DEVELOPMENT AND APPLICATION OF COOPERATIVE THEORY AND MEASUREMENT OF COOPERATIVE PERFORMANCE

Proceedings of a Symposium
Presented at the

Joint Meeting of the
American Agricultural Economics Association
and
Southern Agricultural Economics Association
at
Clemson University
July 27, 1981

ACS Staff Report
Agricultural Cooperative Service
U.S. Department of Agriculture
February 1982

A MODEL FOR THE SHORT-RUN PRODUCTION AND PRICING DECISIONS OF COOPERATIVE ASSOCIATIONS

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This paper presents a model of a cooperative that pays patronage refunds. The analysis is short-run in nature. Each patron, and the cooperative, has some fixed assets. Decisions on capital structure are assumed to have been made prior to this short-run analysis, and some of these decisions place constraints or impose parameters on the short-run analysis.

It is assumed that the objective of the cooperative is to maximize total profits of all member patrons. The purpose of this paper is normative: to present an explanation of how a cooperative should operate if its objective is the one assumed here. The paper presents a positive analysis of any cooperative that has as its objective the one assumed.

THREE SETS OF DECISIONS

Two keys that help in understanding the analysis in this paper are understanding the distinction drawn between price and refund, and the sequence of decisions. A price is money that exchanges hands at the time of a transaction between cooperative and patron. A refund is money allocated to a patron after an accounting period.

Three sets of decisions are made in this model. The cooperative makes two sets of decisions, one on last year's business and one on this year's business. (a)

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When the cooperative has full information on its costs and revenues from last year's operations, it determines its net savings and the patronage refunds on last year's business that are to be allocated this year. (b) It plans this year's operations and determines its current prices and levels of operations. It is through these two sets of decisions that the cooperative affects members' profits. The third set of decisions is the members' current-year decisions on their current operations. It is assumed that when the patrons make these decisions, they have full knowledge of the prices they will receive in the current year because these have already been determined by the cooperative (and other firms with which the patron may do business), but the patrons do not know the patronage refunds that they will receive next year for business they do with the cooperative in the current year. When patrons make their decisions, therefore, they have only a limited knowledge of what per-unit refunds on the current year's patronage will be.

PATRON SUBMODEL

The typical member patron attempts to maximize expected profit π

$$(1)\quad \pi = \sum_{i \varepsilon X_C} p_i a_i + \sum_{i \varepsilon X_O} p_i a_i - \sum_{i \varepsilon Y_C} p_i a_i - \sum_{i \varepsilon Y_O} p_i a_i + pvpr.$$

(For convenient reference, symbols used in discussing the multiproduct cooperative are defined in table 1.) The present value of the patronage refunds that the member firm expects to be allocated can be expressed

$$(2)\quad pvpr = [s + (1 - s)/(1 + d)^{\tau}] \sum_{i \varepsilon C} r_i^* a_i.$$

It is assumed that each r_i^* is a function of current and past values of r_i.

Maximization of (1) subject to the member patron's production function produces a set of first-order conditions. We assume that the corresponding bordered Hessian matrix of second-order partial derivatives is negative definite.

Table 1. Definition of Symbols Used in Patron and Multiproduct Cooperative Models

Symbol	Definition
π	Expected profit of a member patron, see eq. (1)
p_i	Price paid or received for i-th product or factor
a_i	Amount of i-th product or factor purchased or sold by a member patron
X_C	Set of outputs sold by patrons to cooperative
X_O	Set of outputs sold by patrons to firms other than the cooperative
Y_C	Set of variable inputs purchased by patrons from the cooperative
Y_O	Set of variable inputs purchased by patrons from firms other than the cooperative
pvpr	Present value of patronage refunds on current business that a member expects to be allocated, see eq. (2)
s	Proportion of patronage refunds paid in cash
r^*_i	Member patron's expected per-unit patronage refund on i-th product or factor
d	Member patron's discount rate
τ	Number of years that (1 - s) proportion of patronage refunds is deferred
C	Set of all products in X_C or Y_C
r_i	Actual per-unit patronage refund on i-th product or factor
X	Set of all products in X_C or X_O
Y	Set of all products in Y_C or Y_O
P_X	Vector of prices of products in set X
P_Y	Vector of prices of products in set Y
R^*_C	Vector of values of r^*_i

Table I. Continued.

