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Calibration of agricultural risk programming models using positive mathematical programming

Xuan Liu , Gerrit Cornelis van Kooten and Jun Duan[†]

Mathematical programming models of farmers' cropping decisions must first be calibrated before they can be used to examine agricultural producer responses to policy changes. In this paper, we compare three calibration approaches for disentangling the risk parameter from the parameters of the cost function: one assumes a logarithmic utility function, while the others employ an exponential utility function. Historical crop insurance data for southern Alberta, Canada, are used to assess the calibration performance of the three approaches, and sensitivity analysis is implemented to test whether the changes in the optimal land allocation caused by the changes in the values of the parameters are practically reasonable. Only one of the three models is of practical use for policy analysis because it can recover the true values of the parameters and the results of sensitivity analysis are reasonable.

Key words: agricultural policy analysis, calibration of farm management models, expected utility, decreasing and constant absolute risk aversion.

1. Introduction

Many mathematical programming, equilibrium and agent-based farm models have been developed to study the efficacy of agricultural business risk management (BRM) policies in reducing farmers' exposure to risk, and the effect that BRM programs have on land and other input use, outputs and incomes. In a recent survey of 202 studies that developed and used farm models in policy analysis, Reidsma *et al.* (2018) found that nearly 70 per cent used a mathematical programming (MP) approach and an increasing portion of these applied positive mathematical programming to calibrate the model parameters. Many farm management MP models assume that a producer varies land uses or crop activities to maximise her expected utility, where utility is modelled as the expected gross margin (= revenue – certain variable costs) minus the variance of the gross margin multiplied by a risk aversion parameter (denoted ϕ). In such models, ϕ is important for investigating farmers' economic decisions and evaluating the effectiveness of agricultural support programs. Given its importance in such models, the parameter ϕ must be calibrated along with the parameters of the cost function so that the

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MP replicates the observed land allocation before one can use the model to examine the impacts of a new policy (e.g. Howitt 1995, 2005; Paris 2011).

It has been challenging, however, to calibrate models that maximise expected utility (EU) rather than expected gross margins. The reason is that the means for calibrating ϕ along with the cost function parameters remain unsettled in the literature. Several approaches that calibrate both the risk coefficient and cost function parameters have been proposed by different researchers (e.g. Petsakos and Rozakis 2011, 2015; Cortignani and Severini 2012; Louhichi *et al.* 2018). By examining and comparing these calibration methods, it is possible to identify their strengths and shortcomings, which can further contribute to the improvement of empirical analyses related to policy decisions.

In this paper, therefore, we compare several calibration methods, focusing on their strengths and weaknesses, and evaluating their performance in recovering the values of the cost function parameters and the risk aversion parameter ϕ . We begin in the next section by briefly reviewing methods of model calibration and recent efforts to calibrate the risk aversion coefficient in farm BRM models using positive mathematical programming. In Section 3, we provide a detailed discussion of three models for calibrating the risk aversion coefficient. We compare differences in model specification, approaches to calibration and where they are applicable. Then, in Section 4, we apply our models to arable farms with mixed crop portfolios, using sensitivity analysis to test the performance of the methods and determine their robustness. Our application is to arable farms in Vulcan County in the province of Alberta, Canada. Finally, we provide some conclusions in Section 5.

2. Calibrating agricultural business risk management models: background

A major challenge of agricultural BRM modelling relates to calibration. One early approach to calibration is referred to as the historical crop-mix approach, which is used primarily for aggregate- or sector-level analysis (McCarl 1982; Önal and McCarl 1989, 1991). It does not find the explicit economic cost function but assumes that observed past crop choices are optimal; thus, it constrains farmers' crop allocations so they resemble past choices – the historical mixes. The procedure assumes that observed farmers' choices are extreme points or corners on the convex constraint set (*viz.*, a simplex algorithm for solving LP problems). It is argued that the optimal solution for crops at the aggregate level is a weighted average of extreme points at the farm level representing individual farms' optimal plans. That is, farmers' risk attitudes are implicitly addressed because the observed optimal crop portfolio chosen by farmers does not consist solely of a single crop – the one with the largest gross margin. To employ this approach for policy analysis, the aggregated MP model would need to determine the weights associated with each farm to obtain the crop choices at the aggregate level; the sum of the weights is constrained to equal 1.

7One shortcoming of this approach is that future choices are constrained by the historical ranges. As a solution to this issue, Chen and Önal (2012) suggest that it is possible to include new crops that have not previously been planted by adding synthetic (or simulated) mixes of the decision variables to the historical mixes. The optimisation procedure then chooses the weights, which are again constrained so the sum of the historical plus synthetic weights equals 1.

Positive mathematical programming (PMP) is now the preferred approach for calibrating farm management models because PMP can be used to estimate crop-specific marginal cost functions and, thereby, exactly replicate farmers' observed crop allocations (Mérel and Bucaram 2010; Mérel *et al.* 2011). PMP was first developed by Howitt (2005) (hereafter referred to as 'standard PMP') to address land-use allocation problems in agriculture (e.g. Röhm and Dabbert 2003), although PMP has increasingly been adapted for use in trade modelling and other resource management settings (Weintraub *et al.* 2007; Paris *et al.* 2011; Heckeley *et al.* 2012; Mérel and Howitt 2014; Johnston and van Kooten 2017).

While the calibration of crop-specific cost functions using PMP is generally considered to be straightforward, significant challenges remain (Heckeley and Wolff 2003; Heckeley *et al.* 2012). The standard PMP as introduced by Howitt (2005) requires specification of a strictly diagonal quadratic cost matrix, implying that there are no substitutionary or complementary effects among crops. Gradually, the PMP method has been extended by employing external information, such as supply elasticities, and the principle of maximum entropy (ME) to obtain parameter estimates for the entire cost matrix (Paris and Howitt 1998).

Moreover, Heckeley and Wolff (2003) argue that the estimates of the parameters obtained by following the standard PMP procedure can be inconsistent because the first-order conditions imposed in the first step with linear cost functions and in the last step with nonlinear cost functions are generally not compatible. Hence, they propose to estimate directly the parameters of the desired mathematical programming model by a generalised maximum entropy (GME) approach that relies on the first-order conditions (FOCs) to the MP. Prior information can also be included to influence the estimation results even in situations with limited data while ensuring computational stability.

One important challenge with agricultural BRM models relates to their calibration when risk attitudes are to be explicitly included in the analysis – when expected utility rather than net revenue (gross margin) is to be maximised. The challenge is to estimate the risk aversion coefficient and cost function parameters simultaneously within the PMP calibration framework. Several approaches are used in the literature for the calibration of ϕ ; these can be categorised into two groups based on different assumptions about the utility function (Louhichi *et al.* 2010; Jeder, Sghaier and Louhichi 2011; Jeder *et al.* 2014; Petsakos and Rozakis 2015, 2011; Cortignani and Severini 2012; Louhichi *et al.* 2018).

