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Stata tip 134: Multiplicative and marginal interaction effects in nonlinear models

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1 Introduction

In Stata tip 87, Buis (2010) demonstrated how to use Stata to calculate multiplicative interaction effects in nonlinear models. As Buis notes, multiplicative interaction effects, such as odds ratios from a `logit` model (see [R] **logit**), are often easily obtained from standard Stata output without additional programming. Buis contrasts this with marginal interaction effects, which require additional postregression programming for correct computation (Ai and Norton 2003)—although the `margins` commands in Stata have greatly simplified this computation and can produce correct marginal effect calculations with only a few lines of additional postregression commands (Karaca-Mandic, Norton, and Dowd 2012). Buis also explains that multiplicative and marginal interaction effects each answer different questions; thus, it is important for analysts to have both in their toolkit.

Nevertheless, we have observed numerous authors misinterpret Buis (2010), citing the article as justification for presenting only multiplicative interaction effects, claiming they are easier to calculate or interpret. For example, Doidge, Karolyi, and Stulz (2013) state, "We, therefore, report the regression coefficients, but interpret them in terms of odds ratios which are simpler to interpret when there are interaction terms in the model (see, for example, Buis, 2010; Kolasinski and Siegel, 2010)." Similarly, Vaidyanathan (2011) states, "While scholars continue to debate how to interpret interaction effects in nonlinear models, Buis 2010 argues that using multiplicative effects, such as odds ratios, overcomes most difficulties, and I follow him in this regard."

In this Stata tip, we present a simple stylized example illustrating the starkly different conclusions that marginal and multiplicative interaction effects can imply. We argue that unless analysts have a strong theoretical preference, they should routinely calculate and present both marginal and multiplicative interaction effects after fitting nonlinear models.

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2 Multiplicative and marginal interaction effects on probability and odds scales

For illustration, we focus on the `logit` estimator with a binary outcome variable y, which is modeled as a function of two interacted binary explanatory variables, x and z. In other words, $y = f(\beta_1 x + \beta_2 z + \beta_{12} x \times z)$. However, the implications of this article directly generalize to all nonlinear models, including count data models such as Poisson (see [R] **poisson**) or negative binomial (see [R] **nbreg**), which are often interpreted multiplicatively.

We define and compare four ways of representing an interaction effect after fitting a `logit` model. These include multiplicative and marginal interaction effects on both the probability scale and the odds scale. Define p as the predicted probability for a binary dependent variable y, conditional on values of binary x and z and their interaction. The multiplicative interaction effect of changes in both x and z on p can be represented on the probability scale as

$$\frac{p_{x=1,z=1}/p_{x=1,z=0}}{p_{x=0,z=1}/p_{x=0,z=0}}$$

One can easily calculate this in Stata using the `nlcom` command after `margins` following the logistic regression:

```
. logit y i.x##i.z
. margins x#z, post
. nlcom (_b[1.x#1.z] / _b[1.x#0.z]) / (_b[0.x#1.z] / _b[0.x#0.z])
```

In contrast with the above multiplicative interaction effects, marginal interaction effects are represented as a difference in differences on the probability scale:

$$\left(p_{x=1,z=1} - p_{x=1,z=0}\right) - \left(p_{x=0,z=1} - p_{x=0,z=0}\right)$$

One can calculate this marginal interaction effect on the probability scale using a one-line postestimation command:

```
. margins, dydx(z) at(x=(0 1)) contrast(atcontrast(r._at)) post
```

It is also common in some fields to present effects on the odds $p/(1-p)$ scale rather than probability scale p. If p is close to zero, then the results on both scales are usually similar, but more generally they may differ. The multiplicative interaction effect on the odds scale is

$$\frac{\frac{p_{x=1,z=1}}{1-p_{x=1,z=1}} \Big/ \frac{p_{x=1,z=0}}{1-p_{x=1,z=0}}}{\frac{p_{x=0,z=1}}{1-p_{x=0,z=1}} \Big/ \frac{p_{x=0,z=0}}{1-p_{x=0,z=0}}}$$

