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Options on temporary water allocation rights and their pricing

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Abstract

This paper presents a methodology for defining the spot price of temporary water allocation rights for trading zones within the water market in the southern Murray–Darling Basin situated in Australia. The historical spot price is then used to calibrate a stochastic process depicting the dynamics of the water price, allowing the computation of prices of options on the underlying water price with the aim of producing reference prices to catalyse an options trading market. The most suitable stochastic model representing the water price dynamics is selected through comparing the option prices generated from four different models. Using the selected stochastic model, the most liquid trading zone in the Murray–Darling Basin water market (Zone 7) is used to demonstrate how the methodologies developed in the paper are used to calibrate the log-mean stochastic model representing the stochastic spot price dynamics and compute prices for call and put options on the underlying water spot prices. Sensitivities of the water options prices to market input data can be calculated from the formulae provided in the paper. The results presented in this work can serve as a reference tool by industry practitioners and the farming community in using options for effective risk management of water resources.

KEYWORDS

Murray–Darling Basin, option pricing, options on water, water rights

JEL CLASSIFICATION

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1 | THE CONTEXT

One of the effects of climate variability is an increase in the frequency and severity of droughts in countries such as Australia and the United States (Gutzler & Robbins, 2011). A crucial component for maintaining successful production of agricultural crops is the guaranteed supply of irrigation water. When droughts occur, many farmers look to buy irrigation water from water markets to supplement (reduced) rainfall. However, in times of drought, the spot and forward market for water can be very volatile and uncertain because different risk factors can create large impacts on the water price; for example, any unexpected rainfall would significantly reduce the demand for water while if the drought becomes protracted, already scarce water resources can be depleted quickly resulting in significant price jumps. Because planting crops requires long lead times, the uncertainty of potential large price jumps of irrigation water is further exacerbated.

A cost-efficient way for managing the risk of large changes in water prices is to use derivatives products of the water price. This is a well-established practice in the oil and grain markets where futures (forwards) and other derivatives (such as options) are used as standard risk management instruments in addition to the commodity itself. These additional instruments allow market participants to hedge their risk exposure to the underlying commodity price. Globally, water markets are currently in their infancy. Nevertheless, increasing demands for hedging instruments by farmers are leading to new products, such as the commencement of futures trading on the Chicago Mercantile Exchange (CME) in the United States (Chipman, 2020; Stafford, 2020), in addition to calls for a more versatile water market in the Murray–Darling Basin (Long & Jasper, 2019).

The southern Murray–Darling Basin water market is one of the more advanced markets globally; it was established in 1994 and has a governance structure, which regulates water usage and water quality. Currently, the Australian water market consists of a spot and forward market, but the forward market is not very liquid. A recent inquiry into the operations of the water market in the Murray–Darling Basin by the Australian Competition and Consumer Commission (ACCC) specifically refers to using futures and derivatives to hedge risk (ACCC, 2020) as a way to enhance operations and market efficiency. Recently, some brokers have begun to provide services to trade options on water without considering how to generate a fair option value (ACCC, 2020).

Globally, only a small number of water markets exist; Australia and the United States have well-established regional water spot markets in the southern Murray–Darling Basin and California, respectively. Indeed, recently in California, the CME started trading water futures (Chipman, 2020). Work focussing on modelling water prices has therefore concentrated on these markets. However, the water price modelling effort has been mainly on using hydrological modelling to establish the price of water. To determine the fair price of options on the underlying water price, the stochastic dynamics of the traded market price of water must also be taken into account.

One of the first papers to consider options in a water setting was Michelsen and Young (1993), who examined water supply options, which have a multi-exercise feature. While the authors investigated the benefits of purchasing such an water option, they did not value the option by using a model to depict the underlying water price. Other works such as Villinski (2004); Hansen et al. (2006) in a US setting and Schreider (2009); Cui and Schrieder (2009); Fleming et al. (2013); Page and Hafi (2007); Williamson et al. (2008); Leroux and Crase (2010); and Heaney and Hafi (2005) in an Australian setting have used conventional Black–Scholes (BS)-like models to value multi-exercise options.

Villinski (2004) investigated a multi-exercise option priced using stochastic dynamic programming and adopted two models for the underlying water price: a geometric Brownian motion model and a mean reversion model. The authors calibrated their models to 18 months of water prices from Texas (312 trades, 78 weeks, 55 weeks with a price, 23 without).

Williamson et al. (2008) and Fleming et al. (2013) used the BS and Skewness-and-Kurtosis-amended BS models to value options in a similar setting to Michelsen and Young (1993) and found that these models were not suitable when the volatility of the underlying water price was high.

Cui and Schrieder (2009) and Schreider (2009) used the BS model with jumps to model underlying spot price and derived analytic formulae to price a standard European option on water rights in an Australian context. Plausible water dynamics and option prices were observed, but the model was not calibrated to traded water prices.

