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Credit Market Imperfections, Urban Land Rents and the Henry George Theorem

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Credit Market Imperfections, Urban Land Rents and the Henry George Theorem

Abstract

This paper investigates the credit market impact on urban land rents and the tax policy implications. We introduce a borrowing cost and a down-payment requirement in the canonical urban land use model. We first show that both imperfections lower equilibrium land prices in the most attractive city locations. This downward effect is more likely to occur when land is scarce and cities are large and endowed with inefficient transport infrastructures. Only the down-payment requirement generates utility differentials among homogeneous households (*symmetry-breaking*). We further show that the Henry George Theorem, which posits that a confiscatory tax on land rents is sufficient to finance public goods, needs to be amended as aggregate land rents are lower than public expenditures. Depending on the nature of mortgage market imperfections, we derive optimal tax schedules.

Keywords: credit constraint, land use, Henry George Theorem, land taxation, local public good.

JEL Classification: H20, R14, R21, R50.

Marché du Crédit Imparfait, Valeur Foncière Urbaine et le Théorème d'Henry George

Résumé

Cet article étudie l'impact du marché du crédit sur les prix du foncier et ses implications de politiques fiscales. Nous introduisons un coût à l'emprunt et une contrainte d'apport personnel dans le modèle standard d'économie urbaine. Ces imperfections s'avèrent réduire les prix de la terre dans les localisations les plus attractives. Cette baisse est d'autant plus forte que les terres sont rares et les villes peuplées et dotées d'infrastructures de transport inefficaces. La contrainte d'apport personnel peut générer des écarts d'utilités d'équilibre entre des ménages initialement homogènes. Le théorème d'Henry George, selon lequel une taxe confisquant les rentes foncières suffit à financer les biens publics, doit être amendé en présence de contraintes de crédit. Pour chaque type d'imperfections du marché du crédit, nous proposons un système de taxation optimale.

Mots-clés: contrainte de crédit, utilisation des terres, théorème d'Henry George, fiscalité foncière, bien public local.

Classification JEL: H20, R14, R21, R50.

1. Introduction

As many countries increasingly struggle to finance their welfare states, debates on which tax base to target to raise revenues are growing. Recently, the interest in rent taxation has resurged among economists (Schwerhoff *et al.*, 2020). For example, the possibility of taxing land has been evoked in France (Bonnet *et al.*, 2021, Trannoy and Wasmer, 2022). The idea of taxing land is supported by many appealing arguments. Since urban land value mainly captures benefits independent of landowners' efforts (*e.g.*, presence of public facilities and amenities, accessibility to jobs), the return to land can be viewed as an economic rent. In addition, land cannot be moved, and its supply is not responsive to its price. Land thus represents an inelastic tax base on which a tax can be levied to finance local public goods without generating behavioral responses. In the case of France, the aggregate value of the real estate is sizable, amounting to € 7000 billion in 2019, that is, six times the French GDP (Trannoy and Wasmer, 2022), which implies that even a low tax rate can significantly increase revenues.

The idea of land taxation is at the heart of the "single tax movement" and has been formalized in the Henry George Theorem (HGT hereafter) (Stiglitz, 1977, Arnott and Stiglitz, 1979). The HGT posits that, under certain conditions, aggregate differential land rents equal expenditure on local public goods, so that taxing differential land rents is sufficient to finance local public goods (Arnott and Stiglitz, 1979). The validity of the HGT builds on the land rents formation and how rents capitalize locations' characteristics. The model allows for distortions in the economy (Arnott, 2004). Yet, no extension of the HGT has taken explicitly into account distortions originating from the credit market. This absence needs to be tackled as credit market imperfections are a powerful driver of the housing market: Limits as down-payment requirements impact households' ability to purchase a home and, therefore, real estate prices (Ortalo-Magné and Rady, 2006). This issue has become even more salient in the aftermath of the financial crisis as banks tightened mortgage credit availability (Acolin *et al.*, 2016).

This paper investigates how credit market imperfections influence urban land rents and the implications for tax policy. We introduce two features in the standard urban land use model: a borrowing cost and a down-payment requirement. The borrowing cost implies borrowers pay a higher interest rate than the lenders' interest rate. This cost is then proportional to the money borrowed to purchase land. The down-payment requirement implies that households cannot borrow any amount of money to purchase land. Households' ability to pay then depends on their wealth at the time of the purchase. Importantly, our model's credit market imperfections are location-dependent. The possibility of incurring the borrowing cost or being credit constrained depends on land prices which vary within the city. Consequently, a household might not need to borrow when prices are low enough, thus not incurring the borrowing cost. Moreover, a household can be constrained in the most attractive and expensive city locations while being unconstrained in less attractive locations.

We show that both the borrowing cost and the down-payment requirement generate a downward pressure on equilibrium land prices. Yet these two credit market imperfections have a different impact on the urban equilibrium and households' utility. The borrowing cost is capitalized in land prices while the down-payment requirement cap households' ability to pay at a given location. Therefore, only the down-payment requirement distorts land capitalization, since households' willingness to pay does not match equilibrium land price. Such a configuration is more likely to occur when cities are endowed with a high population and inefficient transport infrastructures or/and when land is scarce. Additionally, when the credit constraint is binding, the land price gradient is lower than households' marginal utility to move marginally closer to the Central Business District (hereafter CBD). Hence, credit-constrained households enjoy a higher utility than non-constrained households. In other words, only the down-payment requirement gives rise to *symmetry-breaking* (Matsuyama, 2006), leading *ex-ante* homogeneous households to enjoy different utility levels.

We further discuss the validity of the HGT in the presence of credit market imperfections. Since there is a downward pressure on market prices, aggregate land rents are lower than public expenditures. Therefore, the single tax on land rents is insufficient to finance local public goods. We show that the choice of the appropriate tax schedule depends on the nature of credit market imperfections. Regarding the borrowing cost, we must distinguish the two possible sources of this imperfection. When the borrowing cost arises from the market power of lenders (*e.g.*, banks), the land value is shared between landowners and banks. Therefore, an additional tax on banks' profit associated with housing loans is needed to achieve the first-best outcome. While if borrowers' moral hazard causes the borrowing cost, then a tax to finance monitoring costs and a lump sum tax are needed to reach a more efficient outcome. Under a down-payment requirement, since credit-constrained households extract a surplus to the detriment of landowners, a location-dependent tax on top of the land tax is necessary to restore the HGT. Given that this location-dependent tax demands much information on households' willingness to pay for land, a second-best policy can be implemented involving a property tax, mortgage interest deduction, and down-payment subsidy.

Literature review. First, we contribute to the theoretical literature on urban land use, whose origins can be traced back to Alonso (1964), Mills (1967), and Muth (1988). In the simplest version of the monocentric city model, households decide where to reside while weighing, on the one hand, commuting costs and, on the other hand, land prices.

One prediction of the model, which has been dubbed the Alonso-Muth condition, states that when a city resident decides to move marginally away from the city center, the housing expenditures decrease exactly as much as the increase in commuting costs. The original model has been extended in many ways (see Duranton and Puga, 2015, and Fujita, 1988, for a comprehensive review). However, no model has ever featured credit market imperfections that limit households' ability to pay. We show that when the credit constraint is binding, the Alonso-Muth condition on the land gradient is not met, thus breaking the indifference condition where city residents must be indifferent among all locations.