Symbol	Definition
q_{iC}	Total quantity of factor or product i sold or purchased by member patrons
q_{iO}	Total quantity of product of factor i sold or purchased by from cooperative by nonmember patrons
Π	Sum of expected profits of all member patrons, see eq. (6)
PVPR	Present value of all allocated patronage refunds on current business, see eq. (7)
s'	$s + (1 - s)/(1 + d_C)^{\tau}$
d_C	Cooperative's discount rate
q_i	$q_{iC} + q_{iO}$ for $i\varepsilon C$
NS	Cooperative's net savings, see eq. (8)
FCC	Total fixed costs of cooperative firm
Z	Set of products produced by cooperative and sold outside the cooperative association
V	Set of variable factors purchased by the cooperative from outside the cooperative association
MTPR	Members' total private revenues, see discussion of eq. (8)
MTPC	Members' total private costs, see discussion of eq. (8)
TCR	Total collective revenues, see discussion of eq. (8)
TCC	Total collective costs, see discussion eq. (8)
Φ	Cooperative's implicit production function
Q_Z	Vector of quantities of goods in set Z
Q_{YC}	Vector of quantities of good in set Y_C
Q_{XC}	Vector of quantities of goods in set X_C
Q_V	Vector of quantities of goods in set V

Then the first-order conditions can be solved to obtain the member's output supply and input demand functions. Each member's product supply and input demand functions can be represented

(3) $a_i = a_i\ (P_X,\ P_Y,\ R_C^*)\ i\varepsilon X,\ Y.$

Substituting values of a_i from (3) to (1) yields an expression for the member's maximum profit. The partial derivatives of this expression with respect to $p_i\ (i\varepsilon X_C,\ Y_C)$ show how the variations in the cooperative's prices affect the member's maximum level of profit.

By summing the individual supply functions for product i across all member patrons, an aggregate member supply function is obtained. In a similar manner, aggregate member demand functions can be obtained. The sum of functions (3) over all members can be represented as

(4) $q_{iC} = q_{iC}\ (P_X,\ P_Y,\ R_C^*)\ i\varepsilon X,\ Y.$

By using first-order conditions for nonmember patrons (who do not receive patronage refunds), aggregate nonmember supply and demand functions can be determined. These functions can be represented

(5) $q_{iO} = q_{iO}\ (P_X,\ P_Y)\ i\varepsilon X_C,\ Y_C.$

They are like (4) but do not include patronage refunds.

COOPERATIVE SUBMODEL

The cooperative's objective is assumed to be maximization of actual profits earned on current operations of all members. The cooperative's objective function can be obtained by summing expression (1) over all members and modifying the pvpr terms. If expression (1) is summed over all member patrons and (for convenience) r_i^* is now taken as a weighted average of individual member's values of r_i^*, then the member's total pvpr can be written $s' \sum_{i\varepsilon C} r_i^* q_{iC}$. Maximization of the cooperative's objective function is used to determine the cooperative's prices. To use this same maximization process to determine values of r_i would be inconsistent with the purpose of patronage refunds and with the practices actually followed in determining refunds. It would mean that the cooperative would simultaneously set the prices it will pay (or charge) on each transaction and the refunds it will pay after the accounting period is over. In actual practice, values of r_i are not known at the time of the transactions. These values are not determined until the end of the accounting period, after the cooperative has made and executed its decisions and all patrons have made and executed their decisions. The cooperative does know, however, that $\sum_{i\varepsilon C} r_i q_{iC} = NS$, i.e., that total patronage refunds actually paid on this year's operations will equal this year's NS. The cooperative's objective function is, then, obtained by summing expression (1) over all member patrons and replacing $s' \Sigma r_i^* q_{iC}$ by $s'NS$ and can be written

$$(6) \quad \Pi = \sum_{i\varepsilon X_C} p_i q_{iC} + \sum_{i\varepsilon X_O} p_i q_{iC} - \sum_{i\varepsilon Y_C} p_i q_{iC} - \sum_{i\varepsilon Y_O} p_i q_{iC} + PVPR.$$