The first assumes that wealth W is normally distributed and that the utility function is a negative exponential function of W as follows: $U(W) = 1 - e^{-\varphi W}$. For this functional form, the constant absolute risk aversion coefficient (CARA) can simply be derived as $\varphi = -U''(W)/U'(W)$ (McCarl and Spreen 2003). Then, maximising the expectation of the negative exponential utility function is equivalent to maximising the certainty equivalent (CE) subject to technical constraints, where $CE = \mu^{-1/2} [U''(W)/U'(W)] \sigma^2 = \mu^{-1/2} \varphi \sigma^2$; μ and σ^2 are the mean and variance of the distribution of wealth. The second option assumes a logarithmic utility function: $U(W) = \ln(W)$. Its absolute risk aversion coefficient is decreasing with wealth: $\varphi = -(-W^{-2})/(W^{-1}) = 1/W$, and the corresponding relative risk aversion coefficient is 1 ($= W \times \varphi$).

The majority of studies that maximise expected utility employ an exponential utility function, which implicitly assumes CARA. Cortignani and Severini (2012) extend an ME approach proposed by Heckelevi and Wolff (2003) to estimate simultaneously all the parameters for a farm-level model within a PMP framework, including the parameters of the quadratic cost functions and the CARA coefficients. The objective is to maximise expected utility subject to resource constraints. The authors estimate the model's parameters using time-series data from a single farm, with the error terms expressed as the deviation between the observed and optimal land allocations. Meanwhile, the expected values of the own- and cross-land supply elasticities for all crops are required to obtain the decision set. We do not consider their approach here because we lack a suite of supply price elasticities of land in crops for Alberta (or even Canada).

The EU's Farm System SIMulator (FSSIM) employs another method for deriving the risk aversion parameter. The approach is to vary φ in an iterative fashion until the simulated land allocation comes closest to duplicating the observed crop allocation. If the calibration in the first step is not exact (which is the common result because risk attitudes alone cannot fully explain crop choices), the value of φ determined in the first step is assumed fixed, with the cost function then calibrated in a second step in the same way as the standard PMP method with elasticity adjustment (Louhichi *et al.* 2010; Jeder *et al.* 2014). However, because the marginal crop from the expected utility perspective may not be the least profitable crop in the standard PMP framework, directly applying this method cannot guarantee perfect recovery of the observed land allocation (Liu *et al.* 2018). Hence, calibration of the cost function parameters must be modified to achieve perfect recovery.

With the availability of farm-level data and the increasing demand to assess the impacts of policies on different farms simultaneously for accommodating heteroscedasticity, the individual farm model for the EU's Common Agricultural Policy (IFM-CAP) was developed. The first version of the IFM-CAP model was used for policy assessments in 2018 (see the technical report by Louhichi *et al.* 2018). One feature distinguishing the model from the others discussed in the paper is that the IFM-CAP model is built on the EU's Farm Accountancy Data Network (FADN) with all farms in the 2012 dataset

individually modelled. To calibrate all parameters, IFM-CAP employs data from the 2007–2012 farm-level FADN, official statistics and datasets from other models, such as the Common Agricultural Policy Regional Impact. Due to the lack of such data in Canada and the different levels of model complexity, the IFM-CAP model is not considered for further comparison in this paper.

Arata *et al.* (2014, 2017) propose to make use of the primal and dual specifications of the farmer's expected utility maximisation problem (hereafter referred to as *Arata-CARA*). They combine the 1st and 2nd steps of the standard PMP approach to derive the calibrated objective function and constraints. Instead of ME estimation, their procedure includes a least-squares estimator that is based on the errors on the marginal cost functions. Then, they simultaneously calibrate the CARA coefficient, shadow prices of land and the parameters of the cost functions. This is explained further in the next section [see Equations (11) through (15) below].

In contrast to approaches that seek to estimate a CARA coefficient, Petsakos and Rozakis (2015, 2011) assume linear cost functions and a logarithmic utility function, which leads to a decreasing absolute risk aversion (DARA) parameter that is a convex function of wealth. They then apply an ME method within the PMP framework to recover the parameters. One drawback is that the method requires a choice of an appropriate level of initial wealth. If the initial wealth is too small, the DARA coefficient is highly sensitive to the farmer's cropping choice, which leads to situations where the observed land allocation cannot be recovered. Their approach based on a logarithmic utility function (and thereby DARA) is discussed in detail at the beginning of the next section.

3. Agricultural business risk management modelling

In this section, we describe in more detail three approaches for modelling risk aversion on the part of agricultural decision-makers and compare the results from these models. We classify the models into two groups based on their assumptions about the utility function and risk attitude.

3.1 Logarithmic utility function and DARA

The method proposed by Petsakos and Rozakis (2015, 2011) begins by assuming a logarithmic utility function defined as:

$$U(W) = \ln \left[W^0 + E \left(\sum_{k=1}^K R_k \right) \right], \quad (1)$$

where W^0 denotes the (representative) farmer's initial wealth, E is the expectation operator, R_k represents the total gross margin from crop k , and

the farmer can choose from K crops. $U(W)$ has the DARA property because the risk aversion coefficient is derived as:

$$\phi = \frac{-U''(W)}{U'(W)} = \frac{1}{W^0 + E\left[\sum_{k=1}^K R_k\right]} \quad (2)$$

Petsakos and Rozakis (hereafter P&R) also assume separate linear cost functions at the farm level for each crop for simplicity, so they do not need to calibrate the parameters of a nonlinear cost function.

P&R's intuition is that, when we use regional-level prices, yields, accounting costs and the related variance–covariance matrix of gross margins to solve a farmer's optimisation problem, the derived optimal land allocation deviates from the observed farm-level land allocation. To replicate the observed land choices, the unobservable farm-level values of the above parameters must be recovered. Hence, to calibrate the DARA model, we need to solve algebraically the nonlinear expected utility maximisation problem twice to derive two sets of the FOCs using observed farm-level land allocations. The first problem is defined by equations (3) through (6) below and solved using the observed *regional-level* values of prices, yields, accounting costs and the related variance–covariance (VC) matrix of the gross margin. The second problem is defined by Equations (7) through (9), but it requires the unobserved *farm-level* values of prices, yields and the corresponding VC matrix. The FOCs associated with the above two MPs are equal to zero at the observed farm-level land allocations because the observed allocations are assumed to be optimal, as evident from Equation (10). The calibration process described later will use Equation (10) as a key constraint to derive farm-level values of prices, yields, costs and the VC matrix.

The first problem with the regional-level data can be approximated by the following MP:

$$\text{Maximise CE} = W^{f,o} + E\left[\sum_{k=1}^K r_k^R x_k^f\right] - \frac{1}{2} \frac{\sum_{k=1}^K \sum_{i=1}^K \left[x_k^f \times S_{k,i}^R \times x_i^f\right]}{W^{f,0} + E\left[\sum_{k=1}^K r_k^R x_k^f\right]} \quad (3)$$

$$\text{Subject to : } \sum_{k=1}^K x_k^f \leq \bar{X} [\psi] \quad (4)$$

$$x_k^f \leq x_k^{f,o} + \varepsilon_k[\lambda_k], \forall k, \quad (5)$$

$$x_k^f \geq 0, \forall k. \quad (6)$$

The superscript R indicates regional-level data, while f refers to data at the farm level; $W^{f,o}$ represents the farmer's initial level of wealth. The term $E[r_k^R]$ is the expected regional-level gross margin (\$/ac) from planting crop k , x_k^f

denotes the number of acres at the farm level allocated to crop k , and \bar{X} represents the total area (acres) the farmer allocates to crop production. The nonrisk component of the utility function can be written as:

$$E\left[\sum_{k=1}^K (r_k^R x_k^f)\right] = E\left[\sum_{k=1}^K (p_k^R y_k^R - c_k^R) x_k^f\right].$$

In the equation, p_k^R and y_k^R represent, respectively, the regional output price and yield for crop k ; c_k^R is the observed per-unit-area variable cost of producing crop k . As to the risk component, $S_{k,i}^R$ refers to the elements of the regional variance–covariance matrix of the realised per-acre gross margins related to crops k and i . The optimal allocation of land to crops is endogenously determined.

