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Unit-root tests for explosive behavior

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Abstract. We present a new command, `radf`, that tests for explosive behavior in time series. The command computes the right-tail augmented Dickey and Fuller (1979, *Journal of the American Statistical Association* 74: 427–431) unit-root test and its further developments based on supremum statistics derived from augmented Dickey–Fuller-type regressions estimated using recursive windows (Phillips, Wu, and Yu, 2011, *International Economic Review* 52: 201–226) and recursive flexible windows (Phillips, Shi, and Yu, 2015, *International Economic Review* 56: 1043–1078). It allows for the lag length in the test regression and the width of rolling windows to be either specified by the user or determined using data-dependent procedures, and it performs the date-stamping procedures advocated by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015) to identify episodes of explosive behavior. It also implements the wild bootstrap proposed by Phillips and Shi (2020, *Handbook of Statistics: Financial, Macro and Micro Econometrics Using R*, Vol. 42, 61–80) to lessen the potential effects of unconditional heteroskedasticity and account for the multiplicity issue in recursive testing. The use of `radf` is illustrated with an empirical example.

Keywords: st0659, radf, unit root, date-stamping explosive behavior, rolling window, lag length, wild bootstrap

1 Introduction

The study of the dynamic properties of economic and financial variables occupies a central position in the econometric modeling of time series. One specific type of behavior in which there has always been a great deal of interest, particularly in times of crises or distress, is when the series under consideration exhibits what appears to be explosive behavior. Indeed, analysts have identified several instances in the literature in which variables such as prices appear to increase well beyond the level that could be explained by their fundamentals; see, for example, Garber (2000) for an analysis of famous early bubbles and exuberant behavior.

During the last decade or so, there has been a renewed interest in the application of statistical tests for explosive behavior, mainly because of the appearance of novel theoretical findings by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015). These authors, through the further development of unit-root tests, provide a framework of analysis suitable for testing and date-stamping episodes where explosive behavior might have occurred. Empirical implementation of these new testing strategies is possible thanks to available computer codes in MATLAB;[1] the EViews add-in `Rtadf`, developed by Caspi (2017); the R Core Team (2020) package `exuber`, developed by Vasilopoulos, Pavlidis, and Martínez-García (2020a) for the econometric analysis of explosive time series; and `psymonitor`, developed by Phillips, Shi, and Caspi (2019) for real-time monitoring of asset markets (bubbles and crises).

In this article, we present the community-contributed command `radf`, which tests for explosive behavior in time series. The `radf` command implements the right-tail augmented Dickey and Fuller (1979; ADF) unit-root test and its further developments based on supremum statistics derived from ADF-type regressions estimated using recursive windows (Phillips, Wu, and Yu 2011) and recursive flexible windows (Phillips, Shi, and Yu 2015). Similarly to the other software that is currently available, `radf` supports the implementation of the date-stamping procedures that have been advocated to identify episodes of explosive behavior. In addition, it implements the wild bootstrap proposed by Phillips and Shi (2020) to lessen the potential effects of unconditional heteroskedasticity and multiplicity involved in recursive testing. We believe that the tests computed with the `radf` command constitute a worthwhile addition to the battery of time-series tests currently available to Stata users.

The article is organized as follows. Section 2 provides an overview of the tests for explosive behavior supported by the `radf` command. Section 3 describes the command, while section 4 illustrates the use of the command with an empirical example. Section 5 concludes.

2 Tests for explosive behavior

In this section, we offer an overview of the tests for explosive behavior that are provided by the `radf` command. The reader interested in a thorough and rigorous presentation is referred to Fuller (1976); Dickey and Fuller (1979); Phillips, Wu, and Yu (2011); and Phillips, Shi, and Yu (2015). Following the notation employed by Phillips, Shi, and Yu (2015), `radf` calculates three tests that are based on the ADF regression with a constant,

$$\Delta y_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \delta^i_{r_1,r_2} \triangle y_{t-i} + \varepsilon_t \tag{1}$$

where Δ is the first-difference operator, y_t is the realization of the time series of interest at time t, k is a scalar that denotes the number of lags of the dependent variable that are included to account for residual serial correlation, and r_1 and r_2, respectively, denote

1. See the website of Shuping Shi at https://sites.google.com/site/shupingshi/home/codes.

the starting and ending points used for estimation. With T as the total number of time periods in the sample, r_1 and r_2 are expressed as fractions of T such that $r_2 = r_1 + r_w$, where r_w is the window size of the regression, also expressed as a fraction of T. The number of observations used to estimate (1) is denoted $T_w = \lfloor Tr_w \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function that gives the integer part of the argument. The error term is ε_t.

The unit-root null hypothesis is given by $H_0 : \beta_{r_1,r_2} = 0$, while the alternative is that the series of interest exhibits explosive behavior; that is, $H_1 : \beta_{r_1,r_2} > 0$. The ADF t statistic required to test $H_0 : \beta_{r_1,r_2} = 0$ in (1) is denoted $\text{ADF}_{r_1}^{r_2}$. In this setting, the command `radf` calculates the two statistics that were studied by Phillips, Wu, and Yu (2011). The first is the right-tailed ADF statistic based on the full range of observations, that is, when $r_1 = 0$ and $r_2 = 1$ (that is, $r_w = 1$). The resulting statistic is thus denoted ADF_0^1. Monte Carlo simulation results reported by Phillips, Wu, and Yu (2011) confirm earlier findings by Diba and Grossman (1988) and Evans (1991) in the sense that standard unit-root tests based on the full sample can detect explosive behavior as long as the episodes of bubbles (or exuberance) do not collapse periodically.[2] To deal with the cases where bubbles collapse periodically, Phillips, Wu, and Yu (2011) further consider a second statistic that is based on the supremum t statistic that results from a forward recursive estimation of (1),

$$\text{SADF}(r_0) = \sup_{r_2 \in [r_0,1]} \text{ADF}_0^{r_2} \tag{2}$$

where the window size r_w expands from the smallest sample window width r_0, which provides the first t statistic of the recursion, to the last available observation. The supremum ADF (SADF) t statistic given in (2) is the second test statistic computed by the `radf` command.

In their empirical illustration, Phillips, Wu, and Yu (2011) select the optimal number of lags of the test regression (k) following the general-to-specific methodology, which involves setting $k = k_{\max}$ and testing the statistical significance of $\delta_{r_1,r_2}^{\max}$ based on the standard normal distribution (see Campbell and Perron [1991]; Hall [1994]; and Ng and Perron [1995]). The `radf` command supports the choice of the optimal number of lags with this data-dependent procedure, based on significance levels of 5% and 10%, as well as with the statistical Akaike information criteria (AIC) of Akaike (1974) and the Schwarz information criteria (SIC) of Schwarz (1978).