The symbol q_{iC} is shorthand for $q_{iC}(P_X, P_Y, R_C^*)$ from (4). Patronage refunds are obtained from net savings and

$$(7) \quad PVPR = [s + (1 - s)/(1 + d_C)^{\tau}]\ NS = s'NS.$$

Net savings are determined from

$$(8) \quad NS = \sum_{i\varepsilon Y_C} p_i q_i + \sum_{i\varepsilon Z} p_i q_i - \sum_{i\varepsilon X_C} p_i q_i - \sum_{i\varepsilon V} p_i q_i - FCC.$$

In expression (6) q_{iC} for $i\varepsilon X_C$ is total amount of product i sold to the cooperative by member patrons. In (8) q_i for $i\varepsilon X_C$ is total amount of product i sold to the cooperative by all patrons. Similar distinctions apply to Y_C.

The sum of the first two terms on the right-hand side (RHS) of (6) can be labeled member's total private revenues (MTPR). The sum of the next two terms can be called member's total private costs (MTPC). They represent total sales revenues and total costs of all members if there are no patronage refunds. The sum of the first two terms on the RHS of (8) can be labeled total collective revenues (TCR). They are received by the cooperative firm for the actions it undertakes on behalf of all members. The sum of the last three terms on the RHS of (8) is total collective costs (TCC). Thus (6) can be expressed

$$\Pi = MTPR - MTPC + s'(TCR - TCC).$$

Some novel features of this objective function need to be noted. (A) Each transaction of a member with the cooperative appears twice, once with a positive sign and once with a negative sign. A member's sale to the cooperative increases MTPR and TCC . A member's purchase from the cooperative increases MTPC and TCR . (B) Each transaction with a nonmember appears but once in (8): in NS . (C) Each transaction of a member with a firm other than the cooperative appears once in (6) and not at all in (8). (D) A consequence of A, B, and C is that maximizing NS is not the same as maximizing Π.

The production function of the cooperative is written as

$$(9) \quad \Phi\,(Q_Z, Q_{YC}, Q_{XC}, Q_V) = 0.$$

It is assumed that (9) possesses continuous first- and second-order partial derivatives, that its partial derivatives with respect to outputs are nonnegative, its partial derivatives with respect to inputs are nonpositive, and it is subject to diminishing returns such that all one-product production functions obtained from (9) by fixing the values of all other outputs are strictly concave.

The Lagrangean function can be expressed

(10) $L = \sum_{i\varepsilon X} p_i q_{iC} - \sum_{i\varepsilon Y} p_i q_{iC} + s'NS + \lambda\ \Phi\ (Q_Z, Q_{YC}, Q_{XC}, Q_V)$

where λ is a Lagrange multiplier.

First-order conditions follow. We assume that the second-order condition for maximization of Π is satisified.

For $j\varepsilon X_C$

(11) $$\partial L/\partial p_j = q_{jC} + \sum_{i\varepsilon X} p_i\ (\partial q_{iC}/\partial p_j) - \sum_{i\varepsilon Y} p_i\ (\partial q_{iC}/\partial p_j)$$
$$- s'\ [q_j + \sum_{i\varepsilon X_C} p_i\ (\partial q_i/\partial p_j) - \sum_{i\varepsilon Y_C} p_i\ (\partial q_i/\partial p_j)]$$
$$+ \sum_{i\varepsilon C} \lambda\ (\partial \Phi/\partial q_i)\ (\partial q_i/\partial p_j) = 0$$