Our result on the land rent gradient can also shed light on the empirical counterpart of urban land use literature that has tried to estimate and quantify the rent gradient for house prices and land prices. As reviewed by Duranton and Puga (2015), there is no clear evidence on whether the gradient of unit house prices is negative as predicted by the theory.[1] As for the gradient of land prices, there seems to be a consensus about a negative relationship between land prices and distance from the city center with only a few exceptions (for instance, McDonald and Bowman, 1979). In general, the functional form of the distance function is fundamental in the estimation process since even the simplest monocentric city model does not have sharp predictions on the value of the rent gradient in different city locations. This distance function will depend on the transportation costs' shape, utility function, and income sorting. Our model proposes another factor that should be considered when estimating the rent gradient: the presence of credit market imperfections.

Second, our model speaks to the public finance literature focusing on the HGT. The idea that taxing land rents is an efficient and just way to finance local public goods was already present in Henry George's famous book *Progress and Poverty*. This idea was formalized by Arnott and Stiglitz (1979), who demonstrated that differential land rents equal expenditures on local public goods when the city's population is optimal. Thus, a 100% tax on land rents would be sufficient to finance public goods. The basic model has also been extended to account for distortions in the land price formation (Arnott, 2004). In our case, the distortion arises from the credit market. We show that the aggregate differential land rent is lower than public expenditures with credit market imperfections. The HGT needs then to be amended by considering the possibility of introducing other tax instruments.

Finally, our contribution is also related to the optimal taxation literature with land. Stiglitz (2015) studies the case where land is a productive input but disregards land as a

[1] This may be due to many empirical challenges: Among others, the correlation of houses' characteristics with distance to the city center and the presence of more than one city center.

locational space. If Eerola and Määttänen (2013) address the question of housing taxation, they develop a dynamic model in which housing has no land component. Bonnet *et al.* (2021) extend their approach by introducing land in the housing sector, and discuss its implications in terms of optimal taxation. However, unlike our framework, they assume that the credit market is perfect, and that there is neither spatial heterogeneity nor public goods. In a spatial framework, Bureau (2017) considers the imperfect mobility of households yielding incomplete capitalization of public facilities' benefits into land rents. In his case, there is no credit market imperfection, and the value created by the facilities is shared between landowners and the less mobile residents. In our context, land value is shared between landowners, mobile residents, and lenders, depending on the nature of credit market imperfections.

The remainder of the paper is structured as follows. Section 2 presents the land use model. Section 3 describes the residential equilibrium when households are free to move within and across cities. This section highlights how the addition of credit market imperfections influences the land rent formation and households' utility. Section 4 presents a revised version of the HGT and the policy implications for taxation. Section 5 discusses the robustness of our results. Concluding remarks follow.

2. The model

2.1. Space and preferences

We consider a system of cities where the number of cities $\mathcal{N}$ is endogenous and the total number of homogeneous households $\mathcal{L}$ is given. Locations within each city are heterogeneous and vertically differentiated, that is, locations are more or less attractive places to live. The heterogeneity dimension stems from the disutility of commuting and/or monetary costs (including opportunity costs of time) associated with distance to jobs, amenities attributes, or services. All things being equal, households prefer residential locations implying short trips.[2]

In order to ease the presentation of the model, let us consider a linear, closed, and monocentric city defined over the one-dimensional space $\mathbb{R}_+$. Locations differ only with respect to the accessibility to the Central Business District (CBD) located at $x = 0$. The commuting costs between households' residences and the CBD are given by $\kappa(x)$, which increases

[2] French households devote 13.5% of their expenditure to transportation (Combes *et al.*, 2018). For a typical New Yorker, the opportunity cost of the time spent in commuting represents from three to six weeks of work and, on average, four weeks of work for a resident of Greater Paris (Proost and Thisse, 2019). Moreover, commuting is perceived by individuals as one of their most stressful and unpleasant activities (Kahneman *et al.*, 2004).

with distance x to the CBD and is common to all cities.[3]

Land is owned by absentee landlords who sell their property to households at the price $p(x)$. The amount of land available at each location x is normalized to 1. Finally, the opportunity cost of land, common to all cities, is given by the constant $R_A \geqslant 0$, so that the differential land rent for a landlord owning land at a location x is $p(x) - R_A$.

Each city is inhabited by L households. Each household purchases one unit of housing so that $x = L$ represents the city limit and $\kappa(L) > \kappa(x)$ when $x \in [0, L]$. Although the results we present in section 3 can be generalized to the case in which the housing size is variable, it is convenient to assume that the housing size used by each consumer is fixed and normalized to one as in Arnott and Stiglitz (1979) and Arnott (2004). We develop the case with endogenous housing size in section 5. We also assume that land and housing are perfect substitutes. Introducing a construction industry or intermediaries (*e.g.*, real estate companies) does not change our findings. Households have the same utility function $u(c, g)$, which increases in the consumption of the private good c and the quantity of local public good g. Without loss of generality, we assume that $g = G$, where G stands for local public expenditures. We do not consider the case in which the local public good is congestible, namely, in our model, g is not a decreasing function of the number L of users (this case is discussed in Fujita and Thisse, 2002).

All households have the same level of wealth y and earn the wage w. The wealth y is exogenously given and could be a bequest from households' parents. The wage is earned in a perfectly competitive industry. The production function, common to all cities, is given by $f(L)$ where $f(0) = 0$ and $f(.)$ is an increasing function of the number of households living in the city (labor is the only input). For simplicity, the industry operates under constant returns to scale so that $f(L) = \varphi L$, where $\varphi > 0$ is the labor productivity. The output market is perfectly competitive, and there are no transportation costs so that the output price does not vary across cities and is normalized to one. Similarly, urban local markets are perfectly competitive so that the urban wage is given by $w = \varphi$.[4]

2.2. Credit markets

We consider a sequence of events to account for wealth as the key variable determining access to credit. First, households endowed with y choose their residential location x and

[3] Our results remain valid if the model is extended to a map formed by streets, roads, highways, and railway junctions modeled using a topological network, with locations characterized by distance to various job centers, service facilities, and exogenous amenities. In addition, our results hold if distance enters the utility function directly instead of the budget constraint. See section 5.

[4] Note that $w = f(L)/L$ with diminishing marginal product of labor if we assume local profits are equally distributed among workers.

pay $p(x)$. Second, they work in the CBD and earn $w - \kappa(x)$, consume the composite good $c(x)$, and pay a lump-sum tax t. Solving backward, households compete for location, anticipating the impact on their private consumption. If $p(x) > y$, households need to borrow. Even though the model is static, we distinguish between flows (w, $\kappa(x)$, and t) and stocks (y and $p(x)$) to determine households' status (*e.g.*, borrowers or savers).

Following Galor and Zeira (1993), we assume a first credit market imperfection capturing enforcement and supervision costs incurred by lenders to avoid defaults from borrowers. A simple way to formalize this imperfection is to introduce an additional cost ζ proportional to the amount borrowed. Implicitly, we assume, without loss of generality, that lenders' interest rate is normalized to 0, while individuals who borrow an amount $p(x) - y$ pay an interest rate $\zeta > 0$.[5] Thus, households' budget constraint writes

$$\varphi + [y - p(x)] = c + t + \kappa(x) + \mathbf{1}_{\{p(x)>y\}}\zeta\left[p(x) - y\right], \tag{1}$$

where $\mathbf{1}_{\{p(x)>y\}}$ is a dummy variable that indicates if the household borrows. Given that there is only one consumption good, households consume all their income net of the per capita cost of the public good, housing expenditures, and commuting costs.

The second market imperfection we assume is a down-payment requirement that ties households' ability to borrow to their wealth (Rosenthal *et al.*, 1991, Stein, 1995, Ortalo-Magné and Rady, 2006). This amounts to assuming a credit constraint written as $\lambda p\left(x\right) \geqslant p\left(x\right) - y$ or, equivalently,

$$y \geqslant (1-\lambda)p\left(x\right), \tag{2}$$

with $0 \leqslant \lambda < 1$. Households can borrow only up to a fraction of the house/land value $\lambda p\left(x\right)$. Stated differently, purchasing a house requires a private wealth y at least equal to $(1-\lambda)p\left(x\right)$.

