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A Lagrange multiplier test for the mean stationarity assumption in dynamic panel-data models

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Abstract. In this article, we describe the `xttestms` command, which implements the Lagrange multiplier test proposed by Magazzini and Calzolari (2020, *Econometric Reviews* 39: 115–134). The test verifies the validity of the initial conditions in dynamic panel-data models, which is required for consistency of the system generalized method of moments estimator.

Keywords: st0714, xttestms, panel data, dynamic model, generalized method of moments estimation, initial conditions, test of overidentifying restrictions, Lagrange multiplier test

1 Introduction

The availability of panel data allows advantages with respect to cross-section or time-series data (see, for example, Baltagi [2021]). In particular, panel data allow researchers a better assessment of economic dynamics at the unit (individual, firm, etc.) level.

Dynamic panel-data models are widely used in empirical studies, and various methods have been proposed in the literature for estimation of the parameters of interest (Hsiao, Pesaran, and Tahmiscioglu 2002; Gouriéroux, Phillips, and Yu 2010; Kiviet 1995), with the generalized method of moments (GMM) being the most widely applied method in empirical research (Hansen 1982). Two different approaches are customarily adopted in empirical analysis: the difference GMM estimator and the system GMM estimator (Arellano and Bond 1991; Arellano and Bover 1995; Blundell and Bond 1998). A third GMM estimator is also available, by Ahn and Schmidt (1995), based on a nonlinear set of moment conditions, which has so far seen limited application in the empirical literature (only recently available in Stata; see Kripfganz [2017]).

The three approaches differ in terms of the moment conditions exploited in the estimation process. The difference GMM estimator and the nonlinear estimator by Ahn and Schmidt (1995) exploit the same set of model assumptions, but the latter efficiently includes all moment conditions spanning from these assumptions. On the contrary, the system GMM relies on an additional assumption underlying the initial observation, therefore allowing a wider set of moment conditions. When the condition is satisfied, the system GMM estimator outperforms the other GMM estimation methods in terms of asymptotic efficiency and small-sample bias (Blundell and Bond 1998).

In this article, we describe the command `xttestms` for computation of the Lagrange multiplier (LM) test for verifying the assumption underlying the system GMM estimation. The test has been proposed by Magazzini and Calzolari (2020), who show its better performance with respect to customarily used testing procedures.

The article proceeds as follows. In the next section, we set up the model assumptions and notation and briefly describe available GMM estimation procedures. In section 3, we introduce the LM test, and in section 4 we describe the syntax of the `xttestms` command. In section 5, we present the results of a Monte Carlo simulation. We provide two examples in section 6 and conclude in section 7.

2 Dynamic panel-data models

The baseline dynamic panel-data model can be written as ($i = 1, \ldots, N$, $t = 1, \ldots, T$)

$$y_{it} = \rho y_{it-1} + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it} \tag{1}$$

y_{it} is an observation on the dependent variable for unit i at time t, $\mathbf{x}_{it}$ is a vector of independent variables, and $\boldsymbol{\theta} = (\rho, \boldsymbol{\beta}')'$ is a vector of unknown parameters. The constant term and lags of the independent variables can be included in $\mathbf{x}_{it}$; time dummies are customarily included in $\mathbf{x}_{it}$ to account for the effect of economy-wide variables. Further lags of the dependent variables can also be included in the model. For notational convenience, we assume that y_{i0} and $\mathbf{x}_{i0}$ are observed. The error term ε_{it} can be decomposed into two sources of variation:

$$\varepsilon_{it} = u_i + e_{it}$$

u_i is an individual component, and e_{it} an idiosyncratic noise assumed to be uncorrelated with u_i and uncorrelated over time: $E(u_i e_{it}) = 0$ and $E(e_{it}e_{is}) = 0$ for every t and $s \neq t$. Heteroskedasticity is allowed in e_{it}. Independent variables are allowed to be freely correlated with the individual component u_i, whereas different assumptions can be made about the correlation between $\mathbf{x}_{it}$ and e_{is} at different time periods ($s = 1, \ldots, T$). Let us denote $x_{k,it}$ the kth component of $\mathbf{x}_{it}$; we can distinguish three cases:

- $x_{k,it}$ is strictly exogenous if $E(e_{it}|x_{k,i0}, \ldots, x_{k,iT}, u_i) = 0$.
- $x_{k,it}$ is a predetermined variable if $E(e_{it}|x_{k,i0}, \ldots, x_{k,it}, u_i) = 0$.
- $x_{k,it}$ is endogenously determined if $E(e_{it}|x_{k,i0}, \ldots, x_{k,it-1}, u_i) = 0$.