For $j\varepsilon Y_C$

(12) $$\partial L/\partial p_j = \sum_{i\varepsilon X} p_i\ (\partial q_{iC}/\partial p_j) - \sum_{i\varepsilon Y} p_i\ (\partial q_{iC}/\partial p_j) - q_{jC}$$
$$+ s'\ [q_j - \sum_{i\varepsilon X_C} p_i\ (\partial q_i/\partial p_j) + \sum_{i\varepsilon Y_C} p_i\ (\partial q_i/\partial p_j)]$$
$$+ \sum_{i\varepsilon C} \lambda\ (\partial \Phi/\partial q_i)\ (\partial q_i/\partial p_j) = 0$$

For $j\varepsilon Z$

(13) $$\partial L/\partial q_j = s'\ [p_j + q_j\ (\partial p_j/\partial q_j)] + \lambda\ (\partial \Phi/\partial q_j) = 0$$

For $i\varepsilon V$; $j\varepsilon Y_C$, Z

(14) $$\partial L/\partial q_{ij} = s'\ [- p_i - q_i\ (\partial p_i/\partial q_i)] + \lambda(\partial \Phi/\partial q_{ij}) = 0$$

(15) $\partial L/\partial\lambda = \Phi\ (Q_Z,\ Q_{YC},\ Q_{XC},\ Q_V) = 0$

No attempt is made in this paper to interpret these conditions for multiproduct marketing or supply cooperatives. Instead, attention is directed to simple models of marketing cooperatives because of their importance in the literature on cooperative theory. (For convenience, symbols used in these models are defined in table 2.) Simple models of supply cooperatives are similar to the models of marketing cooperatives discussed here, and these, as well as multiproduct cooperatives, are discussed in Royer.

SINGLE-PRODUCT MARKETING COOPERATIVE

In this model, the cooperative purchases a product (x) produced by single-product member patrons. This product is used by the cooperative in the production of output z which is sold outside the cooperative association. The cooperative does not supply its patrons with any inputs. These must be purchased from sources outside the cooperative association (set Y_0). In addition, the cooperative must purchase some of its inputs from sources outside the cooperative association (set V).

If we assume that all patronage refunds are paid in cash ($s = 1$), the objective function of the cooperative can be written

(16) $\Pi = p_x q_x - \sum_{i \varepsilon Y_0} p_i q_i + p_z q_z\ (q_x,\ Q_V,\ F) - p_x q_x - \sum_{i \varepsilon V} p_i q_i - FCC$

where

(17) $q_z = q_z\ (q_x,\ Q_V,\ F)$

represents the cooperative's production function.

Table 2. Definition of Symbols Used in Models of Single-Product Cooperatives

Symbol	Definition
x	Product handled for patrons by single-product marketing cooperative
p_x	Price paid by cooperative for x
q_x	Total quantity of x handled by cooperative
z	Product produced from x by cooperative
p_z	Price received by cooperative for z
q_z	Quantity of z produced by cooperative
MC_x	Marginal cost to member patrons of producing x
MC_z	Marginal cost to cooperative of producing z
ATC_z	Average total cost to cooperative of producing z
AR_z	Average revenue for z
MR_z	Marginal revenue for z
CS	Cooperative surplus
SRNR	Short-run net returns function
q_{xm}	Quantity of x produced by typical member patron
atc_x	Average total cost to typical member patron of producing x
r_x	Per-unit patronage refund on x
F	Set of fixed factors of cooperative

The first-order condition for a maximum, which can also be obtained by simplifying (11)[1], is

$$(18)\quad \partial\Pi/\partial q_x = - \sum_{i\varepsilon Y_0} p_i\ (\partial q_i/\partial q_x) + [p_z + q_z\ (\partial p_z/\partial q_z)]\ (\partial q_z/\partial q_x) = 0.$$

This is equivalent to stating that for a maximum, the marginal increase in the cost of member patrons from producing x should equal its marginal revenue product in the cooperative.

