



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

# Extending the Kitagawa–Oaxaca–Blinder decomposition approach to panel data

Hannes Kröger  
Socio-Economic Panel (SOEP) Study  
German Institute for Economic Research (DIW)  
Berlin, Germany  
hkroeger@diw.de

Jörg Hartmann  
Georg-August-Universität Göttingen  
Göttingen, Germany  
joerg.hartmann@sowi.uni-goettingen.de

**Abstract.** The Kitagawa–Oaxaca–Blinder decomposition approach has been widely used to attribute group-level differences in an outcome to differences in endowment, coefficients, and their interactions. The method has been implemented for Stata in the popular `oaxaca` command for cross-sectional analyses. In recent decades, however, research questions have been more often focused on the decomposition of group-based differences in change over time, for example, diverging income trajectories, as well as decomposition of change in differences between groups, for example, change in the gender pay gap over time. We review five existing methods for the decomposition of changes in group means and contribute an extension that takes an interventionist perspective suitable for applications with a clear before–after comparison.

These decompositions of levels and changes over time can be implemented using the `xtoaxaca` command, which works as a postestimation command for different regression commands in Stata. It is built to maximize flexibility in modeling and implements all decomposition techniques presented in this article.

**Keywords:** `st0640`, `xtoaxaca`, decomposition, longitudinal data, panel data, Oaxaca, Blinder, Kitagawa

## 1 Introduction

The decomposition of group differences in means (Kitagawa 1955; Oaxaca 1973; Blinder 1973) is a popular tool when researchers seek to attribute such differences to differences in the groups' characteristics and an unexplained part. As such, scholars have applied such decompositions to a variety of topics such as gender income inequality (Blau and Kahn 2017), happiness (Arrosa and Gandelman 2016), or obesity (Taber et al. 2016). This approach has also seen numerous extensions over the last decades to suit researchers' needs, such as its application to distributional parameters other than the mean (Freeman 1980, 1984), to nonlinear models (Fairlie 2005; Bauer and Sinning 2008), to quantile regression (Machado and Mata 2005), to selection models (Neuman

and Oaxaca 2003), and to other topics (for an overview, see Fortin, Lemieux, and Firpo [2011]). In large parts of the applied literature, these kinds of decompositions are known as Oaxaca–Blinder decompositions, after two of the three scholars who pioneered these approaches (Oaxaca 1973; Blinder 1973). We refer to this way of decomposing group mean differences as the Kitagawa–Oaxaca–Blinder (KOB) approach to reference the earliest and often overlooked contribution to this literature as well (Kitagawa 1955).

As researchers became increasingly interested in research questions involving developments over time, further extensions were developed to decompose the changes in mean group differences between two points in time (Smith and Welch 1989; Wellington 1993; Makepeace et al. 1999; DeLeire 2000; Kim 2010). These decomposition techniques are based on principles similar to those in the original KOB decomposition and have been primarily used in repeated cross-sectional studies on income gaps. None of those approaches mentioned above have been coherently implemented in Stata. We therefore propose the `xtoaxaca` command. It implements extensions of the original KOB decomposition that focus on the decomposition of changes between groups across time and makes decompositions available for pooled cross-section and panel data.

The command enables a user-friendly implementation of five existing decomposition methods for change (Smith and Welch 1989; Wellington 1993; Makepeace et al. 1999; DeLeire 2000; Kim 2010) and retains the possibility of applying it to panel data instead of only repeated cross-sectional regression models. It provides a generalization of the existing `oaxaca` command (Jann 2008) to longitudinal and panel data. It also includes a modified version of an existing decomposition approach (Wellington 1993), which is suitable for easy interpretation of the results under an interventionist perspective. This approach is aimed at before–after comparisons such as settings of interventions, policy changes, or (natural) experiments with a posttreatment follow-up. We believe that, in many instances, this perspective is applicable to numerous applied research settings in social sciences and has a more natural interpretation than the existing five decompositions of group differences in change over time.

The purpose of this article is to introduce both the `xtoaxaca` command for Stata and the new decomposition approach for change. In the next section, we introduce the concepts of longitudinal decompositions of levels and change of mean group differences over time. Both decompositions are implemented in the `xtoaxaca` command. We then elaborate on cross-sectional decomposition of levels—recapitulating the original KOB decomposition—and apply these principles to the decomposition of levels over time using longitudinal and panel data. The fourth section presents a discussion of different approaches to the decomposition of change. Thereafter, in the fifth section, we argue that an interventionist perspective on the decomposition of change has a crucial advantage in terms of interpretation for applied research. We then describe in the sixth section the syntax and options of the new `xtoaxaca` command. We present three empirical examples in the seventh section. The eighth section summarizes the main limitations of the `xtoaxaca` command. The last section concludes and provides an outlook for future research.

## 2 Decomposition of levels and change

Generally speaking, there are two ways in which we can use mean decomposition techniques with longitudinal data.

The first way of exploiting longitudinal data examines the contribution of past changes or events to levels of outcome differences between two groups  $A$  and  $B$  at a single time  $t_u$ . The second way is the decomposition of group differences in change in an outcome between two groups  $A$  and  $B$  between times  $s$  and  $t$ . We address both approaches in turn but highlight the importance of the latter.

Using longitudinal data to determine the contribution of past changes, we ask a typical research question:

1. How much of the difference in an outcome  $Y$  between groups  $A$  and  $B$  at time  $t$  is due to the differences in the incidence of a past event  $X$ , its different effects, or its different cumulative effects over the last  $n$  years?

This type of question utilizes longitudinal data by accounting for individuals' past experiences. For instance, we may ask to what extent differences in past unemployment spells and their cumulative impact affect the gender wage gap at time  $t$ . In figure 1, this would translate into the decomposition of the outcome difference at  $t_u$ ,  $\Delta Y_t$ . It would account for the different incidences and effects of events at  $r$  and  $u$  to explain the level difference  $\Delta Y_t$ . Analytically, this type of question is still cross-sectional in nature, and we can examine it using the traditional KOB decomposition.<sup>1</sup>

In the following, we denote the repeated decomposition of group differences over time as the longitudinal decomposition of levels over time. Decomposing levels is a distinct approach from the second way of using longitudinal data for decomposition, the decomposition of change.

---

1. Possibly accounting for panel attrition (Oaxaca and Choe 2016).

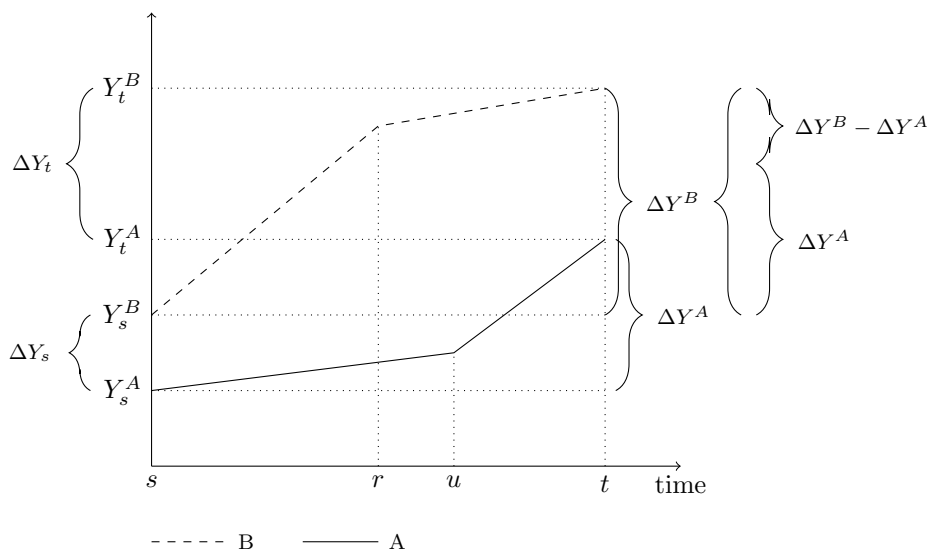


Figure 1. Decomposition of changes over time

If we seek to decompose the change in mean group differences over time, we compare the mean group differences between groups *A* and *B* between two points in time, *s* and *t*, and ask what factors narrowed or widened the outcome difference over time. For example, the wage gap between men and women may have decreased over the last 10 years. A researcher might ask whether this occurred because of compositional changes (differential changes in endowments of the groups) or changes in the contribution of coefficients of the two groups. Thus, the second kind of longitudinal question can be expressed as follows:

2. How much of the change in differences in an outcome *Y* between groups *A* and *B* and between times *t* and *s* is due to changes in the groups' composition or the effects of the explanatory variables?

In figure 1, this amounts to decomposing the change in group differences between time *s* and *t*,  $\Delta Y^B - \Delta Y^A$ .

As has been shown (Smith and Welch 1989; Wellington 1993; Makepeace et al. 1999; DeLeire 2000; Kim 2010), this type of question can be answered with repeated cross-sectional data. In this article, we demonstrate that these existing approaches to the decomposition of change can be easily generalized to the use of panel regression models as well. We argue, however, that the existing approaches are not always easy to interpret when asking a set of research questions that falls under what we call the interventionist perspective. Therefore, we argue that they can be usefully complemented by a new approach to the decomposition of change, which we lay out in detail in section 5.

### 3 Decomposition of levels

#### 3.1 The KOB decomposition for cross-sectional data

Before we review the existing approaches to the decomposition of change, it is useful to recapitulate what the original KOB decomposition does using cross-sectional data (Kitagawa 1955; Oaxaca 1973; Blinder 1973). We will also introduce the notation we will use throughout the article. We start with a basic linear regression model for an outcome  $Y$  and two groups  $A$  and  $B$ :

$$\mathbf{Y}_t^l = \mathbf{X}_t^l \boldsymbol{\beta}_t^l + \boldsymbol{\epsilon}_t^l, \quad E(\boldsymbol{\epsilon}^l) = 0, \quad \text{cov}(\mathbf{X}, \boldsymbol{\epsilon}) = 0 \quad l \in [A, B]$$

$\mathbf{X}$  represents the matrix of covariates, including the unity vector, while  $\boldsymbol{\beta}$  contains the  $k - 1$  coefficients and the constant,  $t$  denotes the time, and  $\boldsymbol{\epsilon}$  is the error term. The basic KOB decomposition applies to data with one time point and divides the mean outcome difference between the two groups into a part that is explained by differences in the groups' characteristics and an unexplained part (twofold approach). Given that the decompositions of change we introduce in section 4 are often based on the assumption that there are group differences in the coefficients (threefold decomposition), we illustrate our notation on this approach. Given the outcome difference

$$\begin{aligned} \Delta Y_t &= E(\mathbf{Y}_t^A) - E(\mathbf{Y}_t^B) \\ &= E(\mathbf{X}_t^A) \boldsymbol{\beta}_t^A - E(\mathbf{X}_t^B) \boldsymbol{\beta}_t^B \end{aligned}$$

and given that  $E(\mathbf{X}_t^l \boldsymbol{\beta}_t^l + \boldsymbol{\epsilon}_t^l) = E(\mathbf{X}_t^l \boldsymbol{\beta}_t^l)$ , the outcome difference can be decomposed into

$$\begin{aligned} \Delta Y_t &= E_t + C_t + I_t \\ E_t &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} \boldsymbol{\beta}_t^B \\ C_t &= E(\mathbf{X}_t^B) (\boldsymbol{\beta}_t^A - \boldsymbol{\beta}_t^B) \\ I_t &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} (\boldsymbol{\beta}_t^A - \boldsymbol{\beta}_t^B) \end{aligned} \tag{1}$$

$E_t$  is defined as the part of the difference that is due to differences in the groups' characteristics at time  $t$  (endowments effect).  $C_t$  is the part of the difference that is due to differences in the coefficients at time  $t$ .  $I_t$ , finally, is the part of the difference at time  $t$  that is due to the interaction of the groups' different characteristics and coefficients.

The presented decomposition in (1) is a threefold decomposition from the viewpoint of group  $B$ , meaning that  $E_t$  is weighted by  $B$ 's coefficients and that  $C_t$  is weighted by  $B$ 's characteristics. While this suffices to represent the basic principle of the KOB decomposition, other decompositions such as a twofold decomposition or a decomposition from the viewpoint of group  $A$  are possible (compare with Jann [2008] and Fortin, Lemieux, and Firpo [2011]).

From the viewpoint of group  $A$ , we would weight the differences in characteristics and coefficients with the characteristics and coefficients of group  $A$  instead of group  $B$ :

$$\begin{aligned}\Delta Y_t &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} \beta_t^A + E(\mathbf{X}_t^A) (\beta_t^A - \beta_t^B) \\ &\quad + \{E(\mathbf{X}_t^B) - E(\mathbf{X}_t^A)\} (\beta_t^B - \beta_t^A)\end{aligned}$$

For the twofold decomposition, which is the one originally devised by Oaxaca and Blinder, the outcome difference (with group  $A$  as the reference) is decomposed by

$$\Delta Y_t = \{E(\mathbf{X}_t^B) - E(\mathbf{X}_t^A)\} \beta_t^A + E(\mathbf{X}_t^B) (\beta_t^B - \beta_t^A)$$

This is also implemented in `xtoaxaca` but not discussed in detail in this article. Normalization of explanatory variables (Yun 2005) has also been implemented as explained in the next section, 3.2.

### 3.2 Normalization of categorical variables

As has been noted in the literature (Jann 2008; Kim 2010; Yun 2005), there is an identification problem when categorical variables are used for decomposition. A widely used solution is to normalize the coefficients of categorical variables by subtracting the variable-specific mean of the coefficient from each category of the variable-specific coefficients and adding all subtracted means to the intercept for decomposition purposes. This yields a new set of coefficients for the decomposition defined as

$$\tilde{\beta}_{t,j}^l = \beta_{t,j}^l - \frac{\sum_{i=1}^c \beta_{t,j,i}^l}{c}$$

In this notation,  $j$  indicates the  $j$ th categorical variable and  $c$  the  $c$ th category within the  $j$ th categorical variable, with  $\beta_{t,j,1}$  constrained to zero for identification in the original model. The time-specific intercept is then defined as

$$\tilde{\beta}_{t,0}^l = \beta_{t,0}^l + \sum_{i=1}^j \frac{\sum_{h=1}^c \beta_{t,i,h}^l}{c} = \beta_{t,0}^l + \bar{\beta}_t^l \quad (2)$$

As can be seen, the basic principle of the original KOB decomposition is to get counterfactual estimates for the outcome, for example, group  $B$ , assuming it had the same endowments or coefficients as group  $A$ . This reasoning is retained in the decomposition of levels and change over time as well.

