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ON THE PRICING OF CROSS CURRENCY FUTURES OPTIONS FOR CANADIAN GRAINS AND LIVESTOCK

by

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WORKING PAPER 02/07

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February 21, 2002

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On the Pricing of Cross Currency Futures Options for Canadian Grains and Livestock

Abstract

This paper explores the problem of pricing an option on the cash commodity in Canadian dollars when the commodity is priced relative to a U.S. futures market. A general options pricing model is developed that separates out the value of a quantos risk and basis risk. The paper uses daily data for cattle, corn and soybeans in Ontario, and the model is employed to price the option on the cash commodity with basis risk and the option on a quantos, without basis risk. The relationship between the pricing model and over-the-counter options and market revenue insurance is also discussed.

Introduction

The purpose of this paper is to investigate the pricing of a Canadian dollar denominated commodity option when the commodity price is heavily influenced by futures markets in the United States. The general problem revolves around the cash price relation in equation (1).

(1) $P_t = F_t{*}E_t + B_t$

where P is the cash price, F is the price of a commodity futures contract traded on a U.S. exchange, E is the exchange rate ($CDN/$U.S.) and B is the basis differential between the cash and adjusted futures price. When a call or put option is priced relative to the boundary condition MAX[$F_T{*}E_T$ – X,0] or MAX[X – $F_T{*}E_T$,0] the option value is a quantos on the futures contract. A quantos is a derivative product involving two currencies. The terms of the underlying futures contract in our context is in $U.S. but the payoff is made in terms of $CDN. When the call or put option is priced relative to the boundary condition MAX[P_T – X, 0] or MAX[X – P_T , 0] it is called an option on the cash commodity. The option on the cash commodity includes the quantos option as a special case and differs in value from the quantos option because of the basis. The purpose of this paper is to derive the equilibrium options prices for both option types and to illustrate how the options can be used in practice to price over-the-counter risk management products or in the case of public policy, market revenue insurance[1].

. The paper is motivated by several factors. First, because of Canada's geographic and competitive position relative to the U.S., its local cash markets are heavily influenced by U.S. markets. Braga (1996) discusses several over-the-counter options sold in Canadian dollars but priced to U.S. and exchange rate risk. These include the cattle options pilot program, which sold put options on live cattle. The contracts were written in 10,000 lbs increments. Producers would buy the options via telephone and electronic funds transfer. The price was set to the market three times per day. The expiry months for the live cattle options matched the months of the underlying futures contracts. The cattle options pilot project did not price basis risk. Using a similar structure, Saskatchewan offered a minimum price hog contract , written by CIBC-Wood Gundy, on the Canadian dollar value of CME live hog futures (Braga, 1996). In addition to this quantos value, a forward basis was added to the $CDN futures floor to establish a minimum cash price. Essentially a buyer of the put would receive the higher of the cash price guarantee or the pooled (pork) board price. Turvey and Romain (2000) examined the use of options to protect the price volatility faced by further milk processors. Under 5a and 5b pricing the price of milk for further processing is heavily weighted on the U.S. base formula price (BFP). In order for further processors to reduce market price risk a quantos based on the U.S. BFP futures contract and the exchange rate was examined. Turvey and Romain (2000) examined ordinary European type options as well as Asian options (options on the average) but do not provide explicit formulae for pricing the options since they used Monte Carlo simulations to price the options. Turvey (1992) used a modified Black-Scholes model to price an option on the cash commodity. He assumed that cash price risk could be spanned or hedged by some other (but unknown) asset or security and therefore did not fully account for the market price of risk, nor did he give any consideration to the influence of U.S. futures prices and foreign exchange on the option value.

A second motivator is the recent focus in the U.S. on market driven revenue insurance products. Motivated in part by Canada's experience with the Gross Revenue Inurance Plan

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[1] The problem of hedging across currencies is not new. The Nikei 250 index traded on the CME is a quantos product since its value is derived from the Nikkei 250 on the Tokyo exchange, except that it is denominated in $U.S. rather than yen. Wei (1997) , Willmot (1998) ,Hull (1997) and others develop models to price these contracts, but there is a gap between pricing stock index futures contracts and commodity contracts in foreign currencies. This paper is dedicated to filling that gap.

