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Uniform emission taxes, abatement, and spatial disparities*

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This paper examines how a commitment to a common tax on greenhouse gas (GHG) emissions affects spatial distribution of production and welfare between countries. To focus on the long-term effects of firm relocation, the paper employs a two-country model of monopolistic competition with a variable abatement technology. In the model, the elasticity of substitution between GHG emissions and the conventional input is a key parameter. If the elasticity of substitution is smaller than one (in the substitutable area), the relative number of firms and the relative welfare in the large country monotonically increase with uniform tax rates. Meanwhile, if the elasticity of substitution is larger than one, they both follow inverted U-shaped curves in response to tax rates. Nevertheless, for any level of taxes, they both are necessarily higher than those in the case without taxes. This suggests that uniform emission taxes widen international disparities of firm location and welfare.

Key words: abatement, emission tax, firm relocation, spatial disparities.

1. Introduction

The main source of the global climate problem is the fact that each individual does not consider the costs she/he imposes on others when she/he emits greenhouse gases (GHGs). To solve the problem, therefore, it is necessary for GHG emissions to be appropriately priced to internalise the costs. Importantly, the pricing of GHG emissions should *not* rely on the voluntary actions of countries. This is because each country has an incentive to be a free-rider by pricing emissions as low as possible to retain the competitiveness of its emission-intensive industries. Recent studies on cooperation clarified that cooperative strategies with a *common commitment* can promote cooperation levels among selfish players (e.g. Bolton and Ockenfels 2000; Hauser *et al.* 2014). A common commitment is a commitment assuring all parties that they will only be required to follow a rule if all others are required to follow the same rule (the "I will if you will" approach). Negotiating a common emission price embodies countervailing force against narrow self-interest by

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automatically incentivising all negotiating parties to internalise the externality (Weitzman 2014). Hence, a common price commitment is thought to be a prime candidate for a way to solve the global warming problem (e.g. Cramton *et al.* 2015a, b; Gollier and Tirole 2015; Weitzman 2015).

Despite the desirable properties of common price commitments, such agreements have not prevailed in the world. For example, in 1991, the Commission of the European Communities (EC) submitted a communication titled. A community strategy to limit carbon dioxide emissions and to improve energy efficiency (SEC (91) 1744 final), in which a harmonisation of carbon tax rates among member countries was proposed. In fact, the differences in carbon tax rates among EC members were large at that time (Pearson and Smith 1991); nevertheless, the proposal was never adopted.

Similar situations could be observed in a series of Conferences of the Parties (COP) under the United Nations Framework Convention on Climate Change (UNFCCC). In the Kyoto Protocol at COP3 in 1997, developed countries were required to reduce the emission of GHGs to some percentage below 1990 levels. While the Protocol was expected to be translated into a uniform price on emissions, such ideas of a common price commitment were abandoned after the Copenhagen COP15 in 2009. The new negotiation mechanism including developing countries, called “pledge and review”, confirmed at the Paris COP21 in 2015, is still far from a common commitment, since the new mechanism largely relies on independent and individual commitments from the participating countries (e.g. Cramton *et al.* 2015a; Gollier and Tirole 2015; Cramton *et al.* 2017).

Hoel (1993) pointed out that one of the difficulties with an agreement harmonising emission taxes is the associated distribution of cost across countries; he concluded that an agreement with such distributional properties would be infeasible unless it was supplemented with some kind of side payments between countries. According to Kverndokk (1993), richer countries tend to have lower abatement costs due to characteristics of their base industries and higher energy efficiency. Several papers clarified that poorer people tend to consume more emission-intensive goods, so that emission taxes will expand disparities in real income among different income groups (Metcalf 1999; Dinan and Rogers 2002; Cremer *et al.* 2003; Parry 2004). In addition, Gollier and Tirole (2015) pointed out that poor people may face large discount rates, and hence, the social cost of emissions will be smaller in low-income countries.

The present paper shows that a uniform harmonised tax could affect spatial disparities via firm relocation, which changes the market accessibility in individual countries and thus influences welfare via consumer prices. Different from the aforementioned papers featuring disparities among countries with different tastes and/or technologies, this paper focuses on disparities of firm location and welfare, which is measured by a real income per capita throughout the paper,[1] among countries that are similar but differ in their size. The findings suggest that a common price (tax) commitment is difficult to achieve without transfers, not only among rich and poor countries,[2] but also among developed countries, such as European Union countries.

Some papers have investigated the relationship between environmental policies and firm location (e.g. Markusen *et al.* 1993, 1995; Pflüger 2001; Venables 2001). More recently, Zeng and Zhao (2009) and Ishikawa and Okubo (2017) examined environmental issues by considering firm relocation using the footloose capital (FC) models (Martin and Rogers 1995). Forslid *et al.* (2018) analysed a similar issue by incorporating firm heterogeneity and abatement activities. Their focus, however, was on firm relocation in reaction to environmental policies on a *local* level, which is *not* internationally harmonised. Put differently, while analysis of spatial disparities affected by introducing a common emission tax is required, there has never been such analysis in the literature. This is a research gap which should be filled. The present study fills the gap. Hence, this paper examines how firm location is affected by a common emission tax commitment.

Abatement costs can be incorporated into the model in several ways (see Copeland and Taylor 2003, Bourne and Philippidis 2018, for a review). One representative approach is assuming that an abatement sector produces abatement goods, and each firm buys the good to reduce GHGs. This approach is often used in studies of computable general equilibrium (CGE) (e.g. Gerlagh *et al.* 2002; Dellink *et al.* 2004; Dellink and van Ierland 2006). The other representative approach is treating emissions as an input to the production function, which can be substituted by the conventional inputs of each firm. Although this approach is also found in CGE studies (e.g. Hyman *et al.* 2002), it tends to be preferred by theoretical ones (Copeland and Taylor 1994; Copeland 1996; Ishikawa and Kiyono 2006), since it simplifies analysis. I also take this approach in the current paper.

As referred to above, this paper theoretically examines how the introduction of uniform taxes on GHG emissions affects spatial disparities, focusing on the long-term effects of firm relocation. To accomplish this, we employ a simple two-country model with a variable abatement technology. Specifically, this paper relies on an FC model with endogenous wage differentials (Takatsuka and Zeng 2012), which is based on the framework of the New Economic Geography (NEG) (e.g. Krugman 1980; Fujita *et al.* 1999; Baldwin *et al.* 2003). There are two sectors, namely the emitting sector producing differentiated varieties of goods under monopolistic competition and the non-emitting sector producing a homogeneous good under perfect competition. To describe the abatement technology, the output of varieties of goods is

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[1] In the present paper, the real income is defined as a nominal income deflated by a price index, net of utility loss due to environmental degradation (i.e. GHG emissions).

[2] The Green Fund was established at COP15 and reinforced at COP21 to provide much of the financing needed by low- and middle-income countries to meet the Paris pledges.

expressed by a function of the conventional input and the amount of GHG emissions during production, following Copeland and Taylor (1994), Copeland (1996), and Ishikawa and Kiyono (2006). In the model, the elasticity of substitution between GHG emissions and the conventional input is a key parameter. The main results are as follows. If the elasticity of substitution is smaller than one (in the substitutable area), the relative number of firms and the relative welfare in the large country monotonically increase with tax rates. Meanwhile, if the elasticity of substitution is larger than one, the relative number of firms and the relative welfare in the large country follow inverted U-shaped curves in response to tax rates. Nevertheless, it is shown that for any level of taxes, the relative number of firms and the relative welfare in the large country are necessarily higher than those in the case without taxes. Since larger countries have a more-than-proportionate share of firms and higher welfare (i.e. a higher real income per capita) due to the home market effect (Krugman 1980; Takatsuka and Zeng 2012), our results suggest that common emission taxes widen the international disparities caused by firm location and welfare.

When labour and GHG emissions are mainly used as variable inputs in the production of emitting sectors, the cost (wage) differential among countries plays an important role in the above results. From the viewpoint of firm location, lower-wage countries have an advantage and a disadvantage under a harmonised emission tax. The advantage is *the lower cost of abatement activities*. If GHG emissions can be reduced by employing additional labour, lower wages are attractive for firms, since abatement costs are relatively low. Meanwhile, the disadvantage is *a relative rise in marginal production costs* in the country. This is because uniform taxes impose an identical charge on firms with additional emissions in all countries involved in the agreement if abatement activities are ignored.[3] Firm relocation due to emission taxes is determined by the balance of the two opposing forces.

It is noteworthy that the cost differential among countries is generated by the immobility of labour. In reality, however, a mobile factor (i.e. capital) is often used not only for production but also for abatement. If mobile capital and GHG emissions are used as the main variable inputs in the production of emitting sectors and GHG emissions can be reduced by using additional capital, the above mechanism of firm relocation collapses since the cost differential among countries vanishes due to international mobility of capital. Section 5 nevertheless shows that our main results qualitatively remain regardless of the introduction of a mobile factor as a variable input. In that case, changes in capital rent drive firm relocation instead of the international difference in marginal costs.

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[3] A similar mechanism is pointed out by Takatsuka (2014). However, his focus is not on environmental issues with abatement activities but on the comparison between *ad valorem* and unit taxes.

The remainder of this paper is organised as follows. Section 2 presents the base model and shows the equilibrium under GHG emission taxes. Section 3 shows some findings related to the effects of taxes on variety production. The findings are building blocks for the main results in the subsequent section. Section 4 then shows the effects of emission taxes on firm location and relative welfare. Section 5 extends the model by introducing capital as a variable input. Finally, Section 6 is the conclusion.

2. The model

Consider an economy that consists of two countries (large country N and small country S), two sectors (emitting sector M and non-emitting sector A) and two factors (labour and capital). As mentioned above, this study clarifies the relationship between common emission taxes and firm location by using the NEG framework. In this framework, firm location of a differentiated-good sector characterised by increasing returns to scale (IRS) and monopolistic competition is endogenously determined (Krugman 1980). Therefore, as in the related literature (e.g. Zeng and Zhao 2009; Ishikawa and Okubo 2017; Forslid *et al.* 2018), we assume that the emitting sector M produces differentiated varieties of goods under IRS and monopolistic competition, while the non-emitting sector A produces a homogeneous good under constant returns to scale (CRS) and perfect competition. Throughout the paper, the good produced by sector M is called good M and the good produced by sector A is called good A.

There are L^w workers in the economy, and each of them owns one unit of labour and one unit of capital. We denote the amounts of labour in N as L and the counterpart in S with an asterisk (*), so that $L^w = L + L^*$ holds. The same notation is applied for other variables. Since country N is larger than country S, it holds that $L/L^w \equiv \theta \in (1/2, 1)$. Labour is mobile among sectors but immobile between countries, while capital is mobile between countries. Furthermore, it is assumed that the consumption share of good A is sufficiently large so that the homogeneous good is produced in each country for any equilibrium.

