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ORIGINAL ARTICLE

Size matters: Optimal management of dynamic systems with varying size

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Abstract

Many natural and economic systems are managed to deliver the highest benefits to society but are subject to regime shifts. We specifically consider the variability of the size of the system itself as a key driver of a regime shift. We address the question of how the optimal management of these systems should vary with its size. Put simply, certain management options might work when the system is of a given size, while others might be preferred when the system has grown or shrunk. In this paper, we develop a model that allows us to analyse the effect of the size of the system on its optimal management. We apply this model to a case of water pollution in a reservoir/lake that varies in size over time: sometimes the lake is deep and sometimes it is shallow. Numerical simulations were conducted to compare optimal management of the reservoir with and without explicitly modelling its size variation. The findings show that the overall social costs of optimally managing pollution are significantly smaller when the variability in size is taken into account. This is due to differences in the timing and magnitude of the optimal control. The key implication is that the variability of the size of a system should be explicitly considered in this type of management problems.

KEYWORDS

numerical simulations, pollution control, size variability, system dynamics

JEL CLASSIFICATION

C61, C88, Q52, Q53, Q57, Q58

1 | INTRODUCTION

This paper deals with a class of management problems for natural or economic systems. The systems of interest provide benefits to society, which are dependent on the internal system dynamics in a sense that the system can be in a 'good' or a 'bad' state, and the benefits are much greater when the system is in a 'good' state as opposed to it being in a 'bad' state. The internal system dynamics depend on a number of factors, but the focus of this paper is on how those dynamics depend on the variation of the size, or the magnitude, of the system itself. We show that whether the system has a 'large' or 'small' size at any given time affects profoundly the state of the system and consequently the benefits that society derives from it.

The systems that we have in mind are not only many natural systems (e.g. lakes, forests, grasslands and fish populations) but also various social systems such as cities, countries, and the financial and economic systems within those countries. Of crucial significance is that the systems that we are interested in are 'managed', in a sense that society has appointed an agency to manage these systems. The remit of the agency is to try to keep the systems in as good a state as possible, so that society can derive maximum benefits. To pursue this objective, the agency undertakes management practices of various kinds. For example, in the case of lake water pollution – as in the empirical case study to be discussed later – those management practices involve various pollution abatement measures. Another example is the case of a financial system, where regulatory agencies aim to maintain the good state of the system by imposing various regulations on the agents that compose the system (e.g. financial institutions and individual borrowers). The main objective of this paper was to show that a management strategy that might be 'optimal' for a system of a given size may be significantly suboptimal when the size of the system changes. Consequently, treating a system that does change in size as if it did not entails unnecessarily high costs to society.

One strand of the literature relevant to the problem at hand has focussed on the economics of water pollution. This work has been epitomised by 'the shallow lake model' that is widely used to analyse the dynamics of nutrient accumulation in lakes, especially in shallow lakes (Brock & Starrett, 2003; Carpenter et al., 1999; Carpenter & Cottingham, 1997; Carpenter & Lathrop, 2008; Dechert & O'Donnell, 2006; Grass et al., 2015, 2017; Grune et al., 2005; Kossioris et al., 2008; Ludwig et al., 2003; Maler et al., 2003; Scheffer et al., 1993; Wagener, 2003). The popularity of this model is due to its usability as a general approach to analyse problems faced by many ecological-economic systems with similar characteristics. However, the focus on shallow lakes limits its practical applicability to many situations of water reservoir nutrient pollution because many natural or man-made reservoirs do not conform well to the characteristics of shallow lakes. In particular, many man-made reservoirs in lower latitudes on the Northern Hemisphere and throughout the Southern Hemisphere are designed as deep lakes, but due to natural variation in the hydrological cycle (e.g. seasonal rainfall) fluctuate between being deep lakes and being very shallow. The implications of such variation in the size of a lake for the dynamics of its system in relation to nutrient pollution and its optimal management have not been studied explicitly before.

Another strand of the literature is related to the size and regulation of the financial sector in an economy. Santomero and Seater (2000) determine an optimal size of the financial sector based on a general equilibrium model. Seater (2001) investigates the optimal regulation of the financial sector. However, neither of these papers investigates how the optimal regulation of the financial sector may be related to the size of the sector. A study by Bossone and Lee (2004) considers the effect of the size of a financial system on the efficiency of financial intermediaries but does not consider the effect on the choice of control instruments or their intensity.

Methodologically, the present paper relates to the literature exploring regime shifts in dynamic systems. Extant literature in this area has analysed regime shifts in groundwater resources (de Frutos Cachorro et al., 2019; Esteban et al., 2021), in fisheries (Costello et al., 2019) and mineral resources (Nkuiya, 2020). These follow the rich tradition of research on regime shifts in environmental and resource economics (Polasky et al., 2011). The present paper contributes to this literature by explicitly showing the difference between the costs of managing an environmental problem in a system that experiences a regime shift but is managed as if it were not and the costs of managing the problem in a system that experiences a regime shift and it is managed with full consideration of the shift. The differences in the overall social costs that we find are striking, which implies that ignoring regime shifts in choosing management actions can be very costly.

The present paper deals with a specific driver of a regime shift: the magnitude, or the size of the managed system itself and how it varies over time. In the case of a water reservoir, on which we focus in the empirical study, we consider the volume of water stored in it. While the reservoir has a maximum capacity for water storage, the actual amount of water stored in it at any given time can range widely. Consequently, at some time the freshwater system in the reservoir might be quite large (deep lake) or quite small (shallow lake). Explicitly accounting for the variable size of the system adds important insights about its optimal management.

The findings of the paper have significance at conceptual and applied levels. At a conceptual level, the findings highlight the importance of considering the size of the system and the variation in size for the overall system dynamics and its management. This has a general significance, as examples of managed systems with varying sizes can be found in ecology, demographics, economics and finance. At an applied level, the significance is in providing guidelines for the optimal management of systems whose size varies substantially over time. Specifically, the empirical application developed here is on water quality management. The economic implications of improved management can be considerable in areas like the Mediterranean (Naselli-Flores (2003)), or in Australia, where due to seasonal or other regular or irregular weather patterns (e.g. El Nina and La Nina cycles), the volume of water in reservoirs can change from very low to very high and back to very low in a short period of time.