### 3.3 Longitudinal data using nonpanel regression models

The use of the KOB decomposition with nonpanel regression models is unproblematic with longitudinal data as long as time is measured discretely. Discretely in this context means that observations are categorized together to have been observed at the same undifferentiated time point, for example, in waves of a cohort of panel study. In this

case, the analysis is identical to a repeated cross-sectional approach. However, the assumption can also be that the time variable is (quasi)continuous. In this case, we have to define a bandwidth ( $b$ ) around the time points to allow for an estimation of the endowments part. The wider the bandwidth is, the more reliable the estimate is because more data points fall into the bandwidth around the time point (and the smaller the standard errors become). Broadening the bandwidth comes at the price of losing sensitivity to time-dependent changes in the endowment. For nonparametric modeling, this is similar to the tradeoff that has to be made between bias and variance (Härdle et al. 2004, 28). We estimate

$$E(\mathbf{X}_{t-b;t+b}^l) \quad (3)$$

The subscript  $t - b$  refers to the lower bound and  $t + b$  refers to the upper bound of the interval on the time variable that is used to estimate the mean of the variables. In the case of the continuous-time variable, the decomposition components become a function of the chosen bandwidth  $b$ .

This raises the question of whether the coefficients, like the endowment, can also become dependent on the chosen bandwidth. This has to be decided in line with the choice of the functional form of the time variable in the regression models used for the decomposition. Even when a bandwidth of some kind is used for the endowments, a parametric form can be chosen for the coefficients over time. However, the time variable can be constructed to reflect the bandwidth around prespecified points in time. In such a case, the coefficients can be estimated nonparametrically for each of these time intervals separately, and the decomposition can be done for each of these time intervals. Under these circumstances, coefficients and endowments would be treated analogously.

For simplicity's sake, we leave out the bandwidth in the index for the remainder of the article, but note that it is theoretically necessary and practically possible to set the bandwidth in cases in which time is assumed to be continuous.

### 3.4 Longitudinal data using panel regression models

Using panel data, we can also estimate  $\beta$  from a panel regression model. Because panel regressions model time-constant individual error terms, a decomposition using panel regression models must account for empirical group differences in these time-constant, unobserved variables. Thereby, the time-constant individual error terms  $\mathbf{u}^l$  become part of the decomposition of group-level differences.

$$\mathbf{Y}_t^l = \mathbf{X}_t^l \beta_t^l + \mathbf{u}^l + \epsilon_t^l, \quad E(\epsilon_t^l) = 0, \quad \text{cov}(\mathbf{X}_t, \epsilon_t) = 0 \quad l \in [A, B]$$



Accounting for the time-constant error terms adds the differences in the expectation of  $\mathbf{u}^l$  as a fourth component  $U$  to the decomposition. This component is not time dependent. It only comprises differences between groups in the time-constant error terms.<sup>2</sup>

$$\begin{aligned}\Delta Y_t &= E_t + C_t + I_t + U \\ E_t &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} \beta_t^B \\ C_t &= E(\mathbf{X}_t^B)(\beta_t^A - \beta_t^B) \\ I_t &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} (\beta_t^A - \beta_t^B) \\ U &= \{E(\mathbf{u}^A) - E(\mathbf{u}^B)\}\end{aligned}$$

Accordingly, a decomposition using panel regression models attributes parts of the differences between groups to unobserved factors that do not change within the period of observation.

### 3.5 Model assumptions

#### Selection and causal identification

Note that any results produced by decompositions of levels or change rely on the assumptions made in the original regression models. This pertains especially to the causal interpretation of the results. Following a counterfactual interpretation of causality in the social sciences (Morgan and Winship 2015), the estimators for the explanatory variables would need to be unbiased. Only then could the results of any of the decomposition approaches presented here (including the original KOB decomposition) be given interpretations like “how much would the gap between group  $A$  and group  $B$  be reduced if group  $A$  had the same endowments as group  $B$ ?” Panel regression models offer some advantages when it comes to arguing that the assumptions for causal interpretation are fulfilled but still rest on certain assumptions that are often not realistic in applied research (Firebaugh, Warner, and Massoglia 2013).

For example, from a more technical perspective, note that in a standard random-effects model (which includes the grouping variable  $\mathbf{l}$  as a covariate), the assumption is that  $\text{cov}(\mathbf{l}, \mathbf{u}) = 0$ . This implies that  $E\{E(\mathbf{u}^A) - E(\mathbf{u}^B)\} = 0$ . If our model conforms to this assumption, we should therefore expect that the time-constant error terms cannot contribute to the explanation of differences in group levels of the outcome. If we see strong deviations in an empirical estimation of  $E\{E(\mathbf{u}^A) - E(\mathbf{u}^B)\} = 0$ , this might indicate a misspecified model leading to biased coefficient estimates, which in turn could lead to a biased decomposition.

---

2. Note that in applied research, an unbalanced panel might mean that taking the expectation of the time-constant error term at different time points  $t$  can lead to time-varying results if the dropout is related to the time-constant error terms.

### Panel dropout

With longitudinal data, panel dropout is a serious issue that may affect the estimation of coefficients (Oaxaca and Choe 2016) as well as the estimation of the time-specific endowment component. If the results are to be interpreted causally, endogeneity problems resulting from panel dropout have to be solved when designing the panel regression models before the decomposition is applied.

It is possible for dropout rates to differ between the groups under study. This can also affect the estimation of the endowments in (3). To avoid biased endowment estimators, one must construct weights that account for the effects of differential panel dropout and applied them in the estimation of the endowments. How these are constructed is beyond the scope of this article; however, we recommend standard procedures from the literature on survey research (Kalton and Flores-Cervantes 2003; Deming and Stephan 1940; Kim and Kim 2007), which can be implemented in Stata using `survwgt` (Winter 2002).

### Functional form of the time variable

In the modeling process, one can either specify time nonparametrically, estimating an interaction of each time point with the group variable and each decomposition variable, or assume a certain functional form like linear growth. The decomposition will rely on these assumptions made in the modeling process. If the functional form is chosen incorrectly, this will also affect the decomposition, and the results will consequently be biased. This is important not only for the overall growth of the dependent variable but also for the change in the effect of the decomposition variables over time. A nonparametric approach is less statistically efficient but has much weaker assumptions than any parametric function and might therefore be preferred if researchers are uncertain in this regard.

## 3.6 Decomposition in a multilevel framework

Because all panel models can be understood as a special case of multilevel models (with time points nested within units), we believe that `xtoaxaca` can also be used to decompose levels and differences<sup>3</sup> between clusters or higher-level units. Thus, the time variable needs to reflect the cluster variable (for example, countries) and should be used in categorical interaction with the desired variables in the model. Differences between units in a multilevel framework and differences between time points can both be seen as a form of difference-in-differences decomposition.

The interpretation of the decomposition of levels over different clusters does not deviate from repeated cross-sectional KOB decompositions, and so using `xtoaxaca` would not contribute much extra benefit over using `oaxaca`. However, decomposition of difference in differences between groups over clusters or higher-level units might be of

---

3. The multilevel analogue to change.

interest (Blau and Kahn 1992). The interpretation then refers to whether differences between clusters or higher-level units in the group differences in the outcome can be attributed to cluster differences in the group differences in endowments, coefficients, or their interactions.

## 4 Decomposition of change

Regardless of whether we have repeated cross-sectional or panel data, given two groups  $A$  and  $B$  for which we have data for at least two points in time,  $t$  and  $s$  with  $t > s$ , the change in the outcome difference between the two groups and between the two points in time is given by

$$\Delta Y = \Delta Y_t - \Delta Y_s \quad (4)$$

Alternatively, changes in outcome differences between two groups and two points in time can be expressed as the difference of group differences over time:

$$\begin{aligned} \Delta Y &= \Delta Y_t - \Delta Y_s \\ &= (E(\mathbf{Y}_t^A) - E(\mathbf{Y}_t^B)) - (E(\mathbf{Y}_s^A) - E(\mathbf{Y}_s^B)) \\ &= E(\mathbf{Y}_t^A) - E(\mathbf{Y}_t^B) - E(\mathbf{Y}_s^A) + E(\mathbf{Y}_s^B) \\ &= E(\mathbf{Y}_t^A) - E(\mathbf{Y}_s^A) - E(\mathbf{Y}_t^B) + E(\mathbf{Y}_s^B) \\ &= E(\mathbf{Y}_t^A) - E(\mathbf{Y}_s^A) - (E(\mathbf{Y}_t^B) - E(\mathbf{Y}_s^B)) \\ &= \Delta Y^A - \Delta Y^B \end{aligned}$$

Essentially, changes over time can therefore be expressed as the difference between two KOB decompositions at different time points.

Several approaches for the decomposition of change in group differences over time exist. We cover the five most prominent examples.<sup>4</sup> These decompositions of change have been applied to both points using repeated cross-sectional data. The generalization of the decomposition of levels to continuous time and panel data introduced in the previous section applies in the same way to the decomposition of change as it does to the decomposition of levels.

### 4.1 Simple subtraction method (SSM)

The simplest decomposition of change is a simple subtraction of the decomposition components of the original KOB decomposition at time  $s$  from the components at time  $t$  and is defined in our notation as SSM:

---

4. There are  $2 \times 2 \times 2 = 8$  (endowments and coefficients per 2 groups per 2 time points) estimates that make up (4). Decompositions can take any number of possible differences and products as part of the decomposition and combine them to form a new kind of decomposition.

$$\begin{aligned}
\Delta Y &= \Delta Y_t - \Delta Y_s \\
&= E_t + C_t + I_t - (E_s + C_s + I_s) \\
&= (E_t - E_s) + (C_t - C_s) + (I_t - I_s) \\
&= \underbrace{\{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\}}_{E_t} \beta_t^B - \underbrace{\{E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B)\}}_{E_s} \beta_s^B \\
&\quad + \underbrace{E(\mathbf{X}_t^B)}_{C_t} (\beta_t^A - \beta_t^B) - \underbrace{E(\mathbf{X}_s^B)}_{C_s} (\beta_s^A - \beta_s^B) \\
&\quad + \underbrace{\{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\}}_{I_t} (\beta_t^A - \beta_t^B) - \underbrace{\{E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B)\}}_{I_s} (\beta_s^A - \beta_s^B)
\end{aligned}$$

This method is straightforward to calculate, applied for example in DeLeire (2000). The endowment part can be interpreted as the part in the change in the gap that is due to changes in the endowments given changes in the evaluation (coefficient) in the reference group over time. The coefficient part is the part in the change in the gap that is due to changes in the coefficients given changes in the evaluation (endowment) in the reference group over time. The interaction part is the difference in the interactions of group differences in coefficients and endowments. Similarly to the original KOB, this component is difficult to interpret and might often be treated as the substantively unexplained part.

The approach has also attracted criticism because it does not estimate the unique contribution of coefficient changes and changes in the variable distributions over time (Kim 2010). As Kim (2010) shows, the coefficient differences at each time point are weighted by the mean distribution of the endowments at their respective time and, because the endowments likely change over time, the coefficient effect captures these changes. Similarly, the endowment effect contains interactions between the coefficient and endowment changes. This kind of criticism applies differently for all the decompositions presented here, except for the one by Kim (2010).

## 4.2 Smith and Welch (1989)

Smith and Welch (1989) propose a fourfold decomposition of change that is defined in our notation as SW (Smith and Welch 1989, 529):

$$\begin{aligned}
\text{i} &= [\{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} - \{E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B)\}] \beta_s^B \\
\text{ii} &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} (\beta_s^A - \beta_s^B) \\
\text{iii} &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B)\} (\beta_t^B - \beta_s^B) \\
\text{iv} &= E(\mathbf{X}_t^A) \{(\beta_t^A - \beta_t^B) - (\beta_s^A - \beta_s^B)\}
\end{aligned}$$

The components can be given the following interpretation:<sup>5</sup>

- i. *Main effect*: The component estimates the predicted change in the outcome between the two groups that can be attributed to the two groups are changing in the endowments (valued at base time  $s$ ) between time  $t$  and  $s$ .
- ii. *Group interaction*: The second component describes the part of change in the endowment of the group that is valued differently at time  $s$ . Therefore, a secular rise in endowments gives a higher benefit to the group with the higher return to this endowment at time  $s$ .
- iii. *Time interaction*: This component takes the endowment differences at the second time point and attributes change to the change in the coefficient of group  $B$ . This would mean that higher returns to an endowment benefit the group with higher endowments at time point  $t$ .
- iv. *Group-time interaction*: The last component attributes change to a change in the differences in the coefficients (returns to endowments) given the initial level of group  $A$ . If group  $A$  were the disadvantaged group, reduction in the differences to the return to their endowments would close the overall gap between the groups.

### 4.3 Wellington (1993)

Wellington (1993) proposes a simple twofold decomposition of change in differences in labor market returns. Her decomposition is defined in our notation as WL (Wellington 1993, 393):

$$\begin{aligned} \text{WL1} &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} \beta_t^A - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} \beta_t^B \\ \text{WL2} &= E(\mathbf{X}_s^A) (\beta_t^A - \beta_s^A) - E(\mathbf{X}_s^B) (\beta_t^B - \beta_s^B) \end{aligned}$$

Wellington (1993, 393–394) gives the following description of the two components:

- WL1. The portion of the change in the gap that can be accounted for by changes in the means if the returns to the independent variables were constant at  $t$  (not at baseline  $s$ ).
- WL2. The portion of the change in the gap that can be explained by changes in the coefficients (including the constant term) over the period, evaluated at the groups' baseline ( $s$ ) means.

This approach is the one that is closest to our own addition (see next subsection) to the set of possible decomposition approaches, but there is a slight but significant difference, as we discuss in section 5.