Workers are assumed to hold the same preference, which is described by a Cobb–Douglas utility for the two types of goods with a constant elasticity of substitution (CES) subutility on the varieties of good M:[4]

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[4] Since M is a CES function, each monopolistic competitive firm sets a constant price markup (Dixit and Stiglitz 1977, Section I). I take this assumption because it makes the present model tractable.

$$U = \mathcal{E}M^{\mu}A^{1-\mu}, \tag{1}$$

where:

$$M \equiv \left[\int_{0}^{n^{w}} c(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}},$$

$\mathcal{E}$ is the environmental factor, which is explained in detail below, n^{w} is the number of varieties in sector M, and $c(i)$ is the consumption of variety i. Parameter $\sigma > 1$ is the elasticity of substitution between any two varieties of good M, and $\mu \in (0,1)$ is the expenditure share on good M.

This paper assumes that emissions harm consumers but do not generate production externalities such as in Copeland and Taylor (1994), Copeland (1996), and Ishikawa and Kiyono (2006). Furthermore, emissions have global effects, which are captured by $\mathcal{E} = \mathcal{E}(e^{w}, l_{G}, l_{G}^{*}) \in (0,1]$ in the utility function of Eq. (1) and are common among countries, where e^{w} is the total GHG emissions in the economy. Each government (sector G) uses all tax revenue, explained below, to employ labour, and the labour employed in sector G in N and S is denoted as l_{G} and l_{G}^{*}, respectively. In addition, the labour employed in sector G is used for research and development (R&D) investment for adaptation; hence, the negative effect of emissions can be reduced by the R&D investment.[5] It is assumed that R&D spillovers are perfectly transmitted between countries, so that the R&D investment also has global effects.[6] Specifically, we assume that:

$$\mathcal{E}(0, l_{G}, l_{G}^{*}) = 1, \frac{\partial \mathcal{E}}{\partial e^{w}} < 0, \frac{\partial \mathcal{E}}{\partial l_{G}} > 0, \frac{\partial \mathcal{E}}{\partial l_{G}^{*}} > 0. \tag{2}$$

$\mathcal{E}$ represents reduction in utility due to GHG emissions. If the economy has no emissions ($e^{w} = 0$), consumers obtain full utility levels ($\mathcal{E} = 1$) from their consumption. However, utility levels decrease with the total GHG emissions in the economy ($\partial \mathcal{E}/\partial e^{w} < 0$) due to global warming, even if consumers keep their amount of consumption constant. However, such a discount due to emissions can be mitigated by R&D investment for adaptation in each country ($\partial \mathcal{E}/\partial l_{G} < 0$, $\partial \mathcal{E}/\partial l_{G}^{*} < 0$).[7]

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[5] As long as $\mathcal{E}$ is common among countries, the results of the present paper are unchanged even when labour in sector G is used for other policies.

[6] As shown in Section 4, when the labour demand of sectors G and M, including labour for abatement and adaptation, increases in a country, sector A elastically supplies a labour force to the two sectors, so that the wage rate is unchanged.

[7] The specification of utility function of Eq. (1) is consistent with Ishikawa and Kiyono (2006), who use a general form of utility function with the environmental factor $\mathcal{E}$. While $\mathcal{E}$ can be introduced as an additive term (e.g. Copeland and Taylor 1994), I use a Cobb–Douglas form for convenience in the following analysis. Similar modelling of adaptation can be seen in Buob and Stephan (2011).

As in most related papers, we assume Samuelson's iceberg transport costs. Specifically, to deliver one unit of good M (respectively, good A) to the other country, $\tau_M > 1$ (respectively, $\tau_A > 1$) units of the good must be shipped. In other words, $\tau_M - 1$ (respectively, $\tau_A - 1$) units of the good are lost in transit as transport costs.

Sector M emits GHGs to produce varieties of goods, and emissions are generated only during production of good M. Although the two countries do not impose emission taxes in the initial state, a uniform emission tax, $t \geq 0$, for one unit of GHG emissions is introduced to reduce GHG emissions by an agreement between the two countries. As discussed in Introduction, the purpose of this paper was to examine how a common price (tax) commitment affects spatial disparities among countries of different sizes. Hence, the uniform tax rate is exogenously given, and we compare firm location and welfare in the case with taxes to those in the case without taxes.[8] By doing so, we see how transfers should be done to enable a common price commitment between the countries.

The government in each country uses all tax revenue in the country to employ labour, which is used for R&D investment for adaptation, as mentioned above. The reasons for assuming this are as follows. First, analysing the tax effects on firm location is simplified by this utilisation of tax revenue because the income effect on the demand for good M is excluded. In addition, as shown later, the trade direction is *not* affected by emission taxes under the assumption; thus, we obtain some clear-cut results. Second, the assumption itself is also realistic to some extent. For example, in the third period (2013–2020) of the European Union Emissions Trading Scheme (EU ETS), at least 50% of tradable-permit revenue is expected to be used for measures against global climate change and for energy policies (Morotomi 2010, p. 185) rather than for some redistribution policies directly affecting welfare. Third, analysing the tax effects on welfare is also simplified because the environmental factor $\mathcal{E}$ is common among countries due to the global effects of R&D investment for adaptation. When we consider the distributional effect of a common tax commitment, this utilisation of tax revenue may provide a benchmark. If the benchmark result is unfavourable from the viewpoint of welfare disparities, we could conclude that some portion of tax revenue should be used for reducing disparities instead of R&D investment for adaptation.

We normalise the price of good A in S as $p_A^* = 1$ and denote the price of good A in N as p_A. One unit of labour produces one unit of good A. Thus, wages in N and S are:

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[8] As shown in Appendix A6, by specifying $\mathcal{E}$ and the world welfare and setting parameter values, I numerically derived the optimal tax rates maximising world welfare. In addition, it is shown that the initial state with $t = 0$ is a Nash equilibrium under the same setting.

$$w = p_A \text{ and } w^* = p_A^* = 1,$$

respectively. Denote R and R^* as the capital returns of firms in N and S, respectively, and $\overline{R} \equiv \max\{R, R^*\}$.[9] Then, the total expenditures in N and S are:

$$E = \left(w + \overline{R}\right)L \text{ and } E^* = \left(w^* + \overline{R}\right)L^*, \tag{3}$$

respectively.

The production technology is symmetric among all firms in sector M. Each firm needs a fixed input of one unit of capital. To incorporate abatement costs into the model, this paper assumes that GHG emissions are used as a variable input, which can be substituted by the conventional inputs as mentioned in Introduction. The theoretical literature specifying abatement technology usually builds on a simple Cobb–Douglas formation (e.g. Forslid *et al.* 2018). However, this paper uses a more general CES abatement technology, thereby showing that the elasticity of substitution plays a key role in spatial disparities.

In reality, mobile capital is often used not only for production but also for abatement. For simplicity, however, mobile capital is neither used for marginal input nor used for abatement in the model until Section 5. To describe how GHG emissions can be substituted by production input (i.e. labour), output x of each variety is expressed by the following function of labour input, l, and the amount of GHG emitted during production, e:

$$x = \min\left\{\left[\alpha e^{\rho} + (1-\alpha)\left(\frac{l}{m}\right)^{\rho}\right]^{\frac{1}{\rho}}, \frac{l}{m}\right\}, \tag{4}$$

where $\alpha \in (0,1)$, $\rho \in (-\infty, 1)$ and $m > 0$.[10] The meaning of each parameter is explained in the next paragraph. It is noteworthy that firm (variety) name i is omitted because all firms are symmetric although their products are differentiated. This is applied in the following equations.

To understand Eq. (4), the following two cases are considered separately: the case of $l > me$ and the case of $l \leq me$. In the first case, where labour input (l) is large relative to GHG emissions (e), Eq. (4) is reduced to the first term in { }. This is a CES function of e and l, where α is the share parameter and $1/(1-\rho) \in (0,\infty)$ is the elasticity of substitution between the two inputs. Intuitively, ρ represents a measure of substitutability between the two inputs. Specifically, emissions and labour input are gross substitutes if $\rho > 0$, while

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[9] Note that $\overline{R} = R$ (respectively, $\overline{R} = R^*$) holds at equilibrium if $n > 0$ (respectively, $n^* > 0$).

[10] Similar specifications are found in Copeland and Taylor (1994), Copeland (1996), and Ishikawa and Kiyono (2006). Here, only labour and GHG emissions are used as variable input. Mobile capital is introduced not only for production but also for abatement in Section 5.

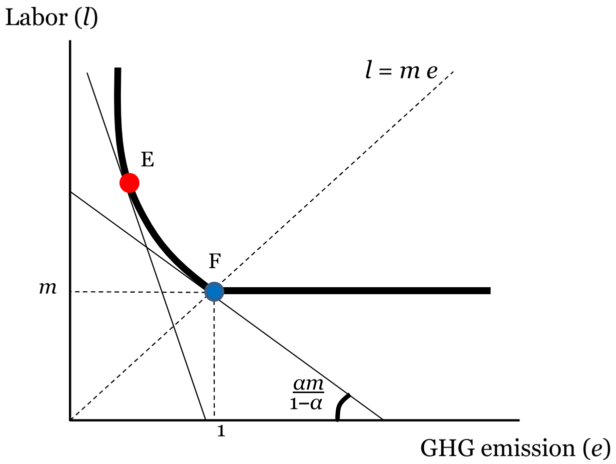

Figure 1 Unit isoquant of variety production. [Colour figure can be viewed at wileyonlinelibrary.com]

they are gross complements if $\rho < 0$. In the case of $l > me$, GHG emissions can be reduced by increasing labour input, which implies that labour is used for emission abatement as well as production. The ability to substitute emissions for labour must, however, have a limit. In fact, when $l \leq me$, we have $x = l/m$. This implies that m is the minimum number of workers required to produce one unit of output, and labour is not used for emission abatement in this case.

The unit isoquant of production function of Eq. (4) is depicted in Figure 1. Starting at a point such as E and moving towards point F, which is on the line $l = me$, the marginal rate of substitution of emissions for labour converges to $\alpha m/(1-\alpha)$. Thus, if the tax–wage ratio is lower than $\alpha m/(1-\alpha)$, each firm chooses (l, e) at point F, that is $(l, e) = (m, 1)$, to produce one unit of its variety. In other words, emission abatement is inactive. This situation includes the unregulated case (the tax–wage ratio is zero). Meanwhile, if the tax–wage ratio is higher than $\alpha m/(1-\alpha)$, each firm chooses (l, e) at a point above $l = me$, such as E, to produce one unit of its variety. When the price of emissions (i.e. tax) is high, each firm tries to reduce GHG emissions by abatement activities. Hence, firms employ more workers when the price of emissions is higher.