The shadow prices associated with the constraints are indicated in square brackets in Equations (4) and (5). Constraint (4) restricts the farmer's cultivated area to that available. Equation (5) constitute the calibration constraints and are needed to derive the shadow values (prices) of the various crops, λ_k , which are then used in Equation (10) below to recover farm-level values. In constraints (5), $x_k^{f,o}$ is the observed number of acres planted to crop k and ε_k is added to each of the calibration constraints to prevent degeneracy that could occur because constraints (4) and (5) are related.

The corresponding farm-level MP for the expected utility maximisation problem is defined as:

$$\text{Maximise CE} = W^{f,0} + E\left[\sum_{k=1}^K r_k^f x_k^f\right] - \frac{1}{2} \frac{\sum_{k=1}^K \sum_{i=1}^K [x_k^f \times S_{k,i}^f \times x_i^f]}{W^0 + E\left[\sum_{k=1}^K r_k^f x_k^f\right]} \quad (7)$$

$$\text{Subject to : } \sum_{k=1}^K x_k^f \leq \bar{X} [\psi^f] \quad (8)$$

$$x_k^f \geq 0, \forall k. \quad (9)$$

The definitions of the parameters and variables are the same as before except: (i) everything is at the farm level (e.g. $r_k^f = p_k^f y_k^f - c_k^f$); and (ii) $c_k^f = c_k^R + q_k^f$, where c_k^R and q_k^f represent, respectively, the explicit and implicit marginal costs of producing crop k , which are equal to the average costs as the cost function is assumed to be linear. In this way, q_k^f is defined as the differences between the farm-level real costs, c_k^f (\$/ac), and the observed regional accounting costs, c_k^R (\$/ac).

One can then equate the two sets of FOCs from the above two models with each other as follows (Petsakos and Rozakis 2015, p.539):

$$\begin{aligned} E(p_k^R y_k^R - c_k^R) - \frac{\sum_{i=1}^K [S_{k,i}^R \times x_i^{f,o}]}{W^{f,0} + E[(p_k^R y_k^R - c_k^R) x_k^{f,o}]} + \frac{1}{2} \frac{\left\{ \sum_{k=1}^K \sum_{i=1}^K [x_k^{f,o} \times S_{k,i}^R \times x_i^{f,o}] \right\} \times E(p_k^R y_k^R - c_k^R)}{\left\{ W^{f,0} + E[(p_k^R y_k^R - c_k^R) x_k^{f,o}] \right\}^2} - \lambda_k = \\ E(p_k^f y_k^f - c_k^f) - \frac{\sum_{i=1}^K [S_{k,i}^f \times x_i^{f,o}]}{W^{f,0} + E[(p_k^f y_k^f - c_k^f) x_k^{f,o}]} + \frac{1}{2} \frac{\left\{ \sum_{k=1}^K \sum_{i=1}^K [x_k^{f,o} \times S_{k,i}^f \times x_i^{f,o}] \right\} \times E(p_k^f y_k^f - c_k^f)}{\left\{ W^{f,0} + E[(p_k^f y_k^f - c_k^f) x_k^{f,o}] \right\}^2}, \forall k. \end{aligned} \quad (10)$$

As described by Petsakos and Rozakis (2011) Petsakos and Rozakis (2015), a maximum entropy (ME) approach can then be applied to obtain the values of

the farm-level prices, yields, variance–covariance matrix (\mathbf{S}^f), and hidden or implicit marginal costs (\mathbf{q}^f) for one representative farm in the region. For their ME process, it is necessary to have information on the initial wealth of the farm, time-series data on regional-level prices and yields, along with the constant accounting variable costs and a regional-level variance–covariance matrix of gross margins across time. Meanwhile, the farm’s technology and land allocations are also assumed constant during the period used for calibration. It is not possible to use cross-sectional data from multiple farms unless farms (i) face different prices and yields while (ii) using the same technology, (iii) have the same land constraint and (iv) allocate land across crops in exactly the same way.

3.2 Exponential utility and CARA

3.2.1 Dual approach

The key features of the method proposed by Arata *et al.* (2014, 2017) include the following: (i) the parameters of the cost functions, the risk aversion coefficients of more than one farm and the shadow prices for total land and land in various crops are all simultaneously calibrated in one step; (ii) The farm-level cost functions are defined in quadratic form with symmetric full matrices; and (iii) Farmers are assumed to have different risk attitudes; their cost functions share the same cost matrix but have different intercepts. Hence, cross-sectional data containing the base-year information for a group of farmers who share the same technology are required. Time-series data are not suitable because no farmer is supposed to change her risk attitude over time nor do the intercept terms on the crop cost functions change from year to year.

The mathematical programming model used to calibrate the parameters is as follows:

Minimise

$$\sum_{f=1}^F \psi_f \bar{X}_f + \sum_{f=1}^F \sum_{k=1}^K \left[\frac{1}{2} \alpha_{f,k}^2 + c_{f,k} y_{f,k} x_{f,k}^o + \lambda_{f,k} (y_{f,k} x_{f,k}^o + \varepsilon_{f,k}) \right. \\ \left. + \varphi_f \sum_{i=1}^K (x_{f,k}^o \times S_{k,i} \times x_{f,i}^o) - p_{f,k} y_{f,k} x_{f,k}^o \right] \quad (11)$$

subject to

$$\mathbf{c}_f + \varphi_f \mathbf{S} \mathbf{x}_f^o + \psi_f \mathbf{J} + \boldsymbol{\lambda}_f \geq \mathbf{p}_f \quad (12)$$

$$\mathbf{c}_f + \boldsymbol{\lambda}_f = \mathbf{Q} \mathbf{x}_f^o + \boldsymbol{\alpha}_f \quad (13)$$

$$\mathbf{Q} = \mathbf{L} \cdot \mathbf{D} \cdot \mathbf{L}' \quad (14)$$

$$\Psi_f, \varphi_f, \boldsymbol{\lambda}_f \geq 0 \quad (15)$$

The objective function is developed from the mathematical problem in the first step of the standard PMP and its corresponding dual problem. Constraint (12) represents the condition that marginal cost in terms of utility is

greater than or equal to price with the optimal choice. Equations (11) and (12) differ slightly from those in Arata *et al.* (2014); as discussed below, our notation and definitions of the parameters and variables differ somewhat from those used by these authors. Constraint (13) is a combination of the FOCs from the first- and second-step standard PMP equations; it shows the relationship between the linear cost and the quadratic cost for each crop.

The parameters for the F farms are calibrated via the above model. The subscript f represents the f^{th} farm; \mathbf{L} and \mathbf{D} are Cholesky decomposition matrices of \mathbf{Q} – the quadratic component of the cost functions; Ψ_f represents the land shadow price for the f^{th} farm. The vectors \mathbf{J} , λ_f , \mathbf{p}_f and α_f are all $K \times 1$; the elements of \mathbf{J} are all 1; and the other vectors represent the shadow prices, the expected product prices and the intercepts of the cost functions for K crops on the f^{th} farm, respectively. Other variables and parameters are defined as previously. \mathbf{Q} , Ψ_f , φ_f , λ_f and α_f are to be simultaneously calibrated. Then, estimated parameters for each farm are implemented using the farm risk management model defined below by Equations (16) through (18) individually for policy analysis.