Recently, He et al. (2020) have examined pricing European call and put options for use in the Shaanxi Province in Northwest China using a BS-like model for the underlying where the Wiener process is replaced by a Liu process (Liu, 2009) to account for the uncertainty due to regulatory effects.

In this paper, we consider several different stochastic processes to model the underlying temporary water allocation spot price and assess the value of European option prices. Additionally, we present an operational stochastic model for pricing European options on temporary water allocation rights calibrated to the spot market in the southern Murray–Darling Basin. The option prices generated from this model can be used as initial indicative fair prices that market participants can rely on in order for an options market to catalyse.

The remainder of the paper is structured as follows. We present methodology for determining a consistent spot price for water rights in Section 3, and then we present the stochastic model to represent the spot price dynamics along with the calibration procedure and analytic formulae for European options in Section 4. Numerical results from the model calibrated to actual spot price data from the Murray–Darling Basin are presented in Section 5, while Section 6 summarises the results and provides the conclusions.

2 | INTRODUCTION TO THE MURRAY–DARLING BASIN WATER MARKET

The Murray–Darling Basin in Australia is a major interconnected network of rivers extending across four states from Southern Queensland, through New South Wales, Victoria and South Australia. These rivers supply irrigation water to a major proportion of agriculturally productive land, in addition to ground water and rainfall. The work presented in this paper focusses on the Southern Basin, encompassing the Murray, Murrumbidgee, Goulburn, Campaspe, Loddon and Lower Darling rivers across three states (NSW, VIC and SA; ACCC, 2020).

To ensure a fair and equitable distribution of water across the Basin, temporary water access rights are granted through a system regulated by a government agency: the Murray–Darling Basin Authority. These rights give the holder the entitlement to access water from the Murray–Darling Basin to irrigate their property. The holder can either use the water allocation themselves or choose to trade the temporary access rights with other parties; the water access rights in the Murray–Darling Basin are therefore not tied to a particular tract of land.

Each year at the beginning, and at various stages within the irrigation season, state governments determine the total volume of water available for allocation to individual water access rights, dependent on rainfall and the flow of water through the river systems among other considerations. It is possible that while the individual holder is entitled to access water, the allocation given to the holder can even be 0 megalitres (0 ML). For typical allocations, temporary water allocation rights holders can find that their allocation is depleted before the end of the year, in which case they can buy access to further allocation on the spot market. Alternatively, if the holder has excess unused water, they can sell access to this water on the spot market. These trades are complicated by geography and hydrology, and the Southern Basin is split into a number of trading zones, each with their own spot price. To avoid these and other complications, this work will only consider option contracts localised to each trading zone with the assumption that each trading zone will have its own options market. Figure 1 presents the geography of the different trading zones in the Southern Basin. The estimates of the value of the water trading market vary from approximately $1.5 billion per annum on average (ACCC, 2020) to an annual turnover of $6 billion in 2020/2021 Commonwealth of Australia (Bureau of Meteorology, 2021).

3 | METHODOLOGY FOR COMPUTING THE WATER SPOT PRICE

For clarity and ease of reading, we will now refer to the temporary water allocation rights price as the water rights price or simply the water price. The historical traded water prices for Zone 7 in the Southern Basin, one of the most liquid trading zones, are presented in Figure 2. While a clear price signal is present, there can be many trades at very different prices executed on the same trading day, including some for 0.00$, and some at thousands of dollars per megalitre (ML). These 'free' trades can sometimes be attributed to family transfers (Sanders et al., 2019), or related party transactions. Furthermore, each trade is made for a fixed but different water volume; these volumes can range from 1 to hundreds of megalitres. Another complication is that the daily record of these traded prices only becomes available to the public after the fact—there are no real-time trading price data.

In a well-functioning market, a necessary condition for determining the fair value of a derivative of an asset is for the underlying asset to have a transparent and verifiable price signal that all parties can agree upon. The trading data exhibited in Figure 2 do not satisfy this condition. However, a single value spot price derived from the daily traded price records can be a useful barometer of the water price level in the market. There are several standard statistical methods that can be used to process the recorded prices and define a single spot