(GRIP) in the early 1990's, , revenue insurance contracts in the U.S. are offered under the CRP program on a wide range of commodities (see Stokes 2001, Stokes et al 1997, Tirupattur et al 1997, Kang and Brorsen 1995). These revenue products consider randomness in yields as well as prices. As support payments for agriculture in Canada wanes, there are opportunities to offer over the-counter market driven revenue contracts where the fundamental price movements are governed by U.S. commodity markets. Should Canada move forward with this paradigm for exchange traded commodities, it must consider the joint randomness in prices and exchange rates, as well as basis risk, and this requires a different model than might be used in the U.S. This paper is dedicated to developing and exploring such a model.

Background

We begin by examining an option on the cash price as specified in equation 1. We then back out of this general model the price of an option on the quantos alone. The model makes several assumptions. The first is that randomness in F, E and B follow geometric Brownian motions of the forms

(2) $dF = F[\mu_1 dt + \sigma_1 dw_1]$,

(3) $dE = E[\mu_2 dt + \sigma_2 dw_2]$, and

(4) $dB = B[\mu_3 dt + \sigma_3 dw_3]$,

where the μ represent the natural growth rates, the σ represent the standard deviations of the percentage change of the stochastic variables, the dw represent Wiener processes of the form $\varepsilon t^{.5}$, and $\varepsilon \sim N(0,1)$ are standard normal deviates. The second assumption is that both F and E are tradable so that a risk free portfolio can be constructed in these two variables. Wei (1997) provides a proof of this using risk neutral valuations and Willmot (1998) provides a proof using arbitrage. The third assumption is that the basis, B, is a non traded variable so that its risk neutral growth rate, in the context discussed in Cox and Ross (1976) and Cox, Ingersoll and Ross (1985), is given by $\mu_3 - \lambda_3\sigma_3$ where λ_3 is the market price of risk and $\lambda_3\sigma_3$ is the risk premium required to accept basis risk in the market place[2]. Furthermore, we assume that the market price

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[2] We calculate the basis on an adjusted versus unadjusted basis. Braga (1990) states that many elevators in Ontario fix the basis as if it were at parity with the U.S. dollar and do not make any basis adjustment for exchange rates. Unadjusted basis variability appears to exist in the short term, but in the long-term Braga (1990) finds that the adjusted basis is more consistent with market transmissions.

of risk is given by the equilibrium risk premium from the security market line of the Sharpe-Lintner capital asset pricing model. That is, the risk neutral growth rate for basis is given by μ_3 - $\beta[R_m - r]$ where β represents the systematic relationship between basis and the return on the market portfolio, R_m , and r is the risk free rate. (Other measures of the market price of risk and market risk premium can be used, but they must be consistent with the equilibrium pricing of risk.)

To obtain a closed form solution for the European option we must ensure that dP/P follows a path of geometric Brownian motion. We assume that this Brownian motion is of the form

(5) $dP = P[\mu_4 dt + \sigma_4 dw_4]$.

Using the Brownian motions in (2)-(4) and applying Ito's Lemma to (1) yields

(6) $dP = (EF(\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2) + B\mu_3)dt + EF(\sigma_1 dw_1 + \sigma_2 dw_2) + B\sigma_3 dw_3$

which is not normally distributed for dP/P. To convert this to a normally distributed random walk, i.e. a Brownian motion, we divide both sides by P to get

(7) $dP/P = ((EF/P)(\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2) + (B/P)\mu_3)dt + (EF/P)(\sigma_1 dw_1 + \sigma_2 dw_2) + (B/P)\sigma_3 dw_3$.

We can now define

(8) $\mu_4 = ((EF/P)(\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2) + (B/P)\mu_3)$

as the natural growth rate in the cash price and

(9) $\sigma_4^2 = (EF/P)^2 (\sigma_1^2 + \sigma_2^2 + \rho_{12}\sigma_1\sigma_2) + (B/P)^2 \sigma_3^2 + (EF)B/P^2 (\rho_{13}\sigma_1\sigma_3 + \rho_{23}\sigma_2\sigma_3)$

as its variance. The Brownian motion for the cash price can be viewed as a portfolio comprised of the weighted average contributions from futures and exchange rate risk and basis risk. Equation (7) also includes components for covariate risk between basis, futures and the exchange rate. It must include these terms because the optimal hedges are geared towards eliminating the marginal risks of S and E only. Since the covariance effects are unhedged then the deduction can be viewed as the market price of risk for taking on covariate risk. We can use this property to derive a closed form solution for the cash price hedge, with and without basis. But before doing this we must recognize that neither cash, the Canadian dollar denominated value of the U.S. futures contract, or the basis are traded variables and must be valued in equilibrium using the risk neutral growth rate.

