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Testing for slope heterogeneity in Stata

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Abstract. In this article, we introduce a new community-contributed command, `xthst`, to test for slope heterogeneity in panels with many observations over cross-sectional units and time periods. The command implements such a test, the delta test (Pesaran and Yamagata, 2008, *Journal of Econometrics* 142: 50–93). Under its null, slope coefficients are homogeneous across cross-sectional units. Under the alternative, slope coefficients are heterogeneous in the cross-sectional dimension. `xthst` also includes two extensions. The first is a heteroskedasticity- and autocorrelation-consistent robust test along the lines of Blomquist and Westerlund (2013, *Economics Letters* 121: 374–378). The second extension is a cross-sectional-dependence robust version. We discuss all tests and present examples using an economic growth model. A Monte Carlo simulation shows that the size and the power behave as expected.

Keywords: st0627, xthst, parameter heterogeneity, fixed effects, pooled OLS, mean-group estimator, cross-section dependence, heterogeneity, common correlated random effects

1 Introduction

Today, panel data are widely used for empirical studies in several research areas, and the benefits of panel data are well known (Baltagi 2013, 6). Linear regression is undoubtedly the workhorse in empirical research, and graduate textbooks like Angrist and Pischke (2009, 86) encourage researchers to use regression models. Standard panel-data regression models like fixed effects (FEs) and random effects all assume that the parameter of interest is homogeneous. Incorrectly ignoring slope heterogeneity might bias the results; see, for example, Pesaran and Smith (1995). Whether the homogeneity assumption holds needs to be clarified before turning to the underlying empirical question.

One possibility for testing slope homogeneity is to apply the F test on the difference of the sum of squared residuals from a pooled ordinary least squares (OLS) and a cross-section unit-specific OLS regression (Baltagi 2013, 64). The main drawback from the latter test is the homoskedastic error variance assumption. In addition, the F test assumes a fixed number of cross-sectional units (N), and the test is shown to perform poorly unless $T > N$ (Bun 2004), with T the number of time periods. Such $T > N$ panels are relatively rare and often not used in the empirical literature. Pesaran, Smith, and Im (1996) proposed a Hausman-type test for $N > T$ comparing the FEs estimator and cross-section unit-specific OLS, but the procedure is not applicable to panel-data models with only strictly exogenous regressors or autoregressive models (Pesaran and Yamagata 2008).

In this article, we introduce a new community-contributed command, `xthst`, that implements a test for slope heterogeneity in the large N and T case, where N can be relatively large to T. The command implements the test presented by Pesaran and Yamagata (2008), producing a normally distributed test statistic under the null hypothesis of homogeneous slope coefficients. The concept of the test is to compare the distance between coefficients obtained by a pooled FEs regression and by a cross-sectional unit-specific regression. The difference is weighted by the unit-specific standard errors and thus allows for residual heteroskedasticity. Blomquist and Westerlund (2016) proposed a heteroskedasticity- and autocorrelation-consistent (HAC) extension, which is included in the `xthst` command. In addition, `xthst` offers a cross-sectional-dependence (CSD) robust test statistic that partials out cross-section averages, inspired by Pesaran (2006) and Chudik and Pesaran (2015b). The latter technique has not been derived in theory; however, Monte Carlo simulations show that this approach seems promising.

To our knowledge, there are two recent studies developing tests for slope heterogeneity when the errors are cross-sectional dependent. Using the same framework as Pesaran and Yamagata (2008), Ando and Bai (2015) presented a test that uses the initial idea of the interactive-effect estimator (Bai 2009). In the latter setup, the number of unknown common factors causing CSD needs to be known or estimated. Choosing a different approach, Blomquist and Westerlund (2016) developed a bootstrap-based test.

For the remainder of this article, we use the following notation: $x_{i,t}$ refers to a scalar. Lowercase bold letters, such as $\mathbf{x}_{i,t}$, denote a vector, usually in a $1 \times k$ dimension, where k refers to the number of regressors. Matrices are denoted in bold and uppercase, such as $\mathbf{X}_i$. Cross-sectional units are denoted by i or j and time periods by t. The number of cross-sectional units is N, whereas the number of time periods is T. Finally, squared brackets describe the floor of a number, for example, $\lfloor T^{1/3} \rfloor = \lfloor 100^{1/3} \rfloor = \lfloor 4.64 \rfloor = 4$.

This article is arranged as follows. In section 2, we review and discuss the econometric theory for the different tests. In section 3, we describe the `xthst` syntax and the available options. In section 4, we present examples using economic growth models. In section 5, we give a detailed description of the Monte Carlo simulation setup alongside results used for assessing the finite sample properties. We close the article with a conclusion. Detailed simulation results can be found in appendix A.1.

2 Econometric theory

Consider the classical panel-data model with heterogeneous slopes

$$y_{i,t} = \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + \varepsilon_{i,t} \tag{1}$$

where $i = 1, \ldots, N$ represents the cross-sectional dimension and $t = 1, \ldots, T$ the time dimension. μ_i is a unit-specific constant. $\boldsymbol{\beta}_{1i}$ is $k_1 \times 1$, $\boldsymbol{\beta}_{2i}$ is $k_2 \times 1$, and both are vectors of unknown slope coefficients with $k = k_1 + k_2$ being the total number of regressors. $\mathbf{x}_{1i,t}$ is a $k_1 \times 1$ vector, and $\mathbf{x}_{2i,t}$ a $k_2 \times 1$ vector containing strictly exogenous regressors. The null hypothesis of interest is formulated as

$$H_0\colon \boldsymbol{\beta}_{2i} = \boldsymbol{\beta}_2 \text{ for all } i \tag{2}$$

against the alternative:

$$H_A\colon \boldsymbol{\beta}_{2i} \neq \boldsymbol{\beta}_2 \text{ for some } i$$

Only coefficients in $\boldsymbol{\beta}_{2i}$ are of interest and tested for slope heterogeneity. The remaining coefficients $\boldsymbol{\beta}_{1i}$ are assumed to be heterogeneous, $\boldsymbol{\beta}_{1i} \neq \boldsymbol{\beta}_1$. In the extreme case that all coefficients are under inspection, $\mathbf{x}_{1i,t}$ reduces to zero variables and $k = k_2$. For clarity, we use the subset notation for the remainder of the article.

2.1 The standard delta test

Based on a standardized version of Swamy's (1970) test, Pesaran and Yamagata (2008) proposed a test for slope homogeneity for panel data with large N and T. The test assumes that $\varepsilon_{i,t}$ and $\varepsilon_{j,s}$ are independently distributed for $i \neq j$ or $t \neq s$, or both, but allows for a heterogeneous variance. The test statistic is given by

$$\widetilde{\Delta} = \frac{1}{\sqrt{N}} \left(\frac{\sum_{i=1}^{N} \widetilde{d}_i - k_2}{\sqrt{2k_2}} \right) \tag{3}$$

where the statistic, under H_0 in (2), is asymptotically $\widetilde{\Delta} \sim \mathcal{N}(0,1)$. In (3), $\widetilde{d}_i$ is defined as the weighted difference between the cross-sectional unit-specific estimate and the pooled estimate,

$$\widetilde{d}_i = \left(\widehat{\boldsymbol{\beta}}_{2i} - \widetilde{\boldsymbol{\beta}}_{2WFE}\right)' \frac{\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i}}{\widetilde{\sigma}_i^2} \left(\widehat{\boldsymbol{\beta}}_{2i} - \widetilde{\boldsymbol{\beta}}_{2WFE}\right)$$

where $\mathbf{X}_{2i} = (\mathbf{x}_{2i,1}, \ldots, \mathbf{x}_{2i,T_i})'$, $\mathbf{M}_{1i} = \mathbf{I}_{T_i} - \mathbf{Z}_{1i}(\mathbf{Z}_{1i}'\mathbf{Z}_{1i})^{-1}\mathbf{Z}_{1i}'$, and $\mathbf{Z}_{1i} = (\boldsymbol{\tau}_{T_i}, \mathbf{X}_{1i})$ with $\boldsymbol{\tau}_{T_i}$ being a $T_i \times 1$ vector of 1s, representing the constant. The coefficients $\widehat{\boldsymbol{\beta}}_{2i}$ and $\widehat{\boldsymbol{\beta}}_{2WFE}$ are defined as

$$\widehat{\boldsymbol{\beta}}_{2i} = (\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i})^{-1}\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i,$$

$$\widehat{\boldsymbol{\beta}}_{2WFE} = \left(\sum_{i=1}^{N} \frac{\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{X}_{2i}}{\widetilde{\sigma}_i^2} \right)^{-1} \sum_{i=1}^{N} \frac{\mathbf{X}_{2i}'\mathbf{M}_{1i}\mathbf{y}_i}{\widetilde{\sigma}_i^2}$$

where $\mathbf{y}_i = (y_{i,1}, \ldots, y_{i,T_i})$,

$$\widetilde{\sigma}_i^2 = \frac{\left(\mathbf{y}_i - \mathbf{X}_{2i}\widehat{\boldsymbol{\beta}}_{\mathrm{FE}}\right)' \mathbf{M}_{1i} \left(\mathbf{y}_i - \mathbf{X}_{2i}\widehat{\boldsymbol{\beta}}_{\mathrm{FE}}\right)}{T_i - 1}$$

and

$$\widehat{\boldsymbol{\beta}}_{\mathrm{FE}} = \left(\sum_{i=1}^{N} \mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{X}_{2i}\right)^{-1} \sum_{i=1}^{N} \mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{y}_i$$

The regressors that are not of interest, including the constant μ_i, are assumed to be heterogeneous, collected in $\mathbf{Z}_{1i}$, and partialed out using the projection matrix $\mathbf{M}_{1i}$. The asymptotic properties of $\widetilde{\Delta}$ are based on $(N,T) \xrightarrow{j} \infty$, such that $\sqrt{N}/T^2 \to 0$. The results presented by Pesaran and Yamagata (2008) also hold if (1) is changed to a standard first-order autoregressive model. However, for the latter, the N and T are required to jointly go to infinity with the same speed; thus, $(N,T) \xrightarrow{j} \infty$ and $N/T \to \kappa$.

