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# Contract farming problems and games under yield uncertainty*

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This study investigates whether a group of independent agricultural producers willingly forms a coalition to jointly cope with yield uncertainty in contract farming. The agricultural producers' cooperative game problem in contract farming is formulated as a two-stage stochastic linear program. Using the strong duality theory of stochastic linear programs, we not only prove that the core of agricultural producers' cooperative game is nonempty but also provide a simple way to compute a profit allocation policy in the core. We establish the convexity of agricultural producers' cooperative game so that the game has population monotonicity, which gives agricultural producers an incentive to expand their current coalition. We then analyze the agricultural producers' cooperative game with a concave cost structure, which may exhibit economies of scale for production. Taking advantage of the proposed stochastic duality approach, the agricultural producers' cooperative game with a concave cost structure is also shown to have a nonempty core.

**Key words:** contract farming, cooperative game, forward contract, yield uncertainty, profit allocation.

## 1. Introduction

An agricultural cooperative is a cooperative where agricultural producers pool their production and/or resources in certain areas of activity (Cobia 1989). The first agricultural cooperatives were created in seventeenth-century Europe on the Military Frontier, where the wives and children of the border guards lived together in organised agricultural cooperatives. The first civil agricultural cooperatives were also created in Europe in the second half of the nineteenth century. They spread later to North America and the other continents and became one of the tools of agricultural development around the world. New Zealand has a strong history of agricultural cooperatives, dating back to the late nineteenth century, and the first was the small Otago Peninsula Co-operative Cheese Factory. In Canada, the most important cooperatives of this kind were wheat pools. In India, there are networks of cooperatives at the local, regional, state and national levels that assist in

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agricultural marketing (Vadivelu and Kiran 2013). Cooperatives provide a method for agricultural producers to form a coalition, through which the group of agricultural producers can achieve better outcomes, typically being financial, in comparison with producing alone. As reported in the New York Times, agricultural producers in the New York State have benefited greatly from cooperation by coordinating production and sharing demand (Shattuck 2014).

It is well known that the process of agricultural production tends to accompany uncertainty due to unexpected factors such as weather conditions, natural disasters, pest infestations and technological innovations, all of which play significant roles in agricultural production. Thus, assessing the main sources of uncertainty in agriculture has become increasingly essential for agricultural producers. Among the sources of uncertainty, yield uncertainty is a typical feature of the agricultural production industry (Dillnon, 1971; Newbery and Stiglitz, 1981; Moschini and Hennessy, 2001). Yield uncertainty leads to the realized amount of output from a given bundle of inputs being unknown to agricultural producers at the end of the period, and may result in immeasurable losses for agricultural producers. Therefore, it is important for agricultural producers to propose feasible approaches to mitigate the negative effects of yield uncertainty (Anderson *et al.,* 1977).

Contract farming was employed by the Japanese colonial state for sugar production in the period after 1885 and by the US banana companies in the early part of the twentieth century.

In the 1930s and the 1940s, contract farming was widely used by the seed industry in Western Europe and by the vegetable canning industry in North America. By the late twentieth century, contract farming had become an integral part of the food and fibre industry in North America, Japan and Western Europe (Rehber, 1998). In contract farming, corporations could engage in forward contracts with agricultural producers to account for risks from the supply and/or demand side. The forward contract has become attractive to many agricultural producers because the arrangement offers both an assured market and access to production support, and is also of interest to buyers who can potentially benefit from the reduced risks of purchase price and supply availability. Thus, many cooperatives have used forward contracts for agricultural production for decades, and its popularity has increased in recent years (Eaton and Shepherd, 2001). Moreover, such cooperatives have been widespread globally (e.g. in America, Argentina, Bangladesh, Brazil, China, Honduras, South Africa, Tanzania, Thailand and Turkey) (Silva and Rankin, 2013). In Turkey, cooperatives were started as a government initiative and include sugar-beet producers' cooperatives, tea producers' cooperatives and village development cooperatives. In these agricultural cooperatives, forward contracts have been used widely along with other procurement methods. Note that the sugar-beet producers' cooperative is the first implementation of a forward contract in Turkey. In the United States, agricultural cooperatives play a significant role in the agro-food system. Bargaining cooperatives are special for US agriculture. This type of cooperative has adopted the forward contract as a mechanism because it plays a major role in determining trade terms between agricultural producers and buyers (Marcus and Frederick, 1994; Rehber, 1998).

Under uncertainty, the previous researches (e.g. Anderson *et al.,* 1977; Moschini and Hennessy, 2001; Tonsor, 2018; Vigani and Kathage, 2019) mainly considered how a typical individual agricultural producer could best hedge production risks. Nevertheless, there are few studies looking at collaboration strategies and the risk-pooling effect for agricultural producers to mitigate production risks using the cooperative game theory in agricultural production. To the best of our knowledge, there are also a limited number of studies on agricultural cooperatives (e.g. Sexton, 1990; Saitone and Sexton, 2009). However, these studies are mainly empirical analysis and neglect the design of profit/cost allocation policy for agricultural producers. Naturally, the cooperative strategy raises two important and challenging questions in this paper. First, for a group of independent agricultural producers in contract farming, how should they cooperate? Second, how do the members of a coalition share the benefit and what possible outcomes can be achieved? As we know, motivating each member in a coalition to agree on how to share common profit is identified as one of the major challenges to collaborative commerce.

In this paper, we investigate whether a group of independent agricultural producers facing yield uncertainty willingly forms a coalition to jointly offer their aggregate output as a single entity in contract farming. The agricultural producers' cooperative game problem is formulated as a two-stage stochastic program. The goal of this study is to analyze the profit allocation problem among members in an agricultural producers' cooperative game setting. Thus, how to fairly allocate the total profit becomes a key task in this study to ensure that the agricultural producers' coalition is stable. The notion of fairness in this paper is captured by the concept of the core in cooperative game theory (Shapley, 1971; Owen, 1975; Samet and Zemel, 1984), which is generally considered to be a basic solution concept in the cooperative game. A profit allocation policy in the core implies that no subset of players would be better off once they deviate from the cooperation.

The main results of this study are summarized as follows. First, in contract farming, the optimization problem corresponding to the agricultural producers' cooperative game under uncertainty is formulated as a two-stage stochastic linear program. We not only provide a constructive proof to the non-emptiness of the core of agricultural producers' cooperative game, but also provide a simple way to compute a core allocation policy using the strong duality theory of a stochastic linear program; that is, a core allocation policy can be defined by any given optimal dual solution. Second, the agricultural producers' cooperative game is established to be convex so that the game has population monotonicity, which gives every member an incentive to expand their current coalition. Finally, we illustrate the proposed stochastic duality approach to the agricultural producers' cooperative game with a concave cost structure, which may exhibit economies of scale for production. Interestingly, the core of the resulting game with a concave cost structure is also shown to be nonempty.

The remainder of this study is organised as follows. In Section 2, we briefly review the related literature. In Section 3, we provide a brief review of certain key results from cooperative game theory and formulate the agricultural producers' cooperative game problem. For individual agricultural producers, a non-cooperative case is considered as a benchmark in Section 4. In Section 5, we focus on the agricultural producers' cooperative game with linear and concave effort cost structures, respectively. In Section 6, we provide illustrations with numerical analyses and conduct sensitivity analyses. Finally, we conclude this paper and provide some directions for future research.

## 2. Literature review

The review of the related literature is structured into three streams in terms of their contextual similarity with our study. The first group focuses on uncertainty and risk in agricultural production. The second group investigates contract farming as a mechanism in the broader context of agriculture. The third group comprises papers that share similar methodologies with ours.

The first group of literature analyzes the effects of uncertainty and risk in agricultural economics. Anderson *et al.* (1977) investigated agricultural production problem under uncertainty using decision theory. Newbery and Stiglitz (1981) investigated commodity price stabilisation and analyzed uncertainty and risk issues in agriculture. Moschini and Hennessy (2001) focused on agricultural production decisions under uncertainty and risk using expected utility theory and the notion of risk aversion. He and Zhang (2008) analyzed the effects of random yield in agricultural supply chains and examined how random yield affects supply chain performance. Recently, Chavas (2018) conducted an in-depth demonstration of the roles of risk and uncertainty in agricultural production. Tonsor (2018) explored the roles of past experiences and question farming in production decision-making under uncertainty and found that producers use the best outcome experienced as a reference point in their decision-making process. Vigani and Kathage (2019) estimated the impact of the adoption of several portfolios of risk management strategies in different farming systems with different levels of risk using survey data from French and Hungarian farms. Iyer *et al.* (2020) presented a systematic review of the extensive body of research on the measurement of farmers' risk preferences across Europe and discussed the remaining challenges for further research. For a comprehensive review, see Ward *et al.* (2019) and Ramsey (2020). The aforementioned literature mainly analyzed the optimal decision of a typical individual agricultural producer to hedge the risk created by uncertainty in agriculture. In contrast, our study explores collaboration strategies and the risk-pooling effect for agricultural producers to mitigate production risks using the cooperative game theory in agricultural production. More importantly, using the duality theory, we provide a simple way to compute fair allocation policies whether agricultural producers are facing economies of scale or not. To the best of our knowledge, there are also a limited number of studies focusing on agricultural cooperatives (e.g. Sexton, 1990; Saitone and Sexton, 2009). However, these studies are mainly empirical analysis and neglect the design of allocation policies for members of cooperatives.

