by economists, financial analysts as well as traders. Yet, it has become apparent that the validity of this formula applies only under restrictive assumptions. Evidence of the limitations of the Black-Scholes formula has accumulated over time (e.g., Fortune, 1996; Bates, 2003; Dionne, 2013). This has stimulated much interest in investigating the factors affecting option pricing as well as the validity of assumptions made in the Black-Scholes model (Hull, 2022; Bates, 2022).

The Black-Scholes model makes a number of assumptions about the distribution and dynamics of the futures price. The validity of these assumptions has been questioned in previous literature. This paper revisits these issues in three directions.[1] First, the Black-Scholes model assumes a lognormal distribution. There is strong evidence that the distributions of prices tend to have "fat tails", implying that the Black-Scholes model tends to underestimate the likelihood of rare events (e.g., Christoffersen et al., 2012; Hull, 2022). Also, the presence of price limits on agricultural futures contracts affected trading behavior as well as the shape of the price distribution (He and Serra, 2022). These factors suggest that the Black-Scholes formula cannot provide an accurate representation of option pricing. This argument has stimulated research on the determinants of option prices under less restrictive distributional assumptions (e.g., Fang and Lai, 1997; Doan et al., 2010; Hull, 2022).[2]

Second, acquiring market information is not costless and heterogeneity of information among traders can affect commodity and financial markets (Stambaugh et al., 2015). This heterogeneity can stimulate herd behavior among traders, possibly leading to "bubbles" and dynamic price instability (Bikhchandani and Sharma, 2001; Barberis et al., 2018; Hommes et al., 2021). Such effects are not captured in the Black-Scholes model. They indicate a need to explore the nature and implications of price dynamics in the upper tail of the price distribution (where bubbles occur).

Third, the Black-Scholes model assumes the absence of liquidity constraints. Liquidity constraints and their effects on commodity and financial markets have been documented (e.g., Brunnermeier and Pedersen, 2009; Rytchkov, 2014; Christoffersen et al., 2021). This includes the role of margin requirements that can affect prices: under unexpected price declines, margin calls can induce liquidity-constrained traders to sell, putting additional downward pressure on prices (Chowdhry and Nanda, 1998; Mendoza, and Smith, 2002). McKenzie et al. (2022) characterized implied volatility functions (IVF) from agricultural option markets and found that short-term hedging pressure is one potential explanation driving their IVFs skew, whereby the market fear of price decline creates large hedging demand for out-of-the-money (OTM) puts and then pushes up IVFs. Again, these effects are not captured in the Black-Scholes model. They suggest a need to explore the nature and implications of price dynamics in the lower tail of the price distribution (when margin calls occur).

Our paper studies the nature and dynamics of market price volatility and its implications for option pricing. In the process, we investigate the validity of key assumptions made in the Black-Scholes model. To address these issues, our analysis relies on a quantile autoregression (QAR) model (Koenker, 2005; Koenker and Xiao, 2004, 2006). Our QAR approach provides a flexible representation of price volatility and its dynamics. First, a quantile regression approach offers a nonparametric estimate of the price distribution. By permitting arbitrary shapes of the distribution function, it allows the investigation of the validity of the lognormality assumption in the Black-Scholes model. Second, a QAR model allows for price dynamics that can vary across quantiles (Koenker and Xiao, 2004, 2006). This is very important for our analysis: a QAR approach allows us to investigate the nature of dynamics in the tails of the price distributions. This includes possibly different price dynamics in the upper tail versus the lower tail. Third, the QAR approach allows for drifts in price dynamics. While the Black-Scholes model allows for drift in the mean log price (Black and Scholes, 1973; Black, 1976; Merton, 1990), its assumption of constant price volatility does not allow for changes in higher moments. As such, Black-Scholes pricing neglects the possibility of having quantile-specific drifts. Fourth, relying on a QAR representation of price volatility and its dynamics, we explore the validity of the Black-Scholes model and its ability to explain and predict option prices. This allows us to investigate the empirical relevance of the assumptions made in the Black-Scholes model.

The usefulness of our approach is illustrated in an empirical application to option prices in the US soybean market. Options strategies have been important components in efficient portfolios of soybean marketing activities since the 1980s, and provided soybean producers with higher, less risky expected returns (Frank et al., 1989; Urcola et al., 2011). Applied under risk neutrality,[3] we evaluate empirically how the Black-Scholes model fails to provide an accurate representation of option pricing. Our analysis identifies which assumptions in the Black-Scholes model are more problematic. First, in a way consistent with previous research, we find strong evidence of "fat-tails" in the distribution of the futures price. This reflects a departure from lognormality, indicating that the "implied volatility" in the Black-Scholes formula cannot be a sufficient statistic for price volatility. Second, we do not find statistical evidence of departure from the Black-Scholes assumptions around the median of the distribution. However, we uncover strong statistical evidence against several of the Black-

Scholes assumptions in the upper tail and lower tail of the distribution, stressing the importance of going beyond simple mean-variance regression analysis in the evaluation of financial markets. Third, we find statistical evidence that price dynamics differ across quantiles. The evidence points to two issues: 1) the presence of local instability (corresponding to a departure from a "unit root" process) in the upper tail of the price distribution; and 2) the presence of local stability and quantile-specific drifts in the lower tail of the price distribution. To the extent that dynamic pricing patterns reflect the behavior of market participants, these results provide indirect evidence that traders' responses to price shocks vary with market conditions (with "overreaction" to positive price shocks and "underreaction" to negative price shocks). In turn, such patterns can affect option pricing in two significant ways: 1) they can influence how traders view the distribution of future prices; and 2) such influences are not captured in the Black-Scholes model.

Fourth, our analysis examines the validity and role of key assumptions made in the Black-Scholes model and its associated option pricing. With a focus on three assumptions (lognormality, instability in the upper tail, and quantile-specific drift in the lower tail), we evaluate which source is more relevant and problematic for Black-Scholes pricing. We find that the departure from lognormality (an issue extensively discussed in previous literature) is the least problematic part of the Black-Scholes model. The evidence indicates that failing to capture local instability in the upper tail is more serious than a failure to capture "fat tails" in the price distribution. The most serious problem with the Black Scholes model arises in the representation of price dynamics in the lower tail. We interpret these results as indirect evidence that traders' behavior and traders' varying response to shocks (with different response to positive shocks versus negative shocks) play an important role in option pricing. We note that our findings are consistent with the presence of herd behavior (affecting price dynamics in the upper tail) and liquidity constraints (contributing to slow price adjustments in response to unanticipated adverse shocks). With this interpretation in mind, our analysis suggests that these factors may play an important role in explaining why the Black-Scholes formula fails to provide an accurate representation of option prices.

The rest of the paper is organized as follows. Section 2 reviews the literature on option pricing. Section 3 presents an economic model of the factors affecting price dynamics. Section 4 examines how a quantile autoregression (QAR) approach can provide a refined econometric representation of price volatility and its dynamics. An application to the US soybean market is presented in sections 5 and 6. Sections 7 and 8 discuss the econometric implications for dynamic futures price volatility and the economic implications for option pricing. Finally, section 9 concludes.

2. Option Pricing

As noted in the introduction, the last few decades have seen rapid growth in option markets. Options include call options, giving a market participant the right, but not the obligation, to purchase an asset under specific terms. They also include put options that give a market participant the right to sell an asset under specific terms. The development of option markets has created new opportunities to manage risk in commodity and financial markets around the world (Dionne, 2013, Hull, 2022). For instance, McKenzie et al. (2022) pointed out that market fear of price declines creates large hedging demand for out-of-the-money (OTM) puts when they analyze short-term hedging in agricultural markets.

An options contract includes an exercise price, or strike price, stating the price at which a given asset is to be exchanged. It also specifies an expiration date. In an American option, the contract can be exercised at any time before and including the expiration date. In contrast, in a European option, the contract can be exercised only on the expiration date. The economics of option pricing has been analyzed by Black and Scholes (1973), Black (1976) and Merton

(1990). Our analysis below focuses on call options[4] applied to a futures contract.

The pricing of a European call option[5] on a futures contract at time $t \in [0, T]$ with strike price K is

$$C(t) = e^{r\,(T-t)}\, E\{\max[F(T) - K, 0]\}, \tag{1}$$

where E_t denotes the risk-neutral expectation conditional on the information available at time t, $e^{r\,(T-t)}$ is a discount factor that depends on the risk-free interest rate r, and $F(T)$ is the price of the futures contract at maturity time T. In the Black-Scholes model, an explicit formula for option pricing is derived under specific assumptions (Black and Scholes, 1973; Black, 1976; Merton, 1990). The Black-Scholes model assumes that the price of the underlying asset follows a geometric Brownian motion, where $\frac{dF(t)}{F(t)} = \mu\, dt + \sigma\, dW(t)$, where $W(t)$ is a standard Weiner process and σ is the instantaneous standard deviation of $\log(F(t))$. In this context, $F(t)$ has a lognormal distribution with $var(\ln(F(t)) = \sigma^2\, t$. Letting $F(t)$ be the futures price at time t and following Black (1976), the pricing of a European call option on a futures contract is given by the formula:[6]

$$C(t) = e^{-r(T-t)}\, [F(t)\, N(d_1) - K\, N(d_2)], \tag{2}$$

where $d_1 = \frac{\ln\left(\frac{F}{K}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{(T-t)}}$, $d_2 = d_1 - \sigma\sqrt{(T-t)}$ and $N(\cdot)$ is the standard cumulative normal distribution function. The Black-Scholes formula (2) or (2') has been widely used by options traders mostly because it is easy to calculate (Hull, 2022). By does it provide an accurate representation of option pricing? Evidence of the limitations of the Black-Scholes formula have accumulated over time (Fortune, 1996; Dionne, 2013). A simple piece of evidence comes from defining "implied volatility" as the value of σ that solves equation (2) given observations on C, F, K, r and T. In a given option market, the Black-Scholes formula can be tested by comparing the implied volatility for different strike prices K and different maturities T. The Black-Scholes model predicts that this implied volatility would be the same for all strike prices and all maturities. Yet, there is strong evidence that implied volatility varies with the strike prices (Hull, 2022), indicating that the Black-Scholes formula fails to provide an accurate representation of option pricing.

This result has stimulated much interest in investigating which assumptions make the Black-Scholes formula inappropriate. Several directions of inquiry have been explored (Hasbrouck, 1993; Jondeau and Rockinger, 2003; Wang and Tomek, 2007). As noted in the introduction, our analysis focuses on three sets of issues. First, the Black-Scholes model assumes that the asset value has a lognormal distribution. This seems restrictive in the sense that some risk distributions exhibit "fat tails" in which case relying on the Black-Scholes formula may underestimate tail risk and misrepresent the value of rare events (Fang and Lai, 1997; Doan et al., 2010; Hull, 2022). Fat-tailed models provide superior option pricing performance, particularly during medium and high volatility periods (Christoffersen et al., 2012). Existing literature use GARCH models with Compound Poisson jump component (Duan, 1995; Heston and Nandi, 2000; Fleming and Kirby, 2003), infinite-activity Levy jump processes (Ornthanalai, 2014), and stochastic volatility models to allow for fat tails and asymmetric distribution (Barone-Adesi et al., 2008). These models provide possible explanations for some well-documented systematic biases associated with Black-Scholes model.

Second, accessing and processing information about market conditions can vary across traders. In turn, this information can affect asset prices. In situations where most traders are well-informed, market prices would be quick to respond to new information, as stated in the efficient market hypothesis (Fama,1970, 2013). However, heterogeneity in information processing can allow poorly informed traders that participate in the market to affect prices. For instance, Buraschi and Jiltsov (2006) illustrate how heterogeneous beliefs among market traders can better account for smirks, resulting in greater demand and relatively higher prices for OTM puts than Black-Scholes predictions. Such effects can vary depending on market conditions and generate asymmetries in the price distribution (Stambaugh et al. 2015). One scenario is when some traders copy the behavior of others, leading to herd behavior. For example, when some traders are bullish and other traders copy them, the market may see increased speculative demand for the asset, putting upward pressure on prices and possibly creating a "bubble" characterized by dynamic instability exemplified by price booms (Bikhchandani and Sharma, 2001; Barberis et al., 2018; Hommes et al., 2021). Such effects are more likely to occur in the upper tail of the price distribution where positive shocks drive market booms.

Third, the Black-Scholes formula assumes that trading is costless and that traders do not face liquidity constraints. The presence of trading cost or liquidity constraints can affect market prices (Brunnermeier and Pedersen, 2009; Christoffersen et al., 2021) and possibly invalidate the Black-Scholes formula. For example, margin requirements are expected to affect the functioning of financial markets (Rytchkov, 2014). Margin requirements are set to help guarantee that contract holders will not default on their obligations. However, these requirements can also affect traders: a margin call would be issued by a brokerage firm to traders who see a significant decline in the asset price used as collateral. In the presence of liquidity constraints, margin calls can induce these traders to sell, thus putting additional downward pressure on the asset price (Chowdhry and Nanda, 1998; Mendoza, and Smith, 2002). And these effects are more likely to occur in the presence of price declines, i.e., in the lower tail of the price distribution. These arguments indicate that liquidity constraints can affect both the distributions of asset prices and their dynamics.

This discussion indicates the need to relax three key assumptions made in the Black-Scholes model: 1) lognormality; 2) a unit-root process; and 3) no quantile-specific drifts. Doing so requires a flexible representation of the price distribution and of price dynamics, including both the upper tail of the price distribution (where bubbles can occur) as well as the lower tail (where the effects of liquid constraints may be observed). In the recent literature, capturing the distributions of prices or returns of commodities as well as financial assets has attracted significant attention (e.g., Isengildina-Massa et al., 2010; Hua and Manzan, 2013; Li and Chavas, 2023). Introducing time variation in the conditional distribution (e.g., through the conditional variance in GARCH models) has been partially successful in explaining several empirical features of returns, such as fat tails. While GARCH models assume a parametric form for the latent variance of returns, a more flexible approach comes from quantile autoregression (QAR) models that do not require a parametric specification of the distribution (Koenker and Xiao, 2004, 2006; Barunik and Cech, 2021). For example, Zikes and Barunik (2014) use linear quantile regressions to investigate how the conditional quantiles of futures returns and volatility of the S&P 500 and WTI Oil futures contracts vary. The model works on integrated variance, upside and downside semi-variance and is found to perform better in the right tail of the distribution. While the use of quantile regression is not new estimating the conditional distribution of spot and futures prices, to the best of our knowledge, the QAR model has not been used in the analysis of option pricing. Relying on a QAR representation of price volatility and its dynamics, this paper explores the validity of the Black-Scholes model and its ability to explain and predict option prices. These issues are further discussed below.