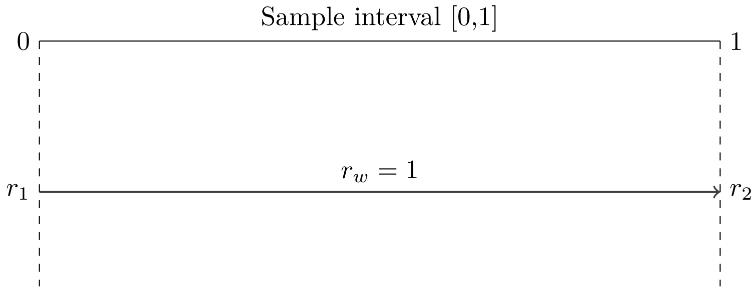
2. At this point, some words of caution seem warranted concerning the interpretation of the test statistics computed by `radf`. Indeed, it is worth reiterating that the testing strategies put forward by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015) focus on the right tail of the unit-root test distributions, which means that the null hypothesis remains a unit root with an implicit alternative of an explosive root. The consequences of repeated collapsing bubbles is a question of power, which is important of course, but then so is the question of test size. One could imagine a unit-root possibility with structural breaks in the deterministic components causing size distortions, which has been studied extensively in the unit-root versus stationary-alternatives literature. We leave the study of the effects of structural breaks on right-tail unit-root tests for future work.

However, Phillips, Shi, and Yu (2015) indicate that when the sample period is characterized by successive episodes of bubbles, one potential limitation of the recursive approach suggested by Phillips, Wu, and Yu (2011) is that it provides consistent estimates of the origination and ending dates of the first bubble but not subsequent ones. To overcome this limitation, Phillips, Shi, and Yu (2015) put forward the generalized supremum ADF (GSADF) test, which is the third statistic produced by `radf`. As the name implies, the GSADF test involves a much more extensive set of regressions, in which the first observation used for estimation varies from 0 to $r_2 - r_0$, while the last observation varies from r_0 to 1. More formally,

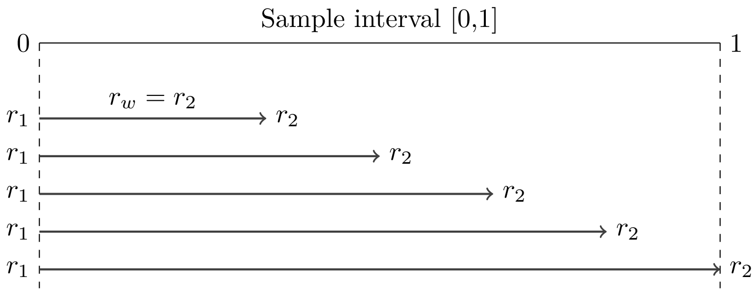
$$\text{GSADF}(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} \text{ADF}_{r_1}^{r_2}$$

For practical purposes, Phillips, Shi, and Yu (2015) recommend implementing the GSADF test by setting a low value of k, say, 0 or 1, and determining the minimum window size using the rule $r_0 = (0.01 + 1.8/\sqrt{T})$. The `radf` command permits the user to set the number of lags k, while the minimum window size is determined automatically based on the rule already described. Although the command also permits the user to set the minimum window size according to personal preferences, it warns that the critical values have been derived for the window size set by the rule.

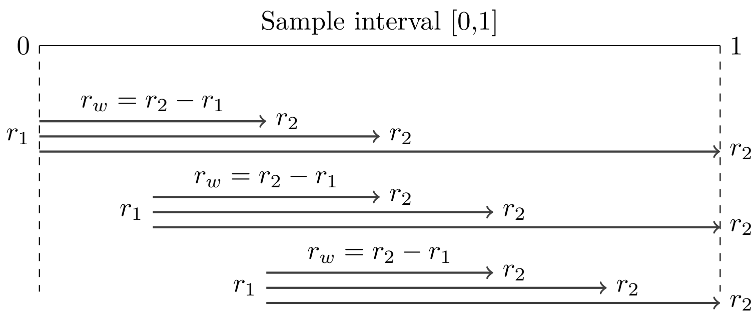
The sample sequences described before are summarized graphically in figure 1 (from Phillips, Shi, and Yu [2015]).



(a) Full sample



(b) Recursive window



(c) Recursive flexible window

Figure 1. Sample sequences and window widths supported by the `radf` command

Inference for the right-tail ADF, SADF, and GSADF statistics requires critical values computed using Monte Carlo simulations. In the case of the ADF_0^1 and $\text{SADF}(r_0)$ statistics, the underlying data-generating process is the well known random walk without a drift,

$$y_t = y_{t-1} + \epsilon_t$$

where $\epsilon_t \sim \text{NID}(0, \sigma^2)$ (NID denotes normally and independently distributed). As for the $\text{GSADF}(r_0)$ statistic, the corresponding critical values are based on a random walk where the drift component is asymptotically negligible,

$$y_t = T^{-1} + y_{t-1} + \epsilon_t$$

where $\epsilon_t \sim \text{NID}(0, \sigma^2)$. For computational convenience, the `radf` command takes advantage of a large set of critical values already available in the R Core Team (2020) package `exuber`; see Vasilopoulos, Pavlidis, and Martínez-García (2020a) and Vasilopoulos et al. (2020b).[3] More specifically, the critical values that we incorporated in `radf` are the 90%, 95%, and 99% critical values provided by `exuber`, which were obtained using 2,000 replications, seed equal to 123, initial window size given by $r_0 = 0.01 + 1.8/\sqrt{T}$, and $T = 6, 7, 8, \ldots, 600, 700, 800, \ldots, 2000$ observations. For $600 < T \leq 2000$, the sample size is used to interpolate between the critical values. For $T > 2000$, the critical values that are included in the summary results table correspond to those for $T = 2000$.

The testing strategies based on recursive window and recursive flexible window estimation of (1) provide useful guidance to date-stamp in real time the episodes of explosive behavior if the null hypothesis is rejected. Following the discussion in Phillips, Shi, and Yu (2015), let us suppose that one is interested in assessing whether any particular observation, say, r_2, belongs to a phase of explosive behavior. These authors recommend performing a SADF test on a sample sequence where the endpoint is fixed at the observation of interest r_2, and expands backwards to the starting point, r_1, which varies between 0 and $(r_2 - r_0)$. In this frame, the backward SADF (BSADF) statistic is defined as the supremum of the resulting sequence of ADF statistics, that is,

$$\text{BSADF}_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \text{ADF}_{r_1}^{r_2} \tag{3}$$

The statistic $\text{BSADF}_{r_2}(r_0)$ is then compared with the corresponding critical values of the $\text{SADF}(r_0)$ for $\lfloor r_2 T \rfloor$ observations. Phillips, Shi, and Yu (2015) indicate that this identification procedure is more general than the earlier suggestion in Phillips, Wu, and Yu (2011), which sets $r_1 = 0$ in (3) and therefore is more effective at identifying episodes of multiple bubbles. The sample sequences and window widths for date-stamping recommended by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015) are illustrated in figure 2, top and bottom, respectively, (from Phillips, Shi, and Yu [2015]).