The second group of literature investigates contract farming as a mechanism in agricultural production. Both Huh and Lall (2013) and Musshoff and Hirschauer (2009) illustrated that contract farming has always played a potential role in reducing production risks and affecting the decisions of agricultural producers. Burer *et al.* (2008) examined contract practices between suppliers and retailers in the agricultural seed industry. In recent years, Mishra *et al.* (2018) assessed the impact of smallholders' perceived production risks on the adoption of contract farming using farm-level data and endogenous switching regression methods. Furthermore, Mishra *et al.* (2019) found that contract agricultural producers appear to show lower inefficiency and lower production risk. For a comprehensive review of contract farming, see Bellemare and Novak (2017), Carrer *et al.* (2019), Bernard *et al.* (2019), Xia *et al.* (2019) and Falkowski *et al.* (2019). Compared to the aforementioned literature on contract farming that mainly uses empirical analysis, to the best of our knowledge, our study is the first to incorporate contract farming as a mechanism in a cooperative game setting. Interestingly, we find that contracts play an important role in the design of allocation policies.

The third group of literature may be in a different context from our study, but shares methodological similarities. One of the approaches adopted in this study is the cooperative game theory, and there are many theoretical studies on this issue (Deng and Papadimitriou, 1994; Meca *et al.,* 2004; Aydinliyim and Vairaktarakis, 2013; Liu *et al.,* 2016). The cooperative game theory has also been widely used in various fields (Huang *et al.,* 2002; Slikker *et al.,* 2005; Özen *et al.,* 2008; Roshanaei *et al.,* 2017; Toyasaki and Wakolbinger, 2019). In this paper, the notion of fairness is captured by the concept of the core in cooperative game theory. Roughly speaking, a profit allocation policy in the core indicates that no subgroup of players will be better off by deviating from the whole group under this allocation scheme (Owen, 1975; Samet and Zemel, 1984). Another approach adopted in this study is the duality theory. Owen (1975) proved that the core is nonempty for a class of linear production games using the duality theory of linear programs. The duality approach provided by Owen (1975) has been a significant tool used in analyzing cooperative games and has been applied widely in a variety of fields (Tamir, 1992; Goemans and Skutella, 2004; Chen et al., 2019). Motivated by the work of Owen (1975), Chen and Zhang (2009) took a stochastic programming duality approach to the inventory centralization games. In this study, we extend the stochastic duality approach to the agricultural producers' cooperative game under uncertainty in contract farming and propose fair allocation policies for agricultural producers.

## 3. Problem formulation

This section first presents some fundamental concepts of cooperative game theory adopted in this paper and then formulates the agricultural producers' cooperative game problem facing yield uncertainty.

### 3.1 Cooperative game theory

Here, we briefly introduce some basic concepts and results of the cooperative game theory applied throughout this paper. It should be noted that a cooperative game is also usually called a coalitional game. In recent years, with the development of game theory, cooperative games have become popular and have received increasing attention from researchers. In addition, the cooperative game theory has been widely applied in various subjects, including engineering and communication networks, management, social sciences and economics (Myerson, 1991; Saad *et al.,* 2009).

Let $N = \{1, 2, ..., n\}$ be the set of players. Establishing a cooperative game $(N, \pi)$ with a transferable payoff such that the achieved profit of coalition $N$ can be allocated to each player in any possible way. The function $\pi : 2^N \rightarrow R$ denotes the value of coalition $N$ and represents the maximum of the total obtainable profit of coalition $N$. For any subset $S \subseteq N$, the pair $(S, \pi)$ is referred to as a subgame. Moreover, a coalition containing all players is commonly known as the grand coalition, and all possible coalitions $S \subseteq N$ are listed in the set $2^N$ of $N$.

It is well known that an effective allocation policy will motivate independent players to collaborate, which will bring about an increased profit for each member of the coalition. Conversely, an invalid or unimplemented allocation policy may lead some players to leave the current coalition to join a more profitable one. Consequently, designing an effective allocation policy for members of a coalition is of great importance. In this study, among several allocation policies, we are particularly interested in the core allocation policy. Before introducing the definition of the core, we first define an imputation.

**Definition 1:** Given an allocation $l = (l_1, l_2, ..., l_n)$ for the coalition $N$, if it satisfies $\sum_{i \in N} l_i = \pi(N)$ and $l_i \geq \pi(\{i\})$, $\forall i \in N$, then, the vector $l = (l_1, l_2, ..., l_n)$ is called an imputation

From Definition 1, we know that, on the one hand, the allocated profit for each member of a coalition adds up to the value of the coalition, that is $\sum_{i\in N} l_i = \pi(N)$. On the other hand, all players receive higher profits than what can be achieved by acting alone, that is $l_i \geq \pi(\{i\})$. The notion of the core is considered to be a basic solution concept and is defined as the set of imputations. When generalising this idea to apply to every coalition, the definition of the core is achieved.

**Definition 2:** An allocation $l = (l_1, l_2, ..., l_n)$ is in the core of the cooperative game $(N,\pi)$ if $\sum_{i\in N} l_i = \pi(N)$ and $\sum_{i\in S} l_i \geq \pi(S)$ for any subset $S \subseteq N$.

A core is the set of profit allocations where no subgroup of players has an incentive to leave the grand coalition to participate in other smaller coalitions. In other words, no players will be better off by deviating from the grand coalition. We now pay attention to the non-emptiness of the core and introduce the definition of a balanced game.

**Definition 3:** A cooperative game $(N,\pi)$ is balanced if $\sum_{S\in 2^N} f(S)\pi(S) \leq \pi(N)$ for any balanced map $f: 2^N \to [0,1]$ and $\sum_{S\in 2^N} f(S) 1_{\{i\in S\}} = 1$, where $1_{\{\cdot\}}$ denotes the indicator function.

Notably, Bondareva (1963) showed that the core of a cooperative game is nonempty if and only if the game is balanced. The non-emptiness of a core means that there is at least one profit allocation considered advantageous by all players. Finally, we present the definition of a stable allocation.

**Definition 4:** If an allocation $l = (l_1, l_2, ..., l_n)$ belongs to the core of a cooperative game, then the allocation is stabilizing.

### 3.2 Agricultural producers' cooperative game problem

The biological lags that characterize agricultural production mean that the inputs have to be committed to production far in advance of the harvest output being realized. Consider a group of $n$ independent agricultural producers denoted by $N = \{1, 2, ..., n\}$. In this study, agricultural producers are assumed to be expected profit maximisers. Due to uncontrollable elements, such as weather conditions, pest infestations and innovative technologies, every agricultural producer faces yield uncertainty. Therefore, the realized yield $y_i$ of each agricultural producer $i \in N$ is a stochastic function associated with the effort level $q_i \in (0, q_i^0)$, where the upper bound $q_i^0$ denotes the maximal production level of each agricultural producer $i \in N$. We assume that the realized yield is proportional to the production effort level in any given scenario; that is, $y_i(q_i,\omega) = u_i(\omega)q_i$, $\omega\in\Omega$, where $u_i(\omega)$ is a non-negative random variable. The yield uncertainty is represented in terms of random experiments with outcomes denoted by $\omega$ and the set of all outcomes is denoted by $\Omega$.

In agricultural production, the inputs are committed to production far in advance of the realization of the final products. Thus, the demand and market price of outputs are not known with certainty every time making a production decision. For this purpose, we take contract farming into account. Considering the commonly used forward contract in agriculture, where the agricultural producers and buyers (or contract providers) commit to both the delivery quantity and the price, $(Q,p)$, at the beginning of the planning horizon. In particular, the individual production capacities of every agricultural producer are assumed to be small relative to the entire market so that individual producers are considered to be price takers. Hence, the forward price $p$ is assumed to be fixed and known. A shortage penalty price $b(\omega)$ is associated with each unit for unsatisfied contract, and a salvage price $v(\omega)$ is associated with each unit for excess contract at the end of the period. The penalty and salvage prices represent the inherent volatility of the market and can be difficult to forecast, so they are modelled as random variables. It is appropriate to point out that the yield factor $u(\omega)$ is independent of the penalty price $b(\omega)$ and salvage price $v(\omega)$ in general.