Risk Neutral Valuations and the Market price of Risk

In equilibrium, options prices can be obtained by assuming that all agents in the economy behave as if they are risk neutral (Cox and Ross, 1976). When pricing options, it is the risk neutral growth rate that matters, not the natural rate. Hence, we must convert the natural growth rates μ_1, μ_2, and μ_3, into their risk neutral equivalents. In doing so we solve for the risk neutral valuation that will substitute for μ_4 in equation (7).

In general, we can define the risk neutral growth rate under the Cox-Ross method as

(10) $r^* = \mu - \lambda\sigma$

Where r^* is the risk-neutral growth rate, μ is the natural growth rate, λ is the market price of risk of the state variable, and σ is its volatility. If hedging can eliminate σ, then the market price of risk is zero and the variable will be priced assuming that μ = r, the risk free rate. If σ cannot be eliminated then $r^* \neq r$, but by adjusting for the market risk premium, $\lambda\sigma$, the risk neutral or risk adjusted growth rate, r^* , can be used instead of μ to price the derivative. When the risk neutral growth rate is used, the cashflows from the derivative can be discounted at the risk free rate.

Because both the Canadian dollar exchange rate and the commodity price can be hedged we can use arbitrage arguments for determining the risk neutral valuations for μ_1 and μ_2 . If F is a traded commodity then arbitrage arguments hold that at some future time T, $\ln(F_T/F_0) = (r_f - \delta)$ where r_f is the risk free rate in the foreign country hosting the forward market for F, and δ is the storage costs associated with a storable commodity. Thus,to avoid arbitrage $\mu_1 =(r_f - \delta)$. In terms of μ_2 we note that to avoid arbitrage, the ratio of exchange defined by E can only grow by the interest rate differential $(r – r_f)$ where r is the risk free rate in domestic currency. Thus $\mu_2 = (r – r_f)$. Having defined the risk neutral growth rates for S and E we now have the risk neutral growth rate for FE as $(r -\delta + \rho_{12}\sigma_1\sigma_2)$.

The risk-neutral growth rate for the basis is given by

(10) $B_T = B_0 e^{\mu-\lambda\sigma}$

Because the market price of risk is an unknown we have to consider how a complete market in equilibrium would price the observed risk. We use the Sharpe-Lintner capital asset pricing model (CAPM) to set the market risk premium $\lambda\sigma = \beta[R_m – r]$ where β measures the degree of covariance (systematic risk) between the commodity basis and the return on the market portfolio,

R_m. Although other measure of the market risk premium can be used, the CAPM model is a convenient expression of equilibrium asset pricing because it represents the market value of systematic risk in a framework that is independent of risk preferences, and does not require (at least in the context of Roll, 1977) that the underlying state variable to be traded. If basis has zero correlation with the market portfolio ($\beta = 0$) then the risk neutral growth rate will simply be its natural growth rate. If the natural growth rate in basis is also zero then simply put, the expected value of the basis at option expiration is deemed to be equal to the initial basis value.

Given the various expressions for the risk neutral components for the three variables making up the cash price, we can express the cash price dynamic as

(11) $dP/P = (\omega(r_d - \delta + \rho_{12}\sigma_1\sigma_2) + (1-\omega)(\mu_3 - \beta[R_m - r]))dt + \omega (\sigma_1 dw_1 + \sigma_2 dw_2) + (1-\omega)\sigma_3 dw_3.$

where $\omega = (EF/P)$ and $(1-\omega) = B/P$. We will use this risk neutral growth rate to price options on the cash and options on the future in the next section.

Options on the Cash and Options on the Futures

Given the risk-neutral growth rate in (11) a generalized option formula can be used to price call and put options. These formulas are consistent with the formulas used by Constantinides (1978) and McDonald and Siegel (1984) and can also be found in Baxter and Rennie (1998) and Trigeorgis (1999). For a strike price on the cash commodity, X, the equilibrium call option value subject to the boundary condition $Max[(F_T E_T + B_T) - X,0]$ is given by

(12) $f_c = P_0 e^{(\theta - r)T} N(d_1) - X e^{-rT} N(d_2)$

Where P_0 is the cash price, r is the domestic risk free rate,

$\theta = (\omega(r - \delta + \rho_{12}\sigma_1\sigma_2) + (1-\omega)(\mu_3 - \beta[R_m - r])),$