---

5. See Smith and Welch (1989, 529–530) for their interpretation of the change in the race gap in wages.

#### 4.4 A threefold extension of WL (interventionist)

There is another useful way in which the change in gaps can be decomposed. This is an extension of the WL decomposition, which takes the form of a threefold decomposition.

$$\Delta Y^A - \Delta Y^B = \Delta Y = \underbrace{\Delta E + \Delta I}_{\text{WL1}} + \underbrace{\Delta C}_{\text{WL2}}$$

The three components are named analogously to the original KOB decomposition. To obtain the endowments effect, we allow only the groups' composition to vary over time and hold the coefficients constant at their initial group-specific levels at time  $s$ .

$$\begin{aligned} \Delta E &= E(\mathbf{X}_t^A)\beta_s^A - E(\mathbf{X}_s^A)\beta_s^A - E(\mathbf{X}_t^B)\beta_s^B + E(\mathbf{X}_s^B)\beta_s^B \\ &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\}\beta_s^A - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\}\beta_s^B \end{aligned} \quad (5)$$

As can be seen in (5), we obtain the endowments component by subtracting the groups' compositional changes over time weighted by their initial coefficients. The endowments component then answers the following question: Given the initial differences in coefficients, how much does the gap between groups change because of the changes in the endowments between both points (if the coefficients do not change)?

Similar to the endowments effect, the component attributable to a change in coefficients is obtained by fixing the groups' endowments so that  $E(\mathbf{X}_t^l) = E(\mathbf{X}_s^l)$ ,

$$\begin{aligned} \Delta C &= E(\mathbf{X}_s^A)\beta_t^A - E(\mathbf{X}_s^A)\beta_s^A - E(\mathbf{X}_s^B)\beta_t^B + E(\mathbf{X}_s^B)\beta_s^B \\ &= E(\mathbf{X}_s^A)(\beta_t^A - \beta_s^A) - E(\mathbf{X}_s^B)(\beta_t^B - \beta_s^B) \end{aligned}$$

which denotes the change of the difference due to a change in coefficients (including the constant) over time between the groups given the groups' initial differences in endowments at  $s$ . The coefficient component answers this question: Given the initial differences in endowments, how much does the gap between groups change because of changes in the coefficients (if the endowments do not change)?

The interaction between the change in endowments and coefficients is the last component of the decomposition:

$$\Delta I = \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\}(\beta_t^A - \beta_s^A) - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\}(\beta_t^B - \beta_s^B)$$

As with the original KOB decomposition, it is difficult to give this component a straightforward interpretation on its own. Additionally, note that the subcomponent of  $\Delta C$  that is attributable to a change in the intercept is usually also a kind of residual, unexplained by the (change in)  $\mathbf{X}$  variables in the model.

We can show that our suggested approach is a direct extension of Wellington (1993). First, our  $\Delta C$  component is exactly the same as WL2 of the Wellington decomposition.

$$\begin{aligned}\text{WL2} &= E(\mathbf{X}_s^A) (\beta_t^A - \beta_s^A) - E(\mathbf{X}_s^B) (\beta_t^B - \beta_s^B) \\ &= \Delta C\end{aligned}$$

In addition, if we add up the endowment and interaction term of the interventionist decomposition, we get the first part of the Wellington decomposition.

$$\begin{aligned}\Delta E + \Delta I &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} \beta_s^A - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} \beta_s^B + \\ &\quad \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} (\beta_t^A - \beta_s^A) - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} (\beta_t^B - \beta_s^B) \\ &= \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} \beta_t^A - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} \beta_t^B \\ &= \text{WL1}\end{aligned}$$

## 4.5 Makepeace et al. (1999)

Another well-known approach aims at partially mirroring the twofold cross-sectional KOB decomposition into explained and unexplained in the decomposition of change. The authors further divide the explained and unexplained components into a component related to change in endowments (pure) and one aspect related to change in coefficients (price) (Makepeace et al. 1999, 539). Their decomposition is defined in our notation as MPJD:

$$\begin{aligned}\Delta E_{\text{pure}} &= \left[ \{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} \right] \beta_t^A \\ \Delta E_{\text{price}} &= \{E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B)\} (\beta_t^A - \beta_s^A) \\ \Delta U_{\text{pure}} &= E(\mathbf{X}_t^B) \left\{ (\beta_t^A - \beta_s^A) - (\beta_t^B - \beta_s^B) \right\} \\ \Delta U_{\text{price}} &= \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\} (\beta_s^A - \beta_s^B)\end{aligned}$$

## 4.6 Kim (2010)

While many decomposition approaches were developed with particular research questions in mind, Kim (2010) develops the most analytical approach. It yields five components, of which two can be attributed purely to the change in endowments and coefficients. He argues that all methods discussed so far in this article confuse or at least conflate the pure change in endowment and the pure change in the coefficients with interactions of such changes with initial (or current) level differences in coefficients and endowments. We agree with the analysis but argue in section 5 that for the sake of interpretability, this might be a desirable property of a decomposition.

To better understand how the KIM decomposition (Kim 2010, 629) decomposes group differences over time, we need to take the intercepts apart from the rest of the coefficients in this subsection. So far, decompositions have used the standard matrix notation of

multiple regression. This means that the time-specific intercepts  $\beta_{t,0}^l$  are part of the coefficient vectors,

$$\beta_t^l = (\beta_{t,0}^l, \beta_{t,1}^l, \dots, \beta_{t,d}^l)' = \begin{pmatrix} \beta_{t,0}^l \\ \tilde{\beta}_t^{*l} \end{pmatrix}$$

with  $d$  being the number of decomposition variables used in the model. Accordingly, the means matrices so far have contained a unity vector that is multiplied with the intercepts:

$$E(\mathbf{X}_t^l) = \{1, E(\mathbf{X}_{t,1}^l), \dots, E(\mathbf{X}_{t,d}^l)\} = \{1, E(\mathbf{X}_t^{*l})\}$$

Kim (2010) uses a different approach by explicitly distinguishing between intercepts and covariates. Therefore, the notation for the coefficients uses  $\tilde{\beta}_t^{*l}$  to represent the coefficient vector without the intercept and  $E(\mathbf{X}_t^{*l})$  as the means vector without the 1. Additionally, the decomposition uses the normalization of categorical variables proposed by Yun (2005) (see section 3.2). After normalization, the vector of coefficients may contain normalized categorical and nonnormalized continuous variables. We distinguish categorical variables by counting them first in the set of coefficients until the total number of categorical variables  $j$  is reached. Continuous variables start with an index of  $j + 1$  until they reach  $d$ , which is the total number of decomposition variables. Such an explicit distinction between continuous and categorical variables is not necessary for other sections of the article. The normalized (demeaned) vector of coefficients thus contains

$$\tilde{\beta}_t^l = \left( \tilde{\beta}_{t,0}^l, \tilde{\beta}_{t,1,c}^l, \dots, \tilde{\beta}_{t,j,c}^l, \beta_{t,j+1}^l, \dots, \beta_{t,d}^l \right)'$$

with  $j$  being the number of categorical variables,  $c$  indexing the categories of each categorical variable,  $\tilde{\beta}_{t,0}^l$  being the intercept for the demeaned coefficients [see (2)],  $\tilde{\beta}_{t,1,c}^l, \dots, \tilde{\beta}_{t,j,c}^l$  being the demeaned coefficients of the categorical variables, and  $\beta_{t,j+1}^l, \dots, \beta_{t,d}^l$  being the regular coefficients of the continuous variables. If we now take the normalized vector of coefficients and leave out the intercept  $\tilde{\beta}_{t,0}^l$ , we get  $\tilde{\beta}_t^{*l}$ .

With the definition of normalization in section 3.2 and  $\tilde{\beta}_t^{*l}$  being the normalized coefficient vector without the intercept, it follows that

$$\begin{aligned} E(\mathbf{X}_t^{*l})\tilde{\beta}_t^{*l} + \beta_{t,0}^l + \bar{\beta}_t^l &= \{1, E(\mathbf{X}_t^{*l})\} \left( \beta_{t,0}^l, \tilde{\beta}_t^{*l} + \bar{\beta}_t^l \right) \\ &= E(\mathbf{X}_t^l)\beta_t^l \end{aligned}$$



Taking these definitions into account, we define the five-part KIM decomposition in our notation as follows:

$$\begin{aligned}
D1 &= \{(\beta_{t,0}^A - \beta_{s,0}^A) - (\beta_{t,0}^B - \beta_{s,0}^B)\} + \left\{ \left( \bar{\beta}_t^A - \bar{\beta}_s^A \right) - \left( \bar{\beta}_t^B - \bar{\beta}_s^B \right) \right\} \\
D2 &= \left( \frac{E(\mathbf{X}_t^{*A}) + E(\mathbf{X}_s^{*A}) + E(\mathbf{X}_t^{*B}) + E(\mathbf{X}_s^{*B})}{4} \right) \left\{ \left( \tilde{\beta}_t^{*A} - \tilde{\beta}_s^{*A} \right) - \left( \tilde{\beta}_t^{*B} - \tilde{\beta}_s^{*B} \right) \right\} \\
D3 &= \left( \frac{\{E(\mathbf{X}_t^{*A}) - E(\mathbf{X}_s^{*A})\} + \{E(\mathbf{X}_t^{*B}) - E(\mathbf{X}_s^{*B})\}}{2} \right) \\
&\quad \left( \frac{\left( \tilde{\beta}_t^{*A} + \tilde{\beta}_s^{*A} \right)}{2} - \frac{\left( \tilde{\beta}_t^{*B} + \tilde{\beta}_s^{*B} \right)}{2} \right) \\
D4 &= \left[ \{E(\mathbf{X}_t^{*A}) - E(\mathbf{X}_s^{*A})\} - \{E(\mathbf{X}_t^{*B}) - E(\mathbf{X}_s^{*B})\} \right] \left( \frac{\tilde{\beta}_t^{*A} + \tilde{\beta}_s^{*A} + \tilde{\beta}_t^{*B} + \tilde{\beta}_s^{*B}}{4} \right) \\
D5 &= \left( \frac{\{E(\mathbf{X}_t^{*A}) + E(\mathbf{X}_s^{*A})\}}{2} - \frac{\{E(\mathbf{X}_t^{*B}) + E(\mathbf{X}_s^{*B})\}}{2} \right) \\
&\quad \left( \frac{\left( \tilde{\beta}_t^{*A} - \tilde{\beta}_s^{*A} \right) + \left( \tilde{\beta}_t^{*B} - \tilde{\beta}_s^{*B} \right)}{2} \right)
\end{aligned}$$

Following Kim (2010), we can give the following descriptions of the five components:

- D1 *Intercept effect*: This is purely the difference in differences between group and overall intercepts.
- D2 *Pure coefficient effect*: This component measures how much the gap between groups changes because of changes in the coefficients if there were no differences in the endowments at all, neither between groups nor over time.
- D3 *Coefficient interaction effect*: This component measures how much the gap between groups changes because of the average change in endowment combined with the difference in the averaged coefficient. It is supposed to capture the aspect of initial level differences in coefficients, which affect the change in the gap in interaction with changes in endowments.
- D4 *Pure endowment effect*: This component is the analog to D2 for endowments. It measures how much the gap between groups changes because of changes in the endowments if there were no differences in the coefficients at all, neither between groups nor over time.
- D5 *Endowment interaction effect*: This component is the analog to D3 but reverses the role of endowment and coefficients. It measures how much the gap between groups changes because of the average change in coefficients combined with the difference in the averaged endowments. It is supposed to capture the aspect of

initial level differences in endowments, which affect the gap in interaction with changes in coefficients.

#### 4.7 Panel models and time-constant error terms

As mentioned in section 3.4, decompositions can also attribute parts of group differences in levels of the outcome to factors that are time constant within the period of observation. The same can be done for change over time ( $\Delta U$ ). However, this makes sense only if we have an unbalanced panel. If the panel is balanced, the expectations of the time-constant error terms cannot change over time and cannot contribute anything to the decomposition of change between groups.

If we see substantial contributions of the time-constant error terms, the data suffer from group-specific panel attrition, which contributes to a change in the group differences in the outcome. In this case, we can add a component  $\Delta U$  to all the previous five decompositions as well as to the interventionist decomposition method introduced in the next section.

$$\Delta U = \{E(\mathbf{u}_t^A) - E(\mathbf{u}_s^A)\} - \{E(\mathbf{u}_t^B) - E(\mathbf{u}_s^B)\}$$

#### 4.8 The relationship between the different types of decompositions

Because all decomposition approaches decompose the same differences in change over time, each decomposition can be expanded and transformed into any other of the existing decompositions. Nevertheless, there are some direct relationships that are worth mentioning and that are also depicted in figure 2.<sup>6</sup>

The six decomposition methods can be divided into a heuristic based on a combination of two characteristics. The first one is simply the number of components used. Here we see between two and five components. The second characteristic divides the methods into those that conduct decompositions groupwise across time and those that conduct decompositions timewise across groups. Timewise across groups means that the differences between groups at one time point are subtracted from the differences between groups from another time point. In contrast, groupwise across time means that the differences between time points within one group are subtracted from the differences between time points within the other group.

WL, KIM, and the interventionist approach fall clearly into the groupwise-across-time category, while SSM is a timewise-across-group approach. For MPJD and SW, not all components of their decomposition follow this logic. For MPJD, the pure components are groupwise across time, and for SW the components i and iv are timewise across group.

---

6. We are very grateful to an anonymous reviewer for pointing out some of these relationships.

In section 4.4, we have already shown that WL can be further divided to yield a threefold decomposition that we label interventionist and that we argue has a certain desirable property in contrast with all other approaches, which we elaborate on in section 5.

The simple subtraction method does timewise subtractions across groups of the components of endowments, coefficients, and interactions. If we were to exchange time points with groups, we would end up with a groupwise subtraction across time. This is exactly what is done in the interventionist perspective. So if we were to substitute  $t = A$  and  $s = A$ , the equations would be

$$\begin{aligned}\Delta E &= E_A - E_B = \{E(\mathbf{X}_A^t) - E(\mathbf{X}_A^s)\} \beta_A^s - \{E(\mathbf{X}_B^t) - E(\mathbf{X}_B^s)\} \beta_A^s \\ \Delta C &= C_A - C_B = E(\mathbf{X}_A^s) (\beta_A^t - \beta_A^s) - E(\mathbf{X}_B^s) (\beta_B^t - \beta_B^s) \\ \Delta I &= I_A - I_B = \{E(\mathbf{X}_A^t) - E(\mathbf{X}_A^s)\} (\beta_A^t - \beta_A^s) - \{E(\mathbf{X}_B^t) - E(\mathbf{X}_B^s)\} (\beta_B^t - \beta_B^s)\end{aligned}$$

These are the same components as in the interventionist perspective, and after changing groups with time points, the SSM could also be reduced to the twofold decomposition of Wellington (1993).