If the typical member patron does not expect to receive a patronage refund, its supply curve is its marginal cost curve above the average variable cost curve, represented by mc in the left panel of figure 1. Then the supply curve facing the cooperative is the horizontal sum of the supply curves of the member patrons, represented by MC in the right panel of the figure. The optimum price is p_x, determined by the intersection of MRP and MC. The quantity supplied by the typical member patron will be q_{xm}, and the total quantity supplied by member patrons will be q_x.[2]

If the typical member expects to receive a patronage refund, its supply curve, represented by s in figure 1, will lie to the right of its marginal cost curve. The supply curve facing the cooperative, represented by S in the figure, will be the horizontal sum of the supply curves of the member patrons but will lie to the right of MC.

If it is assumed that all member patrons have the same expectations, each of their individual supply curves will be an equal distance below their marginal cost

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[1]For the interpretation of the $\lambda\ (\partial\Phi/\partial q_i)$ in (11), see Royer, pp. 161-166.

[2]According to Clark (pp. 38-39), total economic welfare is maximized at the quantity at which marginal cost is equal to average revenue. In this case, the average revenue to the member patrons from x is equivalent to its marginal revenue product in the cooperative. Thus, according to Clark's criterion, q_x is the quantity at which total economic welfare is maximized.

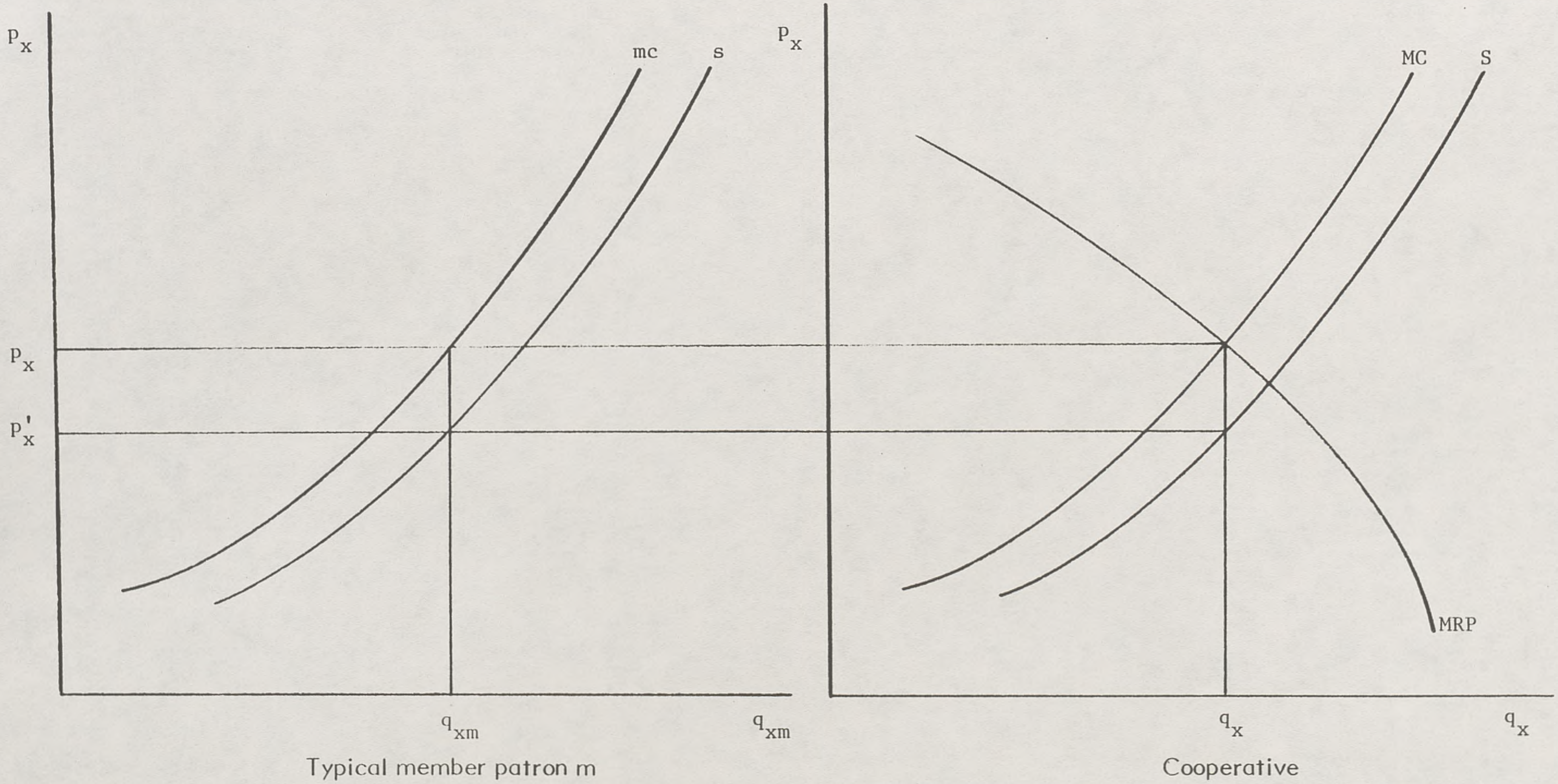

Figure 1. Single-product marketing cooperative

curves. The optimal price will be p'_x for at this price, q_x will be supplied and the marginal cost of the product will equal its marginal revenue product in the cooperative. If all members do not have the same expectations, their individual supply curves will not be equidistant from their marginal cost curves, and maximization of total member profits will be more difficult.

The argument that the quantity at which the marginal cost of the product equals its marginal revenue product in the cooperative is the optimal quantity can be made in terms of producer and consumer surpluses. Producer surplus can be defined as the difference between what producers of the product (member patrons) actually receive and what they would be willing to receive for a given quantity, a measure of the net benefit or