To apply GMM estimation to (1), one usually applies the first-difference transformation to remove the individual component u_i:

$$\Delta y_{it} = \rho \Delta y_{it-1} + \Delta \mathbf{x}'_{it}\boldsymbol{\beta} + \Delta e_{it}$$

The model can be recursively written as

$$\Delta y_{it} = \rho^{t-1}\Delta y_{i1} + \sum_{s=0}^{t-2} \rho^s \left(\Delta \mathbf{x}'_{it-s}\boldsymbol{\beta} + \Delta e_{it-s}\right) \tag{2}$$

Under the assumption of uncorrelated e_{it}, lags two or more of y_{it} can be used as instruments for Δy_{it-1}. Therefore, the following moment conditions can be considered, leading to the difference GMM estimator (Arellano and Bond 1991):

$$E(y_{it-j}\Delta e_{it}) = 0 \;\; (t = 2, \ldots, T;\, j \geq 2) \tag{3}$$

As for $\mathbf{x}_{it}$, different assumptions about the correlation between $\mathbf{x}_{it}$ and e_{is} lead to different sets of moment conditions that are exploited in estimation (Blundell, Bond, and Windmeijer 2001):

- strictly exogenous $x_{k,it}$:
$$E(\Delta x_{k,it}\Delta e_{it}) = 0 \tag{4}$$
- predetermined $x_{k,it}$:
$$E(x_{k,it-j}\Delta e_{it}) = 0 \quad \text{for} \quad j \geq 1 \tag{5}$$
- endogenously determined $x_{k,it}$:
$$E(x_{k,it-j}\Delta e_{it}) = 0 \quad \text{for} \quad j \geq 2 \tag{6}$$

The nonlinear estimator adds the following moment conditions (Ahn and Schmidt 1995):

$$E(\Delta\varepsilon_{it}\varepsilon_{iT}) = 0 \quad \text{for every } t < T \tag{7}$$

The system GMM estimator includes additional moment conditions based on the level equation (Blundell and Bond 1998), particularly

$$E(\Delta y_{it-1}\varepsilon_{it}) = 0 \tag{8}$$

Blundell and Bond (1998) argue that these additional moment conditions encompass the nonlinear moment conditions (7). Magazzini and Calzolari (2020) show that, in the simple dynamic case with no additional regressors, the additional moment conditions (8) would correspond to the moment conditions in (7) plus the moment condition on the initial observation $E(\Delta y_{i1}\varepsilon_{i2}) = 0$.

Additional moment conditions based on the level equation are also available for predetermined or endogenously determined $x_{k,it}$, respectively:

$$E(\Delta x_{k,it}\varepsilon_{it}) = 0 \tag{9}$$

for predetermined $x_{k,it}$ or

$$E(\Delta x_{k,it-1}\varepsilon_{it}) = 0 \tag{10}$$

for endogenously determined $x_{k,it}$.

3 Testing mean stationarity

When the model parameters are estimated using the maximum likelihood method, the (asymptotically equivalent) "trio" of Wald, likelihood ratio, and LM testing is well established among practitioners. Such a "trio" strategy has also been developed in a GMM context to test the value of model parameters or the validity of moment conditions (Newey and West 1987; Newey and McFadden 1994; Ruud 2000).

The focus in this section is on testing the validity of the initial moment condition, which is required for consistency of the system GMM estimator. This is customarily assessed by relying on the Hansen and difference-in-Hansen tests, which are based on the difference between the minimized GMM criterion functions of a restricted and an unrestricted model (Blundell and Bond 1998; Blundell, Bond, and Windmeijer 2001). In this context, Magazzini and Calzolari (2020) propose an alternative approach based on an LM strategy, which is shown to have better power with respect to the strategy customarily used in empirical analysis. This LM strategy treats the system GMM estimator as the restricted estimator in an "augmented" set of moment conditions in which additional parameters are included to account for departure from the mean stationarity assumption.

Consider the simple autoregressive model, in which $\boldsymbol{\beta} = \mathbf{0}$, in (1). Throughout this section, it is assumed that the conditions for the application of the difference and nonlinear GMM estimators are satisfied so that moment conditions in (3) and (7) are satisfied. Under the null hypothesis that the initial moment condition assumption holds, the system GMM estimator is obtained by jointly exploiting the moment conditions in (3) and (8). However, if the moment conditions in (8) are not satisfied, one can introduce additional parameters and write [$t = 2, \ldots, T$; see (10) in Magazzini and Calzolari (2020)]

$$E(\Delta y_{it-1}\varepsilon_{it}) + \psi_{t-1} = 0$$

GMM estimation can be applied for the estimation of the parameters ρ, ψ_1, …, ψ_{T-1} to produce an unrestricted estimator. The system GMM estimator can be obtained under the null hypothesis

$$H_0 : \psi_1 = \psi_2 = \cdots = \psi_{T-1} = 0$$

Any strategy in the "trio" may be applied for this purpose: a Wald or an LM strategy would be equivalent to the difference-in-Hansen test comparing system and difference GMM that is usually adopted (Magazzini and Calzolari 2020).