3. In appendix A, we illustrate the use of `radf` to compute Monte Carlo critical values.

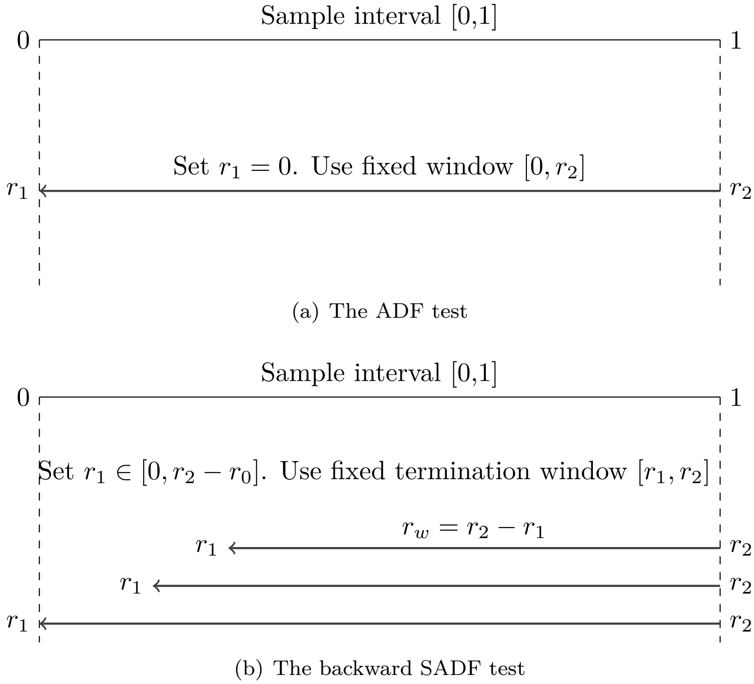


Figure 2. Sample sequences and window widths for date-stamping strategies

The `radf` command permits the user the options to generate and graph the variables that contain the sequences of t ratios and their associated critical values for date-stamping the episodes of explosive behavior. Because the critical values for the BSADF statistic are only available for $T \leq 600$, these series are not available for larger selected samples.

As a further development, in recent work Phillips and Shi (2020) recommend a wild bootstrap procedure to lessen the potential effects of unconditional heteroskedasticity and to account for the multiplicity issue in recursive testing. This bootstrap scheme can be implemented as an option with the `radf` command, which provides 90%, 95%, and 99% bootstrap critical values for the three tests described before.

3 The radf command

The command `radf` calculates the right-tail ADF test statistics for explosive behavior proposed by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015). The command permits the user to compute the statistics in which the number of lags of the dependent variable in the test regressions can be determined either by the user or endogenously by using a data-dependent procedure. Options to generate and graph the resulting sequences of recursive and flexible recursive statistics allow the identification of episodes of explosive behavior.

3.1 Syntax

Before using the command `radf`, and similar to many other Stata time-series commands, it is necessary to `tsset` the data.

`radf` *varname* [*if*] [*in*] [`,` `prefix(`*prefix*`)` `maxlag(`*integer*`)` `criterion(`*string*`)` `window(`*integer*`)` `bs` `seed(`*integer*`)` `boot(`*integer*`)` `print` `graph`]

varname may not contain gaps but can contain time-series operators. `radf` does not support the `by:` prefix.

3.2 Options

The command `radf` supports the following options:

`prefix(`*prefix*`)` can be used to provide a "stub" with which variables created in `radf` will be named if no more than 600 observations are in the specified sample. If this option is given, four Stata variables will be created for the appropriate range of dates: *prefix*_`SADF`, *prefix*_`BSADF`, *prefix*_`BSADF_95`, and *prefix*_`Exceeding`. These variables record the SADF and BSADF statistics, with the third variable displaying the 95% critical values for the BSADF statistic, which vary over the estimation period. The fourth variable is an indicator, set to 1 in each period when the BSADF statistic exceeds its 95% critical value. The `prefix()` option must be specified to enable the `graph` option.

`maxlag(`*integer*`)` sets the number of lags to be included in the test regression to account for residual serial correlation. If not specified, `radf` sets the number of lags following Schwert (1989), with the formula $\text{maxlag} = \text{int}\{4(T/100)^{0.25}\}$, where T is the total number of observations. In either case, the number of lags is reported in the output. If `maxlag()` is given and the window width is set by the data-dependent procedure, they may conflict. In this case, `radf` reduces `maxlag()` so that each ADF regression has positive degrees of freedom.

`criterion(`*string*`)` can be used to specify `AIC`, `SIC`, `GTS05`, or `GTS10` as alternatives to the default value of `FIX`. By default, `radf` computes the ADF regressions based on a fixed `maxlag()`, either one determined by a data-dependent procedure or one specified by the user. Alternatively, the command can determine the optimal number of lags according to the Akaike or Schwarz information criteria, denoted AIC and SIC, respectively, or by following the general-to-specific (GTS) algorithm advocated by Hall (1994) and Ng and Perron (1995). The idea of the GTS algorithm is to start by setting an upper bound on p (denoted $p_{\max}$), estimating (1) with $p = p_{\max}$, and testing the statistical significance of $\delta_{p_{\max}}$. If this coefficient is statistically significant, using, for example, significance levels of 5% (referred to as $\text{GTS}_{0.05}$) or 10% (referred to as $\text{GTS}_{0.10}$), one selects $p = p_{\max}$. Otherwise, the order of the estimated autoregression is reduced by 1 until the coefficient on the last included lag is found to be statistically different from 0.

`window(`*integer*`)` can be used to select a different window width. The initial window width used to compute both the SADF and the GSADF statistics takes the default value of $r_0 = 0.01 + 1.8/\sqrt{T}$ recommended by Phillips, Shi, and Yu (2015). However, because the critical values have been developed for the default window width, a warning is provided if the window width is set by the user, showing the default width and the selected width.

`bs` computes right-tail Monte Carlo critical values for the 90th, 95th, and 99th percentiles based on the wild bootstrap advocated by Phillips and Shi (2020), using 199 replications. To set a different number of replications, use the `boot()` option. Also, notice that the bootstrap critical values cannot be replicated unless `bs` is used along with option `seed()`.

`seed(`*integer*`)` sets the initial seed for random-number generation.

`boot(`*integer*`)` sets the number of replications to perform the wild bootstrap advocated by Phillips and Shi (2020).

`print` specifies that detailed results are to be printed showing the ADF statistics and lag lengths for each of the regressions being estimated.