Therefore, with a simple calculation of cost minimisation, the marginal cost (MC) of producing varieties of good M in N is:[11]

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[11] When ρ tends to 0 (a Cobb–Douglas case), the second equation of Eq. (5) is rewritten as $\left\{\left[\frac{(1-\alpha)}{\alpha}\right]^{\alpha}+\left[\frac{\alpha}{(1-\alpha)}\right]^{1-\alpha}\right\}t^{\alpha}(mw)^{1-\alpha}$.

$$\mathrm{MC} = \begin{cases} mw + t & \text{if } \frac{t}{w} \in \left[0, \frac{\alpha m}{1-\alpha}\right) \\ \left[\left(\frac{t}{\alpha^{\frac{1}{\rho}}}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{mw}{(1-\alpha)^{\frac{1}{\rho}}}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} & \text{if } \frac{t}{w} \geq \frac{\alpha m}{1-\alpha} \end{cases}, \tag{5}$$

and the total cost of producing x units of varieties of good M in N is expressed as:

$$C(x) = R + \mathrm{MC}x.$$

Furthermore, if abatement is active, labour input and GHG emissions per firm in N are:

$$l = \left(\frac{1-\alpha}{m^{\rho}} \frac{\mathrm{MC}}{w}\right)^{\frac{1}{1-\rho}} x, \tag{6}$$

$$e = \left(\alpha \frac{\mathrm{MC}}{t}\right)^{\frac{1}{1-\rho}} x, \tag{7}$$

respectively. Similar equations hold for firms in S.

Let p be the price in N of a variety made in N; p^* the price in S of a variety made in S; $\overline{p}$, the price in N of a variety made in S; and $\overline{p}^*$, the price in S of a variety made in N. Capital owners decide the optimal output and prices to maximise their rent revenues. The monopolistic competition framework of Dixit and Stiglitz 1977, Section I, implies that:

$$p = \frac{\sigma \mathrm{MC}}{\sigma - 1}, p^* = \frac{\sigma \mathrm{MC}^*}{\sigma - 1}, \overline{p} = \frac{\sigma \mathrm{MC}^*}{\sigma - 1} \tau_M, \overline{p}^* = \frac{\sigma \mathrm{MC}}{\sigma - 1} \tau_M. \tag{8}$$

From Eq. (1), the supply of each differentiated variety produced in N and S is:

$$x = \mu \frac{p^{-\sigma}}{P^{1-\sigma}} E + \tau_M \mu \frac{(\overline{p}^*)^{-\sigma}}{(P^*)^{1-\sigma}} E^* \text{ and } x^* = \mu \frac{(p^*)^{-\sigma}}{(P^*)^{1-\sigma}} E^* + \tau_M \mu \frac{\overline{p}^{-\sigma}}{P^{1-\sigma}} E, \tag{9}$$

where P and P^* are the price indices of varieties of good M defined by:

$$P = \left[n p^{1-\sigma} + n^* (\overline{p})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, P^* = \left[n (\overline{p}^*)^{1-\sigma} + n^* (p^*)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{10}$$

and n and n^* are the numbers of firms in N and S, respectively. On the other hand, using Eq. (1), the demand of good A in N and S is:

$$d_A = \frac{(1-\mu)E}{p_A} \text{ and } d_A^* = (1-\mu)E^*, \tag{11}$$

respectively.

In the model, rents to capital equal the difference between revenues and labour costs. The pure profit earned by each firm becomes zero, and thus, the output in N and S is:

$$x = \frac{(\sigma-1)R}{\mathrm{MC}} \text{ and} x^* = \frac{(\sigma-1)R^*}{\mathrm{MC}^*}, \tag{12}$$

respectively. Let:

$$\gamma \equiv \left(\frac{\mathrm{MC}}{\mathrm{MC}^*}\right)^{1-\sigma} \tag{13}$$

and $\phi_M \equiv \tau_M^{1-\sigma} \in (0,1)$. Since ϕ_M is an inverse measure of transport costs τ_M, ϕ_M is often called the trade freeness of good M. Substituting Eqs. (3), (8), (9), (10) and (12) into the market-clearing conditions for varieties of good M produced in N and S, we obtain:

$$\frac{\mu}{\sigma}\gamma\left[\frac{(w+R)L}{n\gamma+n^*\phi_M} + \frac{(w^*+R)L^*\phi_M}{n^*+n\gamma\phi_M}\right] = R, \tag{14}$$

$$\frac{\mu}{\sigma}\left[\frac{(w+R)L\phi_M}{n\gamma+n^*\phi_M} + \frac{(w^*+R)L^*}{n^*+n\gamma\phi_M}\right] = R^* \tag{15}$$

after simplification. It is noteworthy that if firms fully agglomerate in country N (respectively, S), only Eq. (14) (respectively, (15)) holds; otherwise, Eqs. (14) and (15), and $R = R^*$ hold.

On one hand, in the interior equilibrium, n, n^* and R can be expressed by:

$$n = \frac{\mu L^w}{R\sigma}\frac{\theta(w+R)\gamma + (1-\theta)(w^*+R)\gamma\phi_M^2 - [w^*+R+(w-w^*)\theta]\phi_M}{(\gamma-\phi_M)(1-\gamma\phi_M)}, \tag{16}$$

$$n^* = \frac{\mu L^w\gamma}{R\sigma}\frac{(1-\theta)(w^*+R) + \theta(w+R)\phi_M^2 - [w^*+R+(w-w^*)\theta]\gamma\phi_M}{(\gamma-\phi_M)(1-\gamma\phi_M)}, \tag{17}$$

$$R = R^* = \frac{[\theta w + (1-\theta)w^*]\mu}{\sigma-\mu}, \tag{18}$$

respectively, from Eqs. (14) and (15), and the fact that $R = R^*$. On the other hand, if firms fully agglomerate in country N, we have $n = L^w$, and R is equal to Eq. (18) again. Eqs. (16) and (17) are important, since they express the relationship between firm location and the relative marginal costs (MC/MC*). This relationship is established later (Lemma 5).

The supply of good A in each country is equal to the labour force in A, and thus, we have:

$$s_A = L - nl - \frac{nte}{w}, s_A^* = L^* - n^*l^* - n^*te^*, \tag{19}$$

where nte/w and n^*te^* are the amount of labour employed by the government in N and S for R&D investment for adaptation, respectively. Therefore, from Eqs. (11) and (19), the import of good A in countries N and S is expressed as:

$$\text{IM}_A \equiv d_A - s_A = \frac{(1-\mu)E}{p_A} - L + nl + \frac{nte}{w}, \tag{20}$$

$$\text{IM}_A^* \equiv d_A^* - s_A^* = (1-\mu)E^* - L^* + n^*l^* + n^*te^*,$$

respectively. Country N is the importer of good A if $\text{IM}_A>0$, while N is the exporter of good A if $\text{IM}_A<0$.

If N imports (respectively, exports) A, $w = p_A = \tau_A$ (respectively, $w = p_A = 1/\tau_A$) holds, since domestic A and imported A should have identical prices. By this equation and Eqs. (16), (17) and (18), all endogenous variables are determined. Even if good A is not traded, endogenous variables are determined in a similar way. In that case, however, neither $w = p_A = \tau_A$ nor $w = p_A = 1/\tau_A$ holds. Instead, Eq. (20) can be used to determine w. Since the domestic prices of A must be cheaper than the import prices of A in this case, it holds that $p_A = w < \tau_A$ in N and $p_A^* = 1 < w\tau_A$ in S, leading to $w \in (1/\tau_A, \tau_A)$. Taking both results into account, we have:

$$w \in \left[\frac{1}{\tau_A}, \tau_A\right]. \tag{21}$$

Finally, using Eq. (1), the indirect utility of workers in N and S is obtained as:

$$V = \frac{\mathcal{E}(w + \overline{R})}{p_A^{1-\mu}P^\mu} \text{ and } V^* = \frac{\mathcal{E}(1 + \overline{R})}{(P^*)^\mu}, \tag{22}$$

respectively.[12]

In the case without emission taxes, the present model is equivalent to the FC model of Martin and Rogers (1995), with trade costs of good A, which was first analysed by Takatsuka and Zeng (2012). They showed that there is a threshold value $\hat{\tau}_A$ of transport costs for the homogeneous good, so that *(i)*

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[12] For simplicity, a constant multiplier, $\mu^\mu(1-\mu)^{1-\mu}$, is omitted in each equation.

the larger country imports good A if $\tau_A < \hat{\tau}_A$*; otherwise, good A is not traded; and (ii) the larger country is always the net exporter of good M, and it always holds that* $n/(n + n^*) > \theta$ (Takatsuka and Zeng 2012, Theorem 1). This implies that, without emission taxes, the larger country has a more-than-proportionate share of firms and a higher wage.[13] To guarantee that good A is traded at least without taxes, we assume that $\tau_A < \hat{\tau}_A$ holds in what follows.

3. Tax effects on production in sector *M*

This section shows some findings related to the effects of emission taxes on production in sector M. They are also building blocks for the main results presented in the next section.

Although wages are endogenously determined in the model, we assume that they are fixed here. Thus, since N is the importer of good A from our assumption, it holds that $w = \tau_A$ (>1). The next section shows that country N always imports good A from S, irrespective of tax rates, which implies that the equation of $w = \tau_A$ endogenously holds. According to Figure 1, as explained below Eq. (4), if and only if the tax–wage ratio t/w (respectively, t/w^*) is higher than $\alpha m/(1 - \alpha)$, abatement is active in country N (respectively, S). Since $w = \tau_A > 1 = w^*$, the following three phases are possible:

$$\text{Phase I} : t \in \left[0, \frac{\alpha m}{1-\alpha}\right), \text{and abatement is inactive in both countries;}$$

$$\text{Phase II} : t \in \left[\frac{\alpha m}{1-\alpha}, \frac{\alpha m \tau_A}{1-\alpha}\right), \text{and abatement is active only in S;}$$

$$\text{Phase III} : t \in \left[\frac{\alpha m \tau_A}{1-\alpha}, \infty\right), \text{and abatement is active in both countries.}$$

Appendix A1(i) shows the tax effects on labour input (l) and output per firm (x). Figure 2 depicts the labour input of each firm in N as a function of the tax rate t. The figure shows that labour input necessarily decreases in t when abatement is inactive; otherwise, it depends on the substitutability between GHG emissions and labour, that is the parameter ρ. Specifically, labour input increases (respectively, decreases) in t if $\rho > 0$ (respectively, $\rho < 0$). This implies that, when tax rates are higher, more (respectively, less) labour is used to reduce emissions if labour and emissions are gross substitutes (respectively, complements). As in Figure 2, labour input comes back to the level without taxes as $t \to \infty$ if $\rho > 0$. In other words, labour used for abatement fully compensates for the decrease in labour used for production in this case.