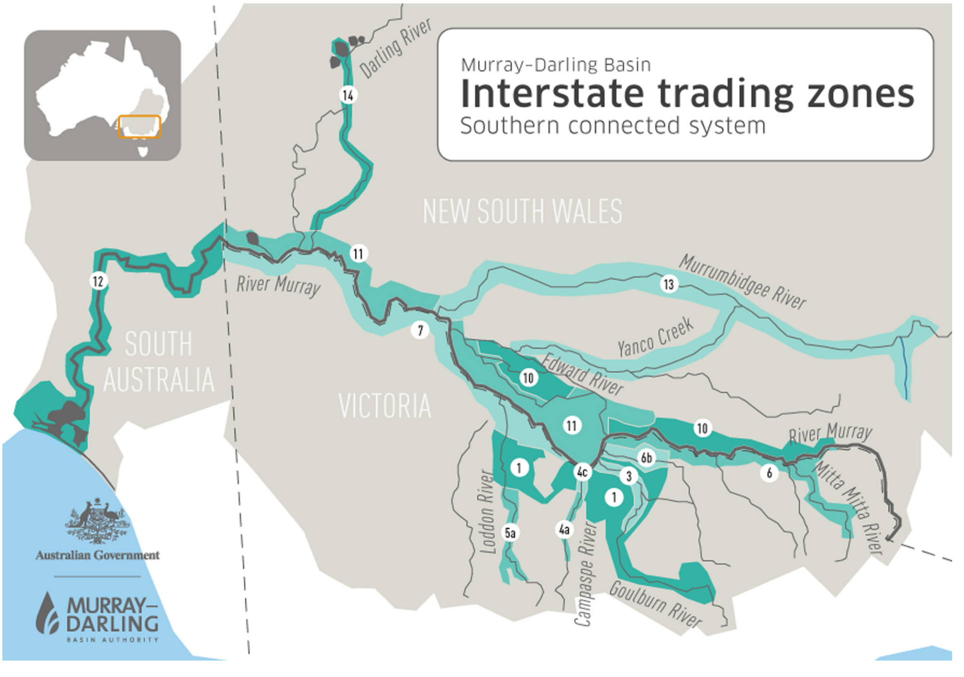

FIGURE 1 Graphic illustrating the trading zones within the Southern Murray–Darling Basin (MDBA, 2017). [Colour figure can be viewed at wileyonlinelibrary.com]

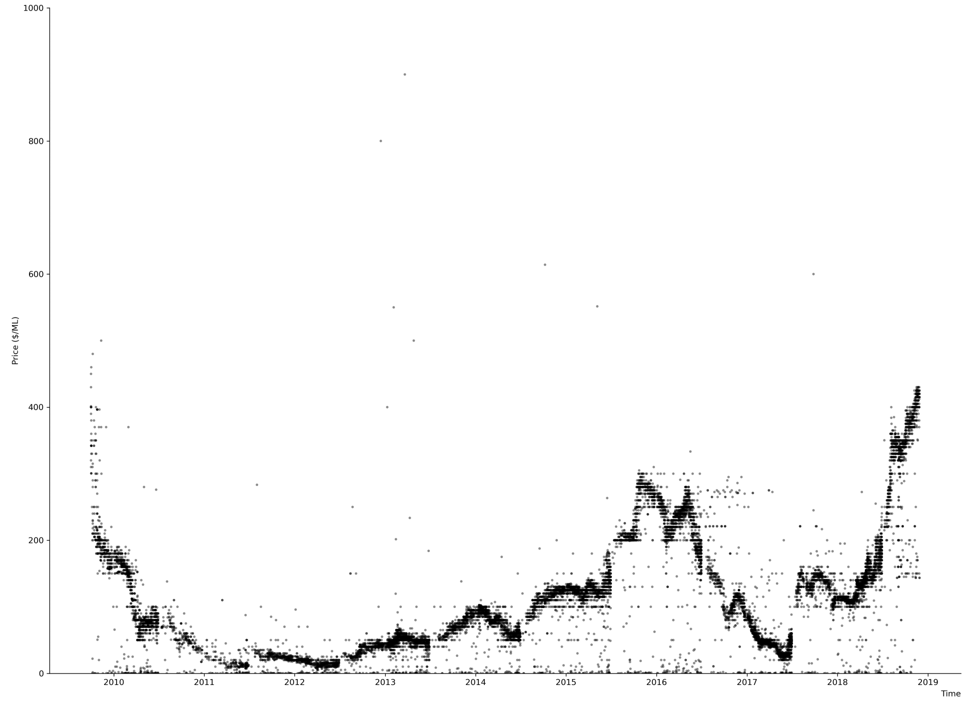

FIGURE 2 Raw traded water rights prices from Zone 7 over the time period of 2009 to 2019. There can be a large range of traded prices on a given trading day, including trades with prices close to $0.00.

price for water rights. Common methods include taking the mean or median value over a time window after removing outliers defined as being outside a threshold such as two standard deviations from the mean, or utilising a smoothing spline such as in the generalised additive model (GAM) methodology (Sanders et al., 2019). However, these methods neglect the volume of water in transactions, focussing only on the price value. This leaves open the potential for price manipulation, for example, through the execution of trades with very small volume but of very high prices. Additionally, when creating a theoretical spot price from transacted water prices, the computational procedure to generate the spot price should be easily understood and can be readily reproduced by market participants without confusion. For the transacted water prices, some GAMs can produce smoother features, but such models can be opaque and difficult to communicate to non-technical audiences. In contrast, a discrete volume-weighted pricing methodology produces simple and transparent spot prices that can be readily comprehended by non-technical people, which is essential for creating trust in a market environment.