`graph` specifies that the time series of the SADF and BSADF statistics, which can be saved as variables with the `prefix()` option, should be graphed along with their 90% and 95% critical values. The graphs will be saved with names specified by the `prefix()` option as *prefix*`_SADF.gph` and *prefix*`_BSADF.gph`. The `graph` option is not available if more than 600 observations are included in the specified sample, and it requires the use of the `prefix()` option.

3.3 Stored results

`radf` stores the following in `r()`:

Scalars

`r(ntests)`	number of ADF tests
`r(gsadfstat)`	GSADF statistic
`r(sadfstat)`	SADF statistic
`r(adfstat)`	ADF statistic
`r(window)`	maximum window width
`r(maxlag)`	maximum lag order in test
`r(N)`	number of observations in full sample test
`r(nobs)`	number of observations available

Macros

`r(cmd)`	`radf`
`r(cmdline)`	command line
`r(last)`	last observation in window
`r(first)`	first observation in window
`r(varname)`	variable name

Matrices

`r(radfstats)`	matrix of test statistics and critical values, 3×4 without bootstrap critical values
`r(radfstats)`	matrix of test statistics and critical values, 3×7 with bootstrap critical values

4 Empirical application

Housing is certainly the most important asset in the portfolio of many individuals and families. For this reason, there is considerable interest in following the dynamic path of property prices so that knowledge can be gained regarding the specific periods of time when they might be viewed as reaching levels that compromise affordability. Indeed, many economists and financial analysts alike feel that property prices are prone to suffering bubble-like phenomena. Before the advent of these tests for explosive behavior, the researcher applying unit-root tests to property price series would have undoubtedly favored the unit-root hypothesis. Then attention would turn to testing the existence of long-run equilibrium relationships with real income (for example, Holly, Pesaran, and Yamagata [2010]) or with other property prices (for example, Holly, Pesaran, and Yamagata [2011]). The findings here are suggestive of episodes of bubbles when applied to certain intervals over sup criteria.

We use data from the International House Price Database of the Federal Reserve Bank of Dallas, which contains quarterly price information on 23 countries that dates back to the first quarter of 1975; see Mack and Martínez-García (2011) for methodological details on the database. To carry out our empirical illustration, we downloaded the data release for the first quarter of 2015 directly from the R console, following the steps described in section 5.2 of Vasilopoulos, Pavlidis, and Martínez-García (2020a), and we created a Stata version of the data that was placed with the Boston College Economics Stata datasets.

We begin by loading the dataset using the community-contributed command `bcuse` (Baum 2017) and declaring it as a time series:

```
. bcuse hprices
  (output omitted)
. tsset yq
Time variable:  yq, 1975q1 to 2015q1
        Delta:  1 quarter
```

We would like to test whether the price index series of the United Kingdom and the United States, respectively named `uk` and `us`, each contain a unit root, against the alternative that they are explosive processes.

Using the default specifications for `criterion()` and `window()` and setting a `maxlag()` of $p = 1$, the results of applying the command `radf` to the two variables of interest over the full range of observations are as follows:

```
. radf uk, maxlag(1)
Right-tail ADF statistics for uk with first observations 1975q3 - 2009q2
Number of obs = 159  lag selection[FIX]  maxlag = 1  window = 24 periods

      |      Test     Tab90     Tab95     Tab99
------+----------------------------------------
 ADF0 |   -0.0907   -0.3562    0.0200    0.8674
 SADF |    2.9384    1.0697    1.3693    1.9305
GSADF |    4.0630    1.8267    2.1139    2.6669
Test: ADF0, SADF (PWY,2011), GSADF (PSY,2015)
Tab : right-tail tabulated critical values for 90, 95, 99 confidence levels
      from Vasilopoulos, Pavlidis, Spavound and Martínez-García (2020)

. radf us, maxlag(1)
Right-tail ADF statistics for us with first observations 1975q3 - 2009q2
Number of obs = 159  lag selection[FIX]  maxlag = 1  window = 24 periods

      |      Test     Tab90     Tab95     Tab99
------+----------------------------------------
 ADF0 |   -1.0302   -0.3562    0.0200    0.8674
 SADF |    4.1461    1.0697    1.3693    1.9305
GSADF |    6.0391    1.8267    2.1139    2.6669
Test: ADF0, SADF (PWY,2011), GSADF (PSY,2015)
Tab : right-tail tabulated critical values for 90, 95, 99 confidence levels
      from Vasilopoulos, Pavlidis, Spavound and Martínez-García (2020)
```

The output indicates that the first observations of the recursive and recursive flexible windows run from the third quarter of 1975 to the second quarter of 2009. Both SADF and GSADF tests provide evidence against the unit-root null hypothesis at the 1% significance level. In the case of ADF, we observe rejection of the null in the case of U.K. prices only at the 10% significance level.

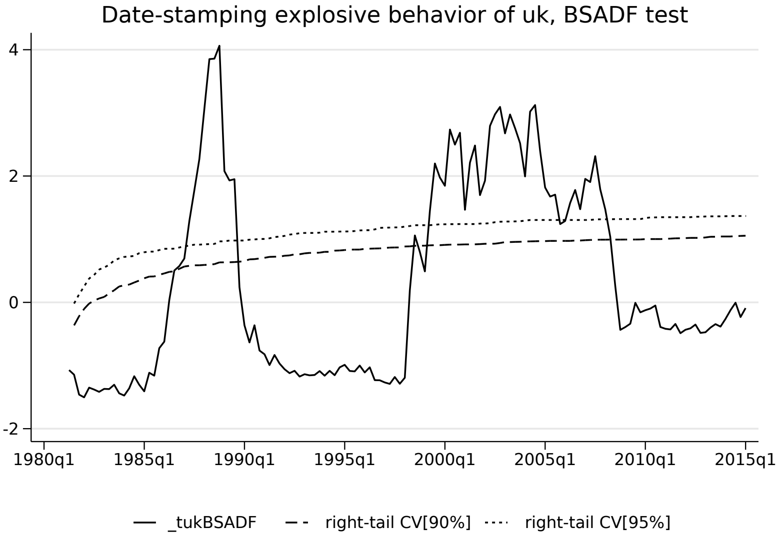
The results reported above are identical to those obtained using the R package `exuber` and the EViews add-in `Rtadf`. In terms of computation time, for the variable `uk`, `radf` runs in 0.24 seconds on Stata version 17 using a laptop computer equipped with processor Intel Core™ i7 CPU @ 2.80 GHz and installed memory (RAM) of 16.0 GB. Using the R package `exuber`, the same results can be obtained in only 0.021 seconds. By contrast, with the EViews add-in `Rtadf`, it takes 0.106, 0.536, and 21.756 seconds to obtain the ADF, SADF, and GSADF statistics, respectively.[4] The speed gain achieved by the R package `exuber` most likely is due to the fact that Vasilopoulos, Pavlidis, and Martínez-García (2020a) use partitioned inversion techniques that avoid the need to invert sequences of matrices when computing the ordinary least-squares estimators, while `radf` is based on Stata's Mata language. All in all, we consider that the speed of `radf` is satisfactory for applied work and also if the user requires the computation of critical values for different values of the number of observations, initial window lengths, and the number of lags.