When faced with yield uncertainty, a group of independent agricultural producers are willing to form a coalition to jointly offer their aggregate output as a single entity in a market (He *et al.,* 2018). The coalition, on the one hand, leverages the complementarity among its constitutive members' outputs to mitigate the risks caused by yield uncertainty by taking advantage of the risk-pooling effect. On the other hand, the coalition gives agricultural producers increased negotiating power with respect to the buyers of agricultural output (e.g. retailers) by offering the aggregate output from multiple agricultural producers, thereby helping offset the price-taker position of individual agricultural producers. In this study, we adopt the following sequence of events. First, the coalition $S\subseteq N$ and a buyer of the agricultural output jointly agree on the terms of a delivery quantity $Q$ and a forward price $p$ for the output at the beginning of the period. Second, each agricultural producer $i$ of coalition $S$ makes a production effort level decision $q_i$. Third, the yield factors are realized; that is, the final output of coalition $S$ is obtained. Finally, the penalty and salvage prices are implemented at the end of the period.

Based on the above chronology, the delivery quantity $Q = (Q_i)_{i\in S}$ included in the forward contract and the effort level $q = (q_i)_{i\in S}$ are the decision variables. For any coalition $S\subseteq N$, the goal is to maximize the expected total profit of coalition $S$, which consists of the total revenues from the contract and the sale of excess yield, minus the corresponding cost including the effort cost and the penalty cost. Next, how to fairly allocate the aggregate profit among members of coalition $S$ to ensure the stability of coalition $S$ becomes a central problem.

We consider a complete information setting where both the production effort costs and yield factors are common knowledge to all agricultural producers. This is a standard assumption in the literature on the forward contract (Popescu and Seshadri, 2013). To conclude this section, we provide a summary of the notation mainly used throughout the paper in Table 1. Other symbols are defined as required.

## 4. Benchmark: non-cooperative case

This section focuses on the non-cooperative case of individual agricultural producers. It not only lays the groundwork for formulating the agricultural producers' cooperative game in the next section, but also provides a simple way to solve the closed-form solutions of decision variables. In the non-cooperative case, every individual agricultural producer $i$ is referred to as a decision maker and chooses an optimal delivery quantity $Q_{1i}^*$ and an optimal production effort level $q_i^*$ to maximize own utility. Define the function as:

$$R(x,b,v) = b\min(x,0) + v\max(x,0). \quad (1)$$

Hence, given the delivery quantity $Q_{1i}$ and effort level $q_i$, the maximum expected profit of an individual agricultural producer $i$ is defined by the following equation:

**Table 1** Summary of notation

| Symbol | Range | Type | Description |
|---|---|---|---|
| $N$ | $(0,+\infty)$ | Exogenous parameter | The total number of agricultural producers |
| $q_i^0$ | $(0,+\infty)$ | Exogenous parameter | The maximal production level of agricultural producer $i$ |
| $q_i$ | $[0,q_i^0]$ | Decision variable | The production effort level of agricultural producer $i$ |
| $\omega$ | $(0,+\infty)$ | Exogenous parameter | The uncertainty in terms of random experiments with outcomes |
| $\Omega$ | $(0,+\infty)$ | Exogenous parameter | The set of all outcomes |
| $u_i(\omega)$ | $[0,1]$ | Exogenous parameter | The random yield factor of agricultural producer $i$ |
| $y_i$ | $[0,+\infty)$ | Endogenous parameter | The realized yield of agricultural producer $i$ |
| $p$ | $(0,+\infty)$ | Exogenous parameter | The forward price per unit |
| $Q_1$ | $(0,+\infty)$ | Decision variable | The delivery quantity to contract in the non-cooperative case |
| $Q_2$ | $(0,+\infty)$ | Decision variable | The delivery quantity to contract in the cooperative case |
| $b(\omega)$ | $(0,+\infty)$ | Exogenous parameter | The random penalty price for unsatisfied contract per unit |
| $v(\omega)$ | $(0,+\infty)$ | Exogenous parameter | The random salvage price for excess contract per unit |

$$\pi(\{i\}) = \max_{q_i \geq 0, Q_{1i} \geq 0} pQ_{1i} - c_i q_i + \mathrm{E}[R(y_i(q_i,\omega) - Q_{1i}, b(\omega), v(\omega))]; \tag{2}$$

that is:

$$\begin{aligned} \pi(\{i\}) \ = & \max_{q_i \geq 0, Q_{1i} \geq 0} pQ_{1i} - c_i q_i \\ & + \mathrm{E}\left[v(\omega)(y_i(q_i,\omega) - Q_{1i})^+\right] - \mathrm{E}\left[b(\omega)(Q_{1i} - y_i(q_i,\omega))^+\right], \end{aligned} \tag{3}$$

where $c_i$ is the unit effort cost. In Equation (3), the first term indicates the revenue by delivering $Q_{1i}$, the second term is the production effort cost, and the third and fourth terms imply the sales from excess output and the penalty cost for unsatisfied contract, respectively. From the definition of $R(x,b,v)$ given in Equation (1), we have $px = pR(x,1,1) = R(x,p,p)$. by substituting $y_i(q_i,\omega)$ with $u_i(\omega)q_i$, and we further observe that:

$$pQ_{1i} = (p\mathrm{E}[u_i(\omega)])q_i - \mathrm{E}[R(u_i(\omega)q_i - Q_{1i}, p, p)].$$

We then apply the above equation to eliminate the term $pQ_{1i}$ in Equation (2), and replace $\mathrm{E}[u_i(\omega)]$ with $\mu_i$. Then, we deduce that:

$$\begin{aligned} \pi(\{i\}) \ = & \max_{q_i \geq 0, Q_{1i} \geq 0} (p\mu_i - c_i)q_i \\ & + \mathrm{E}[(b(\omega) - p)\min(u_i(\omega)q_i - Q_{1i}, 0) + (v(\omega) - p)\max(u_i(\omega)q_i - Q_{1i}, 0)]. \end{aligned} \tag{4}$$

For the sake of presentation, we define:

$$\varphi(\omega) = p - v(\omega) \,\mathrm{and}\, \phi(\omega) = b(\omega) - p, \ \ \omega \in \Omega.$$

Recall that $v(\omega)$ and $b(\omega)$ are the unit salvage price for an excess contract and the unit penalty price for an unsatisfied contract, respectively. Thus, $\varphi(\omega)$ denotes the discount that is offered for selling the surplus, and $\phi(\omega)$ denotes the cost of covering the shortage. That is to say, for every agricultural producer, if the realized output is more than the contract, it pays a per-unit discount cost of $\varphi(\omega)$ for an excess contract; on the other hand, if the realized output is less than the contract, it pays a per-unit penalty cost of $\phi(\omega)$ for an unsatisfied contract. Thus, Equation (4) is further transformed equivalently as:

$$\begin{aligned} \pi(\{i\}) \ = & \max_{q_i \geq 0, Q_{1i} \geq 0} (p\mu_i - c_i)q_i - \mathrm{E}\left[\varphi(\omega)(u_i(\omega)q_i - Q_{1i})^+\right] \\ & - \mathrm{E}\left[\phi(\omega)(Q_{1i} - u_i(\omega)q_i)^+\right], \end{aligned} \tag{5}$$

To avoid trivial cases, we assume that $\varphi(\omega) > 0$, $\phi(\omega) > 0$ for all $\omega \in \Omega$ and $p\mu_i > c_i$. It is clear to see that Equation (5) is non-differentiable due to the existence of yield uncertainty, which poses a huge challenge to solve. For this purpose, we pay close attention to the following equivalent form of Equation (5):

$$\begin{aligned}-\pi(\{i\}) &= \min_{q_i\geq 0, Q_{1i}\geq 0} (c_i - p\mu_i)q_i \\ &\quad + \mathrm{E}\left[\varphi(\omega)(u_i(\omega)q_i - Q_{1i})^+\right] + \mathrm{E}\left[\phi(\omega)(Q_{1i} - u_i(\omega)q_i)^+\right].\end{aligned} \tag{6}$$

We now reformulate Equation (6) as a two-stage stochastic linear programming problem by introducing two non-negative random variables $e_i(\omega)$ and $s_i(\omega)$, which is given by:

$$\begin{aligned}-\pi(\{i\}) = \min_{q_i\geq 0} \ & (c_i - p\mu_i)q_i + \mathrm{E}[f_i(q_i,\omega)] \\ s.t. \quad & q_i \leq q_i^0,\end{aligned} \tag{7}$$

where $f_i(q_i,\omega)$ is defined by:

$$\begin{aligned}f_i(q_i,\omega) = \min\ & \varphi(\omega)e_i(\omega) + \phi(\omega)s_i(\omega) \\ s.t. \quad & u_i(\omega)q_i - Q_{1i} - e_i(\omega) \leq 0,\ \omega\in\Omega, \\ & Q_{1i} - u_i(\omega)q_i - s_i(\omega) \leq 0,\ \omega\in\Omega, \\ & Q_{1i}\geq 0,\ e_i(\omega)\geq 0,\ s_i(\omega)\geq 0,\ \omega\in\Omega,\end{aligned}$$

where $e_i(\omega)$ denotes the surplus of the realized output and $s_i(\omega)$ shows the lost delivery quantity. In the first stage, the constraint implies that the effort level is no more than the maximal production level for individual agricultural producer *i*. In the second stage, for individual agricultural producer *i*, the first constraint implies that the realized yield exceeds the delivery quantity. The second constraint shows that the realized yield may not reach the contract target, which would result in lost sales. For the two-stage gametheoretical model (7), every individual agricultural producer first determines the delivery quantity to contract in the forward market. The forward contract is assumed to be binding so that no parties can renege on the contract agreement. Then, the agricultural producer determines the production effort level.