3. Futures Price Dynamics

Our analysis focuses on options applied to a futures market. This section presents a conceptual analysis of the determinants of futures prices and its dynamics. Consider a futures market where p_t is the price at time t for a futures contract with delivery at some future time $t + \delta$. Let K be the set of agents participating in the futures market. At time t, the k-th market participant trades the quantity Q_{kt} of this futures contract, $k \in K$. The quantity Q_{kt} is defined as a net quantity: it is positive for quantity purchased and negative for quantity sold. The quantity Q_{kt} is chosen by the k-th agent according a decision rule denoted by $Q_{kt} = Q_k\ (p_t, p_{t-1}, x_t, e_t), k \in K$, where x_t represents factors affecting the behavior of market participants and e_t is a vector of random variables representing uncertainty. We assume that the function $Q_k(p_t, \cdot)$ is continuous and strictly monotone. In this context, market equilibrium in the futures market at time t is given by the market clearing condition:

$$\sum_{k \in K} Q_k\ (p_t, p_{t-1}, x_t, e_t) = 0, \tag{3}$$

which has for solution the market-clearing price denoted by

$$p_t = g(p_{t-1}, x_t, e_t). \tag{4}$$

Equation (4) provides a representation of price dynamics in the futures market. Either equation (3) or equation (4) provides a valid representation of the determination of the futures price at time t. In general, such price determination depends on the decision rules of all market participants.

Consider the decision rule $Q_k(p_t, p_{t-1}, x_t, e_t)$ of the k-th market participant, $k \in K$. We expect this decision rule to reflect the information available to the k-th agent along with his/her market position. To see that, consider the case where the k-th agent is a speculator who intend to purchase a futures contract Q_{kt} at price p_t at time t and to sell it at price p_{t+1} at time $(t+1)$. In situations where $Q_{kt} < 0$, this would correspond to the k-th agent selling $|Q_{kt}|$ at price p_t at time t and purchasing it at price p_{t+1} at time $(t+1)$. Denote by $C_k(Q_{kt}, x_t, e_t)$ the cost of the transaction at time t, reflecting information cost, margin deposit requirements as well as brokerage fees. The associated present value of profit at time t is $\pi_{kt} = [\frac{1}{1+r}\ p_{t+1} - p_t]\ Q_{kt}\ - C_k(Q_{kt}, x_t, e_t)$, where $r \in [0, 1)$ is the interest rate. The price p_{t+1} being uncertain at time t, the k-th agent forms expectation about price p_{t+1} based on available information. Assume that the k-th agent is risk neutral and maximizes expected payoff: $E_{kt}[\frac{1}{1+r}\ p_{t+1} -$

$p_t]\, Q_{kt} - C_k(Q_{kt}, x_t, e_t)$, where E_{kt} is the expectation operator based on the information available to the k-th agent at time t. The information available at time t includes (p_t, p_{t-1}, x_t, e_t) as these variables are factors affecting the decision rule for Q_{kt}. When $Q_{kt} \neq 0$ and under differentiability, the optimal decision for Q_{kt} is given by the first-order condition

$$E_{kt}\left(\frac{p_{t+1}}{1+r}\right) = p_t + C'_{kt}, \tag{5}$$

where $C'_{kt} = \frac{\partial C_k(Q_{kt}, x_t, e_t)}{\partial Q_{kt}}$ is the marginal cost of Q_{kt}. Equation (5) states an intertemporal arbitrage condition between the price p_t and p_{t+1}: the expected discounted future price $E_{kt}\left(\frac{p_{t+1}}{1+r}\right)$ is equal to p_t plus the marginal cost C'_{kt}.

Letting the k-th agent's decision rule $Q_k(p_t, p_{t-1}, x_t, e_t)$ be the solution of (5) for Q_{kt}, it follows from equation (5) that trading activities generate an intertemporal price arbitrage in the futures market. The implications of such arbitrage opportunities for the dynamics of p_t depend on the magnitude of the marginal cost C'_{kt}. In the special case where there is no transaction cost and the market participants are risk neutral, then $C'_{kt} = 0$ and equation (5) reduces to

$$E_{kt}\left(\frac{p_{t+1}}{1+r}\right) = p_t, \tag{6}$$

stating that current price p_t is equal to the expected discounted price $E_{kt}\left(\frac{p_{t+1}}{1+r}\right)$. The arbitrage condition (6) has often been treated as a characteristic of an efficient market (e.g., Fama, 1970, 2013).[7] However, comparing (5) and (6), it is clear that this result does not hold under transaction cost (when $C'_{kt} \neq 0$). Under such scenarios, one expects price dynamics to be more complex (e.g., Schwert, 2003; Schiller, 2015). Indeed, C'_{kt} in equation (5) plays the role of a "drift" in intertemporal price arbitrage: when $C'_{kt} > 0$, equation (5) would imply that $E_{kt}\left(\frac{p_{t+1}}{1+r}\right) > p_t$, indicating that transaction cost would be associated with an increase in expected price change. In addition, when the cost function $C_k(Q_{kt}, \cdot)$ is convex, a rise in transaction cost would give the agent a distinctive to trade. These combined effects would affect price dynamics. In particular, in situations where the cost C_{kt} is affected by e_t, such dynamic effects could vary with market shocks e_t. These issues are addressed in our empirical analysis presented below.

4. Econometric methods

This section discusses the econometric method used to model the price distributions of futures prices. Our analysis of futures price p based on equation (4) stating that $p_t = g(p_{t-1}, x_t, e_t)$. Assume that the function $g(\cdot)$ is differentiable, that e_t is a serially independent random vector with distribution function $D(v) = Prob(e_t \leq v)$, and that $D(v)$ is absolutely continuous. In this context, equation (4) is a general autoregressive process representing the stochastic dynamics of p_t.

The conditional distribution function of the price p_t is $F(p_t | \, p_{t-1}, x_{t_t}) = Prob[g(p_{t-1}, x_t, e_t) \leq p_t]$ and its associated quantile function is $Q(q | \, p_{t-1}, x_t) \equiv inf_p \, \{p: F(p | \, p_{t-1}, x_t) \geq q\}$ where $q \in [0, 1]$ is the q-th quantile. Consider the following specification for the quantile function

$$Q(q | \, p_{t-1}, x_t) = \beta_0(q) + \beta_1(q) \, p_{t-1} + \beta_2(q) \, x_t \qquad (7)$$

where the $\beta(q)$'s are parameters to be estimated. Equation (7) constitutes a quantile autoregression (QAR) model providing a flexible representation of the distribution of the price p_t and the underlying price dynamics. Indeed, when $\beta_2 = 0$ and $\beta_1(q)$ does not vary with q, then (7) reduces to a standard autoregression (AR) model commonly used in time series analysis (e.g., Hamilton, 1994; Enders, 2010). Allowing $\beta_0(q)$ to vary with q does not impose a priori restrictions on the shape of the distribution function (e.g., it allows for any skewness or kurtosis).

In addition, allowing for $\beta_1(q)$ to vary with q permits dynamics to affect the variance, skewness or kurtosis of prices. The parameter $\beta_1(q)$ captures the effect of lagged price on p_t

and provides useful information about price dynamics. When $\beta_1(q) = \beta_1$ for all $q \in [0, 1]$, model (7) reduces to an AR model, in which case price dynamics is stable when $|\beta_1| < 1$ and unstable when $|\beta_1| \geq 1$ (Hamilton, 1994; Enders, 2010). This includes as special case a unit root process where $\beta_1 = 1$, as commonly found in the discussion of efficient markets (e.g., Fama, 1970, 2013). But as discussed in section 3, price dynamics can possibly vary depending on stochastic shocks. Such effects are captured in our QAR model as $\beta_1(q)$ in (7) can vary across quantiles. In general, $\beta_1(q)$ measures the speed of price adjustment to p_{t-1} under a shock located at the q-th quantile of the price distribution. This stresses the fact that, unlike standard AR models, model (7) can distinguish between dynamic adjustments due to "positive" versus "negative" shocks. This distinction can be important when markets behave differently in each tail of the price distribution (e.g., price bubbles can be expected to occur only in the upper tail). This indicates the general flexibility of our approach: it allows the investigation of how dynamic stability can vary across market situations.

Following Koenker (2005) and Koenker and Xiao (2006), equation (7) can be estimated by quantile regression. Consider a sample of T observations denoted as (p_t, p_{t-1}, x_t), $t \in \{1, \dots, T\}$. The parameters β's in (7) can be estimated as follows

$$\beta(q) \in argmin_\beta \{\textstyle\sum_{t=1}^{T} \rho_q \, (p_t - Q(q|\, p_{t-1}, x_t)\}, \tag{8}$$

where $\rho_q(w) = w\,[q - I(w < 0)]$, $I(\cdot)$ is an indicator function and $Q(q\,|p_{t-1}, x_t)$ is given in (7). As shown in Koenker and Xiao (2004, 2006), the quantile estimator in (8) provides consistent estimates of the parameters under general conditions. Yet, asymptotic properties of the estimator become non-standard in the presence of a unit root (Koenker and Xiao, 2004, 2006). On that basis, our empirical analysis relies on (8) for estimation, and on bootstrapping in conducting hypothesis testing.

5. Data

To estimate the distributions of futures price and study the linkage with option pricing, we apply our method to the US soybean market. The study relies on weekly futures prices from the Chicago Board of Trade (CBOT) over the last four decades 1980-2019.[8] To construct continuous price sequences, we choose to use the front-month rolling method, which rolls nearby futures contracts on expiration dates in contract months. The four-decade futures price data are used to provide enough information for the QAR model to estimate the underlying time-varying futures price distributions. The soybean options data are collected from *DataStream*. In the empirical section, we evaluate the performance of our pricing model and alternative ones using November 2019 option prices as an example and perform robustness checks using several other contracts. By convention, we construct the options data by applying several excluding filters. First, to avoid contracts with low trading volumes, we exclude options data that are more than six months and less than three weeks to expiration. Figure A1 in Appendix A provides evidence supporting this choice from the perspective of trading volumes. This results in a 22-week option price series used in the empirical analysis. Second, we choose strike prices that are frequently traded, resulting in the inclusion of strike prices ranging from 800 to 1100 cents per bushel for the November 2019 options contract. In option pricing, we also use the weekly interest rate proxied by the one-year treasury constant maturity rate obtained from the FRED database.

The upper panel of Table 1 shows that the soybean futures price has a mean of 711 cents per bushel in the sample period. The soybean futures prices have been very volatile, evidenced by a standard deviation of 274 with a maximum of 1753 and a minimum of 413. Figure A2 in Appendix A reports the trajectory of soybean futures prices in the sample period. It shows that the soybean market experienced several waves of price booms and busts, with the largest fluctuation witnessed during the 2008 global food crisis from 2008. In comparison, the soybean price tends to be less volatile from 2014. The price booms and busts indicate a need to take rare market events into consideration in modeling dynamic price distributions.

The lower panel reports the summary statistics of call option prices of November 2019, classified by ranges of moneyness and different lengths to maturity. Defining S(t) - K as the intrinsic value of a call at time t, a call option is classified as at-the-money (ATM) if S/K belongs to (0.95, 1.05); out-of-the-money (OTM) if less than 0.95; and in-the-money (ITM) if greater than 1.05. We further create six moneyness categories by a finer partition. As for time-to-expiration (TTE), we further classify option contracts as (a) short-term (<25 weeks); (b) medium-term (26-50 weeks); and (c) long-term (>50 weeks). In total, the lower panel of Table 1 reports 18 categories based on the proposed moneyness and maturity classifications. As shown, the option prices decrease in the off-diagonal direction, i.e., larger with OTM shorter-term contracts (averaged at 153.239 for S/K<0.9 and TTE<25) and lower with ITM longer-term ones (averaged at 5.874 for S/K>1.10 and TTE>50 weeks).

6. Econometric estimation

As discussed in Section 4, We employ the QAR approach to investigate the dynamics of US soybean futures prices. Based on previous research (e.g., Sørensen, 2002), our QAR model considers price dynamics as well as X_t (i.e., seasonality).[9] Monthly dummies (M_{1t}, ..., M_{12t}) are introduced to reflect seasonality. The number of price lags n that enter equations (4) were chosen using the Bayesian Information Criterion (BIC). The BIC suggests the QAR (1) model with one lag provides a better fit to the data.

We estimated the time-varying marginal price distributions based on the specified QAR (1) model. Table 2 reports the QAR estimates for the futures price pf for selected quantiles: q = (0.1, 0.3, 0.5, 0.7, 0.9). The goodness-of-fit statistics proposed by Koenker and Machado (1999) show the pseudo R^2 ranges from 0.868 to 0.917 under selected quantiles, indicating a very good fit to the data by our specified QAR model. Two points are worth noting in Table 2. First, the lagged prices are found to be statistically significant at one percent level for all five quantiles. This is a result consistent with conventional findings (e.g., Karali and Power, 2013), which have documented the existence of short-term price memories at the mean. Second, some monthly dummies are statistically significant, e.g., M_6 to M_9 in various quantiles. This points to the important role of seasonality before harvest, which has been discussed in the literature (e.g., Sørensen, 2002). For comparison purposes, Table 2 also plots results from the mean regression, i.e., Autoregressive (AR) model. It shows that seasonality effects (e.g., M_7 and M_8) cannot be captured in the simple AR model: these effects are significant in tails rather than around the mean or median. This indicates that it would be too narrow to focus only on mean effects in modeling price dynamics.

