OPTIMIZATION OF PRIMARY MILK PRODUCTION IN THE HILLY-MOUNTAINOUS REGIONS OF THE REPUBLIC OF SERBIA[1]

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Abstract

Paper presents a model for the optimization of primary milk production in the hilly-mountainous regions of the Republic of Serbia. The goal of creating the model is to demonstrate and analyze the conditions and outcomes of production at the farm, while to find the optimal production structure, considering the organizational, economic, technical, and technological circumstances in which the farm performs its agricultural activities. The model is based on the linear programming optimization method. A mathematical model, or objective function, was established, and constraints were identified. A logical model was created for optimization. The main goal of solving the linear programming problem is to find the maximum or minimum of the objective function. In presented model, the task is to maximize the objective function, what is represented by the farm's net income. By using the linear programming, it is possible to determine the optimal quantities of resources and products to maximize net income, while adhering to resource constraints and other relevant factors.

Key words: Optimization, linear programming, primary milk production.

JEL[5]: Q1, Q13, C61

Introduction

Linear programming (LP) is successfully used for decades in different studies of agroeconomic issues. During these activities, LP continuously proves to be a powerful

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tool with great informational potential for agricultural production organizers. Despite the proven benefits of the LP, it has not found yet its significant practical application in Serbia and wider region. Reasons for this primarily lie in the relative complexity of the process, which involves not only the creation of logical and mathematical models, but also the interpretation and understanding of derived results.

LP is considered as part of larger progressive development that provides to humanity possibility to set general objectives and to outline the steps of deep decisions to be made towards the "best" achievement of its goals facing the practical situations of huge complexity. Dantzig discusses the competences to articulate common objectives and later defining of optimal policy alternatives for practical decisions of large complexity (Dantzig, 2002).

There are specialized software and add-ons, such as Solver in MS Excel that support the LP method. Unfortunately, they are still not sufficiently accessible to wider group of users in the agri-food sector. Additional efforts are needed to facilitate access to these tools, while increasing their practical application.

Literature Review

Agriculture is facing the challenge of enhancing the sustainability of food production, which requires the implementation of new systems and technologies (Springmann et al., 2018; Möhring et al., 2023). Livestock farming is crucial for many economies but often relies on government subsidies to sustain required activities (Kamilaris et al., 2020). However, some researchers (Maksimović Sekulić et al., 2024) emphasize the importance of choosing production structures independent of such subsidies, thereby promoting fair competition and innovation.

Many authors underline the significance of mathematical models and optimization in realizing the biophysical relations within a complex system (Romera et al., 2004; Neal et al., 2007; Addis et al., 2021). Andrić Gusavac (2020) highlights the benefits of applying operations research in agriculture, particularly in optimizing livestock feed. These methods have been proven as useful in better understanding the complexity of agricultural systems and improving their management (Weintraub, Romero, 2006).

Optimization methods, dating back to the 1950s, and firstly proposed by Waugh (1951), include the application of linear programming (LP) for optimizing livestock feed, while Dantzig published his first paper on LP in 1948. (Dantzig, 1948). With the development of information technology, optimization methods increasingly rely on software packages, becoming fundamental tools supported by computers and computer applications, such as the use of Solver tools. After its launching during the February 1991., Microsoft Excel Solver becomes the most widespread and arguably the most widely utilized general-purpose system for optimization modeling (Fylstra et al., 1998). So, optimization tool that use LP will be beneficial for agricultural producers to optimize the resource utilization (use of inputs) and overall farm profitability, as well as for strategic approach in planning, or making better decisions, or better understanding used systems (Addis et al., 2021). Furthermore, increasing productivity in agriculture is crucial for decreasing the regional poverty (Irz et al., 2001; Byerlee et al., 2009; Hoel et al., 2024).