The paper proceeds by outlining the modelling framework in Section 2, followed by Section 3 where the empirical case study is described, and data used are presented. In Section 4, we discuss the methods of numerical simulation used in the empirical analysis. In Section 5, we present the findings, and in Section 6, we draw conclusions.

2 | MODEL OF A LAKE SUBJECT TO POLLUTION AND VARYING SIZE

Here, we provide a sketch of the conceptual model that has been used to derive the rules that are implemented in the numerical simulation study to simulate the regime of the system in each state, and to simulate the regime shift from one state to another. The full model is outlined in Appendix S1. The model is from a perspective of a benevolent social planner that has an objective to maximise the benefits that society derives from a given system, such as benefits derived from a natural or man-made lake as a source of drinking water. The lake is threatened by pollution, for example phosphorus pollution. As pollution increases, water quality is reduced, leading to the reduction in benefits associated with drinking water supply. This reduction in benefits represents the damage costs from pollution. The damage costs are known and can be represented by the following quadratic damage cost function (de Zeeuw & Zemel, 2011; Dockner & Long, 1993):

$$D(P(u(t),t)) = \frac{h}{2}\left(P(u(t),t) - \overline{P}\right)^2 \tag{1}$$

where $P(u(t),t)$ is the stock of pollution in the system; $\overline{P}$ is a threshold point for the pollution in the system; and h is a parameter representing the marginal damage cost defined as: $h\left\{\begin{matrix} =0 \; if \; P(u(t),t) < \overline{P} \\ >0 \; if \; P(u(t),t) \geq \overline{P} \end{matrix}\right\}$ as in Farzin (1996). This representation implies that when the quantity of pollution in the system is less than the threshold, there are no damage costs and that the damage costs are positive when the pollution is at or above the threshold. $u(t)$ denotes the level of pollution abatement undertaken by the management agency. The more the abatement, the less the accumulation of pollution and vice versa. The benefit of abatement is to reduce the damage costs. The implementation of abatement is costly. These costs are represented by an abatement cost function denoted by $C(u(t)) = u(t)^c$, where the parameter c is modelled as an exponent to reflect that marginal abatement costs are increasing at an increasing rate.

The objective of the management agency is to minimise the sum of the damage and abatement costs (Beavis & Dobbs, 1986):

$$\min_{u(t)} \int_0^\infty e^{-\rho t} \left\{ \frac{h}{2} \left(P(u(t),t) - \overline{P} \right)^2 + u(t)^c \right\} dt \tag{2}$$

where ρ denotes the discount rate. This objective function is subject to the state equation of pollution dynamics (modified from Carpenter et al. (1999)):

$$\dot{P}(t) = k\left[I_w(t)\right]^n (a - u(t)) - \varphi(W(t), P(u(t),t)) + \psi(W(t)) \frac{P(u(t),t)^q}{m^q + P(u(t),t)^q} \tag{3}$$

where the first term on the right-hand side is the external pollution load (defined in Equation A7 of the Appendix S1); the second term is the natural loss rate of pollution from the system; and the third term is the internal recycling rate of pollution in the system (defined in Equations A3 and A5 in the Appendix S1).

As we are interested in the effects that the changing size of the system plays in determining the state of the system, we also model the dynamics of the size of the system itself over time. The variability of the size of the lake can be specified as:

$$\dot{W} = I_w(t) - O_w(t) \tag{4}$$

where $\dot{W}$ is the change in the water volume (W) stored in the lake over time, $I_W(t)$ denotes the inflow of water at a given time, and $O_W(t)$ denotes water withdrawals for drinking purposes at a given time.[1] To distinguish between the states of deep and shallow lake, we specify a certain level of water volume stored in the reservoir as a benchmark, denoted by $\overline{W}$. When the volume of water stored in the reservoir at a given time exceeds the benchmark ($W > \overline{W}$), the reservoir has characteristics of a deep lake (i.e. the system is of a large size). When the volume of water stored at a given time is less than the benchmark ($W \leq \overline{W}$), the reservoir has characteristics of a shallow lake (i.e. the system is of a small size). When the reservoir has characteristics of a deep lake, it is capable of assimilating more incoming pollutant and can continue to provide high-quality drinking water supply. By contrast, when the reservoir is characterised as a shallow lake, its ability to absorb the incoming pollutant

[1]There are several other factors that affect the volume of water and consequently the size of the lake, such as evaporation, natural outflows or requirements to release water for environmental flows downstream. These factors are likely to have much smaller effects on the size of the lake at a given time than inflows and drinking water withdrawals. As we do not analyse these factors in this study, we omit them from the state equation (4).

is reduced, and there is an increased release of pollutant that was previously stored in the sediment, further worsening water quality in the reservoir.

Equations (2), (3) and (4) together with the initial conditions for the pollution stock and water volume in the lake $P(0) = P_0$ and $W(0) = W_0$ form an optimal control problem. A current value Hamiltonian for this problem is:

$$H(t, u, P, \lambda) = \frac{h}{2}\left(P(u(t),t) - \overline{P}\right)^2 + u^c + \lambda_1 \dot{P} + \lambda_2 \dot{W} \tag{5}$$

The first-order condition for the optimum with respect to the abatement control is:

$$\frac{\partial H}{\partial u} = cu^{c-1} - \lambda_1 \left(kI_w(t)\right)^n \tag{6}$$

By setting Equation (6) equal to zero, the expression for the control variable can be rewritten as:

$$u = \left[\frac{\lambda_1 \left\{k\left[I_w(t)\right]^n\right\}}{c}\right]^{1/c-1} \tag{7}$$

The expression for u from Equation (7) can be substituted into Equation (3) to give:

$$\dot{P}(t) = k\left[I_w(t)\right]^n \left\{ a - \left[\frac{\lambda_1 \left\{k\left[I_w(t)\right]^n\right\}}{c}\right]^{\frac{1}{c}-1} \right\} - \varphi(W(t), P(u(t), t)) + \psi(W(t)) \frac{P(u(t), t)^q}{m^q + P(u(t), t)^q} \tag{8}$$

The optimality condition for the pollution co-state variable is given by:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial P} = 2\frac{h}{2}\left(P\,(u(t),t) + \overline{P}\right) + \lambda_1 \left(\rho + \varphi - \psi \frac{qm^q P^{q-1}}{(m^q + P^q)^2} \right) \tag{9}$$

Given the free terminal state for this problem, a transversality condition for the pollution co-state variable is (Chiang, 2000, pp. 240–241):

$$\lim_{t \to \infty} \lambda_1(t) = 0 \tag{10}$$

We are not presenting here the first-order condition for the optimum with respect to the drinking water withdrawal control and for the water volume co-state variable, as they are not used in the numerical simulations. These are stated and discussed in the Appendix S1.