Note that each decomposition retains its substantively different interpretation even if each can be transformed into a different decomposition. When one interprets the results, the decompositions should therefore not be treated as 1:1 substitutes for each other.

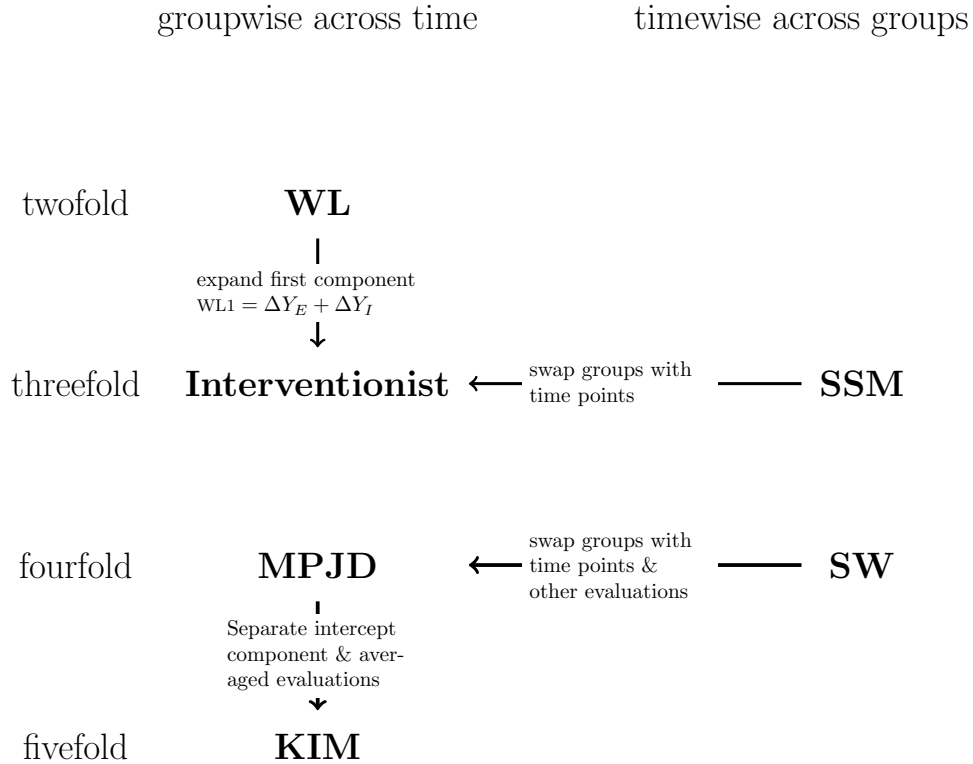


Figure 2. Relationship among decomposition approaches. NOTE: SSM = simple subtraction method (section 4.1), WL = Wellington (1993) (section 4.3), MPJD = Makepeace et al. (1999) (section 4.5), SW = Smith and Welch (1989) (section 4.2), KIM = Kim (2010) (section 4.6)

## 5 An interventionist perspective on the decomposition of change

While all the decomposition approaches that we discussed in the previous section have their uses, we argue that the interventionist approach is best suited to address a certain kind of research question that regularly arises in applied social science research and similar fields like epidemiology or public health (see section 4.4). The premise of this approach is that we take the initial differences in levels between the groups at the reference time point  $s$  as given. We then ask how the difference between the groups could have changed if either the change in endowments or the change in coefficients had been different. This reflects real-world applications in which either an intervention is designed or a (natural) experiment or policy change occurs between  $s$  and  $t$  and is evaluated at time point  $t$ . The initial differences in both coefficients and endowments

at time point  $s$  are seen as inextricably linked to an explanation of change because any change is built on the existing levels at  $s$ . These initial levels are assumed to be beyond intervention and are therefore not subject to counterfactual predictions within the decomposition approach.

There are two combinable types of counterfactual predictions about endowments and coefficients at time  $s$ : 1. across groups and 2. across time (and a combination of both). The former would make statements such as “if group  $A$  had the same endowment as group  $B$  at time point  $s$ ,” while the latter would make statements such as “if group  $A$  already had the coefficients of time  $t$  at time point  $s$ .”

From these two types of counterfactual statements, we can derive two formal requirements for a decomposition approach to conform to the assumptions of our interventionist perspective. First, no component should contain a term that takes group differences at  $s$  (which constitutes a counterfactual prediction at time  $s$  across groups). Instead, only differences of within-group change<sup>7</sup> should be used for the decomposition. Second, changes within groups should be multiplied (valued) only at the initial levels ( $s$ ) or at change ( $t - s$ ) but not at the levels at  $t$  (or any function of the levels, endowments, or coefficients at  $t$ ). If we value at levels of  $t$ , we make a counterfactual prediction at time  $s$  across time.

Except for the interventionist approach, all other decompositions described in section 4 violate these assumptions. They are therefore not applicable under an interventionist perspective.<sup>8</sup> In such a research scenario, it is therefore desirable to use a decomposition that can attribute changes in the gaps to changes in endowments and coefficients given the initial differences in levels in the outcome between groups. We designed the interventionist approach to fill exactly this lack of a decomposition approach to the mean-based decompositions of change in linear models.

Thus, using this decomposition, we seek to answer questions such as how group differences in an outcome would have developed over time had both groups’ characteristics or coefficients changed in the same way. These are counterfactual statements about changes that might have been the result of an intervention, policy change, natural experiment, or any other process or event that occurs between two time points  $s$  and  $t$ . To this end, we need to find a decomposition that does not violate our two interventionist assumptions. This can be achieved by setting the endowments and coefficients at which within-group changes are valued to their groups’ initial values.<sup>9</sup>

---

7. Because we assume that we could manipulate the change through an intervention.

8. For SSM, it is all three components. For SW, it is components i, ii, and iv. For WL, it is component WL1. For MPJD, it is all components. And for KIM, it is components D2–D5 that violate the assumptions.

9. However, by definition and as mentioned above, it does depend on the differences in the initial levels of coefficients and would be regarded as conflating change in endowments with levels of coefficients in Kim’s (2010) analytical perspective.

## 6 The `xtoaxaca` command

The `xtoaxaca` postestimation command provides decomposition techniques after any kind of regression-based growth curve analysis. For the programming of `xtoaxaca`, Stata 14.1 was used. Currently, `xtoaxaca` supports the analysis of stored models fit using `reg`, `xtreg`, or `mixed`.

`xtoaxaca` relies heavily on the use of factor variables. Users are therefore actively encouraged to specify all variables in their regression command explicitly as factor variables (including noninteracted, continuous variables). All variables that are not specified as factor variables are treated as continuous, and the use of dummy variables is not supported.

The exception to this rule is interactions among decomposition variables. These must be created as handmade interaction terms (possibly using dummy variables) and interacted using factor-variable syntax with time and grouping variables. The example in section 7.5 shows how this can be achieved.

The maximum length of variable names for decomposition variables that is supported by `xtoaxaca` is 20.

A longitudinal decomposition using `xtoaxaca` works in two steps:

1. Fit a (growth curve) model. This model should condition on the variables that are used as decomposition variables to explain the gap over time between the groups.
2. `xtoaxaca` takes the model and the dataset as input to decompose the gaps over time using Stata's `margins` command in the background.

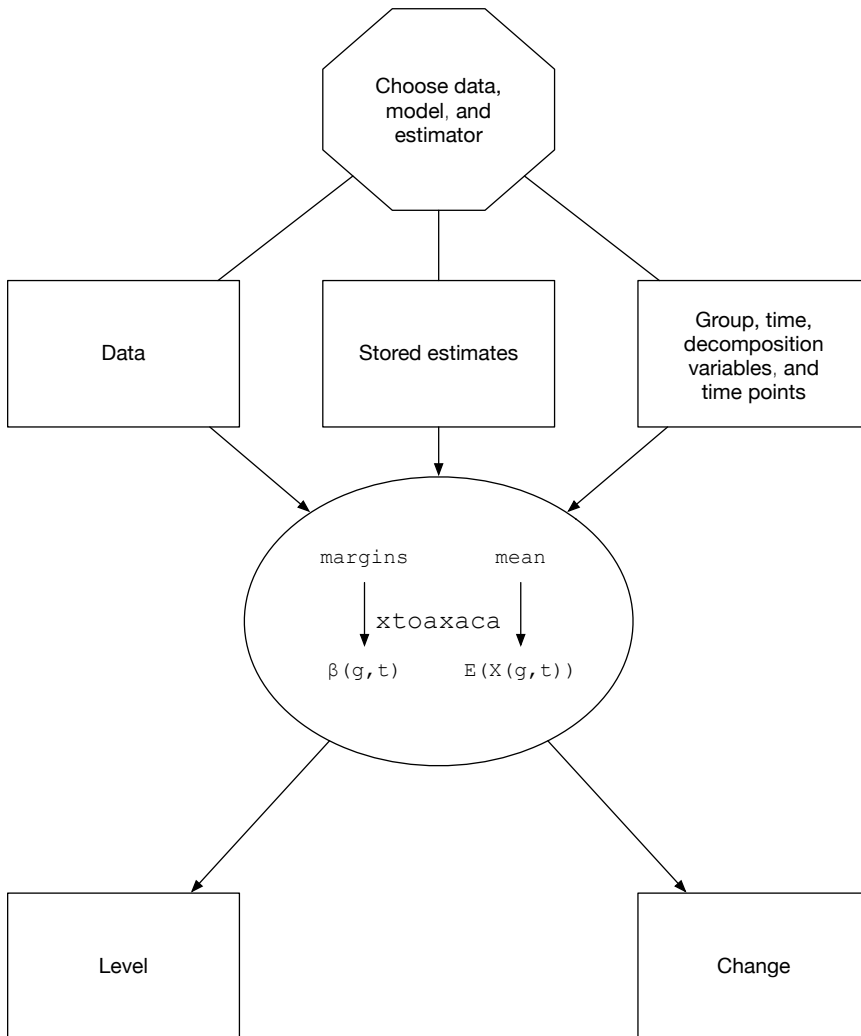


Figure 3. `xtoaxaca` workflow

## 6.1 Syntax

The general syntax is

```
xtoaxaca varlist, groupvar(varname) groupcat(##) timevar(varname)
  times(numlist) model(name) [timeref(#) timebandwidth(#)
  basemodel(name) weights(varname) change(changetype) normalize(varlist)
  noisily detail forcesample twofold(weight|pooled|off)
  resultsdata([pathname]filename[, replace])
  blocks(blockname1 = (varlist1)[, blockname2 = (varlist2)...]) tfweight(#)
  fmt(#) [nolevels | nochange] seed(#) bootstrap(#)]
```

*varlist* contains all decomposition variables and should include all variables interacted with the variable specified in `timevar()`. Otherwise, the decomposition will be incomplete. `estout` (Jann 2004) is required to run `xtoaxaca`.

## 6.2 Options

`groupvar(varname)` specifies the group variable for decomposition. `groupvar()` is required.

`groupcat(##)` identifies the group categories in `groupvar()` between which differences across time will be decomposed. Only two codes are allowed. `groupcat()` is required.

`timevar(varname)` specifies the time variable in the growth curve model. `timevar()` is required.

`times(numlist)` defines the values of `timevar()` at which group differences will be decomposed. `times()` is required.

`model(name)` is the name under which the (growth curve) model is stored. Please ensure that the basic functional form of `timevar()` and possible interactions with the `groupvar()` variable are correctly specified. Please use factor-variable notation for all variables in the model. Therefore, do not use dummy variables to represent categorical variables (both `timevar()` and `groupvar()`, as well as decomposition variables), but use the factor-variable prefix `i.` instead. All variables without a factor-variable prefix are treated as continuous by `xtoaxaca`. `model()` is required.

`timeref(#)` defines the reference time point for which change decomposition will be calculated. This option is required if the option `change()` is specified. It must be one of the time points specified in `times()`.

`timebandwidth(#)` defines the time span around the time points in `times(numlist)` that is used to estimate the time-specific means of the decomposition variables. This option is required only if `timevar()` is specified as a continuous (factor) variable in the models. The default is `timebandwidth(0.1)`.



`basemodel(name)` is the name under which an optional baseline model is stored. This is not necessary for the decomposition. It can be used to ensure that the functional form and interactions of the `timevar()` with the grouping variable are correctly specified. This might be helpful if the code of `model()` contains many complicated interactions that might be prone to errors or typos. Please use factor-variable notation for all variables in the baseline model. Therefore, do not use dummy variables to represent categorical variables (both `timevar()` and `groupvar()` as well as decomposition variables). Instead, use the factor-variable prefix `i..` All variables without a factor-variable prefix are treated as continuous by `xtoaxaca`.

`weights(varname)` specifies the variable containing (longitudinal) weights for the estimation of the endowments (means).

`change(changetype)` specifies the decomposition method of change over time.

`changetype` may be one of the following: `interventionist`, `ssm`, `smithwelch`, `wellington`, `mpjd`, `kim`, `interventionist.twofold`, or `none`. `interventionist` yields the decomposition using the interventionist perspective presented in this article (see section 5). `ssm` gives the simple subtraction method described in section 4.1; `smithwelch` gives the decomposition by Smith and Welch (1989) described in section 4.2; `wellington` gives the decomposition by Wellington (1993) described in section 4.3; `mpjd` gives the Makepeace et al. (1999) decomposition described in section 4.5; `kim` gives the decomposition presented in Kim (2010) described in section 4.6. `interventionist.twofold` gives the decomposition presented in section A.1 in the appendix, which is akin to the original twofold KOB decomposition. The default is `change(none)`, which shows only the decomposition of levels.

`normalize(varlist)` will normalize the categorical variables according to the method by Yun (2005), as described in section 3.2. `varlist` may be any categorical variables from the decomposition `varlist`.

`noisily` yields more output from the in-between estimation steps of, for example, matrices of means and coefficients over time. If you specify `noisily` with `bootstrap()`, you also need to specify `resultsdata()`.

`detail` is the same as `noisily`.