Given that the results presented above reject the unit-root null hypothesis, one might proceed to plot the sequences of t statistics and corresponding critical values to identify the time periods during which episodes of explosive behavior might have taken place. To this end, we run the previous command using the `prefix()` and `graph` options:

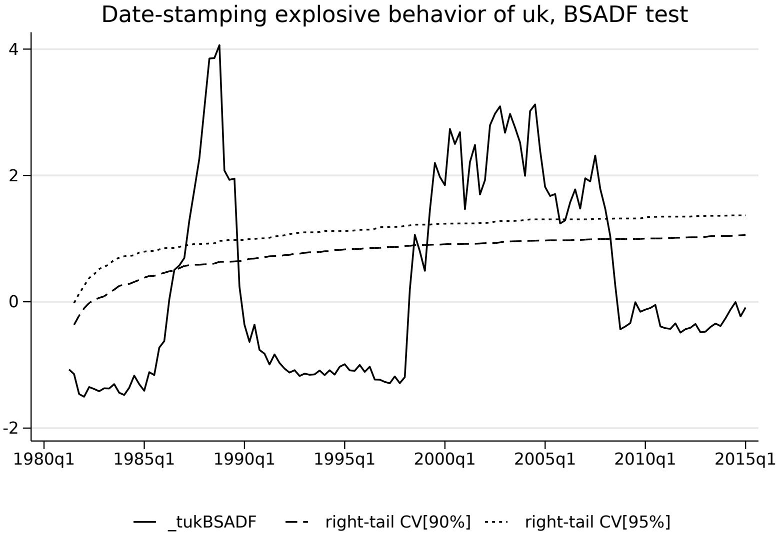
```
. radf uk, maxlag(1) prefix(_t) graph
  (output omitted)
. radf us, maxlag(1) prefix(_t) graph
  (output omitted)
```

The output is omitted to avoid repetition, while the date-stamping plots for the price series in the United Kingdom and the United States are presented in figures 3 and 4, respectively. To facilitate the verification of these results, the variables used to construct the figures are reported in appendix B at the end of the article and are available with the `print` option. Looking at the more powerful BSADF test and using a 5% significance level, the results for the United Kingdom reveal three episodes of explosive behavior in house prices, which can be date-stamped by the indicator variable *prefix*_`Exceeding`: 1987q2–1989q3, 1999q1–2005q3, and 2006q2–2008q1. For house prices in the United States, the identified periods are 1987q1 and 1998q1–2007q2. In terms of the duration of the episodes, Phillips, Shi, and Yu (2015) recommend focusing on those that last more than $\ln(T)$ units of time, which in this illustration is $\ln(161) \approx 5$ quarters. This further refinement, however, is left at the discretion of the user.

4. In the case of the EViews add-in `Rtadf`, runtime improvements of more than 90% can be obtained using a MATLAB parallel computer option (see Caspi [2017, 10]).

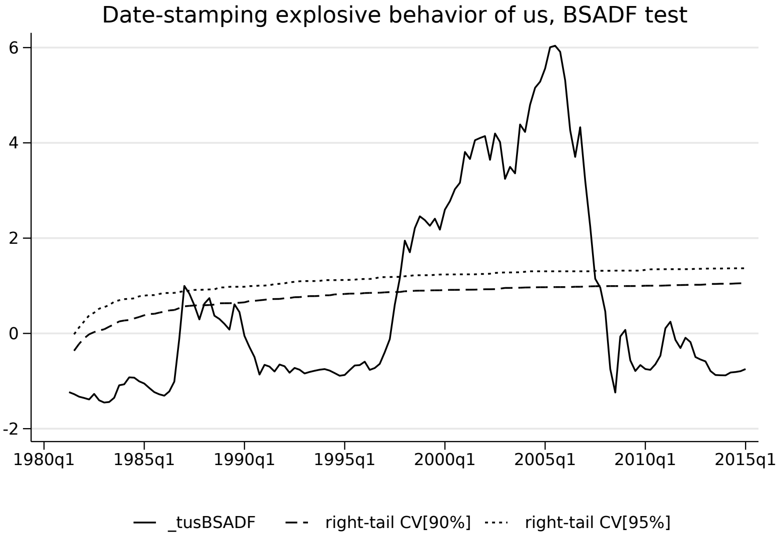


(a) t statistics and critical values from recursive estimation

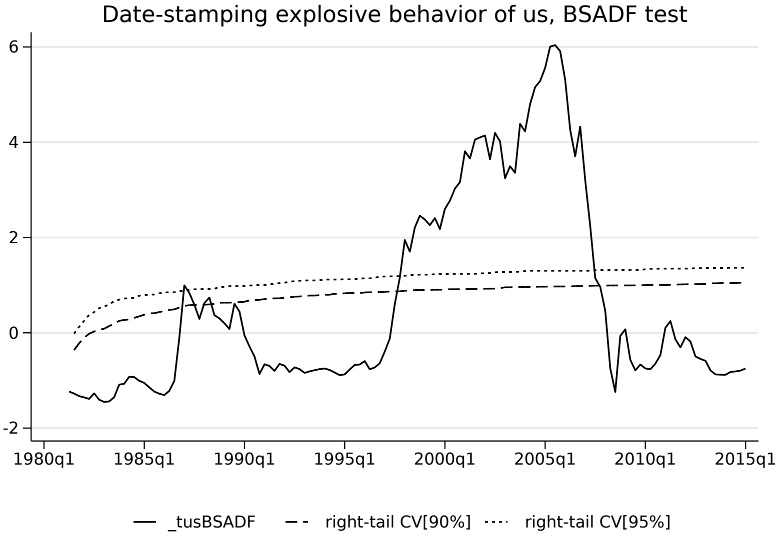


(b) t statistics and critical values from recursive flexible estimation

Figure 3. Date-stamping analysis for the U.K. house price series



(a) t statistics and critical values from recursive estimation



(b) t statistics and critical values from recursive flexible estimation

Figure 4. Date-stamping analysis for the U.S. house price series

Finally, we now illustrate the use of the command `radf` using the options `boot(499)` and `seed(123)`. To save space, we shall present only the results for the United Kingdom.

```
. radf uk, maxlag(1) boot(499) seed(123)
Right-tail ADF statistics for uk with first observations 1975q3 - 2009q2
Number of obs = 159  lag selection[FIX]  maxlag = 1  window = 24 periods

              Test    RTMC90    RTMC95    RTMC99     Tab90     Tab95     Tab99

 ADF0      -0.0907    0.3207    0.6189    1.0457   -0.3562    0.0200    0.8674
 SADF       2.9384    2.5879    2.9414    3.4737    1.0697    1.3693    1.9305
GSADF       4.0630    3.5604    3.9052    4.8017    1.8267    2.1139    2.6669

Test: ADF0, SADF (PWY,2011), GSADF (PSY,2015)
RTMC: right-tail Monte Carlo critical values for 90, 95, 99 percentiles
      based on wild bootstrap with 499 replications
Tab : right-tail tabulated critical values for 90, 95, 99 confidence levels
      from Vasilopoulos, Pavlidis, Spavound and Martínez-García (2020)
```