According to Vico and Rajic (2019), research relying on the application of LP in optimizing agricultural production in this region dates back to the 1960s (Kamenečki, 1963; Dobrenić, 1966; Galev, 1966; Bubica, 1968). It was often applied in creating and analyzing macroeconomic models of agricultural development (Jakovljevski, 1984; Bogdanov, 1994; Rodić, 2001; Ljubanović Ralević et al., 2013; Babovic, Radovic, 2014; Paunovic et al., 2016; Vulević et al., 2018). Vico et al. (2013) optimized production on a cattle farm using minimization of labor as the criterion for optimality. Jandrić (2019) formulated a LP model for optimizing primary milk production.

In previous couple decades there are several researches focused to the optimization of milk production (Eshraga et al., 2011; Sharma et al., 2012; Chen et al., 2020; Gahroui et al., 2021). Therefore, the aim of the paper is to find the maximum of the objective function of the observed dairy production, i.e. to formulate such a model leading to that.

Methodology

The method used for optimizing primary milk production in hilly-mountainous regions of Serbia was linear programming optimization. Initially, a mathematical model was established, including a criterion function, to define the set goals. Then, constraints relevant to milk production in these regions were identified, such as the availability of resources, capacities, and other production conditions. Based on this, a logical model was formed to enable the analysis of different scenarios and their impacts on production. Research data has been collected in the period 2018-2019., within the observed region of Serbia. In the continuation of the paper statements of the criterion function are given as well as the set limitations and that:

Criterion Function

$$(\max) f = \sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} x_j \qquad (1)$$

Constraints

$$\sum_{i=1}^{p}\sum_{j=1}^{q} a_{ijkl} x_j \leq=\geq u_k \text{ while, } k = 1, 2, \ldots, r; l = 1, 2, \ldots, s \quad (2)$$

Non-negativity Condition: i = 1, 2, ..., p; j = 1, 2, ..., q. Indices: p – number of activity groups, q – number of activities in a group, r – number of constraint groups, and s – number of constraints in a group. Activities: x_{ij}, while i = 1, 2, ..., p; j = 1, 2, ..., q. Constraints: u_{kl}, while k = 1, 2, ..., r; l = 1, 2, ..., s. Coefficients in the objective function: c_{ij}, while i = 1, 2, ..., p; j = 1, 2, ..., q. Coefficients in the constraints: quantity of the j^{th} activity in the i^{th} activity group of the l^{th} constraint in the r^{th} constraint group.

This approach allows producers to precisely plan and optimize their resources, maximizing profitability and sustainability of milk production in mountainous areas. Therefore, the research goal would be the formulation of such a model.

Results and Discussion

In optimizing primary milk production in the mountainous regions of the Republic of Serbia, the method of linear programming optimization was used. Key elements of the farming system important for achieving the research objectives were identified and analyzed. The logical model considers activities, constraints, and resources necessary for optimization.

Activity Groups:

- Cattle Farming: Including dairy cows and supporting categories. Calves are sold within fifteen days after birth, except for some female calves retained for herd replacement.
- Crop Production on Own Land: Encompasses different crops, including grass-clover mixtures, cereals, buckwheat, and potatoes for market sale.
- Meal Preparation: Provides animal feed and concentrates from various sources.
- External Inputs for Crop Production: Include fertilizers, pesticides, and other resources.
- Other Costs: Cover various operational expenses.
- Labor Force: Comprises both family and hired labor.
- Final Products: Include dairy and other agricultural products.

Constraint Groups:

- Capacities: Include livestock and storage of agricultural products.

- Biotechnical Constraints: Relate to production processes and methods.
- Market Constraints: Include market demands and limitations.
- Input Balances for Cattle Farming: Include capacities for feed, water, and other resources.
- Input Balances for Crop Production: Cover requirements for fertilizers, pesticides, and other resources.
- Other Costs: Encompass general farm costs.
- Labor Force: Include requirements for labor, including internal and external labor and machinery.
- Mechanization: Relates to the use of equipment and tools for various tasks.
- Final Product Balances: Encompass stocks of final products ready for sale.

The logical model was created based on information gathered from interviews with farmers and advisors, and reflects the real circumstances in which the farm operates, considering new trends in the observed area.