Table A1 in Appendix A formally tests whether the parameter estimates vary across quantiles. The test results find strong evidence that the estimates vary across quantiles, and the upper and lower quantile estimates are significantly different from the median. Table A1 also tests whether seasonality matters. The test results show seasonality significantly affects prices, and such effects are found to differ across quantiles. Compared to the extreme quantiles, the evidence that monthly dummies matter for soybean prices is weaker around median (i.e., $q = 0.5$ and 0.7). The results above indicate that price dynamics and seasonality vary in different parts of the price distribution.

Next, we re-estimate the QAR model in Table 2 for all quantiles, thus providing a basis for evaluating the conditional time-varying distribution functions of soybean futures prices. Figure 1 provides examples of the estimated cumulative distribution functions (CDF) for selected months in three selected years: 1990, 2000, and 2010. We observe obvious changes in the shapes of distributions over time. The changes include many aspects, including the evolving mean, the spread, the asymmetry, and the pattern on tails. As discussed in Section 2, we formally test assumptions embedded in the Black-Scholes, with a focus on three key aspects of the distributions of underlying futures prices: (i) lognormality, (ii) unit root, and (iii) drift.

6.1 Hypothesis testing on lognormality

The Black and Scholes formula relies on the assumption of the lognormality in futures price distribution in deriving the close-from solution for option pricing. The relaxation of this assumption has been intensively studied in various literature (see footnote 2). Our approach offers a new and perhaps more general way to relax the lognormality assumption in a semi-parametric context. Our quantile-based estimates have the advantage of NOT imposing any restrictions on the shape of the distributions of underlying futures prices. Thus, we are able to conduct explicit hypothesis testing on lognormality and to investigate the nature of departures from lognormality, i.e., whether it comes from asymmetry (skewness) and/or fat tail (excess kurtosis).

Table 3 reports normality tests of futures price distributions estimated from our QAR model both for the full sample period and subsamples in four decades. In the full-sample results reported in the second row, the Shapiro tests show that a large proportion (34.7%) of the null hypothesis of lognormal distribution functions are rejected at five percent level. Further analysis shows the departure from lognormality comes both from symmetry and excess kurtosis, as the skewness of -0.098 and the excess kurtosis of 1.153 are found to be statistically different from zero at one percent level. These findings are generally supported by the subsample analysis using data grouped by four decades. Consistently, we still observe that 26.5-40.0% of null hypothesis of lognormal distributions are rejected by Shapiro normality tests. Results of the first three decades further show that "fat tails", instead of asymmetry, are the main driver generating non-normality. Using the estimated price distribution on January 31, 2018, as an example, Figure 1 illustrates a clear discrepancy between theoretical quantiles of normal distribution and sample quantiles from the empirical distribution obtained by our model. The

Q-Q plot shows the obvious departure from normal distribution, especially in lower and upper tails. Next, we will further quantify the pricing performance using both distributions.

6.2 Hypothesis testing on unit root

The second distributional assumption we test is whether future price follows unit root process. Formula (7) shows the coefficient $\beta_1(q)$ corresponds to the dominant root in our QAR (3) model in the q-th quantile. Figure 3 reports the dominant roots across quantiles. It shows that the dominant roots are close to one around the median, which is consistent with previous findings (Wang and Tomek, 2007). In contrast, the dominant roots are less than one in the lower tail, indicating a slow adjustment of speed in that particular neighborhood. More interestingly, the roots are found to be greater than one, which implies instability corresponding to price bubbles (Etienne et al., 2014; Li et al., 2017).

Table 4 reports formal hypothesis testing on the magnitude of the dominant root in selected quantiles (q=0.1, 0.3, 0.5, 0.7, and 0.9). As the unit root test does not have a standard asymptotic distribution, our unit root test relies on bootstrapping (Efron and Tibshirani, 1993). While the test results indicate a failure to reject the "unit root" hypothesis around the median, Table 4 provides strong statistical evidence of departure from a "unit root" in the tails of the price distribution: the dominant roots are statistically smaller than one in the lower tail, but greater than one in the upper tail. This is one of our key findings: using the quantile approach, we find that unit root is not a global property but a local one, with evidence of local stability in the lower tail and evidence of local instability in the upper tail. This result supplements previous research on the stationarity conditions of commodity prices (Wang and Tomek, 2007; Chen et al., 2014). Below we will evaluate and compare the pricing performance with/without the unit root assumption.

6.3 Hypothesis testing on drift

We also test the existence of drifts in futures price distributions. While the Black-

Scholes model allows for a drift in the mean price (Black and Scholes, 1973; Merton, 1990), we examine possible departures from Black-Scholes assumptions by studying the possible presence of quantile-specific drifts. As discussed in section 2, quantile-specific drifts can occur in the presence of transaction cost when the marginal cost C'_{kt} in (5) varies with market conditions (under liquidity constraints). This scenario is applicable in our QAR model when the intercept is non-zero for specific quantiles.

Table 5 reports the hypothesis testing results on drifts in five selected quantiles. As we incorporate monthly dummies in model specification, the drifts vary across months. We estimate and report month-specific drifts in Table 5. The results show no statistical evidence of drift around the median; and similar results apply in the upper tail of the distribution. In contrast, Table 5 shows that the drift terms are significantly different from zero in the lower tail of the price distribution for all months. This is another important finding in our study: the presence of price drift but only in the lower tail of the distribution. As discussed in section 2, we interpret this finding as indirect evidence that transaction costs play a role and affect price dynamics when market experience major negative shocks.

We have performed multi-dimensional robustness checks in our analysis. This includes adding structural breaks and/or excluding seasonality in our QAR model. We found that our main qualitative results remain unchanged. In Appendix B, we considered structural breaks that may play a role in price dynamics. Following previous studies, we incorporated time trends representing the rise of Brazil's factor in soybean market and the role of biofuel production. As reported in Table B3-B5 and Figure B1-B3, our qualitative findings were found to be robust.

7. Evaluating option pricing performance

This section proceeds with the investigation of empirical pricing performance using alternative models, with a focus on evaluating the effects of relaxing three key assumptions made in the Black-Scholes model: (i) lognormality, (ii) unit root and (iii) drift. For this purpose, we consider eight model specifications: QAR model without any restriction (QAR-general), QAR with unit root (QAR-UR), QAR with no drift (QAR-ND), QAR with unit root and no drift (QAR-URND) and Lognormality (LN-general), Lognormality with unit root (LN-UR), Lognormality with no drift (LN-ND), Lognormality with unit root and no drift (LN-URND).

The evaluation was done as follows. For each model, we modified the estimation and conducted a forward simulation of the estimated price distribution. We then used this estimated distribution and equation (1) to obtain predicted option prices. The performance evaluation was then done for each model comparing the predicted and actual option prices for specific contracts.

Specifically, we evaluated model performance based on November 2019 soybean options contracts with 16 most-traded strike prices ranging from 800 to 1100 cents per bushel. Figure 4 reports the predicted option prices using the eight different models. As expected, the QAR-general model performs well in terms of generating predicted prices close to the actual prices, although most models appear to provide satisfactory predictions. Interestingly, Figure 4 shows some heterogeneities of prediction power across strike prices (as further discussed below).

Table 6 reports comparisons of prediction performance evaluated using a Mean Square Error (MSE) criterion. As shown in the last row, the QAR-general model gives the smallest MSE of 23.720 among all eight models considered. The original Black-Scholes model — LN-URND — has a larger MSE of 38.953. By comparing QAR-type models that impose unit root and drift assumptions one at a time, we find that the "no drift" assumption generates the highest prediction error reaching 70.055, followed by URND model at 65.318 and UR model at 35.564. In other words, from the perspective of MSE, the assumption of "no drift" creates the worst predictive performance, followed by the assumption of "unit root" and then by the normality assumption. Note that we also used Mean Absolute Percentage Error (MAPE) as a relative measure of performance for each model in predicting option prices: the main findings were largely consistent with those obtained from using MSE. [10]

To summarize, our evaluation of the predictive power of option price from alternative models gives five important results: 1) the Black-Scholes model (LN-URND) has the worst predictive performance; 2) our QAR-general model has a better predictive performance; 3) departures from log-normality is the least offensive assumption in the Black-Scholes model; 4) departures from a unit-root in the tails of the price distributions contribute to reducing the predictive power of Black-Scholes option pricing; and 5) the largest contributor to poor prediction in the Black-Scholes model is the presence of drifts in the lower tail of the price distribution.

Note that similar rankings are observed from the LN-type model. The LN-general presents the smallest MSE, and the LN-ND predicts the worst among four LN-type models. More valuable comparisons of the pricing performance can be found across contract features. We further calculate the MSE for contracts with different moneyness categorizing them into six groups (<0.9, 0.9-0.95, 0.95-1.00, 1.00-1.05, 1.05-1.10, >1.10). We find that the rankings discussed above remain the same in almost all cases. The only exceptions are the out-of-money contracts (S/K <0.9), where LN-URND and LN-general models give the worst predictive power and QAR-general ranks the second best, possibly reflecting the relatively low trading volumes of those contracts.

The numbers in parenthesis in Table 6 report the relative ratios of MSE values between QAR-general and alternative models. Having those numbers greater than one for almost all cases supports the good predictive power of QAR-general model. Models imposing no-drift restrictions give the worst performance: on average, the ratios are 2.953 and 3.343 for QAR-ND and LN-ND models, respectively. We also find that the predictive performance varies across moneyness categories: MSE tends to decrease with moneyness. This result may be linked to the common findings of "smile" in option pricing literature (Hagan, 2002).

We conducted a series of robustness checks of our evaluation results. First, using Akaike Information Criterion (AIC), we explored alternative model specifications (including structural changes related to biofuel impacts or market shocks) and different lag structure. As reported in Table B6 and Figure B4 in Appendix B, our results on predictive performance remained robust under alternative specifications. Second, we changed the options contracts considered from November 2019 to alternative ones including July 2021 (as shown in Appendix C), October 2019, December 2019, November 2017, and November 2018, respectively. The main results remain unchanged in those alternative settings. These robustness checks support the validity and credibility of our findings.

8. Economic implications

The previous two sections have presented evidence against key assumptions made about the distribution and dynamics of futures price in the Black-Scholes model. This section provides a discussion and economic interpretation of these results.

Based on our QAR approach, we investigated the quantile-specific dynamics of futures prices, with evidence that dynamics differ in the upper tail versus the lower tail of the price distribution. The finding of instability in the upper tail of price distribution indicates the occurrence of black swan events that drive price explosive, as documented in previous literature (e.g., 2008 global food crisis, see Gutierrez, 2013; Etienne et al., 2016). This price instability can be interpreted as corresponding to "irrational exuberance" that contributes to price bubbles (Shiller, 1999, 2015; Akerlof and Shiller, 2010). We see this result as a reflection of heterogeneous information among traders, which can stimulate herd behavior possibly leading to dynamic price instability (Barberis et al., 2018; Hommes et al., 2021). In contrast with previous studies that focused on mean effects, our analysis stresses differing dynamics across quantiles, with instability being likely to occur only in the upper tail of the price distribution.

Second, we uncover evidence of stability (dominant root being less than one) and existence of drifts in the lower tail of the price distribution. Both can be seen as evidence against perfect arbitrage conditions. The former means a decrease in dynamic adjustments to an adverse shock while the latter means location-specific drifts. Note that such results cannot be obtained by conventional “mean-variance” framework, highlighting the usefulness of our QAR approach in capturing important information on the distribution and dynamics of prices. As discussed in section 2, such patterns must reflect the behavior of market participants. In this context, we interpret our findings as indirect evidence that traders tend to make slow adjustments to new information under adverse market shocks.

In the context of commodity trading, this result might point to the role of margin calls which occur when a margin account runs low on funds. Margin requirements, in particular, limit the notional amount of capital that can be invested in the strategies and may force investors to close down positions and realize losses. Although margin calls may be effective in reducing counterparty default risk, they are restrictive and can affect the market position of liquidity-constrained investors (Margrabe, 1978; Santa-Clara and Saretto, 2009; Feng et al., 2014). Existing studies have documented that liquidity issues can affect financial markets (e.g., Christoffersen et al., 2021) and that margin calls can cause asset prices to drop in times of market recessions (Rytchkov, 2013; Kahraman and Tookes, 2020). Our finding of slow price adjustments under adverse market conditions provides indirect evidence supporting such arguments.

By conducting formal statistical testing on some key assumptions imposed in the Black-Scholes model, we evaluated the validity of these assumptions and their impacts on option pricing. Previous studies have questioned the role of log-normality restriction. Our analysis also provides evidence that imposing lognormality helps explain why the Black-Scholes formula fails to provide an accurate representation of option prices. Perhaps more importantly, we find that ignoring quantile-specific drifts and imposing unit root generate a larger bias in option pricing. We show that a failure to capture local instability in the upper tail negatively affects price predictions more than relaxing lognormality. Our empirical analysis also indicates that the most serious problem for the Black-Scholes formula arises in the representation of price dynamics in the lower tail. As discussed above, local instability in the upper tail may reflect herd behavior (as traders respond to favorable shocks); and slow adjustments in the lower tail may reflect traders facing liquidity constraints. More generally, these results can be interpreted as indirect evidence that transaction costs play a role in the functioning of financial markets. Of significant interest is that such effects are found to be especially important under market shocks located in the tails of the price distribution.

9. Conclusion

This study presents a conceptual and empirical analysis of option pricing with a focus on the validity of key assumptions embedded in the Black-Scholes model. Going beyond questioning the lognormality assumption, we investigate the role played by two assumptions made about the nature of price dynamics: quantile-specific departures from a unit root process, and the role of quantile-specific drift. We relied on a Quantile Autoregression (QAR) model to obtain refined futures price distributions under flexible dynamics. In turn, the QAR estimates can be used to compare the performance of option pricing comparing our model and the Black-Scholes model. Going beyond the Black-Scholes model, also allows us to identify factors affecting option pricing.