In the model described by Arata *et al.* (2014), the VC matrix \mathbf{S} is based on farm-level prices, and x_k represents the total yield of the k^{th} crop, which equals the crop yield per acre times the number of acres. The implication is that a farmer can choose each crop's output level. For comparison purposes, we adjust the model used here and define \mathbf{S} as the VC matrix based on gross margins per acre. Hence, a farmer chooses how to allocate land to maximise her utility as represented by the following MP:

$$\text{Maximise: CE} = E \left[\sum_{k=1}^K (p_k y_k - \alpha_k) x_k - \frac{1}{2} \sum_{k=1}^K \sum_{i=1}^K (x_k \times Q_{k,i} \times x_i) \right] - \frac{\varphi}{2} \sum_{k=1}^K \sum_{i=1}^K (x_k \times S_{k,i} \times x_i) \quad (16)$$

$$\text{Subject to : } \sum_{k=1}^K x_k \leq \bar{X} [\psi] \quad (17)$$

$$x_k \geq 0, \forall k. \quad (18)$$

3.2.2 FSSIM-ME approach

The farmer is assumed to maximise her expected utility as defined by the MP represented by Equations (16)–(18),¹ but with one exception: the quadratic

¹ ME is a special case of generalised maximum entropy (GME), which can be used to derive the values of the parameters. If there is prior information on the range of the targeted parameters, the prior information is specified in the form of support values and a related discrete uniform probability distribution. Then, to estimate the parameters, the objective is to maximise the Shannon entropy [Equation (19)] subject to the known constraints [Equations (20) and (21)] and the available data for calculating the probabilities. Where prior information is available, the expected values of estimates of parameters are used (e.g. for α_k , Q_{kk} and λ_k), but parameters such as Ψ and φ are simply determined upon solving the MP.

terms on the cost functions constitute a diagonal matrix with positive diagonal elements. (A diagonal matrix, instead of a full matrix, is assumed because the calibration results would tend to be inconsistent with one observation and lack of information regarding the range of the matrix's elements.) We first calibrate the risk aversion coefficient φ by iteratively varying φ until the simulated land allocation comes closest to duplicating the observed crop allocation. Then, the following ME problem is constructed to calibrate the parameters of the cost functions:

$$\text{Maximise } H = \sum_{k=1}^K \sum_{z=1}^Z \pi_{k,z}^{\alpha_k} \ln(\pi_{k,z}^{\alpha_k}) - \sum_{k=1}^K \sum_{z=1}^Z \pi_{k,z}^{Q_{kk}} \ln(\pi_{k,z}^{Q_{kk}}) \quad (19)$$

subject to

$$p_k y_k - \sum_{z=1}^Z z_{k,z}^{\alpha_k} \pi_{k,z}^{\alpha_k} - \sum_{z=1}^Z z_{k,z}^{Q_{kk}} \pi_{k,z}^{Q_{kk}} \times x_k^o - \psi - \varphi \times \sum_{i=1}^K S_{k,i} \times x_i^o = 0, \forall k \quad (20)$$

$$c_k + \lambda k = \sum_{z=1}^Z z_{k,z}^{\alpha_k} \pi_{k,z}^{\alpha_k} + \sum_{z=1}^Z z_{k,z}^{Q_{kk}} \pi_{k,z}^{Q_{kk}} \times x_k^o, \forall k \quad (21)$$

$$\sum_{z=1}^Z z_{k,z}^{\alpha_k} = 1, \sum_{z=1}^Z z_{k,z}^{Q_{kk}} = 1 \quad (22)$$

$$z_{k,z}^{\alpha_k}, z_{k,z}^{Q_{kk}}, \Psi, \varphi \geq 0; \sum_{z=1}^Z z_{k,z}^{Q_{kk}} \pi_{k,z}^{Q_{kk}} \geq 0 \quad (23)$$

The superscripts α_k and Q_{kk} indicate the intercepts and the diagonal elements of the matrix of the cost functions, respectively, while λ_k represents the crop shadow prices. Following the steps of the standard ME method, for each crop we define discrete support vectors $z_k^{\alpha_k}$ and $z_k^{Q_{kk}}$ for α_k and Q_k . Accordingly, $z_{k,z}^{\alpha_k}$ and $z_{k,z}^{Q_{kk}}$ are the z^{th} elements of their respective support vectors, whose values are based on the gross margins and accounting variable costs for the crops; $\pi_{k,z}^{\alpha_k}$ and $\pi_{k,z}^{Q_{kk}}$ are the endogenous z^{th} elements of the corresponding discrete probability distributions for the above support vectors. The values of p_k , y_k , c_k and x_k^o are from the predefined base-case data. Constraints (20) and (21) are derived from the FOCs of the Lagrange functions for the first and second steps of the PMP approach. Once the values of all $\pi_{k,z}^{\alpha_k}$ and $\pi_{k,z}^{Q_{kk}}$ are obtained, $\sum_{z=1}^Z z_{k,z}^{\alpha_k} \pi_{k,z}^{\alpha_k}$ and $\sum_{z=1}^Z z_{k,z}^{Q_{kk}} \pi_{k,z}^{Q_{kk}}$ are calculated as the expected estimation of α_k and Q_{kk} .

4. Calibration and performance assessment of agricultural BRM models

To assess the performance and robustness of the three approaches discussed in Section 3, we first check their ability to calibrate the true values of the unobserved parameters and then conduct sensitivity analyses to investigate whether the optimal land allocations are sensitive to slight changes in the values of the unobserved parameters.

4.1 Methodology

Initially, we assume that the true values of all unobserved parameters, including the cost function parameters and the risk aversion coefficient, are known for each approach. Then, we obtain three sets of optimal land allocations separately using the true values of the unobserved parameters and the observed farm-level values for the exogenous parameters, including expected prices, yields and the variance–covariance matrix S . Then, for each approach, we use the derived optimal land allocation in the previous step and the observed exogenous parameters to calibrate the cost function parameters and the risk aversion coefficient(s). Finally, we compare the calibrated parameters with the true values to check the ability of the various calibration approaches to recover the true parameter values.

For the sensitivity analyses, we focus on the impacts of the changes in prices and yields on land allocation, because the expected prices and yields are comparable across different approaches since we use the same data source. But the values of ϕ and the cost function parameters are different with each approach due to their different assumptions about the utility function and cost functions, which makes the outputs less comparable. As an illustration, we assume a policy, such as the introduction of a support price, that would increase the revenue from each crop by ten per cent, with a step size of one per cent, while holding the other parameters constant; we also consider a potential decrease in revenue. We obtain an optimal land allocation under each scenario, and compare and discuss the pattern of the changes. Large revenue changes, such as a 50 per cent increase, are not considered because it is not reasonable to hold other parameters constant when expected revenue undergoes such large changes.

To facilitate the presentation and discussion, we refer to Petsakos and Rozakis's (2015, 2011) approach as *P&R-DARA* and Arata *et al.*'s approach as *Arata-CARA*, as noted earlier, and our proposed approach as *FSSIM-ME-CARA*.