However, as noted in Magazzini and Calzolari (2020), according to (2), one can write

$$\psi_{t-1} = \rho^{t-2}\psi_1$$

so that the validity of the system GMM estimator can be assessed only by testing whether the additional parameter ψ_1 is equal to zero in the following "augmented" (in the sense of including additional parameters) set of moment conditions [see (13) in Magazzini and Calzolari (2020)], jointly used in estimation with (3):

$$E(\Delta y_{it-1}\varepsilon_{it}) + \rho^{t-2}\psi_1 = 0 \tag{11}$$

The proposed LM test statistic can be computed as [see (15) in Magazzini and Calzolari (2020)]

$$\text{LM} = N\mathbf{g}_N\left(\widehat{\boldsymbol{\theta}}_{RN}\right)'\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\mathbf{G}}_N\left(\widehat{\mathbf{G}}'_N\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\mathbf{G}}_N\right)^{-1}\widehat{\mathbf{G}}'_N\widehat{\boldsymbol{\Omega}}^{-1}\mathbf{g}_N\left(\widehat{\boldsymbol{\theta}}_{RN}\right)$$

$\mathbf{g}_N(\boldsymbol{\theta})$ is the vector of moment conditions (3) and (11), and $\mathbf{G}_N(\boldsymbol{\theta})$ is its first derivative. Both are evaluated at the restricted estimate, composed of the system GMM estimate of ρ and $\psi_1 = 0$. The matrix $\widehat{\boldsymbol{\Omega}}$ is the efficient weighting matrix used in estimation. The test asymptotically has a χ^2 distribution with a number of degrees of freedom equal to the number of tested parameters—in this case 1 (Ruud 2000; Newey and West 1987).

Compared with the Hansen and difference-in-Hansen strategies customarily adopted in this context, the number of degrees of freedom of the test (the number of parameters to be tested) is reduced, leading to an increased power of the test (Magazzini and Calzolari 2020). Furthermore, when the model includes the lagged dependent variable as a unique regressor, Magazzini and Calzolari (2020) show that the proposed test procedure is asymptotically equivalent to the test based on the difference between the minimized GMM criterion function of the system GMM estimator and the minimized GMM criterion function of the nonlinear GMM estimator by Ahn and Schmidt (1995). For its higher computational simplicity, the LM strategy can be preferred to a Wald-type test statistic or difference-in-Hansen computation; in fact, it can be based on the system GMM estimator (restricted estimator) rather than on the joint estimation of ρ and ψ_1 that would entail minimizing a nonlinear function of the parameters (to produce the unrestricted estimator).

The procedure based on the LM test is easily extended to the case of dynamic models with additional regressors (that is, $\boldsymbol{\beta} \neq \mathbf{0}$). For simplicity, let us consider the case of one additional regressor,

$$y_{it} = \rho y_{it-1} + \beta x_{it} + \varepsilon_{it}$$

with $\varepsilon_{it} = u_i + e_{it}$.

Table 1 summarizes the moment conditions that can be exploited in three cases: 1) x_{it} satisfies strict exogeneity; 2) x_{it} is a predetermined regressor; and 3) x_{it} is treated as an endogenously determined variable. In all cases, the conditions for the application of the difference (and nonlinear) GMM estimator are valid so that moment conditions based on the first-difference equations remain unchanged in the restricted and unrestricted models. On the contrary, absent mean stationarity, departures from the initial condition assumption are modeled by including additional parameters in the set of moment conditions used for system GMM estimation.

When x_{it} satisfies strict exogeneity, system GMM estimation is based on the moment condition in (4) that is added to the moment conditions (3) and (8) to jointly estimate ρ and β. In this case, no additional moment condition is obtained from the level equations. Violation of mean stationarity can therefore be modeled by specifying (11) in place of (8). Thus, computation of the test statistic would proceed as previously described, and the system GMM estimator should not be considered if the null hypothesis H_0: $\psi_1 = 0$ is rejected. Still, the asymptotic distribution of the test statistic is χ^2 with 1 degree of freedom.

In the presence of predetermined or endogenously determined regressors, additional parameters should also be considered to "correct" the moment conditions (9) or (10). We also need to adjust moment condition (11) to account for the inclusion of additional regressors in the equation. For example, for a predetermined regressor, x_{it}, T additional parameters should be considered ($t = 1, \ldots, T$):

$$E(\Delta x_{it} \varepsilon_{it}) + \xi_t = 0 \tag{12}$$

According to the recursive formula (2), the moment conditions related to y_{it-1} should also include the additional parameters ξ_t so that moment conditions in (11) are adjusted according to the following formula:

$$E(\Delta y_{it-1} \varepsilon_{it}) + \rho^{t-1}\psi_1 + \sum_{s=0}^{t-2} \rho^s \xi_t \beta = 0 \tag{13}$$

The system GMM estimator would be obtained as a restricted estimator, and its validity can be assessed by testing the null hypothesis:

$$H_0 : \psi_1 = \xi_1 = \xi_2 = \cdots = \xi_T = 0$$

For an endogenously determined regressor, departure from the mean stationarity assumption can be accommodated by specifying ($t \geq 2$),

$$E(\Delta x_{it-1} \varepsilon_{it}) + \xi_t = 0 \tag{14}$$

Table 1 summarizes the unrestricted and restricted (system) moment conditions, as well as the null hypothesis used to assess the validity of the mean stationarity assumption, in the simpler case of one (strictly exogenous, predetermined, or endogenously determined) regressor (inclusion of additional regressors is straightforward). The moment conditions are developed under the assumption of lack of autocorrelation in the idiosyncratic component e_{it}.