For normally distributed errors, the mean-variance bias-adjusted $\widetilde{\Delta}$ can be expressed in the following way,

$$\widetilde{\Delta}_{\mathrm{adj}} = \sqrt{N} \left(\frac{N^{-1} \sum_{i=1}^{N} \widetilde{d}_i - k_2}{\sqrt{\mathrm{Var}(\widetilde{z}_{i,T_i})}} \right)$$

where

$$\mathrm{Var}\left(\widetilde{z}_{i,T_i}\right) = \frac{2k_2(T_i - k - 1)}{T_i - k_1 + 1}$$

2.2 A HAC robust test

Based on Pesaran and Yamagata (2008), Blomquist and Westerlund (2013) presented a HAC extension. The HAC robust test statistic is given by

$$\widetilde{\Delta}_{\mathrm{HAC}} = \sqrt{N} \left(\frac{N^{-1} S_{\mathrm{HAC}} - k_2}{\sqrt{2k_2}} \right) \tag{4}$$

where

$$S_{\mathrm{HAC}} = \sum_{i=1}^{N} T_i \left(\widehat{\boldsymbol{\beta}}_{2i} - \widehat{\boldsymbol{\beta}}_{2\mathrm{HAC}}\right)' \left(\widehat{\mathbf{Q}}_{i,T_i} \widehat{\mathbf{V}}_{i,T_i}^{-1} \widehat{\mathbf{Q}}_{i,T_i}\right) \left(\widehat{\boldsymbol{\beta}}_{2i} - \widehat{\boldsymbol{\beta}}_{2\mathrm{HAC}}\right)$$

$$\widehat{\boldsymbol{\beta}}_{2\mathrm{HAC}} = \left(\sum_{i=1}^{N} T_i \widehat{\mathbf{Q}}_{i,T_i} \widehat{\mathbf{V}}_{i,T_i}^{-1} \widehat{\mathbf{Q}}_{i,T_i}\right)^{-1} \sum_{i=1}^{N} \widehat{\mathbf{Q}}_{i,T_i} \widehat{\mathbf{V}}_{i,T_i}^{-1} \mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{y}_i$$

where $\widehat{\boldsymbol{\beta}}_{2i}$ again is the OLS estimator for each i, $\mathbf{M}_{1i}$ as described above, and $\widehat{\mathbf{Q}}_{i,T_i} = T_i^{-1}(\mathbf{X}_{2i}' \mathbf{M}_{1i} \mathbf{X}_{2i})$. The HAC correction is done with the following estimator,

$$\widehat{\mathbf{V}}_{i,T_i} = \widehat{\boldsymbol{\Omega}}_i(0) + \sum_{j=1}^{T_i - 1} \kappa(j/B_{i,T_i}) \left\{ \widehat{\boldsymbol{\Omega}}_i(j) + \widehat{\boldsymbol{\Omega}}_i(j)' \right\} \tag{5}$$

where $\widehat{\boldsymbol{\Omega}}_i(j) = T_i^{-1} \sum_{t=j+1}^{T_i} \widehat{\widehat{\mathbf{u}}}_{i,t} \widehat{\widehat{\mathbf{u}}}_{i,t-j}'$ and $\widehat{\widehat{\mathbf{u}}}_{i,t} = (\check{\mathbf{x}}_{2i,t} - \overline{\check{\mathbf{x}}}_{2i,t})\widehat{\varepsilon}_{i,t}$ with $\overline{\check{\mathbf{x}}}_{2i,t} = T_i^{-1} \sum_{t-1}^{T_i} \check{\mathbf{x}}_{2i,t}$, where $\check{\mathbf{x}}_{2i,t}$ is the tth element of $\mathbf{X}_{2i}\mathbf{M}_{1i}$. $\widehat{\varepsilon}_{i,t}$ is an estimated residual from a standard FEs regression using $\mathbf{M}_{1i}$ as the projection matrix. In (5), κ is a kernel function, and B_{i,T_i} its bandwidth parameter. Kernels and bandwidths available in `xthst` are discussed in section 3.2.

2.3 A CSD robust test

Especially in panels with many cross-sectional units and time periods, dependence across cross-sectional units can arise. The literature differentiates between weak and strong CSD (Chudik, Pesaran, and Tosetti 2011). Weak CSD is often approximated by spatial methods. Strong CSD is modeled by a common time-specific factor f_t and factor loading γ_i. The common factors affect all cross-sectional units,

$$\begin{aligned} y_{i,t} &= \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + u_{i,t} \\ u_{i,t} &= \boldsymbol{\gamma}'_i\mathbf{f}_t + \varepsilon_{i,t} \end{aligned} \tag{6}$$

where $\mathbf{f}_t$ is an $m \times 1$ vector of unknown common factors and $\boldsymbol{\gamma}_i$ is an $m \times 1$ vector of unknown factor loadings.

When common factors and explanatory variables are correlated, leaving the factors unaccounted for leads to an omitted variable bias. Especially in the light of testing for slope heterogeneity with a test that compares the distance between the unit-specific and pooled estimator, a bias in the estimated coefficients can have a large effect. The common factors can be approximated either by principal components (Bai 2009) or by cross-sectional averages (CSA) (Pesaran 2006). The approach by Pesaran (2006), the so-called common correlated effects (CCE) estimator, has the advantage that the number of common factors does not need to be known in advance. Therefore, in the remainder, we opt for the latter technique for removing strong CSD.

Chudik and Pesaran (2015b) derive a version for weakly exogenous regressors by adding p_{CSA} lags of the CSA and recommend setting $p_{\text{CSA}} = \lfloor T^{1/3} \rfloor$. Equation (6) with CSA would then be

$$\begin{aligned} y_{i,t} &= \mu_i + \boldsymbol{\beta}'_{1i}\mathbf{x}_{1i,t} + \boldsymbol{\beta}'_{2i}\mathbf{x}_{2i,t} + \sum_{l=1}^{p_{\text{CSA}}} \gamma_{i,l}\overline{\mathbf{v}}_t + \varepsilon_{i,t} \\ \overline{\mathbf{v}}_t &= \frac{1}{N}\sum_{j=1}^{N}(\mathbf{x}_{1j,t}, \mathbf{x}_{2j,t}, y_{j,t}) \end{aligned}$$

where $\overline{\mathbf{v}}_t$ are the CSA and either $\mathbf{x}_{1i,t}$ or $\mathbf{x}_{2i,t}$ include the lag of the dependent variable. The CCE estimator can be applied to a pooled and a mean group model. Therefore, the existing delta test can easily be extended to encompass CSA and give guidance on whether to use a pooled or mean group model.

For the CSD robust delta test, we propose to partial out the CSA to remove strong CSD from the model. Assume that matrix $\overline{\mathbf{V}}_t$ contains the CSA and their lags; then, partialing out is done by

$$\begin{aligned}
\widetilde{\mathbf{V}}_t &= \frac{1}{N}\sum_{j=1}^{N}\left(\mathbf{x}_{1j,t}, \mathbf{x}_{2j,t}, y_{j,t}\right), \quad \overline{\mathbf{V}}_t = \left(\widetilde{\mathbf{V}}_t, \ldots, \widetilde{\mathbf{V}}_{t-p_{\mathrm{CSA}}}\right) \\
\mathbf{M}_{\overline{V}t} &= I_T - \overline{\mathbf{V}}_t\left(\overline{\mathbf{V}}_t'\overline{\mathbf{V}}_t\right)^{-1}\overline{\mathbf{V}}_t' \\
\check{\mathbf{y}}_i &= \mathbf{y}_i\mathbf{M}_{\overline{V}t} \\
\check{\mathbf{X}}_{1i} &= \mathbf{X}_{1i}\mathbf{M}_{\overline{V}t} \quad \text{and} \quad \check{\mathbf{X}}_{2i} = \mathbf{X}_{2i}\mathbf{M}_{\overline{V}t}
\end{aligned}$$

The defactored variables are then used to construct $\widetilde{\Delta}_{\mathrm{CSA}}$ following (3) and (4) for the HAC robust test. Blomquist and Westerlund (2013) show in their Monte Carlo simulations that CSD in residuals does influence the size and power of the HAC robust delta test. Their simulations, however, do not include the case where the variables are cross-sectionally dependent. Our Monte Carlo simulations show that $\widetilde{\Delta}_{\mathrm{CSA}}$ performs well, even if variables are cross-sectionally dependent. The test has not been derived in a more theoretical fashion.

3 The xthst command

3.1 Syntax

`xthst` *depvar indepvars* [*if*] [, `noconstant` `partial(`*varlist_p*`)` `ar` `hac`
`kernel(`*kernel*`)` `bw(`*#*`)` `whitening`
`crosssectional(`*varlist_cr*[, `cr_lags(`*numlist*`)`]`)` `nooutput` `comparehac`]

Data must be `xtset` (see [XT] **xtset**) before using `xthst`. *depvar*, *indepvars*, *varlist_p*, and *varlist_cr* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

depvar is the dependent variable of the model to be tested; *indepvars* are the independent variables. *varlist_p* are the variables to be partialed out; *varlist_cr* are variables added as CSA, calculated by `xthst`.

Options

`noconstant` suppresses the individual heterogeneous constant, μ_i.

`partial(`*varlist_p*`)` requests exogenous regressors in *varlist_p* be partialed out. The constant is automatically partialed out if included in the model. Regressors in *varlist* will be included in $\mathbf{z}_{it}$ and are assumed to have heterogeneous slopes; see section 2.

`ar` allows for an autoregressive [AR(p)] model. The degree of freedom of $\widetilde{\sigma}^2$ is adjusted. It may not be combined with `hac`.

`hac` implements Blomquist and Westerlund's (2013) HAC test. If `kernel()` and `bw()` are not specified, `kernel()` is set to `bartlett`, and the data-driven bandwidth selection is used; see section 3.2. It may not be combined with `ar`.

`kernel(`*kernel*`)` specifies the kernel function used in calculating the HAC test statistic. The available kernels are `bartlett`, `qs` (QS), and `truncated`. It is only allowed with `hac`.

`bw(`*#*`)` sets the bandwidth equal to # for the HAC test statistic, where # is an integer greater than zero. It is only allowed with `hac`. The default is the data-driven bandwidth selection; see section 3.2.

`whitening` performs prewhitening to reduce small-sample bias in HAC estimation. It is only allowed with `hac`.

`crosssectional(`*varlist_cr*[`, cr_lags(`*numlist*`)`]`)` defines the variables to be added as CSA to approximate strong CSD. Variables in *varlist_cr* are partialed out. The suboption `cr_lags(`*numlist*`)` sets the number of lags of the CSA. If not defined, but `crosssectional()` contains a *varlist*, then contemporaneous CSA are added but no lags. `cr_lags(0)` is the equivalent. The number of lags can be variable specific, where the order is the same as defined in `crosssectional()`. For example, if `crosssectional(y x)` and only contemporaneous CSA of `y` but 2 lags of `x` are added, then `cr_lags(0 2)`.

`nooutput` omits output.

`comparehac` compares the standard delta test with the HAC robust version. The standard delta test is first run, then the HAC robust version. Results for both tests are displayed. If the tests disagree, a message is posted. In addition, the base of all variables is tested for CSD using `xtcd2` (Ditzen 2018). If CSD is found, a message is posted. The options `crosssectional()`, `partial()`, and `noconstant` are held constant across both tests. All HAC-related options apply only to the HAC robust run. This option is only for testing purposes and should not replace further testing.

Stored results

`xthst` stores the following in `r()`:

Scalars

`r(bw)`	bandwidth

Macros

`r(cross-sectional)`	variables of which cross-section averages are added
`r(partial)`	variables partialed out
`r(kernel)`	used kernel

Matrices

`r(delta)`	delta and adjusted delta
`r(delta_p)`	p-values of the above

3.2 Kernel and bandwidth for the HAC robust test

Three different kernels for the estimation of the variance–covariance matrix when using the HAC robust test are built into `xthst`. The kernels are the Bartlett, the quadratic spectral (QS), and the truncated kernel. If the bandwidth is not manually chosen, `xthst` opts for a data-dependent selection based on the chosen kernel. The latter follows Newey and West (1994),

$$B_{i,T_i} = \left\lfloor c\left\{\alpha_i(q)^2 T_i\right\}^{1/(2q+1)}\right\rfloor$$

where scalars c and q depend on the type of kernel. When the Truncated kernel is applied, $\kappa = 1$ and $B_{i,T_i} = \lfloor 4(T_i/100)^{1/5}\rfloor$ (Newey and West 1994). For the QS kernel, the parameters are $c = 1.3221$ and $q = 2$, while for the Bartlett kernel, $c = 1.1447$ and $q = 1$; see Andrews (1991) and Andrews and Monahan (1992).

In the QS case, $\alpha_i(2)$ follows Andrews (1991),

$$\alpha_i(2) = \sum_{a=1}^{k_2} \frac{4\widehat{\rho}_{i,a}^2 \widehat{\sigma}_{i,a}^4}{(1-\widehat{\rho}_{i,a})^8} \Big/ \sum_{a=1}^{k_2} \frac{\widehat{\sigma}_{i,a}^4}{(1-\widehat{\rho}_{i,a})^4} \tag{7}$$

Applying an AR(1) model on $\widehat{\widehat{\mathbf{u}}}_{i,t}$ for each i, where $\widehat{\widehat{\mathbf{u}}}_{i,t}$ is $k_2 \times 1$, one obtains the estimated autoregressive coefficient $\widehat{\rho}_{i,a}$ and variance $\widehat{\sigma}_{i,a}^2$, which are used in (7).