4.2 Data

For our analysis and to obtain the values of the exogenous parameters, we choose arable grain farms in Vulcan County located in South Central Alberta (Figure 1). There are 70 municipalities in Alberta with cropland, but we focus on Vulcan County, which is located in the dark brown soil zone and consists of 608 farms, 1.067 million acres of cropland and a population of 3,984 (Government of Alberta, 2017a). Of the 70 municipalities, Vulcan has the

largest area of cropland, the second largest number of cropland acres per person and the third largest average acres per farm. Farmers in the county produce mainly barley, canola, peas, wheat and durum wheat, which are also the most important crops in Alberta. For the *P&R-DARA* model, we identify a representative arable farm in Vulcan County that is of average size and allocates land to crops on the basis of the average cropping pattern in the county, where the distribution of farm sizes and cropping patterns is provided by the Government of Alberta (2017b). The same is true for the *FSSIM-ME-CARA* model. For the *Arata-CARA* model, however, we employ 16 representative (average) farms based on data from 16 townships within Vulcan county and calibrate the parameters for these farms in one step. Because cross-sectional, farm-level data are not available, no other economic, financial or environmental characteristics are considered.

For calibration, although different datasets are applied to different models, the following types of data are required: product prices; yields; variable costs of production; land allocations; and the variance–covariance matrix of gross margins per acre among crops. Total variable costs of production (\$/ac) are obtained from Alberta Agriculture and Forestry (2014) and assumed constant across time and across farms. Yearly crop prices in Alberta are obtained by taking the average of monthly crop prices available from Statistics Canada (2017a). Alberta’s Agricultural Financial Services Corporation (AFSC) provided municipal-level data on average yields, the number of farms and total insured acres of cropland, which are used to calculate yields per acre and the land allocations for all crops for each year.

Table 1 provides the base-case information for the *P&R-DARA* and *FSSIM-ME-CARA* models. The prices and yields are the Olympic averages of the data in Table S1. Further, for the *P&R-DARA* model, the value of the initial wealth level W^0 is set at the average 2015 net worth for farms in the study region, namely \$3,490,636 (Statistic Canada 2017b). Table S1 provides the time-series data of prices and yields for the period 2008 through 2016 for the *P&R-DARA* model. Table S2 provides the data for calibrating the *Arata-CARA* model. Because durum is counted under wheat and not listed separately at the township level, weighted price and production costs calculated according to the land allocations of durum and wheat are used. The assumed true values for the unobserved parameters are reported with the calibration results for comparison in the next subsection.

5. Results and analysis

For each model, we report the true values and the corresponding calibrated values for the parameters of ϕ and the cost functions, followed by the results of sensitivity analysis regarding land allocation. For the *P&R-DARA* model, we also report the true values and the calibration results of the elements of the farm-level variance–covariance matrix \mathbf{S} because this matrix needs to be calibrated under P&R’s approach.

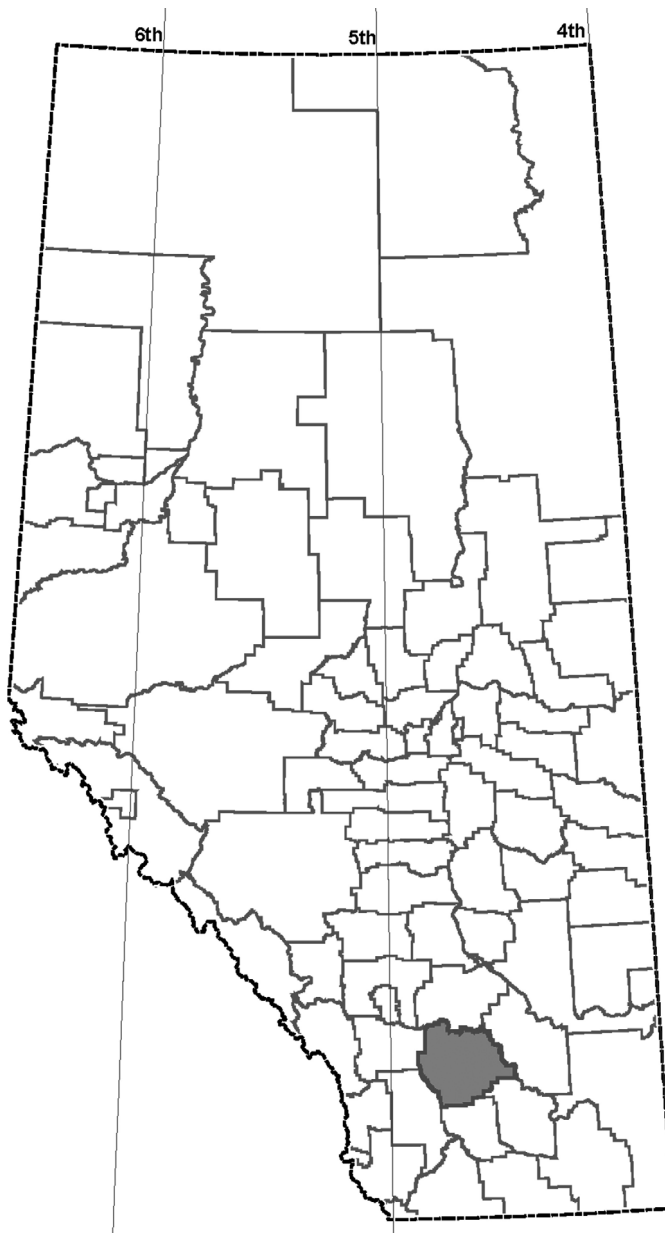


Figure 1 Map of Alberta: Vulcan county is shaded. Source: Cropped from https://education.alberta.ca/media/pdf/SchoolJurisdictionBoundaries/county/Map100_VulcanCounty.png

5.1 P&R-DARA

In this model, φ is not a constant number. Because the farmer's initial wealth is \$3.49 million and the gross margin is around \$0.3 million, the value of φ is about $2.62\text{E-}07$, which is primarily dependent on the assumed initial wealth. The true and calibrated values of farm-level prices, yields, the VC matrix \mathbf{S}

Table 1 Base-case information regarding the representative farm

Item	Barley	Canola	Durum	Peas	Wheat
Price (\$/bushel)	4.18	11.06	7.54	7.72	6.46
Yield (bushel/acre)	65.65	38.74	45.71	41.52	45.52
Variable cost (\$/acre)	110.49	172.70	138.15	135.63	138.15
Gross margin (\$/acre)	163.67	255.88	206.36	184.80	155.96
Land (%)	17.7%	24.9%	11.0%	14.9%	31.6%
Land (acre)	311.2	436.6	192.6	261.2	554.4
Farm size (acre)	1,756				
Region	South Central				
Soil Zone	Dark Brown				

and q – the term that represents the farm’s implicit production costs – are found in Table 2. The *P&R-DARA* model recovers the true values with small differences.