Table 1. Summary of moment conditions exploited in GMM estimation for a general time period t; estimated equation $y_{it} = \rho y_{it-1} + \beta_1 x_{it} + \varepsilon_{it}$ with $\varepsilon_{it} = u_i + e_{it}$ and treating x_{it} as a strictly exogenous, predetermined, and endogenously determined variable

Equation	Restricted set of moment conditions (system GMM)		Unrestricted set of moment conditions		Null hp. H_0
	Case 1: strictly exogenous x				
diff.	$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$\psi_1 = 0$
	$\vdots$	(3)	$\vdots$	(3)	
	$E(y_{i0}\Delta\varepsilon_{it}) = 0$		$E(y_{i0}\Delta\varepsilon_{it}) = 0$		
	$E(\Delta x_{it}\Delta\varepsilon_{it}) = 0$	(4)	$E(\Delta x_{it}\Delta\varepsilon_{it}) = 0$	(4)	
level	$E(\Delta y_{it-1}\varepsilon_{it}) = 0$	(8)	$E(\Delta y_{it-1}\varepsilon_{it}) + \rho^{t-2}\psi_1 = 0$	(11)	
	Case 2: predetermined x				
diff.	$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$\psi_1 = 0$
	$\vdots$	(3)	$\vdots$	(3)	$\xi_1 = 0$
					$\vdots$
	$E(y_{i0}\Delta\varepsilon_{it}) = 0$		$E(y_{i0}\Delta\varepsilon_{it}) = 0$		
	$E(x_{it-1}\Delta\varepsilon_{it}) = 0$		$E(x_{it-1}\Delta\varepsilon_{it}) = 0$		$\xi_T = 0$
	$\vdots$	(5)	$\vdots$	(5)	
	$E(x_{i0}\Delta\varepsilon_{it}) = 0$		$E(x_{i0}\Delta\varepsilon_{it}) = 0$		
level	$E(\Delta y_{it-1}\varepsilon_{it}) = 0$	(8)	$E(\Delta y_{it-1}\varepsilon_{it}) + \rho^{t-1}\psi_1 + \sum_{s=0}^{t-2}\rho^s\xi_t\beta = 0$	(13)	
	$E(\Delta x_{it}\varepsilon_{it}) = 0$	(9)	$E(\Delta x_{it}\varepsilon_{it}) + \xi_t = 0$	(12)	
	Case 3: endogenously determined x				
diff.	$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$E(y_{it-2}\Delta\varepsilon_{it}) = 0$		$\psi_1 = 0$
	$\vdots$	(3)	$\vdots$	(3)	$\xi_2 = 0$
					$\vdots$
	$E(y_{i0}\Delta\varepsilon_{it}) = 0$		$E(y_{i0}\Delta\varepsilon_{it}) = 0$		
	$E(x_{it-2}\Delta\varepsilon_{it}) = 0$		$E(x_{it-2}\Delta\varepsilon_{it}) = 0$		$\xi_T = 0$
	$\vdots$	(6)	$\vdots$	(6)	
	$E(x_{i0}\Delta\varepsilon_{it}) = 0$		$E(x_{i0}\Delta\varepsilon_{it}) = 0$		
level	$E(\Delta y_{it-1}\varepsilon_{it}) = 0$	(8)	$E(\Delta y_{it-1}\varepsilon_{it}) + \rho^{t-1}\psi_1 + \sum_{s=0}^{t-2}\rho^s\xi_t\beta = 0$	(13)	
	$E(\Delta x_{it-1}\varepsilon_{it}) = 0$	(10)	$E(\Delta x_{it-1}\varepsilon_{it}) + \xi_t = 0$	(14)	

The GMM method can accommodate limited violation of the lack of autocorrelation assumption by adjusting the number of lags of the dependent variables and the regressors that can be considered as legitimate instruments. This possibility, as well as the inclusion of further lags of the dependent variables or the regressors, can also be accounted for. Such an instance is considered in the example in section 6.2.

4 The xttestms command

The syntax of the `xttestms` command is the following:

`xttestms` [, showgmm]

The test can be run after a system GMM estimation, using either the command `xtdpdsys` or the command `xtabond2` by Roodman (2009). The `xttestms` command verifies that the additional moment conditions that characterize the system GMM estimator are satisfied using an LM test (Magazzini and Calzolari 2020); that is, it verifies the validity of the mean stationarity assumption for the initial conditions.

Computation of the LM test statistics requires the availability of the matrix of instruments, the matrix of regressors, and the weighting matrix used during estimation. The three matrices are produced when using the `xtabond2` command in estimation with the `svmat` option. This option requires that Mata run in speed-favoring mode. If the option is not specified by the user or the system GMM estimator is obtained using the `xtdpdsys` command, the model is refit to produce the relevant matrices—this step is accomplished using the command `xtabond2` with the `svmat` option. Therefore, one must have `xtabond2` installed before running `xttestms`; this requirement must be verified before running `xttestms`.

The option `showgmm` shows the results obtained when refitting the model.

5 Monte Carlo simulation

We report the results of a Monte Carlo simulation that extends and integrates the results reported in Magazzini and Calzolari (2020).