Unlike the literature cited above, households may or may not face credit-market imperfections depending on the location within the city, as the value of housing varies with distance x. The key mechanism is that a household can be credit constrained in some areas of the city where prices are high, but does not need to borrow any amount of money in less expensive areas. Even though households are *ex-ante* homogeneous (they share the same wealth and the same wage), different types of households can emerge in equilibrium, depending on whether (i) households need to borrow and (ii) the credit constraint is binding.

[5]This extra cost of borrowing ζ could also capture a mark-up set by the banking industry. The nature of the credit market imperfection will play an important role while studying the design of the tax schedule in section 4.

3. Location choices, the bid-rent function and the urban equilibrium

We first consider a market mechanism in which households are free to move within any city and between cities. Hence, the city size L and the residential equilibrium are endogenous. Since households are identical, they must achieve the same utility level regardless of the city in which they live, and regardless of their residential location within each city. As all cities are *a priori* identical, it is sufficient to focus on a representative city.

Households consume all their income, so that the indirect utility can be written as

$$V[c(x), G] = u[\varphi + y - p(x) - t - \kappa(x) - \mathbf{1}_{\{p(x)>y\}}\zeta\left[p(x) - y\right], G].$$

where we used (1). *Without* the down-payment requirement, in equilibrium consumers reach the same utility level regardless of their location, as there is perfect mobility within the city, and they are identical in terms of preferences and income. Formally, $\mathrm{d}V[c(x), G]/\mathrm{d}x = 0$, which implies

$$\Psi'(x) = \frac{-\kappa'(x)}{1 + \mathbf{1}_{\{p(x)>y\}}\zeta}, \tag{3}$$

where $\Psi(x)$ is the bid-rent function that solves the above equilibrium condition (a prime denotes $\mathrm{d}/\mathrm{d}x$). The bid-rent function $\Psi(x)$ is the maximum price per unit of land a household is willing to pay at distance x while enjoying a given utility level. Equation (3) is known as the Alonso-Muth condition in the case of fixed housing consumption. The intuition is straightforward: as distance to the CBD increases, households are willing to pay a lower price to compensate for the higher commuting costs they incur. The bid-rent slope is flatter when households need to borrow, and it decreases with a higher borrowing cost.

By integrating equation (3), we obtain the bid-rent function

$$\Psi(x) = \begin{cases} \Psi^b(x) = \frac{1}{1+\zeta}\left[K^b - \kappa(x)\right] & \text{for } x \text{ such that } \Psi^b(x) > y \\ \Psi^s(x) = K^s - \kappa(x) & \text{otherwise} \end{cases} \tag{4}$$

where the b and s superscripts stand respectively for borrowers and savers, and K^b and K^s represent the constants of integration which do not depend on the distance x. We give below the expressions of K^b and K^s, which are endogenously determined.

As $\Psi(x)$ decreases with distance, households are less likely to face the down-payment requirement, and to pay the borrowing cost when they locate further from the CBD. If the credit constraint is binding, the bid rent offered by credit-constrained households is $y/(1 - \lambda)$ according to (2). In this case, credit-constrained households need to borrow because $y/(1-\lambda) > y$. We assume that the down-payment requirement is binding at the CBD whereas it is slack at the city fringe, that is, when $\Psi^b(0) > y/(1-\lambda) > \Psi^b(L)$. We can define the threshold location $x^b \in [0, L]$ where the down-payment is binding as

$$\frac{K^b - \kappa(x^b)}{1+\zeta} = \frac{y}{1-\lambda}. \tag{5}$$

We also define the threshold location x^s where the households are savers as

$$\frac{K^b - \kappa(x^s)}{1+\zeta} = K^s - \kappa(x^s) = y,$$

where $x^s > x^b$ because $\lambda < 1$. At locations $x \in [0, x^b]$, bid rents are capped by the down-payment requirement and equal to $y/(1-\lambda)$. Therefore, credit-constrained households need to borrow as $y/(1-\lambda) > y$, while in the area $x \in [x^b, x^s]$ households are not constrained but borrow to purchase less expensive housing. Further away, for all $x \in [x^s, L]$, households do not need to borrow. The bid-rent function in the presence of credit market imperfections then writes:

$$\psi(x, y) = \begin{cases} \frac{y}{1-\lambda} & \text{for } 0 < x \leqslant x^b, \\ \frac{1}{1+\zeta}\left[K^b - \kappa(x)\right] & \text{for } x^b \leqslant x \leqslant x^s, \\ K^s - \kappa(x) & \text{for } x^s \leqslant x \leqslant L. \end{cases} \tag{6}$$

Function (6) highlights how households' ability to pay is location-dependent, and how the credit constraint is binding in the most attractive locations (in this case, close to the CBD). In the area $x \in [0, x^b]$, the bid rent depends on households' wealth. It does not capitalize commuting costs and it becomes flat. By contrast, in the area $x \in [x^b, x^s]$, where households satisfy the down-payment requirement and borrow, the bid rent capitalizes both the commuting cost and the cost of borrowing. This highlights that both credit market imperfections are different by nature. Further away, that is for $x \in [x^s, L]$, households do not need to borrow any more, and the bid rent capitalizes only the commuting cost. Furthermore, it is easy to assess that if y is high enough so that the down-payment requirement constrains no household in the city (formally, $x^s \leqslant 0$), we would obtain the standard bid-rent function as in Fujita (1988).[6]

To close the model, land is allocated to the highest bidder. As the wealth distribution is homogeneous, all households have the same bid rent, therefore the equilibrium rent

[6] As mentioned above, the model can be extended to locations characterized by service facilities, and exogenous amenities. Hence, the bid rent would be non-monotonic, reflecting that remote locations endowed with pleasant amenities can be attractive places to live (see Gaigné *et al.*, 2022, for a general study).

function is defined by

$$p^{\star}(x) = \max\left\{\psi(x,y), R_A\right\},$$

where $p^{\star}(x)$ is a continuous function. Note that a city emerges if and only if $y > (1-\lambda)R_A$ or, equivalently, $x^b < L$. If the latter condition is not satisfied, no household would be able to pay more than the agricultural land rent R_A. Hence, when a city exists in equilibrium, a fraction of households does satisfy the down-payment requirement, and we must have $\Psi(L) = R_A$ so that $K^s = R_A + \kappa(L)$. Further, the price function is continuous and satisfies $[K^b - \kappa(x^s)]/(1+\zeta) = R_A + \kappa(L) - \kappa(x^s)$, leading to $K^b = (1+\zeta)\left[R_A + \kappa(L)\right] - \zeta\kappa(x^s)$. Thus, we obtain (see also Figure 1):

$$p^{\star}(x) = \begin{cases} \frac{y}{1-\lambda} & \text{for } 0 < x \leqslant x^b, \\ R_A + \kappa(L) - \frac{\zeta}{1+\zeta}\left[\kappa(x^s) - \kappa\left(x\right)\right] - \kappa(x) & \text{for } x^b \leqslant x < x^s, \\ R_A + \kappa(L) - \kappa(x) & \text{for } x^s \leqslant x < L, \\ R_A & \text{for } x^s \geqslant L. \end{cases} \tag{7}$$