The sensitivity analysis indicates that the optimal land allocation changes in the *P&R-DARA* model, even with one per cent change in revenue; this outcome is not realistic. The changes in land allocation under the scenarios of increasing revenue are provided in Figure 2. The numbers on the horizontal axis represent the revenue level compared to the base case (e.g. 1.0 represents the base-case scenario with 1.05 implying a revenue increase in five per cent). Each stacked bar shows the portions of the land used for each crop for one scenario. When the revenue increases by one per cent from the base case, only canola and durum are planted. All land is used to produce canola once the revenue increases by one more per cent and no other crops are planted beyond a 2 per cent increase in revenue. Alternatively, when revenue declines by 1 per cent, only barley and wheat are produced. If the revenue decreases by 3 per cent compared to the base case, all land is allocated to barley production. The potential reason for the large discrete changes is the inflexibility associated with linear cost functions. Petsakos and Rozakis (2015) state that they use linear cost functions for simplicity. Because they fail to provide technical specifications regarding the calibration process if nonlinear cost functions are used, it is not possible to test the ability of the model to recover the parameters’ true values and conduct sensitivity analysis with a quadratic matrix in the *P&R-DARA* specification.

5.2 Show CARA

For the risk aversion coefficients, the calibrated values resulting from the *Arata-CARA* model are close to the true values. Except for the parameters of the cost functions, the biases of the estimates are large in some cases, especially for the quadratic cost term Q (see Tables 3 and 4). One potential reason for the discrepancies is that \hat{Q} is sensitive to the values of the elements of the decomposition matrices, while no reliable rules exist for the choice of those values in the calibration process.

Table 2 The true values and calibration results, P&R-DARA model

True values					Calibrated values				
$\varphi = 2.66\text{E-}07$									
Barley	Canola	Status	Peas	Wheat	Barley	Canola	Status	Peas	Wheat
Price	4.19	11.07	7.55	6.50	Price	4.19	11.07	7.55	6.50
Yield	67.30	38.82	46.11	47.00	Yield	67.30	38.81	46.11	46.99
q	14.88	99.94	53.90	10.80	q	14.93	99.95	53.92	10.79
True S									
Calibrated S									
Barley	6855	6276	3909	6955	4734	Barley	6868	6277	3913
Canola	6276	7664	2289	7447	4583	Canola	6277	7656	2288
Status	3909	2289	6974	3752	3387	Status	3913	2288	6975
Peas	6955	7447	3752	9677	5628	Peas	6963	7444	3754
Wheat	4734	4583	3387	5628	4179	Wheat	4733	4574	3385
									4168

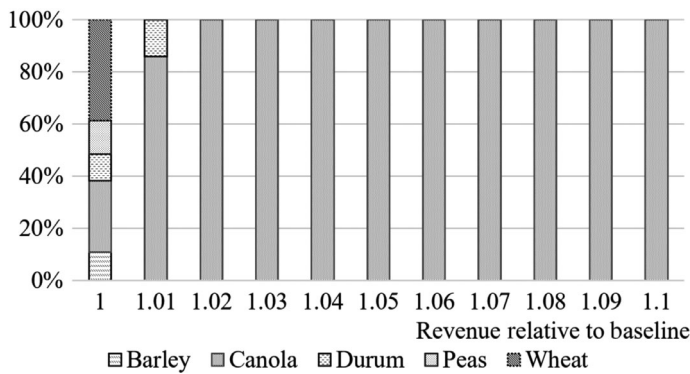


Figure 2 Impact of increases in crop revenues on land allocation, P&R-DARA model.

As to sensitivity, we find the changes in land allocations are reasonable. The land allocations for sixteen farms under two scenarios are provided in Figure 3 on a crop-by-crop basis for the base case and the case where the revenue from each crop increases by 10 per cent. The bars in light grey with dashed borders represent the acres of the crops in the base case. With higher revenues, all farms increase the land allocated to canola by 22.75 acres on average (about 5.4 per cent of the land allocated to canola in the base case), because planting canola increases revenue the most in absolute terms even though all crops’ revenues increase by the same amount in relative terms. Most farms also reduce their production of barley and peas. The change in revenues has the most diverse impacts on the land allocated to wheat: four farms slightly increase the land planted to wheat, but farm 6 reallocates 24.15 acres from wheat to canola, even though this farm allocated the least amount of land (267.7 acres in the base case) to wheat compared to other farms and other crops produced by this farm.

When the revenue from each crop falls by 10 per cent, all farms reduce the production of canola, while farmers’ allocations of land to other crops differ greatly (see Table S3). For example, seven farms decide to reduce wheat plantings, while the other five farms increase plantings of wheat. Four farms even choose to idle part of their land to reduce the production of all crops.

5.3 FSSIM - ME - FACE

With the *FSSIM-ME-CARA* model, the calibrated values of all parameters are very close to their true values, with the exception of the intercept (alpha) of the cost function for peas (see Table 5). For peas, the absolute value of alpha and the ratio of alpha over Q^{kk} (the quadratic term) are both small, indicating that the difference between the true and the calibrated cost function is acceptable.

Similar to Figure 2, the horizontal axis in Figure 4 shows the revenue levels compared to the base case. The acres allocated to each crop (rather than

Table 3 The true values and the calibration results, Arata-CARA model

Farm	True values					Calibrated values				
	φ					φ				
	Alpha					Alpha				
	Barley	Canola	Peas	Wheat		Barley	Canola	Peas	Wheat	
1	0.00E+00	-6.27	52.31	-9.81	2.21	0.00E+00	53.22	-9.35	3.53	
2	6.69E-07	26.40	-2.58	34.23	-58.05	6.40E-07	-2.52	33.56	-57.62	
3	0.00E+00	3.81	202.75	-50.83	31.76	0.00E+00	203.57	-50.24	32.66	
4	0.00E+00	39.42	17.60	-27.29	-29.73	0.00E+00	17.66	-27.70	-29.45	
5	3.03E-05	-18.81	-238.17	82.22	97.49	3.05E-05	-237.21	81.52	96.95	
6	0.00E+00	-5.28	8.37	-72.14	69.06	0.00E+00	8.14	-72.73	68.12	
7	2.17E-05	26.83	18.12	50.43	111.54	2.17E-05	19.18	50.71	112.13	
8	0.00E+00	-31.29	59.39	86.11	68.95	0.00E+00	60.26	86.49	69.73	
9	0.00E+00	-15.65	11.70	64.19	81.32	0.00E+00	12.72	64.51	81.99	
10	0.00E+00	43.58	32.73	8.59	-84.91	0.00E+00	32.71	7.99	-84.53	
11	0.00E+00	-35.46	145.96	46.48	102.70	0.00E+00	130.08	66.57	86.77	
12	3.40E-05	-39.07	-73.82	45.95	36.05	3.42E-05	-73.23	45.05	35.75	
13	0.00E+00	-11.68	52.70	-34.49	30.89	0.00E+00	53.52	-33.95	31.53	
14	0.00E+00	33.34	27.83	-15.84	-3.61	0.00E+00	28.54	-15.34	-2.66	
15	1.09E-05	77.78	5.31	-84.59	1.50	1.09E-05	5.33	-84.97	1.51	
16	9.81E-06	-12.25	-12.77	106.78	-81.76	1.00E-05	-12.54	105.48	-81.74	

Table 4 Q matrix for Arata-CARA model

True Q matrix					Calibrated Q matrix				
	Barley	Canola	Peas	Wheat		Barley	Canola	Peas	Wheat
Barley	1.647	0.014	−0.992	−0.164	Barley	0.799	−0.274	0.924	−0.952
Canola	0.014	0.742	0.111	0.080	Canola	−0.274	1.567	−1.122	0.774
Peas	−0.992	0.111	1.444	−0.133	Peas	0.924	−1.122	1.828	−1.201
Wheat	−0.164	0.080	−0.133	0.488	Wheat	−0.952	0.774	−1.201	1.650