We use Equations (8) and (9) to derive rules (Equations 11–16 further described below) that are implemented in the numerical simulation study to simulate the regime of the system in each state and to simulate the regime shift from one state to another. The details of the numerical simulations are given in Section 4.

3 | DESCRIPTION OF THE CASE STUDY AREA AND DATA

3.1 | The case study area

The empirical case study is drawn from a water quality management problem in a reservoir near Sydney, Australia. The reservoir – Warragamba Dam, also known as Lake Burragorang

– serves as the main drinking water supply for the city of Sydney, with a population of over 5 million people. The reservoir is quite deep and its capacity is large, but the actual volume of water stored in it is variable, so that at certain times it is appropriate to characterise it as a shallow lake and at other times as a deep lake. The main water quality problem is phosphorus pollution. Phosphorus that originates from municipal, sewage, industrial and agricultural sources flows into the reservoir. This external inflow of phosphorus can be potentially controlled through various abatement measures, each of which entails abatement costs.[2]

The reservoir is situated in an area where rainfall is variable. As the water withdrawals for drinking purposes are more or less constant because of the need for regular supply that does not vary much, the dynamics of the volume of water in the reservoir are determined by the inflow. When there are intense inflows, the reservoir is rapidly filled with water. In times of intense inflows, there is also an intense load of phosphorus into the reservoir.

Once the concentration of phosphorus in the reservoir goes beyond a certain threshold, the nutrient is no longer limiting for algae development. As a consequence, profuse algal growth occurs, followed by their die-out, which is paralleled with the release of odorous and bad-tasting chemicals (and in rare occasions chemicals that are toxic to human health). This flip of the state of the lake might be called an 'algal bloom'. Once an algal bloom occurs, additional costs are imposed on the drinking water-supplying agency and on the water consumers in the city. These costs are associated with the reduction in the quality of water (e.g. cost to consumers who have to buy bottled water or have to boil tap water) – and with the extra costs that need to be expended in order to maintain the water quality despite the algal bloom (e.g. extra water treatment cost pertaining to the water supply agency).

To avoid the damage costs, the agency that manages the reservoir can undertake various abatement activities to prevent the system from flipping to an undesirable state.[3] These abatement activities are controlling the inflow of phosphorus in the system but involve substantial costs. The agency has an annual budget to be spent on abatement, which typically does not vary much from year to year. Whereas the annual abatement budget tends to be fixed, we show in this paper that in fact much greater expenditure on controlling phosphorus inflow in the system is warranted in some years than in others because of the changing size of the system. Following an optimal path of expenditure on abatement that is derived based on the changing size of the system allows for managing the system at a lower overall cost to society.

3.2 | Data

The data for phosphorus concentration (in milligram per litre (mg/L)) in the reservoir were obtained from the Sydney Catchment Authority (SCA) (2008). The SCA monitors nutrient levels in the reservoir from six water quality monitoring stations in various locations. Phosphorus concentration data collected from the station near the dam wall were used in the empirical study, as this location is the closest to the outlet pipe where the water is drawn for drinking purposes. Phosphorus concentration data were gathered for a 20-year period, from January

[2]Abatement measures for water pollution in a lake include, among other possibilities, the upgrading of sewage treatment plants located in catchment areas, raising fences to prevent cattle from entering waterways and raising riparian buffer strips.

[3]During the period considered in the empirical analysis, the key relevant regulatory and management agency was the Sydney Catchment Authority (SCA). SCA was subsequently merged into Water NSW: a government agency with a much wider remit in the space of water management and regulation in the state of New South Wales (NSW), Australia.

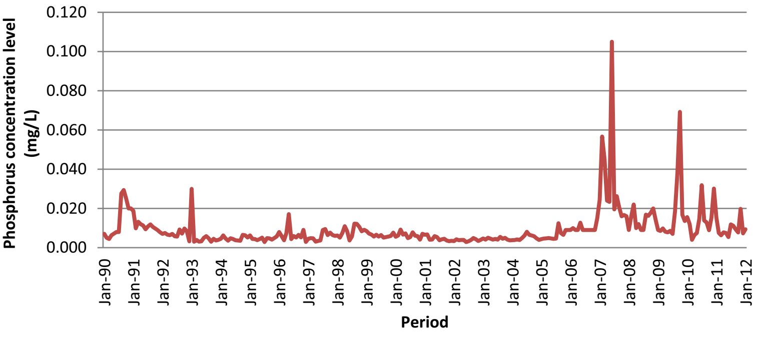

FIGURE 1 Mean monthly phosphorus concentration level in Lake Burragorang, January 1990–January 2012. *Source: Sydney Catchment Authority*. [Colour figure can be viewed at wileyonlinelibrary.com]

1992 to December 2011. Data records were collected on a daily basis. The daily concentration data were converted into monthly data by taking the mean concentration level for each month. These monthly data were used to model changes in the phosphorus concentration level in the lake over time. Figure 1 shows phosphorus concentration in Lake Burragorang from January 1990 to January 2012.