As can be seen, in all cases the simulated bootstrap critical values (columns labeled `RTMC90`, `RTMC95`, and `RTMC99`) are much larger than the corresponding tabulated critical values from Vasilopoulos, Pavlidis, and Martínez-García (2020a) (columns labeled `Tab90`, `Tab95`, and `Tab99`). Using the simulated bootstrap critical values, the unit-root null is rejected at the 10% and 5% levels by the SADF and GSADF tests, respectively.

5 Concluding remarks

In this article, we presented the command `radf`, which calculates the right-tail ADF test statistics for explosive behavior proposed by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015). `radf` permits the user to compute the statistics in which the number of lags of the dependent variable in the test regressions can be determined either by the user or endogenously by using a data-dependent procedure. The options to generate and graph the resulting sequences of recursive and flexible recursive statistics allow the identification of episodes of explosive behavior. Finally, the command `radf` implements a wild bootstrap scheme recently advocated by Phillips and Shi (2020), which aims to lessen the potential effects of unconditional heteroskedasticity and to account for the multiplicity issue in recursive testing.

6 Acknowledgments

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7 Programs and supplemental materials

To install a snapshot of the corresponding software files as they existed at the time of publication of this article, type

```
. net sj 21-4
. net install st0659     (to install program files, if available)
. net get st0659         (to install ancillary files, if available)
```

8 References

Akaike, H. 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19: 716–723. https://doi.org/10.1109/TAC.1974.1100705.

Baum, C. F. 2017. bcuse: Stata module to access instructional datasets on Boston College server. Statistical Software Components S457508, Department of Economics, Boston College. https://ideas.repec.org/c/boc/bocode/s457508.html.

Campbell, J. Y., and P. Perron. 1991. Pitfalls and opportunities: What macroeconomists should know about unit roots. *NBER Macroeconomics Annual 1991* 6: 141–201. https://doi.org/10.1086/654163.

Caspi, I. 2017. rtadf: Testing for bubbles with EViews. *Journal of Statistical Software, Code Snippet 1* 81: 1–16. http://dx.doi.org/10.18637/jss.v081.c01.

Diba, B. T., and H. I. Grossman. 1988. Explosive rational bubbles in stock prices? *American Economic Review* 78: 520–530.

Dickey, D. A., and W. A. Fuller. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 74: 427–431. https://doi.org/10.2307/2286348.

Evans, G. W. 1991. Pitfalls in testing for explosive bubbles in asset prices. *American Economic Review* 81: 922–930.

Fuller, W. A. 1976. *Introduction to Statistical Time Series*. New York: Wiley.

Garber, P. M. 2000. *Famous First Bubbles: The Fundamentals of Early Manias*. Cambridge, MA: MIT Press.

Hall, A. 1994. Testing for a unit root in time series with pretest data-based model selection. *Journal of Business & Economic Statistics* 12: 461–470. https://doi.org/10.1080/07350015.1994.10524568.

Holly, S., M. H. Pesaran, and T. Yamagata. 2010. A spatio-temporal model of house prices in the USA. *Journal of Econometrics* 158: 160–173. https://doi.org/10.1016/j.jeconom.2010.03.040.

———. 2011. The spatial and temporal diffusion of house prices in the UK. *Journal of Urban Economics* 69: 2–23. https://doi.org/10.1016/j.jue.2010.08.002.

Mack, A., and E. Martínez-García. 2011. A cross-country quarterly database of real house prices: A methodological note. Globalization Institute Working Papers 99, Federal Reserve Bank of Dallas. https://www.dallasfed.org/~/media/documents/institute/wpapers/2011/0099.pdf.

Ng, S., and P. Perron. 1995. Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association* 90: 268–281. https://doi.org/10.1080/01621459.1995.10476510.

Phillips, P. C. B., and S. Shi. 2020. Real time monitoring of asset markets: Bubbles and crises. In *Handbook of Statistics: Financial, Macro and Micro Econometrics Using R*, ed. H. D. Vinod and C. R. Rao. Vol. 42, 61–80. Amsterdam: Elsevier. https://doi.org/10.1016/bs.host.2018.12.002.

Phillips, P. C. B., S. Shi, and I. Caspi. 2019. *psymonitor: Real Time Monitoring of Asset Markets*. R package version 0.0.2. https://github.com/itamarcaspi/psymonitor.

Phillips, P. C. B., S. Shi, and J. Yu. 2015. Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review* 56: 1043–1078. https://doi.org/10.1111/iere.12132.

Phillips, P. C. B., Y. Wu, and J. Yu. 2011. Explosive behavior in the 1990s NASDAQ: When did exuberance escalate asset values? *International Economic Review* 52: 201–226. https://doi.org/10.1111/j.1468-2354.2010.00625.x.

R Core Team. 2020. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org.

Schwarz, G. 1978. Estimating the dimension of a model. *Annals of Statistics* 6: 461–464. https://doi.org/10.1214/aos/1176344136.

Schwert, G. W. 1989. Tests for unit roots: A Monte Carlo investigation. *Journal of Business & Economic Statistics* 7: 147–159. https://doi.org/10.1080/07350015.1989.10509723.

Vasilopoulos, K., E. Pavlidis, and E. Martínez-García. 2020a. exuber: Recursive Right-Tailed Unit Root Testing with R. Globalization Institute Working Papers 383, Federal Reserve Bank of Dallas. https://doi.org/10.24149/gwp383.

Vasilopoulos, K., E. Pavlidis, S. Spavound, and E. Martínez-García. 2020b. *exuber: Econometric Analysis of Explosive Time Series*. R package version 0.4.1. https://github.com/kvasilopoulos/exuber.