In this work, we define the spot price as a volume-weighted average of traded prices over the preceding five days where trades were executed. This makes the spot price less prone to price manipulation through volume. In analysing the recorded price data, we have found that if the time window is set to 2 or 3 days, the computed spot prices are still noisy and can oscillate violently from one day to the next. A weekly averaging of the transacted prices removes the highly noisy and fluctuating daily prices, which are regarded as unrealistic and artificial. Using a weekly time window for averaging finds a balance between removing unnecessary noise due to seasonality patterns within each week but still capturing price variations reflecting the balance between demand and supply due to weather or seasonal events over a longer timescale. To ensure further robustness of the computed spot price, we remove outliers defined as the top and bottom 20% of trades over that time period when calculating for the spot price.

It should be noted that the computed spot price can have different values if an alternative smoothing method is used instead, although we do not expect the difference to be very large. The computed spot price is an artificial single parameter calculated from a large number of transacted prices and is not a record of real traded transactions. Therefore, there is no ‘correct’ value for the computed spot price as long as the spot price is accepted as a ‘reasonably good’ indicator of the price values transacted by the majority of market participants.

The process to compute the spot price for water rights can be summarised as follows:

- find the preceding five days over which trades occurred;
- collate all prices and volumes of trades executed in the five-day time period;
- sort trades by price and remove trades in the top and bottom 20% by price; and
- compute the volume-weighted average price of the remaining trades to define the spot price

Defining the total number of trades executed over five days after the removal of outliers as N, the final step in defining the spot price P is defined as.

$$P = \frac{\sum_{i=1}^{N} V_i P_i}{\sum_{i=1}^{N} V_i} \tag{1}$$

Figure 3 displays the spot price calculated in this manner from the raw recorded price data presented in Figure 2. The definition of spot price used in this work does smooth the raw traded data but still captures the inherent price dynamics that are clearly visible in the raw data over time.

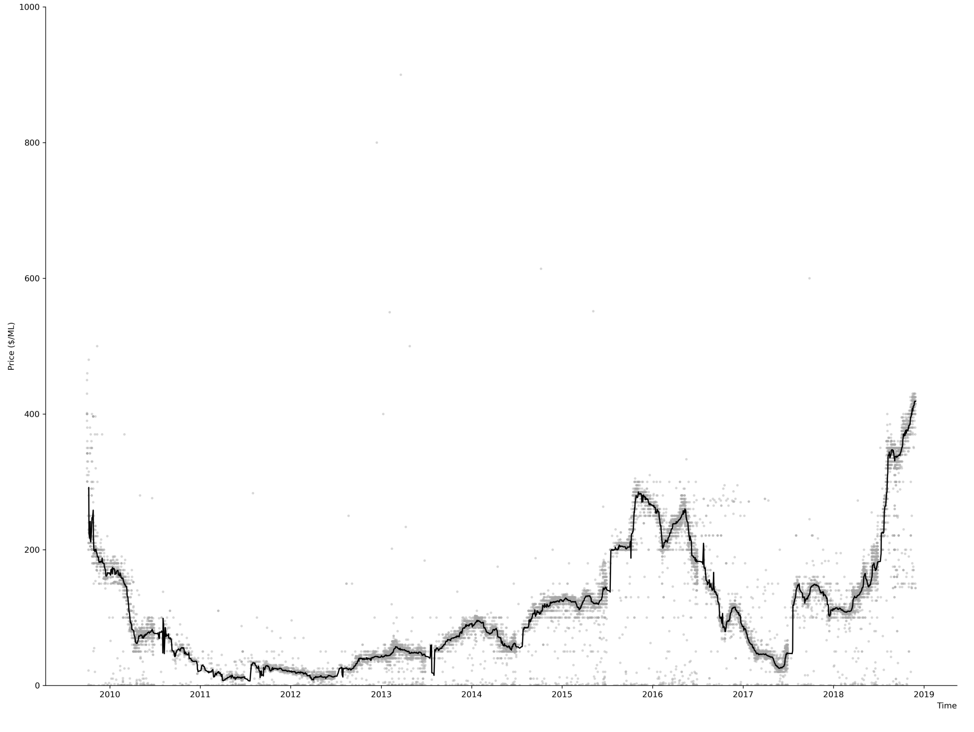

FIGURE 3 Final spot price overlayed on the raw traded water rights prices from Zone 7 over the time period of 2009 to 2019.

4 | STOCHASTIC MODELS OF WATER SPOT PRICE FOR OPTION PRICING

Options are contracts that allow the purchase or sale of an underlying asset (in this case the temporary water allocation right) at a fixed price (termed the strike price) at specified future dates. In this paper, we only consider European options where the transaction (a purchase or sale) can only occur once at the time of expiry of the option. Initially, as simple risk management instruments, only the basic or 'vanilla' options (call options and put options) are to be used to catalyse an options trading market. European call options give the option purchaser the right but not the obligation to buy the underlying asset at the strike price at expiry, while European put options give the option purchaser the right but not the obligation to sell the asset at the strike price at expiry. More information on options can be found in Wilmott (2006).