It is easy to deduce the dual of problem (7) that:

$$\begin{aligned}& \max\ \lambda_i q_i^0 \\ s.t.\ \ & \lambda_i + \mathrm{E}[u_i(\omega)\alpha_i(\omega)] - \mathrm{E}[u_i(\omega)\beta_i(\omega)] \leq (c_i - p\mu_i),\ \omega\in\Omega, \\ & -\mathrm{E}[\alpha_i(\omega)] + \mathrm{E}[\beta_i(\omega)] \leq 0,\ \omega\in\Omega, \\ & -\alpha_i(\omega) \leq \varphi(\omega),\ \omega\in\Omega, \\ & -\beta_i(\omega) \leq \phi(\omega),\ \omega\in\Omega, \\ & \lambda_i\leq 0,\ \alpha_i(\omega)\leq 0,\ \beta_i(\omega)\leq 0,\ \omega\in\Omega,\end{aligned} \tag{8}$$

where $\lambda_i$, $\alpha_i(\omega)$ and $\beta_i(\omega)$, $\omega\in\Omega$ are the dual variables. Denote by $q_i^*$, $Q_{1i}^*$, $e_i^*(\omega)$ and $s_i^*(\omega)$ the optimal solutions to primal problem (7), and $\lambda_i^*$, $\alpha_i^*(\omega)$ and $\beta_i^*(\omega)$ the optimal dual solutions to problem (8).

**1:** The optimal production effort level $q_i^*$ of an individual agricultural producer $i$ is at its upper bound $q_i^0$, while the optimal delivery quantity $Q_{1i}^*$ is determined by the following Newsvendor solution:

$$\mathrm{P}\left(u_i(\omega) > \frac{Q_{1i}^*}{q_i^0}\right) = \frac{\mathrm{E}[\phi(\omega)]}{\mathrm{E}[\phi(\omega)] + \mathrm{E}[\varphi(\omega)]}. \quad (9)$$

See Appendix S1 for the proof of Proposition 1. Proposition 1 shows the optimal delivery quantity and effort level for an individual agricultural producer. Recall that $\varphi(\omega)$ and $\phi(\omega)$ denote the discount to be offered for selling the surplus and the cost of covering the shortage, respectively. Thus, the right-hand side of Equation (9) indicates the proportion of the cost of covering the shortage in the total cost of surplus and shortage. Note that $u_i(\omega)$ is a yield factor, so the left-hand side of Equation (9) shows that the probability that the optimal delivery quantity in a forward contract is less than the realized maximal production level. Moreover, Equation (9) can be reduced to:

$$\mathrm{P}\left(Q_{1i}^* < u_i(\omega) q_i^0\right) = \frac{1}{1 + \mathrm{E}[\varphi(\omega)]/\mathrm{E}[\phi(\omega)]}.$$

Easy to find that the optimal delivery quantity included in the forward contract and the yield uncertainty are negatively correlated, which demonstrates that the yield uncertainty plays a particularly significant role in designing an optimal forward contract. Moreover, after the yield factor is realized, it is clear to see that the optimal delivery quantity mainly depends on the ratio of the discount for selling the surplus to the cost of covering the shortage.

## 5. Agricultural producers' cooperative game

This section mainly focuses on the agricultural producers' cooperative game with a linear effort cost and a concave effort cost, respectively. Using the stochastic programming duality approach, we not only provide a constructive proof to the non-emptiness of the core of the corresponding agricultural producers' cooperative game, but also propose a simple way to compute a core profit allocation policy for each member of a coalition.

### 5.1 The game with a linear effort cost

In this subsection, we focus on the case in which the effort cost function for each agricultural producer in any coalition is assumed to be linear. In practice, this assumption is appropriate for settings where economies of scale for production are negligible. In addition, this subsection serves as a basis for the analysis in the following subsections.

The maximum expected total profit for agricultural producers that cooperate in a coalition $S \subseteq N$ is given by:

$$\pi(S) = \max_{Q_2, q_i \geq 0} pQ_2 - \sum_{i \in S} c_i q_i + \mathrm{E}\left[R\left(\sum_{i \in S} y_i(q_i, \omega) - Q_2, b(\omega), v(\omega)\right)\right]. \qquad (10)$$

By the definition of the function $R$ given by Equation (1), Equation (10) is reformulated as the following stochastic optimization problem:

$$\pi(S) = \max_{Q_2, q_i \geq 0} \sum_{i \in S} (p\mu_i - c_i) q_i - \mathrm{E}\left[\varphi(\omega)\left(\sum_{i \in S} u_i(\omega) q_i - Q_2\right)^+ + \phi(\omega)\left(Q_2 - \sum_{i \in S} u_i(\omega) q_i\right)^+\right]. \quad (11)$$

Like the analysis in Section 4, we prioritize solving the equivalent expression of Equation (11), which is given by:

$$-\pi(S) = \min_{Q_2, q_i \geq 0} \sum_{i \in S} (c_i - p\mu_i) q_i + \mathrm{E}\left[\varphi(\omega)\left(\sum_{i \in S} u_i(\omega) q_i - Q_2\right)^+ + \phi(\omega)\left(Q_2 - \sum_{i \in S} u_i(\omega) q_i\right)^+\right]. \quad (12)$$

By introducing two non-negative random variables $e(\omega)$ and $s(\omega)$, Equation (12) is converted into a two-stage stochastic linear programming problem:

$$\begin{aligned} -\pi(S) = \min_{q_i \geq 0} & \sum_{i \in S} (c_i - p\mu_i) q_i + \mathrm{E}[f(q, \omega)] \\ s.t. \quad & q_i \leq q_i^0, i \in S, \end{aligned} \qquad (13)$$

where $q = (q_i)_{i \in S}$ and $f(q, \omega)$ is defined by:

$$\begin{aligned} & f(q, \omega) = \min \varphi(\omega) e(\omega) + \phi(\omega) s(\omega) \\ s.t. \quad & \sum_{i \in S} u_i(\omega) q_i - Q_2 - e(\omega) \leq 0, \ \omega \in \Omega, \\ & Q_2 - \sum_{i \in S} u_i(\omega) q_i - s(\omega) \leq 0, \ \omega \in \Omega, \\ & Q_2 \geq 0, e(\omega), s(\omega) \geq 0, \ \omega \in \Omega. \end{aligned}$$

The first and second constraints in the second-stage formulation of (13) correspond to the excess yield and lost sale of coalition $S$, respectively. Before giving the dual of problem (13), we first investigate the following two-stage stochastic linear programming problem:

$$\begin{aligned} &\min l_1^{\mathrm{T}} z_1 + \mathrm{E}[h(z_1,\omega)] \\ &s.t. \quad B_{11} z_1 = d_1, z_1 \geq 0, \end{aligned} \tag{14}$$

where for any scenario $\omega \in \Omega$:

$$\begin{aligned} h(z_1,\omega) &= \min\ l_2^{\mathrm{T}} z_2 \\ s.t. \quad B_{12} z_1 + B_{22} z_2 &= d_2(\omega),\ z_2 \geq 0,\ \omega \in \Omega\,. \end{aligned}$$

The dual of problem (14) is obtained as:

$$\begin{aligned} &\max \mathrm{E}\left[d_1^{\mathrm{T}} \lambda_1 + d_2^{\mathrm{T}}(\omega) \lambda_2(\omega)\right] \\ s.t. \quad &B_{11}^{\mathrm{T}} \lambda_1 + \mathrm{E}[B_{12}^{\mathrm{T}} \lambda_2(\omega)] \leq l_1,\ \omega \in \Omega\,, \\ &B_{22}^{\mathrm{T}} \lambda_2(\omega) \leq l_2,\ \omega \in \Omega\,. \end{aligned} \tag{15}$$

It is obvious to see that problem (13) is a special case of problem (14). Notably, problem (14) is feasible when the set $\Omega$ is finite. Additionally, for any $z_1 \in \{z_1 : B_{11} z_1 = d_1, z_1 \geq 0\}$ and $z_2 \in \{z_2 : B_{12} z_1 + B_{22} z_2 = d_2(\omega), \omega \in \Omega\}$, problem (14) is said to have a relatively complete recourse if $\mathrm{E}[l_2^{\mathrm{T}} z_2] < \infty$. Thus, according to the results from Rockafellar and Wets (1976), we know that the optimal objective values of the primal problem (14) and the dual problem (15) are equal if problem (14) is feasible and has a relatively complete recourse. We now apply the above result to the agricultural producers' cooperative game with a linear effort cost. It is easy to see that the dual problem of the two-stage stochastic program (13) is:

$$\begin{aligned} &\max \sum_{i \in S} \lambda_i q_i^0 \\ s.t. \quad &\lambda_i + \mathrm{E}[u_i(\omega)\alpha(\omega)] - \mathrm{E}[u_i(\omega)\beta(\omega)] \leq (c_i - p\mu_i),\ i \in S,\ \omega \in \Omega, \\ &-\mathrm{E}[\alpha(\omega)] + \mathrm{E}[\beta(\omega)] \leq 0,\ \omega \in \Omega, \\ &-\alpha(\omega) \leq \varphi(\omega),\ \omega \in \Omega, \\ &-\beta(\omega) \leq \phi(\omega),\ \omega \in \Omega, \\ &\lambda_i \leq 0,\ i \in S,\ \alpha(\omega) \leq 0,\ \beta(\omega) \leq 0,\ \omega \in \Omega. \end{aligned} \tag{16}$$

We now provide some intuitions on the dual model (16) that describes the profit allocation policy. We suppose that an agricultural producer must satisfy the forward contract in agricultural production. To satisfy the contract, there are two alternative approaches for agricultural producers. In the first approach, the agricultural producer will implement production on their own. This is exactly the primal problem (13). In the second approach, the agricultural producer chooses to outsource production to another entity.