Applied to the US soybean market over the period of 1980-2019, our QAR model generates several new and interesting results regarding the shape and dynamics of the futures price distribution. First, in a way consistent with previous literature, we found evidence against the lognormal distribution assumption in the Black-Scholes model, reflecting the existence of excess kurtosis in the price distribution. Second, we investigated the nature of price dynamics. We found that, while a "unit root" holds locally around the median, it does not hold in the tails of the price distribution. We uncovered evidence of local instability in the upper tail as well as of stability and drifts in the lower tail of the price distributions. We interpret these results as indirect evidence that traders' behavior and traders' varying response to shocks (with different response to positive shocks versus negative shocks) play an important role in market dynamics. Note that our results are consistent with the presence of herd behavior (affecting price dynamics in the upper tail) and liquidity constraints (contributing to slow price adjustments in response to unanticipated adverse shocks).

Using historical options data, our study investigated how relaxing distributional assumptions improves the predictive power of option pricing models. Using an MSE prediction criterion, we examine their empirical pricing performance and find the QAR model without unit root and no drift assumptions presents the best pricing performance with the smallest prediction errors. The evaluation further reveals the rankings of alternative models: we show that models assuming no drift generate the largest pricing error, followed by unit root and lognormality. While we find that the presence of "fat tails" in the price distribution compromises the validity of the Black-Scholes model (as documented in previous research), our analysis shows that a failure to capture local instability in the upper tail is more serious. Furthermore, our empirical analysis indicates that the most serious problem for the Black-Scoles formula arises in the representation of price dynamics in the lower tail. These results suggest that traders' behavior and traders' varying response to shocks play an important role in the functioning of markets. In turn, such behavior affects option pricing in a way that can help explain why the Black-Scholes formula fails to provide an accurate representation of option prices. More generally, we interpret our findings as reflecting the role of transaction costs and their effects on market participants, especially under market shocks located in the tails of the price distribution. To the extent that improving market pricing efficiency and avoiding irrational exuberance are desirable, our findings stress the value of evaluating the dynamics of price distributions with a focus on the response to rare risky events.

This study can be further extended in several ways. First, while our methodology holds under general conditions, our empirical results are specific to a given market and period. There is a need to apply our analysis to different markets and periods. Second, it would be good to explore further the role of nonlinear dynamics, risk aversion and heterogeneous information in the functioning of derivative markets. Finally, the role of transaction costs and their effects on the dynamics of price distributions need further investigations. This includes additional studies on why traders respond differently to rare shocks located in the tails of the price distribution. These seem to be good topics for future research.

References

Andersen, T.G., N. Fusari and V. Todorov. (2015). The Risk Premia Embedded in Index Options" *Journal of Financial Economics* 117: 558-584.

Akerlof, G.A., and Shiller, R.J. (2010). *Animal spirits: How human psychology drives the economy, and why it matters for global capitalism*. Princeton University Press.

Babaoglu, K., Christoffersen, P., Heston, S.L., and Jacobs, K. (2018). Option valuation with volatility components, fat tails, and nonmonotonic pricing kernels. *The Review of Asset Pricing Studies*, 8(2): 183–231.

Bakshi, G., Cao, C., and Chen, Z. (1997). Empirical performance of alternative option pricing models. *The Journal of Finance*, 52(5): 2003-2049.

Barberis, N., Greenwood, R., and Shleifer, A. (2018). Extrapolation and bubbles. *Journal of Financial Economics*, 129: 203-227.

Barberis, N., Jin, L.J., and Wang, B. (2021). Prospect theory and stock market anomalies. *Journal of Finance*, 76(5): 2639-2687.

Barone-Adesi, G., Engle, R.F., and Mancini, L. (2008). A GARCH option pricing model with filtered historical simulation. *Review of Financial Studies*, 21(3): 1223-1258.

Barunik, J., & Cech, F. (2021). Measurement of common risks in tails: a panel quantile regression model for financial returns. *Journal of Financial Markets*, 52, 100562.

Bates, D.S. (2003). Empirical option pricing: A retrospection. *Journal of Econometrics*, 116: 387–404.

Bates, D.S. (2022). Empirical option pricing models. *Annual Review of Financial Economics*, 14(1): 1–21.

Battalio, R., Figlewski, S., and Neal, R. (2017). Exercise boundary violations in American-style options: The rule, not the exception. Kelley School of Business Research Paper NO. 15-10.

Bikhchandani, S., and Sharma, S. (2001). Herd behavior in financial markets. Staff Papers Vol. 47, No. 3, *International Monetary Fund*, 279-310.

Black, F., and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81: 637-654.

Brunnermeier, M.K., and Pedersen, L.H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22: 2201-2238.

Buraschi, A., Jiltsov, A., Bakshi, G., Basak, S., Gomes, F. Gromb, D., Rosenblatt, R., and Uppal, R. (2006).

Model uncertainty and option markets with heterogeneous beliefs. *The Journal of Finance*, 61: 2841–2897.

Carr, P., and Wu, L. (2017). Leverage effect, volatility feedback, and self-exciting market disruptions: disentangling the multi-dimensional variations in S&P 500 index options. *Journal of Financial and Quantitative Analysis*, 52(5): 2119–2156.

Chowdhry, B. and Nanda, V. (1998). Leverage and market stability: The role of margin rules and price limits. *The Journal of Business*, 71(2): 179-210.

Chen, S.L., Jackson, J.D., Kim, H., and Resiandini, P. (2014). What drives commodity prices?. *American Journal of Agricultural Economics*, *96*(5): 1455-1468.

Christoffersen, P., Heston, S., and Jacobs, K. (2006). Option valuation with conditional skewness. *Journal of Econometrics*, *131*(1-2): 253-284.

Christoffersen, P., Jacobs, K., and Ornthanalai, C. (2012). Dynamic jump intensities and risk premiums: Evidence from S&P500 returns and options. *Journal of Financial Economics*, 106: 447–472.

Christoffersen, P., Feunou, B., Jeon, Y., and Ornthanalai, C. (2021). Time-varying crash risk embedded in index options: The role of stock market liquidity. *Review of Finance*. 25: 1261-1298.

Dionne, G. (2013). Risk management: History, definition, and critique. *Risk Management and Insurance Review,* 16(2): 147-166.

Doan, P., Lin, C. T., and Zurbruegg, R. (2010). Pricing assets with higher moments: Evidence from the Australian and US stock markets. *Journal of International Financial Markets, Institutions and Money*, *20*(1): 51-67.

Duan, J. (1995). The GARCH option pricing model. *Mathematical finance*, 5: 13–32.

Enders, W. (2010). *Applied Econometric Time-Series*. 3rd edition. Wiley, New York.

Efron, B., and R.J. Tibshirani. (1993). *An Introduction to the Bootstrap*. Chapman and Hall.

Etienne, X.L., Irwin, S.H., and Garcia, P. (2014). Bubbles in food commodity markets: Four decades of evidence. *Journal of International Money and Finance*, 42: 129-155.

Fang, H., and Lai, T.Y. (1997). Co-kurtosis and capital asset pricing. *Financial Review*, *32*(2): 293-307.

Fama, E. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*. 25(2): 383–417.

Fama, E. (2013). Two pillars of asset pricing. *Prize Lecture for the Nobel Foundation*. Nobel Prize

Committee.

Fleming, J., Kirby, C., and Ostdiek, B. (2006). Stochastic volatility, trading volume, and the daily flow of information. *Journal of Business*, 79: 1551–1590.

Fortune, P. (1996). Anomalies in option pricing: The Black-Scholes model revisited. *New England Economic Review*, March/April, 17-40.

Frank, S.D., Irwin, S.H., Pfeiffer, G.H. and Curtis, C.E. (1989). Further evidence on soybean marketing strategies: The role of options. *Applied Economic Perspectives and Policy*, 11: 213-219.

Gutierrez, L. (2013). Speculative bubbles in agricultural commodity markets. *European Review of Agricultural Economics*, *40*(2), 217-238.

Gruber, P.H., Tebaldi, C., and Trojani, F. (2021). The price of the smile and variance risk premia. *Management Science*, 67: 4056-4074.

Hagan, P.S., Kumar, D., Lesniewski, A.S., and Woodward, D.E. (2002). Managing smile risk. *The Best of Wilmott*, 1, 249-296.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton.

Hao, J., and Zhang, J.E. (2013). GARCH option pricing models, the CBOE VIX, and variance risk premium. *Journal of Financial Econometrics*, 11: 556–580.

Hasbrouck, J. (1993). Assessing the quality of a security market: A new approach to transaction-cost measurement. *The Review of Financial Studies*, *6*(1), 191-212.

He, X., and Serra, T. (2022). Are price limits cooling off agricultural futures markets? *American Journal of Agricultural Economics*, 104(5), 1724-1746.

Heston, S.L. (1993). A Closed-From Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. Review of Financial Studies. 6(2): 327-343.

Heston, S.L., and Nandi, S. (2000). A closed-form GARCH option valuation model. *The Review of Financial Studies*, *13*(5), 585-625.

Hommes, C., Kopányi-Peuker, A., and Sonnemans, J. (2021). Bubbles, crashes and information contagion in large-group asset market experiments. *Experimental Economics, 24*, 414-433.

Hua, J., and Manzan, S. (2013). Forecasting the return distribution using high-frequency volatility measures. *Journal of Banking & Finance*, 37(11): 4381-4403.

Hull, J. and White, A. (1987). The pricing of options on assets with stochastic volatilities. The *Journal of*

Finance, 42(2): 281-300.

Hull, J.C. (2022). *Option, Futures and Other Derivatives,* 11-th Edition, Pearson/Prentice Hall.

Irwin, S.H., Pelly, R.A., and Zulauf, C.R. (1989). An investigation of pricing models for live cattle and feeder cattle options. Staff Paper, Department of Agricultural Economics, Virginia Polytechnic Institute and State University.

Isengildina-Massa, O., Irwin, S. H., & Good, D. L. (2010). Quantile regression estimates of confidence intervals for WASDE price forecasts. *Journal of Agricultural and Resource Economics*, 545-567.

Jensen, M., and Pedersen, L. (2016). Early option exercise: Never say never. Journal of Financial Economics, 121: 278-299.

Jondeau, E., and Rockinger, M. (2003). Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements. *Journal of Economic dynamics and Control*, *27*(10), 1699-1737.

Kahraman, B., & Tookes, H. (2020). Margin trading and comovement during crises. *Review of Finance*, 24(4): 813-846.

Karali, B., and Power, G.J. (2013). Short-and long-run determinants of commodity price volatility. *American Journal of Agricultural Economics*, 95(3), 724-738.

Kim, I.N.J., and Kim, S. (2005). Is it important to consider the jump component for pricing and hedging short-term options?. *Journal of Futures Markets*, 25: 989–1009.

Koenker R. (2005). *Quantile Regression*. Cambridge University Press, Cambridge.

Koenker, R., and Xiao, Z. (2004). Unit root quantile autoregression inference. *Journal of the American Statistical Association*, 99, 775-787.

Koenker, R., and Xiao, Z. (2006). Quantile Autoregression. *Journal of the American Statistical Association*, 101, 980-990.

Li, J., Chavas, J.P., Etienne, X.L., and Li, C. (2017). Commodity price bubbles and macroeconomics: evidence from the Chinese agricultural markets. *Agricultural economics*, 48(6), 755-768.

Li, J., & Chavas, J. P. (2023). A dynamic analysis of the distribution of commodity futures and spot prices. *American Journal of Agricultural Economics*, 105(1): 122-143.

MacBeth, J., and Merville, L. (1979). An empirical examination of the Black-Scholes call option pricing model. *The Journal of Finance*, 34(5): 1173-1186.

Maheu, J.M., and McCurdy, T.H. (2004). News arrival, jump dynamics and volatility components for individual stock returns. *Journal of Finance*, 59(2): 755–793.

McKenzie, A.M., Thomsen, M.R., and Adjemian, M.K. (2022). Characterizing implied volatility functions from agricultural options markets. *American Journal of Agricultural Economics*, 1–20.

Mendoza, E.G., and Smith, K.A. (2002). Margin Calls, Trading Costs, and Asset Prices in Emerging Markets. Working Paper 9286, NBER, Cambridge. MA.

Merton, R.C. (1973). The Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science*, 4: 141-183.

Merton, R.C. (1990). *Continuous-Time Finance*. Blackwell, Cambridge, MA.

Ornthanalai, C. (2014). Lévy jump risk: Evidence from options and returns. *Journal of Financial Economics*, 112: 69–90.

Rytchkov, O. (2014). Asset pricing with dynamic margin constraints. *The Journal of Finance*, *69*(1), 405-452.

Schwert, G.W. (2003). *Anomalies and market efficiency*. Handbook of the Economics of Finance.

Shiller, R. (1999). Human behavior and the efficiency of the financial system. in J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics,* Vol 1, 1305-1340.

Shiller, R. (2015). *Irrational Exuberance*. Princeton University Press.

Sørensen, C. (2002). Modeling seasonality in agricultural commodity futures. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *22*(5), 393-426.

Stambaugh, R. F., J. Yu, and Y. Yuan (2015). Arbitrage asymmetry and the idiosyncratic volatility puzzle. *Journal of Finance*, 70(5): 1903-1948.

Urcola, H.A., and Irwin, S.H. (2011). Are agricultural options overpriced? *Journal of Agricultural and Resource Economics*, 63-77.

Wang, D. and Tomek, W. G. (2007). Commodity prices and unit root tests. *American Journal of Agricultural Economics*, 89 (4): 873–889.

Whaley, R. E. (1986). Valuation of American futures options: Theory and empirical tests. *Journal of Finance*, 41(1):127-150.

Zikes, F., and Baruník, J. (2015). Semi-parametric conditional quantile models for financial returns and realized volatility. *Journal of Financial Econometrics*, 14(1): 185-226.