Option contracts provide the purchaser of the contracts with a possible financial benefit in the future and without any risk of possible loss. Therefore, the purchaser of the option contract needs to pay the option price, often called the option premium, to acquire the option contract. The option price is determined by how the underlying water price is expected to move from the inception time of the option contract until its expiry date. The stochastic movement of the underlying water price is represented by a pricing model that can best capture the dynamics of the stochastic process.

For each trading day, the transacted prices on temporary water allocation rights can range from \$0 to thousands of dollars per ML even when the averaged spot price is around \$100 per ML. The large price discrepancies in a single day can be attributed to transactions between related parties known to each other. Consequently, the large variation of water prices within a trading day should not be viewed as representing the volatility of the underlying water prices. In fact, even the volume-averaged water prices transacted within a single day often exhibit a high degree of fluctuation because the number of transactions can swing significantly depending on the day of the week. By using weekly volume-weighted averaged prices, we can eliminate the impact of price variation caused by intra-weekly patterns.

For the computed water prices, the uncertain seasonal variation should be reflected by a benchmark volatility parameter that a water options market can rely on to value the uncertainty of future water prices. The strong seasonality evident in the water allocation market should be included in the pricing model through a term structure of mean reversion levels (and potentially volatilities), that is, a discretely varying piece-wise constant mean reversion level aligned with the future demand–supply pattern nominally captured by futures or forward prices traded in the market. However, in order to calibrate such a term structure model, the water futures (forwards) market would need to be much more liquid than it is currently in the southern Murray–Darling Basin water market. Without a liquid futures market to provide a clear signal on the expected future price level or seasonality pattern, the best way to catalyse an options market is to begin with a constant mean reversion level, with the expectation that the seasonal market dynamics are reflected in traded option prices after the market has been operational for a sufficient length of time.

4.1 | Model selection

We present our investigation of a number of stochastic models that can potentially capture the main dynamics of the traded water spot prices in the southern Murray–Darling Basin water market. For modelling the water spot prices, mathematical models developed for commodities such as oil (Aba Oud, 2014; Aba Oud & Goard, 2015) should be analysed. We trialled a number of different models, including the BS model, Merton's Jump Diffusion model (Merton, 1976), which is the BS model with jumps, and two mean-reverting models: a mean-reverting log-normal model and an inverse gamma (IvG) model.

Table 1 summarises the stochastic models that have been implemented for modelling the historical water price data.

The parameters for each model are determined through a linear regression of the underlying historical spot price. For the case of the log-mean model given by Equation 2,

$$dS_t = \kappa(\theta - \ln S_t)S_t dt + \sigma S_t dB_t, \tag{2}$$

we can define $X_t = \ln S_t$ as the log-spot price and by applying Ito's Lemma we find the following dynamics for X_t:

$$dX_t = \kappa\left(\theta - \frac{\sigma^2}{2\kappa} - X_t\right)dt + \sigma dB_t. \tag{3}$$

We can then approximate the log-spot process X_t as a linear function.

$$X_t = \beta_0 + \beta_1 X_{t-\Delta t} + \varepsilon_t. \tag{4}$$

By comparing Equations (3) and (4), the parameters of the model can be determined from the regression coefficients such that.

$$\sigma = \frac{\text{std}(\varepsilon_t)}{\sqrt{\Delta t}}, \tag{5}$$

$$k = -\frac{\beta_1}{\Delta t}, \tag{6}$$

and

$$\theta = \frac{\beta_0}{k\Delta t} + \frac{\sigma^2}{2k}. \tag{7}$$

This process is similarly performed for the other models to obtain calibrated model parameters. We then use the calibrated parameters of each model to value a range of options on the water spot price, each with a one-year expiry. The pricing results are shown in Table 2. The prices resulting from the calibrated Jump Diffusion model are little different from the BS values indicating that BS dynamics dominates. We also note that the BS prices are significantly higher than the two mean-reverting processes for a wide range of strike prices. Through consultations with practitioners, prices generated from the BS model were deemed too high as a reference price for the initiation of an options trading market.

TABLE 1 List of stochastic models implemented for water spot price.

Name	Equation
Black–Scholes	$dS_t = rS_t dt + \sigma S_t dB_t$
Merton's Jump Diffusion	$dS_t = (\alpha - \lambda k)S_t dt + \sigma S_t dB_t + (y_t - 1)S_t dN_t$
Inverse gamma	$dS_t = \kappa(\theta - S_t)S_t dt + \sigma S_t dB_t$
Log-mean (LM)	$dS_t = \kappa(\theta - \ln S_t)S_t dt + \sigma S_t dB_t$

TABLE 2 Pricing results for call options.