The entity will charge the agricultural producer and would like to maximize the total charge. Specifically, given the underlying primitive uncertainty $\omega$, the entity will charge a final unit price $\lambda_i$ for the maximal production level $q_i^0$ of agricultural producer $i$, the reason for which is that every agricultural producer will choose to their own maximal production level as an optimal strategy. However, given the agricultural producer's alternative approach, the agricultural producer may impose certain constraints on the charge; that is, the unit price should be no more than the unit production cost; otherwise, the agricultural producer has an incentive to implement production on their own. This is exactly the dual problem (16). As analyzed, the maximal production level $q_i^0$ denotes the optimal production effort level of agricultural producer $i$ and the variable $\lambda_i$ denotes the unit price for the production level, both of which play key roles in the profit issue. Thus, they are sensible variables to use for a profit allocation policy.

Moreover, $\alpha(\omega)$ and $\beta(\omega)$ can be interpreted as scenario-specific shadow costs for over-commitment and under-commitment, respectively. More importantly, for any collection of agricultural producers $S \subseteq N$, $-\pi(S)$ is equal to the optimal value of problem (16) based on the result from Rockafellar and Wets (1976).

### 5.2 Non-emptiness of the core

This subsection provides a simple way to compute an allocation policy in the core of the agricultural producers' cooperative game with a linear effort cost using the strong duality theory of a stochastic linear program. Denote by $(\lambda^*, \alpha^*(\omega), \beta^*(\omega))$ an optimal solution to problem (16) with $S = N$. We define for any $i \in N$:

$$l_i = -\lambda_i^* q_i^0. \tag{17}$$

**2:** The vector $l = (l_1, l_2, \ldots, l_n)$ defined by (17) is an allocation policy in the core of the agricultural producers' cooperative game $(N, \pi)$.

The proof of Proposition 2 is provided in Appendix S1. Proposition 2 demonstrates that there exists at least one allocation policy that is considered advantageous by all members of coalition $N$. The existence of a core profit allocation policy provides an incentive for independent agricultural producers to form a coalition, with the goal of mitigating the risks that yield uncertainty leads to. Moreover, Proposition 2 provides a constructive proof to the non-emptiness of the core of the agricultural producers' cooperative game $(N, \pi)$. More importantly, Proposition 2 provides a simple way to compute a core allocation policy by solving the dual problem of a stochastic linear program. Namely, a core profit allocation policy can be defined by any given optimal dual solution. Interestingly, the stochastic programming duality approach used in this paper is always effective in computing a core allocation policy, at least for problems of moderate sizes, by solving a linear programming problem. With the development of computing technologies, it is possible to solve the large-scale stochastic programming problem more efficiently. In recent years, several feasible methods have been proposed to solve large-scale stochastic programs. For example, Linderoth *et al.* (2006) provided sampling methods to solve large-scale stochastic linear programs.

Proposition 2 shows that the core of the agricultural producers' cooperative game is nonempty. Thus, we gain another interesting result derived from Bondareva (1963)

**Corollary1:** The agricultural producers' cooperative game $(N, \pi)$ is balanced.

By Corollary 1, it is easy to see that the agricultural producers' cooperative game with linear effort cost has a stabilizing allocation policy.

### 5.3 Population monotonicity in the game

As we know, a large cooperative usually cannot be established at once but gradually expands over time. For example, Jesse and Rogers (2006) discussed a famous cooperative, referred to as Ocean Spray, in the cranberry industry that was founded with only three berry growers in 1930. After many years, it grew to have more than 700 members across North America. Intuitively, the expansion of a cooperative needs support from each of its existing members when it is jointly managed. Whereas a fair allocation policy in the core cannot ensure that each current member in a coalition would not be negatively affected by adding a new player, it just gives no incentive for any player to deviate from the coalition.

For this reason, an important question is naturally arisen whether the allocated profit of each member of a coalition increases as the size of the coalition grows, that is to say, when a new player joins an agricultural producers' coalition, and whether every current agricultural producer in the coalition can obtain a higher payoff. To answer this question, we first introduce the concept of the population monotonic allocation scheme (PMAS) defined by Sprument (1990). It is well known that the PMAS motivates independent players to cooperate by forming a larger coalition. Thus, the PMAS is helpful for coalition expansion. Before giving the notion of the PMAS, we define a vector $L = (L_{i,S})_{i \in S, S \subseteq N}$, where the element $L_{i,S}$ denotes the allocated profit of agricultural producer $i$ in coalition $S \subseteq N$.

**Definition 5:** If a vector $L = (L_{i,S})_{i \in S, S \subseteq N}$ satisfies
$\sum_{i \in S} L_{i,S} = \pi(S),\ S \subseteq N,$
and $L_{i,S} \leq L_{i,T},\ i \in S$ and $S \subseteq T \subseteq N$,
then we call the vector $L = (L_{i,S})_{i \in S, S \subseteq N}$ a PMAS.

Definition 5 indicates that each agricultural producer in a coalition can benefit from forming a larger coalition, and the PMAS is in the core of the game $(S,\pi)$. It should be noted that a PMAS is different from the core allocation, the latter of which only needs to satisfy that a smaller coalition will not get more profit. For any two disjointed coalitions $S$ and $T$, if the characteristic function $\pi$ satisfies:

$$\pi(S)+\pi(T)\leq\pi(S\uplus T),$$

then the cooperative game $(N,\pi)$ is superadditive. An interesting property of a superadditive game is that the values of two disjointed smaller coalitions can be improved by combining the two into a bigger one.

**Definition 6:** For all $i\in N$ and $S\subseteq T\subseteq N\setminus\{i\}$, if the inequality $\pi(S\uplus\{i\})-\pi(S)\leq\pi(T\uplus\{i\})-\pi(T)$ holds, then the characteristic function $\pi$ is supermodular.

Under supermodular, if a new player joins a larger coalition, his marginal profit associated with the coalition is greater than that when joining a smaller coalition. Based on the definition of supermodular, we gain an important result corresponding to the agricultural producers' cooperative game $(N,\pi)$.

**Proposition 3:** An agricultural producers' cooperative game $(N,\pi)$ is convex if the characteristic function $\pi$ is supermodular.

Proposition 3 can be proven easily using the result of Shapley (1971), so we omit this proof for brevity. It is important to note that Proposition 3 provides a simple method to prove that a cooperative game is convex by establishing that its characteristic function is supermodular. In recent years, some studies have been conducted on the proof of the convexity of newsvendor games under mild conditions, such as Özen *et al.* (2011) and Montrucchio and Scarsini (2007). Furthermore, Sprument (1990) showed that a convex game always has a PMAS.

Next, we investigate the characteristic function (11) corresponding to the agricultural producers' cooperative game $(N,\pi)$ with a linear effort cost. For convenience of analysis, we consider a simple case in this subsection, where the delivery quantity included in the forward contract is an exogenous random variable, that is $Q_2(\omega)$. In some sense, the forward contract is analogous to a menu contract and this contract form has been widely applied in reality (Corbett *et al.,* 2004; Lau *et al.,* 2008). Let $c_i=c$ for all $i\in N$. Assume that $u_1(\omega),\ldots,u_n(\omega)$ are independent identical distributions and satisfy $\mathrm{E}[u_i(\omega)]=\mu$. Denote by $q^*=\sum_{i\in S}q_i^*$ an optimal solution to (11). Then, Equation (11) is reformulated as:

$$\pi(S) = (p\mu - c)q^* - \mathrm{E}\left[\mu\varphi(\omega)(q^* - X_S(\omega))^+ + \mu\phi(\omega)(X_S(\omega) - q^*)^+\right], \quad (18)$$

where $X_S(\omega) = \sum_{i\in S} Q_{2i}(\omega)/\mu$.

**Definition 7:** Chen et al. 2019. We call a continuous random variable $X$ log-concave if the logarithm of its density function $g(x)$ is concave.