When a city emerges, some households are borrowing limited if and only if $(1-\lambda)\Psi^b(0) > y$ or, equivalently, $(1-\lambda)[R_A + \kappa(L) - \zeta\kappa(x^s)/\left(1+\zeta\right)] > y$. In other words, the down-payment requirement is more likely to be binding under land scarcity (high R_A), large cities (high L), inefficient transport infrastructures (high $\kappa(L)$), and tighter credit constraints (low λ). Notice that the higher the additional cost of borrowing ζ softens the down-payment requirement since it lowers the bid rent. To sum up,

Proposition 1 *When cities are large enough and are endowed with inefficient transport infrastructures or/and land is scarce, the down-payment requirement makes the equilibrium unit price of land ($p^{\star}(x)$) lower than the rent per unit of land that households would be willing to pay ($\Psi^b(x)$) in the most attractive locations.*

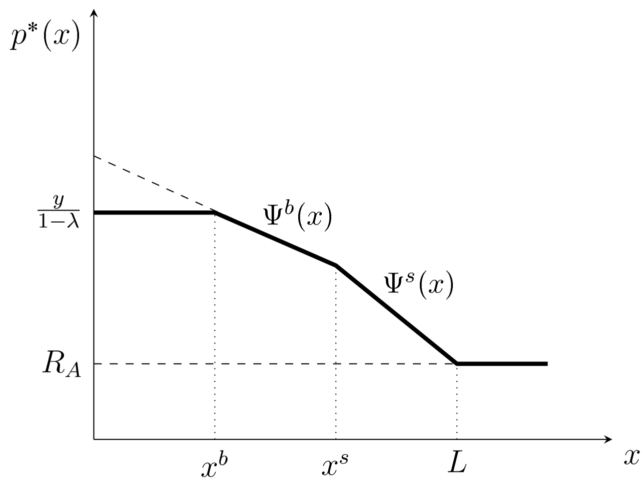

Figure 1: Equilibrium rent with credit market imperfections

Proposition 1 is still valid when housing size is variable (see section 5). A direct consequence of Proposition 1 is that the Alonso-Muth condition is not met in areas where households are constrained. When the housing size is fixed, the gradient of the bid-rent function does not depend on commuting costs. More generally, the bid rent will not capitalize even other urban features as amenities.

We also show that the down-payment requirement can impact households' utility. In a decentralized equilibrium, households' indirect utility function writes:

$$V(x) = \begin{cases} u(\varphi - G/L - \kappa(x) - (1+\zeta)\lambda y/(1-\lambda), G) \equiv V_{\mathcal{C}}(x) & \text{for } x \in \left[0, x^b\right], \\ u(y + \varphi - G/L - K^s, G) \equiv V_{\mathcal{U}} & \text{for } x \in \left[x^b, x^s\right], \\ u(y + \varphi - G/L - K^s, G) = V_{\mathcal{U}} & \text{for } x \in \left[x^s, L\right]. \end{cases}$$

where $\mathcal{C}$ stands for constrained households and $\mathcal{U}$ for unconstrained households. It is straightforward to check that $V_{\mathcal{C}} = V_{\mathcal{U}}$ when $x = x^b$ and $V_{\mathcal{C}}'(x) < 0$, so that $V_{\mathcal{C}} > V_{\mathcal{U}}$ when $x < x^b$. Some households are credit constrained and get to live close to the CBD. Their indirect utility is location-dependent. All households strictly prefer to live in $\left[0, x^b\right]$ because their marginal utility of moving marginally closer to the CBD is higher than the land rent gradient, which is nil since the rent is flat and equal to $y/(1-\lambda)$. In other words, the Alonso-Muth condition is not satisfied in $\left[0, x^b\right]$ as the price, which is capped by the credit constraint, does not increase to capture lower commuting while moving to the CBD. However, only some lucky households live there. The remaining households cannot outbid residents to live closer to the CBD to be better off, for all $x < x^b$. There is credit rationing as some households cannot borrow up to their borrowing limit to live close to the center, and are relegated to areas where they enjoy a lower utility level.[7] By contrast, wherever they borrow or save, unconstrained households are indifferent across all locations within the area $\left[x^b, L\right]$ and enjoy a lower utility compared to credit-constrained households. This result stems from the bid-rent capitalizing the additional cost of borrowing.

The down-payment requirement gives rise to *symmetry-breaking* (Matsuyama, 2000, 2006). For example, in Matsuyama (2006), the credit constraint can lead two individuals having the same wealth to differ *ex-post* as one can borrow to become an entrepreneur while the other does not get this opportunity. In our case, the mechanism is different: There is an implicit transfer of surplus from landlords to constrained households, as the latter pay a lower price than what they would pay without any borrowing limit. This makes constrained households better off compared to unconstrained households. These results can be summarized in the following proposition:

[7] We do not specify any rationing rule which would be inevitably *ad hoc* (Matsuyama, 2006).

Proposition 2 *Unlike the credit market imperfection captured by the borrowing cost, the down-payment requirement generates utility differentials among ex-ante homogeneous households (symmetry-breaking).*

We can now determine the optimal quantity of public good and the equilibrium allocation of population across cities. We consider that a given group of households with a number L, that chooses to form an urban community in order to benefit from the local public good, delegates to a city government the task of maximizing their utility level. The budget constraint of the city is such that $G = tL$. Given $V(x)$, regardless of the status (constrained or unconstrained) and their residential location, the optimal level of public good for each household is such that $(1/L)(\partial u/\partial c) = \partial u/\partial G$. The existence of the public good provides an incentive for city formation. Indeed, the per inhabitant public expenditures G/L decrease as the city size rises. However, a rise in city population implies higher land rents due to land competition. Formally, $\Psi^s(x)$ increases with population size L as $K^s = R_A + \kappa(L)$ where $\kappa(L)$ increases with L. Hence, the relationship between $V_{\mathcal{U}}$ and L follows an inverted-U curve pattern. Since individuals are freely mobile across cities, a spatial equilibrium arises when no household has an incentive to change her city choice. The unconstrained households who reach the same level of utility $V_{\mathcal{U}} = u[y + \varphi - G/L - R_A - \kappa(L), G]$ within each city, must also achieve the same level of utility across cities $V_{\mathcal{U}}^{\star}$. The outside option of workers being the benefit in the countryside $u(y - G - R_A, G)$, the population size of cities in equilibrium is given by $L^{\star}$ that is such that $\varphi - G/L^{\star} - \kappa(L^{\star}) = -G$. Since the relationship between $V_{\mathcal{U}}$ and L follows an inverted-U curve pattern, the resulting equilibrium population level is only stable when utility is decreasing with city size (see Behrens and Robert-Nicoud, 2015). As a result, the population size of cities in equilibrium is too large. In addition, the number of constrained households is identical in each city in equilibrium, and the equilibrium number of cities is given by $\mathcal{N}^{\star} = \mathcal{L}/L^{\star}$ (the integer problem is neglected here).

4. A revised Henry George Theorem

We first determine the optimal solution, and then discuss the tax instruments that allow for the decentralization of the social optimum when the credit market is imperfect.