profit they derive from selling the product. In figure 1, producer surplus is represented by the area below the horizontal line through the equilibrium price and above the marginal cost curve. Consumer surplus can be defined as the difference between what the consumer of the product (the cooperative) would be willing to pay and what it actually pays for a given quantity, a measure of the net benefit or net savings it derives from purchasing the product. In the figure, consumer surplus is represented by the area above the horizontal line through the equilibrium price and below the marginal revenue product curve.

A noncooperative firm might be interested in maximizing consumer surplus. This would be accomplished by operating at the point at which the marginal revenue product curve intersects the marginal factor cost curve instead of where it intersects the marginal cost or supply curve. However, the cooperative attempts to maximize the sum of producer and consumer surpluses.

If the supply curve facing the cooperative is the marginal cost curve, the cooperative maximizes profits of its member patrons by setting a price equal to the marginal revenue product of the product. Unless marginal revenue product is equal to average revenue product, this price by itself will not result in all of producer surplus being distributed to member patrons. A price equal to average revenue product would by itself result in all of producer surplus being distributed to the member patrons, but it would not produce a maximum. The cooperative, however, can set a price equal to marginal revenue product and still distribute all of the producer surplus through use of patronage refunds.[3]

COMPARISON WITH OTHER ANALYSES

Phillips

In the Phillips model, each cooperating firm individually attempts to maximize its profit, and each is treated as a multiplant, vertically integrated firm. The output of the individual plants is assumed to be the raw product input of the cooperative.

As a multiplant firm, each cooperating firm must make decisions concerning the allocation of its productive resources between the cooperative and its individual plant or plants. Within this framework, Phillips attempted to outline a set of rules for the optimum behavior of a member firm, given its objective of maximizing profit. According to Phillips, a member firm maximizes profit by equating the sum of the marginal cost in its individual plant or plants and the marginal cost in the cooperative with marginal revenue.