The data-generating process (DGP) is set as in (1), with the error terms $u_i \sim N(0, \sigma_u^2)$. Heteroskedasticity is introduced in the design by letting $e_{it} = \delta_i \tau_t \nu_{it}$ with $\delta_i \sim U(0.5, 1.5)$, $\tau_t = 0.5 + 0.1\,t$, and $\nu_{it} \sim \chi_1^2 - 1$ (Windmeijer 2005). The regressor x_{it} is generated as

$$x_{it} = \rho_x\, x_{it-1} + \theta_u u_i + \theta_e \eta_{it} + w_{it}$$

with $\rho = \rho_x = 0.5$ and $\beta = 1$.

The parameter θ_u drives the correlation between x_{it} and u_i, whereas the term $\theta_e \eta_{it}$ is defined to drive the correlation between x_{it} and e_{is}. When $\eta_{it} = e_{it}$, the variable x_{it} is endogenously determined; if $\eta_{it} = e_{it-1}$, it is predetermined; and when η_{it} is generated independently from e_{is} $(s = 1, \ldots, T)$, the regressor is strictly exogenous [we consider $\eta_{it} \sim N(0,1)$]. We set $\theta_u = 0.25$ and $\theta_e = -0.1$; $w_{it} \sim N(0, 0.16)$ (see Blundell, Bond, and Windmeijer [2001]).

When we generate the initial observation, the covariance stationary formulation is adopted, but at the initial period, we multiply the individual component in y_{i1} and x_{i1} by, respectively, γ_y and γ_x. Thus, the mean stationary case corresponds to $\gamma_y = \gamma_x = 1$. Otherwise, the assumption is not satisfied.

The sample size is set to $N = 500$ to resemble the number of units in the examples in section 6, and $T = 4, 8$.

Table 2 reports the parameters used in the experiments and the corresponding mean (and standard deviations) of the two-step system GMM estimates of ρ and β obtained in the Monte Carlo replications under different DGPs for the x, that is, strictly exogenous, predetermined, and endogenously determined. The results of estimation reported in table 2 were obtained after `xtdpdsys` (more precisely, to save on computation time, we obtained the estimates using the `xtabond2` command with the `h(2)` option, which produces the same results as `xtdpdsys`). Similar results can be obtained when using the `xtabond2` default weighting matrix in the first step (corresponding to the `h(3)` option; see Windmeijer [2000]; Youssef, El-Sheikh, and Abonazel [2014]; and Kiviet [2007] for a discussion related to the choice of the initial weighting matrix in system GMM).

Table 2. Experiment details and Monte Carlo mean and standard deviation of the system GMM estimator for different DGP for the regressor

				x strictly exo.		x predet.		x endog. det.	
exp.	T	γ_y	γ_x	$\widehat{\rho}$	$\widehat{\beta}$	$\widehat{\rho}$	$\widehat{\beta}$	$\widehat{\rho}$	$\widehat{\beta}$
1	4	1	1	0.501	0.998	0.520	1.060	0.509	1.017
				(0.062)	(0.113)	(0.060)	(0.167)	(0.065)	(0.303)
2	8	1	1	0.501	0.998	0.510	1.034	0.506	0.960
				(0.027)	(0.082)	(0.028)	(0.087)	(0.029)	(0.157)
3	4	1	0.7	0.565	1.019	0.573	1.419	0.559	1.537
				(0.055)	(0.115)	(0.053)	(0.114)	(0.061)	(0.262)
4	4	0.7	1	0.666	1.048	0.657	1.338	0.668	1.225
				(0.040)	(0.117)	(0.031)	(0.110)	(0.043)	(0.276)
5	4	0.7	0.7	0.696	1.053	0.655	1.507	0.661	1.503
				(0.039)	(0.118)	(0.032)	(0.091)	(0.046)	(0.245)
6	8	0.7	0.7	0.613	1.096	0.612	1.349	0.614	1.401
				(0.029)	(0.088)	(0.024)	(0.072)	(0.028)	(0.144)
7	4	0.8	0.8	0.652	1.043	0.629	1.411	0.634	1.398
				(0.043)	(0.117)	(0.036)	(0.101)	(0.047)	(0.254)
8	8	0.8	0.8	0.577	1.058	0.582	1.258	0.585	1.277
				(0.030)	(0.086)	(0.027)	(0.080)	(0.030)	(0.153)
9	4	1.2	1.2	0.570	1.058	0.663	1.371	0.636	1.641
				(0.108)	(0.129)	(0.068)	(0.203)	(0.087)	(0.351)
10	8	1.2	1.2	0.505	1.069	0.539	1.115	0.524	1.227
				(0.031)	(0.089)	(0.032)	(0.097)	(0.034)	(0.188)
11	4	1	1.3	0.454	0.993	0.606	1.074	0.592	1.115
				(0.068)	(0.112)	(0.063)	(0.220)	(0.072)	(0.399)
12	4	1.3	1	0.620	1.084	0.675	1.457	0.644	1.732
				(0.108)	(0.133)	(0.072)	(0.146)	(0.099)	(0.302)
13	4	1.3	1.3	0.706	1.136	0.708	1.505	0.680	1.763
				(0.093)	(0.131)	(0.032)	(0.152)	(0.056)	(0.300)
14	8	1.3	1.3	0.531	1.132	0.575	1.219	0.552	1.437
				(0.034)	(0.093)	(0.032)	(0.098)	(0.034)	(0.186)

For each experiment, we run 10,000 replications using the same seed. The number of Monte Carlo replications has been chosen so that, in the worst-case scenario (corresponding to a rejection rate of 50%), the power of the test can be assessed with a 95% confidence bound of about 1% (Morris, White, and Crowther [2019]).[1] When we assess size at the 5% level of statistical significance, the confidence interval is 5% ± 0.43%. Under a different perspective, Davidson and MacKinnon (1998) suggest considering the Kolmogorov–Smirnov critical value when comparing empirical p-values with theoretical ones. With 10,000 Monte Carlo replications, discrepancies in size of ±1.36% would be ascribed to randomness.