4.1. First-best city

We now expose the first-best benchmark that has been established by Arnott and Stiglitz (1979) and discussed in Arnott (2004). We consider an economy-wide *social planner*. The objective of the social planner is to maximize per capita utility. As all cities are identical, it is sufficient to focus on a representative city. The planner acquires land at price R_A

from the absentee landlords and chooses the optimal values of population L, local public good G, and consumption c. The per capita resource constraint writes

$$y + f(L)/L = c + R_A + G/L + \mathcal{T}(L)/L, \tag{8}$$

where $\mathcal{T}(L) = \int_0^L \kappa(x)\mathrm{d}x$ represents aggregate commuting costs. Increasing the population in the city has two effects: the per capita cost of providing G decreases (economies of scale) but the total commuting costs increase (diseconomies of scale).[8]

The planner maximizes households' utility $u(c, G)$ subject to the resource constraint (8). As $\mathcal{T}'(L) = \kappa(L)$, maximizing $u(y + \varphi - R_A - G/L - \mathcal{T}(L)/L, G)$ with respect to L and G yields the optimal city size, L^o, implicitly given by

$$G = \kappa(L)L - \mathcal{T}(L), \tag{9}$$

and the optimal resources allocated to public good, implicitly given by $(1/L)(\partial u/\partial c) = \partial u/\partial G$. It follows that (i) the first-order condition associated with population size (9) holds whatever the households' preferences, and (ii) the first-order condition associated with the quantity of public good ($\partial u/\partial c = L.\partial u/\partial G$) is identical to that prevailing when households are freely mobile (see section 3). Inserting (9) in (8) yields the optimal consumption of private good

$$c^o = \varphi + y - R_A - \kappa(L^o).$$

Let us now define the shadow price of land $s(x)$ as the resource saved by acquiring an extra unit of land at location x. If a household moves from the city fringe to a new location x, there are two effects on the city's resource constraint: the opportunity cost of land R_A, and the resource saving due to lower commuting costs $\kappa(L) - \kappa(x)$. Therefore, we can define the shadow price of land as $s(x) = R_A + \kappa(L) - \kappa(x)$, with $s'(x) = -\kappa'(x)$ and $s(L) = R_A$. As a result, the aggregate differential shadow land rent is

$$ASLR = \int_0^L [\kappa(L) - \kappa(x)]\mathrm{d}x = \kappa(L)L - \mathcal{T}(L). \tag{10}$$

Given (9) and (10), at the optimal population, public goods expenditures equal the aggregate differential shadow land rent ($G = ASLR$). This is the classical result of the HGT, which also posits that a 100% tax rate on differential land rents will be sufficient to finance local public goods. It can be shown that the HGT remains valid when the lot size is variable and in the presence of local amenities (see Arnott, 2004, and Fujita, 1988, for more details).

[8] The aggregate commuting cost is convex regardless of the structure of commuting cost as long as the commuting cost incurred by a household increases with the distance to jobs. For example, if $\kappa(x) = \kappa x^{\epsilon}$ with $\kappa > 0$ and $\epsilon > 0$, then $\mathcal{T}(L) = \kappa L^{1+\epsilon}/(1+\epsilon)$ so that $\mathcal{T}(L)$ is convex with city size.

Credit market imperfections imply a distortion in the land price formation ($p^\star(x)$ can differ from $s(x)$), and preclude the decentralization of Pareto optimal allocations. As discussed in section 3, credit market imperfections create a difference between what households would be willing to pay and what they can pay. This implies that there will be a wedge between market prices and shadow prices. It is straightforward to check that the aggregate differential *market* land rent ALR, which equals to $\int_0^L [p^\star(x) - R_A]\mathrm{d}x$, is lower than the aggregate differential shadow land rent $ASLR$, as illustrated in Figure 2. First, the differential market land prices in the area hosting the unconstrained households who do not need to borrow ($x \in [x^s, L]$) $p^\star(x) - R_A$ is equal to $s(x) - R_A$. Hence, despite distortions, market and shadow prices of land located at the city fringe coincide. Second, the land price is lower than the shadow price when households need to borrow because the additional borrowing cost (ζ) is negatively capitalized into land prices. This distortion causes a fall in land rent equal to the area S^b in Figure 2. Third, we have shown that the lucky households who are credit constrained at the equilibrium pay a house price that is lower than the price they would pay without the down-payment requirement ($y_t/(1-\lambda) < \Psi^b(x)$ for $0 < x < x^b$). In this case, there is an additional loss of land rent equal to S^c in Figure 2.

Overall, for a given city size, aggregate differential shadow land rents $ASLR$ are greater than aggregate differential market rents ALR. As a result, the HGT needs to be adjusted in our distorted economy. We now discuss tax instruments that allow for the decentralization of the social optimum. First, we consider the case where the imperfection is only due to an additional borrowing cost ($\zeta > 0$ and $\lambda = 1$). Then, we focus our analysis on the case where there is a borrowing constraint ($\lambda < 1$ and $\zeta = 0$). In both cases, the population size of the decentralized city is assumed to be L^o (we disregard policies implemented by the planner to reach this level).

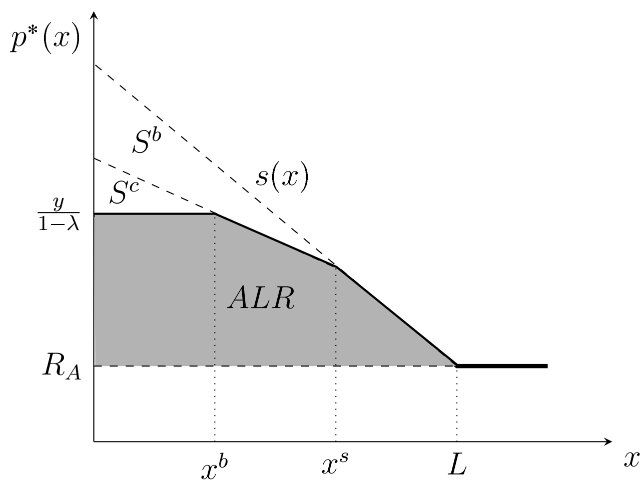

Figure 2: Equilibrium land rent and shadow rent

4.2. Tax schedule under additional borrowing cost

Without loss of generality, we assume that all households have to borrow regardless of their residential location, so that $R_A > y$. In this case, according to section 3, the rent function is $p(x) = R_A + [\kappa(L) - \kappa(x)]/(1+\zeta) \equiv p^m(x)$, which is lower than the shadow rent. Consequently, aggregate differential land rents are lower than aggregate differential shadow rents, namely $ALR^m = ASLR/(1+\zeta) < G$. In this case, a land tax is insufficient to finance local public goods. Moreover, the choice of tax instruments depends on the nature of market imperfections. We consider two cases in which the cost of borrowing reflects either (i) the market power of the banking sector, or (ii) borrowers' moral hazard behavior.

If $\zeta > 0$ reflects market power in the banking industry, a tax on banks' profit is needed. Indeed, the presence of a mark-up on loans yields a transfer of rent from landowners to banks. A higher mark-up generates lower land prices $p^m(x)$ and a rise in profits for lenders, which amounts to $\zeta[p^m(x) - y]$. The planner must confiscate a fraction of lenders' profits associated with the wedge between the price of land and its opportunity cost, equal to $\zeta[p^m(x) - R_A] \equiv \tau^b(x)$, to finance public expenditures. Indeed, we have $ALR^m + \int_0^{L^o} \tau^b(x)\mathrm{d}x = ASLR$. Under this configuration, the consumption of the private good reaches $c^b = \varphi - (1+\zeta)(R_A - y) - \kappa(L^o) < c^o$. To restore the first-best configuration, the government has to implement a lump-sum tax on lenders equal to $\zeta[R_A - y]$, where tax revenues are equally distributed among households. In this case, banks' profits are nil and $c^b = c^o$.