$d_1 = [\ln (P_0/X) + (\theta + .5\sigma^2)t]/\sigma\sqrt{t}$, and

$d_2 = d_1 - \sigma\sqrt{t} = [\ln (P_0/X) + (\theta - .5\sigma^2)t]/\sigma\sqrt{t}.$

Using the put-call parity, the equilibrium put option value is given by

(13) $f_p = Xe^{-rT} N(-d_2) - P_0 e^{(\theta-r)T} N(-d_2)$

The options prices in (12) and (13) are very general but consistent with the equilibrium pricing of options on non-traded assets. The option prices represent a strike relative to the cash value, but they can easily be modified to examine other options types. For example, setting ω=1 eliminates the basis component to the option. This option is a quantos in that it is the price in Canadian dollars on a futures contract written on a U.S. exchange, but with (cash) settlement in Canadian dollars. If this option is used then $P_0 = F_0E_0$ should be calculated. If the commodity is a storable commodity then $\delta \geq 0$, but if it is not storable (e.g. livestock) then $\delta=0$. If $\omega = 0$, then the valuation provides a solution to pricing an option on the basis and $P_0 = B_0$ should be used. For pricing options on the cash price of a commodity where there exists no forward market then by setting $\omega= 0$ and $\mu_3 = \mu_4$ (the natural growth rate in the price of the cash commodity), $\theta = (\mu_4 - \beta[R_m - r])$ becomes the risk neutral growth rate for the cash commodity and the model collapses to one similar to Stokes, Nayda and English (1997). If E=1 and $\omega = 1$, then $\rho_{12}\sigma_1\sigma_2 = 0$, and $\theta = r = r_f$. By setting $P_0 = F_0e^{-rT}$, the equations collapse to Black's (1976) model for pricing futures in its original currency. Finally if the cash price can be spanned by a tradeable asset then for E=1 and $\omega = 1$, $\theta = r$ or r_f , and P_0 equal to the value of the traded security, the formulas collapse to Black and Scholes (1973) in the originating currency.

The Relationship between Canadian Cash and U.S. Futures Prices

Before proceeding to illustrative examples of pricing options on the cash commodity, this section explores in modest detail the relationship between Ontario corn, soybeans, and live cattle prices in Canada, the U.S. futures prices, and the exchange rate. Data for corn, soybeans (Chatham Ontario basis), live cattle (Toronto Ontario basis) and the $CDN/$U.S. exchange rate were obtained from Ridgetown College and cover daily observations from January 3, 1995 through June 30, 2000. All data were date matched and each data series is comprised of 1,396 observations. The data summary is presented in Table 1.

The top panel in Table 1 summarizes data in levels prices and the bottom panel provides a summary in terms of percentage price changes. The former presents an absolute measure of risk while the latter provides a relative measure of risk. Looking first at the levels data, it is notable that the standard deviation of cattle prices is much lower relative to cash prices than corn and soybeans. The coefficient of variation (standard deviation / mean) on livestock is only 0.066 which means that for each dollar of expected price there is only $0.066 of risk. In contrast the coefficient of variation for corn is 0.29 and for soybeans it is 0.17. The coefficient of variation on the exchange rate is only 0.04 so it is clear that much of the risk in the cash markets is due to the variability in the U.S. forward markets and the local basis. In terms of basis the cash prices are on average lower than the Canadian dollar denominated futures prices. However there is still significant variability in basis. The cattle basis of $-4.47 represents only 5% of the average cash cattle price. Likewise the basis for corn (-0.18) represents about 5.5% of the average cash price and the $-0.27 basis for soybeans represents about 3.7% of the cash price. Where the basis differs is in its variance. For cattle the standard deviation is $4.52 but for corn it is $.43, more than twice its expected value, and for soybeans the standard deviation of .34 is almost 26% higher than its mean.

The lower panel of Table 1 provides a somewhat different story. The mean percentage changes actually represent the natural growth rates of the commodities and exchange rate (symbolized by μ in the theory section), and the standard deviation is the volatility measure, σ, referred to in the theory section. The natural growth rates and volatilities have been converted from a daily measure into an annualized measure based on a 250 day trading year. The results show that all three commodities have relatively low growth rates ranging from 1% to –2%.