As expected, the system GMM estimator is consistent if $\gamma_x = \gamma_y = 1$, that is, when the mean stationarity assumption is satisfied. On the contrary, bias emerges when the initial condition is not satisfied (either γ_x or γ_y is not equal to 1 or both are not equal to 1).

In each experiment, we also computed the share of rejections for the test of overidentifying restrictions (Hansen; H), the difference-in-Hansen test that compares the system GMM and the difference GMM estimators (diffH), and the LM test produced by `xttestms` (LM).

1. We thank the managing editor for suggesting this reference.

The size of the proposed test procedure is reported in table 3, showing the share of rejection at the 1–5–10% level of significance (LOS) in the experiments with $\gamma_x = \gamma_y = 1$.

Table 3. Sizes of the test procedures (experiments 1, 2): share of 1–5–10% LOS rejections; test of overidentifying restrictions (Hansen; H), the difference-in-Hansen test that compares the system GMM and the difference GMM estimators (diffH), and the LM test produced by `xttestms` (LM)

T	DGP for x	LOS	H	diffH	LM
4	strict exo.	1%	0.0084	0.0096	0.0107
		5%	0.0475	0.0499	0.0506
		10%	0.9998	0.1019	0.0986
4	predetermined	1%	0.0088	0.0129	0.0145
		5%	0.0477	0.0613	0.0602
		10%	0.0974	0.1213	0.1161
4	endog. det.	1%	0.0078	0.0120	0.0114
		5%	0.0466	0.0561	0.0518
		10%	0.0931	0.1092	0.1008
8	strict exo.	1%	0.0054	0.0107	0.0121
		5%	0.0424	0.0600	0.0591
		10%	0.0945	0.1183	0.1125
8	predetermined	1%	0.0038	0.0131	0.0114
		5%	0.0351	0.0623	0.0524
		10%	0.0864	0.1208	0.1049
8	endog. det.	1%	0.0052	0.0147	0.0127
		5%	0.0378	0.0669	0.0604
		10%	0.0900	0.1267	0.1112

In terms of size, the proposed test procedure generally performs well as diffH (even though small size distortion emerges with $T = 8$). The Hansen test, H, is undersized in experiments with $T = 8$ and endogenous regressors (Hayakawa 2016). When we use `xtabond2` to produce the results (with the default `h(3)` option), similar results are obtained even if diffH and LM exhibit larger overrejection in the experiments with $T = 8$ and endogenous (predetermined or simultaneous) regressors.[2]

Figures 1–3 report the share of rejections at the 5% level in the experiments with γ_y, γ_x, or both different from 1, that is, these experiments in which the mean stationarity assumption is not satisfied (see table 2 for details on the value of γ_y, γ_x, and T across the experiments).

2. Full simulation results are available from the authors upon request.

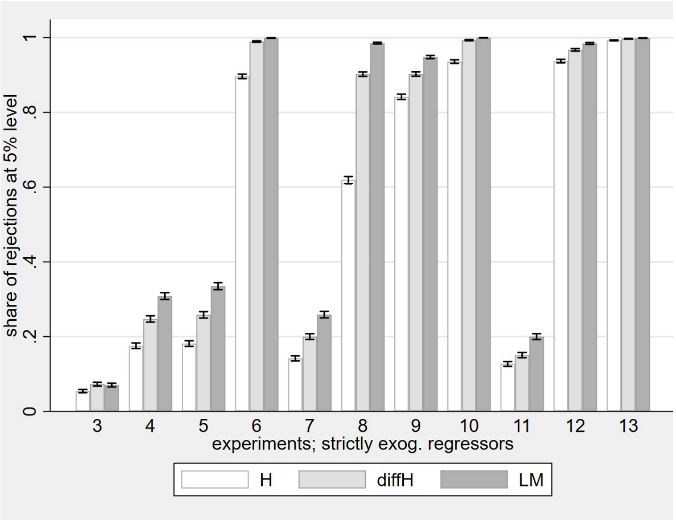

Figure 1. Power of the testing procedures across the Monte Carlo experiments, strictly exogenous regressors (refer to table 2 for details on the value of the parameters)