If the credit market is distorted by moral hazard behavior, a different tax regime has to be implemented. We adopt a second-best approach where the social planner acts under the incentive compatibility constraints of households. First, we need to specify the microeconomic foundation of imperfect lending. Assume that borrowers can evade debt payments by implementing different costly strategies. In order to prevent this, lenders incur a cost z either in monitoring or keeping track of borrowers. As in Galor and Zeira (1993), borrowers can evade by paying a higher amount δz $(\delta > 1)$. The bank has now to decide what interest rate to charge the borrower (ζ), and how much to spend on tracking activities (z). Assuming banks operate under perfect competition with free entry, we have $\zeta[p(x) - y] = z$. Lenders choose z so that the borrower has no incentive to run away. The incentive compatibility constraint for the borrower set by the bank is such that $\delta z = (1+\zeta)[p(x) - y]$. As $\zeta[p(x) - y] = z$, the borrowing rate is $\zeta = 1/(\delta - 1) > 0$, and the consumption of the private good under moral hazard can be rewritten as

$$c^\star = \varphi - \left(1 + \frac{1}{\delta - 1}\right)(R_A - y) - \kappa(L^o) - \frac{G}{L^o}.$$

In this context, we study second-best tax policy in the presence of moral hazard. To achieve the second-best outcome, a lump-sum tax t^h and a tax paid by households $\tau^h(x)$ to finance the monitoring costs, given by

$$\tau^h(x) = \frac{p^h(x) - y}{\delta},$$

are required. The government would have to cover the costs of moral hazard from the proceeds of a tax levied on borrowers. Since lenders do not incur those costs, the borrowing rate equals the lending rate, so that $\zeta = 0$. Under these circumstances, land price becomes

$$p^h(x) = \frac{1}{1 + 1/\delta}[K^h - \kappa(x)] \ ,$$

with $K^h = R_A(1 + 1/\delta) + \kappa(L^o)$ (as $p^h(L^o) = R_A$), and the aggregate land rents are given by $ALR^h = ASLR/(1 + 1/\delta)$. As a result, an additional tax t^h is required so that $ALR^h + t^h L^o = G$. As we must have $ASLR = G$, then $t^h = (G/L^o)/(\delta + 1)$ so that

$$c^h = \varphi - \left(1 + \frac{1}{\delta}\right)(R_A - y) - \kappa(L^o) - \frac{G}{L^o}\frac{1}{\delta + 1},$$

where $c^o > c^h > c^\star$. A location-specific tax on households to finance the costs of moral hazard yields a lower borrowing cost, and allows the price of land to move toward its shadow price. To finance public expenditures, combining a confiscatory tax on land rent and a lump sum tax on households generates higher utility levels. If the cost to evade debt payments is prohibitive ($\delta \to \infty$), then the utility level reaches its optimal level (c^h tends to c^o).

To summarize our discussion,

Proposition 3 *When the borrowing rate is higher than the lending rate, the presence of credit constraints implies that a single 100 % land rent tax is insufficient to finance local public expenditures. In order to reach an efficient outcome, a fraction of banks' profits has to be confiscated in the presence of market power in the banking industry while a combination of lump-sum tax and location-specific tax on households must be implemented in the presence of moral hazard.*

4.3. Tax schedule with down-payment requirement

We study the case where $\zeta = 0$ and a fraction of the population is credit constrained (a necessary condition is $\Psi(0) > y/(1 - \lambda) > R_A$ with $\Psi(x) = R_A + \kappa(L) - \kappa(L)$), which amounts to have $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ with $\underline{\lambda} = 1 - y/R_A$ and $\overline{\lambda} = 1 - y/(R_A + \kappa(L))$. In order to reach the first best, policymakers could implement policies aiming to relax down-payment requirement (*e.g.*, increasing the λ) up to the point in which no household would be constrained, so that $\lambda \geqslant \overline{\lambda}$. In this case, the rent function capitalizes any urban features, and taxing land rents will be sufficient to finance public goods. Nevertheless, the credit constraint is implemented to manage financial market failures, and is not a policy instrument to regulate land markets. Since credit market imperfections might be challenging to tackle, introducing other tax instruments is necessary to reach the first best.

When the credit constraint is binding, the aggregate land rent $\overline{ALR}$ is lower than $ASLR$, since constrained households capture a fraction of aggregate shadow land rents (Proposition 2). Formally, we have

$$\overline{ALR} = \int_0^{x^b} \left[\frac{y}{1-\lambda} - R_A \right] \mathrm{d}x + \int_{x^b}^{L^o} [p^\star(x) - R_A] \mathrm{d}x < G, \tag{11}$$

where $p^\star(x) = s(x)$ when $x > x^b$. In order to reach the first best, the city government should design a tax schedule addressing the credit market imperfection. First, it can confiscate the differential land rent, and levy a tax $\bar{\tau}(x) > 0$ that may vary with consumers' locations in the area $[0, x^b)$ so that $\overline{ALR} + \int_0^{L^o} \bar{\tau}(x)\mathrm{d}x = G$. More formally, $\bar{\tau}(x) = \Psi^s(x) - y/(1-\lambda)$ when $x \in [0, x^b)$, and $\bar{\tau}(x) = 0$ when $x \in [x^b, L]$, such that $p^\star(x) = s(x)$. Note that $\Psi^s(x) - y/(1-\lambda)$ represents the additional disposable income enjoyed by constrained households. Thus, when the city size is optimal, *public expenditures equal the aggregate differential land rent plus a tax on disposable income*. In this case, $c = c^o$.

Nonetheless, this tax schedule is difficult to implement as the government needs a lot of information on the households' willingness to pay for land. Instead, policymakers could subsidize households to meet the down-payment requirement. For example, we can consider a subsidy and interest deduction on outstanding loans financed by the proceeds of a tax on real estate wealth. Without loss of generality, we assume that all households borrow to live in the city, that is, $R_A > y$. In this case, the budget constraint is

$$c = \varphi + y - p(x)[1 + \tau^p(x,y)] - \kappa(x) - \left\{ t^d - \gamma(x,y)[p(x) - y] \right\}$$

where $\tau^p(x,y)$ is a property tax, t^d is a lump-sum tax, and $\gamma(x,y)[p(x,y) - y]$ represents the mortgage interest deduction where $\gamma(x,y)$ is determined by the government. Setting $\tau^p(x,y) = \gamma(x,y)$ in order to avoid behavioral responses to the property tax, the tax revenue from a household living at x is equal to $p(x)\tau^p(x,y) - \gamma(x,y)[p(x) - y] = \gamma(x,y)y$, which can be recycled in a down-payment subsidy. In this case, the borrowing constraint (2) becomes less stringent $\lambda p(x) \geqslant p(x) - [1 + \gamma(x,y)]y$. Hence, $\gamma(x,y)$ is such that

$\gamma(x,y)y = (1-\lambda)p(x) - y > 0$, which implies $c = c^d$ with

$$c^d = \varphi + y - p(x) - \kappa(x) - t^d - [(1-\lambda)p(x) - y].$$

If $\lambda = \overline{\lambda}$, then the government sets $\gamma(x) = 0$ for any given x. As a consequence, the land price collapses to the shadow value of land, $p^\star(x) = s(x)$, and a confiscatory tax is sufficient to fund public expenditures. If $\lambda < \overline{\lambda}$, then the government sets a positive $\gamma(x)$. The bid rent verifies $(2-\lambda)\Psi'(x) = -\kappa'(x)$, so that $p(x) = [K^d - \kappa(x)]/(2-\lambda) \equiv p^d(x) < s(x)$ and $p^d(L^o) = R_A$ implies $K^d = (2-\lambda)R_A + \kappa(L^o)$. It is straightforward to check that the aggregate differential land rent under this tax regime ALR^d is equal to $ASLR/(2-\lambda)$ that is lower than $ASLR$. As a result, a positive lump-sum tax t^d levied by the government is required to finance public expenditures. Hence, t^d is such that $ASLR/(2-\lambda) + L^o t^d = G$. As $ASLR = G$, we obtain

$$t^d = \frac{G}{L^o}\frac{1-\lambda}{2-\lambda}$$

yielding

$$c^d = c^o + y - (1-\lambda)R_A - \frac{G}{L^o}\frac{1-\lambda}{2-\lambda} = c^\star + y - (1-\lambda)R_A + \frac{G}{L^o}\frac{1}{2-\lambda}$$

where $c^\star < c^d < c^o$. A combination of property tax, mortgage interest deduction, and down-payment subsidy is welfare improving regarding to a lump-sum tax, but does not achieve the first best.