Similar results hold for firms in S. In summary, we have the following lemma (see Appendix A1(i) for the proof):

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[13] If good A is tradable, it holds that $w = \tau_A$, since N imports A; otherwise, w is the smallest τ_A that prevents country N from importing A (i.e. $\hat{\tau}_A$).

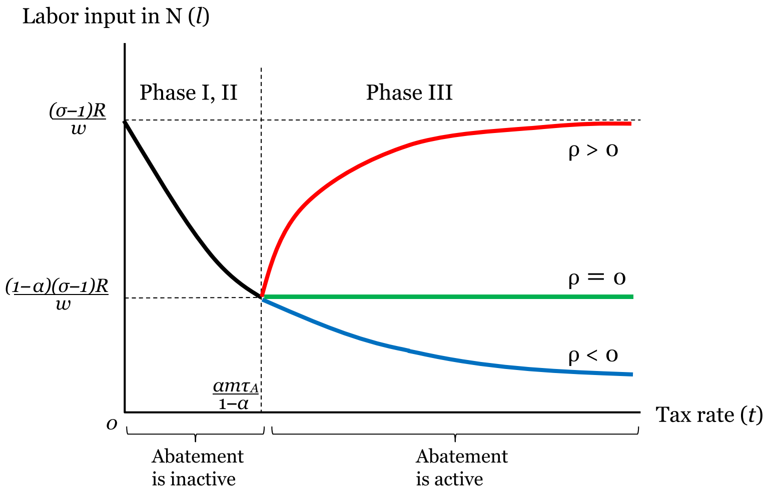

Figure 2 Labour input of a firm in country N. [Colour figure can be viewed at wileyonlinelibrary.com]

Lemma 1: *Assume that wages are fixed. For any firm in any country,*

(i) when $\rho < 0$ (i.e. e and l are gross complements), labour input and output decrease with the tax rate t and converge to zero as $t \to \infty$;

(ii) when $\rho > 0$ (i.e. e and l are gross substitutes), labour input follows a U-shaped curve in response to t and converges to the level without taxes as $t \to \infty$, while output decreases in t and converges to the non-tax level multiplied by $(1-\alpha)^{1/\rho}$ as $t \to \infty$.

Concerning the output per firm, this lemma suggests that when $\rho > 0$, a certain amount of each variety is necessarily produced, even if tax rates are sufficiently high, because abatement activities are effective enough to allow production.

Next, Appendix A1(ii) shows the tax effects on tax payment per firm (te). Figure 3 depicts the tax payment of each firm in N as a function of the tax rate, t. The figure shows that tax payments necessarily increase in t when abatement is inactive, although they depend on ρ when abatement is active. Specifically, tax payments decrease (respectively, increase) in t if $\rho > 0$ (respectively, $\rho < 0$). This implies that when tax rates are higher, GHG emissions decrease more (respectively, less) significantly if labour and emissions are gross substitutes (respectively, complements), since emissions are easily abated by additional labour input. The decreasing rate of GHG emissions is sufficiently higher than the increasing rate of taxes, which results in $\lim_{t\to\infty} te = 0$. Meanwhile, the decrease in emissions is slower when $\rho \leq 0$; thus, the tax payment of each firm remains positive even if $t \to \infty$.

Similar results hold for firms in S. In summary, we have the following lemma (see Appendix A1(ii) for the proof):

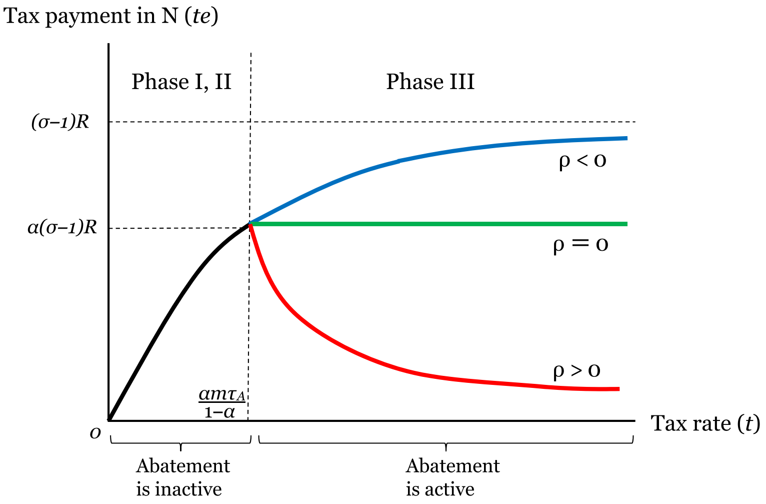

Figure 3 Tax payment of a firm in country N. [Colour figure can be viewed at wileyonlinelibrary.com]

Lemma 2: *Assume that wages are fixed. For any firm in any country,*

(i) when $\rho < 0$ (i.e. e and l are gross complements), tax payments increase in t and converge to $(\sigma - 1)R$ as $t \to \infty$;

(ii) *when $\rho > 0$ (i.e. e and l are gross substitutes), tax payments follow an inverted U-shaped curve in response to t and converge to zero as $t \to \infty$.*

Lemmas 1 and 2 show that labour costs per firm and tax payments per firm both change with t. However, they completely cancel each other out. In fact, we can show that the following lemma holds (see Appendix A2 for the proof):

Lemma 3: *Assume that wages are fixed. For any firm in any country*, *the sum of labour costs and tax payments is a constant, $(\sigma - 1)R$.*

This lemma is useful for examining the trade direction in Section 4. It is also noteworthy that Lemmas 2 (i) and 3 suggest that when $\rho < 0$, labour costs converge to zero as $t \to \infty$. This is consistent with the result of Lemma 1 (i).

Finally, we observe the tax effects on the relative marginal cost in N, that is MC/MC^*. As shown later, this ratio is important for determining the firm location and relative welfare. Appendix A3 shows that the following lemma holds:

Lemma 4: *Assume that wages are fixed.*

(i) When $\rho < 0$ (i.e. e and l are gross complements), MC/MC^* decreases with tax rate t and converges to one as $t \to \infty$.

(ii) When $\rho > 0$ (i.e. e and l are gross substitutes), MC/MC^* follows a U-shaped curve in response to t (i.e. decreases in t up to a tax rate in Phase II and increases thereafter) and converges to the level without taxes as $t \to \infty$.

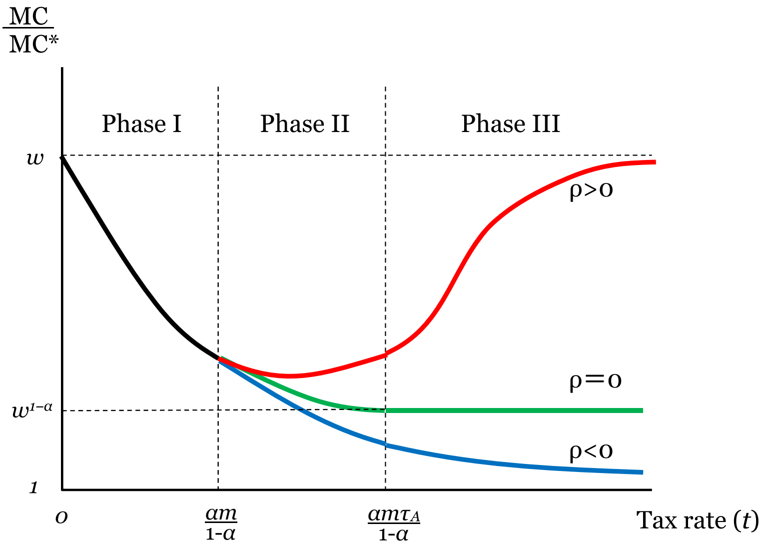

Figure 4 Relative marginal costs in country N. [Col2our figure can be viewed at wileyonlinelibrary.com]

The tax effects on MC/MC^* are represented in Figure 4. This result can be explained by the balance between advantages and disadvantages of the lower-wage country, S. When the elasticity of substitution between the two inputs is low enough, the production function of Eq. (4) resembles a Leontief production function, $x = \min\{e, l/m\}$. This implies that, when ρ is very low, the situations of Phases II and III are similar to that of Phase I; the marginal cost in the lower-wage country relatively increases because of an identical tax payment for additional production in both countries. Thus, MC/MC^* monotonically decreases in t. This is the disadvantage of country S. Meanwhile, when ρ is higher, and thus the two inputs are more substitutable, the advantage of country S dominates. In this case, abatement is more effective for lowering production costs when the emission price is higher. Since abatement costs are cheaper in S due to lower wages, MC/MC^* increases in t for a higher t.

4. Tax effects on firm location and welfare

This section examines how uniform emission taxes affect the relative number of firms and the relative welfare. We especially focus on the interior equilibrium at which the number of firms $n, n^* \in (0, L^w)$ and $R = R^* = \overline{R}$ hold, because the analysis of the corner equilibrium does not add much essential insight.

First, we can show the following lemma on the relationship between the relative marginal cost and firm location (see Appendix A4 for the proof):

Lemma 5: *In the interior equilibrium, n decreases and* n^* *increases with MC/MC** *for fixed wages.*

The message of this lemma is simple and intuitive. A higher MC/MC* suggests a negative effect on the production in N, since it means a relatively higher marginal cost there, which results in a relatively higher price and less demand. Thus, as shown by Lemma 5, the number of firms in N decreases.

Next, it is shown that country N always imports good A from S, irrespective of tax rates; thus, it necessarily holds that $w = \tau_A$ (>1). This can be explained as follows. As reported at the end of Section 2, country N imports good A from S without taxes. Thus, with a *marginal tax*, N still imports good A; thus, $w = p_A = \tau_A$ holds, and R does not change from Eq. (18). Here, we note that the import of good A in N is expressed by:

$$\mathrm{IM}_A = \underbrace{\frac{(1-\mu)(w+R)Lv}{w}}_{\text{demand of } A \text{ in N}} - \underbrace{\left[L - \frac{n}{w}(wl + te)\right]}_{\text{supply of } A \text{ in N}} \tag{23}$$

from Eqs. (3) and (20). The first term of Eq. (23) expresses the demand of A in N, which is constant. The square-bracket term of Eq. (23) represents the supply of A in N, since it is the difference in the total labour force in N (i.e. L) and the employment in sectors M and G of N (i.e. $nl + te/w$). From Lemma 3, the sum of labour costs and tax payments is constant for a firm in each country, which suggests that $wl + te$ is constant in the square-bracket term of Eq. (23). Meanwhile, from Lemmas 4 and 5, we know that MC/MC* decreases, and thus, n increases with a small tax. Hence, the supply of A in N decreases, and the import of good A in N increases. Intuitively, since the relative production costs in N are lowered by the tax, workers in N shift from sector A to sectors M and G, which results in the expanded import of good A.

