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RESEARCH ARTICLE

Determining Economic Optimum Soil Sampling Density for Potassium Fertilizer Management in Soybean: A Case Study in the U.S. Mid-South

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Abstract: Determining the number of samples to collect in a field to develop soil-test K (STK) maps that are sufficiently accurate for profit-maximizing fertilizer rate prescription maps is complex. The decision also hinges on the application method—variable rate or uniform rate (VRT vs. URT). Using a 400 m^2 fishnet grid on a 26.3-ha irrigated soybean field, the authors compared sampling densities ranging from 5 to 60 samples or 5.3 ha/sample to 0.40 ha/sample. Subsequently, the authors simulated yields based on STK maps generated with that range of samples taken to generate i) associated profit-maximizing fertilizer-K rates (K*) that varied by grid with VRT, or ii) a single fertilizer rate based on field-average STK with URT, to compare revenue less fertilizer cost (NR) across VRT, URT, and sampling strategy. With more information, NR increased at a diminishing rate as crop needs could be better matched to fertilizer needs with greater detail in STK maps with VRT. Also, fertilizer use with URT was higher than VRT given the field-specific distribution of STK. Regardless of the sampling strategy, NR was higher for VRT than URT, however, that benefit was smaller than the upcharges for VRT equipment. Marginal benefits from added soil sampling were smaller than their marginal cost leading to an optimal least-cost, 5-sample strategy and URT. Changing one of the 5 sampling locations, however, revealed unreliable field average STK estimates. Since soil samples inform about several macronutrients, splitting soil sampling charges across K and P profitably justified sampling near every 1.5 ha with URT.

Keywords: Soil sampling density; Potassium; Soybean; URT; VRT

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1. Introduction

The profit-maximizing number of soil samples to collect in a field depends on the value gained from collecting that extra information. As such, optimal spatial soil sampling density or economic optimum sampling density (EOSD) translates to an economic and environmental benefit-cost tradeoff analysis. In essence, this research analyzes the benefit of greater spatial knowledge of soil-test potassium (STK) to inform K fertilizer rate and application technology selection at the onset of the growing season. Variable-rate technology (VRT) may be used to tailor in-season fertilizer-K application rates to grid portions of the field to avoid excess/insufficient nutrient application for profit maximization and/or minimization of nutrient runoff. To maximize each field's productivity, VRT equipment precision plays an important role in matching crop nutrient needs to spatial soil nutrient availability that usually needs to be supplemented with fertilizer. Fertilizer rate changes, both along the path and across the operating width—for equipment with section control—are not instantaneous, and may only occur in lumpy increments (i.e. in 5 kg K h^{-1} increments). Thus, with VRT, spatial fertilizer placement may suffer from timing and rate change capability errors. Nonetheless, compared to less complex and lower-cost uniform rate technology (URT), where the field receives the same fertilizer rate across the entire field, field profitability improvement with VRT due to nutrient matching is expected but is also costly. Changes in yield, fertilizer use, and application costs differ between URT and VRT and are impacted by the spatial precision of information available as well as the equipment's ability to match application rate to different crop needs in subsections of a field with varying STK. Quantifying yield and fertilizer use changes leads to a potential benefit estimate that, in turn, needs to be greater than the added cost for soil sampling and an upcharge for VRT compared to URT application equipment, for producers to benefit financially. At the same time, environmental benefits are possible as excess nutrient application in regions of the field where fertilizer may not be needed or could be applied at lesser than URT rates is expected to lead to less nutrient loss (e.g., runoff).

A large body of literature discusses the economic and environmental benefits of VRT adoption [1-5] and the effect of different spatial soil sampling densities and interpolation methods on mapping accuracy [6-8]. The optimum grid size of VRT fertilizer prescription maps has also been evaluated [9]. However, the economic benefit of sampling density or EOSD in site-specific or whole-field management under both design and model-based sampling in a precision agricultural setting has not been evaluated [10].

Soil sampling for nutrient management commenced in the mid-1940s with rapid adoption in North America. Murell et al. [11] documented continued growth in the number of soil samples collected annually between the early 2000s and 2015. Reasons for this growth are both an increase in acreage being sampled and finer spatial soil sampling densities. In Arkansas, the number of client soil samples submitted to the University of Arkansas System Division of Agriculture Marianna Soil Test Laboratory increased by almost 17.8% from 2011 to 2021 [12,13]. In 2011, 60% of the samples were collected using grid sampling. The remainder was collected as field- or area-average. In 2021, 77%, 7.5%, and 1.7% of the samples were collected using 1 ha, 0.8 ha, and 0.4 ha grid sampling, respectively. Farmers use soil test results to inform management practices, and the collected data must be reliably interpreted for spatial fertilizer rate recommendations either at the field scale using URT or the sub-field grid scale using VRT.

Temporal variation in soil-test nutrient holdings is a function of crop rotation, fertilizer application rate, and the farmer's approach to nutrient management. For instance, fertilizer rates can be selected to build sub-optimal soil nutrient levels to the optimum range using a 'build and maintain' approach. Fertilizer rates can also be selected to maximize profitability in the given year of application using a 'sufficiency' approach. Along those lines, Oliver et al. [14] suggested that for the case of K-fertilizer in fields with rice (*Oryza sativa* L.) and soybean (*Glycine max* L.) in rotation, annual profit-maximizing K-fertilizer rates led to similar input use whether or not the value of soil-test K was taken into consideration (long-run) or not (short-run). Further, short-run, profit-maximizing 'sufficiency' rates were lower compared to 'build and maintain' fertilizer rate extension recommendations that are based mainly on yield removal and soil-test K information in the application year. Given minor profitability and yield implications between 'build and maintain' vs. 'sufficiency' approaches, Oliver et al. [14] recommend the use of a short-run profit-maximizing modeling tool for soybean [15] and rice [16] to estimate yields and input use subject to soil-test K information, yield potential, input cost, and output price information.