	Moneyness K / S_0						
Model	**0.5**	**0.7**	**0.9**	**1**	**1.1**	**1.3**	**1.5**
BS	270.64	234.80	206.40	194.30	183.33	164.25	148.21
MJD	269.11	233.37	205.17	193.21	182.40	163.64	147.88
IvG	258.54	220.15	190.35	177.86	166.66	147.39	131.41
LM	243.08	195.27	158.29	143.04	129.56	107.04	89.21

Furthermore, commodity prices are typically assumed to follow mean-reverting processes (Andersson, 2007) reinforcing the selection of the two mean-reverting models that we have implemented. Both the IvG and log-mean (LM) models produce sensible prices; however, the log-mean model admits a closed-form solution for certain option classes and is chosen over the IvG model for this reason.

4.2 | Closed-form valuation of water options

While Monte Carlo methods commonly used for option pricing in the financial markets are generic and straightforward to implement, the benefit of selecting the LM model is that it allows the computation of the European 'plain vanilla' call and put options from closed-form formulae leading to efficient and accurate calculations. The 'plain vanilla' options are characterised by an expiry of T years and a strike price of K. The derivation of the closed-form formulae is straightforward, with the resulting value of a European call option C_0 given by

$$C_0 = S_0\Phi(d) - e^{-rT}K\Phi(d - w), \tag{8}$$

where

$$d = \frac{rT + \ln\frac{S_0}{K} + \frac{1}{2}w^2}{w}, \tag{9}$$

$$w = \sqrt{\frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa T}\right)}, \tag{10}$$

r is an interest rate and $\Phi(d)$ is the standard normal cumulative distribution function:

$$\Phi(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-z^2/2}dz. \tag{11}$$

Similarly, the value of the European put option P_0 with expiry T and strike price K can be obtained through using the formula below:

$$P_0 = e^{-rT}K\Phi(-d + w) - S_0\Phi(-d). \tag{12}$$

Derivatives of these formulae with respect to the input parameters (e.g., the initial spot S_0 and the volatility σ), used to compute sensitivities of the water options prices to input market data, are provided in the Appendix S1 of this paper.

TABLE 3 Calibrated stochastic parameters for Zone 7 data from 2009 to 2018.

Parameter	Value
Spot S_0	418.578
Mean Reversion Speed κ	0.827
Mean Reversion Level θ	5.301
Volatility σ	1.198

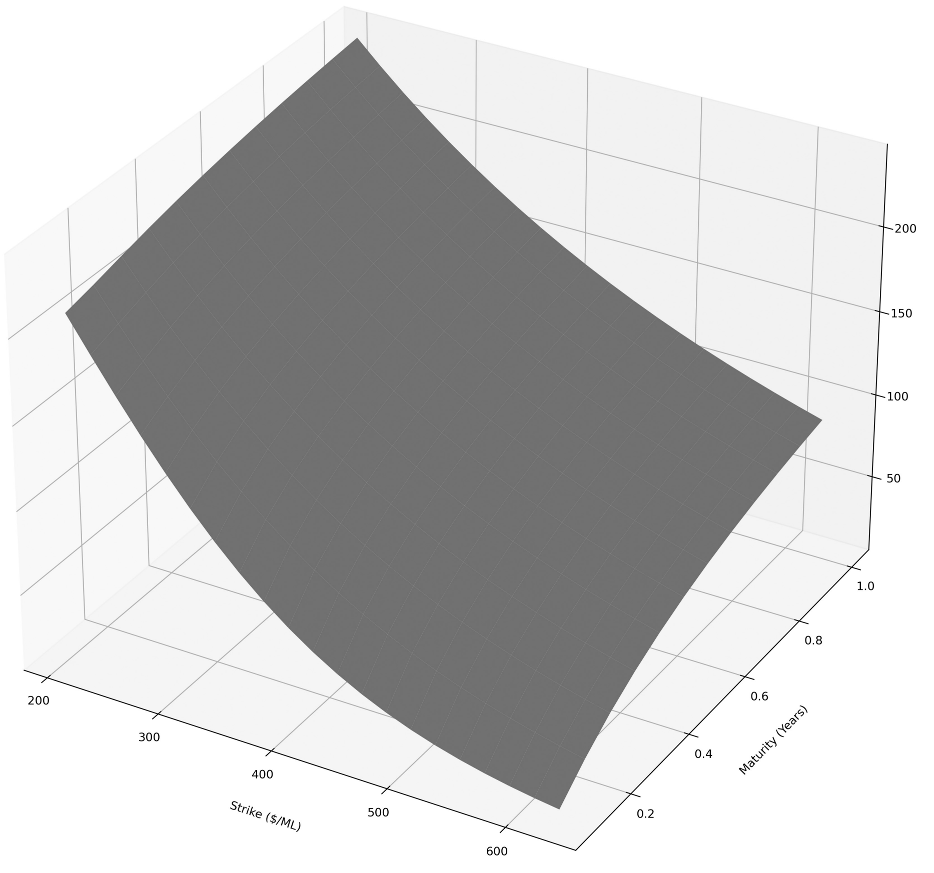

FIGURE 4 Sensitivity analysis of call options prices for different strikes.