To summarize,

Proposition 4 *The presence of a down-payment requirement implies that a single 100% land rent tax is insufficient to finance local public expenditures. If the government had all the available information on households' willingness to pay for land, then a location-dependent tax would reach the first best configuration. Otherwise, a second-best policy involves a lump-sum tax and mortgage subsidies funded by a property tax.*

Two final comments are in order. First, if mortgage markets are characterized by both a borrowing limit $\lambda < 1$ and a mark-up on loans $\zeta > 0$, then the city government has to implement three tax instruments to achieve an efficient outcome: a confiscatory tax on the revenue of landowners corresponding the land rent evaluated at market price (the ALR area in Figure 2), a tax on lenders' profits so that the revenue of this tax equals the area S^b in Figure 2, and a tax on household's disposable income corresponding to the area S^c in Figure 2.

Second, the optimal taxation in our framework also implies that there is no *ex-post* welfare inequality. As discussed above, without taxation, credit constraints generate *ex-post* inequalities in the welfare of *ex-ante* homogeneous households. In fact, some lucky households can reside in attractive places while paying a low price, while those living at the city fringe cannot outbid the lucky ones. According to most theories of justice, it is unfair that two individuals with the same behavior and characteristics (φ and y in our case) enjoy unequal welfare levels. An additional tax on top of the land tax would capture the welfare seized by constrained households and decrease welfare inequalities.

5. Discussion

As mentioned above, our results hold in more general frameworks.

Bid-rent function and amenities. Our model can be extended to locations characterized by service facilities and exogenous amenities. Hence, the bid-rent function could be non-monotonic, reflecting that remote locations endowed with pleasant amenities can be attractive places to live. The key point is that locations within each city are heterogeneous and vertically differentiated, that is, locations are more or less attractive places to live. We can build a location-quality index $\chi(x)$ that subsumes commuting cost $\kappa(x)$ and amenity level (say $a(x)$) into a single scalar (see Gaigné *et al.*, 2022, for a general study). The amenity level $a(x)$ can be characterized by a spatial distribution yielding adjacent nice/not nice sites (*e.g.*, a multi-modal distribution of $a(x)$). For example, consider a framework à la Rosen-Roback as in Moretti (2011) in which the utility function is given by the following quasi-linear utility $u = c + [a(x)]^\rho + \nu(G)$ where $a(x)$ is a measure of local amenities, $\rho > 0$, and ν is an increasing function of G. Assuming $\zeta = 0$ so that $c = w + y - p(x) - \kappa(x) - t$, the expression of bid-rent function is now $\Psi(x) = K - \kappa(x) + [a(x)]^\rho$, where K is independent of x, so that the location-quality index is $\chi(x) = [a(x)]^\rho - \kappa(x)$, which is assumed to be continuously differentiable. The location-quality index χ can take its lowest value at $x = \underline{x}$ and its maximum value at $x = \bar{x}$, where $\underline{x}$ and $\bar{x}$ are located in the interval $[0, L]$ (no land is vacant). In other words, χ is defined over the interval $[\underline{\chi}, \overline{\chi}]$, where $\underline{\chi} = \chi(\underline{x})$ ($\overline{\chi} = \chi(\bar{x})$) is the minimum (maximum) value of χ. Hence, the bid-rent function can be rewritten as $\Psi(\chi) = K + \chi$ with $\Psi(\underline{\chi}) = R_A$ yielding $K = R_A + \underline{\chi}$. In this context, the more attractive place is the location for which $\chi = \bar{\chi}$. The credit constraint can be binding in the areas where χ reaches its highest values. Indeed, as Ψ increases with χ, the bid-rent function intersects once $y/(1 - \lambda)$ along the χ-dimension. Therefore, our analysis reported in section 4 is still valid in the χ-dimension. It is then sufficient to study how χ varies with x to determine the residential locations of credit-constrained households. Even if $\chi(x)$ can have several peaks of different heights along the x-dimension and the bid-rent function may intersect several times $y/(1-\lambda)$ along the x-dimension, it is straightforward to determine the intervals in which the credit constraint is binding along the x-dimension.

Variable housing demand. We now consider an extension of the model in which households can decide the mass of housing units to consume. We assume $\zeta = 0$ without loss of generality. The utility function is given by $u(c, h, G)$, which is increasing in the lot size h. The budget constraint in this case is

$$y + \varphi = c + G/L + p(x)h(x) + \kappa(x),$$

where now $p(x)$ is the per-unit price of housing, and $p(x)h(x)$ represents total housing expenditures. Housing demand impacts also the credit constraint. Indeed, the credit constraint becomes:

$$\lambda p\,(x)\,h(x) \geqslant p\,(x)\,h(x) - y.$$

Households choose both c and h to maximize their utility, so that the first-order conditions are given by $u_h/u_c = p$ (with $\partial u/\partial z = u_z$ with $z = c, h$), which implicitly gives the housing demand $h(x, y)$ (with $h'(x, y) > 0$) and the indirect utility function $V(x) = u(y + \varphi - \kappa(x) - p(x)h(x), h(x), G)$. Households must be indifferent across all locations, so that $V_x(x) = 0$, hence

$$\Psi'(x) = \frac{-\kappa'(x)}{h(x)}.$$

This equation is the Alonso-Muth condition with endogenous housing demand. A marginal increase in commuting costs associated with a longer trip is compensated by the income saved on housing consumption. Assuming that housing is a normal good, households consume more land as the distance to the city center rises. Hence, a consumer paying a lower housing price bears higher commuting costs. However, the compensation is not necessarily exact because the consumption of the private good also changes with the distance x. Each individual residing further away from the city center has a larger consumption of housing and a smaller consumption of the composite good for the utility level to be the same across the city.

In order to add more structure, let us take a Cobb-Douglas utility function $u(c, h, G) = c^{1-\alpha}h^{\alpha}G^{\beta}$. It follows that the demand for housing is

$$h[x, \Psi(x)] = \alpha \frac{y + \varphi - \kappa(x) - G/L}{\Psi(x)}, \tag{12}$$

and the bid-rent function without credit constraint is

$$\Psi(x) = \left[\frac{y + \varphi - \kappa(x) - G/L}{y + \varphi - \kappa(L) - G/L}\right]^{1/\alpha} R_A, \tag{13}$$

as $V(x) = V(L)$ must hold in equilibrium. Using (12), it is straightforward to check that with a Cobb-Douglas function, total housing expenditures $\Psi(x)h(x)$ decrease with distance. Hence, as in section 3, households can be constrained in the city center, and are not constrained in the periphery when a city emerges. However, the cutoff distance below which households are credit constrained also depends on the demand for housing. Indeed, we can define $\widehat{x}$ such that

$$\Psi(\widehat{x})h[\widehat{x}, \Psi(\widehat{x})] = \frac{y}{(1-\lambda)}. \tag{14}$$

In this case, total expenditures are capped. Plugging (12) into (14) implies that $\widehat{x}$ is such that

$$y + \varphi - \kappa(\widehat{x}) - \frac{G}{L} = \frac{1}{\alpha}\frac{y}{(1-\lambda)}. \tag{15}$$