Lawrence et al. [10] stated that at least 7.4 soil samples ha^{-1} are needed to adequately inform about soil-test phosphorus (P) at a five percent precision level. The cost of collecting soil samples and analyzing the soil ranges widely, but for average farmers, meeting the precision level as mentioned above would likely be a burden when valued at \$5.50 per sample or \$40.77 ha^{-1} using the representative mid-southern cost of production information from 2023 as reported by Mississippi State University [17]. However, this cost may need to be adjusted based on multiple end uses of soil sampling information. For example, the cost of soil sampling across multiple macro-nutrients (Nitrogen (N)-P-K) should be allocated to benefits created by individual nutrient applications (N, P, or K) for a proper cost-benefit analysis. Furthermore, soil sampling information may also inform about pH, organic matter, variable rate seeding and/or pest control. Hence, addressing the economic benefit of increasing spatial soil sampling density in the context of farm field net returns is a complex endeavor.

Given this background, we surmise that producers lack information about costs and benefits related to the number of soil samples collected at the field level with attendant implications about how much fertilizer to apply and whether or not to invest in more expensive variable- rather than uniform-rate application. The hypothesis is that soil sampling density and application method lead to different field profitability estimates and are obtained by: i) simulating soybean yield based on STK maps of varying accuracy using decision support software [15]; ii) calculating profit-maximizing K-fertilizer rates by grid; iii) comparing partial field returns across sampling strategy and application method to determine the economically optimal sampling density (EOSD); iv) conduct sensitivity analysis on soil sampling cost, application technology cost differences, fertilizer rate change equipment capability, and impact of sampling location.

2. Conceptual Framework

To quantify the benefits of different spatial soil sampling densities, the law of diminishing marginal returns [18] suggests that producer profit at the field level will increase with more intensive soil sampling because the greater accuracy from site-specific information will more closely match the plant's nutrition needs with the applied fertilizer rate. The expectation is that those benefits will diminish as the number of samples increases. The EOSD is thus reached where the marginal benefit of additional samples is equal to their marginal cost.

3. Materials and Methods

This research collected STK data from a 26.3-ha farm field near Lonoke, Arkansas in the spring of 2021. Historically, various crops, including rice, soybean, and corn (*Zea mays* L) have been grown in this field, with soybean grown in the year prior to sampling. A total of 65 soil samples at a sampling depth of 15.24 cm generated the most 'informed' soil map for the field (Figure 1) at a spatial soil sampling density of approximately 2.5 samples ha^{-1}. Soil sample information was successively removed to create soil maps of less and less accuracy as information was withheld with fewer sampling locations (black dots) in Figure 1 from left to right.

Using inverse distance weighting (IDW), soil maps with a fishnet grid size of 20 m × 20 m (400 m^2) were created to match equipment technology capable of changing application rate every 20 m given field application speeds of up to 4.5 m s^{-1} and anticipatory rate change time requirements of 2 seconds. Using a spin spreader or granular pneumatic application equipment, an operating width of 20 m without section control is relatively standard.

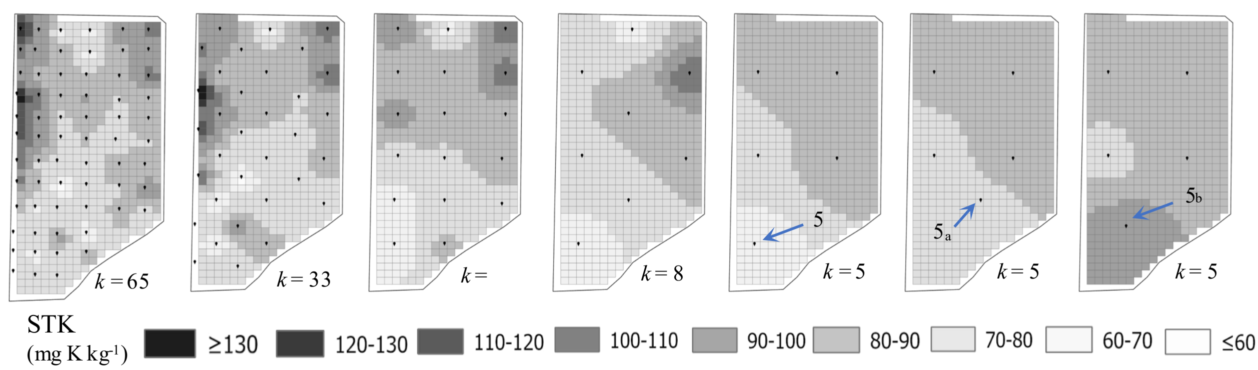

Figure 1. Field STK maps were created using ArcGIS Pro's (ESRI, Redlands, CA, USA) inverse distance weighting interpolation method (radius variable 12, power 2) with 602 – 20 m × 20 m grids at decreasing spatial soil sampling densities from left to right. STK are Mehlich-3 extractable soil K values in the top 0-15.24 cm soil layer in the spring of 2021, Lonoke, AR. Soil sampling strategies vary by the number (*k*) of soil samples taken. Sampling locations are shown with black dots. For the lowest soil sampling density strategy, the selection of the 5th sampling location was labeled for sensitivity analysis.

As such, variable rate application employs profit-maximizing-fertilizer-K application rates (K*) per grid that are based on i) calculated yield response to added K-fertilizer using prior research [15]; ii) estimated soil-test K maps that vary by soil sampling density (Figure 1); iii) 10-year average crop price; iv) fertilizer cost; and v) a user-specified field yield potential as explained in greater detail below. In comparison, the profit-maximizing uniform field rate is calculated using the same information, except that the average soil-test K value for the field rests on values per grid that change with the number of soil samples and every grid receives the same fertilizer rate.