`forcesample` forces the command to accept differences in the current sample in the dataset and the samples used to fit the models. If a `basemodel()` is specified, it also forces `xtoaxaca` to accept differences in sample size between `model()` and `basemodel()`. In normal circumstances, it is not recommended to use this option, and it should be used only if the output is interpreted according to the differences between the samples.

`twofold(weight|pooled|off)` gives the twofold decomposition of the level over time. `weight` allows the user to specify a weight for the coefficients. `pooled` uses the method proposed by Oaxaca and Ransom (1994), which is equivalent to a pooled regression model and accounts for the relative amount of variance in the decomposition variables between the two groups found in the data. The default is `twofold(off)`, which conducts the threefold decomposition.

`resultsdata([pathname] filename [, replace])` saves the main decomposition results in a results dataset that can be used for further presentation of results in tables or graphs.

`blocks(blockname1 = (varlist1) [, blockname2 = (varlist2) ...])` allows the calculation of decomposition in blocks of variables. This is especially useful with the `bootstrap()` option because it will generate block-specific standard errors. If you specify `blocks()`, you also need to specify `resultsdata()`.

`tfweight(#)` specifies the weight that is to be given to the first of the two groups specified in `groupcat(##)`. This is allowed only if `twofold(weight)` is specified.

`fmt(#)` specifies the decimal points that are to be used in the results presentation.

`nolevels` skips the output of the decomposition of levels.

`nocomplete` skips the output of the decomposition of change.

`seed(#)` specifies the seed for the bootstrapping option. The default is the seed currently set in the Stata session.

`bootstrap(#)` estimates standard errors via bootstrapping with `#` iterations. In addition to the normal results, it returns an `e(dec_b)` and an `e(dec_V)` matrix for further processing. If bootstrap clustered standard errors are to be estimated, the clustering has to be specified in the original regression command using Stata's `cluster(varlist)` option. If you specify `noisily` with `bootstrap()`, you also need to specify `resultsdata()`.

### 6.3 Stored results

When one uses the `xtoaxaca` command, the results of the original regression command are retained in Stata's `e()` format. Additionally, the user will find matrices specific to the `xtoaxaca` command that are named after the decomposition method chosen (see table 1).

Table 1. Overview of stored results after `xtoaxaca`

Method	Stored results	
	absolute values	percentages
Levels	$e(E   C   CE)$	$e(pE   pC   pCE)$
SSM	$e(dE   dC   dCE)$	$e(pdE   pdC   pdCE)$
Interventionist	$e(dE   dC   dCE)$	$e(pdE   pdC   pdCE)$
Smith and Welch (1989)	$e(sw\_X)$	$e(psw\_X)$
Wellington (1993)	$e(wl\_X)$	$e(pwl\_X)$
Makepeace et al. (1999)	$e(mpjd\_X)$	$e(pmpjd\_X)$
Kim (2010)	$e(kim\_DX)$	$e(pkim\_DX)$

NOTE:  $X$  stands for the different components within one method.

The `resultsdata()` option allows users to store the results matrices as a dataset as well, which can be useful if they are to be presented graphically. The `resultsdata()` option will also store all draws from the bootstrapping procedure if this has been chosen as an option. Several additional results are also stored by `xtoaxaca` as described in table 2.

Table 2. Overview of additional results stored after `xtoaxaca`

Name	Content
<code>e(catX_coef_mean)</code>	coefficients for category $X$ of group variable
<code>e(catX_endow_mean)</code>	means of decomposition variables for category $X$ of group variable
<code>e(change_model)</code>	change in the gap predicted by the model
<code>e(change_observed)</code>	change in the gap as observed in the dataset
<code>e(means_model)</code>	group means predicted by the model
<code>e(means_observed)</code>	group means as observed in the dataset
<code>e(prefmat   drefmat)</code>	contribution of random effects or fixed effects to the decomposition ( $p$ for percentage, $d$ for change)
<code>e(summary_levels)</code>	summary matrix of level decomposition results
<code>e(summary_change)</code>	summary matrix of change decomposition results

## 7 Example: Increasing household income inequality and composition effects

### 7.1 Example 1—Decomposition of changes in household income between East and West Germany

To demonstrate the capabilities of the `xtoaxaca` command, we take an example from research on income inequality and examine the extent to which the decreasing gap in household income between households in the new and old German federal states can be traced back to their changing household composition. To this end, we use data from the German Socio-Economic Panel (SOEP) v.34.1 (Goebel et al. 2019; Liebig et al. 2018) and compute the logarithmized monthly net equivalent and inflation-adjusted household income. In addition to income data, we use information on the households' composition and employment situation (`hh_emp`). The variable we build from this information captures whether the household consists of a full-time working person, a full-time part-time working person, a not-working person, a full-time and part-time working couple, a couple both working full time, and other constellations.

First, we fit the panel regression model and store its results. Because we are interested in the effects of changing household compositions over time, the model includes a threefold interaction term that includes time, the group variable `hh_east`, and the decomposition variable of interest `hh_emp`. In this way, the model predicts income changes for each group and every household composition at every year. Note that we model time as a categorical variable in this example. However, the `xtoaxaca` command does not assume any functional form of time and that we can also specify time as a continuous variable.

After fitting the model, we run the `xtoaxaca` command. In our example, we specify `hh_emp` as the decomposition variable we are interested in, the group variable (`hh_east`), and the two values of the group variable that we want to compare (0 and 1). Further, we need to specify the time variable (`year`) and the specific time values (2006 2010 2014), the reference time (2006), and the name of the stored fitted model. Finally, we tell `xtoaxaca` that we wish to use the interventionist approach, denoted by `change(interventionist)`.

```
. quietly eststo est1: xtreg hh_inc i.year##i.hh_east##i.hh_emp
. xtoaxaca hh_emp, groupvar(hh_east) groupcat(0 1) timevar(year)
> times(2006 2010 2014) timeref(2006) model(est1) change(interventionist)
WARNING: This is a beta version. Please check the results carefully
and report bugs and suggestions to hkroeger@diw.de
```

#### Decomposition of Levels

##### Summary of level decomposition

	year		
	2006	2010	2014
Level			
non-parame-c	0.199	0.182	0.176
Decomp			
Endowments	0.011	0.010	0.018
Coefficients	0.170	0.151	0.136
Interaction	0.002	-0.002	-0.002
RE	0.017	0.023	0.025
Total	0.199	0.182	0.176
Decomp %			
Endowments	5.404	5.647	9.998
Coefficients	85.200	82.685	77.310
Interaction	0.764	-1.063	-1.324
RE	8.631	12.730	14.017
Total	100.000	100.000	100.000

RE = random effects.

#### Decomposition of Change

##### Summary of changes in the outcome

	year		
	2006	2010	2014
Change			
non-parame-c	0.000	-0.017	-0.023
Decomp			
Endowments	0.000	-0.007	-0.004
Coefficients	0.000	-0.015	-0.024
Interactions	0.000	-0.001	-0.002
RE	0.000	0.006	0.008
Total	0.000	-0.017	-0.023
Decomp %			
Endowments	.	40.920	16.559
Coefficients	.	89.446	105.874
Interactions	.	5.111	10.334
RE	.	-35.478	-32.767
Total	.	100.000	100.000

RE = random effects.

For an explanation of this change decomposition, please see:

Kröger, H., & Hartmann, J. (2020). xtoaxaca - Extending the Kitagawa-Oaxaca-Blinder

> Decomposition Approach to Panel Data. <https://doi.org/10.31235/osf.io/egj79>

By default, `xtoaxaca` displays a table of the decomposition of levels and another table showing the decomposition of change. In the first table, the row `non-parametric` denotes the mean group differences in log household incomes as estimated nonparametrically from the observed data. The rows in the `Decomp` section show the results of the income gaps' decomposition into an endowments part, a coefficient part, an interactions part, and a part that is due to the time-constant error term (`RE`). In our example, household compositions hardly contribute to the gap in household income, and their contribution grows only in 2014. Because the `RE` component is close to 0, our model is also reasonably well specified. However, the `RE` component is nonnegligible compared with the size of the other components. Further, we see that the decomposed parts sum up to the difference predicted by the base model for all years. The lower part of the table displays the relative contribution of the four decomposition effects to the overall gap.

The second table displays the results of the decomposition of change. We see the change in the income gap in comparison with the reference year 2006 in the second and third column. For the observed data, the gap decreased between 2006 and 2014 by 0.023 log incomes. We can now examine the role of changing endowments and coefficients over time. As we can see, the changing household composition decreased the gap by 0.004 log incomes, and the changing coefficients contributed 0.024 log incomes to the narrowing gap between 2006 and 2014. The part that is due to differences between groups in the time-constant error term increased the gap by 0.008 log incomes. While this part is rather small, it still indicates that group-specific panel dropout has a small effect on the results.

## 7.2 Bootstrapping for standard errors

So far, we have decomposed the changes in household incomes between 2006 and 2014 in Germany and have the estimates but no standard errors. The `xtoaxaca` command provides a `bootstrap()` option to estimate the standard errors. Because there have been no attempts to analytically derive standard errors for all decomposition models of change over time, we believe that bootstrapping is a viable alternative. By default, `xtoaxaca` does not bootstrap the standard errors, because it is a potentially time-consuming endeavor.

Below, we see the same example as above with bootstrap standard errors. The point estimates of the decomposition components are by design identical to the previous example. However, we now get a standard error below the point estimate and can now be more confident that the households' composition contributes about 5.4% to the income gap in 2006 and almost 10% in 2014, because the components are statistically significant. However, the size of the component of households' changing composition is small in size and is not statistically significant.

```
. quietly eststo est2: xtreg hh_inc i.year##i.hh_east i.year##i.hh_east##i.hh_emp
. xtoaxaca hh_emp, groupvar(hh_east) groupcat(0 1) timevar(year)
> times(2006 2010 2014) timeref(2006) model(est2) change(interventionist)
> bootstrap(100)
WARNING: This is a beta version. Please check the results carefully
        and report bugs and suggestions to hkroeger@diw.de
WARNING: Bootstrapping requires the original data set for estimation
> and the regressions need to be repeated 100 times. This may take some time.
(running xtoaxaca_bootstrap_wrapper on estimation sample)
```

Bootstrap replications (100)

```
—|— 1 —|— 2 —|— 3 —|— 4 —|— 5
..... 50
..... 100
```

#### Decomposition of Levels

	year 2006 b/se	2010 b/se	2014 b/se
Outcome			
Observed	0.199*** (0.011)	0.182*** (0.013)	0.176*** (0.013)
Decomp			
Endowments	0.011*** (0.003)	0.010* (0.004)	0.018** (0.006)
Coefficients	0.170*** (0.010)	0.151*** (0.010)	0.136*** (0.011)
Interactions	0.002 (0.002)	-0.002 (0.002)	-0.002 (0.002)
RE	0.017 (0.010)	0.023* (0.011)	0.025* (0.012)
Total	0.199*** (0.011)	0.182*** (0.013)	0.176*** (0.013)
Decomp%			
Endowments	5.404*** (1.503)	5.647* (2.336)	9.998*** (2.948)
Coefficients	85.200*** (4.994)	82.685*** (5.672)	77.310*** (6.486)
Interactions	0.764 (1.060)	-1.063 (0.954)	-1.324 (1.332)
RE	8.631 (4.905)	12.730* (5.517)	14.017* (6.309)
Total	100.000 (.)	100.000 (.)	100.000 (.)

Decomposition of Change

	year		
	2006	2010	2014
	b/se	b/se	b/se
Outcome			
Observed	0.000 (.)	-0.017 (0.012)	-0.023 (0.014)
Decomp			
Endowments	0.000 (.)	-0.007* (0.003)	-0.004 (0.004)
Coefficients	0.000 (.)	-0.015 (0.008)	-0.024** (0.009)
Interactions	0.000 (.)	-0.001 (0.001)	-0.002 (0.002)
RE	0.000 (.)	0.006 (0.008)	0.008 (0.010)
Total	0.000 (.)	-0.017 (0.012)	-0.023 (0.014)
Decomp%			
Endowments	0.000 (.)	40.920 (237.566)	16.559 (102.940)
Coefficients	0.000 (.)	89.446 (283.015)	105.874 (458.214)
Interactions	0.000 (.)	5.111 (43.023)	10.334 (42.913)
RE	0.000 (.)	-35.478 (397.911)	-32.767 (466.494)
Total	0.000 (.)	100.000 (.)	100.000 (.)