As housing size is variable, households need to be indifferent across all city locations even though they are constrained. Indeed, we must have $V_{\mathcal{C}}[x, \widetilde{\Psi}(x)] = V_{\mathcal{U}}[x, \Psi(x)] = V_{\mathcal{U}}(L, R_A)$ in equilibrium, where $\widetilde{\Psi}(x)$ is the bid rent of constrained households. According to (14), $\widetilde{h} = [y/(1-\lambda)](1/\widetilde{\Psi}(x))$ where $\widetilde{h}$ is the housing demand when the credit constraint is binding. It follows that $V_{\mathcal{C}}[x, \widetilde{\Psi}(x)] = V_{\mathcal{U}}(L, R_A)$ implies

$$\widetilde{\Psi}(x) = \left[\frac{y/(1-\lambda)}{y + \varphi - \kappa(x) - G/L}\right]\left[\frac{y + \varphi - \kappa(x) - G/L}{y + \varphi - \kappa(L) - G/L}\right]^{1/\alpha} R_A. \tag{16}$$

Because we have $y/(1-\lambda) \leqslant \Psi(x)h[x, \Psi(x)]$ or, equivalently, $[y/(1-\lambda)]/(y + \varphi - \kappa(x) - G/L) \leqslant \alpha$ when the credit constraint is binding, we have $\widetilde{\Psi}(x) < \Psi(x)$ when $x < \widehat{x}$. The only way to satisfy the credit constraint while keeping utilities constant is to pay a lower price. Therefore, with variable housing size, market rents are still lower than shadow rents when the credit constraint is binding for some households. Hence, *under variable housing size, Proposition 1 is still valid.* In addition, the housing size of constrained households declines. For instance, consider the case in which a household living in the CBD consumes the same space as a household living in the city fringe while paying a lower rent to satisfy (14). In this case, the household living at the city fringe can outbid the household in the CBD while decreasing her housing demand to enjoy a higher utility. Stated differently, constrained households have to reduce the size of their housing while satisfying the credit constraint to equalize utility levels. When households can freely choose the size of their housing, there is no difference in welfare between constrained and unconstrained households. However, the utility level of all households with the same income declines when the credit constraint is binding for some households. As the land price paid by credit-constrained households is lower than the shadow value of land, our results reported in section 4 hold.

Household heterogeneity. Although a thorough study of household heterogeneity is beyond the scope of the present paper, we discuss two types of heterogeneity. First, the willingness to pay for a shorter commute may differ among households (*e.g.*, commuting distance varies for men and women according to Le Barbanchon *et al.*, 2020). The utility of each household i is $u(c_i, G)$ with $c_i = w + y - p(x) - \kappa_i(x) - t$ (note that $\kappa_i(x)$ can also capture the sum of commuting costs borne by the members of household i).[9] In this context, the bid-rent functions, given by $\Psi_i = K_i - \kappa_i(x)$, differ across households (the integration constant K_i is specific to each household). The household with a steeper bid-rent curve (given by $\kappa_i'(x)$) locates closer to the job center (Fujita, 1988, Chapter 2). Hence, the households with the highest willingness to pay for a shorter commute live in the more attractive site, and are more likely to pay the borrowing cost and to be credit constrained. In all cases, households' bid-rent functions will be impacted by credit market imperfections and, therefore, the aggregate land rent will be lower than the aggregate shadow land rent. Consequently, tax instruments to be implemented to achieve the first-best outcome are the same as the ones presented in section 4.

Second, we discuss the case where the initial wealth y differs across households to grasp the intuition of the consequence of wealth inequalities on the urban equilibrium. As an example, let us consider a wealth distribution with a share of rich households ϕ endowed with $\overline{y}$, while the rest of households $1 - \phi$ receive an initial wealth $\underline{y} < \overline{y}$. Assume $\overline{y}$ is set so that rich households are not credit constrained, and do not need to borrow at any location x, while poor households can be borrowing limited, that is $0 < x^b(\underline{y}) < x^s(\underline{y}) < L$. In this setting, $\overline{y}$ households can outbid $\underline{y}$ households at any $x \in [0, x^s(\underline{y})]$ to be better off. Indeed, the slope of the $\overline{y}$ households' bid-rent, $\Psi^{s\prime}(x) = -\kappa'(x)$, is greater (in absolute value) than the one of the $\underline{y}$ households whether they borrow, $\Psi^{b\prime}(x) = -\kappa'(x)/(1+\zeta)$, or they are borrowing limited, whose bid-rent slopes are nil. As in Fujita (1988), households with steeper bid-rents bid away the households with flatter bid-rents. Hence, credit market imperfections generate spatial sorting of heterogeneous households. Further, as each household buys one unit of land, if $\phi L < x^b(\underline{y})$, then all rich households live in the most attractive sites. However, under this configuration, they do not pay $\Psi^s(x) = K^s - \kappa(x)$ with $K^s = R_A + \kappa(L)$. The bid rent of each $\overline{y}$ household is $\Psi\left(x, \overline{K}\right) = \overline{K} - \kappa(x)$, where $\overline{K}$ is such that $\overline{K} - \kappa(\phi L) = \underline{y}/(1-\lambda)$ with $\Psi\left(x, \overline{K}\right) < \Psi^s(x)$. Hence, the gain captured by a rich household arising from the credit constraint increases with a lower level of wealth owned by the poorest households and with a lower mass of rich households. A fraction of poor households can live in $x \in (\phi L, x^b)$ and pay a lower price $\underline{y}/(1-\lambda)$. The other poor households occupy remote sites and pay $\Psi^b(x)$ in $\left(x^b, x^s\right)$ and $\Psi^s(x)$ in (x^s, L). The bid-rent formation process generates transfers among the different types of households. In a companion paper (Brunetti *et al.*, 2022), we study the characteristics of the urban equilibrium depending on the wealth distribution. Even

[9]For example, the household may be composed by two working individuals, and the choice of the residential location can be the result of a joint optimization while preferences might differ between individuals. However, we disregard the within-household bargaining process.

though the wealthiest households are not credit constrained, the equilibrium price of land they pay is lower than the shadow value of land. Hence, our results of section 4 remain valid.

6. Conclusion

This paper investigates the impact of credit market imperfections on the urban equilibrium and the consequences of tax policy. We introduce two credit market imperfections in the standard urban land use model: households face a borrowing cost proportional to the amount borrowed, and a down-payment requirement to obtain a mortgage. There are two main results to be highlighted: First, when cities are endowed with a high population size and an inefficient transport infrastructure or/when land is scarce, the down-payment requirement distorts the land capitalization mechanism (the Alonso-Muth condition does not hold) and yields symmetry-breaking. By contrast, the borrowing cost is capitalized in the rent function. Second, the Henry George Theorem does not apply when the credit market is imperfect. Indeed, credit market imperfections lower the equilibrium land price. Hence, aggregate differential land rents are lower than total public expenditures. As a result, a 100% land tax rate is insufficient to finance local public goods, and it needs to be complemented with other types of taxation. We show that the combination of taxes necessary to finance local public goods depends on the nature of credit market imperfections. When the borrowing cost reflects credit institutions' market power, a tax on a fraction of banks' profits is needed. When the borrowing cost is generated from borrowers' moral hazard and banks' monitoring costs, a combination of a tax to finance monitoring costs and a lump sum tax is necessary. Finally, as the down-payment requirement lowers the equilibrium land price paid by households residing in attractive locations, increasing their utility, a location-dependent tax will restore the HGT. However, such a tax instrument is difficult to implement as public authorities must observe the maximum bid rent of households. A second-best policy involving a property tax, mortgage interest deduction, and down-payment subsidy can be implemented to address the inefficiency generated by a down-payment requirement.

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