To assess profitability changes across soil sampling strategy and application method (URT vs. VRT), field partial returns are calculated from estimated field yield times crop price minus the sum of i) fertilizer cost driven by application rate(s); ii) technology-dependent fertilizer application cost; and iii) soil sampling charges impacted by number of soil samples used. Comparison of field partial returns across sampling strategy, VRT and URT, will allow identification of the EOSD and application method as the one with the highest field partial returns.

Using the leftmost soil map in Figure 1 as the baseline, successive removal of information, as shown in Figure 1 had 33, 17, 8, and 5 soil samples remaining. This led to spatial soil sampling densities ranging from 2.47 samples ha^{-1} to 1.25, 0.65, 0.30, and 0.19 samples ha^{-1} or taking a soil sample roughly every 0.4, 0.8, 1.5, 3.3, and 5.3 ha from left to right, respectively. The latter three sampling densities are most common in Arkansas and the highest sampling density of 1 sample per 0.4 ha is considered by industry experts to be the highest sampling density a commercial crop producer or custom applicator would entertain to gain accurate field information.

The most used sampling design for many field studies is systematic sampling using transects or grids [19]. While statisticians often criticize systematic sampling designs, they are considered the most economically efficient way of collecting or analyzing information in commercial agricultural settings [20]. The STK data from each sampling density were interpolated to a fishnet grid of 20 m × 20 m using IDW with a power parameter of 2. To simplify the analysis, grids not fully included in the field boundary were excluded from the analysis as was a detailed field path analysis. As such, the field size was reduced from 26.3-ha to 24.08-ha with 602 grids comprising the field unit analyzed.

Inverse distance weighting and Kriging methods were considered as possible options for interpolation. However, semivariogram analysis (Kriging) could prove site-specific, and, as such, IDW would be more comparable across sampling density scenarios. Also, with the successive elimination of soil sampling locations, we strived to maintain more or less equal distances between sampling locations so as not to require knowledge of semivariogram parameters [6]. Finally, numerous agronomic software tools (e.g., Agstudio, Ag Leader, and Trimble Inc.) use the IDW method as their primary interpolation method to create prescription maps for seeding and fertilizer inputs [21]. In that sense, IDW conforms to what might happen when performing actual field applications.

3.1 Field Profit Estimation

Calculating soybean field partial returns as a function of yield-driven soybean revenue less operating expenses for soil sampling, fertilizer, and fertilizer application charges will vary with soil sampling density, resultant soil map information, and whether or not fertilizer is applied using VRT or URT. To obtain grid-based yield estimates, a recently published decision aid that simulates yield based on STK and K-fertilizer application was used [15]. Their tool was developed using field trial information from 2004 to 2019 involving 374 individual treatment means from 86 site-years of fertilizer-K response trials with 4 to 5 K-rate treatment comparisons to zero-K control treatments per site year. To make the tool usable across fields, yield response to K-fertilizer was estimated using relative yield by indexing K rate treatment yields relative to the yield-maximizing K rate treatment (RY = 100) for each trial. Using that relative yield response to fertilizer rate, the decision aid requires entry of a field's yield potential (*YP*) to estimate soybean yields that are achieved with varying K-fertilizer rates. The profit-maximizing K-fertilizer rate thus is a function of yield response, STK, crop price, and fertilizer cost. Hence, grid-level yield estimates ($\widehat{Y}_{ij}$) in response to STK and fertilizer application (*K*) were possible using Popp et al.'s [15] coefficient estimates by grid (*i*) when using soil maps that varied by soil sampling strategy (*j*) based on the number of soil samples collected (*k*) as follows:

$$\begin{aligned}\widehat{Y}_{ij} = (&60.013 + 0.354 \cdot STK_{i65} - 7.615 \cdot 10^{-4} \cdot STK_{i65}^2 \\ &+ 0.558 \cdot K_{ij} - 1.896 \cdot 10^{-3} \cdot {K_{ij}}^2 \\ &- 5.150 \cdot 10^{-3} \cdot STK_{i65} \cdot K_{ij} \\ &+ 1.673 \cdot 10^{-5} \cdot STK_{i65} \cdot {K_{ij}}^2 \\ &+ 1.114 \cdot 10^{-5} \cdot {STK_{i65}}^2 \cdot K_{ij} \\ &- 3.614 \cdot 10^{-8} \cdot {STK_{i65}}^2 \cdot {K_{ij}}^2)/100 \cdot YP/25\end{aligned} \quad (1)$$

where the part of the equation in parentheses represents the relative yield index estimate based on the field trials and division by 25 accounts for the number of 400 m^2 grids ha^{-1} for a yield estimate per grid. Note that while

K_{ij} will vary by grid and sampling density, the 'most informed' STK_{i65} (left-most field map in Figure 1) is used regardless of sampling density to develop yield estimates.

As in Popp et al. [15], the profit-maximizing fertilizer application rate K* (in kg K ha^{-1}) is obtained by setting the marginal cost of added fertilizer-K equal to the marginal revenue the added fertilizer delivers as follows:

$$K_{ij}^{*} = \frac{\left[\frac{c_K}{\frac{YP}{100} \cdot P_S} - (0.558 - 5.150 \cdot 10^{-3} \cdot STK_{ij} + 1.114 \cdot 10^{-5} \cdot STK_{ij}^2)\right]}{[2 \cdot (-1.896 \cdot 10^{-3} + 1.673 \cdot 10^{-5} \cdot STK_{ij} - 3.614 \cdot 10^{-8} \cdot STK_{ij}^2)]} \quad (2)$$

Ten-year average Arkansas soybean price (P_S = \$0.398 kg^{-1}) and fertilizer-K cost (c_K = \$1.094 kg^{-1}) were used to avoid unusually high or low values [22,17]. Fertilizer cost was transformed from muriate of potash fertilizer (500 g K kg^{-1}) cost information as reported by Mississippi State University to \$ kg^{-1} K and is deemed representative of mid-Southern US prices a producer would pay. Importantly, K_{ij}^* are developed using STK_{ij} that varies from STK_{i65} as less information is available to make progressively less accurate field STK maps (Figure 1 moving from left to right) for VRT fertilizer rate recommendations that vary by grid.