For the Bartlett kernel case, $\alpha_i(1)$ is estimated according to Newey and West (1994),

$$\alpha_i(1) = \frac{2\sum_{s=1}^{r} s\widehat{\sigma}_{i,s}}{\widehat{\sigma}_{i,0} + 2\sum_{s=1}^{r}\widehat{\sigma}_{i,s}}$$

where $r = \lfloor 4(T_i/100)^{2/9}\rfloor$ and $\widehat{\sigma}_{i,s} = (T_i-1)^{-1}\sum_{t=j+1}^{T_i} \widehat{\widehat{\mathbf{u}}}_{i,t}\widehat{\widehat{\mathbf{u}}}_{i,t-j}$.

The `xthst` command offers prewhitening to reduce the small-sample bias in a HAC estimation, in line with Blomquist and Westerlund (2013). Applying the prewhitening option replaces $\widehat{\widehat{\mathbf{u}}}_{i,t}$ from above with $\widehat{\widehat{\mathbf{u}}}_{i,t}^* = \widehat{\widehat{\mathbf{u}}}_{i,t} - \widehat{\boldsymbol{\rho}}_i'\widehat{\widehat{\mathbf{u}}}_{i,t-1}$, where $\widehat{\boldsymbol{\rho}}_i$ are the coefficients from the AR(1) model on $\widehat{\widehat{\mathbf{u}}}_{i,t}$ for each i.

4 Examples

In this section, we carry out several examples, all drawing on a growth model using the Penn World Tables 8.0 (Feenstra, Inklaar, and Timmer 2015). We restrict the dataset to 48 years between 1960 and 2007 and 93 countries. The Penn World Tables include data until 2011, but data from 2008 onward are excluded because of the financial crisis. First, we give an example for the standard $\widetilde{\Delta}$ test; then, we give examples for testing a subset of coefficients, the HAC, and the CSD robust extensions.

4.1 Standard $\widetilde{\Delta}$ test

In this section, we want to test whether the coefficients of a cross-country growth regression are homogeneous or heterogeneous. To do so, we fit an economic growth model along the lines of Mankiw, Romer, and Weil (1992), Islam (1995), and Lee, Pesaran, and Smith (1997). The dependent variable is real gross domestic product (GDP) per capita growth in logarithms, `log_rgdpo`. The explanatory variables are human capital, `log_hc`, physical capital, `log_ck`, and population growth added with break-even investments of 5%, `log_ngd`. All variables are in logarithms.

For a first exemplified model, we assume a static model; hence, no lag of the dependent variable occurs. We want to test whether any of the slope coefficients are homogeneous or heterogeneous. The command line and output are

```
. use xthst_sample_dataset

. xtset
       panel variable:  id (strongly balanced)
        time variable:  year, 1960 to 2007
                delta:  1 unit

. xthst d.log_rgdpo log_hc log_ck log_ngd
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
─────────────────────────────────────

         Delta          p-value
          6.328           0.000
 adj.     6.694           0.000
─────────────────────────────────────
Variables partialled out: constant
```

`xthst` automatically assumes a heterogeneous constant. The delta test statistic is sufficiently large to reject the null of slope homogeneity. Therefore, when running this model, one should use an estimator allowing for heterogeneous slopes, such as the mean group estimator.

In the next step, we add the first lag of GDP growth, so the regression model is an actual growth model. We extend the command line from above with `L.d.log_rgdpo`:

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
─────────────────────────────────────

         Delta          p-value
          2.957           0.003
 adj.     3.171           0.002
─────────────────────────────────────
Variables partialled out: constant
```

Once again, we can comfortably reject the null at a level of 5%. However, we note that the value of the test statistic decreased.

4.2 Testing a subset of coefficients

If the assumption is that all variables except the lag of GDP growth are heterogeneous, the `partial(`*varlist_p*`)` option can be used. In this case, all variables in *varlist_p* are partialed out and assumed to be heterogeneous:

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd,
> partial(log_hc log_ck log_ngd)
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
─────────────────────────────────────
         Delta          p-value
          2.324           0.020
 adj.     2.409           0.016
─────────────────────────────────────
Variables partialled out: log_hc log_ck log_ngd constant
```

The test confirms that the coefficient of the lag of GDP growth is heterogeneous. The test statistic decreased in comparison with the model above.

4.3 Allowing for heteroskedastic and serially correlated errors

In a dynamic macrodataset, it is likely that errors exhibit serial correlation. To account for autocorrelation in the residual, one can use the option `hac` to use the HAC robust standard errors following Blomquist and Westerlund (2013):

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd, hac
Testing for slope heterogeneity
(Blomquist, Westerlund. 2013. Economic Letters)
H0: slope coefficients are homogenous
─────────────────────────────────────
         Delta          p-value
         12.203           0.000
 adj.    13.086           0.000
─────────────────────────────────────
HAC Kernel: bartlett
with average bandwith 3
Variables partialled out: constant
```

The test for slope homogeneity becomes heteroskedastic robust by using a HAC robust estimator for the variance, which relies on a kernel function with a given bandwidth `bw()`. The default is to use a Bartlett kernel with automatically selected bandwidth following Andrews and Monahan (1992) and Newey and West (1994). Besides the Bartlett kernel, `xthst` supports the QS and truncated kernels. The kernels can be set with the option `kernel()` and the bandwidth with `bw()`. To use the QS kernel with bandwidth 5, type

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd, hac bw(5) kernel(qs)
Testing for slope heterogeneity
(Blomquist, Westerlund. 2013. Economic Letters)
H0: slope coefficients are homogenous
-------------------------------------

         Delta          p-value
         -1.843           0.065
 adj.    -1.977           0.048
-------------------------------------

HAC Kernel: quadratic spectral (QS)
with average bandwith 5
Variables partialled out: constant
```

The Monte Carlo simulations in section 5 show that the performance of the delta test crucially depends on the assumption on the residuals, in particular whether autocorrelation is present. To guide the user to obtain the optimal settings, the option `comparehac` compares the results from the standard delta test with its HAC robust equivalent. If the results with respect to the confidence level specified by `c(level)` disagree, a warning is shown. In addition, the variables are tested for CSD if `xtcd2` (Ditzen 2018) is installed. The options `noconstant`, `crosssectional()`, and `partial()` are applied to both tests.

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd, comparehac
Testing for slope heterogeneity
H0: slope coefficients are homogenous
-------------------------------------

         Delta          p-value
          2.957           0.003
 adj.     3.171           0.002
-------------------------------------

         Delta (HAC)    p-value
         -0.534           0.593
 adj.    -0.573           0.567
-------------------------------------

Tests disagree. Autocorrelation might occur.
See helpfile for further info.
HAC Settings:
       Kernel: quadratic spectral (QS)
       with average bandwith 45
Variables partialled out: constant
Cross Sectional dependence in base variables detected:
    D.log_rgdpo LD.log_rgdpo log_hc log_ck log_ngd
See helpfile for xthst and xtcd2 for further info.
```

In the example above, we find that the variables contain CSD that needs to be accounted for. The standard delta test and the HAC robust version lead to different results. `xthst` can guide the user to the correct specification, but further testing for autocorrelation is left to the user.

4.4 Accounting for CSD

In large panels, CSD is likely, is mostly unobserved, and, if untreated, leads to biased and inconsistent regression estimates. A popular method to approximate strong CSD is to add CSA as further covariates. This estimator is known as the CCE estimator (Pesaran

2006; Chudik and Pesaran 2015b).[1] In Stata, the community-contributed command `xtdcce2` (Ditzen 2018) introduced the CCE estimator.

Along those lines, `xthst` can take out strong CSD by approximating it with CSA, thus comparing a CCE pooled and mean group estimator. The CSA are partialed out and can be defined by the option `crosssectional(`*varlist_cr* [`, lags(`*numlist*`)`]`)`. *varlist_cr* is a variable list containing the variables from which the CSA are derived. The optional *numlist* defines the number of lags of the CSA. If not defined, only the base of the CSA, the contemporaneous CSA, is added.

We can use `xtcd2` or `xtcse2` (Ditzen 2018, 2019) to test for weak CSD and estimate the strength of it. The result implies strong CSD for all variables, urging the inclusion of CSA. We follow the theory in Chudik and Pesaran (2015b) and add $\lfloor T^{1/3} \rfloor = \lfloor 38^{1/3} \rfloor = \lfloor 3.36 \rfloor = 3$ lags.