In the following, the cases of $\rho < 0$ and $\rho > 0$ are examined separately. When $\rho < 0$, MC/MC* monotonically decreases with tax rate t from Lemma 4 (i). Thus, the above fact confirms that N continues to import good A, and *wages are unchanged* for *any t*.

Even when $\rho > 0$, MC/MC* monotonically decreases in t up to a tax rate in Phase II (Lemma 4 (ii)), suggesting that a similar result holds if t is less than the threshold tax rate, as in the case of $\rho < 0$. When t exceeds the threshold tax rate, MC/MC* increases monotonically. This implies that n starts to decrease monotonically, and thus, according to Eq. (23), the import of good A in N also decreases with t. Appendix A5 shows that, nevertheless, the import of good A in N is still positive for a sufficiently large t. Therefore, we conclude that even when $\rho > 0$, N continues to import good A, and *wages are unchanged* for *any t*.

Lemma 6: *For any tax rates, country N imports good A from S, and* $w = \tau_A$ *holds.*

For the case of $\rho < 0$, we obtain the following proposition:[14]

Proposition 1: *(Effects of emission taxes: the case of* $\rho < 0$*). When* $\rho < 0$ *(i.e. e and l are gross complements),*

(i) the export of good A from S to N increases with tax rate t, and the wages are unchanged;

(ii) the relative marginal cost, MC/MC^**, decreases in t;*

(iii) the relative number of firms, $n/(n+n^*)$*, and the relative welfare,* V/V^**, increase in t;*

(iv) the output and the tax payment per firm in each country converge to zero and $(\sigma - 1)R$*, respectively, as* $t \to \infty$*.*

The claims in the proposition, except for the relative welfare, are obtained via Lemmas 1-6. In a straightforward manner, the relative price index, P/P^*, is expressed by

$$\frac{P}{P^*} = \left[\frac{n(\mathrm{MC})^{1-\sigma}\phi_M + n^*(\mathrm{MC}^*)^{1-\sigma}}{n(\mathrm{MC})^{1-\sigma} + n^*(\mathrm{MC}^*)^{1-\sigma}\phi_M}\right]^{\frac{1}{\sigma-1}} = \left[\frac{n\gamma\phi_M + n^*}{n\gamma + n^*\phi_M}\right]^{\frac{1}{\sigma-1}}$$

from Eqs. (8) and (10). Thus, we know that P/P^* is decreased by taxes, since $\mathrm{MC}/\mathrm{MC}^*\left(=\gamma^{\frac{1}{1-\sigma}}\right)$ decreases, and n/n^* increases. Therefore, according to Eq. (22),

$$\frac{V}{V^*} = \frac{(w+R)/\left(w^{1-\mu}P^{\mu}\right)}{(1+R)/(P^*)^{\mu}} = \frac{w+R}{w^{1-\mu}(1+R)}\left(\frac{P^*}{P}\right)^{\mu}$$

must increase.

This proposition suggests that if $\rho < 0$, emission taxes necessarily accelerate firm agglomeration in the larger country, where the relative welfare is improved, as compared to the case without taxes. Since substitutability between emissions and labour (i.e. ρ) is low, country S does not have a great advantage with regard to abatement costs, even though the wage is lower in that country.[15] Instead, a disadvantage (identical tax payments for additional emissions in the lower-wage country) is dominant; thus, firms relocate to the larger country.

We obtain the following proposition for the case of $\rho < 0$:

Proposition 2: *(Effects of emission taxes: the case of* $\rho > 0$*). When* $\rho > 0$ *(i.e. e and l are gross substitutes) and the tax rate rises from zero to a sufficiently high value,*

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[14] When ρ tends to 0 (a Cobb–Douglas case), (i)-(iii) in Proposition 1 still hold in Phases I and II, while all variables in (i)-(iii) are unchanged in Phase III.

[15] It is noteworthy that country S is always the lower-wage country, since $w = \tau_A > 1 = w^*$ according to Lemma 6.

(i) S continues to export good A to N, and the wages are unchanged;
(ii) the relative marginal cost, MC/MC^**, follows a U-shaped curve, and it is maximised at* $t = 0, \infty$*;*
(iii) the relative number of firms, $n/(n + n^*)$*, and the relative welfare,* V/V^**, follow inverted U-shaped curves, and both are minimised at* $t = 0, \infty$*; and.*
(iv) the output and the tax payment per firm in each country converge to the non-tax level multiplied by $(1 - \alpha)^{1/\rho}$ *and zero, respectively, as* $t \to \infty$*.*

Through Propositions 1 and 2, we find that the two cases, $\rho < 0$ and $\rho > 0$, contrast with each other. First, the relative number of firms and relative welfare in the large country monotonically increase with tax rates when $\rho < 0$, while they follow inverted U-shaped curves when $\rho > 0$. As mentioned below Lemma 4, this is due to the balance of an advantage and a disadvantage of the lower-wage country. When $\rho > 0$, the advantage in S, *the lower cost of abatement activities*, is dominant if tax rates are high. Meanwhile, when $\rho < 0$, the disadvantage in S is dominant. In other words, because of *a relative rise in marginal production costs*, firms relocate to the large country.

Second, as the tax rate becomes sufficiently high, the output becomes zero, and the tax payment converges to a positive value when $\rho < 0$. This implies that, when emissions and labour are gross complements, governments take the initiative to improve the environment, since the tax revenue is used for R&D investment for adaptation, while the abatement activities of firms vanish. Meanwhile, when $\rho > 0$, the output remains positive, and the tax payment becomes zero as $t \to \infty$. Hence, when emissions and labour are gross substitutes, private firms take the lead in improving the environment via abatement activities if t is high enough.

Despite these differences in the two cases, we have a common result: for any level of taxes, the relative number of firms and relative welfare in the large country are higher than those in the case without taxes. In fact, if $\rho > 0$, they follow inverted U-shaped curves in response to tax rates, but they are not below the non-tax level. They converge at the non-tax level as $t \to \infty$. As in Eqs. (16) and (17), the firm number in each country is determined by wages and the relative marginal cost. On one hand, wages are unchanged, since the large country continues to import good A from S. On the other hand, if $\rho > 0$, the relative marginal cost comes back to the level without taxes as $t \to \infty$ (Lemma 4 (ii)). As mentioned in Section 3, when $\rho > 0$, GHG emissions significantly decrease with the increase in the tax rate, and the marginal costs of production (including abatement) become proportionate to the wage in each country again.

5. Capital-using abatement technology

As discussed above, the cost (wage) differential among countries plays an important role in our results. The cost differential is generated by the immobility of labour. In reality, however, a mobile factor (i.e. capital) is often used not only for production but also for abatement. If mobile capital and

GHG emissions are used as the main variable inputs in the production of emitting sectors and GHG emissions can be reduced by using additional capital, the cost differential among countries vanishes due to international mobility of capital. This section nevertheless shows that our main results basically hold even in that case.

5.1 Extending the model

In the FC model, capital is usually used as a fixed input in sector M, as in the present model, so that such capital is often interpreted as firm-specific input, such as patents (e.g. Baldwin *et al.* 2003). We also follow this interpretation, and we introduce a different type of capital as variable input, such as machinery. In this section, the former type of capital is called as *knowledge* capital (its return is denoted by R), while the latter type is called *physical* capital (its return is denoted by r). Specifically, there are three types of factors (labour, knowledge capital and physical capital), and each worker owns one unit of each type of factor. Both types of capital are freely mobile among countries.

Instead of Eq. (4), the output, x, of each variety is described as a linearly homogeneous function of composite input, $k^{\beta}l^{1-\beta}$, and e:

$$x = \min\left\{ \left[\alpha e^{\rho} + (1-\alpha)\left(\frac{k^{\beta}l^{1-\beta}}{m} \right)^{\rho} \right]^{\frac{1}{\rho}}, \frac{k^{\beta}l^{1-\beta}}{m} \right\}, \tag{24}$$

where k is physical capital input and $\beta \in 0,1)$ is capital share, while other symbols are the same as in (4). Our base model is the case of $\beta = 0$. From cost minimisation, the price of a composite of k and l in N is:

$$p_c = \left\{ \left[\frac{(1-\beta)}{\beta} \right]^{\beta} + \left[\frac{\beta}{(1-\beta)} \right]^{1-\beta} \right\} r^{\beta} w^{1-\beta}.$$

When labour is replaced with the composite, the interpretation of Eq. (4) holds for Eq. (24). For example, MC expressed by Eq. (5) can be applied to the general model by replacing w with p_c. If capital share β approaches one, the cost differential among countries vanishes, because we have $\lim_{\beta \to 1} p_c = r$.

Since individual income includes physical capital return r, if we replace w with $w + r$, the market-clearing conditions of Eqs. (14) and (15) and the equilibrium values of Eqs. (16), (17), (18) and (22) hold. In this general model, a new endogenous variable, that is physical capital return r, is added. The market-clearing conditions of physical capital, $nk + n^{*}k^{*} = L^{w}$, determine the equilibrium physical capital return r. Finally, other assumptions are essentially the same as in the basic model examined in the previous sections.

As in Section 3, if we assume that wages are fixed (i.e. $w = \tau_A (>1)$), we have three possible phases. Although r could change with t, it holds that $p_c > p_c^{*}$

for a given t, since $w > w^* = 1$. Thus, when t increases from zero, the following three phases emerge, in order:

$$\text{Phase I} : t \in \left[0, \frac{\alpha m p_c^*}{1-\alpha}\right), \text{and abatement is inactive in both countries;}$$

$$\text{Phase II} : t \in \left[\frac{\alpha m p_c^*}{1-\alpha}, \frac{\alpha m p_c}{1-\alpha}\right), \text{and abatement is active only in S;}$$

$$\text{Phase III} : t \in \left[\frac{\alpha m p_c}{1-\alpha}, \infty\right), \text{and abatement is active in both countries.}$$

Different from the base model, the general model makes it difficult to obtain analytical results. The source of difficulty is the endogeneity of r. In the base model, firm relocation is driven only by the relative marginal cost $\mathrm{MC}/\mathrm{MC}^*$, which is driven only by tax rate t. This simple structure allows us to obtain analytical results. Meanwhile, in the general case, the relative number of firms and the relative marginal cost are both indirectly driven by r, which may change with t, because r impacts individual income as well as production costs. Thus, instead of obtaining analytical results, this paper executes numerical simulations to check the robustness of our results. In fact, numerical simulations are often used to complement analytical results in the field of New Economic Geography (e.g. Fujita *et al.* 1999), since analysis in the field tends to be complex. Needless to say, numerical simulations have an advantage. Qualitative analyses never provide quantitative information about spatial disparities generated by a common price commitment, but numerical simulations can do.