The threshold value for phosphorus concentration that triggers the development of algal bloom was set to 0.05 mg/L because algal blooms have only occurred in this reservoir when the concentration exceeded 0.05 mg/L (Figure 1). This threshold value was determined based on the data provided by the SCA. The data confirmed that historically when the concentration level was below 0.05 mg/L, no algal bloom incidents were reported in the reservoir. However, when phosphorus concentration exceeded 0.05 mg/L, for example in June 2007 and in June 2010, algal blooms developed near the dam wall.

Inflow data were obtained from the Hydstra database system, which is the SCA's hydrological database (Rod Mcinnes (SCA), pers.comm., 2013). The inflow data (in megalitres per day, ML/day) collected for this study were recorded from a gauge station in Fowlers Flat. Collected data were converted into monthly data to coincide with phosphorus concentration data.

The level of water stored in the dam (in gigalitres, GL), which is an indicator of the size of the system, was recorded on a weekly basis as per the data obtained from the SCA website and then aggregated to obtain monthly averages. A benchmark was used in the model to differentiate between two possible states of the lake in terms of its size: a deep lake or a shallow lake. The benchmark value was set at 1300 GL. This is the point where the volume of water is at 50 per cent of the total capacity of the reservoir, i.e. the lake is half-full. This value was used by the SCA for their operations management of the reservoir.[4] Consequently, when the recorded volume of water stored in the reservoir is greater than 1300 GL, we treat it as a deep lake, and when the stored water level is below this value, we treat it as a shallow lake.

The data for the phosphorus inflow from the catchment into the reservoir were obtained from the SCA for point sources (e.g. sewage treatment plants) and from the information published in McNamara and Cornish (2005) for nonpoint, agricultural sources. The total quantity of phosphorus in the lake at any given time was calculated by multiplying the phosphorus concentration at that time by the amount of water stored in the reservoir at that time.

[4]When the water stored in the reservoir fell below 50% capacity, it triggered a change in management of the reservoir by SCA, including drawing water from other reservoirs and implementing some water use restrictions.

4 | NUMERICAL SIMULATION METHODS

The numerical simulations were performed by using the MATLAB software, version 2013b, and its BVP4c solver. This solver is suitable for solving optimal control problems that represent boundary value problems (BVPs; Mathworks, 2022). The system of differential equations used in this study represents a two-point BVP where *ex ante* information must be specified at two points: initial conditions for the state variables ($P(0)$ and $W(0)$) and the transversality condition for the co-state variable in Equation (10).

When there are many parameters in the model, as is a case here, a continuation approach can be implemented in the following way: the BVP problem is solved for a shorter time interval, and in the following step, the solution generated for the shorter interval is used as an initial value for the next interval. Then, the process of extending the length of the time interval can be implemented gradually until the desired length of estimation is reached. In this study, the continuation approach was implemented using the fourth-order Runge–Kutta method to numerically solve the differential equations and generate numerical simulations with the BVP4c solver.

Eight scenarios were created by varying parameter values in the model to derive and compare optimal management of the reservoir with and without explicitly considering its variation in size. For each scenario, one of the key model parameters was changed while the others were held constant. Each scenario was characterised with one of the two possible initial states: a good state, initially characterised by a deep lake and low concentration of phosphorus; and a bad state, initially characterised by a shallow lake and high phosphorus concentration. Simulations were conducted for a range of values for the initial conditions for the two state variables – pollutant concentration and the size of the system (Table 1). Parameter values were based on the data pertinent to the case study. The parameters and their values used in the model are listed in Table 2.

Based on Equations (8) and (9), the following equations were used to simulate the dynamics of the system under the various possible states of the lake at any given point of time with respect to its size, W (deep or shallow lake), and with respect to the phosphorus concentration, P (below or above the threshold level):

$$\dot{P}(t) = k\left[I_w\right]^n \left\{ a - \left[\frac{\lambda_1 \left\{ k\left[I_W\right]^n \right\}}{c} \right]^{1/c-1} \right\} - \bar{b}P(t) + \underline{r}\frac{P(t)^q}{m^q + P(t)^q}, \text{if } \mathrm{W} > \overline{W} \tag{11}$$

$$\dot{P}(t) = k\left[I_w\right]^n \left\{ a - \left[\frac{\lambda_1 \left\{ k\left[I_W\right]^n \right\}}{c} \right]^{1/c-1} \right\} - \underline{b}P(t) + \bar{r}\frac{P(t)^q}{m^q + P(t)^q}, \text{if } \mathrm{W} \leq \overline{W} \tag{12}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \left[-2\frac{h}{2}P - \lambda_1\left(-\bar{b} + \underline{r}\frac{qm^q P^{q-1}}{(m^q + P^q)^2} \right) \right] \; if\ W > \overline{W}\ and\ P \geq \overline{P} \tag{13}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \left[-2\frac{h}{2}P - \lambda_1\left(-\underline{b} + \bar{r}\frac{qm^q P^{q-1}}{(m^q + P^q)^2} \right) \right] \; if\ W \leq \overline{W}\ and\ P \geq \overline{P} \tag{14}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \left[-0 - \lambda_1\left(-\bar{b} + \underline{r}\frac{qm^q P^{q-1}}{(m^q + P^q)^2} \right) \right] \; if\ W > \overline{W}\ and\ P < \overline{P} \tag{15}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \left[-0 - \lambda_1\left(-\underline{b} + \bar{r}\frac{qm^q P^{q-1}}{(m^q + P^q)^2} \right) \right] \; if\ W \leq \overline{W}\ and\ P < \overline{P} \tag{16}$$

TABLE 1 Simulated scenarios with different parameter and starting values for water volume and phosphorus concentration in the lake.

Scenario	Treatment of the size of the system	Starting value for water volume (GL)[a]	Starting value for phosphorus concentration (mg/L)[b]
1	Constant	1420	0.02
2	Constant	1420	0.068
3	Constant	1220	0.02
4	Constant	1220	0.068
5	Variable	1420	0.02
6	Variable	1420	0.068
7	Variable	1220	0.02
8	Variable	1220	0.068

[a]The starting values for the volume of water stored in the reservoir were chosen at approximately 10% above and below the threshold value of 1300 GL.