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A Generating critical values with radf

In this appendix, we illustrate the use of `radf` to tabulate finite-sample right-tail critical values for the ADF, SADF, and GSADF statistics. The example below tabulates critical values for $T = 100$ observations, window size determined as recommended by Phillips, Shi, and Yu (2015), and an asymptotically negligible drift component as in (2):

```
clear all
local nobs = 100
local wsize = floor(`nobs'*(0.01 + 1.8/sqrt(`nobs')))
local drift = 1/`nobs'
set obs `nobs'
set seed 123
generate t = _n
quietly generate u = 0
quietly generate y = `drift'
quietly tsset t

tempname sim_cv

postfile `sim_cv' tadf tsadf tgsadf using results, replace

forvalues i = 1/2000 {

     quietly replace u = rnormal(0,1) in 1/`nobs'

     quietly replace y = `drift' + L.y + u in 2/`nobs'

     quietly radf y, maxl(0) win(`wsize')
     scalar tadf = r(adfstat)
     scalar tsadf = r(sadfstat)
     scalar tgsadf = r(gsadfstat)

     post `sim_cv' (tadf) (tsadf) (tgsadf)
}

postclose `sim_cv'

use results, clear

quietly sum tadf, detail
display "ADF   90% " %8.2f r(p90) " 95% " %8.2f r(p95) " 99% " %8.2f r(p99)
quietly sum tsadf, detail
display "SADF  90% " %8.2f r(p90) " 95% " %8.2f r(p95) " 99% " %8.2f r(p99)
quietly sum tgsadf, detail
display "GSADF 90% " %8.2f r(p90) " 95% " %8.2f r(p95) " 99% " %8.2f r(p99)
```

B Date-stamping results

Table 1. t_{SADF}, t_{BSADF}, and t_{BSADF} 95% critical values

	United Kingdom			United States			
Date	t_{SADF}	t_{BSADF}	Exceeding	t_{SADF}	t_{BSADF}	Exceeding	t_{BSADF} 95% cv
1981q2	−1.072	−1.072	.	−1.233	−1.233	.	.
1981q3	−1.145	−1.145	0	−1.274	−1.274	0	−0.018
1981q4	−1.460	−1.460	0	−1.328	−1.328	0	0.127
1982q1	−1.533	−1.504	0	−1.355	−1.355	0	0.252
1982q2	−1.427	−1.350	0	−1.386	−1.386	0	0.378
1982q3	−1.455	−1.381	0	−1.331	−1.270	0	0.435
1982q4	−1.491	−1.417	0	−1.404	−1.404	0	0.520
1983q1	−1.431	−1.369	0	−1.451	−1.451	0	0.551
1983q2	−1.428	−1.373	0	−1.465	−1.439	0	0.595
1983q3	−1.349	−1.304	0	−1.471	−1.351	0	0.664
1983q4	−1.485	−1.440	0	−1.495	−1.089	0	0.697
1984q1	−1.512	−1.475	0	−1.525	−1.069	0	0.720
1984q2	−1.381	−1.361	0	−1.547	−0.922	0	0.726
1984q3	−1.180	−1.169	0	−1.570	−0.929	0	0.735
1984q4	−1.316	−1.304	0	−1.594	−1.006	0	0.780
1985q1	−1.423	−1.409	0	−1.617	−1.051	0	0.794
1985q2	−1.127	−1.114	0	−1.635	−1.145	0	0.804
1985q3	−1.173	−1.160	0	−1.643	−1.232	0	0.804
1985q4	−0.849	−0.726	0	−1.656	−1.278	0	0.828
1986q1	−0.861	−0.622	0	−1.610	−1.307	0	0.848
1986q2	−0.428	0.037	0	−1.441	−1.218	0	0.848
1986q3	−0.172	0.504	0	−1.425	−1.012	0	0.850
1986q4	−0.174	0.575	0	−1.321	−0.102	0	0.869
1987q1	−0.078	0.694	0	−1.212	0.997	1	0.889
1987q2	0.407	1.287	1	−1.211	0.836	0	0.898
1987q3	0.780	1.775	1	−1.241	0.591	0	0.914
1987q4	1.172	2.270	1	−1.307	0.294	0	0.914
1988q1	1.056	3.063	1	−1.141	0.624	0	0.918
1988q2	1.645	3.851	1	−1.057	0.742	0	0.923
1988q3	2.938	3.858	1	−1.200	0.372	0	0.924
1988q4	2.836	4.063	1	−1.196	0.303	0	0.966
1989q1	1.657	2.078	1	−1.222	0.202	0	0.966
1989q2	1.760	1.931	1	−1.263	0.080	0	0.979
1989q3	1.858	1.950	1	−0.944	0.611	0	0.979

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Table 1 (*continued*)

	United Kingdom			United States			
Date	t_{SADF}	t_{BSADF}	Exceeding	t_{SADF}	t_{BSADF}	Exceeding	t_{BSADF} 95% cv
1989q4	0.150	0.241	0	−1.056	0.447	0	0.979
1990q1	−0.411	−0.362	0	−1.292	−0.051	0	0.979
1990q2	−0.687	−0.635	0	−1.382	−0.287	0	0.993
1990q3	−0.369	−0.363	0	−1.486	−0.496	0	0.996
1990q4	−0.806	−0.763	0	−1.696	−0.864	0	1.004
1991q1	−0.862	−0.822	0	−1.599	−0.659	0	1.004
1991q2	−1.069	−0.993	0	−1.633	−0.696	0	1.014
1991q3	−0.848	−0.833	0	−1.716	−0.799	0	1.034
1991q4	−1.016	−0.968	0	−1.616	−0.653	0	1.041
1992q1	−1.129	−1.058	0	−1.653	−0.690	0	1.052
1992q2	−1.210	−1.121	0	−1.771	−0.825	0	1.073
1992q3	−1.142	−1.083	0	−1.707	−0.724	0	1.083
1992q4	−1.315	−1.175	0	−1.754	−0.762	0	1.094
1993q1	−1.214	−1.138	0	−1.828	−0.841	0	1.099
1993q2	−1.241	−1.155	0	−1.813	−0.809	0	1.099
1993q3	−1.201	−1.150	0	−1.801	−0.784	0	1.099
1993q4	−1.286	−1.088	0	−1.791	−0.762	0	1.103
1994q1	−1.213	−1.161	0	−1.790	−0.749	0	1.119
1994q2	−1.231	−1.084	0	−1.828	−0.780	0	1.119
1994q3	−1.209	−1.153	0	−1.879	−0.833	0	1.119
1994q4	−1.276	−1.029	0	−1.931	−0.888	0	1.121
1995q1	−1.312	−0.988	0	−1.928	−0.873	0	1.122
1995q2	−1.291	−1.086	0	−1.842	−0.770	0	1.124
1995q3	−1.304	−1.093	0	−1.750	−0.673	0	1.129
1995q4	−1.344	−1.000	0	−1.752	−0.665	0	1.139
1996q1	−1.321	−1.110	0	−1.702	−0.593	0	1.143
1996q2	−1.353	−1.028	0	−1.846	−0.766	0	1.143
1996q3	−1.288	−1.231	0	−1.830	−0.727	0	1.155
1996q4	−1.305	−1.233	0	−1.816	−0.639	0	1.179
1997q1	−1.282	−1.268	0	−1.780	−0.393	0	1.182
1997q2	−1.312	−1.291	0	−1.740	−0.120	0	1.184
1997q3	−1.194	−1.182	0	−1.584	0.603	0	1.187
1997q4	−1.300	−1.289	0	−1.451	1.157	0	1.187
1998q1	−1.202	−1.193	0	−1.210	1.948	1	1.200
1998q2	−1.036	0.191	0	−1.215	1.703	1	1.209
1998q3	−0.963	1.059	0	−1.012	2.211	1	1.222
1998q4	−1.170	0.799	0	−0.852	2.458	1	1.222