Actually, log-concave probability distributions cover a wide variety of common distributions, including logistics, exponential, normal and uniform distributions, and have been widely applied in economics (Bagnoli and Bergstrom, 2005). Recall that $X_i(\omega) = Q_{2i}(\omega)/\mu$, $i\in N$. We now present the main result of this subsection.

**Proposition 4:** The agricultural producers' cooperative game $(N,\pi)$ is a convex game if $X_1,\ldots,X_n$ are independent continuous random variables with log-concave distribution functions.

Appendix S1 shows the proof of Proposition 4. Proposition 4 illustrates that the agricultural producers' cooperative game $(N,\pi)$ is a convex game under some mild conditions, which means that an individual agricultural producer's marginal contribution to a coalition increases if he joins a larger coalition. After acquiring the convexity of the agricultural producers' cooperative game $(N,\pi)$, we present another main result of this subsection.

**Corollary 2:** If $X_1,\ldots,X_n$ are independent continuous random variables with log-concave distribution functions, the agricultural producers' cooperative game $(N,\pi)$ always has population monotonicity.

Corollary 2 can be directly proven from the results of Sprument (1990), so we omit this proof for brevity. Corollary 2 illustrates that no member of an existing coalition can benefit from forming a smaller coalition. Under population monotonicity, each time a new agricultural producer joins a coalition, every current member of the coalition will be allocated a larger benefit, which gives every agricultural producer of the current coalition an incentive to expand their coalition to an ever-larger one. As we know, a larger coalition has greater powers in dealing with the risks that yield uncertainty brings about, so independent agricultural producers should choose to form a larger coalition to mitigate these risks.

### 5.4 The game with a concave effort cost

This subsection focuses on the agricultural producers' cooperative game $(N,\pi)$ with a concave effort cost. The structure of the concave effort cost may reflect the production economy in the process of agricultural production. When a production diseconomy or production economy exists, the agricultural producer's marginal production effort cost is nonlinear, and so, it may increase or decrease with the volume of production. This is described by the functional form $c_i(\cdot) = aq_i + bq_i^2$, and production diseconomy (economy) occurs when $b>0 (b<0)$. This concave cost structure has been applied widely in reality and often arises when suppliers provide incremental discounts to additional units beyond some threshold. Moreover, the concave cost structure has also been widely applied in other fields; see Chen and Zhang (2009) for more details. We note that the strong duality theory proposed by Rockafellar and Wets (1976) may not be directly applied to the concave cost structure.

Denote by $c_i(\cdot)$ the concave effort cost function for each agricultural producer $i \in N$. For the sake of analysis, we assume that the effort cost function $c_i(\cdot)$ is continuous and differentiable. Notably, the agricultural producers' cooperative game $(N, \pi)$ with a concave effort cost analyzed in this subsection is similar to (13), except for the structure of the effort cost function. Therefore, the characteristic profit function $\pi(S)$ with a concave effort cost structure, for any coalition $S \subseteq N$, is described by the following two-stage stochastic programming problem:

$$\begin{aligned} -\pi(S) = \min_{q_i \geq 0} & \sum_{i \in S} c_i(q_i) + \mathrm{E}[F(q, \omega)] \\ s.t. \quad & q_i \leq q_i^0, i \in S, \end{aligned} \tag{19}$$

where $q = (q_i)_{i \in S}$ and $F(q, \omega)$ is defined by:

$$\begin{aligned} F(q, \omega) = & \min \varphi(\omega) e(\omega) + \phi(\omega) s(\omega) - \sum_{i \in S} p \mu_i q_i \\ s.t. \quad & \sum_{i \in S} u_i(\omega) q_i - Q_2 - e(\omega) \leq 0, \ \omega \in \Omega, \\ & Q_2 - \sum_{i \in S} u_i(\omega) q_i - s(\omega) \leq 0, \ \omega \in \Omega, \\ & Q_2 \geq 0, \ e(\omega) \geq 0, \ s(\omega) \geq 0, \ \omega \in \Omega. \end{aligned}$$

It is clear that the first-stage formulation of problem (19) mainly contains the concave effort cost. Nevertheless, the other terms are all included in the second-stage formulation, which differs from the two-stage stochastic linear programming problem (13). Owing to the concave effort cost structure, solving the dual problem of (19) is challenging. To do this, we first solve the dual problem of generalising two-stage problem (14). Define:

$$\Phi^* = \min_{t_1 \in \Theta} l_1(t_1) + \mathrm{E}[h(t_1, \omega)], \tag{20}$$

where $\Theta = \{z_1 : B_{11} z_1 = d_1, z_1 \geq 0\}$ and $l_1(\cdot)$ is a concave function and satisfies $l_1(0) = 0$. We introduce two variables $\rho_1(z_1)$ and $\rho_2(z_1, \omega)$, $\forall z_1 \in \Theta$ such that:

$$l_1(z_1) - (\nabla l_1(z_1))^{\mathrm{T}} z_1 \geq \mathrm{E}\left[d_1^{\mathrm{T}} \rho_1(z_1) + d_2^{\mathrm{T}}(\omega)\rho_2(z_1,\omega)\right].$$

Moreover, for any given $z_1 \in \Theta$, we define $\lambda_1$ and $\lambda_2(\omega)$ so that:

$$\lambda_1 \leq \rho_1(z_1) + \xi_1(z_1); \lambda_2(\omega) \leq \rho_2(z_1,\omega) + \xi_2(z_1,\omega), \omega \in \Omega,$$

where $\xi_1(z_1)$ and $\xi_2(z_1,\omega)$, $\omega \in \Omega$ are the dual variables of the minimum problem $\min_{t_1 \in \Theta} (\nabla l_1(z_1))^{\mathrm{T}} t_1 + \mathrm{E}[h(t_1,\omega)]$. Hence, the dual of the two-stage stochastic programming problem (20) is given by:

$$\begin{aligned}
\Phi^* = \max\ & \mathrm{E}[d_1^{\mathrm{T}} \lambda_1 + d_2^{\mathrm{T}}(\omega)\lambda_2(\omega)] \\
s.t.\ \ & \lambda_1 \leq \rho_1(z_1) + \xi_1(z_1),\ \forall z_1 \in \Theta, \\
& \lambda_2(\omega) \leq \rho_2(z_1,\omega) + \xi_2(z_1,\omega),\ \forall z_1 \in \Theta,\ \omega \in \Omega, \\
& \mathrm{E}\left[d_1^{\mathrm{T}} \rho_1(z_1) + d_2^{\mathrm{T}}(\omega)\rho_2(z_1,\omega)\right] \leq l_1(z_1) - (\nabla l_1(z_1))^{\mathrm{T}} z_1,\ \forall z_1 \in \Theta,\ \omega \in \Omega, \\
& B_{11}^{\mathrm{T}} \xi_1(z_1) + \mathrm{E}\left[B_{12}^{\mathrm{T}} \xi_2(z_1,\omega)\right] \leq \nabla l_1(z_1),\ \forall z_1 \in \Theta,\ \omega \in \Omega, \\
& B_{22}^{\mathrm{T}} \xi_2(z_1,\omega) \leq l_2,\ \forall z_1 \in \Theta,\ \omega \in \Omega,
\end{aligned} \tag{21}$$

where $(\lambda, \rho, \xi)$ is the decision variable of the dual problem (21). We now turn our attention to the agricultural producers' cooperative game with a concave effort cost function given by problem (19). It is obvious that problem (19) is a special case of problem (20). Thus, the dual problem of the two-stage stochastic programming problem (19) is obtained immediately:

$$\begin{aligned}
& \max \sum_{i \in S} \lambda_i q_i^0 \\
& s.t.\ \ \lambda_i \leq \xi_i(z) + \rho_i(z),\ \forall z \in Z,\ i \in S, \\
& \sum_{i \in S} \rho_i(z) q_i^0 \leq \sum_{i \in S} \left(c_i(z_i) - c_i'(z_i) z_i\right),\ \forall z \in Z, \\
& \xi_i(z) + \mathrm{E}[u_i(\omega)\delta(z,\omega)] - \mathrm{E}[u_i(\omega)\sigma(z,\omega)] \leq c_i'(z_i) - p\mu_i,\ \forall z \in Z,\ i \in S,\ \omega \in \Omega, \\
& -\mathrm{E}[\delta(z,\omega)] + \mathrm{E}[\sigma(z,\omega)] \leq 0,\ \forall z \in Z,\ \omega \in \Omega, \\
& -\delta(z,\omega) \leq \varphi(\omega),\ \forall z \in Z,\ \omega \in \Omega, \\
& -\sigma(z,\omega) \leq \phi(\omega),\ \forall z \in Z,\ \omega \in \Omega,
\end{aligned} \tag{22}$$

where $Z = \{q : 0 \leq q \leq q^0\}$ and $(\lambda,\ \rho(z),\ \xi(z),\ \delta(z,\omega),\ \sigma(z,\omega))$, $\omega \in \Omega$ are the decision variables of the dual problem (22). Denote by $(\lambda^*, \rho^*(z), \xi^*(z), \delta^*(z,\omega), \sigma^*(z,\omega))$ an optimal solution to the dual problem (22) with $S = N$. Define for $i \in N$:

$$l_i = -\lambda_i^* q_i^0. \tag{23}$$

**5:** The vector $l = (l_1, l_2, ..., l_n)$ defined by (23) is a profit allocation policy in the core of the agricultural producers' cooperative game $(N, \pi)$ with a concave effort cost, if $\rho^*(z) \leq 0$ for any $z \in Z$.