Uniform fertilizer rate recommendations by sampling strategy were calculated similarly,

$$UK_j^{*} = \frac{\left[\frac{c_K}{\frac{YP}{100} \cdot P_S} - (0.558 - 5.150 \cdot 10^{-3} \cdot \overline{STK}_j + 1.114 \cdot 10^{-5} \cdot \overline{STK}_j^2)\right]}{[2 \cdot (-1.896 \cdot 10^{-3} + 1.673 \cdot 10^{-5} \cdot \overline{STK}_j - 3.614 \cdot 10^{-8} \cdot \overline{STK}_i^2)]} \quad (3)$$

except $\overline{STK}_j$ are the simple averages of the field STK map derived STK_{ij} that, in turn, are a function of the number of soil samples used and lead to one fertilizer rate for the entire field.

Plugging K_{ij}^* from Equation (2) into Equation (1) as K_{ij}, field level partial returns using VRT are:

$$FPR_{j,VRT} = \sum_{i=1}^{n} (Y_{ij,VRT}^{*} \cdot P_S - K_{ij}^{*}/25 \cdot c_K - C_{VRT}/25) - FSSC_j \quad (4)$$

where n = 602 is the number of grids in the field, C_{VRT} = \$5 ha^{-1} are added VRT application charges in comparison to uniform rate application, and $FSSC_j$ are field soil sampling charges that depend on the number of samples taken at different sampling densities (k = 65, 33, 17, 8 and 5 samples in the field) at the cost of \$5.50 per sample (SSC) as reported by Mississippi State University [17]. Dividing fertilizer cost and C_{VRT} by 25 again adjusts for the number of grids ha^{-1}.

By the same token, field-level partial returns using URT were calculated with Y_{ij}^* estimates from Equation (1) using UK_j^* from Equation (3):

$$FPR_{j,URT} = \sum_{i=1}^{n} (Y_{ij,URT}^{*} \cdot P_S - UK_j^{*}/25 \cdot c_K) - FSSC_j \quad (5)$$

3.2 Sensitivity Analyses on Technology Soil Sampling Density-Related Charges

Since the cost difference between application charges for VRT vs. URT fertilizer application can vary substantially, a breakeven C_{VRT} upcharge for VRT compared to URT fertilizer application was calculated by subtracting revenue less fertilizer cost per field across the two application technologies as that net revenue difference is the maximum C_{VRT} a producer would pay to be as profitable with VRT compared to URT:

$$BEC_{j,VRT} = \sum_{i=1}^{n} \left(Y_{ij,VRT}^{*} \cdot P_S - \frac{K_{ij}^{*}}{25} \cdot c_K\right) - \sum_{i=1}^{n} \left(Y_{ij,URT}^{*} \cdot P_S - \frac{UK_j^{*}}{25} \cdot c_K\right) \quad (6)$$

In addition, as soil sampling charges (SSC) may vary not only by the charge of individual soil samplers but also by different factors: field size, crop, and number of nutrient content analyses in the report, breakeven SSC was calculated for different sampling densities. Breakeven represents the maximum a producer could pay per soil sample to adopt a particular soil sampling strategy j to achieve the same level of profitability regardless of the number of soil samples collected. It was calculated by solving for the SSC per soil sample that makes each FPR_j across sampling strategy equal and is different when more expensive VRT compared to URT is employed as follows:

$$BESSC_{j,VRT} = \frac{FSSC_j - \left(\max_j FPR_{j,VRT} - FPR_{j,VRT}\right)}{k} \quad (7)$$

$$BESSC_{j,URT} = \frac{FSSC_j - \left(\max_j FPR_{j,URT} - FPR_{j,URT}\right)}{k} \quad (8)$$

The numerator represents the maximum to pay for soil sampling to be indifferent between the most profitable sampling strategy (max FPR) and their alternative. As such, it is the strategy-relevant field soil sampling charges less the amount of profit lost by choosing a sub-optimal sampling strategy, a disadvantage that can only be justified if paying less per sample. Recall that $FSSC = SSC \cdot k$.

3.3 Sensitivity Analyses on Sampling Location and Application Rate by Grid

As the importance of a particular soil sample taken in a field influences a more significant portion of the soil map with fewer samples taken per field, the location of individual sample points also increases in importance. As shown in Figure 1, the effect of a location change for one of the sample points is used to exemplify this issue in an irregularly shaped field where this issue may be more prominent than in a square or rectangular field.

Finally, the assumption to this point was that the application equipment could change the grid application rate to match K* recommendations exactly. What if the equipment could only change K* in 5.6 kg K ha^{-1} or 11.2 kg KCl ha^{-1} muriate of potash fertilizer increments as the equipment moves from grid to grid? How would this technology limitation impact economic performance and recommendations?