[b]The starting values for the phosphorus concentration were chosen to be at some distance above and below the critical concentration of 0.05 mg/L. The unequal distance to this critical point is justified based on the nonlinear nature of the effect of the increased concentration, as the marginal increases from a low base have much lower negative effects than corresponding marginal increases from a higher base.

TABLE 2 Range of parameter values used in the numerical simulations of phosphorus concentration and water volume stored in Lake Burragorang (Warragamba Dam).

Parameter	Description	Range of values
k	Slope of the transfer coefficient	0.0005
n	Exponent of the transfer coefficient	−0.36
c	Parameter for marginal abatement cost function	1.5
$\bar{b}$	Natural phosphorus degradation rate (deep lake)	0.07
$\underline{b}$	Natural phosphorus degradation rate (shallow lake)	0.035
$\bar{r}$	Maximum phosphorus recycling rate (deep lake)	0.0006
$\bar{r}$	Maximum phosphorus recycling rate (shallow lake)	0.006
q	Exponent that determines the steepness of sigmoid curve	[8,10]
m	Oxygen depletion rate in the reservoir	0.25
ρ	Discount rate	0.0016
h	Parameter for the marginal damage cost function	0.003

These equations characterise the necessary conditions for optimality, which are used to paste the behaviour of the variables of interest across different regimes.[5] Specifically, Equations (13–16) refer to the different regimes of the co-state variable and are used to paste the change in the value of the co-state over time across those regimes.

At any given point of time, and dependent on the simulated values of the W and P at that point of time, one of the equations (11) and (12) was combined with one of the equations (13) to (16), and with the initial condition for the pollutant stock, the initial condition for water volume in the lake and the transversality condition for the co-state variable (Equation 10), to numerically simulate the behaviour of the system and to find the optimal level of abatement.

[5]Parameters $\bar{b}$, $\underline{b}$, $\bar{r}$ and $\underline{r}$ are the values that function φ and ψ stated in Equation (3) can take. A detailed description of these functions and their values is provided in the Appendix, in Equations (A3) and (A5).

For example, when the lake can be characterised as a deep lake $\left(W > \overline{W}\right)$ and the phosphorus concentration in the lake is below the critical point $\left(P < \overline{P}\right)$, the dynamics of the pollution variable represented in Equation (11) and the dynamics of the co-state variable represented in Equation (15) were used jointly to start the simulation. However, if and when the phosphorus concentration in the reservoir, which was tracked continuously within the simulation, crossed the critical point $\left(P \geq \overline{P}\right)$ Equation (13) replaced Equation (15), whereas Equation (11) was still used to simulate the dynamics of the pollution variable. On the contrary, whenever the lake can be characterised as a shallow lake ($W \leq \overline{W}$), the dynamics of the pollution variable were represented by Equation (12) instead of Equation (11).

The numerical simulations described above were used to produce output at monthly time intervals over 20 years (i.e. $t = 240$). For each month, output was recorded on phosphorus concentration in the reservoir, optimal quantity of phosphorus abated and the associated costs of abatement and damage. All simulations achieved convergence towards the optimal solution.

5 | FINDINGS

Results of the simulated pollutant concentration and abatement are presented in Figures 2–9, and the results of the costs are presented in Table 3.

Scenarios 1–4 simulate the dynamics of the reservoir under the assumption that the size of the lake does not change over time. These are contrasted by Scenarios 5–8, where the size of the lake is explicitly modelled as varying through time. The pairs of scenarios: 1 and 5, 2 and 6, 3 and 7, and 4 and 8, are directly contrasting in terms of the treatment of the dynamics of the size of the lake, with the first in the pair modelled as static size and the second modelled as dynamically changing size, all other things being equal.

In Scenario 1, the system is assumed to be in a good state initially, where the phosphorus concentration is below the threshold level and the lake is initially fairly full with water, that is, it can be characterised as a deep lake. The simulation results show that for Scenario 1, a significant amount of abatement is optimally implemented over the entire time horizon to control the phosphorus load entering the lake. As a result of intensive abatement throughout the simulation period, a substantial amount of phosphorus load is prevented from entering the lake. The

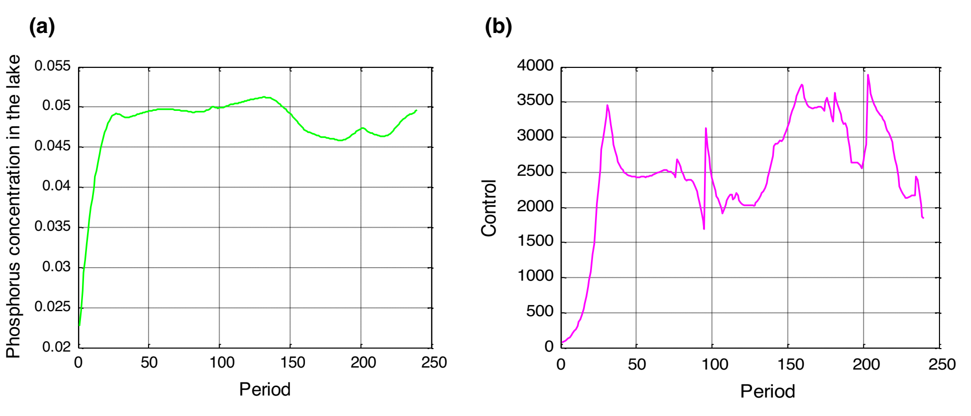

FIGURE 2 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 1. [Colour figure can be viewed at wileyonlinelibrary.com]