```
. xthst d.log_rgdpo L.d.log_rgdpo log_hc log_ck log_ngd,
> crosssectional(d.log_rgdpo log_hc log_ck log_ngd, cr_lags(3))
Testing for slope heterogeneity
(Pesaran, Yamagata. 2008. Journal of Econometrics)
H0: slope coefficients are homogenous
-------------------------------------
          Delta          p-value
          5.286            0.000
 adj.     5.994            0.000
-------------------------------------
Variables partialled out: constant
Cross Sectional Averaged Variables: D.log_rgdpo log_hc log_ck log_ngd
```

5 Monte Carlo

In this section, we assess the finite sample properties using a Monte Carlo simulation. We focus on the size and the power of the delta test. The simulation setup follows Pesaran and Yamagata (2008) and Blomquist and Westerlund (2013), but we add further CSD via the independent and dependent variables.

The data-generating process (DGP) for the simulation with k regressors is

$$y_{i,t} = \mu_i + \sum_{l=1}^{k} \beta_{l,i} x_{i,l,t} + u_{i,t}$$

$$x_{i,l,t} = \mu_i \left(1 - \rho_{x,i,l}\right) + \rho_{x,i,l} x_{i,l,t-1} + \left(1 - \rho_{x,i,l}\right)^{\frac{1}{2}} v_{i,l,t}$$

1. For a summary on CSD, see Chudik and Pesaran (2015a).

The error component $u_{i,t}$ contains serial correlation, if $\rho_{u,i} > 0$, and is heteroskedastic in all specifications. The error components of the independent and identically distributed (i.i.d.) variables, $v_{i,t}$, are white noise with a unit-specific variance and are generated as

$$\begin{aligned} u_{i,t} &= \rho_{u,i} u_{i,t-1} + \sqrt{1-\rho_{u,i}^2}\,(\gamma_{u,i} f_t + e_{i,t}) \\ e_{i,t} &\sim N(0,\sigma_{i,e}^2) \text{ with } \sigma_{i,e}^2 = \frac{k\chi^2(2)}{2} \\ v_{i,l,t} &= \gamma_{x,i,l} f_t + \epsilon_{i,l,t} \\ \epsilon_{i,l,t} &\sim \text{i.i.d. } N(0,\sigma_{\epsilon,i,l}^2) \text{ with } \sigma_{\epsilon,i,l} \sim \text{i.i.d.}\chi^2(1) \end{aligned}$$

The autocorrelation coefficients of the independent variables are generated as $\rho_{x,i,l} \sim$ i.i.d. $U(0.05, 0.95)$. The generation of CSD follows Chudik and Pesaran (2015b) and is introduced by the terms $\gamma_{x,i,l} f_t$ and $\gamma_{u,i} f_t$. The common factors f_t are generated as $f_t = \rho_f f_{t-1} + \xi_t$, $\xi_t \sim$ i.i.d. $N(0, 1-\rho_f^2)$. ρ_f is varied between 0 (no CSD) and 0.8 (CSD). The generation of CSD via the independent and dependent variables is an extension to the existing literature. Pesaran and Yamagata (2008) do not consider any CSD in the DGP in their Monte Carlo simulations. In Blomquist and Westerlund (2013), CSD enters via the error component. The factor loadings $\gamma_{x,i,l}$ and $\gamma_{u,i}$ are centered on a common mean,

$$\begin{aligned} \gamma_{u,i} &= \gamma_u + \eta_{u,i} & \gamma_{x,i,l} &= \gamma_{x,l} + \eta_{x,i,l} \\ \eta_{u,i} &\sim \text{i.i.d.} N(0,\sigma_{\gamma,l}^2) & \eta_{x,i,l} &= \text{i.i.d. } N(0,\sigma_{x,\gamma,l}^2) \\ \gamma_u &= \sqrt{\frac{1}{m} - \sigma_{y,\gamma,l}^2} & \gamma_{x,l} &= \sqrt{l\left(\frac{2}{m(m+1)} - \frac{2}{m+1}\sigma_{x,\gamma,l}^2\right)} \\ \sigma_{\gamma,l}^2 &= 0.2^2 & \sigma_{x,\gamma,l}^2 &= \sigma_{y,\gamma,l}^2 = 0.2^2 \end{aligned}$$

where $m = k$ is the number of regressors.

For serial correlated errors, the autocorrelation coefficients of $u_{i,t}$ are generated as $\rho_{u,i} \sim$ i.i.d. $U(0, \rho_u)$, whereas ρ_u is varied between 0 (no serial correlation) and 0.7 (serial correlation).

The main focus of the Monte Carlo simulation exercise will lie on the coefficient $\beta_{l,i}$. Under the null hypothesis of homogeneous slopes, the coefficients are set to unity, $\beta_{l,i} = 1$. Under the alternative, the first $N/2$ coefficients are set to unity; the remaining coefficients are drawn from a normal distribution:

$$\begin{aligned} \beta_{l,i} &= 1 & &\text{for } i = 1,\ldots,\frac{N}{2} \text{ and } l = 1,\ldots,k \\ \beta_{l,i} &\sim N(1, 0.04) & &\text{for } i = \frac{N}{2}+1,\ldots,N \text{ and } l = 1,\ldots,k \end{aligned}$$

For simplicity, it is assumed that all k coefficients are the same; hence, $\beta_{l,i} = \beta_{1i}$. Under the alternative, the coefficients are generated as $\beta_{l,i} \sim N(1, 0.04)$ for $i > (N/2), l =$

$1, \ldots, k$. We vary the number of coefficients between $k = 1$ and $k = 4$. In the special case of $k = 4$, the first h coefficients are generated as heterogeneous even under the null hypothesis. These coefficients are then partialed out. We vary h between 0 and 1. The unit-specific FE is generated as $\mu_i \sim N(1, 1)$.

In the simulations, we observe 4 cases, one without serial correlation and cross-sectional dependence (specification 1), one with either serial correlation or CSD (specifications 2 and 3), and a combination of both (specification 4). To make things easy, we focus on simulations with one regressor. Results with four regressors are available in appendix A.1 and described in more detail in Bersvendsen and Ditzen (2020).

5.1 Tests

We are comparing the results for the standard delta test ($\widetilde{\Delta}$) and for the HAC ($\widetilde{\Delta}_{\mathrm{HAC}}$) and CSD robust versions ($\widetilde{\Delta}_{\mathrm{CSA}}$). For the CSD robust version, only contemporaneous values of the CSA are added, $p_{\mathrm{CSA}} = 0$. Following Blomquist and Westerlund (2013), the HAC robust delta test performs best with prewhitening and the QS kernel. To save space, we focus on the prewhitened delta test with the QS kernel, $\widetilde{\Delta}_{\mathrm{HAC(QS)+Whitening}}$. We also use a mix of the HAC and CSA robust tests, $\widetilde{\Delta}_{\mathrm{HAC(QS)+Whitening+CSA}}$.

For the specification without CSD and serially correlated errors, we expect the standard delta test to perform best (specification 1 in the tables; see table 1 in appendix A.1 and sections 1–5 in figure 1). For specification 2 (sections 11–14) and specification 3 (sections 6–10), the HAC robust and CSD robust tests, respectively, should show the best size and power. Pesaran and Yamagata (2008) find that an increase in the number of regressors leads to lower performance of the tests.

5.2 Results

We present the simulation results using nested loop plots (Rücker and Schwarzer 2014). The corresponding tables can be found in appendix A.1. For all simulations, the number of cross-sectional units and time periods is varied between 20 and 200. The main focus is on the size and the power of the test. We present the size as the rejection frequencies in percent if the hypothesis is true, that is, the number of times the delta test falsely rejects the hypothesis of homogeneous slope coefficients. The power of the test is the rejection frequency if the hypothesis is false, meaning when the true coefficients are heterogeneous. The size and power are evaluated at a level of 5%.

Figure 1 displays the simulation results. The upper third of the figure shows the size, the middle the power. The different parameter settings are displayed in the lower third. For better readability, we omit the grid lines for different numbers of cross-sectional units. Each section marked with 1 to 20 represents a given parameterization with a fixed number of time periods, autocorrelation, and CSD; the number of cross-sectional units increases from 20 to 200 in each block.

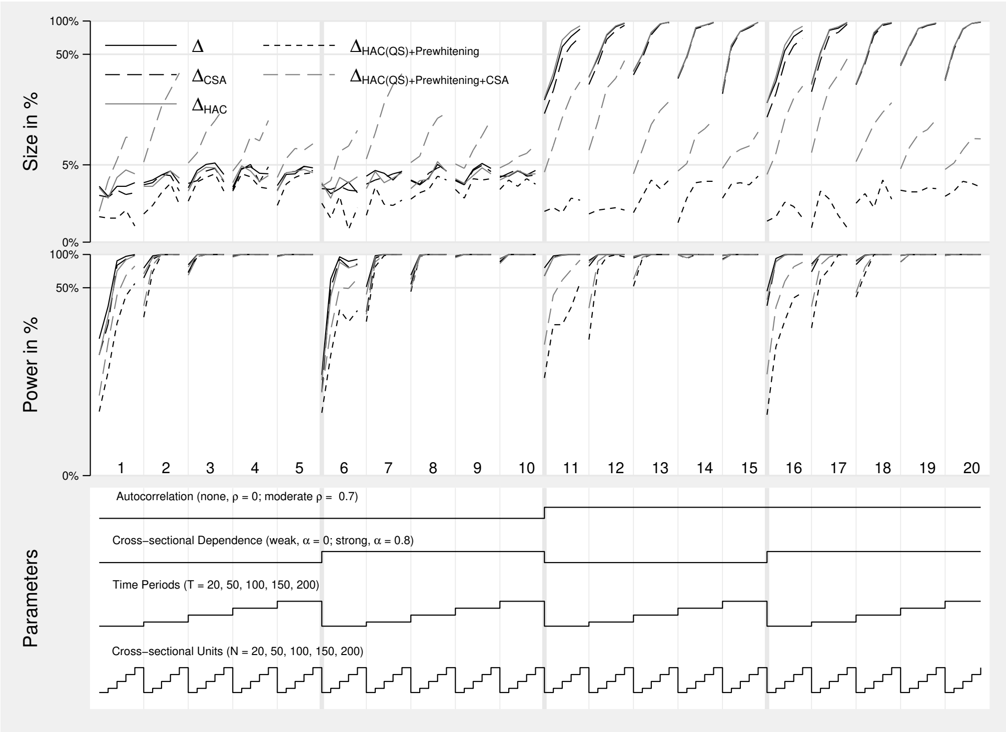

Figure 1. Nested loop plot of Monte Carlo simulation results. The vertical axis is scaled in logarithms. Each section, marked 1 to 20, represents a parameterization with a fixed number of time periods (T), degree of CSD (α), and autocorrelation (ρ). The number of cross-sectional units is increased from 20 to 200. Size and power are evaluated at a level of 5%. Δ is the standard delta test, described in (3), Δ_{HAC} is the HAC robust version from (4), Δ_{CSA} is the CSD robust test from section 2.3. $\Delta_{\text{HAC(QS)+Prewhitening}}$ is the HAC robust test with QS kernel and prewhitening, and $\Delta_{\text{HAC(QS)+Prewhitening+CSA}}$ is the HAC and CSD robust test with CSA, QS kernel, and prewhitening.

In the case of no autocorrelation (sections 1–10), all tests except $\widetilde{\Delta}_{\text{HAC(QS)+Prewhitening+CSA}}$ are below the 5% line and slightly undersized. Within an increase in time periods, the size of the test moves closer to the nominal level of 5%.

The power of the standard delta test performs correspondingly to the size. For combinations of N and T with N or T, or both, being small, the power is lower. For large N and T, the power reaches 100%. In particular, an increase in the number of periods T leads to a better power. Our results for the power and size confirm the findings in Pesaran and Yamagata (2008).

When one adds CSD and no autocorrelation (sections 6–10), the standard delta test and the CSD robust delta test behave similarly for all combinations of N and T. There are two potential reasons for this. First, the CSA might take out some of the heterogeneous variation. Second, the bias of the pooled and mean group estimator is of a similar magnitude and direction. This applies to the infeasible standard OLS estimator that $\widetilde{\Delta}$ is based on as well as to the CCE-type estimators that $\widetilde{\Delta}_{\text{CSA}}$ is based on, as shown in Pesaran (2006, table I). These results highlight that the differences of the two tests might come at low cost and gain. However, the correct method to be applied for datasets with CSD is the $\widetilde{\Delta}_{\text{CSA}}$ test. We have not outlined the proof for the latter. Surprisingly, the $\widetilde{\Delta}_{(\text{QS})+\text{Prewhitened}}$ is marginally outperformed by the $\widetilde{\Delta}_{\text{CSA}}$, which is slightly closer to the nominal value of 5%.

Sections 11–15 show results with serially correlated errors $\rho_u = 0.7$. The standard test performs badly in terms of the size, reaching almost under 100% for all combinations of N and T. The cross-sectional robust version of the test performs somewhat better; however, it is still oversized. This result underlines the importance of a serial correlation robust version of the delta test.

The HAC robust delta test is heavily oversized; even the small-sample-adjusted test never reaches the nominal value of 5%. The equivalence holds for the power of the test. The test lacks power in small samples, especially when the number of time periods is small. However, when one uses the QS kernel with prewhitening, the HAC robust test is superior. This finding is in line with the simulation results in Blomquist and Westerlund (2013). In their simulations, the HAC robust test performs best in panels with serial correlation. Therefore, we strongly encourage users to apply this test, despite its shortcomings in size when applied to a wrong model.

The DGP in sections 16–20 contains CSD and serially correlated errors. As with serial correlation, the standard delta test and its CSD robust counterpart are oversized. The HAC robust delta with the QS kernel and prewhitening test performs surprisingly well but is slightly undersized. To encompass this, we use an additional testing procedure, $\Delta_{\text{HAC+CSA}}$, that first takes out strong CSD by partialing out the CSA and then uses the HAC robust delta test. While this test is oversized, the power of the test is much better than the one of the HAC robust delta test with QS kernel and prewhitening.

We present further Monte Carlo simulation results in appendix A.1 that confirm findings in Pesaran and Yamagata (2008) and Blomquist and Westerlund (2013). We find that the Bartlett kernel leads to oversized test statistics. The truncated kernel suffers from an oversize for small N and large T panels, but for large N and T panels, the size comes close to its nominal value. As found in Blomquist and Westerlund (2013), prewhitening leads to much better results for the QS kernel. Once again, the test lacks power in small samples. In general, the results strongly suggest to use the QS kernel in combination with prewhitening.

In further simulations, we extend the DGP to include four regressors. In general, the results are similar to those with only one regressor. However, the power of the tests is below those with only a single regressor. Our findings are in line with Blomquist and Westerlund (2013).

As a final exercise, we also check results with four regressors of which one is heterogeneous $h = 1$. Both the standard delta test and the CSD robust test have a size above their nominal value; however, in most cases, it is well below 10%. The result is expected because it is harder for the test to identify the correct heterogeneous slope coefficients. This translates into a lower power. However, for large combinations of N and T, the size and power are in acceptable regions around 5%, respectively around 90%. The oracle test, which partials out the correct variable, performs reasonably well. This implies that if a variable is known to have a heterogeneous slope parameter, partialing it out works well.

In general, the simulations confirm results established in the literature for the standard and HAC robust delta test. The correct choice of the test is crucial, and results can vary hugely. In particular, autocorrelation has a strong influence, especially on the size of the test. The extension that takes out CSD works well and can be used if CSD is suspected.

6 Conclusion

This article introduced and discussed `xthst`, a community-contributed command for Stata. `xthst` implements tests for slope heterogeneity in panels with many periods over time (T) and cross-sectional units (N). Three different tests were considered: the standard delta test following Pesaran and Yamagata (2008), a HAC robust version following Blomquist and Westerlund (2013), and a CSD robust version. `xthst` supports different kernel estimators for HAC robust variance estimators. The bandwidth can be chosen by hand or by a data-driven method. We gave several examples testing slope homogeneity in an economic growth model. We showed that all three tests behave as expected using a Monte Carlo simulation. However, when the test is applied incorrectly, test results vary hugely. Therefore, a careful assessment of the properties of the data, in particular with respect to CSD and autocorrelation, is essential. While the Monte Carlo results for the CSD robust test are promising and show that the method works, the formal derivation of a test is left for further research. `xthst` can help the user find the optimal settings with the option `comparehac`.

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8 Programs and supplemental materials

To install a snapshot of the corresponding software files as they existed at the time of publication of this article, type