The following assumptions were made in creating the model:

- The farm has five hectares of its own arable land, with the possibility of leasing additional land at annual cost.
- The areas under pastures are not a constraint as there is assumed to be sufficient available land.
- One dairy cow is kept in production for eight lactations, and herd replacement is done through internal reproduction.
- The production of roughage feed is ensured by sowing grass-clover mixtures.
- The age of first calving for pregnant heifers is 26 months.
- The summer-feeding period lasts for 215 days and it is based on grazing with concentrates, while the winter-feeding period lasts for 150 days and is based on hay from artificial meadows with concentrates.

These assumptions establish the framework for optimization and the creation of a sustainable model of primary milk production in hilly and mountainous areas of Serbia.

At the heart of the logical model is one dairy cow, which, along with its supporting categories, forms the structural unit or activity “cattle farming” in the mathematical model. From one dairy cow, three products are obtained: milk, calves, and beef (which is obtained when adult cattle become unproductive for breeding).

These products are sold on the market, while some female calves are kept for herd replacement. Inputs such as animal feed and additional inputs used in livestock production can be purchased from the market or produced at the farm. For example, the production of animal feed within farms' crop production provides internal inputs for livestock.

Some final products from crop production, such as potatoes and buckwheat, can be sold on the market. The market, with its requirements, can limit the minimum and maximum quantities of plant products the farm can produce and sell. Additionally, the structure of crop production partly depends on biotechnical constraints, such as crop rotation.

Plant and livestock production represent the production capacities available to the farm. These two types of production share certain resources, such as labor and machinery. Some capacities are specific to plant production, such as arable land, while others are related to livestock farming, such as the stalls for dairy cattle. These distinctions between capacities and resources help to optimize production and the sustainability of the farm.

Activities in the model of optimizing primary milk production: After establishing the logical model, analysis of each individual element of the system was performed. As a result of this process, activities that will be included in the criterion function of the mathematical model were identified. All activities can be categorized into ten groups: 1. Cattle farming, 2. Plant production on farms' land, 3. Plant production on rented land, 4. Forage, 5. Purchased animal feed, 6. External inputs for plant production, 7. Other costs, 8. Internal labor, 9. External labor, and 10. Final products.

Activities from the second group, namely plant production on rented land, deserve further explanation. Since farm is limited by its own arable land, but has the possibility for using other land sources through rental agreements, the model treats production on owned and rented land separately. Production on rented land involves additional costs (rental expenses), so it is necessary to treat these activities separately. Otherwise, both types of production would be equated in the model, which would not reflect the real circumstances.

The same situation applies to internal and external labor. Due to the methodological approach in calculating coverage margins, internal labor is not represented as a cost, while this is the case with the external labor. Forage is recognized as a separate activity because it can be obtained by combining different grains or processing grains individually.

Constraints in the model of primary milk production: Similar to the dairy farm optimization model, the model for optimizing primary milk production includes the establishment of multiple groups of constraints. Based on further considerations of the logical model, constraints and technical coefficients were defined. In defining such a mathematical model, the constraints can be categorized into two main groups: capacity constraints and constraints that are linked to activities (so-called “balances”).

A more detailed classification of the predefined constraints reveals eight groups: capacity constraints, biotechnical constraints, market constraints, balances of inputs for cattle farming, balances of inputs for plant production, labor, mechanization, and balances of final products.

Capacity Constraints:

1. The maximum capacity of the barn is limited to 12 places for dairy cattle, or $X_1 \leq 12$
2. Own arable land - the farm has available 5 ha of arable land, or $X_2 + X_3 + X_4 + X_5 + X_6 + X_7 \leq 5$

Biotechnical Constraints:

3. Maximum area under potatoes - on farms’ arable land, potatoes can occupy up to one quarter of the total area, or $X_2 \leq 1.25$
4. Maximum share of cereals - cereals on farms’ land can be sown on up to 50% of the total area, or $X_3 + X_4 + X_5 + X_7 \leq 2.50$
5. Maximum share of grass-legume mixtures – similarly to cereals, grass-legume mixtures can be sown on up to 50% of farms’ land, or $X_6 \leq 2.50$