Continued on next page

Table 1 (*continued*)

	United Kingdom			United States			
Date	t_{SADF}	t_{BSADF}	Exceeding	t_{SADF}	t_{BSADF}	Exceeding	t_{BSADF} 95% cv
1999q1	−1.124	0.490	0	−0.764	2.378	1	1.222
1999q2	−0.825	1.446	1	−0.678	2.260	1	1.222
1999q3	−0.563	2.199	1	−0.524	2.408	1	1.229
1999q4	−0.668	1.976	1	−0.487	2.179	1	1.236
2000q1	−0.595	1.846	1	−0.250	2.599	1	1.236
2000q2	−0.176	2.736	1	−0.087	2.775	1	1.238
2000q3	−0.258	2.497	1	0.114	3.027	1	1.238
2000q4	−0.040	2.685	1	0.278	3.162	1	1.240
2001q1	−0.422	1.466	1	0.685	3.810	1	1.240
2001q2	0.159	2.213	1	0.723	3.661	1	1.240
2001q3	0.331	2.484	1	1.017	4.055	1	1.240
2001q4	−0.062	1.698	1	1.177	4.102	1	1.248
2002q1	0.311	1.926	1	1.345	4.142	1	1.248
2002q2	1.100	2.795	1	1.328	3.643	1	1.253
2002q3	1.304	2.978	1	1.674	4.196	1	1.270
2002q4	1.387	3.095	1	1.724	4.018	1	1.277
2003q1	1.207	2.675	1	1.593	3.244	1	1.280
2003q2	1.601	2.975	1	1.813	3.497	1	1.280
2003q3	1.543	2.758	1	1.862	3.359	1	1.282
2003q4	1.524	2.520	1	2.586	4.386	1	1.284
2004q1	1.281	1.993	1	2.348	4.229	1	1.295
2004q2	2.275	3.020	1	2.743	4.801	1	1.304
2004q3	2.355	3.125	1	3.439	5.157	1	1.304
2004q4	1.767	2.397	1	3.319	5.285	1	1.304
2005q1	1.502	1.818	1	3.599	5.561	1	1.304
2005q2	1.505	1.675	1	4.017	6.008	1	1.304
2005q3	1.610	1.706	1	4.088	6.039	1	1.304
2005q4	1.201	1.240	0	4.146	5.912	1	1.304
2006q1	1.289	1.289	0	4.030	5.313	1	1.304
2006q2	1.575	1.575	1	3.404	4.266	1	1.304
2006q3	1.779	1.779	1	3.051	3.705	1	1.304
2006q4	1.475	1.475	1	3.565	4.328	1	1.304
2007q1	1.955	1.955	1	2.692	3.203	1	1.304
2007q2	1.905	1.905	1	1.935	2.244	1	1.305
2007q3	2.315	2.315	1	1.007	1.146	0	1.313
2007q4	1.790	1.790	1	0.788	0.963	0	1.315
2008q1	1.472	1.472	1	0.321	0.460	0	1.315

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Table 1 (*continued*)

	United Kingdom			United States			
Date	t_{SADF}	t_{BSADF}	Exceeding	t_{SADF}	t_{BSADF}	Exceeding	t_{BSADF} 95% cv
2008q2	1.038	1.038	0	−0.759	−0.747	0	1.315
2008q3	0.257	0.257	0	−1.278	−1.241	0	1.318
2008q4	−0.541	−0.435	0	−0.305	−0.065	0	1.318
2009q1	−0.461	−0.391	0	−0.166	0.075	0	1.318
2009q2	−0.384	−0.338	0	−0.800	−0.565	0	1.318
2009q3	−0.008	−0.007	0	−1.025	−0.789	0	1.318
2009q4	−0.159	−0.158	0	−0.925	−0.664	0	1.318
2010q1	−0.125	−0.124	0	−1.014	−0.748	0	1.334
2010q2	−0.099	−0.098	0	−1.036	−0.765	0	1.345
2010q3	−0.051	−0.050	0	−0.923	−0.648	0	1.345
2010q4	−0.447	−0.390	0	−1.055	−0.467	0	1.348
2011q1	−0.476	−0.418	0	−1.248	0.107	0	1.348
2011q2	−0.487	−0.429	0	−1.282	0.246	0	1.348
2011q3	−0.369	−0.342	0	−1.193	−0.135	0	1.348
2011q4	−0.565	−0.488	0	−1.196	−0.307	0	1.348
2012q1	−0.487	−0.434	0	−1.266	−0.091	0	1.349
2012q2	−0.458	−0.411	0	−1.267	−0.183	0	1.349
2012q3	−0.381	−0.351	0	−1.222	−0.496	0	1.356
2012q4	−0.556	−0.483	0	−1.230	−0.547	0	1.356
2013q1	−0.540	−0.473	0	−1.237	−0.587	0	1.360
2013q2	−0.441	−0.399	0	−1.189	−0.789	0	1.362
2013q3	−0.373	−0.345	0	−1.175	−0.873	0	1.362
2013q4	−0.428	−0.384	0	−1.174	−0.878	0	1.362
2014q1	−0.274	−0.261	0	−1.177	−0.880	0	1.364
2014q2	−0.123	−0.123	0	−1.111	−0.821	0	1.366
2014q3	−0.005	−0.005	0	−1.098	−0.810	0	1.367
2014q4	−0.248	−0.232	0	−1.078	−0.792	0	1.367
2015q1	−0.091	−0.091	0	−1.030	−0.748	0	1.367

NOTE: The right-tail 95% critical value of t_{SADF} is that of the Dickey–Fuller distribution and is equal to 0.02. The right-tail 95% critical values of t_{BSADF} were obtained from `exuber` based on a window size given by $r_0 = 0.01+1.8/\sqrt{T}$. Exceeding is an indicator variable equal to 1 when t_{BSADF} is above its 95% critical value and equal to 0 otherwise. This variable can be used to date-stamp the periods of explosive behavior.