The proof of Proposition 5 is similar to that of Proposition 2, so we omit it for brevity. Proposition 5 demonstrates that the core of the agricultural producers' cooperative game $(N, \pi)$ with a concave effort cost is nonempty and provides a simple way to compute a core allocation policy when facing economies of scale for production. Particularly, Proposition 5 requires the condition that $\rho^*(z) \leq 0$ for any $z \in Z$. We would like to point out that this condition is just a sufficient condition and ensures that any optimal dual solution to the grand coalition $N$ is a feasible solution to the dual problem of coalition $S \subseteq N$.

**3:** The agricultural producers' cooperative game $(N, \pi)$ with a concave effort cost is balanced.

Based on the result proposed by Proposition 5, Corollary 3 can be easily obtained from Bondareva (1963). Thus, when facing economies of scale of production, the agricultural producers' coalition still has a stabilizing allocation policy.

So far, we have analyzed a non-cooperative case and a cooperative case, respectively. Next, we discuss the implications of expanding the model to include agricultural producers with different strategies and provide some insights and conjectures on this point. For a group of $n$ independent agricultural producers denoted by $N = \{1, ..., n\}$, let $RS \subseteq N$, we assume that $r$ agricultural producers choose a cooperative strategy denoted by $R = \{1, ..., r\}$, and $n - r$ agricultural producers choose a non-cooperative strategy denoted by $N - R = \{r+1, ..., n\}$. In this case, the maximum expected total profit of $n$ agricultural producers is given by:

$$\begin{aligned}\pi(R, \{N-R\}) = \max_{Q_2, q_i, Q_{1i} \geq 0} \sum_{i \in R} (p\mu_i - c_i) q_i - \mathrm{E}\left[\varphi(\omega)\left(\sum_{i \in R} u_i(\omega) q_i - Q_2\right)^+ + \phi(\omega)\left(Q_2 - \sum_{i \in R} u_i(\omega) q_i\right)^+\right] \\ + \sum_{i \in N-R} \left((p\mu_i - c_i) q_i - \mathrm{E}\left[\varphi(\omega)(u_i(\omega) q_i - Q_{1i})^+\right] - \mathrm{E}\left[\phi(\omega)(Q_{1i} - u_i(\omega) q_i)^+\right]\right).\end{aligned}$$

We now provide some insights and conjectures on this case where agricultural producers have more than one strategy. First, denote by $L_{i \in R}$ the allocated profit of agricultural producer $i \in R$ in the case of more than one strategy, we have $L_{i \in R} \leq l_i$, where $l_i$ is the allocated profit of agricultural producer $i$ in coalition $N$ based on the population monotonicity. Namely, if $n$ agricultural producers choose more than one strategy, the allocated profit for agricultural producer $i \in R$ is no more than that for agricultural producer $i \in N$ in the case of only cooperative strategy. Second, let $\Pi(\{i\})_{i \in N-R}$ be the obtainable profit of agricultural producer $i \in N-R$ in the case of more than one strategy, we have $\Pi(\{i\})_{i \in N-R} = \pi(\{i\})$, where $\pi(\{i\})$ is the obtainable profit of agricultural producer $i \in N$ in the case of only non-cooperative strategy. Finally, for a group of $n$ independent agricultural producers, we argue that $\pi(N) \geq \pi(R, \{N-R\}) \geq \sum_{i \in N} \pi(\{i\})$. In other words, the expected total profit of $n$ agricultural producers in the cooperative strategy is the highest. On the contrary, in the non-cooperative strategy, the expected total profit is the lowest. We would like to point out that the corresponding calculations are very complicated, so we leave the full calculations for future research.

## 6. Numerical analysis

This section presents some numerical analyses to illustrate our results and approach proposed in this paper. We first provide a core profit allocation policy for three agricultural producers of a coalition to illustrate the proposed stochastic duality approach. Then, we conduct sensitivity analyses for the three important parameters $u(\omega)$, $\varphi(\omega)$ and $\phi(\omega)$ on the profit allocation policy.

### 6.1 A core profit allocation policy for a coalition

Consider a coalition involving three independent agricultural producers denoted by $N = \{1,2,3\}$. Set the forward price $P = 5$, the unit cost of the effort level $c = \{1.0, 1.5, 2.0\}$ and the maximal production level $q^0 = \{10, 20, 30\}$. Denote by a set $\Omega = \{\omega_1, \omega_2, \omega_3\}$ the random experiments with the outcomes that three agricultural producers face, so that $\omega_1$ occurs with probability $\delta_1 = 0.2$, $\omega_2$ occurs with probability $\delta_2 = 0.3$ and $\omega_3$ occurs with probability $\delta_3 = 0.5$. The yield factor is given by

$$(u_1(\omega_i), u_2(\omega_i), u_3(\omega_i)) = \begin{cases} (0.4218, 0.9157, 0.7922) if \;\; i = 1, \\ (0.9157, 0.4218, 0.4218) if \;\; i = 2, \\ (0.7922, 0.7922, 0.9157) if \;\; i = 3. \end{cases}$$

Let $\varphi(\omega) = 1$ and $\phi(\omega) = 1$. A core profit allocation policy is obtained for the three agricultural producers of coalition $N$, which is listed in Table 2.

It is easy to verify that the allocated profit of each agricultural producer is 27.3882, 38.8496 and 46.2384, and the total profit of coalition $N$ is 112.4762. This result illustrates that the proposed stochastic programming duality approach to the agricultural producers' cooperative game is feasible.

Here, we explore another possible aspect where the three independent agricultural producers do not choose to cooperate, but produce alone to find

**Table 2** A core profit allocation policy

| | Producer 1 | Producer 2 | Producer 3 |
|---|---|---|---|
| The optimal dual solution $\lambda_i^*$ | −2.7388 | −1.9425 | −1.5413 |
| The core allocation policy $l_i$ | 27.3882 | 38.8496 | 46.2384 |

an optimal strategy for themselves. In the non-cooperative case, it is easy to derive that:

$\pi(\{1\}) = 26.6472$, $\pi(\{2\}) = 37.8616$ and $\pi(\{3\}) = 46.2384$.

One can then find that $l_1 \geq \pi(\{1\})$, $l_2 \geq \pi(\{2\})$ and $l_3 \geq \pi(\{3\})$, which illustrates that not only the allocated profit of each agricultural producer in the cooperative case is at least as good as that obtained by playing alone in the non-cooperative case, that is $l_i \geq \pi(\{i\})$, but also the coalition can improve their expected total profit by jointly offering their aggregate output, that is $\pi(N) \geq \sum_{i \in N} \pi(\{i\})$. Note that, in the following discussion, the parameters concerning the agricultural producers' cooperative game are set to the same as those mentioned above, apart from some special instructions.

## 6.2 Discussion for parameter $u(\omega)$

The parameter $u(\omega)$ in an agricultural producers' cooperative game model reflects the yield uncertainty caused by uncontrollable factors such as weather conditions, technological innovations and natural hazards. We now investigate the impact of the yield factor $u(\omega)$ on the core allocation policy for the three agricultural producers.

Figure 1 shows that the yield factor $u(\omega)$ plays an important role in the profit allocation policy for the three agricultural producers. It is clear to see that the allocated profit of each agricultural producer increases linearly with the change in the yield factor $u(\omega)$, which is in accordance with the role that the yield factor $u(\omega)$ plays in the agricultural producers' cooperative game $(N, \pi)$. Hence, yield uncertainty brings about a huge challenge to agricultural producers. To deal with this challenge, forming a willing coalition for independent agricultural producers is an appropriate choice by taking advantage of the risk-pooling effect. In particular, when $u(\omega)$ is extremely small and may even be close to 0, it would lead to a terrible situation where agricultural producers of coalition $N$ seriously violate the forward contract and thus receive a low profit, as is shown in Figure 1. On the other hand, if $u(\omega)$ is close to 1, the realized yield of each agricultural producer is roughly equal to their own effort level, and thus, a better payoff will be obtained.