Market Constraints:

6. Maximum production of potatoes - based on practical experience, the constraint on maximum potato production is set at 2 ha, or $X_2 + X_8 \leq 8$
7. Maximum production of buckwheat - given to previous experience regarding sales, the maximum buckwheat production is limited to 3 ha, or $X_5 + X_{11} \leq 3$

Balance of Inputs for Dairy Farming:

8. Cereal balance for fodder - based on information related to feeding of cows with concentrated feed, it has been found that in the diet for milking cows and breeding heifers, cereals in the form of fodder are used in addition to concentrated feed mixtures with 18% protein. The fodder itself

may consist of one or more different cereals. Its composition depends on the structure of plant production, which is defined by the competitiveness of different lines of plant production. Additionally, the model allows the purchase of part or all of the raw materials on the market, or $-3{,}000X_3 - 2\ 800X_4 - 3{,}000X_7 - 3{,}000X_9 - 2{,}800X_{10} - 3{,}000X_{13} + X_{14} - X_{15} - X_{16} - X_{17} = 0$

9. Fodder balance - annual fodder requirement is 1,100 kg. As previously explained, the model includes the "dairy farming" activity, i.e. representing a milking cow with its categories. So, this constraint needs to include the corresponding share of fodder needs related to feeding heifers. It is important to consider that milking cows are used for eight lactations in practice. Based on practical information and additional calculations, the value of technical coefficient is presented as 1,220 kg: $-1\ 220X_1 + X_{14} = 0$
10. Hay balance - only coarse fodder used for feeding animals in the winter period is artificial meadow hay. A daily quantity of 20 kg per milking cow is planned. Additionally, the corresponding portion of hay used for breeding heifers must be added, or $-3{,}600X_1 + 9{,}000X_6 + 9{,}000X_{12} = 0$
11. Concentrate balance with 18% protein - in addition to fodder, the concentrated part of the diet also consists of certain amount of ready-made concentrated feed mixtures with 18% protein. The mixture is given in different amounts over the year, depending on the stage of lactation and pregnancy. As like in previous case, calculation of technical coefficient reflecting the consumption of concentrates per structural unit was carried out, considering the needs for a milking cow and corresponding portion of needs for a breeding heifer, or $-600X_1 + X_{18} = 0$

Balance of Inputs for Plant Production:

12. Balance of potato planting material, or $-2{,}500X_2 - 2{,}500X_8 + X_{19} = 0$
13. Balance of barley seed, or $-300X_3 - 300X_9 + X_{20} = 0$
14. Balance of oat seed, or $-180X_4 - 180X_{10} + X_{21} = 0$
15. Balance of buckwheat seed, or $-150X_5 - 150X_{11} + X_{22} = 0$
16. Balance of clover-grass seed mixtures - in defining this technical coefficient, it is necessary to consider that the average exploitation period of land sown under the clover-grass mixtures lasts for five years. This requires the need to adjust the "load" of one hectare under clover-grass mixtures with the necessary inputs for seeding (establishment). The need for seed during the sowing of one hectare is 40 kg of seed, but this amount

will not be used as a technical coefficient in the model, or $-8X_6 - 8X_{12} + X_{23} = 0$