3.4 Statistical Analysis

To assess differences in estimated STK, fertilizer application rate, and field partial returns, fishnet grid-based estimates were randomly assigned to four replicates. Analysis of variance was used to investigate differences in the average, standard deviation, minimum and maximum STK and K* values between sampling strategies. The sampling strategy was the explanatory variable, or treatment effect, and separate linear models were fitted for each descriptive statistic. For each model, the number of degrees of freedom for the treatment effect and residual error were 6 and 21, respectively. Analysis of variance was also used to investigate differences in field-level returns for URT and VRT at the different sampling densities. The main effects of sampling strategy and K fertilizer application method and their two-way interaction were considered as explanatory variables. The number of degrees of freedom was 6 for the main effect of the sampling distribution, 1 for the main effect of K fertilizer application method, 6 for the two-way interaction, and 42 for the residual error. The null hypothesis was that there were no significant differences in field partial return between sampling strategy and application method combinations. The null hypothesis was evaluated at P = 0.05, and post-hoc analysis was computed when appropriate using multiple pairwise comparisons. Statistical differences between treatment pairs were summarized using the compact letter display and the method established by Gramm et al. [23].

4. Results and Discussion

To benefit from VRT, the yield and fertilizer use benefits from minimizing under- and over-fertilization at the grid level in comparison to URT, must outweigh the added cost. Table 1 and Figure 1 highlight this issue by indicating changes in the fishnet grid estimates of STK and their field average, minima, and maxima across sampling densities. With more information comes more significant variability in STK, as shown in the standard deviation estimates. Hence the potential for fertilizer rate mismatch, given spatially varying STK, decreases as more information is obtained.

Also, the choice of soil sampling location can significantly impact the average STK, as shown in the last three columns of the rows with STK information. Pending the choice of one sample location 5, 5_a, or 5_b (Figure 1), field soil map information changed, leading to average field STK that successively increased for 5, 5_a, and 5_b.

Recall that profit-maximizing K* (K_{ij}^* for VRT and UK_j^* for URT) varies indirectly with STK or the more STK available in the soil, the less fertilizer K* is needed to maximize profit as evident in Table 1. In addition, K_{ij}^*, using Equation (2), varies by grid and by soil sampling strategy under VRT, and hence variance in grid STK_{ij} translated to larger variance in K_{ij}^* as sampling density increased. Additionally, it is interesting with URT that the profit-maximizing fertilizer rates, UK_j^*, were all larger than the average K_{ij}^*, a result that is likely due to the non-normal spatial distribution of STK_{ij}, as shown in the field STK maps (Figure 1).

Regarding sample point selection with the least-cost soil sampling strategy with 5 soil samples, K* successively decreased with greater STK when moving from sample points 5 to 5_a and 5_b. While the change in STK is small, it does impact the profit-maximizing K* more so than across all the other soil sampling strategies. Hence, the selection of location leads to random outcomes, a finding that relates to Lawrence et al.'s [10] findings in terms of soil map precision.

Using the field STK map information from Figure 1, the profit-maximizing K_{ij}^* were mapped in Figure 2, with the expected yield, input use, and financial implications highlighted in Table 2. As expected, yield variability increased with greater sampling density, given that K* and STK were more variable with the greater number of soil samples collected. At the same time, using the URT-based UK_j^*, led to more uniform yields than experienced with VRT. Since both yield estimates were calculated using the same, highest-information STK field map, spatial yield variability was mainly a function of VRT fertilizer use. The impact is small and would likely not be observable visually in the field by the producer. While yield variance was different, average yields were more or less the same and increased with lesser sampling density as average STK decreased and thereby fertilizer use increased, driving yields higher with lesser sampling density.

At the same time, the direct relationship between sampling density and average STK in the field is likely random and field-specific (Table 1). Note, for example, that this direct relationship between STK and sampling density changed numerically when reducing the number of samples from 8 to 5 and more or less significantly so when choosing different sampling points for the fifth soil sample with the least-cost sampling strategy occurring where $k = 5$.

Table 1. Estimated marginal means for the average, standard deviation, minimum, and maximum Mehlich-3 extractable soil-test K (STK) values in the top 15.24 cm soil layer and their resultant profit-maximizing K fertilizer rates (K*) using 10-yr average soybean price (P_S = \$0.40 kg^{-1}), fertilizer K cost (c_K = \$1.09 kg^{-1} K), and 5,044 kg ha^{-1} yield potential (YP) at decreasing soil sampling density from left to right in a 24.08-ha field near Lonoke, AR, 2021.

	Soil sampling strategy (j)[1]						
# of samples (k)	65	33	17	8	5	5_a	5_b
Statistic	*STK* in mg K kg^{-1}						
Average STK_{ij}	$83.5^{b,2}$	82.8^{b}	81.2^{c}	78.5^{d}	79.2^{d}	81.0^{c}	85.9^{a}
Standard Deviation	10.2^{a}	9.7^{ab}	8.8^{bc}	8.2^{cd}	7.2^{d}	3.6^{e}	4.4^{e}
Minimum	62.4^{b}	61.7^{b}	61.4^{b}	60.5^{b}	60.3^{b}	74.4^{a}	76.4^{a}
Maximum	125.3^{a}	122.6^{a}	104.2^{b}	104.7^{b}	87.5^{c}	87.4^{c}	95.7^{bc}
	K* in kg K ha^{-1}						
Average K_{ij}^{*3}	100.0^{c}	100.6^{c}	102.1^{b}	104^{a}	103.7^{a}	102.7^{b}	98.9^{d}
Standard Deviation	9.5^{a}	8.9^{a}	6.7^{b}	6.0^{bc}	4.6^{cd}	2.6^{d}	3.6^{d}
Minimum	44.2^{b}	46.8^{b}	81.1^{a}	80.4^{a}	97.8^{a}	97.9^{a}	90.4^{a}
Maximum	125.3^{a}	122.6^{a}	104.7^{b}	104.2^{b}	95.7^{bc}	87.5^{c}	87.4^{c}
UK_j^{*4}	101.0^{c}	101.5^{c}	102.7^{b}	104.6^{a}	104.1^{a}	102.8^{b}	99.2^{d}

Notes:

[1] See Figure 1 for soil sampling locations with varying soil sampling strategies j leading to STK_{ij} per grid i, and resultant profit-maximizing K_{ij}^* or uniform rate UK_j^*.