Table 1. Summary statistics

Panel A: Futures price				
Variables	Mean	St. Dev	Max	Min
pf	711.392	273.642	1753.000	413.500

Panel B: Option price				
Moneyness	Weeks-to-expiration			# of observations
S/K	1-25 weeks	26-50 weeks	>50 weeks	Subtotal
	153.239	147.844	111.211	
< 0.90	(30.303)	(11.638)	(13.006)	
	{203}	{28}	{19}	{250}
	101.022	115.014	74.681	
0.90-0.95	(19.594)	(13.754)	(16.369)	
	{124}	{52}	{49}	{225}
	79.032	82.230	44.033	
0.95-1.00	(16.685)	(12.734)	(17.956)	
	{126}	{61}	{53}	{240}
	58.050	55.069	23.898	
1.00-1.05	(13.110)	(9.868)	(16.301)	
	{137}	{54}	{54}	{245}
	47.155	37.236	13.007	
1.05-1.10	(8.560)	(8.773)	(11.372)	
	{111}	{54}	{56}	{221}
	29.281	18.140	5.462	
> 1.10	(9.057)	(8.355)	(5.874)	
	{195}	{151}	{169}	{515}

Note: The futures price data in panel A involve weekly observations over the period January 7, 1980 – May 1, 2019, while the Nov2019 soybean option contract data in panel B cover the period October 18, 2017 – October 23, 2019. The unit of futures and options prices are in USD per contract. Standard deviations are presented in parentheses; the number of observations in each category are presented in braces.

Table 2. QAR and AR estimates of futures price pf

Variables	AR	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
pf_1	0.995*** (0.002)	0.978*** (0.005)	0.991*** (0.003)	1.000*** (0.002)	1.004*** (0.003)	1.011*** (0.004)
m_1	0.000 (0.004)	-0.008 (0.006)	-0.003 (0.004)	-0.001 (0.002)	0.002 (0.003)	0.000 (0.006)
m_2	0.002 (0.004)	0.001 (0.006)	0.001 (0.003)	0.002 (0.003)	0.002 (0.003)	-0.001 (0.005)
m_3	0.004 (0.004)	0.007 (0.005)	0.002 (0.003)	0.001 (0.003)	0.005*** (0.003)	0.004 (0.005)
m_4	0.004 (0.004)	0.006 (0.006)	0.002 (0.003)	0.001 (0.003)	0.005 (0.003)	0.002 (0.005)
m_5	0.001 (0.004)	-0.001 (0.006)	-0.003 (0.003)	-0.003 (0.003)	0.003 (0.003)	0.009 (0.006)
m_6	-0.007*** (0.004)	-0.019*** (0.007)	-0.010*** (0.004)	-0.006*** (0.003)	0.003 (0.003)	0.010*** (0.006)
m_7	-0.005 (0.004)	-0.032*** (0.008)	-0.014*** (0.005)	-0.006 (0.005)	0.004 (0.004)	0.027*** (0.011)
m_8	0.000 (0.004)	-0.014*** (0.006)	-0.006 (0.004)	-0.005 (0.004)	0.002 (0.004)	0.016*** (0.007)
m_9	-0.006*** (0.004)	-0.013*** (0.007)	-0.011*** (0.004)	-0.007*** (0.003)	0.000 (0.003)	0.005 (0.005)
m_{10}	0.003 (0.004)	-0.003 (0.006)	-0.001 (0.003)	-0.001 (0.003)	0.006*** (0.003)	0.016*** (0.008)
m_{11}	0.003 (0.004)	-0.002 (0.005)	0.002 (0.004)	0.001 (0.002)	0.003 (0.003)	0.006 (0.005)
$intercept$	0.033*** (0.015)	0.113*** (0.031)	0.047*** (0.020)	-0.001 (0.013)	-0.015 (0.017)	-0.040 (0.024)
Adjusted R-square	0.990	0.868	0.895	0.912	0.917	0.913

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. For the "goodness of fit", we report the adjusted $R2$ for OLS estimates and the Pseudo-$R2$ proposed by Koenker and Machado (1999) for quantile estimates. The sampling period is 7 January 1980 to 01 May 2019.

Table 3. Normality test of futures prices pf under alternative scenarios

Sample	Years	Skewness	Excess Kurtosis	Shapiro test: percentage of non-normal distributions	# of observations
Full sample	1980-2019	-0.098*** (0.010)	1.153*** (0.021)	34.678	2033
Subsamples	1980-1990	-0.015 (0.018)	1.161*** (0.043)	35.893	521
	1990-2000	0.020 (0.018)	1.168*** (0.044)	35.824	522
	2000-2010	-0.022 (0.021)	1.202*** (0.044)	39.961	508
	2010-2019	-0.395*** (0.015)	1.077*** (0.039)	26.501	482

Note: Column 1 and 2 show the skewness test with null hypothesis of $SK = 0$ and excess kurtosis test with null hypothesis of $K = 0$. Statistical significance is represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. Standard deviations are presented in parentheses. The proportions of non-normal distributions (column 3) of futures price pf are obtained by Shapiro test, using 5% critical value. The sample period is from January 7, 1980 to May 1, 2019.

Table 4. Modulus of the dominant roots

Dominant roots	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
β_1	0.979*** (0.005)	0.991*** (0.003)	1.000 (0.002)	1.004 (0.003)	1.011*** (0.004)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping. And using bootstrapping, statistical significance of the test for a unit root ($H_0: \beta_1 = 1$) is represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table 5. Hypothesis tests on drifts across months for selected quantiles

Month	Drift	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
January	df_1	0.105***	0.044***	-0.002	-0.013	-0.040
		(0.034)	(0.022)	(0.014)	(0.019)	(0.025)
February	df_2	0.114***	0.048***	0.002	-0.013	-0.041***
		(0.037)	(0.019)	(0.014)	(0.017)	(0.024)
March	df_3	0.119***	0.050***	0.000	-0.011	-0.036
		(0.033)	(0.020)	(0.014)	(0.016)	(0.024)
April	df_4	0.119***	0.050***	0.000	-0.010	-0.038
		(0.034)	(0.020)	(0.013)	(0.018)	(0.025)
May	df_5	0.112***	0.044***	-0.004	-0.013	-0.031
		(0.033)	(0.020)	(0.014)	(0.019)	(0.025)
June	df_6	0.094***	0.037***	-0.007	-0.012	-0.030
		(0.036)	(0.020)	(0.015)	(0.017)	(0.024)
July	df_7	0.081***	0.033	-0.007	-0.012	-0.014
		(0.036)	(0.021)	(0.016)	(0.017)	(0.027)
August	df_8	0.099***	0.041***	-0.006	-0.013	-0.024
		(0.034)	(0.020)	(0.014)	(0.016)	(0.028)
September	df_9	0.100***	0.037***	-0.007	-0.015	-0.035
		(0.037)	(0.019)	(0.015)	(0.017)	(0.026)
October	df_{10}	0.110***	0.046***	-0.002	-0.009	-0.024
		(0.033)	(0.019)	(0.015)	(0.015)	(0.023)
November	df_{11}	0.111***	0.049***	0.000	-0.012	-0.035
		(0.031)	(0.019)	(0.016)	(0.018)	(0.026)
December	df_{12}	0.113***	0.047***	-0.001	-0.015	-0.040
		(0.030)	(0.019)	(0.014)	(0.017)	(0.025)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table 6. Option pricing performance analysis with MSE criteria across contracts

Moneyness	QAR				Under lognormality			
	[1] QAR-general	[2] QAR-UR (unit root)	[3] QAR-ND (no drift)	[4] QAR-URND	[5] LN-general	[6] LN-UR (unit root)	[7] LN-ND (no drift)	[8] LN-URND
< 0.90	64.132	167.945	116.876	108.384	92.253	90.214	110.781	33.968
		(2.619)	(1.822)	(1.690)	(1.438)	(1.407)	(1.727)	(0.530)
0.90-0.95	52.739	86.377	162.179	158.584	50.67	76.976	178.691	66.388
		(1.638)	(3.075)	(3.007)	(0.961)	(1.460)	(3.388)	(1.259)
0.95-1.00	45.548	67.845	168.664	154.75	52.879	74.671	192.703	72.091
		(1.490)	(3.703)	(3.398)	(1.161)	(1.639)	(4.231)	(1.583)
1.00-1.05	38.748	47.383	113.494	97.081	52.265	43.418	149.411	56.966
		(1.223)	(2.929)	(2.505)	(1.349)	(1.121)	(3.856)	(1.470)
1.05-1.10	15.892	16.345	43.477	34.760	24.905	19.184	56.743	33.021
		(1.029)	(2.736)	(2.187)	(1.567)	(1.207)	(3.571)	(2.078)
> 1.100	4.292	4.658	7.834	10.553	4.471	9.825	7.499	18.615
		(1.085)	(1.825)	(2.459)	(1.042)	(2.289)	(1.747)	(4.337)
Avg	23.720	35.564	70.055	65.318	28.528	34.840	81.453	38.953
		(1.499)	(2.953)	(2.754)	(1.203)	(1.469)	(3.434)	(1.642)

Note: The ratios of the MSE of alternative models relative to the QAR model are presented in parentheses. The sampling period is May 8, 2019, to October 2, 2019.

Figure 1. Selected futures price distributions in each quarter of 1990, 2000 and 2010

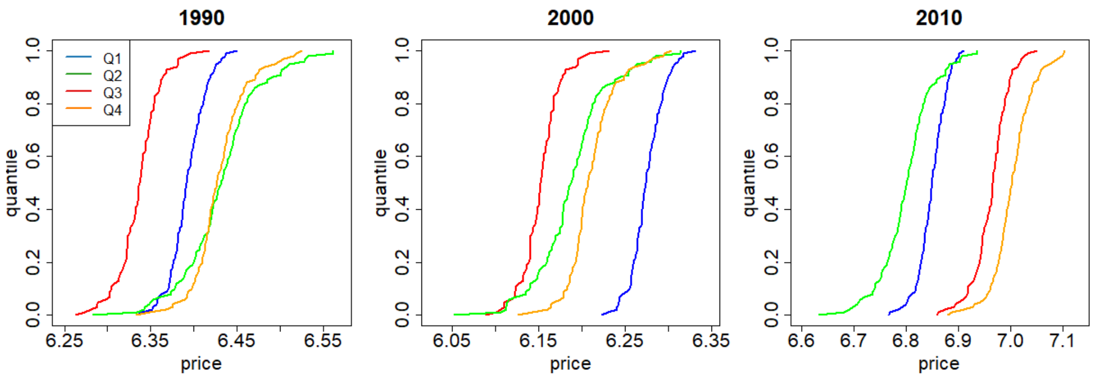


Figure 2. The normality test of futures price with Q-Q plot on a stochastic day, e.g., Jan 31, 2018

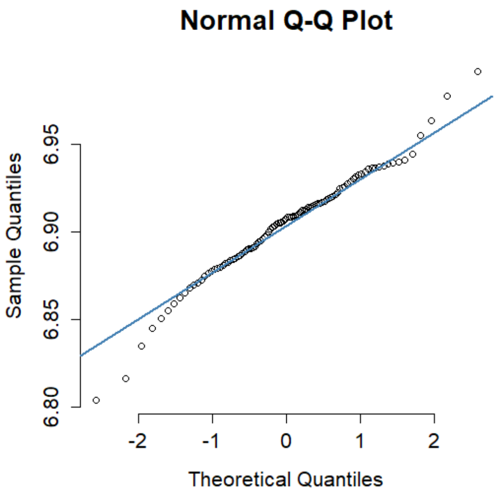


Figure 3. Dominant roots of futures price

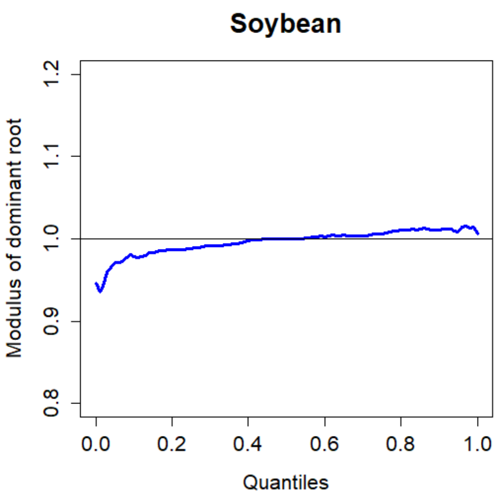
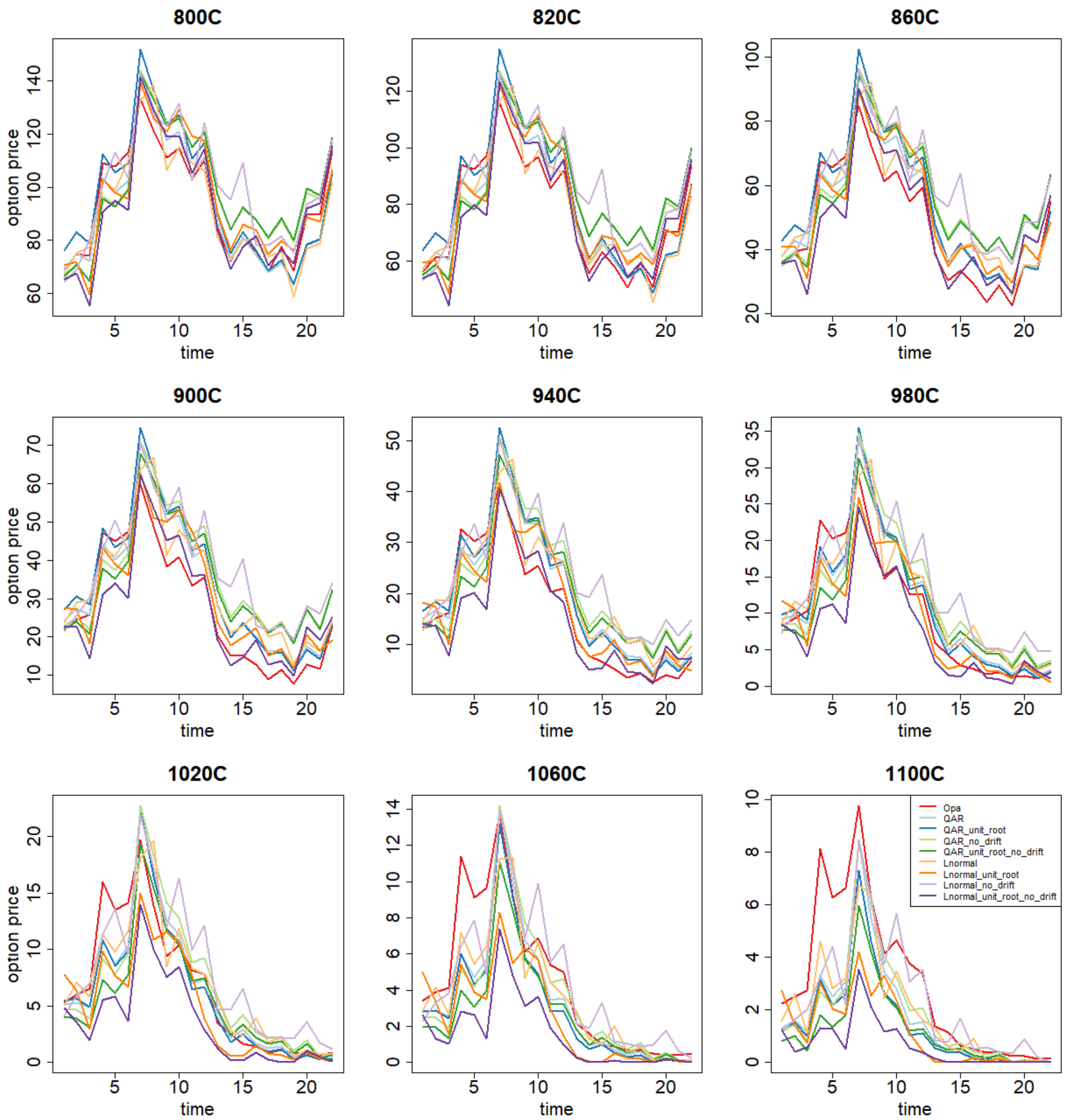