The volatility for cattle is larger in the cash market than the futures market in either $U.S. (.15) or Canadian (.16) dollars. In contrast, the volatilities for corn and soybeans cash are .19 and .17 respectively, which are lower than the .24 and .23 found for the original futures prices when converted to Canadian dollars. One would expect that a price driven by the U.S. market would have a higher volatility in the cash market than the futures market, but in the case of corn and soybeans this does not appear to be the case. The implication of this is that the basis is positively correlated with the futures price (in both currencies) for livestock, but negatively correlated for corn and soybeans. Further investigation, however revealed that the covariance between basis and the futures price (using levels data) for all three commodities was sample dependent and would oscillate between positive and negative covariance in an apparently unpredictable way. This in fact should not be surprising since the basis captures many local economic factors that are not reflected in the corresponding futures prices. These factors include supply and demand imbalances, convenience yields and availability of storage facilities (Hull, 1997). The practice of using an unadjusted basis by some Ontario elevators to establish cash market prices may also have an influence on this result (Braga, 1990). The implication in terms of options pricing for the cash commodity is that care should be taken to ensure that the economic conditions at the time of writing the option should be consistent with the sample data used to calculate option values.

The natural growth rates and volatilities of basis are presented in the last two column of the lower panel in Table 1. Under the model assumptions, the variability and natural growth rate in cash prices are determined by the combined risk of futures and foreign exchange and the local basis. Risk neutral rates and volatilities for futures and exchange rates can be determined directly from the sample data under the log normal assumption, but the risk neutral return of basis and basis risk are not so easily calculated since these are mathematically residual measures that can have positive or negative values. To overcome these problems, we extract the implied long term natural growth rate and volatility of basis from the sampling distributions of cash, futures and exchange rates. To determine the natural growth rate we note from equation (8) that

(14) $\mu_4 = (\omega\,(\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2) + (1\text{-}\omega)\,\mu_3)$

where μ_4 is the mean percentage change in cash prices, $\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2$ is the mean percentage change of the product FE, ω is the mean of the ratio FE/P and μ_3 is the natural growth rate in basis implied by the natural rates of cash and futures prices. The natural growth rate in basis is then given by

(15) $\mu_3 = (\ \mu_4 - (\omega\,(\mu_1 + \mu_2 + \rho_{12}\sigma_1\sigma_2)\ )\ /\ (1\text{-}\omega)$

where are all terms on the right hand side observable and measurable.

We can extract the volatility of basis by examining the relationship between the cash and futures prices. There are a number of ways of doing this but a simple approach is to use the formula

(16) $\sigma_3\ = (\sigma_1^{\,2} - \phi^2\sigma_2^{\,2}\ )^{.5}$

where σ_1 and σ_2 are the sample standard deviations of the cash price and the product FE respectively and ϕ is the slope from a least squares regression of cash prices against the FE product. Since $\phi^2\sigma_2^{\,2}$ measures the proportion of variance in cash prices explained by futures and exchange, then the residual variance measured by $(\sigma_1^{\,2} - \phi^2\sigma_2)$ must be that attributable to basis risk. The computed value of ϕ for cattle, corn and soybeans are -.117, .556 and .604 respectively. The low negative values of ϕ for cattle suggests that day-to-day, the percentage change in cash price does not closely follow percentage changes in the futures price, even though there is a systematic pattern using levels data. In contrast, cash price changes for corn and soybeans are more predictable, and this may explain why, as discussed below, the basis volatility for cattle is so much higher than for corn and soybeans.

On a relative measure, the mean percentage changes in basis are consistent with measures for cash and futures[3]. All three natural rates are negative, which indicates over the sampling horizon that basis has been decreasing over time. The volatilities associated with basis are .39, .13 and .11 for live-cattle, corn and soybeans respectively. The largest relative swings in basis are found for cattle, which explains why the cash price volatility in Table 1 is so much higher than the associated futures prices. In contrast the basis volatilities for corn and soybeans are lower than the corresponding volatilities in cash and futures. In terms of options pricing, the results indicate that basis risk will be far more significant in the pricing of cash market options on cattle than cash market options on corn and soybeans.