5 | PRICING RESULTS OF WATER OPTIONS

For the example shown in this paper, we use the computed spot prices of the most active trading zone (Zone 7 Murray) to calibrate the LM model. Using trading data from 2009 to 2018, we obtain the calibrated parameters displayed in Table 3. The volatility found for this data set is 120%, with a mean level of 5.301, or $200/ ML.

From the calibrated parameters provided in Table 3, we can evaluate call and put options on the underlying water spot price for any strike value. Figure 4 shows prices for call options with different strikes at different maturities (in years), while Figure 5 shows the prices of put options for the same range of strikes and maturities. The call and put prices as shown in both figures exhibit the typical call/put option price trends for different strikes and maturities, similar to standard financial 'plain vanilla' call/put options.

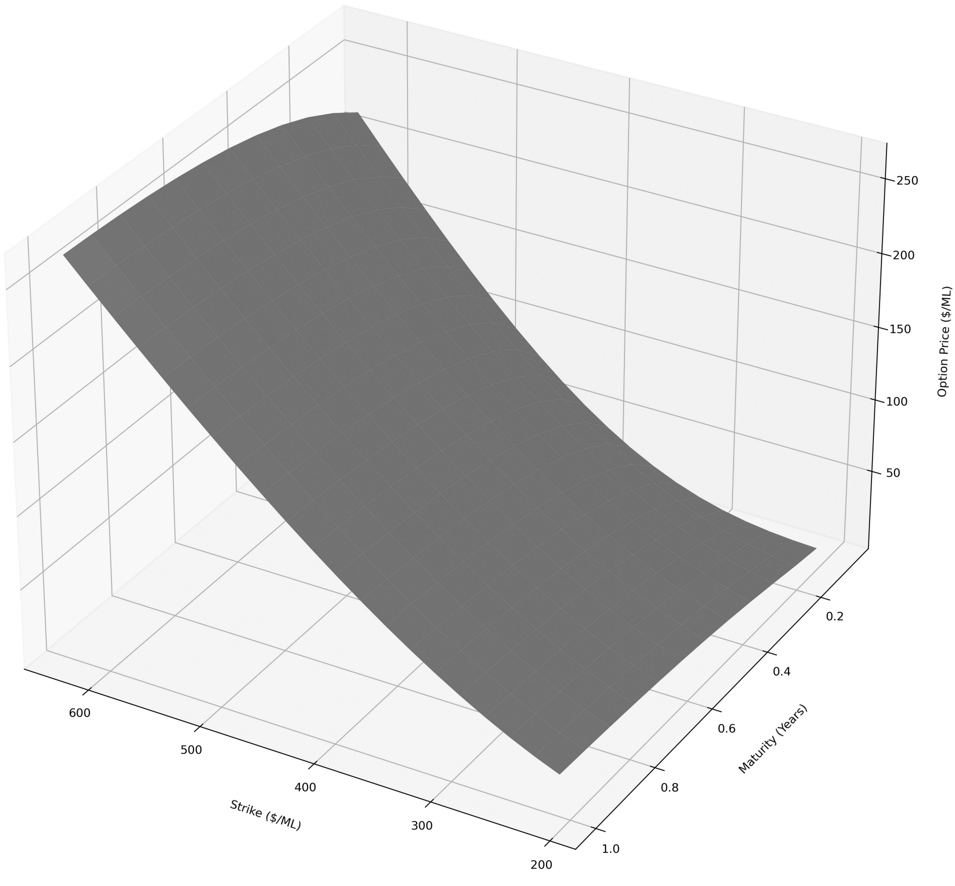

FIGURE 5 Sensitivity analysis of put options prices for different strikes.

For a spot price of \$418.58/ML, a call option with a strike price equal to this spot price and a 3-month expiry has a value of \$91.50. The buyer of such a call option pays around 22% of the current spot price to achieve the flexibility of buying the temporary water allocation at the current price but in 3 months' time if the price at that time is higher. Alternatively, if the price in 3 months' time is lower, the buyer of this call option can simply purchase the water allocation from the open market where the price is lower than the strike price. Essentially, the call option gives the buyer an extra 3 months to lock in the current spot price but without the obligation to purchase the water.

Such a call option provides a typical farmer with the tool to secure the cost of water anticipated for irrigation in 3 months' time, but if the water price drops in the meantime, the farmer can still benefit from the reduced price. Additionally, the farmer has gained the flexibility of buying less water if more rain occurs or the crop requires less water due to other factors.

Conversely, the seller of the call water option can be the farm land owner with a water allocation right and is happy to sell at the current spot price but is happy to accept an extra 22% of the price now for the possibility of selling the water at a lower price on the spot market if the future water spot price in 3 months' time is below today's current spot price.