5.2 Results

Numerical simulations suggest that the main results in the base model hold even if capital share β is so high that the cost differential among countries is negligible. Specifically, N imports good A from S for any t, so that $w > \tau_A = 1$, and (i) when $\rho < 0$, the relative number of firms and the relative welfare in N monotonically increase in tax rates; (ii) when $\rho > 0$, they both follow inverted U-shaped curves; and (iii) they both are necessarily higher than those in the case without taxes. Although I have obtained such results under various parameter combinations, the present paper shows numerical examples with $\sigma = 5$, $\mu = 0.5$, $\alpha = 0.5$, $m = 1$, $\theta = 0.6$, $L^w = 1$, $\phi_M = 0.5$ and $\tau_A = 1.01$ below.[16]

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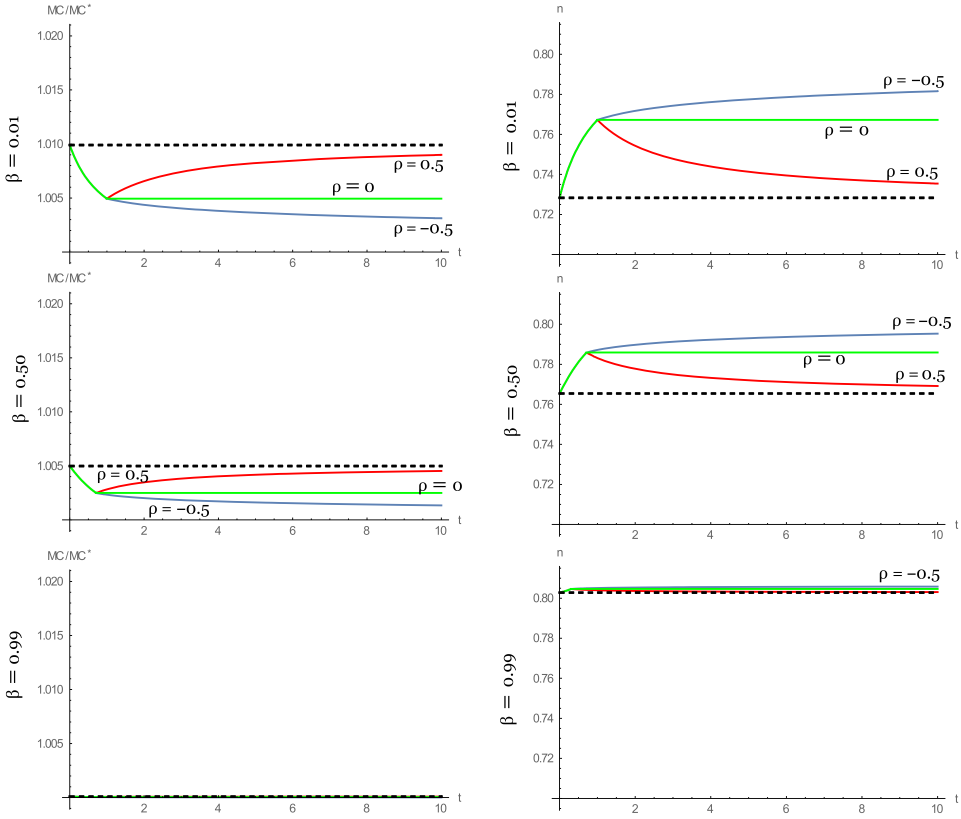
[16] Recent "Quantitative Spatial Economics" (e.g. Redding and Rossi-Hansberg 2017) set σ equal to around 5. I set $\mu = 0.5$ to guarantee the assumption that the consumption share of good A (i.e. $1 - \mu$) is sufficiently large so that the homogeneous good is produced in each country for any equilibria. Also, I set $\tau_A = 1.01$ to guarantee the assumption that country N is the importer of good A without taxes. Although other parameter values are arbitrary, they are not essential for the results.



(a) Relative marginal costs in N **(b)** Firm share in N

Figure 5 Numerical examples: different β. [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 5 shows how some key variables change when the tax rate, t, increases, and indicates that (i)-(iii) hold.[17] The four panels of the figure feature (a) relative marginal costs (i.e. MC/MC*), (b) the relative number of firms in N (i.e. $n/(n+n^*)$), (c) relative welfare (i.e. V/V^*) and (d) physical capital rent (i.e. r). Each panel has three graphs: the top, middle and bottom correspond to the case of $\beta = 0.01$, $\beta = 0.50$ and $\beta = 0.99$, respectively. The case of $\beta = 0.01$ is almost equivalent to the base model with $\beta = 0$. In fact, each panel reproduces the results obtained in the previous sections.[18]

When β approaches one, the relative marginal cost converges to one, irrespective of the value of ρ (see the bottom graph of Panel (a)), since there is no production-cost differential among countries due to the mobility of physical capital. Although the relative marginal cost MC/MC* remains

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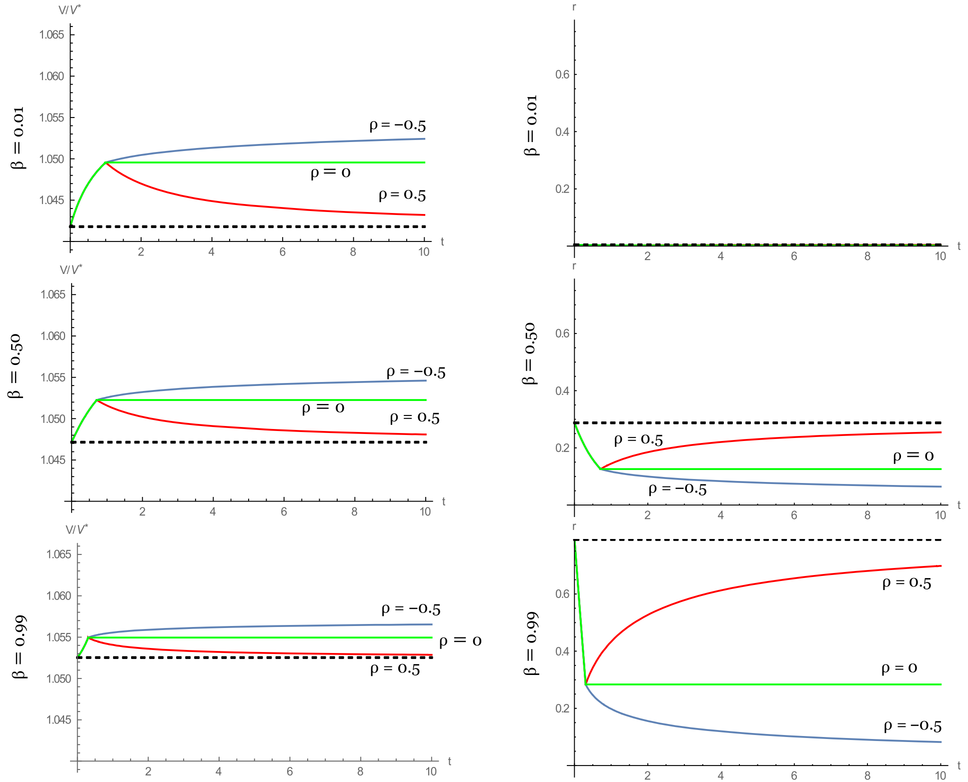
[17] If $\mathcal{E}$ in the utility function of Eq. (1) and world welfare are specified, optimal uniform tax rates can be obtained. A numerical example is shown in Appendix A6.

[18] For example, Panel (a) corresponds to Figure 4 for the base model. Note that the interval of tax rates for Phase II is very narrow in the examples. In the case of $\beta = 0.01$, Phase II emerges for $t \in [0.9951, 1.0049)$.



(c) Relative welfare in N **(d)** Physical-capital rent

Figure 5 (continued) [Colour figure can be viewed at wileyonlinelibrary.com]

around one, the relative number of firms and relative welfare could change, corresponding with a rise in tax rates. This is because, instead of the relative marginal costs, physical capital rent, r, drives the firm location and relative welfare. When r falls, the relative expenditure in the higher-wage country (i.e. N), E/E^*, increases, since capital income is common among countries, while wage income is not. Thus, when r falls, country N attracts more firms since its market becomes relatively larger. At that time, the relative welfare in N is raised more significantly, not only from this firm-relocation effect but also from the direct effect of relative expenditure. When r rises, the opposite movement occurs.

The bottom of Panel (d) shows how r changes corresponding to a rise in t when β is high enough. Specifically, r monotonically decreases if $\rho < 0$. This is because the demand for physical capital is necessarily decreased by taxing emissions in the case that capital and emissions are gross complements, even if abatement is active. Meanwhile, r evolves as a U-shaped curve if $\rho > 0$, since the demand for physical capital is increased by taxing emissions in the case that capital and emissions are gross substitutes, if abatement is active.[19] Such movement of r drives the firm location and relative welfare as depicted in the bottom graphs in Panels (b) and (c), respectively, even if the international difference in marginal costs vanishes.

5.3 Values of ρ

Our results show that, regardless of the use of mobile capital as variable inputs, the effects of emission taxes depend on whether emissions and the conventional input (i.e. labour and capital) are gross substitutes ($\rho > 0$) or gross complements ($\rho < 0$). Hyman *et al.* (2002) treat non-CO_2 GHG emissions as additional input in the production process, and a CES function is used to characterise the substitutability between emissions and the use of a composite input. Then, the elasticity of substitution between emissions and conventional inputs is calibrated for each type of GHG and for each industry. According to their calibration (p. 180, table 3), the observed values of the elasticity of substitution are very low. For example, in the case of methane (CH_4), the observed value is from 0.20 to 0.30. In terms of ρ, it is from -49 to -2.3. In the case of nitrous oxide (N_2O), the observed value is from 0 to 1; thus, ρ is negative again.

It is noteworthy, however, that they focus on non-CO_2 GHG abatement.[20] Since non-CO_2 GHGs account for only 24% of total GHG emissions (Intergovernmental Panel on Climate Change 2015), their finding that ρ is negative could not be simply extended to the relationship between general GHG emissions and conventional inputs.