Figure 4. Selected option pricing shapes under alternative scenarios



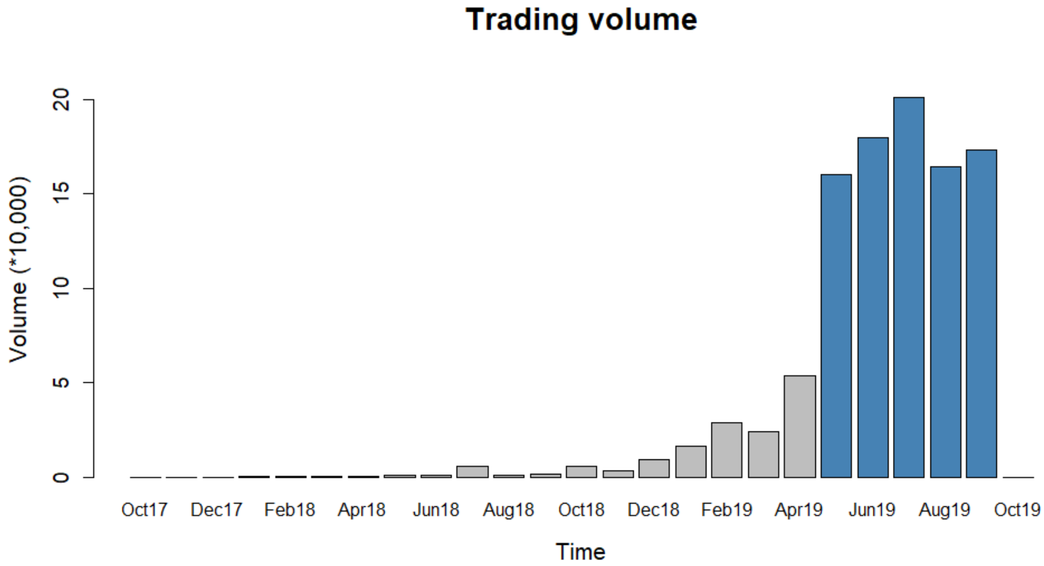
Appendix A:

Table A1. Hypothesis testing results for quantile effects and seasonality

Testing items	Estimated method		F value
Same coefficients across quantiles	QAR	$q = 0.1, 0.3, 0.5, 0.7, 0.9$	4.503***
		$q = 0.1$	5.881***
		$q = 0.3$	2.863***
		$q = 0.5$	/
		$q = 0.7$	2.747***
		$q = 0.9$	6.015***
Seasonality	AR		2.192**
	QAR	$q = 0.1$	4.413***
		$q = 0.3$	3.230***
		$q = 0.5$	1.644*
		$q = 0.7$	0.641
		$q = 0.9$	2.682***

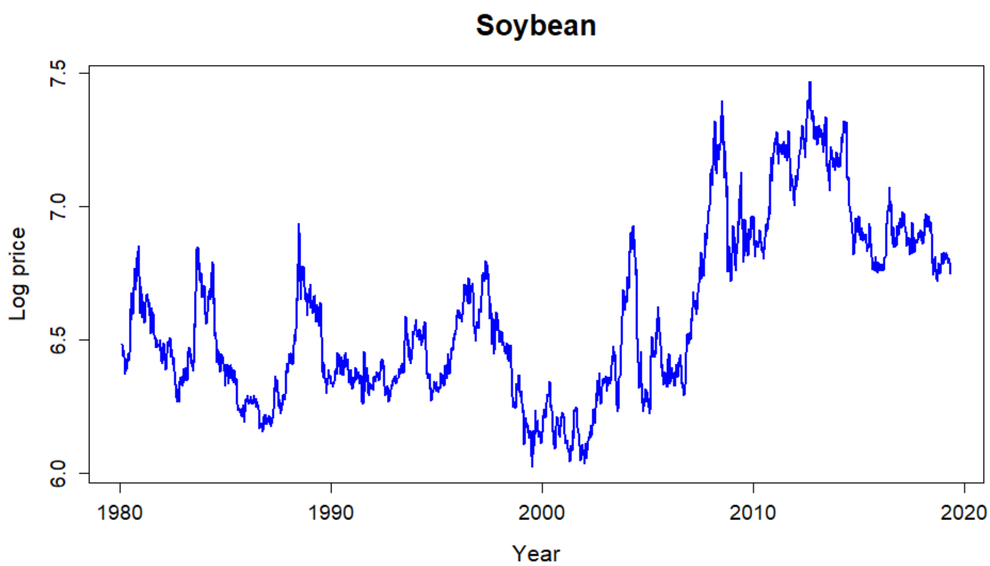
Note: Asterisks indicate the significance level: * at the 10% level, ** at the 5% level, and *** at the 1% level.

Figure A1. Trading volume of soybean Nov '19 option contract



Note: The option pricing models in this paper select a 22-week period from May 8, 2019 to October 2, 2019, which is almost covered in blue bar.

Figure A2. Trajectory of soybean futures prices, 1980-2019



Appendix B: Robustness and specification checks, adding structure breaks

Table B1. Summary statistics

Panel A: Futures price				
Variables	Mean	St. Dev.	Max	Min
pf	711.392	273.642	1753.000	413.500

Panel B: Option price				
Moneyness	Weeks-to-expiration			# of observations
S/K	1-25 weeks	26-50 weeks	>50 weeks	Subtotal
< 0.90	153.239 (30.303) {203}	147.844 (11.638) {28}	111.211 (13.006) {19}	{250}
0.90-0.95	101.022 (19.594) {124}	115.014 (13.754) {52}	74.681 (16.369) {49}	{225}
0.95-1.00	79.032 (16.685) {126}	82.230 (12.734) {61}	44.033 (17.956) {53}	{240}
1.00-1.05	58.050 (13.110) {137}	55.069 (9.868) {54}	23.898 (16.301) {54}	{245}
1.05-1.10	47.155 (8.560) {111}	37.236 (8.773) {54}	13.007 (11.372) {56}	{221}
> 1.10	29.281 (9.057) {195}	18.140 (8.355) {151}	5.462 (5.874) {169}	{515}

Note: The futures price data in panel A involve weekly observations over the period January 7, 1980 – May 1, 2019, while the Nov2019 soybean option contract data in panel B cover the period October 18, 2017 – October 23, 2019. The unit of futures and options prices are in USD per contract. Standard deviations are presented in parentheses; the number of observations in each category are presented in braces.

Table B2. QAR and AR estimates of futures price pf

Variables	AR	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
pf_1	0.990***	0.958***	0.984***	1.000***	1.007***	1.017***
	(0.003)	(0.007)	(0.005)	(0.004)	(0.004)	(0.005)
m_1	0.000	0.001	-0.002	0.000	0.001	0.006
	(0.004)	(0.007)	(0.004)	(0.003)	(0.004)	(0.005)
m_2	0.002	0.004	0.000	0.003	0.002	0.000
	(0.004)	(0.006)	(0.004)	(0.003)	(0.003)	(0.004)
m_3	0.004	0.005	0.002	0.001	0.002	0.006
	(0.004)	(0.006)	(0.003)	(0.003)	(0.003)	(0.005)
m_4	0.004	0.010*	0.003	0.001	0.004	0.003
	(0.004)	(0.006)	(0.004)	(0.003)	(0.003)	(0.004)
m_5	0.001	-0.002	-0.002	-0.004	-0.001	0.014**
	(0.004)	(0.005)	(0.004)	(0.003)	(0.004)	(0.007)
m_6	-0.006*	-0.012	-0.010**	-0.006**	0.000	0.007
	(0.004)	(0.007)	(0.004)	(0.003)	(0.004)	(0.006)
m_7	-0.004	-0.025***	-0.012**	-0.006	0.001	0.025**
	(0.004)	(0.009)	(0.005)	(0.004)	(0.004)	(0.012)
m_8	0.000	-0.011	-0.008*	-0.005	0.002	0.017***
	(0.004)	(0.008)	(0.004)	(0.003)	(0.004)	(0.005)
m_9	-0.006*	-0.006	-0.009**	-0.005	-0.001	0.007
	(0.004)	(0.006)	(0.005)	(0.004)	(0.003)	(0.005)
m_{10}	0.003	0.002	-0.001	-0.002	0.006	0.013**
	(0.004)	(0.005)	(0.003)	(0.004)	(0.004)	(0.006)
m_{11}	0.003	0.001	0.002	0.001	0.002	0.013**
	(0.004)	(0.005)	(0.003)	(0.002)	(0.003)	(0.005)
tt	0.000	0.000	0.000	0.000	0.000	-0.001*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
tt_1	0.001*	-0.001	0.000	0.000	0.002**	0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
tt_2	-0.001	0.004***	0.001	-0.001	-0.003***	-0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$intercept$	0.066***	0.241***	0.092***	0.000	-0.031	-0.079**
	(0.022)	(0.045)	(0.030)	(0.025)	(0.025)	(0.035)
Adjusted R-square	0.990	0.871	0.896	0.912	0.917	0.915

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. For the "goodness of fit", we report the adjusted $R2$ for OLS estimates and the Pseudo-$R2$ proposed by Koenker and Machado (1999) for quantile estimates. The sampling period is 7 January 1980 to 01 May 2019.

Table B3. Normality test of futures prices pf under alternative scenarios

Sample	Years	Skewness	Excess Kurtosis	Shapiro test: percentage of non-normal distributions	# of observations
Full sample	1980-2019	-0.098*** (0.010)	1.153*** (0.021)	34.678	2033
Subsamples	1980-1990	-0.015 (0.018)	1.161*** (0.043)	35.893	521
	1990-2000	0.020 (0.018)	1.168*** (0.044)	35.824	522
	2000-2010	-0.022 (0.021)	1.202*** (0.044)	39.961	508
	2010-2019	-0.395*** (0.015)	1.077*** (0.039)	26.501	482

Note: Column 1 and 2 show the skewness test with null hypothesis of $SK = 0$ and excess kurtosis test with null hypothesis of $K = 0$. Statistical significance is represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. Standard deviations are presented in parentheses. The proportions of non-normal distributions (column 3) of futures price pf are obtained by Shapiro test, using 5% critical value. The sample period is from January 7, 1980 to May 1, 2019.

Table B4. Modulus of the dominant roots

Dominant roots	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
β_1	0.959*** (0.007)	0.984*** (0.005)	1.000 (0.004)	1.007* (0.004)	1.015*** (0.005)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table B5. Hypothesis tests on drifts across months for selected quantiles

Month	Drift	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
January	df_1	0.242***	0.000	0.001	-0.007	0.003
		(0.047)	(0.000)	(0.003)	(0.004)	(0.004)
February	df_2	0.245***	0.000	-0.001	-0.010**	0.002
		(0.046)	(0.000)	(0.003)	(0.005)	(0.003)
March	df_3	0.246***	0.000	-0.004	-0.008**	0.001
		(0.042)	(0.000)	(0.003)	(0.004)	(0.004)
April	df_4	0.251***	0.000	-0.005	-0.008	-0.001
		(0.045)	(0.000)	(0.003)	(0.005)	(0.003)
May	df_5	0.239***	0.000	0.001	-0.002	0.003
		(0.048)	(0.000)	(0.003)	(0.005)	(0.003)
June	df_6	0.229***	0.000	0.009**	0.001	0.002
		(0.050)	(0.000)	(0.004)	(0.005)	(0.004)
July	df_7	0.216***	0.000	0.011**	0.000	0.001
		(0.047)	(0.000)	(0.005)	(0.005)	(0.004)
August	df_8	0.230***	0.000	0.007*	-0.001	0.000
		(0.047)	(0.000)	(0.004)	(0.004)	(0.004)
September	df_9	0.235***	0.000	0.008*	-0.002	0.003
		(0.050)	(0.000)	(0.004)	(0.004)	(0.003)
October	df_{10}	0.243***	0.000	-0.008*	-0.004	-0.003
		(0.047)	(0.000)	(0.004)	(0.004)	(0.004)
November	df_{11}	0.242***	0.000	-0.012***	-0.008**	0.000
		(0.047)	(0.000)	(0.004)	(0.003)	(0.003)
December	df_{12}	0.241***	0.000	-0.009*	-0.006*	0.002
		(0.049)	(0.000)	(0.005)	(0.003)	(0.003)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table B6. Option pricing performance analysis with MSE criteria across contracts

Moneyness	QAR				Under lognormality			
	[1] QAR-general	[2] QAR-UR (unit root)	[3] QAR-ND (no drift)	[4] QAR-URND	[5] LN-general	[6] LN-UR (unit root)	[7] LN-ND (no drift)	[8] LN-URND
< 0.90	28.383	144.622	174.864	143.610	103.096	133.643	176.372	48.989
		(5.095)	(6.161)	(5.060)	(3.632)	(4.709)	(6.214)	(1.726)
0.90-0.95	63.420	82.996	216.140	188.225	72.032	112.930	257.244	64.768
		(1.309)	(3.408)	(2.968)	(1.136)	(1.781)	(4.056)	(1.021)
0.95-1.00	50.572	70.537	237.954	186.247	56.216	121.749	280.249	66.181
		(1.395)	(4.705)	(3.683)	(1.112)	(2.407)	(5.542)	(1.309)
1.00-1.05	44.310	54.631	179.640	122.384	54.746	79.476	227.593	47.441
		(1.233)	(4.054)	(2.762)	(1.236)	(1.794)	(5.136)	(1.071)
1.05-1.10	21.371	21.855	82.735	45.441	29.928	35.790	100.676	22.776
		(1.023)	(3.871)	(2.126)	(1.400)	(1.675)	(4.711)	(1.066)
> 1.100	5.169	4.448	11.448	8.275	5.735	9.104	14.366	12.410
		(0.861)	(2.215)	(1.601)	(1.109)	(1.761)	(2.779)	(2.401)
Avg	26.326	36.359	103.424	78.028	33.473	53.570	124.100	33.052
		(1.381)	(3.929)	(2.964)	(1.271)	(2.035)	(4.714)	(1.255)

Note: The ratios of the MSE of other seven models to the best performance model (i.e., QAR model) are presented in parentheses. The sampling period is May 8, 2019 to October 2, 2019.