For the buyer of a put option with strike value being the same as the spot price (the case of Moneyness $K/S_0 = 1$ in Table 5), the buyer might be the farm land owner of water allocation rights and the put option provides the buyer with a tool to lock in the current spot price for

TABLE 4 Pricing surface for call options.

	Moneyness K / S_0						
Maturity (months)	**0.5**	**0.7**	**0.9**	**1**	**1.1**	**1.3**	**1.5**
3	218.94	156.48	109.67	91.50	76.28	53.05	37.04
6	229.98	175.86	134.73	118.18	103.85	80.68	63.20
9	237.65	187.55	149.08	133.35	119.55	96.75	78.98
12	243.08	195.27	158.29	143.04	129.56	107.04	89.21

TABLE 5 Pricing surface for put options.

	Moneyness K / S_0						
Maturity (months)	**0.5**	**0.7**	**0.9**	**1**	**1.1**	**1.3**	**1.5**
3	7.05	27.27	63.13	86.30	112.42	171.86	238.53
6	15.52	43.06	83.57	107.85	134.34	192.82	256.99
9	20.66	51.20	93.36	117.94	144.46	202.29	265.16
12	23.58	55.41	98.06	122.62	148.96	206.08	267.87

sale in 3 months' time. For the cost of $86.30 or around 21% of the spot price of today, the buyer of the put option has guaranteed a minimum sale price at the current spot price, but also can capture the benefit if the spot price moves higher in 3 months' time.

Conversely, for a seller of this put option, the seller can be a farmer who needs to purchase water for cropping in 3 months' time, and is happy to receive an upfront $86.30 (21% of the spot price) in cash now but is willing to accept possible negative outcomes if the spot price in 3 months' time becomes: (1) lower by more than 21% or (2) higher by more than 21% from the contracted strike price. As long as the spot price does not change by more than 21% within the next 3 months, the farmer would benefit from selling such a put option.

We note that the option prices presented in Tables 4 and 5 obey call-put parity.

As the presented possible use of such water call and put options demonstrates, water call and put options can be utilised by farmers as flexible instruments to manage their specific needs for water resources when the future water spot price is uncertain. Climate change may introduce even more uncertainty into the availability of water resources as reflected by the high volatility in water spot price shown in Figure 3. With highly volatile water prices, water derivatives such as the call and put options presented here could increasingly be adopted as effective products for farmers to manage their exposure to climate risk.

6 | CONCLUSION

In this paper, a methodology for defining and computing the spot prices of temporary water allocation rights for each trading zone within the complex interconnected network of the southern Murray–Darling Basin water market is provided. Using this methodology, the calculated historical spot price is then used to calibrate the stochastic model representing the dynamics of the water price process. For the southern Murray–Darling Basin water market, four types of stochastic models were initially chosen for the potential of capturing the underlying water spot price dynamics. Through analysis of option prices generated from each of the four selected models for a range of strike prices, the LM stochastic model was identified as the most suitable for representing the dynamics of the spot price process.

The LM model should be independently calibrated to the historical spot prices of each trading zone of the southern Murray–Darling Basin water market, and, for the calibrated log-mean model, we have implemented and presented the closed-form formulae for calculating 'plain vanilla' call and put options on the underlying water spot price of each trading zone.

The most active trading zone in the southern Murray–Darling Basin water market is used to demonstrate how the methodologies developed in the paper are used to: (1) calculate and create the spot prices for traded water, (2) calibrate the LM stochastic model representing the calculated spot price dynamics and (3) compute the prices for call and put options on the underlying water spot price. Lastly, sensitivities of the water options prices to market input data can be calculated from the formulae provided in the Appendix S1 of this paper. It should be noted that though the numerical results presented in this paper are for the most active trading zone, the methodologies presented here can be applied to other trading zones without requiring modification. Because this paper is focussed on the implemented methodologies, numerical results for other trading zones are not included.

In the paper, the LM options pricing model is demonstrated to produce call and put option prices that have been viewed by industry participants as within acceptable and reasonable price levels for users (e.g., farmers) in managing water price risk. We have also provided some examples of using water options for a farmer to manage their requirement for water resources and accept possible losses due to specific movements in the market water price.

In summary, in this paper, we have presented (1) a methodology for generating the daily spot price of temporary water allocation rights from recorded daily prices that can range from $0 to thousands of dollars per ML, (2) a LM stochastic model representing the dynamics of the generated water spot prices, (3) the establishment of a methodology for valuing water options in the Australian Murray–Darling water market and (4) examples of real-time generation of call and put water option prices from live-streamed spot prices. The paper can serve as a reference for industry practitioners and the farming community in adopting water derivatives as a new product for effective risk management of water resources.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from RuralCo, a subsidiary of Nutrien. Restrictions apply to the availability of these data, which were used under licence for this study. Data are available from the authors with the permission of RuralCo.

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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