## 6.3 Discussion for parameter $\varphi(\omega)$

The parameter $\phi(\omega)$ denotes the penalty cost of covering the shortage. Thus, the size of the parameter $\phi(\omega)$ has a direct impact on the profit allocation

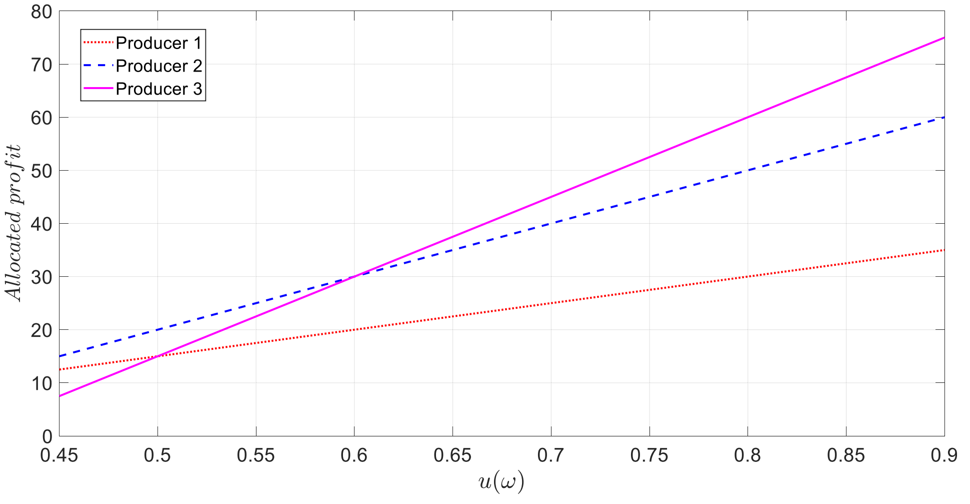


**Figure 1** Allocation policy under different yield factors. [Colour figure can be viewed at wileyonlinelibrary.com]

policy for the three agricultural producers. The maximal production level is set to $q^0 = \{10, 15, 18\}$. In this case, the results are shown in Figure 2.

Figure 2 shows that the allocated profits of agricultural producers 1, 2 and 3 decrease as the penalty cost $\phi(\omega)$ increases. In other words, the larger the penalty cost $\phi(\omega)$, the smaller the allocated profits of agricultural producers 1, 2 and 3, which is consistent with our results. This is because an increase in the penalty cost $\phi(\omega)$ leads to a decrease in the profits of coalition $N$. Thus, reducing the penalty cost would benefit all producers in practice. Moreover, it is easy to see that the profit's change for producer 3 is larger than that for

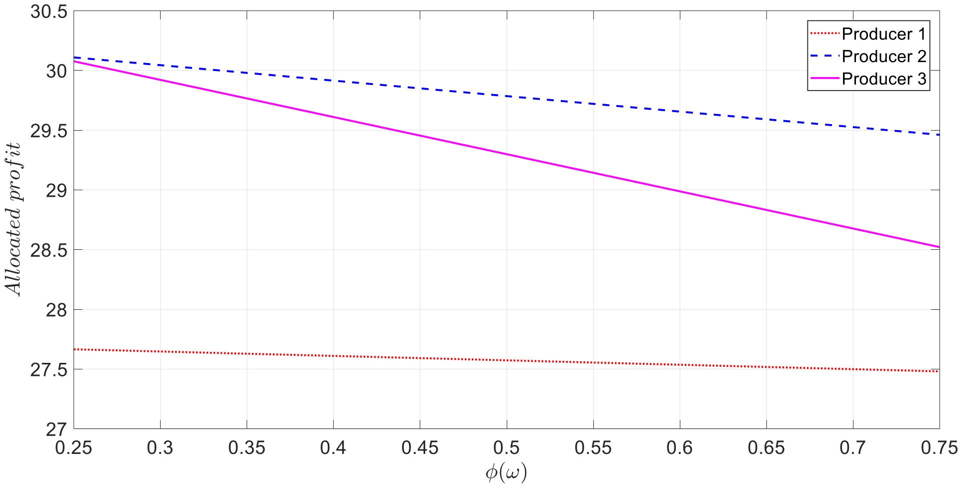


**Figure 2** Allocation policy under different values of $\phi(\omega)$. [Colour figure can be viewed at wileyonlinelibrary.com]

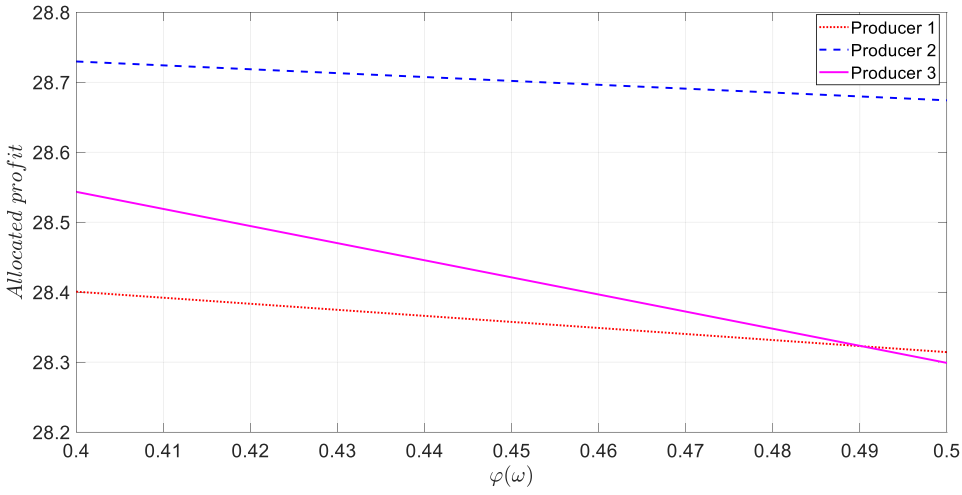


**Figure 3** Allocation policy under different values of $\varphi(\omega)$. [Colour figure can be viewed at wileyonlinelibrary.com]

producers 1 and 2 as the penalty cost $\phi(\omega)$ increases, the reason for which is that producer 3 has the highest production cost. Consequently, the profit of the producer with a high production cost would be seriously affected by a change in the penalty cost.

### 6.4 Discussion of parameter $\varphi(\omega)$

The parameter $\varphi(\omega)$ represents the discount to be offered for selling the surplus. Consequently, the allocated profit of each agricultural producer in coalition $N$ is affected substantially by different values of $\varphi(\omega)$. For this purpose, Figure 3 investigates the relationship between the allocation policy and parameter $\varphi(\omega)$.

From Figure 3, we can observe that the allocated profits of agricultural producers 1, 2 and 3 decrease when the discount cost $\varphi(\omega)$ increases. The reason for this phenomenon is that the discount cost $\varphi(\omega)$ has a negative effect on the profits of coalition $N$. Recall that $\varphi(\omega) = p - v(\omega)$, where $v(\omega)$ denotes the salvage value. Thus, a high discount cost (or a low salvage value) would result in small profits for agricultural producers. Furthermore, we also observe that the profits' changes for both producers 1 and 2 are smaller than those for producer 3. The reason for this is similar to that analysed in subsection 6.3. Thus, as the discount cost (or the salvage value) changes, producers with low production costs would be slightly affected in practice.

## 7. Conclusion

In this study, we propose a two-stage stochastic programming model for studying the agricultural producers' cooperative game problem that explicitly accounts for yield uncertainty in contract farming. Using the cooperative game theory and stochastic duality theory, we propose different profit allocation policies for agricultural producers of a cooperative under different situations.

First, we examine the individual optimal decisions in the non-cooperative case. Second, a constructive proof is provided to show the non-emptiness of the core of the agricultural producers' cooperative game, and a simple way for computing a profit allocation policy in the core is provided. Third, we establish that the agricultural producers' cooperative game has population monotonicity, which gives an incentive for coalition expansion. Finally, we find that the agricultural producers' cooperative game with a concave cost structure also has a nonempty core, and we also provide a way to compute the corresponding allocation policy.

We generate some implications to help agricultural producers obtain better profits. For example, agricultural producers should try their best to improve their maximal production levels and reduce their yield uncertainties. At the same time, agricultural producers should raise their unit salvage prices and lower their unit penalty prices. Our study formalizes why agricultural producers should form a cooperative, how they should cooperate, and how to design a fair profit allocation policy for members of a cooperative. One of the main contributions of this study is to provide a simple and stylized model for agricultural producers' cooperation in the contract farming context. Thus, our model not only can serve as a reference point for future empirical fieldwork but also can be a useful building block for more complicated models. Another main contribution of this study is to provide a unified and parsimonious framework of profit allocation for studying the farmers' cooperative faced with uncertainty in contract farming. Hence, our proposed allocation mechanism can be applied to a variety of farmers' cooperatives around the world. The allocation mechanism is proven to be fair, and so, it plays an important role in the sustainable development of farmers' cooperatives. Interestingly, taking advantage of the allocation mechanism, every agricultural producer in a cooperative can improve its own strategy to obtain a better profit.

There are several interesting directions for future research in relation to this study. First, we only consider a common single product with respect to all agricultural producers in this study. Thus, it may be interesting to consider multiple products in this context. Second, it would also be interesting to extend the stochastic duality approach to analyze our model under the assumption that agricultural producers are competing with each other. Finally, this paper considers a complete information setting in which production effort costs are common knowledge to all agricultural producers. Hence, it would be interesting to conduct research into the design of a fair allocation policy when the setting is characterised by an incomplete information.

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## Supporting Information

Additional Supporting Information may be found in the online version of this article:

**Appendix S1.** Derivations and Proofs.