[2] Same letter(s) across sampling strategy j for a particular statistic (within a row) indicate no statistically significant differences at P = 0.05 for all models.

[3] See Equation (2) for the calculation of K_{ij}^* that vary by strategy and grid.

[4] See Equation (3) for the calculation of UK_j^* that vary by strategy only and is uniform across grids.

While yield results (Table 2) were somewhat random and more or less numerically invariant between VRT and URT, fertilizer use (Table 1) within a sampling strategy was always numerically less with VRT than URT and a direct result of a better match between spatial STK changes that dictated changes in K*. The fertilizer use difference between VRT and URT got smaller with less accurate soil mapping. Combining yield and fertilizer use effects, we measured the benefits from added soil sampling. A noticeable trend shows more or less stable field net revenue (revenue less fertilizer cost varied ≤ \$4 across sampling strategy, k = 65 at \$44,391 and k = 5 at \$44,387) for URT and a greater range of \$39 (k = 65 at \$44,415 and k = 5_b at \$44,376) with VRT across sampling strategy. Again, this is likely field-specific. Nonetheless, added information impacts VRT more than URT as URT applies only a slightly different UK* across sampling strategies whereas VRT results in a multitude of K* changes across grids based on the prescription maps (Figure 2). Hence, added soil map accuracy mainly benefited VRT profitability as expected.

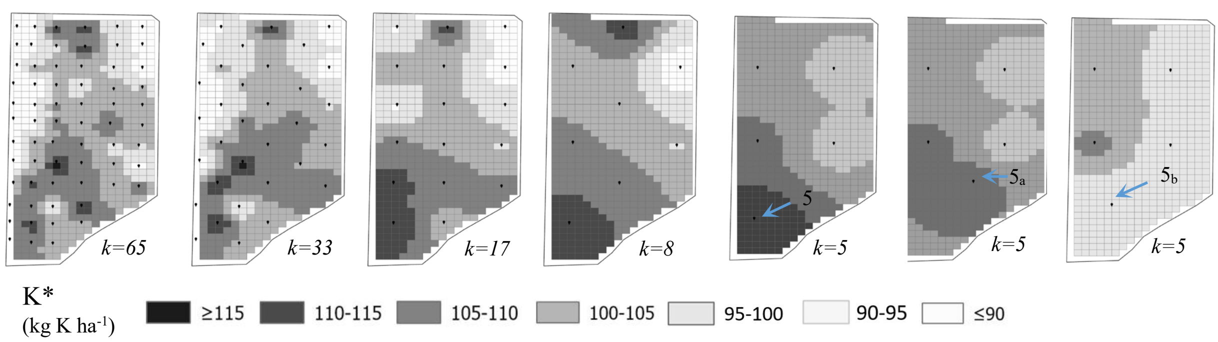

Figure 2. Grid-level profit-maximizing fertilizer-K rates (K*) for each of 602-400 m^2 grids with decreasing sampling density from left to right, Lonoke, AR, 2021. Soil sampling strategies vary by the number (k) of soil samples taken. Sampling locations are shown with black dots. For the lowest soil sampling density strategy, the selection of the 5th sampling location is labeled for sensitivity analysis.

Table 2. Estimated soybean yields (Y), field revenue ($Y\cdot P_S$), a \$5 ha^{-1} upcharge for variable rate technology (VRT) vs. uniform rate technology (URT), and soil sampling cost (SSC) of \$5.50 per sample for comparison of field partial returns (FPR) by application technology using 10-yr average soybean price (P_S = \$0.40 kg^{-1}), fertilizer K cost (c_K = \$1.09 kg^{-1} K), and 5,044 kg ha^{-1} yield potential (YP) and soil sampling strategyin a 24.08-ha field near Lonoke, AR, 2021.

	Soil sampling strategy (*j*)[1]						
# of samples (*k*)	65	33	17	8	5	5_a	5_b
Description	*Soybean average yield (standard deviation) in kg ha^{-1}*						
Y_{VRT}	4,913 (20)	4,914 (20)	4,917 (16)	4,921 (16)	4,920 (12)	4,918 (7)	4,906 (12)
Y_{URT}	4,913 (1.8)	4,914 (1.5)	4,917 (0.9)	4,922 (0.2)	4,921 (0.1)	4,918 (0.8)	4,908 (2.8)
	Field revenue in \$						
$REV_{VRT} = Y_{VRT}\cdot P_S$	\$47,048	\$47,059	\$47,086	\$47,128	\$47,120	\$47,095	\$46,981
$REV_{URT} = Y_{URT}\cdot P_S$	\$47,050	\$47,064	\$47,094	\$47,140	\$47,129	\$47,097	\$47,000
	Field fertilizer-K expense in \$						
$FC_{VRT} = K_{ij}^{*}\cdot c_K$	\$2,632	\$2,649	\$2,687	\$2,740	\$2,732	\$2,704	\$2,606
$FC_{URT} = UK_{j}^{*}\cdot c_K$	\$2,659	\$2,673	\$2,703	\$2,753	\$2,741	\$2,706	\$2,610
	Field revenue less fertilizer cost in \$						
REV_{VRT} - FC_{VRT}	\$44,415	\$44,410	\$44,399	\$44,388	\$44,388	\$44,391	\$44,376
REV_{URT} - FC_{URT}	\$44,391	\$44,391	\$44,390	\$44,387	\$44,388	\$44,390	\$44,389
	Field VRT upcharge & soil sampling cost in \$						
C_{VRT}	\$120	\$120	\$120	\$120	\$120	\$120	\$120
FSSC	\$358	\$182	\$94	\$44	\$28	\$28	\$28
	Partial field return in \$						
FPR_{VRT}[2,3]	\$43,938[j]	\$44,108[h]	\$44,186[g]	\$44,224[e]	\$44,240[d]	**\$44,243[d]**	\$44,228[d]
FPR_{URT}	\$44,033[i]	\$44,209[f]	\$44,297[c]	\$44,343[b]	\$44,361[a]	**\$44,363[a]**	\$44,362[a]
	Breakeven upcharge for VRT in \$ for field						
BEC_{VRT}[3]	\$24	\$19	\$9	\$1	\$0	\$1	-\$14
	Breakeven soil sampling charge in \$ per sample						
$BESSC_{VRT}$[3]	\$0.80	\$1.39	\$2.08	\$3.08	\$4.81	\$5.50	\$2.39
$BESSC_{URT}$	\$0.44	\$0.86	\$1.63	\$3.04	\$5.08	\$5.50	\$5.35