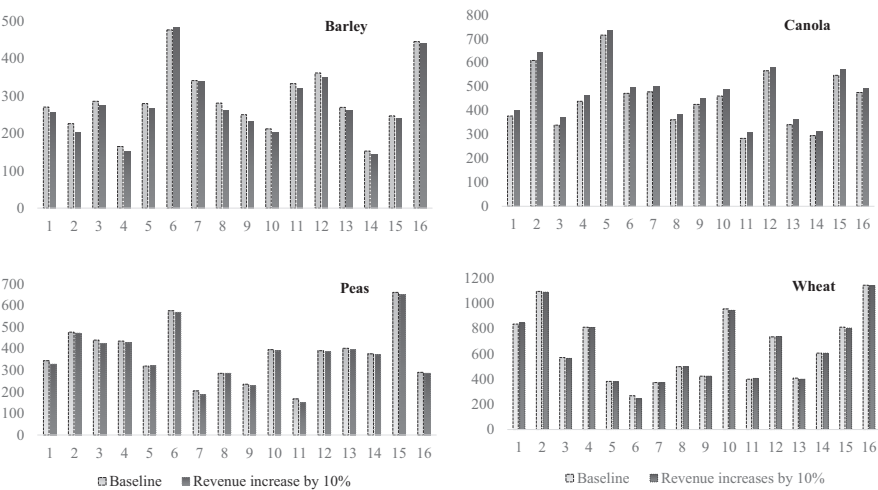


Figure 3 Impact of increases in crop revenues on land allocation, Arata-CARA Model (Farm Identifier Number on the Horizontal Axis).

proportions) are provided on the vertical axis. As crop revenue increases, more land is allocated to canola while less is allocated to barley and wheat. Meanwhile, more durum and less peas are produced, but the changes are small. For example, if revenue increases by 10 per cent, the land allocations to canola and durum are increased by 13.13 and 1.91 acres, respectively, while the land allocated to barley, peas and wheat decreases by 8.24, 0.59 and 6.22 acres, respectively. When revenue decreases, the impacts are just the opposite and all land continues to be allocated to crop production.

6. Discussion and conclusion

Agricultural economists frequently construct mathematical programming models to investigate the effects of various policy levers on farm management decisions. Instead of maximising the expected net returns (or gross margins), economists tend to maximise the expected utility of net returns because farmers are also concerned about the variance of returns when making crop allocation decisions. Given the focus on crop allocation decisions, in this study, we examined a simplified version of a farm management MP that leaves aside

Table 5 The true values and the calibration results, FSSIM-ME-CARA model

True values					Calibrated values					
$\varphi = 5.10\text{E-}06$					$\varphi = 5.10\text{E-}06$					
	Barley	Canola	Status	Peas	Wheat	Barley	Canola	Status	Peas	Wheat
Alpha	-16.89	-13.50	50.72	0.55	-73.05	-16.79	-13.54	50.37	0.73	-72.53
Q^{kk}	0.62	0.78	1.06	0.83	0.49	0.62	0.78	1.06	0.83	0.49

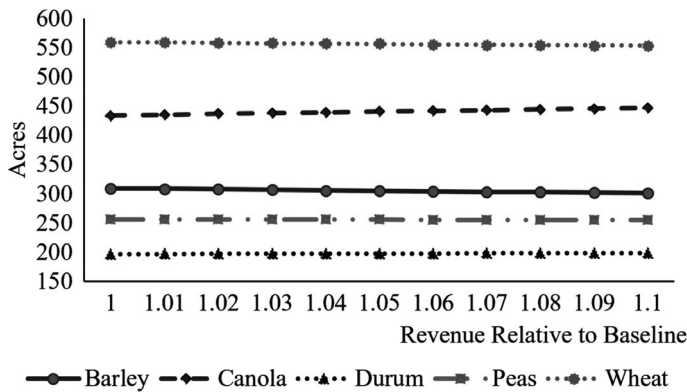


Figure 4 Impact of increases in crop revenues on land allocation, FSSIM-ME-CARA model.

biophysical and other constraints that might impact farmers’ crop choices. Where such constraints can be identified, they can simply be carried along in the various stages of the calibration without changing the results presented here. However, when these biophysical constraints are not included, the calibrated cost function and risk aversion parameters reflect the impact that such considerations would have on crop allocations. Thus, the PMP calibration procedure itself takes into account, for example, the need of farmers to plant crops in rotation to manage pathogens and other pests.

Mathematical programming models must be calibrated before they can be used to analyse the effects of new policy initiatives. Hence, the ability to calibrate the needed risk aversion and cost function parameters, and the reliability of the calibration results are key elements for assessing calibration approaches. In this study, three approaches for explicitly calibrating the risk aversion coefficients for an agricultural business risk management model were compared using an application from a grain-producing region in western Canada.

Given a logarithmic utility function, linear crop-specific cost functions and unobserved farm-level values, the *P&R-DARA* model uses a maximum entropy procedure that is able to recover the hidden marginal crop production cost (q_k) for a representative farm, along with the farm-level prices, yields and the variance–covariance matrix. However, a sensitivity analysis determined that very small changes in the calibrated parameters can lead to big changes in land allocation. Therefore, concerns are raised about the use of linear cost functions, as required by this calibration approach. It will be necessary for the future to assess the model’s performance with nonlinear cost functions, once the related technical specifications become available.

The *Arata-CARA* model employs a primal/dual approach and assumes an exponential utility function and quadratic cost functions with a full matrix. While the *P&R-DARA* model uses regional-level time-series data to calibrate parameters for one farm, the *Arata-CARA* model relies on farm-level, cross-

sectional data to calibrate parameters for more than one farm simultaneously. As a result, *Arata-CARA* leads to different calibrated values of the CARA coefficient and the intercept component of the cost functions for each of the 'observed' farms even though they share the same technology. The main drawback of this approach is that it cannot recover the true values of the parameters, especially for the quadratic terms (Q) of the cost function. The imprecision in the estimates is mainly due to the high sensitivity of the estimates to the decomposition of the Q matrix and the lack of a rule to eliminate unreasonable estimates. One potential improvement is to obtain and incorporate prior information regarding the range of the parameters into the calibration process.

Finally, the *FSSIM-ME-CARA* model starts with an iteration process to find the value of the risk aversion coefficient. Then, it uses a maximum entropy method to estimate the parameters of a quadratic cost function with a diagonal matrix for a single representative farm. In this regard, the *P&R-DARA* and *Arata-CARA* models have one advantage in that they use information from multiple data points in the calibration process. The *FSSIM-ME-CARA* model relies on only one observation. We can improve the reliability of the calibration results by taking an average of the calibrated estimates as the parameter values, and use that for policy purposes, if farm-level data for more than one farm with similar size and crop mix are available.

In summary, due to the high sensitivity of land allocation to the changes in parameters, the use of the *P&R-DARA* model for policy analysis is not empirically practical. The *Arata-CARA* model can be employed, but only after prior information about the range of the parameter values is incorporated into the calibration process to ensure an appropriate level of accuracy and precision of the estimates. At present, the *FSSIM-ME-CARA* approach is perhaps the most practicable to use for policy purposes. However, more research on this topic is required.

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Data availability statement

The data used to create the final set of results that supports the findings of this study are available in the supplementary material of this article.