6. Conclusion

This paper examines how a harmonisation of taxes on GHG emissions affects spatial disparities of firm location and welfare. To this aim, we employ a two-country model of monopolistic competition with variable abatement technology, where GHG emissions are treated as an input to the production function and can be substituted by the conventional inputs such as labour and capital. Two countries are almost symmetric but differ only in their size. It is shown that, then, the elasticity of substitution between GHG and conventional input is a key parameter. Regardless of the elasticity of substitution, however, uniform emission taxes always widen international disparities of firm location and welfare. The results suggest that such a

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[19] If abatement is active, physical capital input per firm in N is $k = \left(\frac{\beta}{1-\beta}\frac{w}{r}\right)^{1-\beta}\left(\frac{1-\alpha}{m^{\rho}}\frac{\mathrm{MC}}{p_c}\right)^{\frac{1}{1-\rho}}x$. From this equation, other things being equal, we can show that $\partial k/\partial t \lesseqgtr 0$ if abatement is active and $\rho \lesseqgtr 0$.

[20] According to Hyman *et al.* (2002), CO_2 emissions are usually estimated in proportion to the activity levels of the coal, oil and gas industries in CGE models.

harmonisation may be difficult; hence, it will be infeasible without some kind of transfer between countries. This is consistent with the fact that the 'pledge and review' mechanism confirmed at the Paris COP21 is still far from the idea of common price commitment (e.g. Gollier and Tirole 2015; Cramton *et al.* 2017). Different from the literature featuring disparities among countries with different tastes and/or technologies (e.g. Kverndokk 1993; Metcalf 1999; Dinan and Rogers 2002; Cremer *et al.* 2003; Parry 2004; Gollier and Tirole 2015), this paper suggests that addressing the distributional problem (i.e. transfers) is required even among similar developed countries.

Throughout the present paper, it is assumed that a common price commitment is implemented as an internationally harmonised, but nationally retained, emission tax. According to Ranson and Stavins (2016), 36 countries had cap and trade (C&T) systems operating or scheduled in 2014. In addition, most of them have established or proposed at least one linkage with another C&T system. If the caps were freely traded as allowances between linked countries, there would be one uniform price of emissions among the countries. This suggests that a common price commitment can be also implemented as linking C&T systems. In fact, without uncertainty, emission taxes and C&T systems are equivalent, since, for a given price target, it is always possible to determine the supply of permits (i.e. cap) that will support this equilibrium price.[21] Flachsland *et al.* (2009) and Ranson and Stavins (2016) qualitatively examined the impacts of linking C&T systems. Although they argue that the distributional impacts will depend on marginal abatement costs in the linked countries and how the allowances are allocated, they do not refer to the effects via firm relocation. Meanwhile, Rose *et al.* (2018) clarified the effects of linking C&T systems in helping to implement the GHG reduction pledges at the Paris COP21 by using numerical simulations. Again, the effects via firm relocation were not considered, while they debated the equity issues between highest- and lowest-income countries. I believe that, hence, the present paper will contribute to the literature on linking C&T systems.

In the present model, there are only two countries, and they form an agreement requiring the harmonisation of emission taxes. In reality, however, there are other countries not bound by the agreement. Interactions with a third country are not considered in the paper. In addition, if the present model is applied in a domestic (regional) context, we would need to consider interregional labour mobility as well as the rest of the world. To address such situations, extending the present model to a two-country and four-region framework would then be a useful, interesting and challenging task for further research.

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[21] With uncertainty, however, they are different. Gollier and Tirole (2015), Stiglitz (2015), Weitzman (2015), and Cramton *et al.* (2015b) examined which would likely have an advantage from various viewpoints.

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Appendix

A1. Tax effects on labour input, output and tax payment

(i) First, we consider the tax effects on labour input in each firm. Without emission taxes, labour input is $(\sigma - 1)R/w$ from Eqs. (5), (12), and the fact that each firm uses m units of labour to produce one unit of its variety (i.e. point F in Figure 1). If taxes are imposed but emission abatement is inactive in N, we have $l = mx = mR(\sigma - 1)/(mw + t)$, where the second equality is from the first equation of Eq. (5) and Eq. (12). Since the capital return R is independent of taxes from Eq. (18), we immediately know that t decreases in t. This is because a rise in the tax rate increases marginal costs, and thus each firm reduces production for saving production costs, as shown in Eq. (12).

On the other hand, if emission abatement is active in N, we obtain the following labour-input equation for each firm in N:

$$l = \underbrace{\left(\frac{1-\alpha}{m^{\rho}}\frac{\mathrm{MC}}{w}\right)^{\frac{1}{1-\rho}}}_{\text{labor input per output}} \underbrace{\frac{(\sigma-1)R}{\mathrm{MC}}}_{\text{output}}, \tag{25}$$

from Eqs. (6) and (12). The first and the second terms in this equation show labour input per output and output, respectively. From the first term, we know that labour input per output (l/x) *increases* in t. This is because labour use is more reasonable when the emission price becomes higher relative to the wage. This tendency is more significant when the elasticity of substitution between emissions and labour is higher (i.e. ρ is higher). Meanwhile, as mentioned above, each firm *reduces* production to save production costs when marginal costs are higher. This is captured by the second term of Eq. (25). Thus, the tax effects on labour input via the first and the second terms of Eq. (25) are opposite each other. From Eq. (25), we know that the positive (respectively, negative) effect through the first (respectively, second) term is dominant if and only if $\rho > 0$ (respectively, $\rho < 0$) and the two effects cancel each other out when $\rho = 0$. Furthermore, since it holds that

$$\lim_{t\to\infty} \mathrm{MC}^{\frac{\rho}{1-\rho}} = \begin{cases} 0 & \text{if } \rho < 0 \\ 1 & \text{if } \rho = 0 \\ \left(\frac{mw}{(1-\alpha)^{\frac{1}{\rho}}}\right)^{\frac{\rho}{1-\rho}} & \text{if } \rho > 0 \end{cases} \tag{26}$$

from Eq. (5), we obtain the following equation:

$$\lim_{t\to\infty} l = \begin{cases} 0 & \text{if } \rho < 0 \\ (1-\alpha)(\sigma-1)R/w & \text{if } \rho = 0\,. \\ (\sigma-1)R/w & \text{if } \rho > 0 \end{cases} \tag{27}$$

It is noteworthy that Eqs. (26) and (27) hold even if $w \neq \tau_A$ since w is finite from Eq. (21).[22]

From Eq. (5), we obtain $\lim_{t\to\infty} \mathrm{MC} = mw/(1-\alpha)^{1/\rho}$ if $\rho > 0$; otherwise, $\lim_{t\to\infty}\mathrm{MC} = \infty$. Since MC monotonically increases in t, output of Eq. (12) monotonically decreases in t and:

$$\lim_{t\to\infty} x = \begin{cases} 0 & \text{if } \rho \leq 0 \\ \frac{(1-\alpha)^{\frac{1}{\rho}}(\sigma-1)R}{mw} & \text{if } \rho > 0 \end{cases}.$$

Without taxes, output of a firm in N is $(\sigma-1)R/(mw)$ from Eqs. (5) and (12). Thus, if $\rho > 0$, output per firm converges to the non-tax level multiplied by $(1-\alpha)^{1/\rho} < 1$ as $t \to \infty$.

(ii) When abatement is inactive, the GHG emission of each firm in N (e) is equal to output (x). Hence, tax payment of each firm in N is $te = tx = t(\sigma-1)R/\mathrm{MC}$ from Eq. (12). This monotonically increases in t since MC/t decreases in t from Eq. (5).

If abatement is active, it holds that:

$$e = \left(\alpha \frac{\mathrm{MC}}{t}\right)^{\frac{1}{1-\rho}} \frac{(\sigma-1)R}{\mathrm{MC}} \tag{28}$$

from Eqs. (7) and (12). Since MC increases and MC/t decreases in t, e monotonically decreases in t again. In addition, the GHG emission of each firm converges to zero as $t \to \infty$.[23] In addition, we have

$$te = \left(\frac{\mathrm{MC}}{t}\right)^{\frac{\rho}{1-\rho}} \alpha^{\frac{1}{1-\rho}} (\sigma-1)R \tag{29}$$

from Eq. (28). This implies that tax payment of each firm decreases (respectively, increases) if and only if $\rho > 0$ (respectively, $\rho < 0$). This is because GHG emissions are more (respectively, less) significantly abated by additional labour input if $\rho > 0$ (respectively, $\rho < 0$). Furthermore, we obtain

$$\lim_{t\to\infty} te = \begin{cases} (\sigma-1)R & \text{if } \rho < 0 \\ \alpha(\sigma-1)R & \text{if } \rho = 0 \\ 0 & \text{if } \rho > 0 \end{cases} \tag{30}$$

by using Eqs. (5) and (26).

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[22] The limit equations obtained in the following also hold even if $w \neq \tau_A$.

[23] Note that $\lim_{t\to\infty}\mathrm{MC}^{\rho/(1-\rho)}$ is finite from (26).

A2. Proof of Lemma 3

When abatement is inactive, we have

$$\underbrace{wl}_{\text{labour costs}} + \underbrace{te}_{\text{tax payments}} = wmx + tx = (mw + t)\tfrac{(\sigma-1)R}{\mathrm{MC}} = (\sigma - 1)R,$$

which suggests that the sum of labour costs and tax payments is constant. Meanwhile, from Eqs. (25) and (29), when abatement is active, we have:

$$\begin{aligned}\underbrace{wl}_{\text{labour costs}} + \underbrace{te}_{\text{tax payments}} &= \left[\left(\tfrac{\mathrm{MC}}{w}\right)^{\frac{\rho}{1-\rho}}\left(\tfrac{1-\alpha}{m^{\rho}}\right)^{\frac{1}{1-\rho}} + \left(\tfrac{\mathrm{MC}}{t}\right)^{\frac{\rho}{1-\rho}}\alpha^{\frac{1}{1-\rho}}\right](\sigma - 1)R \\ &= \mathrm{MC}^{\frac{\rho}{1-\rho}}\left[t^{\frac{\rho}{\rho-1}}\alpha^{\frac{1}{1-\rho}} + (mw)^{\frac{\rho}{\rho-1}}(1-\alpha)^{\frac{1}{1-\rho}}\right](\sigma - 1)R \\ &= (\sigma - 1)R,\end{aligned}$$

where the last equality is from Eq. (5). The sum of labour costs and tax payments is constant again. This is because variable costs are equal to fixed costs multiplied by $(\sigma - 1)$ under the Dixit–Stiglitz framework of monopolistic competition.

A3. Proof of Lemma 4

The three phases referred to at the beginning of Section 3 are examined in order.