Notes:

[1] See Figure 1 for soil sampling locations with varying soil sampling strategies *j* leading to STK_{ij} per grid *i*, and resultant profit-maximizing K_{ij}^{*} or uniform rate UK_{j}^{*}.

[2] Same letter(s) across sampling strategy *j* and application technology indicate no statistically significant differences at P = 0.05 for all models.

[3] See Equations (4) and (5) for calculating partial field returns (FPR). See Equation (6) for the maximum field cost for variable rate technology application of fertilizer, or its breakeven cost, and see Equations (7) and (8) for the maximum soil sample charge per sample allowable before switching to the profit-maximizing sampling strategy.

On the cost side of added information, field soil sampling charge differences across sampling strategies varied considerably more (k = 65 at \$358 and k = 5 at \$28 or a range of \$330) than the benefits or field revenue less fertilizer cost numbers (\$4 URT and \$39 VRT). As such, cost savings with lesser sampling led to the most profitable field partial returns as highlighted with bold lettering in the FPR rows per application technology in Table 2. For both VRT and URT, the economic optimum sampling density (EOSD) was to collect 5 samples.

The breakeven C_{VRT} (Equation 6) increased with greater information as expected and was highest at \$24 with most information used. However, none of the sampling strategies led to greater field partial returns with VRT than URT. Hence the variation of STK in this field would not justify the use of VRT as the added upcharge for VRT application of \$120 for the field is greater than the maximum benefit attained by more precisely matching field nutrient availability with crop needs at the grid level.

Similar to the breakeven VRT upcharge results, the breakeven price for soil sampling showcased that soil sampling charges needed to decrease to justify increased accuracy in STK values. Given soybean production, the cost of soil sampling may be allocated across 2 macronutrients: P and K. The cost per nutrient per soil sample for K would thus drop to $5.50/2 samples or $2.75 per sample collected. At this cost, the EOSD is somewhere between 17 and 8 samples or sampling every 1.5 to 3.3 ha, as the most one could afford for sampling to not be worse off, or the $BESSC_{URT}$ with 17 samples was $1.63 per sample, and the $BESSC_{URT}$ with 8 samples was $3.04 per sample.

Repeating this analysis with lesser equipment accuracy (assuming fertilizer rate changes in increments of 5.6 kg K ha^{-1} by grid), results are summarized in Table 3 and show remarkably similar findings when compared to Table 2. Again, VRT is not profitable; however, with the aforementioned breakeven cost for SSC at $2.75 per sample, the EOSD is now much closer to 8 samples than 17 samples at higher equipment accuracy. Also, as profit-maximizing UK_j^* reacted to changes in average field map STK in a much more lumpy manner, given the 5.6 kg K ha^{-1} increment, field fertilizer expenses of either $2,633 or $2,765 were observed.

Now URT fertilizer expense was no longer always higher with URT than with VRT as in Table 2. With that loss in equipment accuracy, the justification for more precise STK maps thus expectedly is slightly lower.

Table 3. Estimated soybean yields (Y), field revenue ($Y \cdot P_S$), a $5 ha^{-1} upcharge for variable rate technology (VRT) vs. uniform rate technology (URT), and soil sampling cost (SSC) of $5.50 per sample for comparison of field partial returns (FPR) by application technology using 10-year average soybean price (P_S = $0.40 kg^{-1}), fertilizer K cost (c_K = $1.09 kg^{-1} K), and 5,044 kg ha^{-1} yield potential (YP) and soil sampling strategy in a 24.08-ha field near Lonoke, AR, 2021, using grid-based K* rate at nearest 5.6 kg K ha^{-1}.