This is even simpler to calculate in Stata using the `or` option in the `logit` command, which is one reason why it is among the most commonly reported variants:

```
. logit y i.x##i.z, or
```

Finally, although rarely used, it is also possible to calculate marginal interaction effects on an odds scale (this is the marginal effect variant presented in Buis [2010]):

$$\left(\frac{p_{x=1,z=1}}{1-p_{x=1,z=1}}-\frac{p_{x=1,z=0}}{1-p_{x=1,z=0}}\right)-\left(\frac{p_{x=0,z=1}}{1-p_{x=0,z=1}}-\frac{p_{x=0,z=0}}{1-p_{x=0,z=0}}\right)$$

```
. margins x#z, expression(exp(xb())) post
. lincom (_b[1.x#1.z] - _b[1.x#0.z]) - (_b[0.x#1.z] - _b[0.x#0.z])
```

Similar Stata code can be written to calculate interaction effects after probit, Poisson, and negative binomial models.

3 A simple cautionary example

To illustrate the potential danger of reporting only one variant of the above interaction effect calculations, we present an example in which each of these variants implies different conclusions. Our simple example has just four data points (see panel A of figure 1). When $x = 0$, as z increases from 0 to 1, the probability p of a positive outcome rises from 0.05 to 0.10. When $x = 1$, as z increases from 0 to 1, the probability of a positive outcome rises from 0.10 to 0.19. Panel B of figure 1 shows the same four data points transformed to the odds scale.

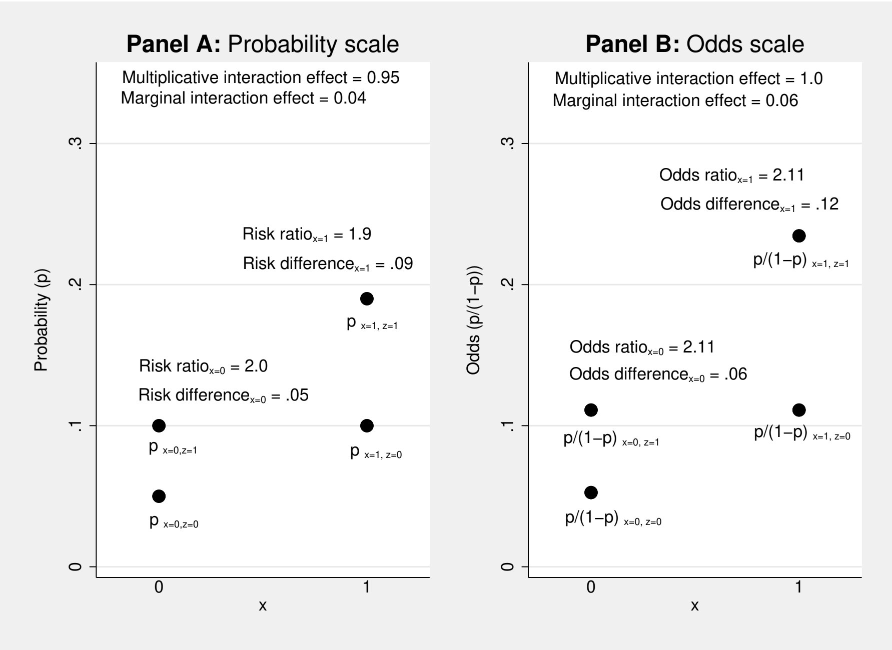

Figure 1. Multiplicative and marginal interaction effects

The multiplicative interaction effect on the probability scale is 0.95 (final column of table 1, panel A), which implies that as x increases, the effect of z decreases. By contrast, the multiplicative effect on the odds scale is 1.0 (final column of table 1, panel B), which implies that as x increases, the effect of z is unchanged. Finally, the marginal effect on the probability scale is 0.04 (middle column of table 1, panel A), which implies that as x increases, the effect of z increases. The marginal effect on the odds scale in panel B shows a similar increasing effect in this example, although in other examples it can differ meaningfully from the marginal effects on the probability scale.