17. Balance of triticale seed, or $-300X_7 - 300X_{13} + X_{24} = 0$
18. Balance of NPK fertilizers - required quantities of NPK mineral fertilizers are determined according to the standard technology for each sown crop. In the case of clover-grass mixtures, the calculated coefficient implies the annual requirement together with the corresponding portion of the requirement in the year of sowing, or $-400X_2 - 250X_3 - 200X_4 - 150X_5 - 200X_6 - 250X_7 - 400X_8 - 250X_9 - 200X_{10} - 150X_{11} - 200X_{12} - 250X_{13} + X_{25} = 0$
19. Balance of urea - requirements for urea were calculated similarly as the previous case, or $-200X_2 - 150X_3 - 100X_4 - 100X_5 - 130X_6 - 150X_7 - 200X_8 - 150X_9 - 100X_{10} - 100X_{11} - 130X_{12} - 150X_{13} + X_{26} = 0$
20. Balance of calcium ammonium nitrate (KAN) - it is assumed in the model that KAN is used just in potato production, or $-100X_2 - 100X_8 + X_{27} = 0$
21. Balance of diesel (fuel) - requirements for diesel are presented through technical coefficients based on technological charts, or $-10X_1 - 300X_2 - 150X_3 - 100X_4 - 100X_5 - 130X_6 - 150X_7 - 200X_8 - 150X_9 - 100X_{10} - 100X_{11} - 130X_{12} - 150X_{13} + X_{28} = 0$
22. Balance of other variable costs - accepted approach to creating the model involves some inputs as separate activities in the criterion function. This is the case with concentrated feed in cattle production, as well as seeds, mineral fertilizers, and diesel in crop production. Given the research goals, there is no need for an additional analytical presentation of inputs in crop and cattle production, as this would only increase the model without improving the quality of obtained solutions.
23. Therefore, one aggregate activity called "other variable costs" is defined in the criterion function. This activity represents a combined value of inputs for each production. In cattle production, it encompasses the costs of artificial insemination, treatment and care of livestock, electricity, hygiene products, advisory services, and other consumable materials and services. In crop production, it covers the costs of soil chemical analysis, pesticides, binders, bags, and other consumable materials and services, or $-30{,}000X_1 - 50{,}000X_2 - 15{,}000X_3 - 12{,}000X_4 - 10{,}000X_5 - 18{,}000X_6 - 15{,}000X_7 - 70{,}000X_8 - 35{,}000X_9 - 32{,}000X_{10} - 30{,}000X_{11} - 38{,}000X_{12} - 35{,}000X_{13} + X_{29} = 0$
24. Labor Force 23-34 - labor balance includes a group of twelve constraints

where technical coefficients link the activities of production lines with activities related to internal labor and those concerning external labor. With the adopted approach, where activities related to external (paid) labor are specifically defined in the objective function, the model can independently determine the need for external labor during problem-solving. The model considers that internal labor is unpaid, while external labor represents a cost. This cannot be achieved through a synthetic treatment of labor, or $-18X_1 + X_{30} + X_{42} = 0$, $-18X_1 + X_{31} + X_{43} = 0$, $-18X_1 + X_{32} + X_{44} = 0$, $-18X_1 - 20X_2 - 7X_3 - 7X_4 - 7X_5 - 1.7X_6 - 7X_7 - 20X_8 - 7X_9 - 7X_{10} - 7X_{11} - 1.7X_{12} - 7X_{13} + X_{33} + X_{45} = 0$, $-18X_1 - 8X_2 - X_3 - X_4 - X_5 - X_6 - X_7 - 8X_8 - X_9 - X_{10} - X_{11} - X_{12} - X_{13} + X_{34} + X_{46} = 0$, $-18X_1 - 7X_2 - X_3 - X_4 - X_5 - 16X_6 - X_7 - 7X_8 - X_9 - X_{10} - X_{11} - 12X_{12} - X_{13} + X_{35} + X_{47} = 0$, $-18X_1 - 5X_2 - 0.5X_3 - 0.5X_4 - 0.5X_5 - 0.5X_6 - 0.5X_7 - 5X_8 - 0.5X_9 - 0.5X_{10} - 0.5X_{11} - 0.5X_{12} - 0.5X_{13} + X_{36} + X_{48} = 0$, $-18X_1 - 3X_2 - 5X_3 - 5X_4 - 5X_5 - 16X_6 - 5X_7 - 3X_8 - 5X_9 - 5X_{10} - 5X_{11} - 16X_{12} - 5X_{13} + X_{37} + X_{49} = 0$, $-18X_1 - 125X_2 - 125X_8 + X_{38} + X_{50} = 0$, $-18X_1 - 5X_2 - 5X_3 - 5X_4 - 5X_5 - X_6 - 5X_7 - 5X_8 - 5X_9 - 5X_{10} - 5X_{11} - 1X_{12} - X_{13} + X_{39} + X_{51} = 0$, $-18X_1 + X_{40} + X_{52} = 0$, $-18X_1 + X_{41} + X_{53} = 0$