For an explanation of this change decomposition, please see:

Kröger, H., & Hartmann, J. (2020). xtoaxaca - Extending the Kitagawa–Oaxaca–Blinder > Decomposition Approach to Panel Data. <https://doi.org/10.31235/osf.io/egj79>

### 7.3 Blocks of variables

One can also combine two or more variables to blocks and get the standard error via bootstrapping for their combined contribution to the decomposition of both levels and change. The relevant option is `blocks()`, and the `resultsdata()` option needs to be specified as well. Below is an example of code and output for variable blocks. The results for the variables are shown after all other output is given. Here the contribution of all categories of the variables `hh_emp` and `hh_edu` is combined into the block `socio`. The variable `hh_married` is treated separately. One can specify more than one block of variables.

```
. quietly eststo est3: xtreg hh_inc i.year##i.hh_east
> i.year##i.hh_east##i.(hh_emp hh_married hh_edu), fe
. xtoaxaca hh_emp hh_married hh_edu, groupvar(hh_east) groupcat(0 1) timevar(year)
> times(2006 2010 2014) timeref(2006) normalize(hh_emp hh_married hh_edu)
> model(est3) change(interventionist) bootstrap(10) detail
> blocks(socio = (hh_emp hh_edu)) resultsdata(results_example1.dta, replace)
(output omitted)
```



Detailed Decomposition of Levels - Variable Blocks

Level Component: C

	2006	2010	2014
	b/se	b/se	b/se
hh_married-n	-0.009 (0.005)	-0.004 (0.007)	0.003 (0.005)
hh_married_1	0.010 (0.006)	0.005 (0.008)	-0.004 (0.005)
socio	-0.119* (0.051)	-0.109** (0.039)	-0.075 (0.070)
Intercept	0.244*** (0.070)	0.216** (0.067)	0.167* (0.074)
Total	0.126*** (0.036)	0.108** (0.041)	0.092* (0.038)

Level Component: CE

	2006	2010	2014
	b/se	b/se	b/se
hh_married-n	0.001 (0.001)	0.000 (0.001)	-0.000 (0.000)
hh_married_1	0.001 (0.001)	0.000 (0.001)	-0.000 (0.000)
socio	0.035*** (0.010)	0.030** (0.010)	0.021 (0.011)
Intercept	0.000 (.)	0.000 (.)	0.000 (.)
Total	0.037*** (0.011)	0.031** (0.011)	0.020 (0.011)

Level Component: E

	2006	2010	2014
	b/se	b/se	b/se
hh_married-n	-0.000 (0.000)	0.000 (0.001)	0.000 (0.000)
hh_married_1	-0.000 (0.000)	0.000 (0.001)	0.000 (0.000)
socio	-0.057*** (0.016)	-0.060*** (0.012)	-0.037** (0.014)
Intercept	0.000 (.)	0.000 (.)	0.000 (.)
Total	-0.058*** (0.016)	-0.060*** (0.012)	-0.036** (0.014)

## Percentages of Level Component: C

	2006 b/se	2010 b/se	2014 b/se
hh_married-n	-4.304 (2.681)	-2.344 (3.883)	1.971 (2.623)
hh_married_1	5.018 (3.089)	2.694 (4.299)	-2.363 (3.109)
socio	-59.852* (23.887)	-59.522** (20.890)	-42.358 (40.748)
Intercept	122.507*** (34.509)	118.227*** (35.711)	94.820* (44.731)
Total	63.368*** (19.209)	59.055** (21.464)	52.069* (21.899)

## Percentages of Level Component: CE

	2006 b/se	2010 b/se	2014 b/se
hh_married-n	0.491 (0.335)	0.211 (0.480)	-0.175 (0.248)
hh_married_1	0.491 (0.335)	0.211 (0.480)	-0.175 (0.248)
socio	17.445** (5.487)	16.477** (6.298)	11.835 (7.070)
Intercept	0.000 (.)	0.000 (.)	0.000 (.)
Total	18.427** (5.962)	16.898* (6.843)	11.486 (6.963)

## Percentages of Level Component: E

	2006 b/se	2010 b/se	2014 b/se
hh_married-n	-0.064 (0.231)	0.000 (0.277)	0.128 (0.182)
hh_married_1	-0.064 (0.231)	0.000 (0.277)	0.128 (0.182)
socio	-28.714*** (8.371)	-32.669*** (7.227)	-20.913* (8.747)
Intercept	0.000 (.)	0.000 (.)	0.000 (.)
Total	-28.842*** (8.651)	-32.669*** (7.341)	-20.657* (8.659)

## 7.4 Example 2—An intervention

As a further example, we simulate a dataset with a group variable (`group`), two time points, and two binary intervening variables, of which the first is exogenous (`int1`) and the second is endogenous to the first one (`int2`). The purpose of this example is to demonstrate how `xtoaxaca` can decompose change in group differences over time using multiple intervening and exogenous decomposition variables similar to experimental settings. For instance, an exogenous treatment effect, such as a policy change, could lead to increases in one group's endowments, and we can then ask whether the treatment has any effect on the changing group differences.

As with the previous example, the first step involves fitting the model, which now includes two interaction terms: the interaction of the group variable with time and the first decomposition variable and the interaction of the group variable with time and the second decomposition variable. These two interaction terms are included so that the `xtoaxaca` command can estimate the groups' counterfactual trajectories. In the second step, we call the `xtoaxaca` command followed by the two decomposition variables and specify the interventionist decomposition method (`interventionist`).

```
. use xtoaxaca_example2, clear
. quietly eststo m: xtreg dep i.time##i.group##i.int1 i.time##i.group##i.int2,
> i(id)
```

```
. xtoaxaca int1 int2, groupvar(group) groupcat(0 1) timevar(time) times(1 2)
> timeref(1) model(m) change(interventionist)
WARNING: This is a beta version. Please check the results carefully
and report bugs and suggestions to hkroeger@diw.de
```

Decomposition of Levels

Summary of level decomposition

	time	
	1	2
Level		
non-parame-c	1.623	3.770
Decomp		
Endowments	0.034	-0.005
Coefficients	1.711	3.824
Interaction	-0.122	-0.049
RE	-0.000	-0.000
Total	1.623	3.770
Decomp %		
Endowments	2.086	-0.132
Coefficients	105.445	101.424
Interaction	-7.531	-1.292
RE	-0.000	-0.000
Total	100.000	100.000

RE = random effects.

Decomposition of Change

Summary of changes in the outcome

	time	
	1	2
Change		
non-parame-c	0.000	2.147
Decomp		
Endowments	0.000	-0.097
Coefficients	0.000	2.265
Interactions	0.000	-0.021
RE	0.000	0.000
Total	0.000	2.147
Decomp %		
Endowments	.	-4.510
Coefficients	.	105.482
Interactions	.	-0.972
RE	.	0.000
Total	.	100.000

RE = random effects.

For an explanation of this change decomposition, please see:

Kröger, H., & Hartmann, J. (2020). xtoaxaca - Extending the Kitagawa–Oaxaca–Blinder  
> Decomposition Approach to Panel Data. <https://doi.org/10.31235/osf.io/egj79>

As can be seen from the output, the changing endowments cause the gap to decrease by 4.5% over time, while the changing coefficients increase the gap by 105%. Thus, we conclude that the increasing gap between the groups over time is not caused by their changing endowments. On the contrary, they decrease the gap over time, while all the increase in the gap can be attributed to changing coefficients. If the role of the changing return to the decomposition variables is to be investigated, results using the `detail` option should be used (as illustrated in the last example in section 7.5) because the intercept is part of the coefficient block. A change in the intercept does not usually have a meaningful substantive interpretation with respect to the intervening variables and therefore might be considered an unexplained part within the coefficient component. Note also that because the example uses simulated data, the contribution of the time-constant error term is now exactly zero as expected by model assumptions.

## 7.5 Example 3—Interaction of decomposition variables

In the last example, we showed how a regression model has to be set up if interactions of decomposition variables are to be used in `xtoaxaca`. For this purpose, we distinguish between three types of interactions: Categorical-categorical, continuous-continuous, and categorical-continuous interactions.

### Categorical-categorical interactions

For these kinds of interactions, we recommend generating a new variable that contains all combinations of the two categorical variables.

```
egen newvar = group(var1 var2)
```

Then, we can use this new variable (`newvar`) as a decomposition variable in both the regression and the `xtoaxaca` command.

### Continuous-continuous interactions

For these kinds of interactions, the key is to create interaction terms by hand and then combine them with the standard Stata factor-variable notation of the group and time variable.

```
. use xtoaxaca_example, clear
. xtset id
    panel variable:  id (balanced)
. * Create interaction term for a squared term
. replace exp = exp-10
(10,000 real changes made)
. generate exp2 = exp*exp
. * for interactions among decomposition variables (squared experience)
. * a handmade interaction term is used
. quietly eststo est3: xtreg inc i.group##i.time##c.(exp exp2)
> i.group##i.time##i.edu
```

```
. xtoaxaca edu exp exp2, groupvar(group) groupcat(0 1) timevar(time) nolevels
> times(4 2 1) model(est3) timeref(2) change(interventionist) detail normalize(edu)
WARNING: This is a beta version. Please check the results carefully
and report bugs and suggestions to hkroeger@diw.de
```

---

```
Group variable:          group (0,1)
Decomposition variables:  edu exp exp2
Times:                   time (4 2 1)
```

---

```
[ Progress ] Calculating endowment means for each time ...
[ Progress ] Calculating mean outcomes ...
[ Estimating random effects ] Estimating mean of random effects for both groups
> ...
[ Progress ] Calculating coefficients ...
[ Progress ] Running level decomposition ...
[ Progress ] Running Interventionist decomposition ...
[ Progress ] Display output ...
```

---

Outcome

Mean predicted outcome differences between the groups (empirical values)

	Time		
	4	2	1
Group A	3663.107	2983.06	2715.14
Group B	6524.738	4533.923	3951.633
Diff	-2861.632	-1550.863	-1236.493

---

Endowments

Endowment means for each time (Group (group) 0, variables: edu exp exp2)

	Time		
	4	2	1
0.edu	.0914697	.1438849	.1819116
1.edu	.3329908	.4110997	.4635149
2.edu	.3103803	.307297	.270298
3.edu	.2651593	.1377184	.0842754
exp	-4.367934	-4.75334	-4.971223
exp2	24.46249	27.72148	29.62076
Intercept	1	1	1

---

Endowment means for each time (Group (group) 1, variables: edu exp exp2)

	Time		
	4	2	1
0.edu	.0681597	.2755599	.5423564
1.edu	.2375852	.4333009	.3417722
2.edu	.3446933	.2259007	.1051607
3.edu	.3495618	.0652386	.0107108
exp	2.057449	.661149	-.0467381
exp2	15.46933	11.35054	10.73418
Intercept	1	1	1

---

Coefficients

## Group A coefficients

	Time r1	r2	r3
0.edu	-235.8521	83.33642	238.1277
1.edu	-78.87413	22.97487	67.34441
2.edu	76.95988	-28.07654	-98.34574
3.edu	237.7663	-78.23475	-207.1264
exp	28.87513	-4.020149	-14.08591
exp2	.9153148	.5256663	.278332
Intercept	3727.745	2947.345	2606.376

## Group B coefficients

	Time r1	r2	r3
0.edu	-312.1004	93.12871	297.7939
1.edu	-98.5876	55.99646	72.83483
2.edu	108.9784	-35.06983	-137.1094
3.edu	301.7096	-114.0553	-233.5193
exp	79.22486	-32.17301	-65.14026
exp2	.2232951	-.2076859	-.0671643
Intercept	6259.948	4522.989	3779.826

Decomposition of Change

## Summary of changes in the outcome

	time 4	2	1
Change			
non-parame-c	-1,310.769	0.000	314.370
Decomp			
Endowments	93.165	0.000	-41.610
Coefficients	-1,073.681	0.000	431.704
Interactions	-330.253	0.000	-75.724
RE	0.000	0.000	0.000
Total	-1,310.769	0.000	314.370
Decomp %			
Endowments	-7.108	.	-13.236
Coefficients	81.912	.	137.324
Interactions	25.195	.	-24.088
RE	0.000	.	0.000
Total	100.000	.	100.000

RE = random effects.

## Total change: Endowments

	time		
	4	2	1
0.edu	14.947	0.000	-21.677
1.edu	9.165	0.000	6.330
2.edu	4.079	0.000	-3.196
3.edu	22.458	0.000	-2.038
exp	43.374	0.000	-21.899
exp2	-0.858	0.000	0.870
Intercept	0.000	0.000	0.000
Total	93.165	0.000	-41.610

Note: Change in group differences over time if only the groups' endowments had > changed.

## Total change: Coefficients

	time		
	4	2	1
0.edu	65.738	0.000	-34.125
1.edu	25.111	0.000	10.944
2.edu	-0.263	0.000	1.457
3.edu	16.395	0.000	-9.957
exp	-230.013	0.000	69.642
exp2	5.910	0.000	-8.451
Intercept	-956.559	0.000	402.194
Total	-1,073.681	0.000	431.704

Note: Change in group differences over time if only the groups' coefficients had > changed.

## Total change: Interactions

	time		
	4	2	1
0.edu	-67.314	0.000	-48.718
1.edu	-22.299	0.000	3.867
2.edu	-16.788	0.000	-9.720
3.edu	-77.940	0.000	0.374
exp	-142.867	0.000	-21.144
exp2	-3.045	0.000	-0.383
Intercept	0.000	0.000	0.000
Total	-330.253	0.000	-75.724

Note: Change in group differences over time attributable to the interaction of > change in endowments and change in coefficients.



## Total Percentages of change: Endowments

	time		
	4	2	1
0.edu	-1.140	.	-6.896
1.edu	-0.699	.	2.013
2.edu	-0.311	.	-1.016
3.edu	-1.713	.	-0.648
exp	-3.309	.	-6.966
exp2	0.065	.	0.277
Intercept	0.000	.	0.000
Total	-7.108	.	-13.236

## Total Percentages of change: Coefficients

	time		
	4	2	1
0.edu	-5.015	.	-10.855
1.edu	-1.916	.	3.481
2.edu	0.020	.	0.464
3.edu	-1.251	.	-3.167
exp	17.548	.	22.153
exp2	-0.451	.	-2.688
Intercept	72.977	.	127.937
Total	81.912	.	137.324

## Total Percentages of change: Interactions

	time		
	4	2	1
0.edu	5.135	.	-15.497
1.edu	1.701	.	1.230
2.edu	1.281	.	-3.092
3.edu	5.946	.	0.119
exp	10.899	.	-6.726
exp2	0.232	.	-0.122
Intercept	0.000	.	0.000
Total	25.195	.	-24.088

For an explanation of this change decomposition, please see:

Kröger, H., & Hartmann, J. (2020). *xtoaxaca* - Extending the Kitagawa-Oaxaca-Blinder > Decomposition Approach to Panel Data. <https://doi.org/10.31235/osf.io/egj79>

The first difference from the previous examples is the additional output that is generated by using the `detail` option. It provides not only the components of the decomposition but also the estimates for the endowments and coefficients for each variable and group. Note that for the interpretation of each variable and its contribution, separate assumptions about the reference category or value have to be made (see section 3.2 or Yun [2005]; Jann [2008]).

The second difference is that this example shows interactions of two continuous decomposition variables. If interactions of decomposition variables are used, note that the individual contribution of each interacted variable is now conditional on the value

of the other variable. This means that in our example, we interacted experience with itself to get a squared term in the regression equation. The contribution of labor market experience (`exp`) now varies along the values of labor market experience.

For this particular example, we could state that the difference in change between groups A and B and time points 4 and 2 is explained about 20% by the difference in change in coefficients in the experience variable at the 15 years of experience in the sample (value of 0). If labor market experience is not centered at a meaningful value, this might not be a particular useful result.

### **Categorical-continuous interactions**

The strategy described in section 7.5 for continuous-continuous interactions technically also works for categorical-continuous interactions. There are two limitations, however. First, normalization (3.2) for the categorical variable is not possible. This also implies that we cannot interpret the results for the decomposition by Kim (2010) as originally intended, because this decomposition relies on normalization. Second, we believe that it is overall very difficult to interpret the detailed output for such a decomposition. Users might consider performing the decomposition separately for the categorical variable they wish to interact with the continuous variable to ease interpretation.