```
. net sj 21-1
. net install st0627     (to install program files, if available)
. net get st0627         (to install ancillary files, if available)
```

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A Appendix

A.1 Monte Carlo simulation

Table 1. $\rho_f > 0$ indicates CSD, $\rho_u > 0$ indicates serial correlation in the errors, and h is the number of heterogeneous coefficients under the null. When $k = 1$, then $h = 0$.

Specification	ρ_f	ρ_u	Table	Specification	ρ_f	ρ_u	h	Table
$k = 1$				$k = 4$				
1	0	0	Table 2	5	0	0	0	Table 7
2	0.8	0	Tables 3 & 4	6	0.8	0	0	Table 8
3	0	0.7	Table 5	7	0	0.7	0	Table 9
4	0.8	0.7	Table 6	8	0.8	0.7	0	Table 10
				9	0	0	1	Table 11

Results with (k=1)

Table 2. Specification 1—Size and power of delta test with no CSD and heteroskedastic normal independent and identically distributed (i.i.d.) errors. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. One exogenous regressor. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	3.20	3.45	3.10	3.15	2.95	17.30	75.35	80.70	98.60	97.75
	(5.05)	(3.70)	(3.30)	(3.30)	(3.10)	(20.05)	(76.15)	(80.80)	(98.65)	(97.75)
50	2.60	3.65	4.45	4.65	4.10	34.05	97.10	99.65	100.00	100.00
	(3.70)	(4.30)	(4.80)	(4.90)	(4.25)	(38.55)	(97.25)	(99.65)	(100.00)	(100.00)
100	3.20	4.05	5.05	4.80	4.15	87.60	100.00	100.00	100.00	100.00
	(5.15)	(4.75)	(5.50)	(5.20)	(4.30)	(89.75)	(100.00)	(100.00)	(100.00)	(100.00)
150	3.20	4.40	5.20	4.20	4.85	96.15	100.00	100.00	100.00	100.00
	(4.65)	(5.00)	(5.70)	(4.40)	(5.10)	(97.10)	(100.00)	(100.00)	(100.00)	(100.00)
200	3.45	3.40	4.10	4.15	4.70	99.85	100.00	100.00	100.00	100.00
	(4.95)	(4.30)	(4.40)	(4.50)	(4.80)	(99.85)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(QS)+Prewhitened}}$										
20	1.70	1.80	2.35	2.90	2.15	3.80	27.10	68.35	96.00	96.45
	(2.55)	(2.40)	(2.50)	(2.95)	(2.20)	(5.75)	(28.85)	(68.75)	(96.25)	(96.55)
50	1.65	2.15	3.50	4.15	3.35	8.65	69.30	94.65	99.95	100.00
	(2.60)	(2.45)	(3.75)	(4.30)	(3.60)	(11.40)	(70.80)	(94.80)	(99.95)	(100.00)
100	1.65	2.80	3.85	3.90	3.85	23.90	99.40	100.00	100.00	100.00
	(2.70)	(3.15)	(4.35)	(4.10)	(3.90)	(29.60)	(99.45)	(100.00)	(100.00)	(100.00)
150	1.95	3.40	4.15	3.15	4.15	42.30	99.65	100.00	100.00	100.00
	(3.05)	(3.95)	(4.45)	(3.25)	(4.30)	(49.10)	(99.70)	(100.00)	(100.00)	(100.00)
200	1.40	2.20	2.90	3.05	4.25	54.60	100.00	100.00	100.00	100.00
	(2.60)	(2.45)	(3.10)	(3.15)	(4.30)	(61.25)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{CSA}}$										
20	2.65	3.30	3.40	3.00	3.25	12.40	61.35	69.25	96.25	95.85
	(4.25)	(4.20)	(3.85)	(3.35)	(3.40)	(15.10)	(62.75)	(69.95)	(96.35)	(95.85)
50	2.55	3.45	3.60	4.55	4.30	23.55	90.40	99.05	100.00	100.00
	(4.35)	(4.40)	(4.20)	(4.60)	(4.50)	(28.55)	(91.25)	(99.05)	(100.00)	(100.00)
100	2.95	4.10	4.40	5.05	4.35	79.70	100.00	100.00	100.00	100.00
	(5.40)	(5.10)	(4.70)	(5.25)	(4.75)	(83.65)	(100.00)	(100.00)	(100.00)	(100.00)
150	2.70	4.00	4.65	3.50	4.85	90.75	100.00	100.00	100.00	100.00
	(4.10)	(4.90)	(5.15)	(3.65)	(4.95)	(92.75)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.80	2.90	3.50	4.80	4.40	98.95	100.00	100.00	100.00	100.00
	(4.55)	(3.75)	(4.00)	(5.05)	(4.50)	(99.30)	(100.00)	(100.00)	(100.00)	(100.00)

Table 3. Specification 2—Size and power of delta test with no CSD and serially correlated errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\mathrm{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\mathrm{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\mathrm{adj}}$ are given in parentheses. One exogenous regressor. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	19.30	31.25	35.10	31.00	23.50	75.65	88.15	95.80	99.90	98.55
	(22.90)	(32.80)	(35.70)	(31.50)	(23.85)	(78.25)	(88.85)	(95.90)	(99.90)	(98.60)
50	30.00	46.30	49.65	48.10	60.40	96.95	99.55	100.00	99.70	100.00
	(35.10)	(47.90)	(50.40)	(48.65)	(60.60)	(97.70)	(99.55)	(100.00)	(99.70)	(100.00)
100	58.55	74.40	78.80	86.90	80.45	99.35	100.00	100.00	100.00	100.00
	(63.10)	(75.95)	(79.30)	(87.25)	(80.70)	(99.60)	(100.00)	(100.00)	(100.00)	(100.00)
150	72.55	88.60	94.05	91.50	88.80	99.95	100.00	100.00	100.00	100.00
	(76.35)	(89.60)	(94.20)	(91.65)	(89.00)	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)
200	84.45	95.25	97.50	97.30	97.15	100.00	100.00	100.00	100.00	100.00
	(87.05)	(95.40)	(97.50)	(97.50)	(97.25)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{HAC(QS)+Prewhitened}}$										
20	1.90	1.80	1.85	1.50	2.50	7.65	17.00	51.60	99.45	91.20
	(2.55)	(2.10)	(1.95)	(1.75)	(2.60)	(9.40)	(18.50)	(52.10)	(99.45)	(91.30)
50	2.05	1.95	2.60	2.50	3.35	23.15	72.30	92.05	92.10	99.70
	(3.80)	(2.30)	(2.80)	(2.60)	(3.50)	(27.60)	(74.15)	(92.40)	(92.25)	(99.70)
100	1.85	2.00	3.65	3.45	3.45	23.20	95.00	100.00	100.00	100.00
	(2.70)	(2.65)	(3.85)	(3.75)	(3.55)	(28.55)	(95.80)	(100.00)	(100.00)	(100.00)
150	2.50	2.05	3.05	3.55	3.30	33.25	99.30	100.00	100.00	100.00
	(3.85)	(2.65)	(3.45)	(3.70)	(3.45)	(38.85)	(99.50)	(100.00)	(100.00)	(100.00)
200	2.40	1.95	3.65	3.55	3.95	55.35	95.15	100.00	100.00	100.00
	(3.65)	(2.45)	(3.95)	(3.60)	(4.00)	(60.70)	(95.55)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{CSA}}$										
20	14.55	26.65	32.55	30.15	21.95	63.95	78.45	92.90	99.95	97.70
	(18.55)	(28.10)	(33.35)	(30.70)	(22.15)	(68.05)	(79.30)	(92.95)	(99.95)	(97.75)
50	23.50	40.70	46.10	46.70	57.25	93.90	99.00	99.75	99.55	100.00
	(28.55)	(42.90)	(47.00)	(47.55)	(57.45)	(95.20)	(99.20)	(99.75)	(99.55)	(100.00)
100	45.70	68.20	75.10	85.25	79.75	98.55	100.00	100.00	100.00	100.00
	(51.80)	(70.90)	(75.80)	(85.55)	(80.00)	(99.00)	(100.00)	(100.00)	(100.00)	(100.00)
150	60.10	81.55	93.25	90.10	86.80	99.75	100.00	100.00	100.00	100.00
	(65.00)	(83.00)	(93.55)	(90.50)	(87.05)	(99.75)	(100.00)	(100.00)	(100.00)	(100.00)
200	71.90	91.45	96.55	96.80	96.45	100.00	100.00	100.00	100.00	100.00
	(75.55)	(92.30)	(96.75)	(96.95)	(96.55)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Table 4. Specification 2—Size and power of delta test with no CSD and serially correlated errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. One exogenous regressor. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}_{\text{HAC(Bartlett)}}$										
20	20.05	30.75	34.70	31.00	23.20	64.35	84.50	94.90	100.00	98.40
	(23.85)	(32.35)	(35.15)	(31.60)	(23.50)	(68.70)	(85.15)	(94.90)	(100.00)	(98.40)
50	33.50	48.00	49.55	47.45	60.15	93.15	99.35	100.00	99.60	100.00
	(38.95)	(49.30)	(50.55)	(47.90)	(60.45)	(94.55)	(99.40)	(100.00)	(99.65)	(100.00)
100	67.35	75.65	78.90	87.35	80.75	98.85	100.00	100.00	100.00	100.00
	(71.30)	(77.15)	(79.80)	(87.75)	(81.25)	(99.05)	(100.00)	(100.00)	(100.00)	(100.00)
150	81.05	90.30	94.35	91.95	88.85	99.85	100.00	100.00	100.00	100.00
	(85.40)	(90.95)	(94.45)	(92.05)	(89.45)	(99.85)	(100.00)	(100.00)	(100.00)	(100.00)
200	90.40	96.25	97.90	97.45	97.25	100.00	100.00	100.00	100.00	100.00
	(92.95)	(96.70)	(98.00)	(97.50)	(97.30)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(QS)}}$										
20	1.00	2.25	2.65	3.35	4.60	6.25	18.95	43.25	99.60	92.15
	(1.55)	(2.85)	(3.10)	(3.55)	(4.75)	(7.95)	(20.50)	(44.10)	(99.60)	(92.30)
50	1.20	3.05	4.55	4.60	5.60	23.50	82.75	94.40	94.60	99.70
	(2.10)	(3.60)	(4.85)	(4.70)	(5.85)	(28.95)	(84.00)	(94.60)	(94.70)	(99.70)