25. Capacities of internal labor 35-46 - as was previously mentioned, when creating the model, it was assumed that the farm has two family members who are permanently engaged in agricultural production, or $X_{30} \leq 400$, $X_{31} \leq 400$, $X_{32} \leq 400$, $X_{33} \leq 400$, $X_{34} \leq 400$, $X_{35} \leq 400$, $X_{36} \leq 400$, $X_{37} \leq 400$, $X_{38} \leq 400$, $X_{39} \leq 400$, $X_{40} \leq 400$, $X_{41} \leq 400$
26. Mechanization 47-58 - available mechanization Labor - model assumes that the farm has one medium-sized tractor with required equipment, allowing the performing of activities for both, crop and livestock production. In defining the available monthly mechanization labor capacity, all circumstances that influence availability were considered. This information was collected through interviews with agricultural producers and advisors. The farm does not own a combine harvester, so it relies on external services during the harvest period, or $X_1 \leq 120$, $X_1 \leq 120$, $X_1 \leq 120$, $2X_1 + 9X_2 + 4X_3 + 4X_4 + 4X_5 + 1.5X_6 + 4X_7 + 9X_8 + 4X_9 + 4X_{10} + 4X_{11} + 1.5X_{12} + 4X_{13} \leq 140$, $2X_1 + 5X_2 + 1.5X_3 + 1.5X_4 + 1.5X_5 + 1X_6 + 1.5X_7 + 5X_8 + 1.5X_9 + 1.5X_{10} + 1.5X_{11} + 1X_{12} + 1.5X_{13} \leq 140$, $2X_1 + 5X_2 + X_3 + X_4 + X_5 + 6X_6 + X_7 + 5X_8 + X_9 + X_{10} + X_{11} + 6X_{12} + X_{13} \leq 170$, $2X_1 + 5X_2 + 5X_8 \leq 170$, $2X_1 + 5X_3 + 5X_4 + 5X_5 + 6X_6 + 5X_7 + 5X_9 + 5X_{10} + 5X_{11} + 6X_{12} + 5X_{13} \leq 170$, $2X_1 + 20X_2 + 20X_8 \leq 150$, $2X_1 + 4X_2 + 4X_3 + 4X_4 + 4X_5 + 1.5X_6 + 4X_7 + 4X_8 + 4X_9 + 4X_{10} + 4X_{11} + 1.5X_{12} + 4X_{13} \leq 130$, $X_1 \leq 120$

27. Balances of Final Products 59-63 - model assumes that the farm can deliver five final products to the market, i.e. milk, calves, culled dairy cows, potatoes, and buckwheat. The expected milk yield is 3,700 liters per dairy cow annually. Calves are sold up to 15th day after calving. When defining the technical coefficient for calves, the needs for herd replacement were considered, as well as the fact that the fertility index is approximately 90%, or $3{,}700X_1 - X_{54} = 0$, $0.8X_1 - X_{55} = 0$, $75X_1 - X_{56} = 0$, $18{,}000X_2 + 18{,}000X_8 - X_{57} = 0$, $1{,}600X_5 + 1{,}600X_{11} - X_{58} = 0$

Objective Function in the Optimization Model of Primary Milk Production

Solving a LP task implies finding the maximum or minimum of the objective function. The specific goal in a given task depends on the research objective. In this model, the task is to maximize the objective function, which represents the net income. The following Table (Table 1.) provides an overview of the used input prices in the model.