## **8 Limitations**

We focus on continuous outcomes and linear models but believe that the general approach can be generalized to nonlinear models as well, as it has been for the cross-sectional KOB decomposition (Bauer and Sinning 2008; Jann 2008). Furthermore, applying regression with recentered influence functions in the modeling step might also be a way to circumvent the current restrictions to linear models of `xtoaxaca` (Essama-Nssah and Lambert 2012; Firpo, Fortin, and Lemieux 2018), as exemplified in the community-contributed command `oaxaca_rif` (Rios-Avila 2020).

Our decomposition approach is further limited to mean decompositions. Longitudinal decompositions of or using other distributional statistics (for example, percentiles, variances) might also be useful as user-friendly programs (Fortin, Lemieux, and Firpo 2011; Blau and Kahn 1992; Juhn, Murphy, and Pierce 1993).

## **9 Conclusion**

We provided a systematic extension of the KOB decomposition to longitudinal (and multilevel) data. We reviewed five central approaches to the decomposition of change. We noted that none of them are directly useful for the evaluation of an intervention, policy changes, or natural experiments. We proposed an extension of the Wellington (1993) decomposition of change over time from an interventionist perspective. We introduced the `xtoaxaca` command, which implements the decomposition of levels and

change (all six variants) over time in a user-friendly postestimation command for Stata. We are open to suggestions from users of `xtoaxaca` concerning new functions and other improvements so that we can update and improve it regularly.

## 10 Acknowledgments

We would like to thank Benita Combet and Christoph Halbmeier for helpful feedback on earlier versions of this article. We would also like to extend our special thanks to Benita Combet for extensive testing of earlier versions of `xtoaxaca`. All remaining mistakes remain, of course, the sole responsibility of the authors.

Hannes Kröger was funded by the Deutsche Forschungsgemeinschaft (DFG, German Science Foundation) 415809395, 427279591, 40965412 and the German Federal Ministry of Education and Research (BMBF, grants: 01UJ1911BY; 01NV1601B).

## 11 Programs and supplemental materials

To install a snapshot of the corresponding software files as they existed at the time of publication of this article, type

```
. net sj 21-2
. net install st0640      (to install program files, if available)
. net get st0640         (to install ancillary files, if available)
```

To install the current version of the software files, type

```
. net install xtoaxaca, from(https://gitlab.com/jhart/xtoaxaca/-/raw/master)
> replace
```

## 12 References

- Arrosa, M. L., and N. Gandelman. 2016. Happiness decomposition: Female optimism. *Journal of Happiness Studies* 17: 731–756. <https://doi.org/10.1007/s10902-015-9618-8>.
- Bauer, T. K., and M. Sinning. 2008. An extension of the Blinder–Oaxaca decomposition to nonlinear models. *Advances in Statistical Analysis* 92: 197–206. <https://doi.org/10.1007/s10182-008-0056-3>.
- Blau, F. D., and L. M. Kahn. 1992. The gender earnings gap: Learning from international comparisons. *American Economic Review* 82: 533–538.
- . 2017. The gender wage gap: Extent, trends, and explanations. *Journal of Economic Literature* 55: 789–865. <https://doi.org/10.1257/jel.20160995>.
- Blinder, A. S. 1973. Wage discrimination: Reduced form and structural estimates. *Journal of Human Resources* 8: 436–455. <https://doi.org/10.2307/144855>.

- DeLeire, T. 2000. The wage and employment effects of the Americans with Disabilities Act. *Journal of Human Resources* 35: 693–715. <https://doi.org/10.2307/146368>.
- Deming, W. E., and F. F. Stephan. 1940. On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. *Annals of Mathematical Statistics* 11: 427–444. <https://doi.org/10.1214/aoms/1177731829>.
- Essama-Nssah, B., and P. J. Lambert. 2012. Influence functions for policy impact analysis. In *Inequality, Mobility and Segregation: Essays in Honor of Jacques Silber*, ed. J. A. Bishop and R. Salas, 135–159. Bingley, UK: Emerald.
- Fairlie, R. W. 2005. An extension of the Blinder–Oaxaca decomposition technique to logit and probit models. *Journal of Economic and Social Measurement* 30: 305–316. <https://doi.org/10.3233/JEM-2005-0259>.
- Firebaugh, G., C. Warner, and M. Massoglia. 2013. Fixed effects, random effects, and hybrid models for causal analysis. In *Handbook of Causal Analysis for Social Research*, ed. S. L. Morgan, 113–132. Dordrecht: Springer. [https://doi.org/10.1007/978-94-007-6094-3\\_7](https://doi.org/10.1007/978-94-007-6094-3_7).
- Firpo, S. P., N. M. Fortin, and T. Lemieux. 2018. Decomposing wage distributions using recentered influence function regressions. *Econometrics* 6: 28. <https://doi.org/10.3390/econometrics6020028>.
- Fortin, N., T. Lemieux, and S. Firpo. 2011. Decomposition methods in economics. In *Handbook of Labor Economics*, vol. 4A, ed. O. Ashenfelter and D. Card, 1–102. Amsterdam: Elsevier.
- Freeman, R. B. 1980. Unionism and the dispersion of wages. *ILR Review* 34: 3–23. <https://doi.org/10.1177/001979398003400101>.
- . 1984. Longitudinal analyses of the effects of trade unions. *Journal of Labor Economics* 2: 1–26. <https://doi.org/10.1086/298021>.
- Goebel, J., M. M. Grabka, S. Liebig, M. Kroh, D. Richter, C. Schröder, and J. Schupp. 2019. The German Socio-Economic Panel (SOEP). *Journal of Economics and Statistics* 239: 345–360. <https://doi.org/10.1515/jbnst-2018-0022>.
- Härdle, W. K., M. Müller, S. Sperlich, and A. Werwatz. 2004. *Nonparametric and Semiparametric Models*. Berlin: Springer.
- Jann, B. 2004. estout: Stata module to make regression tables. Statistical Software Components S439301, Department of Economics, Boston College. <https://ideas.repec.org/c/boc/bocode/s439301.html>.
- . 2008. The Blinder–Oaxaca decomposition for linear regression models. *Stata Journal* 8: 453–479. <https://doi.org/10.1177/1536867X0800800401>.
- Juhn, C., K. M. Murphy, and B. Pierce. 1993. Wage inequality and the rise in returns to skill. *Journal of Political Economy* 101: 410–442. <https://doi.org/10.1086/261881>.

- Kalton, G., and I. Flores-Cervantes. 2003. Weighting methods. *Journal of Official Statistics* 19: 81–97.
- Kim, C. 2010. Decomposing the change in the wage gap between white and black men over time, 1980–2005: An extension of the Blinder–Oaxaca decomposition method. *Sociological Methods & Research* 38: 619–651. <https://doi.org/10.1177/0049124110366235>.
- Kim, J. K., and J. J. Kim. 2007. Nonresponse weighting adjustment using estimated response probability. *Canadian Journal of Statistics* 35: 501–514. <https://doi.org/10.1002/cjs.5550350403>.
- Kitagawa, E. M. 1955. Components of a difference between two rates. *Journal of the American Statistical Association* 50: 1168–1194. <https://doi.org/10.1080/01621459.1955.10501299>.
- Liebig, S., J. Goebel, D. Richter, C. Schröder, J. Schupp, M. Kroh, C. Bartels, K. Erhardt, A. Fedorets, A. Franken, M. Giesselmann, M. Grabka, P. Krause, H. Kröger, S. Kühne, M. Metzger, J. Nebelin, D. Schacht, P. Schmelzer, C. Schmitt, D. Schnitzlein, R. Siegers, and K. Wenzig. 2018. Socio-Economic Panel (SOEP), data from 1984–2017, version 34.1. <https://doi.org/10.5684/soep.v34>.
- Machado, J. A. F., and J. Mata. 2005. Counterfactual decomposition of changes in wage distributions using quantile regression. *Journal of Applied Econometrics* 20: 445–465. <https://doi.org/10.1002/jae.788>.
- Makepeace, G., P. Paci, H. Joshi, and P. Dolton. 1999. How unequally has equal pay progressed since the 1970s? A study of two British cohorts. *Journal of Human Resources* 34: 534–556. <https://doi.org/10.2307/146379>.
- Morgan, S. L., and C. Winship. 2015. *Counterfactuals and Causal Inference: Methods and Principles for Social Research*. 2nd ed. New York: Cambridge University Press.
- Neuman, S., and R. L. Oaxaca. 2003. Gender versus ethnic wage differentials among professionals: Evidence from Israel. *Annales d'Économie et de Statistique* 71/72: 267–292. <https://doi.org/10.2307/20079055>.
- Oaxaca, R. 1973. Male–female wage differentials in urban labor markets. *International Economic Review* 14: 693–709. <https://doi.org/10.2307/2525981>.
- Oaxaca, R. L., and C. Choe. 2016. Wage decompositions using panel data sample selection correction. IZA Discussion Paper No. 10157, Institute of Labor Economics (IZA). <http://ftp.iza.org/dp10157.pdf>.
- Oaxaca, R. L., and M. R. Ransom. 1994. On discrimination and the decomposition of wage differentials. *Journal of Econometrics* 61: 5–21. [https://doi.org/10.1016/0304-4076\(94\)90074-4](https://doi.org/10.1016/0304-4076(94)90074-4).

- Rios-Avila, F. 2020. Recentered influence functions (RIFs) in Stata: RIF regression and RIF decomposition. *Stata Journal* 20: 51–94. <https://doi.org/10.1177/1536867X20909690>.
- Smith, J. P., and F. R. Welch. 1989. Black economic progress after Myrdal. *Journal of Economic Literature* 27: 519–564.
- Taber, D. R., W. R. Robinson, S. N. Bleich, and Y. C. Wang. 2016. Deconstructing race and gender differences in adolescent obesity: Oaxaca–Blinder decomposition. *Obesity* 24: 719–726. <https://doi.org/10.1002/oby.21369>.
- Wellington, A. J. 1993. Changes in the male/female wage gap, 1976–1985. *Journal of Human Resources* 28: 383–411. <https://doi.org/10.2307/146209>.
- Winter, N. 2002. `survwgt`: Stata module to create and manipulate survey weights. Statistical Software Components S427503, Department of Economics, Boston College. <https://ideas.repec.org/c/boc/bocode/s427503.html>.
- Yun, M.-S. 2005. A simple solution to the identification problem in detailed wage decompositions. *Economic Inquiry* 43: 766–772. With Erratum, *Economic Inquiry* 44: 198. <https://doi.org/10.1093/ei/cbi053>.

### About the authors

Hannes Kröger is the leader of the junior research group Social and Psychological Determinants of Mental Health in the Life Course (SocPsych-MH) at the Socio-Economic Panel Study (SOEP) at the German Institute for Economic Research (DIW) in Berlin. His research interests include health inequalities and applied statistical methods for social science research.

Jörg Hartmann is a senior researcher at the University of Göttingen. His main research interests comprise the causes and consequences of social inequality, right-wing populism, and immigrant integration, as well as quantitative methods.

## A Appendix

### A.1 Twofold decomposition of change

We can also derive an analogue to the twofold KOB within the interventionist perspective. This is based on a model that assumes a valuation of the endowments at a nondiscriminatory coefficient vector.

$$\begin{aligned}\Delta Y_2^l &= E(\mathbf{Y}_t^l) - E(\mathbf{Y}_s^l) \\ &= E(\mathbf{X}_t^l)\boldsymbol{\beta}_t - E(\mathbf{X}_s^l)\boldsymbol{\beta}_s \\ \boldsymbol{\beta}_t &= W\boldsymbol{\beta}_t^A + (I - W)\boldsymbol{\beta}_t^B\end{aligned}$$

The common coefficient is defined as a weighted average of the two group-specific coefficients. Weights can be defined similarly to the implementation of the original KOB twofold decomposition, as described by Jann (2008, 455–457).

In our adaption of the twofold decomposition for change, we have two components. The first relates to the differences in change in endowments given the same coefficients at time  $s$   $\Delta Y_{E2}$ . The second component is the remaining unexplained part  $\Delta Y_U$ .

$$\begin{aligned}
\Delta Y_{E2} &= [\{E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A)\} - \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B)\}] \beta_s \\
&= E(\mathbf{X}_t^A) \beta_s - E(\mathbf{X}_s^A) \beta_s - E(\mathbf{X}_t^B) \beta_s + E(\mathbf{X}_s^B) \beta_s \\
&= E(\mathbf{X}_t^A) W \beta_s^A + E(\mathbf{X}_t^A) (I - W) \beta_s^B - E(\mathbf{X}_s^A) W \beta_s^A - E(\mathbf{X}_s^A) (I - W) \beta_s^B \\
&\quad - E(\mathbf{X}_t^B) W \beta_s^A - E(\mathbf{X}_t^B) (I - W) \beta_s^B + E(\mathbf{X}_s^B) W \beta_s^A + E(\mathbf{X}_s^B) (I - W) \beta_s^B \\
\Delta Y_U &= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) W \beta_s^B - E(\mathbf{X}_s^A) (I - W) \beta_s^A \\
&\quad + \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) - E(\mathbf{X}_s^A)\} W \beta_s^A \\
&\quad + \{E(\mathbf{X}_s^A) + E(\mathbf{X}_t^B) - E(\mathbf{X}_t^A)\} (I - W) \beta_t^B
\end{aligned}$$

The first component mirrors the cross-sectional endowment component of the twofold decomposition. We immediately see, however, that it violates the formal condition that no term is to be restricted across groups (coefficient  $\beta_s$ ). It therefore imagines a situation in which there would not have been differences in the coefficients at time  $s$ , something that is beyond a potential intervention occurring after  $s$ . Still, for specific purposes, researchers might find this decomposition useful as well if change is to be decomposed twofold, similar to the twofold level decomposition in the original KOB approach.

We can also show that together the two components add to the total change over time.