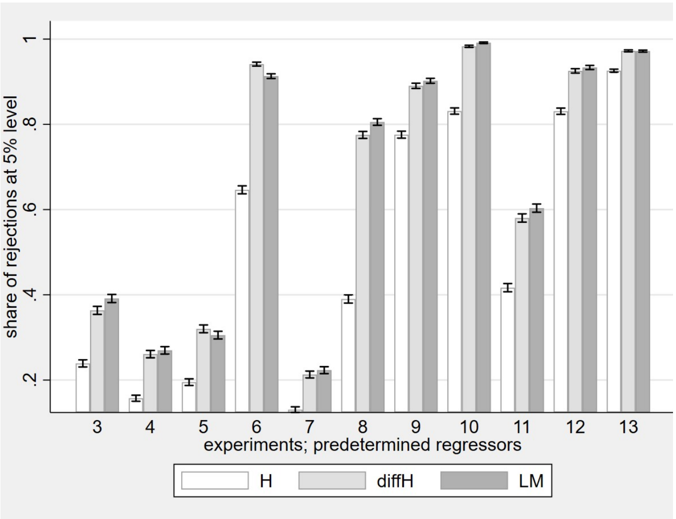

Figure 2. Power of the testing procedures across the Monte Carlo experiments, predetermined regressors (refer to table 2 for details on the value of the parameters)

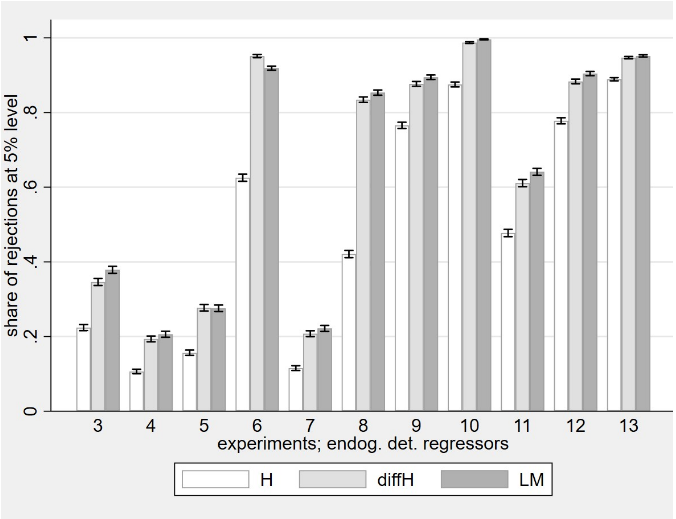

Figure 3. Power of the testing procedures across the Monte Carlo experiments, endogenously determined regressors (refer to table 2 for details on the value of the parameters)

In terms of power, the LM test outperforms diffH and H in the case of strictly exogenous regressors, while no clear-cut advantage emerges with predetermined or endogenously determined regressors. Overall, LM behaves similarly to diffH in the case of correlation between the idiosyncratic component and the regressor, either predetermined or simultaneous. In all cases, diffH and LM outperform H.

6 Examples

We propose two examples to show how to implement the `xttestms` command on real data.

In all cases, before applying the proposed testing procedure, the researcher should check whether the assumptions for the application of the difference GMM estimator are satisfied. The LM test procedure by Magazzini and Calzolari (2020) *uniquely* checks the validity of the mean stationarity (initial condition) assumption needed for consistency of the system GMM estimator.

6.1 A labor equation

We first consider the data used in Cameron and Trivedi (2005, chap. 21–22), taken from Ziliak (1997). The data include information on a sample of 532 individuals over the years 1979–1988. Please refer to the cited references for additional details. The variable of interest is `lnhr`, the logarithm of annual hours worked. We first consider a simple dynamic model with no regressors so that only the first lag of `lnhr` is included in the specification, besides a constant term and time dummies. Then the variable `lnwg`, the natural logarithm of hourly wage, is included among the regressors, treated as a strictly exogenous variable.

6.1.1 Dynamic model with no regressors

Consider the model

$$\texttt{lnhr}_{it} = \mu + \rho\,\texttt{lnhr}_{it-1} + \tau_t + u_i + e_{it}$$

The system GMM estimator can be obtained by using

```
. infix 2 firstlineoffile 1: lnhr 1-16 lnwg 17-33 kids 34-50 age 51-67
>       2: agesq 1-16 disab 17-33 id 34-50 year 51-67
>       using "https://cameron.econ.ucdavis.edu/mmabook/MOMprecise.dat"
(5,320 observations read)
. tabulate year, generate(dyear)
  (output omitted)
. sort id year
. xtset id year
  (output omitted)
. xtdpdsys lnhr dyear3-dyear10, twostep vce(robust)
  (output omitted)
```

The Arellano–Bond tests for autocorrelation in first-difference residuals (Arellano and Bond 1991) support the lack of serial correlation in the idiosyncratic disturbance so that identifying conditions for the application of the difference and the nonlinear GMM estimator are satisfied.

The LM test for verifying the condition on initial observations required for consistency of the system GMM estimator can be computed by typing `xttestms`:

```
. xttestms
Number of lags detected in the equation: 1
  lag(s) of lnhr included among the regressors: 1
  LM test of mean stationarity
   Test statistic =  6.82063 with p-value  .009011
   The test statistic has a chi2(1) distribution
```

In the do-file that replicates this example, the value of the test statistic is also obtained using matrix computation. The LM test is χ^2 distributed with 1 degree of freedom [equal to the number of additional parameters; in this case ψ_1 in (11)]. The LM test rejects the mean stationarity condition, so the system GMM method should not be applied for estimation, and difference or nonlinear GMM should be used instead.