Table 1. Input prices in the optimization model of primary milk production

Code	Input	Unit of Measure	Purchase Price (RSD/UM)
X_{15}	Animal feed - barley grain	kg	22.00
X_{16}	Oat grain - purchased	kg	25.00
X_{17}	Triticale grain - purchased	kg	18.00
X_{18}	Concentrate (18% protein)	kg	50.00
X_{19}	Seed potatoes	kg	70.00
X_{20}	Forage barley seed	kg	50.00
X_{21}	Oat seed	kg	50.00
X_{22}	Buckwheat seed	kg	160.00
X_{23}	Grass-legume mixture seed	kg	360.00
X_{24}	Triticale seed	kg	50.00
X_{25}	NPK fertilizer	kg	63.00
X_{26}	Urea	kg	60.00
X_{27}	KAN (calcium ammonium nitrate)	kg	59.00
X_{28}	Diesel fuel	l	155.00
X_{42-53}	External labor	Hour	240.00

Source: According to authors calculations.

The coefficients in the objective function for activities, representing production lines, have a zero value, while coefficients are negative for inputs purchased at the market or positive for final products sold in the market. The prices of final products used in the model could be seen in next table (Table 2.).

Table 2. Prices of final products in the optimization model of primary milk production

Code	Final Products	Unit of Measure	Selling Price (RSD/UM)
X_{54}	Milk	l	28.00
X_{55}	Calves	pcs	25,000.00
X_{56}	Cows removed from milking herd	kg	160.00
X_{57}	Potatoes	kg	40.00
X_{58}	Buckwheat	kg	95.00

Source: According to authors calculations.

Solving the model

Model that directly includes inputs and outputs in the objective function has several advantages compared to approach that includes production lines. The advantages lie in faster and simpler interpretation of results after solving the model, as they are deriving directly from the model without additional calculations. This approach is allowing separate consideration of related products, what is particularly important for post-optimal analysis, as well as for easier experimentation with the initial model, by changing initial parameters. Interpreting the values of activities representing final products directly from the model allows determination of the production structure.

The optimal solution was achieved in the sixtieth iteration. The advantages of using approach that includes inputs and outputs in the objective function, as explained in the previous model, are also present in this case.

The clover-grass mixtures should be sown on a total area of 4.80 ha, including 2.50 ha at the farm's own land and 2.3 ha on rented land. Mentioned yields in total 43,200 kg of hay in two harvests (mowing), what meets the needs of twelve dairy cows and supporting cattle categories. Potatoes should be planted on a total area of 2 ha, including 1.25 ha at the farm's own land. The same area should be sown with buckwheat, while 1.75 ha has to be sown on rented land. Other cereals were not competitive, so the needs for concentrated feeds will be met through market procurement. For these purposes, the farm has to annually purchase 7,200 kg of 18% protein concentrate and 14,640 kg of triticale grain, which is used as crumbled feed for feeding dairy cows and breeding stock.

An overview of the production structure can be easily read from the optimal solution. Annual production provides the market with 44,400 l of fresh raw milk, nine calves, 900 kg of beef from culled breeding cows, 36 t of potatoes, and 4.8 t of buckwheat grain. In this way, the farm can achieve an annual net income of 1,287,536.00 RSD. In next table is visible the optimal production structure (Table 3.).

Table 3. Optimal structure of primary milk production

Cod	Element	Unit of Measure	Quantity
X_1	Cattle heads	heads	12.00
X_2	Potatoes, own land	ha	1.25
X_3	Barley, own land	ha	0.00
X_4	Oats, own land	ha	0.00
X_5	Buckwheat, own land	ha	1.25
X_6	Grass-clover mixtures, own land	ha	2.50
X_7	Triticale, own land	ha	0.00
X_8	Potatoes, leased land	ha	0.75
X_9	Barley, leased land	ha	0.00
X_{10}	Oats, leased land	ha	0.00
X_{11}	Buckwheat, leased land	ha	1.75
X_{12}	Grass-clover mixtures, leased land	ha	2.30
X_{13}	Triticale, leased land	kg	0.00

Source: According to authors calculations.

The largest share in the structure of external variable costs is given to other costs, 30.08%. This is due to the level of detail in the model creation, highlighting only key elements (inputs), while others are shown as aggregate and expressed in value as the activity “other costs”. These include land rent, protective agents, veterinary services, cattle care and treatment costs, protective agents in crop production, and the costs of other materials and external services. Within the sum of external variable costs, the costs of livestock feed account for 27.89%. Clearly, these costs should be added to the costs of hay production and the fact that the grazing period lasts for seven months, indicating that the cost of livestock feed has a greater share in external variable costs, providing a realistic picture of cattle production in hilly and mountainous areas. After livestock feed costs, the large share has also the costs of seed potatoes (15.66%) and diesel (11.52%).