100	0.95	4.00	5.90	6.85	7.20	24.25	98.25	100.00	100.00	100.00
	(1.90)	(4.90)	(6.50)	(7.05)	(7.50)	(30.50)	(98.50)	(100.00)	(100.00)	(100.00)
150	0.75	4.05	8.20	7.50	9.10	32.60	99.80	100.00	100.00	100.00
	(1.65)	(4.75)	(8.70)	(7.85)	(9.45)	(38.65)	(99.80)	(100.00)	(100.00)	(100.00)
200	1.30	5.45	9.70	9.30	9.35	58.85	98.75	100.00	100.00	100.00
	(2.45)	(6.05)	(10.00)	(9.75)	(9.70)	(65.65)	(99.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(QS)+Whitening}}$										
20	1.90	1.80	1.85	1.50	2.50	7.65	17.00	51.60	99.45	91.20
	(2.55)	(2.10)	(1.95)	(1.75)	(2.60)	(9.40)	(18.50)	(52.10)	(99.45)	(91.30)
50	2.05	1.95	2.60	2.50	3.35	23.15	72.30	92.05	92.10	99.70
	(3.80)	(2.30)	(2.80)	(2.60)	(3.50)	(27.60)	(74.15)	(92.40)	(92.25)	(99.70)
100	1.85	2.00	3.65	3.45	3.45	23.20	95.00	100.00	100.00	100.00
	(2.70)	(2.65)	(3.85)	(3.75)	(3.55)	(28.55)	(95.80)	(100.00)	(100.00)	(100.00)
150	2.50	2.05	3.05	3.55	3.30	33.25	99.30	100.00	100.00	100.00
	(3.85)	(2.65)	(3.45)	(3.70)	(3.45)	(38.85)	(99.50)	(100.00)	(100.00)	(100.00)
200	2.40	1.95	3.65	3.55	3.95	55.35	95.15	100.00	100.00	100.00
	(3.65)	(2.45)	(3.95)	(3.60)	(4.00)	(60.70)	(95.55)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(truncated)}}$										
20	5.30	3.60	2.30	1.70	2.70	12.00	20.15	46.85	98.65	88.40
	(6.60)	(4.00)	(2.40)	(1.95)	(2.85)	(15.10)	(21.45)	(47.65)	(98.65)	(88.70)
50	11.75	5.95	3.65	2.40	3.70	37.15	67.80	85.35	87.15	99.55
	(13.65)	(6.60)	(3.75)	(2.55)	(3.85)	(43.35)	(70.20)	(85.90)	(87.35)	(99.55)
100	20.10	9.45	5.40	3.70	3.35	53.50	95.90	99.95	100.00	100.00
	(23.00)	(10.25)	(5.65)	(3.85)	(3.80)	(59.90)	(96.70)	(99.95)	(100.00)	(100.00)
150	30.10	12.55	6.50	3.60	4.05	71.20	99.35	100.00	100.00	100.00
	(34.35)	(13.60)	(6.80)	(3.80)	(4.20)	(76.75)	(99.55)	(100.00)	(100.00)	(100.00)
200	38.80	14.05	8.10	4.30	3.35	88.45	97.75	100.00	100.00	100.00
	(43.70)	(15.70)	(8.50)	(4.45)	(3.50)	(91.35)	(98.05)	(100.00)	(100.00)	(100.00)

Table 5. Specification 3—Size and power of delta test with CSD and heteroskedastic normal i.i.d. errors. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. One exogenous regressor. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	3.35	3.95	3.60	3.65	3.95	8.15	50.85	65.10	97.35	91.45
	(5.40)	(4.20)	(4.05)	(3.90)	(4.10)	(10.25)	(52.25)	(65.70)	(97.35)	(91.55)
50	2.75	4.45	3.30	3.40	3.95	59.90	97.50	99.05	100.00	100.00
	(3.95)	(5.40)	(3.55)	(3.40)	(4.15)	(64.20)	(97.55)	(99.05)	(100.00)	(100.00)
100	3.05	4.20	3.80	4.55	4.40	95.20	100.00	100.00	100.00	100.00
	(4.90)	(4.80)	(4.30)	(4.90)	(4.50)	(96.45)	(100.00)	(100.00)	(100.00)	(100.00)
150	3.50	3.65	5.05	5.15	4.05	86.70	100.00	100.00	100.00	100.00
	(4.60)	(4.35)	(5.35)	(5.40)	(4.35)	(89.50)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.80	4.40	4.40	4.65	4.45	90.40	100.00	100.00	100.00	100.00
	(4.70)	(4.95)	(4.70)	(5.00)	(4.80)	(92.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(QS)+Prewhitened}}$										
20	2.25	1.75	2.45	3.00	3.00	3.70	24.70	46.10	95.25	85.45
	(2.90)	(2.15)	(2.55)	(3.30)	(3.05)	(5.00)	(26.25)	(47.10)	(95.45)	(85.65)
50	1.65	3.15	2.80	2.50	3.65	12.25	77.60	95.45	100.00	100.00
	(2.50)	(3.60)	(3.05)	(2.65)	(3.80)	(15.85)	(79.30)	(95.55)	(100.00)	(100.00)
100	2.60	2.20	3.00	3.75	3.20	31.60	97.90	100.00	100.00	100.00
	(3.60)	(2.65)	(3.20)	(3.95)	(3.45)	(37.90)	(98.30)	(100.00)	(100.00)	(100.00)
150	1.30	2.15	3.95	3.65	3.75	24.60	100.00	100.00	100.00	100.00
	(2.40)	(2.80)	(4.15)	(4.00)	(3.85)	(29.65)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.05	2.40	3.65	3.75	3.35	31.10	99.90	100.00	100.00	100.00
	(2.90)	(2.95)	(4.05)	(3.85)	(3.55)	(37.75)	(99.90)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{CSA}}$										
20	3.05	3.35	3.65	3.60	3.85	5.75	30.30	53.65	94.20	86.45
	(5.30)	(4.05)	(3.85)	(3.80)	(3.95)	(8.05)	(32.00)	(54.50)	(94.20)	(86.50)
50	3.00	3.65	3.35	3.30	4.20	49.70	92.45	98.15	100.00	100.00
	(4.60)	(4.45)	(3.75)	(3.50)	(4.40)	(54.90)	(93.60)	(98.20)	(100.00)	(100.00)
100	3.20	3.85	4.20	4.45	4.55	90.85	100.00	100.00	100.00	100.00
	(5.05)	(5.05)	(4.50)	(4.65)	(4.75)	(93.20)	(100.00)	(100.00)	(100.00)	(100.00)
150	2.95	4.15	4.70	4.95	3.95	77.20	100.00	100.00	100.00	100.00
	(4.70)	(5.00)	(5.20)	(5.20)	(4.40)	(82.05)	(100.00)	(100.00)	(100.00)	(100.00)
200	2.90	4.35	4.95	4.35	4.05	82.00	100.00	100.00	100.00	100.00
	(4.10)	(5.15)	(5.20)	(4.50)	(4.25)	(86.30)	(100.00)	(100.00)	(100.00)	(100.00)

Table 6. Specification 4—Size and power of delta test with CSD and serially correlated errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\mathrm{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\mathrm{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\mathrm{adj}}$ are given in parentheses. One exogenous regressor. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	18.20	27.00	26.90	40.40	29.70	46.55	81.95	87.70	98.50	99.85
	(21.80)	(28.35)	(27.55)	(40.90)	(30.15)	(50.80)	(83.35)	(87.95)	(98.60)	(99.85)
50	28.25	52.00	42.85	59.80	58.05	92.05	99.90	98.80	100.00	100.00
	(33.20)	(53.30)	(43.65)	(60.45)	(58.60)	(93.45)	(99.90)	(98.80)	(100.00)	(100.00)
100	53.85	82.50	77.15	85.50	85.10	99.85	100.00	100.00	100.00	100.00
	(59.85)	(83.40)	(77.60)	(86.00)	(85.40)	(99.90)	(100.00)	(100.00)	(100.00)	(100.00)
150	72.55	86.05	93.55	91.30	95.15	100.00	100.00	100.00	100.00	100.00
	(76.25)	(87.25)	(93.80)	(91.60)	(95.25)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	82.30	96.75	96.65	95.45	98.00	100.00	100.00	100.00	100.00	100.00
	(85.60)	(97.00)	(96.75)	(95.70)	(98.05)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{HAC(QS)+Prewhitened}}$										
20	1.55	1.35	2.25	2.95	2.60	3.55	21.65	41.15	85.60	98.05
	(2.55)	(1.50)	(2.35)	(3.00)	(2.75)	(4.35)	(23.25)	(41.60)	(85.65)	(98.05)
50	1.70	2.90	2.75	2.85	2.90	14.65	59.50	66.95	99.75	100.00
	(2.60)	(3.35)	(3.10)	(2.90)	(3.00)	(18.10)	(61.70)	(67.75)	(99.75)	(100.00)
100	2.30	2.45	2.05	2.85	3.55	25.15	74.10	99.50	99.25	100.00
	(3.65)	(2.65)	(2.55)	(3.10)	(3.55)	(30.65)	(76.15)	(99.50)	(99.30)	(100.00)
150	2.10	1.75	3.65	3.10	3.40	40.50	98.85	100.00	100.00	100.00
	(2.90)	(2.05)	(3.85)	(3.30)	(3.55)	(47.60)	(99.10)	(100.00)	(100.00)	(100.00)
200	1.65	1.35	2.45	3.05	3.15	45.30	98.95	100.00	100.00	100.00
	(2.95)	(1.60)	(2.60)	(3.30)	(3.15)	(51.90)	(99.15)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{CSA}}$										
20	13.50	22.90	26.55	39.35	28.85	34.35	69.65	81.15	97.80	99.50
	(17.20)	(24.30)	(27.40)	(40.00)	(29.00)	(38.40)	(70.80)	(81.65)	(97.90)	(99.55)
50	22.00	44.05	40.95	58.35	55.55	86.05	99.65	97.65	100.00	100.00
	(25.85)	(46.10)	(41.85)	(59.10)	(55.95)	(88.45)	(99.65)	(97.70)	(100.00)	(100.00)
100	42.25	73.55	73.85	83.85	84.25	98.90	100.00	100.00	100.00	100.00
	(47.90)	(75.05)	(74.40)	(84.50)	(84.45)	(99.30)	(100.00)	(100.00)	(100.00)	(100.00)
150	59.85	78.90	91.05	90.90	94.95	99.95	100.00	100.00	100.00	100.00
	(66.10)	(80.85)	(91.65)	(91.15)	(95.05)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	69.60	93.40	95.45	94.50	97.55	100.00	100.00	100.00	100.00	100.00
	(74.25)	(94.15)	(95.55)	(94.60)	(97.65)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{CSA+HAC(QS)+Prewhitened}}$										
20	4.10	6.45	5.35	4.60	4.55	8.25	33.80	45.20	86.10	97.65