*Proof.*

$$\begin{aligned}
\Delta Y_{E2} + \Delta Y_U &= E(\mathbf{X}_t^A) W \beta_s^A + E(\mathbf{X}_t^A) (I - W) \beta_s^B - E(\mathbf{X}_s^A) W \beta_s^A \\
&\quad - E(\mathbf{X}_s^A) (I - W) \beta_s^B - E(\mathbf{X}_t^B) W \beta_s^A - E(\mathbf{X}_t^B) (I - W) \beta_s^B \\
&\quad + E(\mathbf{X}_s^B) W \beta_s^A + E(\mathbf{X}_s^B) (I - W) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) W \beta_s^B - E(\mathbf{X}_s^A) (I - W) \beta_s^A \\
&\quad + \{E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) - E(\mathbf{X}_s^A)\} W \beta_s^A \\
&\quad + \{E(\mathbf{X}_s^A) + E(\mathbf{X}_t^B) - E(\mathbf{X}_t^A)\} (I - W) \beta_t^B \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^A) W \beta_s^A + E(\mathbf{X}_s^B) (I - W) \beta_t^B \\
&\quad - E(\mathbf{X}_s^A) (I - W) \beta_s^A + E(\mathbf{X}_s^B) W \beta_s^B \\
&= \{E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A\} - \{E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B\} \\
&= \Delta Y^A - \Delta Y^B \\
&= \Delta Y
\end{aligned}$$

□

## B Relations between change decompositions and KOB

In this section are the proofs that all decompositions of change presented in section 4 are derivatives of the difference between two KOB decomposition at two time points.<sup>10</sup>

### B.1 Smith and Welch (1989)

Together, the four components fully decompose changes in group differences over time.

*Proof.*

$$\begin{aligned}
\Delta Y &= \text{i} + \text{ii} + \text{iii} + \text{iv} \\
&= \left[ \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B) \right\} - \left\{ E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B) \right\} \right] \beta_s^B \\
&\quad + \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A) \right\} \left( \beta_s^A - \beta_s^B \right) \\
&\quad + \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_t^B) \right\} \left( \beta_t^B - \beta_s^B \right) \\
&\quad + E(\mathbf{X}_t^A) \left\{ \left( \beta_t^A - \beta_t^B \right) - \left( \beta_s^A - \beta_s^B \right) \right\} \\
&= E(\mathbf{X}_t^A) \beta_s^B - E(\mathbf{X}_t^B) \beta_s^B - E(\mathbf{X}_s^A) \beta_s^B + E(\mathbf{X}_s^B) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_s^A - E(\mathbf{X}_t^A) \beta_s^B - E(\mathbf{X}_s^A) \beta_s^A + E(\mathbf{X}_s^A) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_t^B - E(\mathbf{X}_t^A) \beta_s^B - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_t^B) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_t^A) \beta_t^B - E(\mathbf{X}_t^A) \beta_s^A + E(\mathbf{X}_t^A) \beta_s^B \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_s^B - E(\mathbf{X}_t^A) \beta_s^B + E(\mathbf{X}_t^B) \beta_s^B - E(\mathbf{X}_t^B) \beta_s^B \\
&\quad + E(\mathbf{X}_s^A) \beta_s^B - E(\mathbf{X}_s^A) \beta_s^B + E(\mathbf{X}_t^A) \beta_s^A - E(\mathbf{X}_t^A) \beta_s^A \\
&\quad + E(\mathbf{X}_t^A) \beta_t^B - E(\mathbf{X}_t^A) \beta_t^B + E(\mathbf{X}_t^A) \beta_s^B - E(\mathbf{X}_t^A) \beta_s^B \\
&= \left\{ E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A \right\} - \left\{ E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B \right\} \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&= \Delta Y^A + \Delta Y^B
\end{aligned}$$

□

---

10. For the SSM, this is directly shown in section 4.1.



## B.2 Wellington (1993)

As can be shown, WL1 and WL2 fully decompose changes in group differences over time.

*Proof.*

$$\begin{aligned}
\Delta Y &= \text{WL1} + \text{WL2} \\
&= \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A) \right\} \beta_t^A - \left\{ E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) \right\} \beta_t^B \\
&\quad + E(\mathbf{X}_s^A) \left( \beta_t^A - \beta_s^A \right) - E(\mathbf{X}_s^B) \left( \beta_t^B - \beta_s^B \right) \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_t^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_t^B \\
&\quad + E(\mathbf{X}_s^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_s^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&= \left\{ E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A \right\} - \left\{ E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B \right\} \\
&= \Delta Y^A - \Delta Y^B
\end{aligned}$$

□

## B.3 Interventionist

Finally, we can show that the three components added up to give the total difference in change between the two groups:

$$\Delta Y = \Delta E + \Delta C + \Delta I$$

*Proof.*

$$\begin{aligned}
\Delta E + \Delta C + \Delta I &= \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A) \right\} \beta_s^A - \left\{ E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) \right\} \beta_s^B \\
&\quad + E(\mathbf{X}_s^A) \left( \beta_t^A - \beta_s^A \right) - E(\mathbf{X}_s^B) \left( \beta_t^B - \beta_s^B \right) \\
&\quad + \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A) \right\} \left( \beta_t^A - \beta_s^A \right) \\
&\quad - \left\{ E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) \right\} \left( \beta_t^B - \beta_s^B \right) \\
&= E(\mathbf{X}_t^A) \beta_s^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_s^B + E(\mathbf{X}_s^B) \beta_s^B \\
&\quad + E(\mathbf{X}_s^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_s^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&\quad + E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_t^A) \beta_s^A - E(\mathbf{X}_s^A) \beta_t^A + E(\mathbf{X}_s^A) \beta_s^A \\
&\quad - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_t^B) \beta_s^B + E(\mathbf{X}_s^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&= \left\{ E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A \right\} - \left\{ E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B \right\} \\
&= \Delta Y^A - \Delta Y^B \\
&= \Delta Y
\end{aligned}$$

□

### B.4 Makepeace et al. (1999)

Together, the components fully decompose the change in group differences over time.

*Proof.*

$$\begin{aligned}
\Delta Y &= \Delta E_{\text{pure}} + \Delta E_{\text{price}} + \Delta U_{\text{pure}} + \Delta U_{\text{price}} \\
&= \left\{ \left\{ E(\mathbf{X}_t^A) - E(\mathbf{X}_s^A) \right\} - \left\{ E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) \right\} \right\} \beta_t^A \\
&\quad + \left\{ E(\mathbf{X}_s^A) - E(\mathbf{X}_s^B) \right\} \left( \beta_t^A - \beta_s^A \right) \\
&\quad + E(\mathbf{X}_t^B) \left\{ \left( \beta_t^A - \beta_s^A \right) - \left( \beta_t^B - \beta_s^B \right) \right\} \\
&\quad + \left\{ E(\mathbf{X}_t^B) - E(\mathbf{X}_s^B) \right\} \left( \beta_s^A - \beta_s^B \right) \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&\quad + E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_s^A) \beta_s^A + E(\mathbf{X}_t^B) \beta_t^A - E(\mathbf{X}_t^B) \beta_t^A \\
&\quad + E(\mathbf{X}_s^B) \beta_t^A - E(\mathbf{X}_s^A) \beta_t^A + E(\mathbf{X}_s^B) \beta_s^A - E(\mathbf{X}_s^B) \beta_s^A \\
&\quad + E(\mathbf{X}_t^B) \beta_s^A - E(\mathbf{X}_t^B) \beta_s^A + E(\mathbf{X}_t^B) \beta_s^B - E(\mathbf{X}_t^B) \beta_s^B \\
&= E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A - E(\mathbf{X}_t^B) \beta_t^B + E(\mathbf{X}_s^B) \beta_s^B \\
&= \left\{ E(\mathbf{X}_t^A) \beta_t^A - E(\mathbf{X}_s^A) \beta_s^A \right\} - \left\{ E(\mathbf{X}_t^B) \beta_t^B - E(\mathbf{X}_s^B) \beta_s^B \right\} \\
&= \Delta Y^A - \Delta Y^B
\end{aligned}$$

□

## B.5 Kim (2010)

We can show that the KIM decomposition fully decomposes changes in group differences over time.

*Proof.*

$$\begin{aligned}
\Delta Y &= D1 + D2 + D3 + D4 + D5 \\
&= \left\{ \beta_{t,0}^A - \beta_{s,0}^A \right\} - \left\{ \beta_{t,0}^B - \beta_{s,0}^B \right\} + \left\{ \bar{\beta}_t^A - \bar{\beta}_s^A \right\} - \left\{ \bar{\beta}_t^B - \bar{\beta}_s^B \right\} \\
&\quad + \left( \frac{E(\mathbf{X}_t^{*A}) + E(\mathbf{X}_s^{*A}) + E(\mathbf{X}_t^{*B}) + E(\mathbf{X}_s^{*B})}{4} \right) \left\{ \left( \tilde{\beta}_t^{*A} - \tilde{\beta}_s^{*A} \right) - \left( \tilde{\beta}_t^{*B} - \tilde{\beta}_s^{*B} \right) \right\} \\
&\quad + \left( \frac{\{E(\mathbf{X}_t^{*A}) - E(\mathbf{X}_s^{*A})\} + \{E(\mathbf{X}_t^{*B}) - E(\mathbf{X}_s^{*B})\}}{2} \right) \left( \frac{\left( \tilde{\beta}_t^{*A} + \tilde{\beta}_s^{*A} \right)}{2} - \frac{\left( \tilde{\beta}_t^{*B} + \tilde{\beta}_s^{*B} \right)}{2} \right) \\
&\quad + \left[ \{E(\mathbf{X}_t^{*A}) - E(\mathbf{X}_s^{*A})\} - \{E(\mathbf{X}_t^{*B}) - E(\mathbf{X}_s^{*B})\} \right] \left( \frac{\tilde{\beta}_t^{*A} + \tilde{\beta}_s^{*A} + \tilde{\beta}_t^{*B} + \tilde{\beta}_s^{*B}}{4} \right) \\
&\quad + \left( \frac{\{E(\mathbf{X}_t^{*A}) + E(\mathbf{X}_s^{*A})\}}{2} - \frac{\{E(\mathbf{X}_t^{*B}) + E(\mathbf{X}_s^{*B})\}}{2} \right) \left( \frac{\left( \tilde{\beta}_t^{*A} - \tilde{\beta}_s^{*A} \right) + \left( \tilde{\beta}_t^{*B} - \tilde{\beta}_s^{*B} \right)}{2} \right) \\
&= \left\{ \beta_{t,0}^A - \beta_{s,0}^A \right\} - \left\{ \beta_{t,0}^B - \beta_{s,0}^B \right\} + \left\{ \bar{\beta}_t^A - \bar{\beta}_s^A \right\} - \left\{ \bar{\beta}_t^B - \bar{\beta}_s^B \right\} \\
&\quad + \left\{ E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*A} + E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*B} - E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad - \left\{ E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*A} + E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*A} + E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*B} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad - \left\{ E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*A} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*A} + E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*A} + E(\mathbf{X}_t^{*A}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_t^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad - \left\{ E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_t^{*B} - E(\mathbf{X}_s^{*A}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad - \left\{ E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*A} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_t^{*B} - E(\mathbf{X}_t^{*B}) \tilde{\beta}_s^{*B} \right\} / 4 \\
&\quad + \left\{ E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*A} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*A} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*B}) \tilde{\beta}_s^{*B} \right\} / 4
\end{aligned}$$

$$\begin{aligned}
& + \left\{ E(\mathbf{X}_t^{*A})\tilde{\beta}_t^{*A} - E(\mathbf{X}_t^{*A})\tilde{\beta}_s^{*A} + E(\mathbf{X}_t^{*A})\tilde{\beta}_t^{*B} - E(\mathbf{X}_t^{*A})\tilde{\beta}_s^{*B} \right\} / 4 \\
& + \left\{ E(\mathbf{X}_s^{*A})\tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*A})\tilde{\beta}_s^{*A} + E(\mathbf{X}_s^{*A})\tilde{\beta}_t^{*B} - E(\mathbf{X}_s^{*A})\tilde{\beta}_s^{*B} \right\} / 4 \\
& - \left\{ E(\mathbf{X}_t^{*B})\tilde{\beta}_t^{*A} + E(\mathbf{X}_t^{*B})\tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*B})\tilde{\beta}_t^{*B} + E(\mathbf{X}_t^{*B})\tilde{\beta}_s^{*B} \right\} / 4 \\
& - \left\{ E(\mathbf{X}_s^{*B})\tilde{\beta}_t^{*A} + E(\mathbf{X}_s^{*B})\tilde{\beta}_s^{*A} - E(\mathbf{X}_s^{*B})\tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*B})\tilde{\beta}_s^{*B} \right\} / 4 \\
= & \left\{ E(\beta_{t,0}^A - \beta_{s,0}^A) - (\beta_{t,0}^B - \beta_{s,0}^B) \right\} + \left\{ E(\bar{\beta}_t^A - \bar{\beta}_s^A) - (\bar{\beta}_t^B - \bar{\beta}_s^B) \right\} \\
& + E(\mathbf{X}_t^{*A})\tilde{\beta}_t^{*A} - E(\mathbf{X}_s^{*A})\tilde{\beta}_s^{*A} - E(\mathbf{X}_t^{*B})\tilde{\beta}_t^{*B} + E(\mathbf{X}_s^{*B})\tilde{\beta}_s^{*B} \\
= & \left\{ 1, E(\mathbf{X}_t^{*A}) \right\} \left( \beta_{t,0}^A, \tilde{\beta}_t^{*A} + \bar{\beta}_t^A \right) - \left\{ 1, E(\mathbf{X}_s^{*A}) \right\} \left( \beta_{s,0}^A, \tilde{\beta}_s^{*A} + \bar{\beta}_s^A \right) \\
& - \left\{ 1, E(\mathbf{X}_t^{*B}) \right\} \left( \beta_{t,0}^B, \tilde{\beta}_t^{*B} + \bar{\beta}_t^B \right) + \left\{ 1, E(\mathbf{X}_s^{*B}) \right\} \left( \beta_{s,0}^B, \tilde{\beta}_s^{*B} + \bar{\beta}_s^B \right) \\
= & \left\{ E(\mathbf{X}_t^A)\beta_t^A - E(\mathbf{X}_s^A)\beta_s^A \right\} - \left\{ E(\mathbf{X}_t^B)\beta_t^B - E(\mathbf{X}_s^B)\beta_s^B \right\} \\
= & \Delta Y^A - \Delta Y^B
\end{aligned}$$

□