September is the month with the highest labor expenditure, as in addition to own capacities, 66 hours of paid (external) labor have to be hired. This is the month when potatoes are harvested, requiring the labor use in larger volume. After September, the other months with the high labor expenditure are August (313.80 hours) and June (309.80 hours). In these months, the first and second mowing and hay storing is done. April represents the so-called “spring labor peak”. In next table (Table 4.) is observed the structure of external variable costs occurred in milk production.

Table 4. Structure of external variable costs in primary milk production

Code	Element	Costs (RSD)	Share (%)
X_{15}	Purchased feed - barley grain	0.00	0.00
X_{16}	Purchased - oat grain	0.00	0.00
X_{17}	Purchased - triticale grain	263,520.00	11.79
X_{18}	Concentrate 18% protein	360,000.00	16.10
X_{19}	Seed potatoes	350,000.00	15.66
X_{20}	Feed barley seed	0.00	0.00
X_{21}	Oat seed	0.00	0.00
X_{22}	Buckwheat seed	72,000.00	3.22
X_{23}	Grass-clover mixtures seed	13,824.00	0.62
X_{24}	Triticale seed	0.00	0.00
X_{25}	NPK	139,230.00	6.23
X_{26}	Urea	79,440.00	3.55
X_{27}	KAN	11,800.00	0.53
X_{28}	Diesel	257,610.00	11.52
X_{29}	Other costs	672,400.00	30.08
X_{42}-X_{53}	External labor	15,840.00	0.71
Total		2,235,664.00	100.00

Source: According to authors calculations.

A quantitative analysis of the optimal solution can be conducted through a post-optimal analysis. This information is useful for the farmer for both, annual planning and long-term business orientation.

Raw milk, as the main product of cattle farming, has an average selling price of 28 RSD/l. Sensitivity analysis shows that a reduction in the price of milk by 3.94 RSD/l (14.01%) would affect a change in the optimal solution, resulting the decrease in the volume of cattle production. Increase in selling price of raw milk would not affect a change in the optimal solution because the maximum stable capacity has been fully utilized.

Post-optimal analysis, including the assessment of constraint utilization and the so-called “shadow prices” provides valuable information for the farmer. Each additional increase in stables’ capacity for one stall increases the net income by 14,578 RSD, but in this case, the increase can amount to only three stalls (3.67). Beyond that threshold, the second constraint becomes a real constraint.

Additional hectare of planted potatoes would contribute to increase in total net income by 355,400 RSD, but this increase can be achieved for a maximum of around half hectare (0.528 ha). For every additional hectare of planted buckwheat, the total net income of the farm would increase by 68,600 RSD, while by the starting parameters, the maximum increase can be 1.75 ha. Each additional hectare of arable land would increase net income by 20,000 RSD, what is equivalent to rental costs.

Conclusion

The solution of the established linear programming (LP) model for the optimization of primary milk production indicates the need to combine cattle farming with crop production. This approach includes not only the production of roughage on artificial meadows, but also entails the production of other crops for the market. Mentioned combination allows better utilization of farm production capacities.

The results derived from the model show that systematic analysis can encompass resources and production activities in primary milk production, providing a logical model with clearly defined system elements and their mutual interconnection. This creates the conditions for observing primary milk production as a system that can be modeled and subjected to agro-economic analysis using LP. Based on systematic analysis and developed logical model, there was defined mathematical model, considering the specificity of production conditions at the particular farm.

The special value of this research is in development and applying of optimization methods in primary production of milk in observed region. The goal of the model was to maximize the use of all available natural and production resources, thereby enabling the achievement of maximum economic effects. The next research steps could be based on the assessment of influence of certain factors in the development of dairy farming, i.e. in development of model that would optimize that production.

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