The Phillips model was criticized by Aresvik, who argued that the marginal cost that a member firm incurs in the cooperative is not the marginal cost of the cooperative plant but the average cost of the plant. Aresvik also argued that the marginal revenue that a member firm receives from a marketing cooperative is

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[3]Enke presented a similar analysis of producer and consumer surpluses, without discussing patronage refunds, in his model of a consumer cooperative.

not the marginal revenue received by the cooperative but the average revenue received by the cooperative. Thus, according to Aresvik, a member firm participating in a marketing cooperative maximizes profit by equating the sum of the marginal cost in its individual plant or plants and the average cost in the cooperative with the average revenue received by the cooperative. Aresvik did not, however, dispute Phillips' contention that it is the member firms and not the cooperative that are decisionmakers. Instead, he stated that Phillips was correct in indicating that the member units, not the cooperative, are the maximizing units.

Trifon indicated that the analysis of neither Phillips nor Aresvik was correct. He suggested that each member patron maximizes profit by equating the sum of the marginal cost in its individual plant and the marginal cost it incurs in the cooperative with the marginal revenue it receives through the cooperative. However, he argued that by increasing its patronage, an individual patron incurred only a portion of the additional cost to the cooperative while assuming a larger share of the initial costs. Thus the marginal cost the member patron incurs in the cooperative is neither the marginal nor average cost curve of the cooperative. Similarly, the marginal revenue the member patron receives through the cooperative is neither the marginal nor average revenue curve of the cooperative. Trifon also suggested that as each individual member patron independently attempted to maximize profit, there was no guarantee that an equilibrium would be reached.

The model presented here can be specialized to correspond to the Phillips model by assuming that (a) the cooperative serves only member patrons who produce a single product (x), (b) members sell all their output to the cooperative, (c) the cooperative uses this product in the production of a single output (z) which it markets, (d) all patronage refunds are paid in cash ($s = 1$), and (e) production of each unit of z requires one unit of x.

Given these assumptions, the objective function is again expressed by (16), but the first-order condition for a maximum, which can also be obtained by simplifying (11) or (13), is

$$(19)\quad \partial\Pi/\partial q_x = - \sum_{i\varepsilon Y_0} p_i\,(\partial q_i/\partial q_z) + [p_z + q_z\,(\partial p_z/\partial q_z)] - \sum_{i\varepsilon V} p_i\,(\partial q_i/\partial q_z) = 0.$$