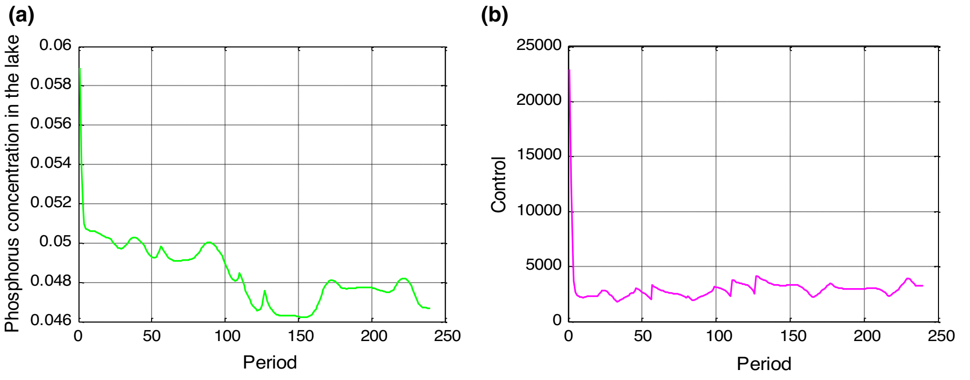

FIGURE 3 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 2. [Colour figure can be viewed at wileyonlinelibrary.com]

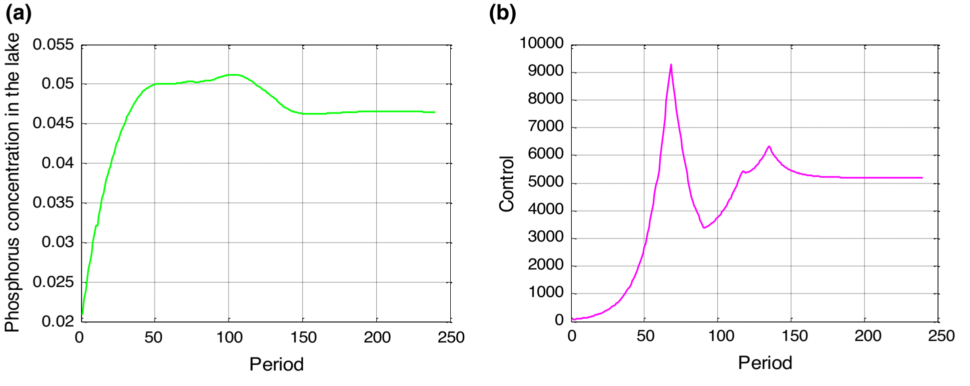

FIGURE 4 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 3. [Colour figure can be viewed at wileyonlinelibrary.com]

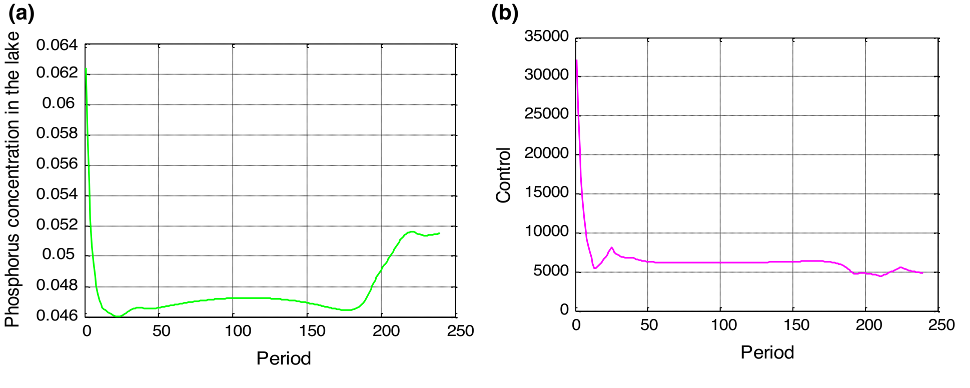

FIGURE 5 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 4. [Colour figure can be viewed at wileyonlinelibrary.com]

implementation of abatement for the entire period helps to keep the concentration level below the threshold point ($P < \overline{P}$), thereby preventing high cost of damages (Figure 2). However, in

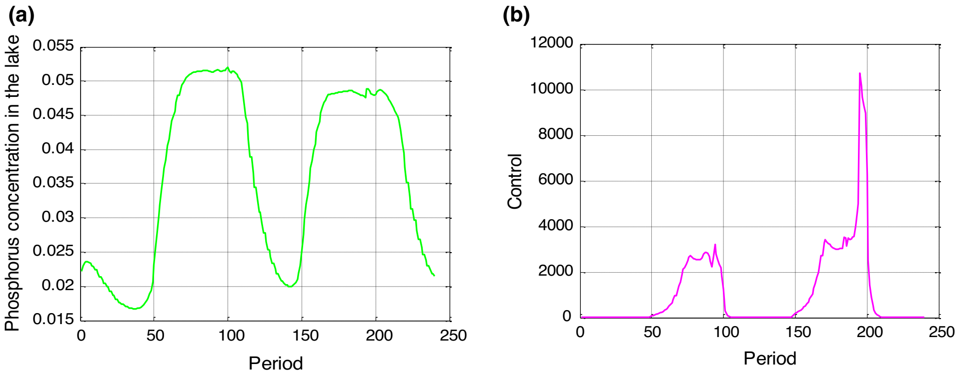

FIGURE 6 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 5. [Colour figure can be viewed at wileyonlinelibrary.com]

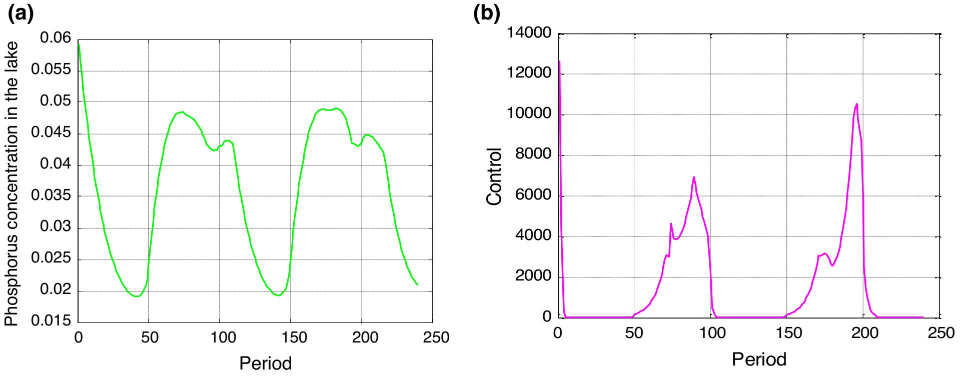

FIGURE 7 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 6. [Colour figure can be viewed at wileyonlinelibrary.com]