In Phase I (i.e. $t \in [0, \alpha m/(1-\alpha))$), we have $\mathrm{MC}/\mathrm{MC}^* = (mw + t)/(m + t)$. Since $w = \tau_A > 1$, we immediately know that $\mathrm{MC}/\mathrm{MC}^*$ decreases in t. The emission tax requires an identical tax payment for additional production in two countries with different marginal costs (wages); thus, the marginal cost in the lower-wage country, S, is relatively increased.

In Phase II (i.e. $t \in [\alpha m/(1-\alpha), \alpha m\tau_A/(1-\alpha))$), we have $\mathrm{MC} = mw + t$, and:

$$\mathrm{MC}^* = \left[\left(\frac{t}{\alpha^{\frac{1}{\rho}}}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{m}{(1-\alpha)^{\frac{1}{\rho}}}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}, \tag{31}$$

which is the counterpart of the second equation in (5) for N. From these equations, we can show that:

$$\frac{\partial}{\partial t}\frac{\mathrm{MC}}{\mathrm{MC}^*} \geq 0 \text{ if and only if } t \geq \frac{\alpha m w^{1-\rho}}{1-\alpha}.$$

Since $w > 1$, we know that $\alpha m w^{1-\rho}/(1-\alpha)$ is always larger than the lower bound of t (i.e. $\alpha m/(1-\alpha)$) in Phase II, while it is smaller than the upper bound of t (i.e. $\alpha m w/(1-\alpha)$) if and only if $\rho > 0$. Thus, we conclude that in Phase II, $\mathrm{MC}/\mathrm{MC}^*$ monotonically decreases in t if $\rho \leq 0$; otherwise, it follows a U-shaped curve.

Finally, in Phase III (i.e. $t \in [\alpha m \tau_A/(1-\alpha), \infty)$), we can show that:

$$\frac{\partial}{\partial t}\frac{\mathrm{MC}}{\mathrm{MC}^*} \geq 0 \text{ if and only if } w^{\frac{\rho}{1-\rho}} \geq 1$$

from Eqs. (5) and (31). Since $w > 1$, this shows that $\mathrm{MC}/\mathrm{MC}^*$ decreases (respectively, increases) in t if and only if $\rho < 0$ (respectively, $\rho > 0$). Furthermore, from Eqs. (5) and (31), we have:

$$\lim_{t\to\infty}\frac{\mathrm{MC}}{\mathrm{MC}^*} = \begin{cases} 1 & \text{if } \rho < 0 \\ w^{1-\alpha} & \text{if } \rho = 0 \\ w & \text{if } \rho > 0 \end{cases}. \tag{32}$$

Without taxes, it holds that $\mathrm{MC}/\mathrm{MC}^* = mw/m = w$. Thus, the above result shows that if $\rho > 0$, $\mathrm{MC}/\mathrm{MC}^*$ comes back to the level without taxes as $t \to \infty$. This is because when $\rho > 0$, GHG emissions are significantly decreased by abatement activities as the tax rate rises, and marginal costs of production (including abatement) become proportionate to the wage in each country again.[24]

A4. Proof of Lemma 5

From Eq. (16), we have:

$$\frac{\partial n}{\partial \gamma} = \mathcal{A}\Big\{\theta(\gamma-\phi_M)^2 w + \big(1-\theta\big)\big(1-\gamma\phi_M\big)^2 w^* + R\Big[\big(1-\gamma\phi_M\big)^2 - \theta \big(1-\gamma^2\big)\big(1-\phi_M^2\big)\Big]\Big\}, \tag{33}$$

where:

$$\mathcal{A} \equiv \frac{\mu L^w \phi_M}{R\sigma(\gamma-\phi_M)^2(1-\gamma\phi_M)^2} > 0.$$

If $\gamma \geq 1$, it is evident that Eq. (33) is positive; otherwise, Eq. (33) is positive again, since we know that Eq. (33) is greater than

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[24] In fact, we have $\lim_{t\to\infty}\mathrm{MC} = mw/(1-\alpha)^{1/\rho}$ and $\lim_{t\to\infty}\mathrm{MC}^* = m/(1-\alpha)^{1/\rho}$ from (5) and (31).

$$\mathcal{A}\left\{\theta(\gamma-\phi_M)^2 w+(1-\theta)(1-\gamma\phi_M)^2 w^*+R(\gamma-\phi_M)^2\right\}>0,$$

from the fact that $\theta < 1$. This suggests that n decreases with MC/MC* by Eq. (13).

A5. Trade direction when $\rho > 0$ and $t \rightarrow \infty$

From Eqs. (27) and (30), it holds that $\lim_{t\rightarrow\infty} l=(\sigma-1)R/w$ and $\lim_{t\rightarrow\infty} te=0$ when $\rho>0$. Since n is a function of w and t from Eq. (16), so n is explicitly denoted by $n(w,t)$. Let $\lim_{t\rightarrow\infty} w=w_\infty$, where w_∞ is finite from Eq. (21). Noting the fact that if $\rho>0$, MC/MC* comes back to the level without taxes (i.e. w) as $t\rightarrow\infty$ (Eq. (32)), we have

$$\lim_{t\rightarrow\infty} n(w,t)=n(w_\infty,0).$$

By using these equations and Eq. (20), we have

$$\begin{aligned}\lim_{t\rightarrow\infty}\mathrm{IM}_A &= \lim_{t\rightarrow\infty}\left[-\mu L+\tfrac{(1-\mu)RL}{w}+nl+\tfrac{nte}{w}\right]\\ &= -\mu L+\tfrac{(1-\mu)R(w_\infty)L}{w_\infty}+n(w_\infty,0)\tfrac{(\sigma-1)R(w_\infty)}{w_\infty}\\ &= -\mu L+\tfrac{R(w_\infty)}{w_\infty}[(1-\mu)L+n(w_\infty,0)(\sigma-1)],\end{aligned}$$

where R is explicitly denoted by $R(w)$ since R is a function of w from Eq. (18).

Eqs. (14) and (15) immediately derive

$$\frac{\left(w+\overline{R}\right)L}{n\gamma+n^*\phi_M}=\frac{R\sigma}{\mu}\frac{R\gamma^{-1}-R^*\phi_M}{1-\phi_M^2},\ \frac{\left(w^*+\overline{R}\right)L^*}{n^*+n\phi_M\gamma}=\frac{\sigma}{\mu}\frac{R^*-R\phi_M\gamma^{-1}}{1-\phi_M^2}.$$

Since both left-hand sides are positive, we obtain a necessary condition for equilibrium:

$$\gamma\in\left(\frac{\phi_M R}{R^*},\frac{R}{\phi_M R^*}\right).$$

When $t\rightarrow\infty$, this is rewritten as $w_\infty\in(1/\tau_M,\tau_M)$, from the fact that $\lim_{t\rightarrow\infty}\mathrm{MC/MC}^*=w_\infty$ (Eq. (32)) and $R=R^*$.

Let

$$\mathrm{IM}_A(w) \equiv -\mu L + \tfrac{R(w)}{w}[(1-\mu)L + n(w,0)(\sigma-1)].$$

Takatsuka and Zeng (2012, Appendix A.2) show that

$$\mathrm{IM}'_A(w) < 0 \text{ for } w \in \left(\tfrac{1}{\tau_M}, \tau_M\right),$$

$$\mathrm{IM}_A(1) > 0, \lim_{w \to \tau_M - 0} \mathrm{IM}_A(w) = -\infty,$$

implying that there exist the following three cases:

$$\begin{cases} \text{(i) } \mathrm{IM}_A(w) > 0 & \text{if } w \in \left(\tfrac{1}{\tau_M}, \hat{\tau}_A\right) \\ \text{(ii) } \mathrm{IM}_A(w) = 0 & \text{if } w = \hat{\tau}_A \\ \text{(iii) } \mathrm{IM}_A(w) < 0 & \text{if } w \in (\hat{\tau}_A, \tau_M) \end{cases},$$

where $\hat{\tau}_A \in (1, \tau_M)$. As discussed above Eq. (21), if N exports A (i.e. IM_A<0), we have $w = p_A = 1/\tau_A < 1$, so that Case (iii) is impossible. If good A is not traded (i.e. IM_A=0), we have $w \in (1/\tau_A, \tau_A)$. Thus, Case (ii) is also impossible since $\tau_A < \hat{\tau}_A$ from our assumption. In sum, Case (i) is only possible, so that N imports A (i.e. IM_A>0) and $w_\infty = p_A = \tau_A \in (1, \hat{\tau}_A)$.

A6. Optimal uniform tax rates

In this appendix, in the setting of Sections 5.1 and 5.2, we obtain the optimal tax rates in terms of world welfare, and we see the characteristics. World welfare, V^w, is defined as the sum of individual welfare: $V^w = LV + L^* V^*$. $\mathcal{E}$ in the utility function of Eq. (1) is specified as follows:

$$\mathcal{E}(e^w, l_G, l_G^*) = \exp\left[-\tfrac{e^w}{1 + l_G + l_G^*}\right],$$

which satisfies the conditions of Eq. (2). In the setting, we can numerically show that the initial state, where the two countries do not impose emission taxes (i.e. $t = 0$), is a Nash equilibrium. Namely, when the foreign country does not impose emission taxes, the home country does not have an incentive to impose emission taxes.

The initial and optimal uniform tax rates and the accompanying V, V^* and V/V^* are shown in Table A1. When $\beta = 0.01$ and 0.50, the optimal tax rate is determined in Phase I; it does not depend on the value of ρ. Meanwhile, when $\beta = 0.99$, the optimal tax rate is determined in Phase III; it depends on the value of ρ. If capital share β in the conventional input is lower (respectively, higher), physical capital is less (respectively, more) useful for abatement; hence, the cost of taxing emissions is higher (respectively, lower). Therefore, the optimal tax rate is lower (respectively, higher). Furthermore, if GHG emissions and conventional inputs are gross complements (respectively, substitutes), abating emissions is more (respectively, less) difficult; hence, the cost of taxing emissions is higher (respectively, lower). Thus, the optimal tax rate increases in ρ.

Table A1 Numerical examples of optimal tax rates

β		0.01	0.50	0.99		
		$\rho = -0.5, 0, 0.5$		$\rho = -0.5$	$\rho = 0$	$\rho = 0.5$
Zero tax case	t	0.000	0.000	0.000	0.000	0.000
	V	0.628	0.713	0.740	0.740	0.740
	V^*	0.603	0.681	0.703	0.703	0.703
	V/V^*	1.042	1.047	1.053	1.053	1.053
Optimal tax case	t	0.150	0.229	0.765	0.893	1.425
	V	0.634	0.726	0.918	0.963	1.067
	V^*	0.607	0.692	0.870	0.913	1.013
	V/V^*	1.044	1.049	1.055	1.055	1.054