Calculating Options Values

This section reports on the options values calculations. We simplify the calculations by assuming first that storage costs, δ, and the market risk index, β are zero. The first assumption is a statement that the options are to be written on crops in the field; a traditional assumption for crop and revenue insurance, and the second assumption simply states that movements in commodity basis are independent of general movements in the market index[4]. This latter assumption also sets the market price of risk to zero so that the risk neutral growth rate for calculating the value of basis risk in the option is the natural rate as presented in the lower part of Table 1. Rather than using the definition of volatility explicitly, the respective measures of volatility can be taken directly from the sample data in Table 1. For example, the cash price volatility for cattle was .39 and for the quantos option it was .16. The risk neutral growth rate for basis was -.23,-.01 and -.35 for cattle, corn and soybeans respectively. The risk free rate was assumed to be 6% and the futures-exchange rate covariance terms used to capture the risk neutral growth rate for the quantos component of the option prices were .0001, -.0004, and -.0002 for cattle, corn and soybeans.

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[3] One might expect that the basis is mean reverting. Least squares estimates of the regression $dB_t = a + b*B_{t-1}$ found that for all three commodities the estimated coefficients on a and b were both negative so that a mean-reverting switch was not possible.

[4] Storage costs are generally regulated by the exchanges. For corn and soybeans traded on the CBOT the maximum storage rates per bushel are .15 cents/day or 4.5 cents per 30 day month. Since the storage costs as a percent depend on the price of the underlying contract it is not reasonable to use the same rate for all commodities. On April 3rd 2000 the Chicago prices of corn and soybeans were $U.S. 2.34 and 5.47 per bushel, so the storage rates (δ) at the annual maximum of $.54/bushel would be .23 and .098 respectively. The effect of these storage costs on the formula prices is not trivial, decreasing the value of a call option and increasing the value of a put.

The results of the model are found in Table 2. The strike prices represent 80%, 100%, and 120% of the cash prices as at April 3rd, 2000. The range of strike prices allows for the calculation of in and out-of-the-money calls and puts. The Table also provides a comparison of the option prices with and without basis risk.

The behaviour of the respective call and put prices are as expected from theory with call prices falling and put prices rising as the strike price increases. At-the-money, the price of the call option on the cash commodity for cattle was $12.27/cwt and for corn and soybeans it was $0.21/bu and $0.52/bu respectively. The interesting result is found with respect to the quantos option that excludes basis. Because the volatility of cattle was lower in the futures market the price of the put and call options at all strike prices are lower. But with a higher volatility in the futures market than the cash markets, the option prices increase for corn and soybeans.

In Table 2 the cash prices for each commodity are $96.38, $2.94, and $7.57 for cattle, corn and soybeans. At- the-money, the price of a put option to protect downside risk in the cash market would be $8.71/cwt, $0.11/bu. and $0.24/bu. Excluding basis, and using only the futures price and exchange rate, provides a different price regime. The corresponding Canadian dollar prices are $104.92/cwt, $3.41/bu. and $7.95/bu. Relative to the option prices calculated for the cash commodities, at-the-money quantos put options would decrease by 64% to $3.15/cwt for cattle and would increase by 64% to $0 .18/bu., and 63% to $0.39/bu. for corn and soybeans. The cattle price option decreased because so much of its volatility was in the basis, whereas the corn and soybean option prices increased because basis risk actually reduced cash market volatility. In either case, the clear conclusion is that basis risk does not have a trivial impact on the pricing of options, even if the proportion of cash price attributable to basis is low.

Relationship to Market Revenue Insurance

The options pricing presented in the previous section represent equilibrium prices that would occur in a Cox-Ross risk neutral world. The put formula can also be used to price market revenue insurance. In Ontario, market revenue insurance is administered by Agricorp, a crown corporation, and is offered to farmers on a per acre basis using adjusted 10 year moving average yields. The market revenue option is priced only to the cash commodity, although some marketing boards do offer price protection based on commodity futures price movements. The reference price used in ontario is the 15-year indexed moving average price. Farmers in Ontario can choose coverage levels equal to 75%, 80% and 90% of the reference price. The per-acre premium is equal to the option value of price protection times the number of bushels/acre as determined by the average yield history.