Table 1. Main and interaction effects in nonlinear models

Panel A: Parameter calculations on the probability scale.

	Predicted probabilities (p)		Marginal effects (p)		Multiplicative effects (p)	
	$z=0$	$z=1$	Risk difference	Interaction effect	Risk ratio	Interaction effect
$x=0$	0.05	0.10	$0.10-0.05=0.05$	$0.09-0.05=0.04$	$\frac{0.10}{0.05}=2.0$	$\frac{1.9}{2.0}=0.95$
$x=1$	0.10	0.19	$0.19-0.10=0.09$		$\frac{0.19}{0.10}=1.9$	

Panel B: Parameter calculations on the odds scale.

	Predicted odds $(\frac{p}{1-p})$		Marginal effects $(\frac{p}{1-p})$		Multiplicative effects $(\frac{p}{1-p})$	
	z = 0	z = 1	Odds difference	Interaction effect	Odds ratio	Interaction effect
$x=0$	0.0526	0.1111	$0.1111-0.0526=0.06$	$0.12-0.06=0.06$	$\frac{0.1111}{0.0526}=2.1$	$\frac{2.1}{2.1}=1.0$
$x=1$	0.1111	0.2346	$0.2346-0.1111=0.12$		$\frac{0.2346}{0.1111}=2.1$	

Thus, in this simple example, the interaction effect could alternatively be interpreted as positive, null, or negative depending on which variant is estimated and reported. As x increases, the effect of z

- decreases because of the multiplicative interaction effect on the probability scale;
- increases because of the marginal interaction effect on the probability scale;
- is zero because of the multiplicative interaction effect on the odds scale; or
- increases because of the marginal interaction effect on the odds scale.

4 Discussion

To provide intuition regarding situations in which results are likely to differ across these variants, let's consider the case in which our example refers to a natural experiment. Suppose that z is an indicator for treatment $(z = 1)$ versus control $(z = 0)$ group and x is a time indicator of preperiod $(x = 0)$ versus postperiod $(x = 1)$. Our example is not well balanced in the preperiod: treatments had double the baseline risk as the controls. When there is such imbalance, it is well known that treatment-effect estimates are sensitive to functional form. In our example, the outcome in the control group doubled over time, an increase of 0.05; the outcome in the treatment group increased by a greater absolute amount of 0.09, but it did not quite double. Hence the different sign of the intervention effect: the marginal effect of 0.04 shows an increase in p in the treatment group relative to absolute growth in the controls, but the multiplicative interaction effect on the probability scale of 0.95 shows a negative effect of the treatment relative to the multiplicative growth in the controls. The preferred solution in the evaluation literature is to choose a different control group that is better matched at the baseline. With better matching, at least the signs (though not the magnitudes) of the interaction effect would be the same for the marginal and multiplicative effects.

Another important point is that our example, if it had included other covariates, would have different effect sizes across observations. In nonlinear models, the magnitudes of the marginal effects are not constant but vary across observations (Ai and Norton 2003). For example, although odds ratios are constant for all observations in a logistic model, marginal effects are typically larger when the underlying probability is close to 50% and smaller when the underlying probability is close to 0 or 1. On the other hand, the magnitude of the odds ratio from a logistic regression is scaled by an arbitrary factor that changes when additional covariates are added to the model, making comparisons of magnitudes impossible (Norton, Dowd, and Maciejewski 2018). However, marginal effects are more robust to changes in model specification. In summary, with a richer dataset, the researcher should be aware that treatment effects will often differ across observations. In this case, one can use the `margins` command to calculate average marginal effects—a summary measure of effect magnitudes and their statistical significance.

Different disciplinary traditions tend to default to different variants of these interaction effects. Setting aside the debates regarding the general merits and drawbacks of each (see Norton and Dowd [2018]; Mustillo, Landerman, and Land [2012]), we argue that it can be misleading to focus on only one variant by default. Thus, we build on Buis (2010) to argue that researchers should estimate both multiplicative and marginal interaction effects and report the sensitivity of key inferences.

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