	(5.65)	(7.40)	(5.65)	(4.75)	(4.80)	(11.10)	(36.25)	(46.25)	(86.25)	(97.70)
50	6.35	13.55	7.85	7.20	5.20	33.35	80.40	76.50	99.75	100.00
	(8.05)	(15.20)	(8.15)	(7.30)	(5.25)	(40.35)	(82.30)	(77.45)	(99.75)	(100.00)
100	12.90	24.95	11.65	9.45	6.85	57.10	95.15	99.90	99.80	100.00
	(16.65)	(27.10)	(12.30)	(9.65)	(6.90)	(62.80)	(95.85)	(99.90)	(99.80)	(100.00)
150	20.00	32.75	14.95	10.40	8.65	77.40	99.90	100.00	100.00	100.00
	(25.10)	(35.05)	(15.95)	(10.70)	(8.70)	(82.25)	(99.95)	(100.00)	(100.00)	(100.00)
200	25.75	45.40	18.25	12.85	8.60	85.00	100.00	100.00	100.00	100.00
	(30.75)	(48.45)	(19.35)	(13.20)	(9.05)	(88.20)	(100.00)	(100.00)	(100.00)	(100.00)

Results with (k=4)

Table 7. Specification 5—Size and power of delta test with no CSD and heteroskedastic normal i.i.d. errors. Bandwidth for $\widetilde{\Delta}_{\mathrm{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\mathrm{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\mathrm{adj}}$ are given in parentheses. Four exogenous regressors. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	2.10	4.30	4.95	4.35	4.50	1.95	4.05	7.40	7.15	6.60
	(5.90)	(5.95)	(5.75)	(4.80)	(4.90)	(5.35)	(5.30)	(8.10)	(7.70)	(6.90)
50	2.35	3.80	4.05	4.90	4.20	2.00	3.85	15.80	23.20	10.95
	(4.90)	(5.60)	(4.80)	(5.40)	(4.60)	(5.05)	(5.30)	(17.15)	(24.05)	(11.25)
100	2.55	3.35	4.40	4.00	4.35	2.45	11.15	26.00	29.15	67.05
	(6.40)	(4.65)	(5.20)	(4.35)	(4.85)	(6.30)	(13.85)	(27.50)	(30.10)	(67.70)
150	1.40	3.40	4.25	4.60	5.80	1.25	6.80	70.10	51.00	93.55
	(4.95)	(5.40)	(4.90)	(5.05)	(6.00)	(4.50)	(8.15)	(71.65)	(52.00)	(93.75)
200	2.75	3.55	4.35	3.85	5.00	3.60	25.05	94.60	53.90	97.50
	(6.35)	(4.80)	(5.20)	(4.30)	(5.40)	(7.75)	(28.75)	(95.20)	(55.10)	(97.55)
$\widetilde{\Delta}_{\mathrm{HAC(QS)}}$										
20	1.95	3.10	3.85	3.80	3.90	2.05	2.75	3.80	4.75	4.80
	(6.65)	(4.20)	(4.30)	(4.20)	(4.45)	(6.30)	(3.85)	(4.50)	(5.40)	(4.95)
50	3.60	2.75	3.20	4.20	4.30	3.35	2.25	4.95	6.60	7.35
	(10.30)	(3.85)	(3.60)	(4.55)	(4.50)	(9.05)	(3.00)	(5.55)	(7.30)	(7.70)
100	8.80	3.35	3.75	4.10	3.80	8.20	1.70	10.25	15.70	39.60
	(20.60)	(5.35)	(4.25)	(4.55)	(4.15)	(18.20)	(2.75)	(11.30)	(16.30)	(40.55)
150	14.15	5.10	3.85	4.00	4.55	11.85	2.25	20.95	18.95	74.10
	(30.85)	(6.65)	(4.55)	(4.60)	(4.90)	(26.15)	(3.35)	(22.30)	(20.05)	(74.70)
200	21.25	5.25	4.90	3.95	4.65	16.85	1.85	37.45	25.45	82.65
	(38.95)	(7.20)	(5.65)	(4.65)	(4.90)	(34.25)	(2.80)	(39.45)	(26.95)	(83.25)
$\widetilde{\Delta}_{\mathrm{CSA}}$										
20	1.85	2.85	4.65	4.25	5.00	1.90	2.95	4.95	5.35	5.30
	(7.20)	(5.65)	(5.85)	(4.90)	(5.60)	(6.75)	(5.25)	(6.30)	(6.00)	(5.80)
50	1.55	2.55	3.50	4.75	3.90	1.55	2.90	7.00	13.80	7.65
	(5.75)	(5.15)	(4.20)	(5.30)	(4.40)	(5.25)	(5.45)	(8.30)	(14.40)	(8.40)
100	1.70	2.85	4.10	4.35	4.95	1.65	3.20	13.00	17.80	50.75
	(6.15)	(4.75)	(4.70)	(5.05)	(5.40)	(6.15)	(6.80)	(14.75)	(19.20)	(51.90)
150	1.75	2.85	4.05	4.00	4.65	1.45	2.75	41.60	31.05	86.85
	(6.20)	(5.70)	(5.25)	(4.50)	(4.95)	(5.70)	(5.40)	(45.10)	(32.35)	(87.50)
200	1.65	2.80	3.50	5.00	5.40	1.60	5.55	75.10	31.95	91.25
	(6.45)	(5.50)	(4.35)	(5.75)	(5.65)	(6.30)	(9.55)	(77.35)	(33.40)	(91.60)

Table 8. Specification 6—Size and power of delta test with no CSD and serially correlated errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. Four exogenous regressors. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	22.70	43.35	62.20	60.60	65.30	24.20	48.20	68.70	76.55	76.20
	(31.95)	(46.90)	(63.50)	(61.15)	(66.05)	(34.20)	(51.80)	(69.85)	(77.30)	(76.90)
50	60.85	92.15	95.90	94.90	95.80	66.65	94.55	97.65	98.35	99.70
	(72.40)	(93.65)	(96.35)	(95.35)	(95.90)	(77.05)	(96.00)	(98.00)	(98.40)	(99.70)
100	91.00	99.55	99.70	99.95	99.70	94.05	99.85	100.00	100.00	100.00
	(95.75)	(99.70)	(99.80)	(99.95)	(99.70)	(97.15)	(99.85)	(100.00)	(100.00)	(100.00)
150	97.90	100.00	100.00	100.00	100.00	98.55	100.00	100.00	100.00	100.00
	(98.90)	(100.00)	(100.00)	(100.00)	(100.00)	(99.50)	(100.00)	(100.00)	(100.00)	(100.00)
200	99.75	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00
	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\text{HAC(QS)}}$										
20	1.00	1.20	1.95	2.40	3.60	1.10	1.10	2.20	6.55	6.25
	(4.25)	(1.85)	(2.25)	(2.55)	(3.95)	(4.10)	(1.50)	(2.40)	(6.90)	(6.65)
50	1.30	1.70	2.35	7.40	4.65	1.35	1.90	3.60	14.85	32.00
	(5.40)	(2.20)	(2.90)	(8.05)	(5.00)	(5.25)	(2.35)	(4.20)	(15.85)	(32.80)
100	4.95	0.65	3.55	6.20	12.80	4.35	1.30	10.90	38.55	53.65
	(14.50)	(1.25)	(4.05)	(6.80)	(13.50)	(13.15)	(1.85)	(12.05)	(40.10)	(54.70)
150	10.40	0.90	7.95	11.25	12.30	8.40	2.25	19.20	39.40	84.30
	(24.30)	(1.70)	(8.90)	(11.90)	(13.05)	(22.05)	(3.25)	(20.65)	(40.85)	(84.75)
200	21.30	1.40	8.40	13.60	15.40	17.70	5.25	44.40	84.00	89.15
	(40.70)	(2.10)	(9.85)	(14.35)	(16.55)	(36.90)	(7.35)	(46.60)	(84.80)	(89.75)
$\widetilde{\Delta}_{\text{CSA}}$										
20	5.50	17.50	53.10	52.55	61.95	5.55	19.60	57.45	66.45	71.45
	(13.90)	(23.65)	(55.35)	(53.65)	(62.70)	(14.55)	(26.05)	(59.30)	(68.10)	(72.20)
50	17.85	59.15	91.50	92.50	94.40	20.25	63.25	94.00	96.55	99.30
	(32.55)	(67.20)	(92.65)	(93.15)	(94.75)	(36.70)	(71.50)	(94.60)	(96.70)	(99.35)
100	39.70	86.85	99.65	99.70	99.80	45.15	91.65	99.80	100.00	100.00
	(61.00)	(91.30)	(99.65)	(99.75)	(99.85)	(65.75)	(94.35)	(99.90)	(100.00)	(100.00)
150	56.40	97.20	100.00	100.00	100.00	63.45	98.90	100.00	100.00	100.00
	(73.40)	(98.65)	(100.00)	(100.00)	(100.00)	(79.25)	(99.35)	(100.00)	(100.00)	(100.00)
200	73.85	99.35	100.00	100.00	100.00	80.05	99.95	100.00	100.00	100.00
	(88.10)	(99.55)	(100.00)	(100.00)	(100.00)	(91.20)	(100.00)	(100.00)	(100.00)	(100.00)

Table 9. Specification 7—Size and power of delta test with CSD and heteroskedastic normal i.i.d. errors. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. Four exogenous regressors. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	1.85	4.60	4.65	5.55	4.40	1.70	5.10	8.95	5.10	8.60
	(5.60)	(6.40)	(5.55)	(5.95)	(4.60)	(5.05)	(6.30)	(9.95)	(5.35)	(8.80)
50	2.00	4.15	5.05	4.35	4.75	1.85	5.60	35.25	27.75	31.00
	(5.75)	(5.65)	(5.65)	(4.95)	(5.15)	(5.00)	(6.65)	(37.25)	(28.90)	(32.00)
100	2.75	4.20	4.25	4.05	4.00	2.40	4.35	16.75	42.80	80.90
	(6.30)	(5.55)	(5.05)	(4.75)	(4.45)	(5.90)	(5.95)	(17.90)	(44.40)	(81.45)
150	1.95	3.80	4.70	5.00	5.25	1.80	9.45	32.60	67.15	65.45
	(5.60)	(5.05)	(5.40)	(5.25)	(5.45)	(5.10)	(11.80)	(34.35)	(68.10)	(66.30)
200	2.30	3.55	4.25	4.70	4.60	2.75	9.15	35.10	84.70	98.60
	(5.65)	(5.00)	(4.85)	(5.15)	(4.85)	(5.80)	(11.35)	(37.30)	(85.25)	(98.65)
$\widetilde{\Delta}_{\text{HAC(QS)}}$										
20	1.75	3.55	3.90	4.45	3.60	1.55	2.55	3.80	4.00	5.35
	(5.70)	(4.85)	(4.55)	(5.00)	(3.85)	(5.60)	(3.70)	(4.50)	(4.45)	(5.60)
50	3.80	3.50	3.90	3.85	4.60	3.75	1.95	9.70	14.45	19.05
	(11.20)	(5.00)	(4.95)	(3.95)	(5.05)	(10.70)	(3.30)	(10.65)	(15.10)	(19.50)
100	8.80	3.95	3.70	3.50	4.10	7.45	2.10	6.35	18.85	53.75
	(21.20)	(5.75)	(4.55)	(3.95)	(4.35)	(19.70)	(3.05)	(7.15)	(19.75)	(54.85)
150	13.50	4.55	4.65	4.60	4.95	11.60	1.85	11.15	33.25	48.15
	(28.85)	(5.90)	(5.70)	(5.20)	(5.20)	(24.15)	(2.75)	(12.20)	(34.85)	(49.15)
200	21.55	5.45	4.10	4.45	4.20	16.85	2.50	9.90	45.80	91.50
	(39.60)	(7.75)	(4.60)	(4.85)	(4.40)	(34.20)	(3.50)	(11.00)	(47.85)	(91.70)