On April 3rd, 2000 the cash market prices for corn and soybeans were $2.94/bu. and $7.57/bu. respectively. The reference prices for market revenue insurance in Ontario in the Spring of 2000 were $2.70/bu. and $6.50/bu. based on the indexed 15 year moving average. At 80% coverage levels the base premium for an average risk farm based on the average yield history was $6.85/acre for corn and $4.60/acre for soybeans. While we are not privy to the exact approach used by Agricorp to determine these values, we can provide some comparison by multiplying our premiums by the actual provincial average yields in 2000, which were 105 bu./acre for corn and 38 bu./acre for soybeans. Based on these averages, the actuarial value of the put options as represented in Table 2 for 80% coverage are $0.38/acre and $0.19/acre for corn and soybeans. The discrepancy in pricing is large. The differences can be attributed to a number of factors. Most important is that the options based framework used in this paper uses the April cash prices as the initial condition and is based upon market possibilities 6 months hence. When support programs use historical price patterns, the support prices often have little bearing on current market prices, for example a $6.50 reference price for soybeans versus a $7.57 market price. Another significant difference is the assumed volatility of the underlying. By setting our initial cash price equal to Agricorp's $2.70/bu. for corn and using the provincial average yield of 105 bu./acre we calculate an implied volatility of .396 which is about twice the .19 value used in our calculation. That is, if we used a volatility of .396 rather than .19 and an initial price of $2.70/bu. our model would also have priced the market revenue insurance at $6.85/acre. To provide a further comparison, the historical volatilities of the corn and soybean futures contracts in April 2000, as calculated by the CBOT and provided on its web site, were .173 and .181 and the volatilities implied by at-the-money options on that date were .2019 and .2124. These are much closer to the values used in this paper than those implied by market revenue premiums, although from time to time historical and implied volatilities have exceeded 40% for both commodities.

Conclusions

The purpose of this paper was to examine the pricing of options on Canadian commodities when those commodities are priced relative to U.S. futures markets. Under these conditions the cash price is comprised of two components. The first component is a quantos priced on the U.S. denominated futures contracts and the U.S./CDN exchange rate. The second component is the basis effect, which captures local supply and demand influences. Using the risk neutral valuation techniques applied to the problem of pricing an option on non-traded assets we derived an options pricing model that included the basic pricing components. In fact it is the non-tradability of the cash commodity price or the Canadian dollar value of a U.S. traded commodity that is at the core of the model developed in the paper. Using historical data for cattle, corn, and soybeans we showed how the model could be applied in practice. We also discussed the relationship between the options calculations and various instruments offered by government and non governmental agencies. The live cattle options pilot program for example sold options on U.S. live cattle futures priced in Canadian dollars, and various other marketing boards and agencies have also developed U.S. market-linked price protection. In terms of direct government support we discussed the relationship between our option pricing model and Ontario’s market revenue insurance program. The generality of our model, and our use of equilibrium concepts, provides a means to price all of these contracts.

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Table 1: Sample Statistics on Commodity Prices, Exchange Rates and Basis

	Average Cash Price $CDN	Standard Deviation Cash Price $CDN	Average Futures Price $U.S.	Standard Deviation Futures Price $U.S.	Average Futures Price $CDN	Standard Deviation Futures Price $CDN	Average Basis $CDN	Standard Deviation Basis $CDN
	Levels Data							
Livestock (cwt)	89.54	5.93	66.11	3.48	94.00	5.97	-4.47	4.52
Corn (bu.)	3.64	1.05	2.70	0.64	3.82	0.79	-0.18	0.43
Soybeans (bu.)	8.60	1.47	6.28	1.15	8.88	1.39	-0.27	0.34
$CDN/$U.S.	1.42	0.06	-	-	-	-	-	-
	Annualized Percentage Change (250 day year)							
Livestock (cwt)	0.01	0.39	-0.01	0.15	-0.01	0.16	-.23	.39
Corn (bu.)	-0.02	0.19	-0.03	0.24	-0.02	0.24	-.01	.13
Soybeans (bu.)	-0.01	0.17	-0.03	0.22	-0.02	0.23	-.35	.11
$CDN/$U.S.	0.01	0.05	-	-	-	-	-	-

Table 2: Calculated Call and Put Option Premia

	With Basis			No Basis		
Coverage %	80%	100%	120%	80%	100%	120%
	Cattle ($/cwt)					
Strike	77.10	96.38	115.66	83.94	104.92	125.90
Call	24.40	12.27	5.39	23.51	6.26	0.47
Put	2.13	8.71	20.53	0.04	3.15	17.73
	Corn ($/bu.)					
Strike	2.35	2.94	3.53	2.73	3.41	4.09
Call	0.67	0.21	0.03	0.78	0.28	0.06
Put	0.0036	0.11	0.50	0.02	0.18	0.63
	Soybeans ($/bu.)					
Strike	6.05	7.57	9.08	6.36	7.95	9.54
Call	1.75	0.52	0.06	1.80	0.62	0.12
Put	0.005	0.24	1.25	0.03	0.39	1.43