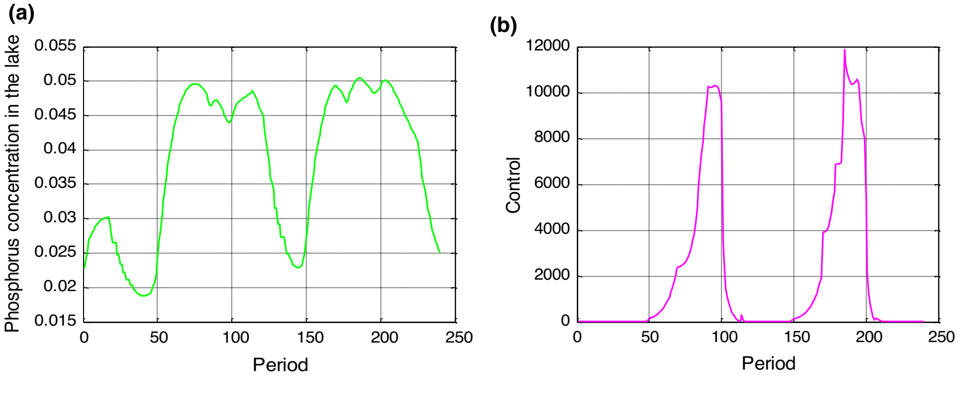

FIGURE 8 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 7. [Colour figure can be viewed at wileyonlinelibrary.com]

the later stages of the planning period, the concentration level in the lake exceeds the threshold point ($P \geq \overline{P}$) for a short period of time. The cost of damages incurred as a result is not great.

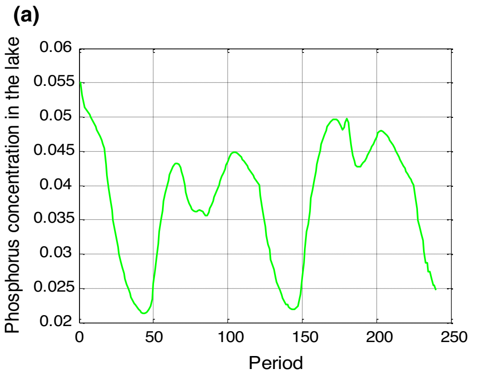

FIGURE 9 (a) Phosphorus concentration (mg/L) and (b) optimal quantity of phosphorus abated (kg) under Scenario 8. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 3 Damage costs, abatement costs and total costs under optimal control solutions for each scenario.[a]

Scenario	Total damage cost (AUD)	Total abatement cost (AUD)	Total cost (AUD)
1	85,102	31,065,000	31,150,102
2	311,930	41,989,000	42,300,930
3	61,132	74,420,000	74,481,132
4	683,710	129,050,000	129,733,710
5	215,730	15,473,000	15,688,730
6	43,371	25,879,000	25,922,371
7	1347	41,359,000	41,360,347
8	95,864	75,198,000	75,293,864

[a]The results of the costs presented in Table 3 were obtained by summing over 240 months the optimal monthly costs of abatement (in AUD/month) derived from simulations (total abatement cost); summing the resulting damage costs over 240 months (total damage cost); and summing total abatement cost and total damage cost to obtain the total cost.

Under Scenario 2, it is assumed that the system is initially in a bad state where the phosphorus concentration is above the threshold point and the lake can be described as a deep lake. When the system starts with high phosphorus concentration, more abatement is optimally applied (Figure 3) to prevent further significant amount of phosphorus load from entering the reservoir. This ensures that the damage costs are kept in check. When the phosphorus concentration in the lake comes down, the optimal level of abatement becomes almost constant. The total cost from pollution (the sum of abatement and damage costs) in this scenario is significantly greater than the cost simulated in the previous scenario (Table 3).

In Scenario 3, the lake can be initially described as a shallow lake, where the volume of water stored in it is below the threshold level ($W \leq \overline{W}$) as shown in Figure 4, and the initial phosphorus concentration is assumed to be below the threshold point ($P < \overline{P}$). In this scenario, the cost of abatement to optimally manage pollution increases significantly in comparison with the two previous scenarios, which is due to the initial state of a shallow lake. A higher level of optimal abatement is applied over time to prevent a significant amount of phosphorus load from entering the reservoir as compared to Scenarios 1 and 2. This is to ensure that damage costs are kept in check. Total phosphorus load abated in this scenario is almost double the load controlled in Scenario 1. The total social cost from pollution in this scenario is greater than the social cost simulated in Scenarios 1 and 2.

Under Scenario 4, the lake is initially in a bad state, with the initial level of phosphorus concentration above the threshold ($P \geq \overline{P}$). In addition, the volume of water in the lake under this scenario is low ($W \leq \overline{W}$), so it is characterised as a shallow lake. Under this scenario, more abatement is optimally undertaken for the entire time period to prevent the incoming phosphorus from entering the lake (Figure 5). As the initial phosphorus concentration in the lake is above the threshold point, a significant level of abatement is applied at the outset to prevent further substantial amount of phosphorus from entering the lake. When the pollutant concentration in the lake is reduced below the threshold point, the level of abatement applied to control the incoming load becomes almost constant. The total social cost from pollution in this scenario is significantly greater than the social cost simulated in the previous three scenarios, which is as expected given the dire initial state of the lake.

With Scenario 5, we start the series of simulations where the size of the lake varies over time. The variations in the inflow of water and consequently the amount of phosphorus loading into the lake were explicitly incorporated in the simulation model. The numerical results derived under Scenarios 5–8 are based on variable water volume and phosphorus load in the lake.