Figure B1. Selected futures price distributions in each quarter of 1990, 2000 and 2010

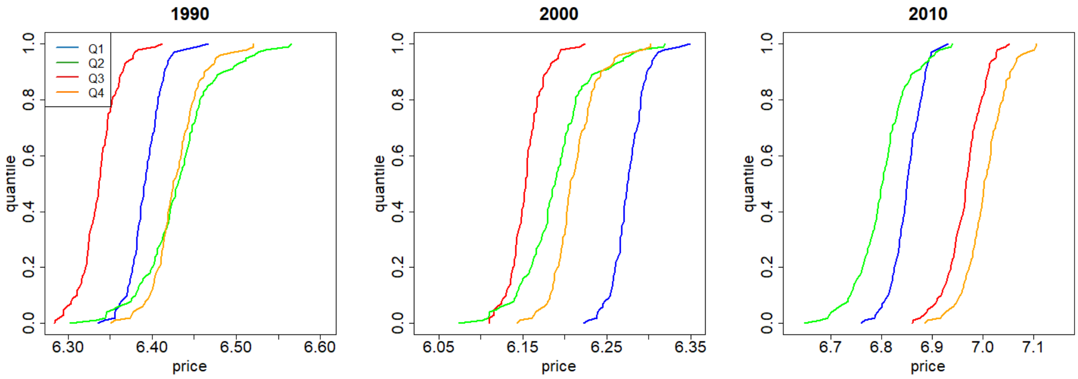


Figure B2. The normality test of futures price with Q-Q plot on a stochastic day, e.g., Jan 31, 2018

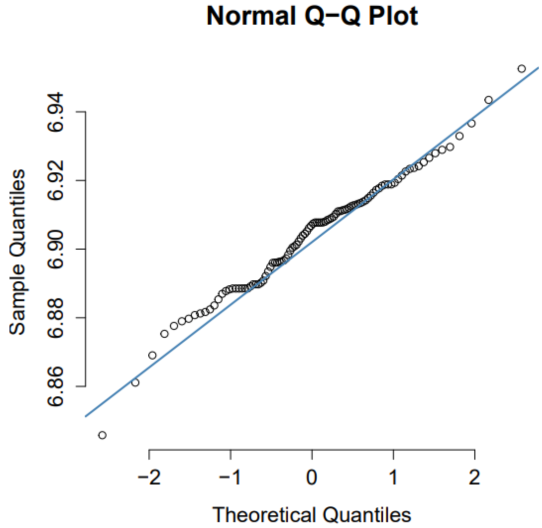


Figure B3. Dominant roots of futures price

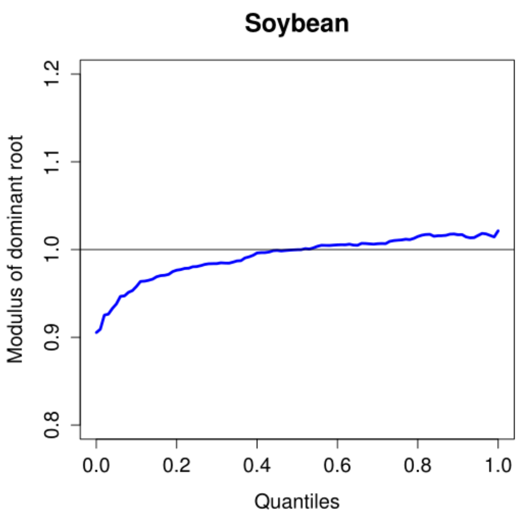
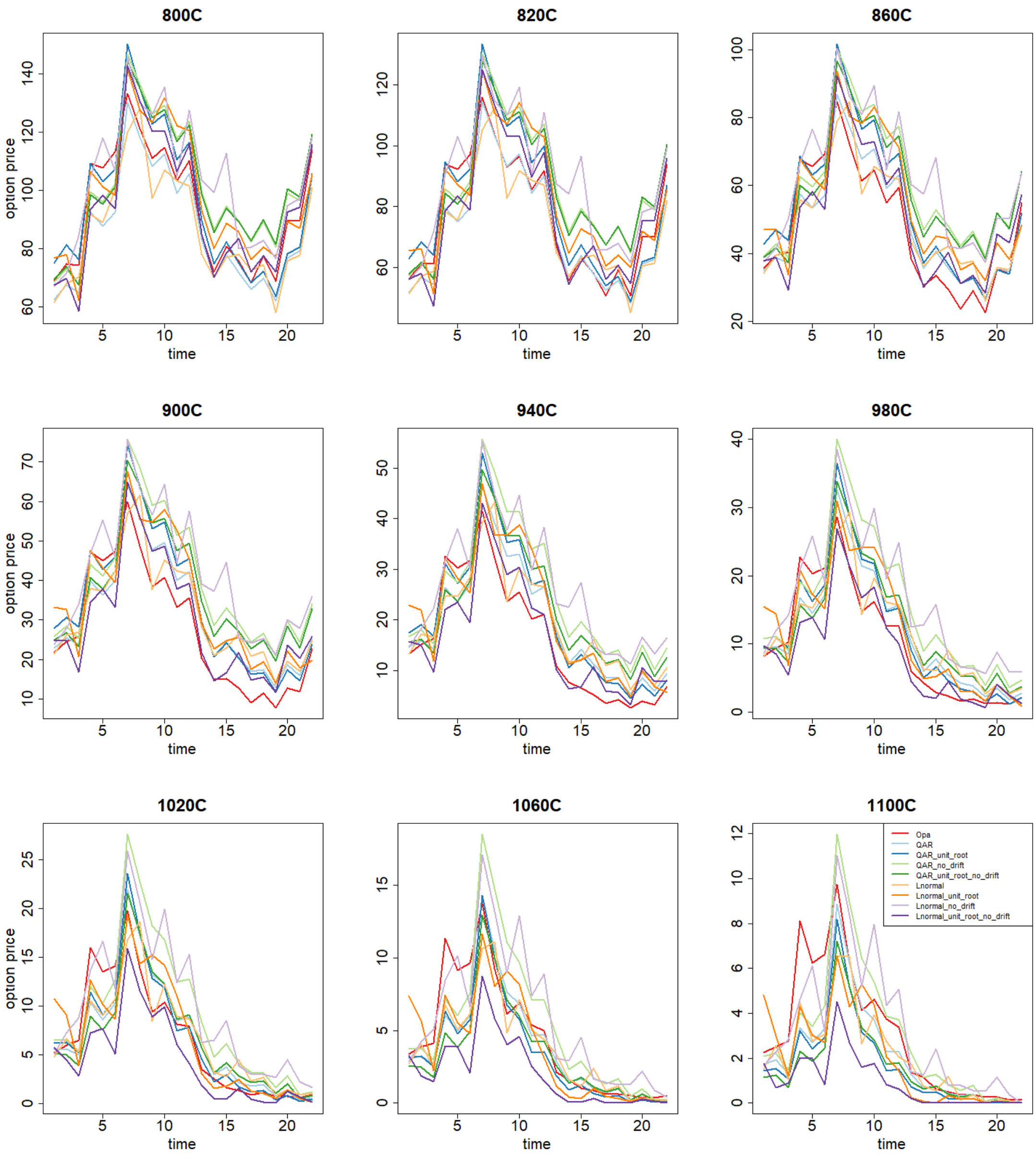


Figure B4. Selected option pricing shapes under alternative scenarios



Appendix C: Robustness of using alternative option contracts checks, e.g., July 2021.

Table C1. Summary statistics

Panel A: Futures price				
Variables	Mean	St. Dev	Max	Min
pf	789.345	281.004	1753.000	413.500

Panel B: Option price				
Moneyness	Weeks-to-expiration			# of observations
S/K	1-25 weeks	26-50 weeks	>50 weeks	Subtotal
	6.207	9.213	8.292	
< 0.90	(4.320)	(7.648)	(8.251)	
	{340}	{515}	{517}	{1372}
	6.207	35.834	27.463	
0.90-0.95	(4.320)	(9.611)	(14.352)	
	{340}	{61}	{104}	{505}
	6.207	52.973	49.397	
0.95-1.00	(4.320)	(11.573)	(18.230)	
	{340}	{50}	{95}	{485}
	6.207	75.302	83.853	
1.00-1.05	(4.320)	(14.511)	(27.909)	
	{340}	{41}	{87}	{468}
	6.207	102.913	123.586	
1.05-1.10	(4.320)	(17.198)	(29.754)	
	{340}	{26}	{77}	{443}
	6.207	175.788	303.322	
> 1.10	(4.320)	(42.864)	(107.670)	
	{340}	{39}	{392}	{771}

Note: The futures price data in panel A involve weekly observations over the period January 7, 1980 – March 11, 2020, while the July2021 soybean option contract data in panel B cover the period March 18, 2020 – June 23, 2021. The unit of futures and options prices are in USD per contract. Standard deviations are presented in parentheses; the number of observations in each category are presented in braces.

Table C2. QAR and AR estimates of futures price pf

Variables	AR	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
pf_1	0.995*** (0.002)	0.979*** (0.005)	0.991*** (0.002)	1.000*** (0.002)	1.004*** (0.002)	1.011*** (0.004)
m_1	-0.001 (0.004)	-0.009 (0.006)	-0.004 (0.003)	-0.002 (0.002)	0.002 (0.003)	-0.002 (0.005)
m_2	0.002 (0.004)	0.001 (0.006)	0.000 (0.003)	0.002 (0.003)	0.003 (0.003)	-0.001 (0.004)
m_3	0.003 (0.003)	0.005 (0.005)	0.002 (0.003)	0.001 (0.003)	0.005* (0.003)	0.004 (0.005)
m_4	0.003 (0.004)	0.006 (0.006)	0.002 (0.003)	0.001 (0.003)	0.005 (0.003)	0.002 (0.005)
m_5	0.001 (0.003)	-0.001 (0.005)	-0.003 (0.004)	-0.003 (0.003)	0.002 (0.004)	0.010* (0.006)
m_6	-0.007* (0.004)	-0.017** (0.008)	-0.011*** (0.004)	-0.006** (0.003)	0.003 (0.003)	0.010 (0.006)
m_7	-0.005 (0.004)	-0.031*** (0.009)	-0.014*** (0.005)	-0.006 (0.004)	0.002 (0.004)	0.023** (0.011)
m_8	-0.001 (0.003)	-0.014** (0.006)	-0.007 (0.004)	-0.006* (0.003)	0.002 (0.003)	0.016** (0.006)
m_9	-0.006* (0.004)	-0.013* (0.008)	-0.009** (0.005)	-0.006** (0.003)	0.000 (0.004)	0.005 (0.005)
m_{10}	0.003 (0.003)	-0.003 (0.006)	-0.001 (0.003)	-0.001 (0.003)	0.006* (0.003)	0.016** (0.008)
m_{11}	0.002 (0.004)	-0.002 (0.005)	0.001 (0.003)	0.001 (0.002)	0.003 (0.003)	0.005 (0.005)
$intercept$	0.033** (0.015)	0.113*** (0.031)	0.048*** (0.016)	0.000 (0.014)	-0.013 (0.015)	-0.039 (0.026)
Adjusted R-square	0.990	0.869	0.916	0.897	0.912	0.913

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. For the "goodness of fit", we report the adjusted $R2$ for OLS estimates and the Pseudo-$R2$ proposed by Koenker and Machado (1999) for quantile estimates. The sampling period is 7 January 1980 to 01 May 2019.

Table C3. Normality test of futures prices pf under alternative scenarios

Sample	Years	Skewness	Excess Kurtosis	Shapiro test: percentage of non-normal distributions	# of observations
Full sample	1980-2020	-0.100*** (0.010)	1.200*** (0.021)	37.754	2066
Subsamples	1980-1990	-0.011 (0.018)	1.195*** (0.043)	35.125	521
	1990-2000	0.026 (0.019)	1.204*** (0.043)	37.165	522
	2000-2010	-0.026 (0.022)	1.242*** (0.043)	42.323	508
	2010-2020	-0.390*** (0.015)	1.160*** (0.039)	36.434	515

Note: Column 1 and 2 show the skewness test with null hypothesis of $S = 0$ and excess kurtosis test with null hypothesis of $K = 0$. Statistical significance is represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01. Standard deviations are presented in parentheses. The proportions of non-normal distributions (column 3) of futures price pf are obtained by Shapiro test, using 5% critical value. The sample period is from January 7, 1980 – March 11, 2020.