$\widetilde{\Delta}_{\text{CSA}}$										
20	1.60	2.75	3.90	4.20	5.15	1.25	2.90	5.35	4.40	6.20
	(6.30)	(5.80)	(4.70)	(4.75)	(5.80)	(5.85)	(5.50)	(6.30)	(4.85)	(6.65)
50	1.80	2.75	4.15	5.30	4.85	1.75	2.95	18.10	15.70	20.95
	(6.50)	(5.35)	(5.25)	(5.85)	(5.15)	(6.30)	(5.55)	(19.60)	(16.80)	(21.65)
100	1.45	3.25	3.85	4.80	4.30	1.25	3.20	9.60	24.80	66.70
	(5.30)	(5.65)	(4.95)	(5.35)	(5.00)	(5.25)	(5.65)	(10.90)	(26.20)	(67.35)
150	2.10	2.45	3.90	3.95	4.75	1.70	3.00	14.35	47.75	48.50
	(6.00)	(4.75)	(4.55)	(4.40)	(5.30)	(5.80)	(5.55)	(16.35)	(49.25)	(49.70)
200	1.80	2.70	4.40	4.10	4.65	1.60	3.80	16.35	65.05	94.75
	(6.30)	(5.90)	(5.25)	(4.70)	(5.10)	(5.75)	(6.00)	(18.45)	(66.60)	(95.00)

Table 10. Specification 8—Size and power of delta test with CSD and serially correlated errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\mathrm{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\mathrm{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\mathrm{adj}}$ are given in parentheses. Four exogenous regressors. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	13.65	46.30	51.35	70.25	62.10	13.80	53.05	70.85	88.50	80.75
	(21.00)	(50.10)	(52.65)	(71.05)	(62.85)	(21.00)	(56.70)	(71.85)	(89.30)	(81.35)
50	60.20	93.05	94.85	95.25	96.60	62.15	95.75	98.15	98.70	99.60
	(72.40)	(94.45)	(95.45)	(95.45)	(96.75)	(74.70)	(96.75)	(98.25)	(98.75)	(99.65)
100	89.00	99.45	99.85	100.00	99.95	90.80	99.75	100.00	100.00	100.00
	(93.50)	(99.55)	(99.85)	(100.00)	(99.95)	(94.95)	(99.85)	(100.00)	(100.00)	(100.00)
150	98.05	100.00	100.00	100.00	99.95	98.75	100.00	100.00	100.00	100.00
	(99.15)	(100.00)	(100.00)	(100.00)	(99.95)	(99.45)	(100.00)	(100.00)	(100.00)	(100.00)
200	99.85	100.00	100.00	100.00	100.00	99.95	100.00	100.00	100.00	100.00
	(99.95)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{HAC(QS)}}$										
20	1.55	0.70	2.10	2.95	3.25	1.45	0.65	3.25	11.35	6.35
	(4.85)	(1.25)	(2.35)	(3.05)	(3.55)	(4.95)	(1.30)	(3.75)	(12.05)	(6.65)
50	2.85	0.65	3.10	4.25	4.60	3.40	0.85	8.85	14.30	17.40
	(10.50)	(1.15)	(3.50)	(4.90)	(4.80)	(9.90)	(1.50)	(10.00)	(15.15)	(18.10)
100	6.05	0.90	4.45	8.85	9.30	5.35	1.40	15.25	34.30	58.05
	(17.15)	(1.60)	(4.90)	(9.75)	(9.70)	(15.25)	(2.30)	(16.55)	(35.55)	(59.50)
150	12.90	1.35	5.10	15.00	12.50	11.75	2.25	30.90	66.90	88.60
	(30.35)	(1.95)	(5.80)	(16.05)	(13.25)	(26.75)	(3.25)	(33.05)	(68.80)	(88.90)
200	17.55	1.30	8.50	11.15	16.90	14.40	3.60	39.85	56.15	79.50
	(33.15)	(2.00)	(9.65)	(12.05)	(17.60)	(30.25)	(5.25)	(42.50)	(57.50)	(80.75)
$\widetilde{\Delta}_{\mathrm{CSA}}$										
20	2.85	17.95	40.45	64.35	57.95	2.90	20.45	55.25	81.85	73.10
	(8.40)	(24.20)	(42.90)	(65.40)	(58.60)	(8.30)	(26.95)	(57.15)	(82.45)	(74.10)
50	17.80	61.90	88.95	92.85	94.90	19.35	65.85	93.80	96.90	98.80
	(33.15)	(70.10)	(90.45)	(93.40)	(95.00)	(35.30)	(73.10)	(94.10)	(97.20)	(98.95)
100	37.55	87.25	99.50	99.85	99.85	41.75	91.15	100.00	100.00	100.00
	(56.85)	(90.75)	(99.55)	(99.85)	(99.85)	(60.10)	(94.20)	(100.00)	(100.00)	(100.00)
150	59.00	95.95	100.00	100.00	100.00	65.25	98.05	100.00	100.00	100.00
	(75.60)	(97.65)	(100.00)	(100.00)	(100.00)	(80.35)	(98.65)	(100.00)	(100.00)	(100.00)
200	72.75	99.35	100.00	100.00	100.00	80.85	99.70	100.00	100.00	100.00
	(86.35)	(99.65)	(100.00)	(100.00)	(100.00)	(91.45)	(99.90)	(100.00)	(100.00)	(100.00)
$\widetilde{\Delta}_{\mathrm{CSA+HAC(QS)+Whitening}}$										
20	99.80	100.00	96.55	94.70	85.10	99.75	99.95	97.65	97.80	90.85
	(100.00)	(100.00)	(96.90)	(95.05)	(85.40)	(99.80)	(99.95)	(98.05)	(97.95)	(91.10)
50	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
150	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
200	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Table 11. Specification 9—Size and power of delta test with no CSD and normally i.i.d. errors with heteroskedasticity. Bandwidth for $\widetilde{\Delta}_{\text{HAC}}$ is automatically selected following Newey and West (1994). Only contemporaneous CSA are added for $\widetilde{\Delta}_{\text{CSD}}$. Results for the small-sample-adjusted $\widetilde{\Delta}_{\text{adj}}$ are given in parentheses. Four exogenous regressors. β_1 is heterogeneous under the null and the alternative. In $\widetilde{\Delta}_{\text{oracle}}$, the correct coefficients are partialed out. For a definition of the DGP, see section 5.

	Size (%)					Power (%)				
N, T	20	50	100	150	200	20	50	100	150	200
$\widetilde{\Delta}$										
20	2.10	3.90	6.20	5.05	4.60	1.60	3.40	8.65	8.35	4.60
	(6.60)	(5.40)	(6.75)	(5.75)	(5.10)	(5.60)	(5.15)	(9.25)	(9.05)	(5.55)
50	1.50	3.60	4.30	7.60	16.90	1.55	4.85	6.45	23.00	59.20
	(4.90)	(4.85)	(4.60)	(8.30)	(17.55)	(4.40)	(6.25)	(7.05)	(24.10)	(59.85)
100	2.05	3.85	4.90	6.10	9.00	2.15	5.55	27.15	27.80	51.25
	(6.00)	(4.95)	(5.65)	(6.55)	(9.35)	(6.00)	(7.30)	(29.40)	(29.35)	(52.00)
150	2.25	3.40	4.20	15.10	12.00	2.20	8.55	28.10	64.15	72.35
	(4.60)	(4.80)	(4.85)	(16.15)	(12.55)	(4.85)	(11.00)	(30.05)	(65.10)	(72.90)
200	2.60	4.35	5.65	14.55	85.50	3.45	18.85	48.85	64.15	99.10
	(6.00)	(6.15)	(6.15)	(15.25)	(85.85)	(7.15)	(22.70)	(50.80)	(65.20)	(99.10)
$\widetilde{\Delta}_{\text{HAC(QS)}}$										
20	1.75	2.40	3.40	3.95	4.10	1.45	2.20	4.20	5.20	4.20
	(6.35)	(4.15)	(3.90)	(4.45)	(4.30)	(5.25)	(3.90)	(4.65)	(5.85)	(4.75)
50	4.35	3.30	2.50	4.55	5.80	3.95	2.45	2.85	11.65	30.35
	(10.25)	(4.30)	(3.00)	(5.25)	(6.15)	(9.90)	(3.05)	(3.55)	(12.10)	(31.20)
100	8.85	3.00	3.05	3.55	5.60	7.40	1.85	11.50	15.55	30.20
	(19.70)	(4.55)	(3.80)	(4.00)	(6.10)	(17.25)	(3.10)	(12.10)	(16.65)	(31.30)
150	11.90	4.05	2.85	4.95	7.05	10.50	2.10	6.80	32.00	53.95
	(25.55)	(5.15)	(3.25)	(5.20)	(7.55)	(22.70)	(2.95)	(7.55)	(33.20)	(55.00)
200	19.50	4.25	3.65	5.85	24.15	16.20	1.90	14.95	34.30	89.10
	(34.30)	(5.35)	(4.05)	(6.65)	(25.35)	(29.30)	(2.80)	(16.80)	(35.15)	(89.75)
$\widetilde{\Delta}_{\text{CSA}}$										
20	2.10	2.85	4.65	4.80	4.80	1.75	2.85	5.75	5.65	5.25
	(7.80)	(6.35)	(6.00)	(5.60)	(5.05)	(7.50)	(5.85)	(6.65)	(6.45)	(5.70)
50	1.15	2.15	3.85	5.50	11.40	1.15	2.30	3.95	13.70	46.30
	(5.65)	(4.80)	(4.70)	(6.05)	(11.70)	(5.20)	(3.95)	(4.80)	(14.70)	(47.05)
100	1.10	2.75	4.30	4.30	7.15	1.15	2.65	12.25	17.00	36.25
	(5.25)	(5.05)	(5.30)	(5.20)	(7.60)	(5.05)	(5.55)	(14.40)	(18.35)	(37.65)
150	1.10	2.45	3.55	10.35	9.15	1.15	3.15	12.55	43.50	55.80
	(5.50)	(4.80)	(4.80)	(11.30)	(9.80)	(5.35)	(5.35)	(14.40)	(45.60)	(57.05)
200	1.25	3.00	4.20	9.75	77.60	1.40	3.60	23.05	44.80	98.30
	(6.20)	(4.55)	(5.20)	(10.65)	(78.50)	(6.10)	(6.85)	(25.45)	(46.50)	(98.35)
$\widetilde{\Delta}_{\text{oracle}}$										
20	2.05	4.35	4.70	4.70	3.90	2.00	4.10	6.10	8.25	4.60
	(5.70)	(5.35)	(5.25)	(5.10)	(4.35)	(5.05)	(5.25)	(7.25)	(8.75)	(5.00)
50	1.85	3.55	4.10	5.15	4.30	2.05	5.15	6.05	16.15	49.15
	(4.60)	(4.80)	(4.60)	(5.60)	(4.85)	(4.25)	(5.95)	(6.65)	(16.65)	(49.65)
100	2.00	3.65	4.40	4.55	5.60	2.40	4.85	25.05	22.70	46.90
	(6.30)	(4.45)	(4.80)	(5.25)	(5.95)	(5.75)	(5.90)	(26.65)	(23.25)	(47.55)
150	2.30	3.45	3.65	4.75	4.00	2.35	10.00	25.90	57.70	56.20
	(4.85)	(4.55)	(4.10)	(5.35)	(4.05)	(4.80)	(11.40)	(27.70)	(58.60)	(56.95)
200	2.75	3.80	4.35	5.00	4.20	3.60	18.80	44.10	55.10	97.95
	(5.15)	(5.05)	(5.00)	(5.70)	(4.60)	(7.25)	(21.10)	(45.95)	(56.10)	(98.00)