References

- Alberta Agriculture and Forestry (2014). AgriProfits\$ Crop Cost and Returns Profiles. Available from URL: [http://www1.agric.gov.ab.ca/\\$department/deptdocs.nsf/all/econ14263](http://www1.agric.gov.ab.ca/$department/deptdocs.nsf/all/econ14263) [accessed 21 December 2017].

- Arata, L., Donati, M., Sckokai, P. and Arfini, F. (2014). Incorporating risk in a positive mathematical programming framework: a new methodological approach. Available from URL: <http://ageconsearch.umn.edu/record/182659> [accessed 6 January 2018].
- Arata, L., Donati, M., Sckokai, P. and Arfini, F. (2017). Incorporating risk in a positive mathematical programming framework: a dual approach, *Australian Journal of Agricultural and Resource Economics* 61, 265–284.
- Chen, X. and Önal, H. (2012). Modeling agricultural supply response using mathematical programming and crop mixes, *American Journal of Agricultural Economics* 94, 674–686.
- Cortignani, R. and Severini, S. (2012). Modelling farmer participation to a revenue insurance scheme by the means of the positive mathematical programming, *Agricultural Economics – Czech* 58, 324–331.
- Government of Alberta (2017a). Alberta regional dashboard- Vulcan County. Available from URL: <https://regionaldashboard.alberta.ca/region/vulcan-county/#/> [accessed 21 July 2018].
- Government of Alberta (2017b). Alberta regional dashboard - Vulcan county - Number of farms. Available from URL: <https://regionaldashboard.alberta.ca/region/vulcan-county/number-of-farms/#/stacked/area/> [accessed 21 July 2018].
- Heckeley, T. and Wolff, H. (2003). Estimation of constrained optimisation models for agricultural supply analysis based on generalised maximum entropy, *European Review of Agricultural Economics* 30, 27–50.
- Heckeley, T., Britz, W. and Zhang, Y. (2012). Positive mathematical programming approaches – Recent developments in literature and applied modelling, *Bio-based and Applied Economics Journal* 1, 1–16.
- Howitt, R.E. (1995). Positive mathematical programming, *American Journal of Agricultural Economics* 77, 329–342.
- Howitt, R.E. (2005). Agricultural and environmental policy models: calibration, estimation and optimization. Available from URL: <https://core.ac.uk/download/pdf/48034443.pdf> [accessed 21 July 2018].
- Jeder, H., Sgħaier, M. and Louhichi, K. (2011). Impact of water pricing policy on the sustainability of the irrigated farming systems: a case study in the South-east of Tunisia, *NewMedit* 10, 50–57 (In French).
- Jeder, H., Sgħaier, M., Louhichi, P. and Reidsma, P. (2014). Bio-economic modelling to assess the impact of water pricing policies at the farm level in the Oum Zessar Watershed, Southern Tunisia, *Agricultural Economics Review* 15, 1–19.
- Johnston, C.M.T. and van Kooten, G.C. (2017). Impact of inefficient quota allocation under the Canada-U.S. Softwood lumber dispute: A calibrated mixed complementarity approach, *Forest Policy and Economics* 74, 71–80.
- Liu, X., Duan, J. and van Kooten, G.C. (2018). The impact of changes in the agristability program on crop activities: a farm modeling approach, *Agribusiness* 34, 650–667.
- Louhichi, K., Kanellopoulos, A., Janssen, S., Flichman, G., Blanco, M., Hengsdijk, H., Heckeley, T., Berentsen, P., Lansink, A.O. and Ittersum, M.V. (2010). FSSIM, a bio-economic farm model for simulating the response of EU farming systems to agricultural and environmental policies, *Agricultural Systems* 103, 585–597.
- Louhichi, K., Espinosa, M., Ciaian, P., Perni, A., Vosough Ahmadi, B., Colen, L. and Gomez y Paloma, S. (2018). The EU-wide individual farm model for common agricultural policy analysis (IFM-CAP vol 1): Economic impacts of CAP greening, European Commission, Joint Research Centre, EUR 28829 EN, <https://doi.org/10.2760/218047>.
- McCarl, B.A. (1982). Cropping activities in agricultural sector models: a methodological proposal, *American Journal of Agricultural Economics* 64, 768–772.
- McCarl, B.A. and Spreen, T.H. (2003). Applied mathematical programming using algebraic systems. Available from URL: <http://agecon2.tamu.edu/people/faculty/mccarl-bruce/books.htm> [accessed 12 November 2016].

- Mérel, P. and Bucaram, S. (2010). Exact calibration of programming models of agricultural supply against exogenous supply elasticities, *European Review of Agricultural Economics* 37, 395–418.
- Mérel, P. and Howitt, R. (2014). Theory and application of positive mathematical programming in agriculture and the environment, *Annual Review of Resource Economics* 6, 451–470.
- Mérel, P., Simon, L.K. and Yi, F. (2011). A fully calibrated generalized constant-elasticity-of-substitution programming model of agricultural supply, *American Journal of Agricultural Economics* 93, 936–948.
- Önal, H. and McCarl, B.A. (1989). Aggregation of heterogeneous firms in mathematical programming models, *European Review of Agricultural Economics* 16, 499–513.
- Önal, H. and McCarl, B.A. (1991). Exact aggregation in mathematical programming sector models, *Canadian Journal of Agricultural Economics* 39, 319–334.
- Paris, Q. (2011). *Economic Foundations of Symmetric Programming*. Cambridge University Press, Cambridge, UK.
- Paris, Q. and Howitt, R.E. (1998). An analysis of Ill-posed production problems using maximum entropy, *American Journal of Agricultural Economics* 80, 124–138.
- Paris, Q., Drogué, S. and Anania, G. (2011). Calibrating spatial models of trade, *Economic Modelling* 28, 2,509–2,516.
- Petsakos, A. and Rozakis, S. (2011). Integrating Risk and Uncertainty in PMP Models. Paper presented at the EAAE Congress, August 30 to September 2, Zurich, Switzerland
- Petsakos, A. and Rozakis, S. (2015). Calibration of agricultural risk programming models, *European Journal of Operational Research* 242, 536–545.
- Reidsma, P., Janssen, S., Jansen, J. and van Ittersum, M.K. (2018). On the development and use of farm models for policy impact assessment in the European union – A review, *Agricultural Systems* 159, 111–125.
- Röhm, O. and Dabbert, S. (2003). Integrating Agri-environmental programs into regional production models: an extension of positive mathematical programming, *American Journal of Agricultural Economics* 85, 254–265.
- Statistics Canada (2017a). Farm product prices, crops and livestock. Available from URL: <http://www5.statcan.gc.ca/cansim/a05?lang=eng&id=0020043&pattern=0020043&searchType=ByValue=1&p2=35> [accessed 20 December 2017].
- Statistics Canada (2017b). Farm financial survey, Canadian and regional agricultural balance sheet. Available from URL: <http://www5.statcan.gc.ca/cansim/a26?lang=eng&id=20071> [accessed 7 January 2018].
- Weintraub, A., Romero, C., Bjørndal, T., Epstein, R. and Miranda, J. (eds) (2007). *Handbook of Operations Research in Natural Resources*. Springer, New York, NY.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Table S1. Time series data for prices and yields, 2008–2016.

Table S2. Township-level data of yields and land allocations in Vulcan County, 2016.

Table S3. Land allocation changes with decreased revenue, Arata-CARA model.

Appendix S1. Additional data and sensitivity analysis results.