Table C4. Modulus of the dominant roots

Dominant roots	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
β_1	0.980 *** (0.005)	0.991 *** (0.003)	1.000 (0.002)	1.004 (0.002)	1.011 *** (0.004)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table C5. Hypothesis tests on drifts across months for selected quantiles

Month	Drift	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
January	df_1	0.105***	0.044***	-0.002	-0.012	-0.041***
		(0.032)	(0.018)	(0.015)	(0.016)	(0.024)
February	df_2	0.114***	0.048***	0.002	-0.011	-0.040
		(0.030)	(0.018)	(0.014)	(0.016)	(0.025)
March	df_3	0.119***	0.050***	0.000	-0.009	-0.035
		(0.032)	(0.018)	(0.015)	(0.017)	(0.025)
April	df_4	0.119***	0.050***	0.000	-0.009	-0.037
		(0.032)	(0.018)	(0.014)	(0.018)	(0.024)
May	df_5	0.112***	0.045***	-0.003	-0.011	-0.029
		(0.033)	(0.018)	(0.015)	(0.015)	(0.022)
June	df_6	0.096***	0.037***	-0.007	-0.010	-0.029
		(0.032)	(0.019)	(0.014)	(0.015)	(0.022)
July	df_7	0.082***	0.034***	-0.006	-0.011	-0.016
		(0.031)	(0.020)	(0.015)	(0.016)	(0.025)
August	df_8	0.099***	0.041***	-0.006	-0.012	-0.024
		(0.033)	(0.019)	(0.014)	(0.017)	(0.025)
September	df_9	0.100***	0.039***	-0.007	-0.014	-0.034
		(0.029)	(0.018)	(0.016)	(0.016)	(0.025)
October	df_{10}	0.110***	0.047***	-0.001	-0.008	-0.023
		(0.034)	(0.018)	(0.012)	(0.016)	(0.022)
November	df_{11}	0.111***	0.049***	0.000	-0.011	-0.034
		(0.035)	(0.019)	(0.013)	(0.015)	(0.024)
December	df_{12}	0.113***	0.048***	0.000	-0.013	-0.039
		(0.030)	(0.020)	(0.014)	(0.015)	(0.024)

Note: Standard errors (presented in parentheses) are obtained using bootstrapping, with statistical significance represented by stars: * = p<0.1; ** = p<0.05; *** = p<0.01.

Table C6. Option pricing performance analysis with MSE criteria across contracts

Moneyness	QAR (1) QAR-general	(2) QAR-UR (unit root)	(3) QAR-ND (no drift)	(4) QAR-URND	Under lognormality (5) LN-general	(6) LN-UR (unit root)	(7) LN-ND (no drift)	(8) LN-URND
< 0.90	35.011	47.202	230.673	106.292	64.768	70.694	345.800	148.375
		(1.348)	(6.589)	(3.036)	(1.850)	(2.019)	(9.877)	(4.238)
0.90-0.95	31.071	39.159	136.804	88.857	48.394	40.704	213.734	159.227
		(1.260)	(4.403)	(2.860)	(1.558)	(1.310)	(6.879)	(5.125)
0.95-1.00	45.403	71.431	292.510	102.411	82.949	133.966	357.336	114.436
		(1.573)	(6.443)	(2.256)	(1.827)	(2.951)	(7.870)	(2.520)
1.00-1.05	35.011	47.202	230.673	106.292	64.768	70.694	345.800	148.375
		(1.348)	(6.589)	(3.036)	(1.850)	(2.019)	(9.877)	(4.238)
1.05-1.10	31.071	39.159	136.804	88.857	48.394	40.704	213.734	159.227
		(1.260)	(4.403)	(2.860)	(1.558)	(1.310)	(6.879)	(5.125)
> 1.100	45.403	71.431	292.510	102.411	82.949	133.966	357.336	114.436
		(1.573)	(6.443)	(2.256)	(1.827)	(2.951)	(7.870)	(2.520)
Avg	39.769	58.033	251.846	102.245	72.014	102.885	338.303	132.184
		(1.459)	(6.333)	(2.571)	(1.811)	(2.587)	(8.507)	(3.324)

Note: The ratios of the MSE of other seven models to the best performance model (i.e., QAR model) are presented in parentheses. The sampling period is May 8, 2019 to October 2, 2019.

Figure C1. Selected futures price distributions in each quarter of 1990, 2000 and 2010

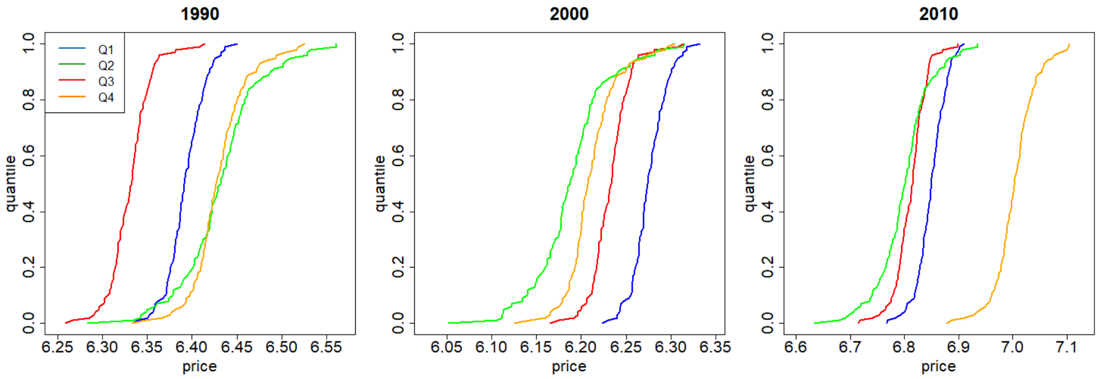


Figure C2. The normality test of futures price with Q-Q plot on a stochastic day, e.g., Jan 31, 2018

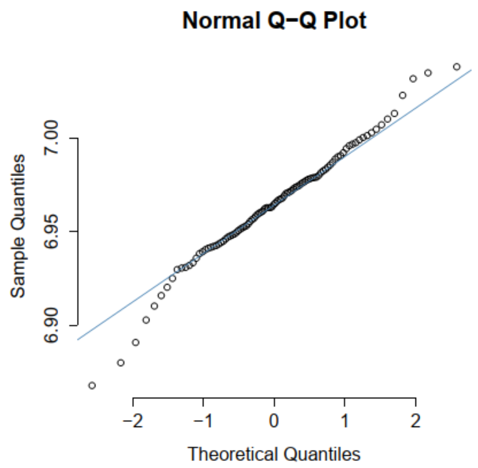


Figure C3. Dominant roots of futures price

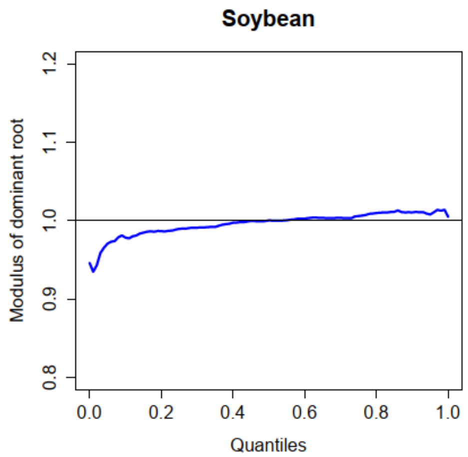
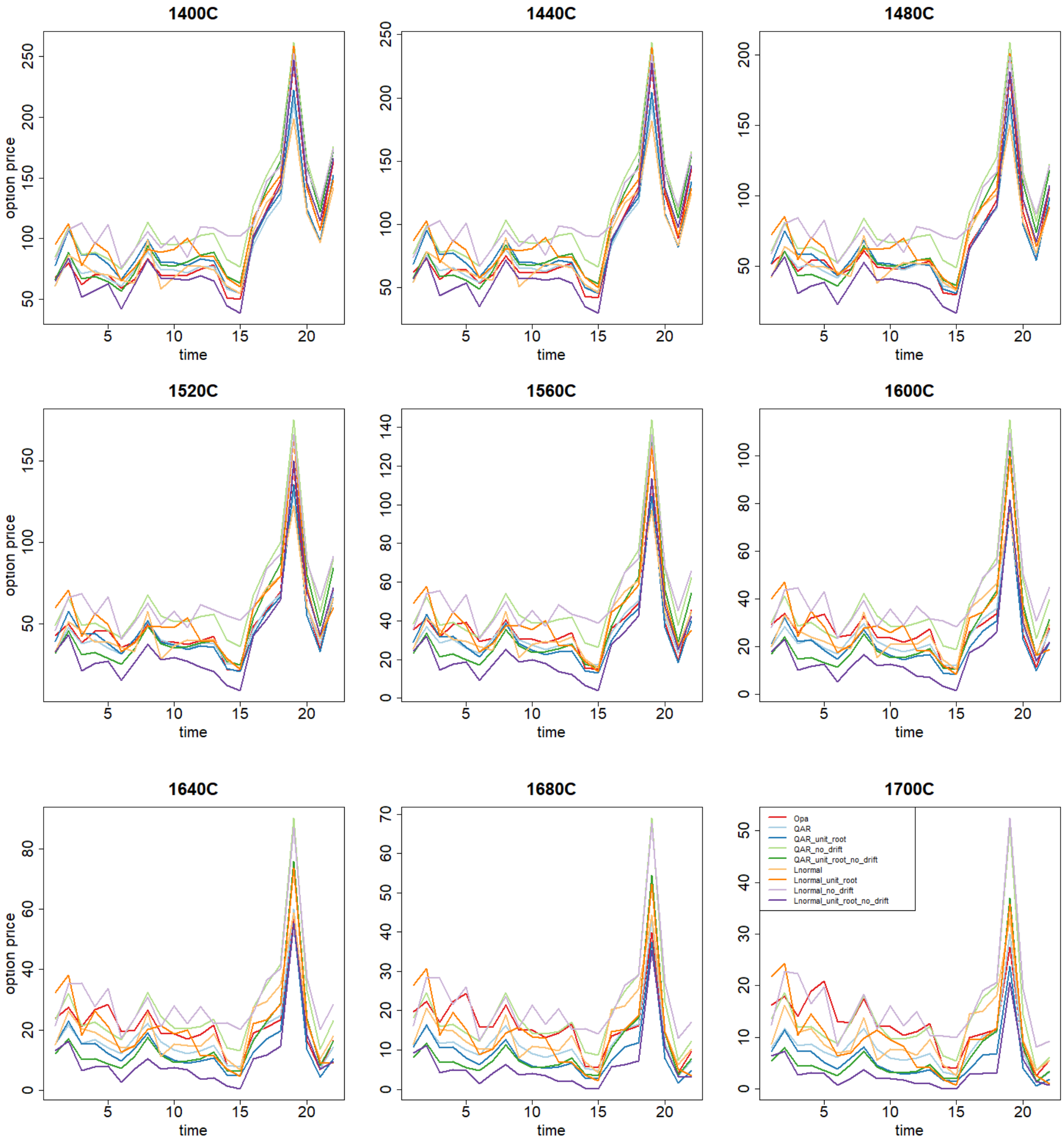


Figure C4. Selected option pricing shapes under alternative scenarios



[1] Other directions (not analyzed in this paper) include the role of traders' risk preferences and probability assessments in the functioning of financial markets (e.g., Barberis et al., 2021). Exploring such directions is beyond the scope of this paper.

[2] Previous research has explored the role of non-Gaussian properties of asset returns (Hull, 2022). This includes stochastic volatility, stochastic interest rates, jump risk, and conditional skewness (e.g., Heston, 1993; Hull and White, 1987; Andersen et al., 2015; Christoffersen et al., 2021). The research has typically used either GARCH-type models (e.g., Heston and Nandi, 2000; Maheu and McCurdy, 2004; Kim and Kim, 2005; Hao and Zhang, 2013) or alternative parameterized distribution functions (e.g., Christoffersen et al., 2006; Carr and Wu, 2017; Babaoglu et al., 2018; Gruber et al., 2021).

[3] By focusing on risk neutrality, our paper does not examine the role of risk preferences. Note that distinguishing between probability assessments and risk preferences can raise difficult identification issues. For example, under the expected utility model, changing a probability can have the same economic effect as changing marginal utility. Under prospect theory, changing the probability of a rare event can have the same effect as changing the weight on this probability. We leave the investigation of these issues to future research.

[4] Our analysis focuses on call options with the understanding that our main conclusions would also apply to put options (see Hull and White (1987) and Bakshi et al., (1997)).

[5] We consider pricing issues for European options for the reason that early exercise has been infrequent in practice (Battalio et al., 2015; Jensen and Pedersen, 2016). Although commodity futures options are American and allow for early exercise, Merton (1973) and MacBeth and Merville (1979) have shown that American call options should not be exercised prematurely. And Whaley (1986) and Irwin et al. (1989) provide evidence that the pricing models for European options provide accurate estimates for American-style agricultural options.

[6] A closely related result for the pricing of a call option on a stock is given by the Black-Scholes formula (Black and Scholes, 1973):

$$C_{BS}(t) = S(t)\, N(d_1) - K\, e^{-r(T-t)}\, N(d_2), \qquad (2')$$

where $S(t)$ is the stock price at time t.

[7] When using discounted prices $\frac{p_{t+1}}{1+r}$, equation (6) implies that prices would follow a random walk process, where expected discounted future prices are equal to the current price.

[8] To obtain meaningful results, we use Wednesday prices to construct weekly futures and options price data.

[9] We also explored the introduction of other variables, including nonlinear dynamics and time-to-maturity effects. However, we found that the corresponding effects were not statistically significant; and that they did not improve the model fit (as measured by the BIC criterion).

[10] The results using MAPE are available from the authors upon request.