	Soil sampling strategy (*j*)[1]						
# of samples (*k*)	65	33	17	8	5	5_a	5_b
Description	*Soybean average yield (standard deviation) in kg ha^{-1}*						
Y_{VRT}	4,913 (21)	4,914 (20)	4,917 (17)	4,921 (16)	4,921 (12)	4,918 (7)	4,907 (13)
Y_{URT}	4,910 (2.3)	4,910 (2.3)	4,923 (0.4)	4,923 (0.4)	4,923 (0.4)	4,923 (0.4)	4,910 (2.3)
	Field revenue in $						
$REV_{VRT} = Y_{VRT} \cdot P_S$	$47,049	$47,056	$47,080	$47,125	$47,124	$47,095	$46,994
$REV_{URT} = Y_{URT} \cdot P_S$	$47,024	$47,024	$47,151	$47,151	$47,151	$47,151	$47,024
	Field fertilizer-K expense in $						
$FC_{VRT} = K_{ij}^* \cdot c_K$	$2,634	$2,647	$2,681	$2,737	$2,739	$2,706	$2,619
$FC_{URT} = UK_j^* \cdot c_K$	$2,633	$2,633	$2,765	$2,765	$2,765	$2,765	$2,633
	Field revenue less fertilizer cost in $						
REV_{VRT} - FC_{VRT}	$44,415	$44,409	$44,399	$44,387	$44,385	$44,389	$44,374
REV_{URT} - FC_{URT}	$44,390	$44,390	$44,386	$44,386	$44,386	$44,386	$44,390
	Field VRT upcharge & soil sampling cost in $						
C_{VRT}	$120	$120	$120	$120	$120	$120	$120
FSSC	$358	$182	$94	$44	$28	$28	$28
	Partial field return in $						
FPR_{VRT}[2,3]	$43,937[j]	$44,107[h]	$44,185[g]	$44,223[e]	$44,237[d]	**$44,241[d]**	$44,226[e]
FPR_{URT}	$44,033[i]	$44,209[f]	$44,292[c]	$44,342[b]	$44,358[a]	$44,358[a]	**$44,363[a]**
	Breakeven upcharge for VRT in $ for field						
BEC_{VRT}[3]	$24	$19	$13	$1	$-1	$3	-$16
	Breakeven soil sampling charge in $ per sample						
$BESSC_{VRT}$[3]	$0.82	$1.45	$2.19	$3.26	$4.78	$5.50	$2.63
$BESSC_{URT}$	$0.42	$0.83	$1.35	$2.86	$4.58	$4.58	$5.50

Notes:

[1] See Figure 1 for soil sampling locations with varying soil sampling strategies *j* leading to STK_{ij} per grid *i*, and resultant profit-maximizing K_{ij}^* or uniform rate UK_j^*.

[2] Same letter(s) across sampling strategy *j* and application technology indicate no statistically significant differences at P = 0.05 for all models.

[3] See Equations (4) and (5) for calculating partial field returns (FPR). See Equation (6) for the maximum field cost for variable rate technology application of fertilizer, or its breakeven cost, and see Equations (7) and (8) for the maximum soil sample charge per sample allowable before switching to the profit-maximizing sampling strategy.

5. Conclusions

The goal of this research was to find an economically optimal sampling density and make a recommendation about whether or not VRT fertilizer application is profitable in comparison to applying fertilizer using URT. Using 65 soil samples collected in a 26.3-ha field dedicated to irrigated soybean production near Lonoke, AR, field STK maps were developed. By successively withholding collected soil sample information, soil map accuracy declined.

Using simulated yields that vary as a function of yield potential, STK and profit-maximizing K-fertilizer rates, field profitability implications of alternative soil map accuracy could be evaluated. This is innovative as profit-maximizing rates involving soybean price and fertilizer cost in addition to STK and yield potential alone have not been evaluated in this context to date. The proposed methods are deemed more informative and representative of what producers may do. Also, conducting this kind of analysis with actual field trials would be cost prohibitive and marred with difficulties as no two fields are the same and the same field can't be used over time given changes in STK.

Findings supported that more information led to superior net revenue (revenue less fertilizer cost) results at diminishing rates as expected with VRT. In comparison, URT used more fertilizer than VRT, given the spatial mismatch that was a function of the field-specific distribution of STK present in the soil before planting. Changes in fertilizer expense and yield implications across sampling strategy or benefits of added soil sampling were much less pronounced than concomitant changes in soil sampling charges. This was especially so at the initial cost of $5.50 per sample to collect P and K information needed for fertilizer rate prescriptions in soybean. Allocating this charge to each macronutrient equally resulted in an optimal economic sampling density between 17 and 8 samples for this field with the assumption that profit-maximizing fertilizer rates could be adjusted from grid to grid to exact needs based on IDW grid estimates of STK. Relaxing equipment accuracy to adjust the fertilizer rate in increments of 5.6 kg K ha^{-1} lowered the economically optimal number of samples to just above 8 samples.

These results supported the use of URT in comparison to VRT, which is similar to Lowenberg-DeBoer and Erickson's [3] findings. The upcharge for reducing spatial mismatch in fertilizer application was considerably larger than the economic benefit derived. Nonetheless, a difference of approximately $100 profit in a field (comparing FPR_{VRT} to FPR_{URT} in Tables 2 or 3 by sampling strategy) may well not be large enough of an economic deterrent for producers not to employ VRT. Further, greater sampling densities are economically justified with VRT than URT regardless of equipment accuracy ($BESSC_{VRT} > BESSC_{URT}$ in Tables 2 or 3 by sampling strategy).

With higher sampling density justified with VRT, the impact of potentially picking a poor soil sampling location at least sampling density (5 vs. 5a vs. 5b in the figures), becomes a moot point. Further work is needed to generalize findings to more fields in hopes of finding a rule of thumb that may help producers decide whether or not to adopt VRT in comparison to URT. At the same time, yield response to K-fertilizer is different by crop. As such, this research ought to be replicated across more crops. Finally, profit-maximizing K-fertilizer rates depend on crop price and fertilizer cost. Additional sensitivity analysis in that vein could be insightful.

Author Contributions

Bayarbat Badarch: initial writing; modeling. Michael Popp: conceptualization; modeling assistance; writing and editing; funding acquisition. Aurelie Poncet: conceptualization; statistical analysis; soil sample data acquisition. Shelby Rider: map generation. Nathan Slaton: conceptualization; editing.

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Data Availability

Data can be obtained from the authors upon request.

Conflict of Interest

The authors declare that they have no known competing interests or personal relationships that could have appeared to influence the work reported in this paper.

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