In Scenario 5, the starting values used for water volume and the phosphorus concentration in the lake are the same as the values used in Scenario 1. The simulation results obtained in Scenario 5 show that explicitly considering the variation in the size of the lake leads to very different optimal abatement management as compared to treating the lake as if it were of constant size. It can be noticed from the simulation results that the total social costs related to the phosphorus pollution in the lake are significantly less when the size changes are explicitly taken into consideration for management purposes (Table 3). This is due to differences in the implementation of optimal abatement across time. When the lake is managed as if it were of constant size, the optimal abatement of phosphorus load does not change much over time (Figure 2). However, the optimal abatement varies over time significantly when the variation in the size of the lake is explicitly considered in the simulation model (Figure 6). In this scenario, most abatement is optimally applied when the lake reduces in size and just before the start of high inflows that rapidly fill it. High inflows transport a significant amount of phosphorus load. As a result, pollution concentration in the lake may raise rapidly and may consequently lead to an algal bloom. Therefore, in this scenario, more abatement is optimally applied when the lake is at a higher risk of developing an algal bloom. By contrast, at times when the lake is full of water and the pollutant concentration is low, there is little or no abatement optimally applied. As a result, the sum of the total abatement cost and total damage cost in this scenario is significantly smaller than the cost simulated under Scenario 1.

Under Scenario 6, as in Scenario 2, the system is assumed to start from a bad state where the pollutant concentration is above the threshold point ($P \geq \overline{P}$) and the lake can be initially described as a deep lake ($W > \overline{W}$) (Table 2). In this scenario, high optimal level of abatement is initially applied as the lake exhibits high phosphorus concentration (Figure 7). As a result, the concentration of pollution falls below the critical point (0.05 mg/L) and remains below, albeit fluctuating, for the remaining time (Figure 7). Consequently, the cost of damages is kept low. Although the lake was initially characterised as a deep lake as in the previous scenario (Scenario 5), there are significant differences between the two scenarios in terms of the optimal abatement cost to control the incoming phosphorus. This is due to the differences in the initial phosphorus concentration level in the reservoir, with Scenario 6 having significantly higher initial phosphorus concentration than Scenario 5. So, even though the cost of damages is low in this scenario, the cost of abatement is quite high, and therefore the total costs of pollution in this scenario are relatively higher than in the previous scenario (Table 3).

In Scenario 7, as in Scenario 3, the lake can be initially described as a shallow lake ($W \leq \overline{W}$). It is assumed that the lake is initially in a good state where the initial value for phosphorus concentration is 0.02 mg/L. The simulation results (Figure 8) show that the total cost to manage pollution is significantly higher when the reservoir is initially characterised as a shallow lake, as compared to the scenarios where the lake was initially characterised as a deep lake (Scenarios 5 and 6; Table 3). However, when compared to Scenario 3, we notice that the abatement cost to control phosphorus load, as well as the total cost of pollution, decreased significantly under Scenario 7 (Table 3). This is because, under this scenario, the variability in the size of the lake is explicitly considered in the model.

In Scenario 8, as in Scenario 4, the lake can be initially described as a shallow lake ($W \leq \overline{W}$), and the initial phosphorus concentration is high (0.068 mg/L). The simulation results show that the optimal level of abatement is high initially, as the lake already has high phosphorus concentration (Figure 9). Given the dire initial conditions under this scenario, the total cost to manage phosphorus pollution is higher as compared to the previous scenarios with variable size of the system (Scenarios 5–7; Table 3). Nevertheless, the overall costs are still substantially lower under Scenario 8 than under Scenario 4, where the size variation was not explicitly considered (Table 3).

An overall pattern that can be noticed from the simulation results is that the total cost to manage phosphorus pollution is significantly higher when the lake is initially characterised as a shallow lake, and the pollution status of the lake starts from a high concentration level. This is as expected. More notably, we notice that when the variation in the size of the system is explicitly considered in the simulation model (Scenarios 5–8), the overall cost of pollution is much lower than in the scenarios that do not include variation in size (Scenarios 1–4). This gives an important insight into the significance of explicitly considering the variation in the size of a system as a driver of regime shift for its optimal management. Our findings show that optimal implementation and timing of controls is considerably different when the dynamics of the changing size are explicitly considered as opposed to when they are not considered. Significant cost savings are achieved by applying more intense control measures at critical times under the management that explicitly considers the variation in the size of the system, as opposed to more or less constant application of control measures over time under the management that does not explicitly consider the variation in the system size.

6 | CONCLUSION

The present paper treats a complexity that pertains to a general class of managed systems, both natural and economic. This is to do with the inherently varying size of many systems that are of benefit to society – lakes, forests and fisheries, but also financial systems within national economies. In this context, the variation in the size of a system is the main driver of a regime shift.

A model of a system of varying size in an optimal control setting was applied to a case of phosphorus pollution in a water reservoir. Comparisons were made using numerical simulations between optimal management of a lake with and without explicitly modelling its size variation. Several conclusions can be drawn based on the findings.

Explicitly considering size variation matters significantly for optimal management and for the associated social costs. When the system does vary in size, managing it as if it does not entails significantly higher costs. To put it differently, the costs of ignoring a regime shift due to size variation can be quite high. Those costs are mostly attributable to higher control cost due to fairly even optimal control effort across time corresponding to the assumption of the constant size of the system, as opposed to highly variable optimal control effort under varying size. When the size of the system is explicitly considered, control costs are invariably reduced, and damage costs also decrease in most of the scenarios, but not all. This implies that optimal management of a system of varying size may entail enduring higher damage costs than it would have been the case if the system were managed as being of constant size. Consequently, the agency has to be prepared to accept somewhat higher damage costs on rare occasions in order to be able to save considerably on control costs. While this is economically rational, it may be politically difficult, as society at large might be averse to such a possibility. Notwithstanding, an argument can be made that in all of our scenarios, the damage costs have been kept at fairly modest levels and that the large savings in abatement costs due to improved management as a result of explicitly considering size variability can easily justify any possible increase in damage costs (e.g. our Scenario 5 vs Scenario 1 in Table 3). In general, our findings suggest that it is beneficial to explicitly consider variation in the size of a system when determining an optimal management strategy.

This paper points to the importance of considering specific attributes of systems to be optimally managed over time. Their own size is perhaps the most obvious attribute to be considered. The implications for management are significant and if applied can lead to more socially desirable outcomes.

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DATA AVAILABILITY STATEMENT

Data and code are available from authors upon request

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SUPPORTING INFORMATION

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