6.1.2 The case of strictly exogenous regressors

We now include the variable `lnwg` in the equation and treat it as a strictly exogenous variable (again, we also test the lack of autocorrelation in the idiosyncratic component using the Arellano–Bond test):

```
. xtdpdsys lnhr lnwg dyear3-dyear10, twostep vce(robust)
  (output omitted)
. xttestms
Number of lags detected in the equation: 1
  lag(s) of lnhr included among the regressors: 1
  LM test of mean stationarity
   Test statistic =  7.02113 with p-value  .008055
   The test statistic has a chi2(1) distribution
```

In this case, there are no additional moment conditions to be tested related to the strictly exogenous variable. The computation of the test can proceed analogously to the previous case with no regressors (the matrices `e(X)` and `e(Z)` used in computation now include, respectively, the additional variable and the additional moment conditions related to `lnwg`).

The initial condition assumption is not satisfied in this case, and the result is also in line with the results of the Hansen and difference-in-Hansen test (with p-values, respectively, of 0.007 and 0.0099). Moreover, we are allowed to treat `lnwg` as a strictly exogenous variable because the p-value of the difference-in-Hansen test for the variables in `ivstyle()` equals 0.166.

6.2 A production function

We now consider `usbal89.dta`, used by Blundell and Bond (2000) and Bond (2002). The balanced panel dataset includes 509 U.S. firms observed over 8 years, 1982–1989.

We consider the preferred specification in Blundell and Bond (2000) and Bond (2002). However, we use the `twostep` option when computing the system GMM estimator. Differently from the previous example, only lags 3 or older can be used as legitimate instruments, and lagged values of the regressors are included in the equation of interest.[3] The proposed test procedure can still be applied by adjusting the way in which we "augment" the moment conditions related to the equation in levels.

We compute the system GMM estimator and then compute the LM test for the validity of the mean stationarity assumption:

3. See "Using Stata to Replicate Table 4 in Bond (2002)", available at https://www.nuffield.ox.ac.uk/people/sites/bond-teaching/.

```
. use http://fmwww.bc.edu/ec-p/data/bond/usbal89, clear
. xi: xtabond2 y l.y n l.n k l.k i.year, gmmstyle(y n k, laglimits(3 .))
> ivstyle(i.year, equation(level)) twostep robust
  (output omitted)
. xttestms
Number of lags detected in the equation: 1
  lag(s) of y included among the regressors: 1
  lag(s) of n included among the regressors: 0 1
  lag(s) of k included among the regressors: 0 1
  LM test of mean stationarity
   Test statistic =  33.3191 with p-value  .000467
   The test statistic has a chi2(11) distribution
```

As in Bond (2002), time dummies are included as instruments in the level equation only. When one uses `xtabond2` to obtain the estimates, it is important to specify lags of the variables using the lag operator (that is, `l.y`, `l.n`, and `l.k`) so that `xttestms` can properly recognize the lags of the variables included in the equation. To check whether all variables are properly treated, one can look at the output of `xttestms`, which includes the number of lags that are specified for each variable and whether these are treated as predetermined or simultaneously determined variables.

According to the LM test, the mean stationarity assumption is not satisfied. This result is in line with the Hansen test and difference-in-Hansen test reported at the bottom of `xtabond2`'s results, being, respectively, 79.45 (χ^2 with 55 degrees of freedom, p-value 0.017) and 41.12 (χ^2 with 15 degrees of freedom, p-value < 0.001).

Note that, if we had used the command `xtdpdsys` to obtain estimation, that is, used a different initial weighting matrix for estimation, we would have obtained a weaker result, with the value of the test being equal to 18.56 (p-value $= 0.069$).

7 Conclusions

In this article, we presented a new command, `xttestms`, to be run after dynamic panel-data estimation. The command computes the LM test for mean stationarity proposed by Magazzini and Calzolari (2020). The testing procedure focuses on assessing the validity of the “level” moment conditions exploited in the context of the system GMM estimator (Blundell and Bond 1998).

Based on Monte Carlo simulations in this article and in Magazzini and Calzolari (2020), the new testing strategy produces advantages with respect to customarily used procedures based on the Hansen and difference-in-Hansen tests. The increase in the power of the test is substantial for strictly exogenous variables, whereas advantages seem to be more limited for predetermined or endogenously determined regressors.

The increase in power is determined by the reduction in the number of degrees of freedom of the proposed testing procedure, which is more limited when endogeneity of the regressors is introduced in the model. Further research might consider the application of the recursive formula to the “level” moment conditions built on the regressors to further improve the performance of the test. Indeed, Blundell, Bond, and Windmeijer (2001) state that mean stationarity of the regressors is critically dependent on the initial observation.

In the context of endogenous regressors, we would suggest using the proposed test procedure jointly with the ones customarily used in the literature (that is, the procedures based on the Hansen test).

8 Programs and supplemental materials

To install a snapshot of the corresponding software files as they existed at the time of publication of this article, type

```
. net sj 23-2
. net install st0714     (to install program files, if available)
. net get st0714         (to install ancillary files, if available)
```

9 References

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