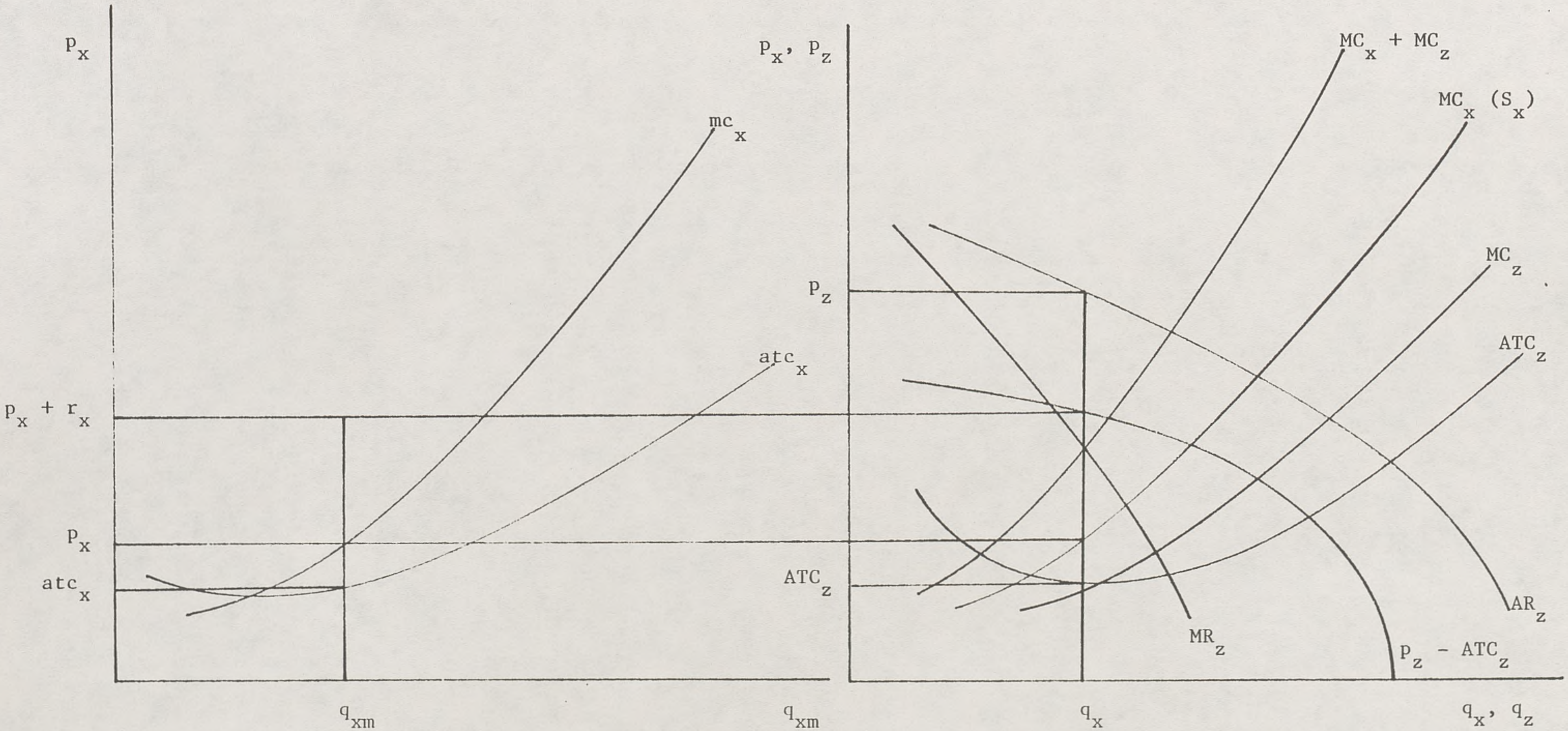
This is equivalent to stating that for a maximum value of Π, the marginal increase in the cost of member patrons from supplying the raw product x plus the marginal cost to the cooperative of producing z should equal the marginal revenue to the cooperative from producing z.

A graphical illustration of this optimality condition is shown in figure 2. Assume the typical member patron expects no patronage refunds so its marginal cost curve (mc_x) is its supply curve and the supply curve facing the cooperative is MC_x. Total member profit is maximized at output q_x where marginal revenue (MR_z) equals the sum of member and cooperative marginal costs ($MC_x + MC_z$). The cooperative will offer members price p_x.

The cooperative will receive price p_z for processed product z. The difference between this price and the average total cost of processing z (ATC_z) is available as per-unit patronage refund r_x. This refund augments the profits of the typical member patron so that his total profit per unit is $p_x + r_x - atc_x$.

Expression (19) can be written as

$$(20)\quad \partial MTPC/\partial q_x + \partial TCC/\partial q_z = \partial TCR/\partial q_z.$$



Typical member patron m

Cooperative

Figure 2. Phillips model of a marketing cooperative

This is almost the same as Phillips' equilibrium condition. A difference arises because his is an equilibrium condition for an individual member where (20) is an equilibrium condition for the entire cooperative association. The two conditions can be reconciled by focusing on the first term on the LHS of (20). This is the marginal cost to all members of increasing output, whereas in Phillips it is the marginal cost to an individual member. Working with finite differences the first term on the LHS can be expressed $\Delta MTPC/\Delta q_x$. Write member i's cost of increasing output by amount Δq_{xi} as $\Delta M_i C/\Delta q_x$, and let $\Delta q_{xi} = f_i \Delta q_x$, $f_i > 0$, $\sum_i f_i = 1$. Suppose

(21) $\Delta M_i C/\Delta q_{xi} = f_i \ \Delta MTPC/\Delta q_x$

that is, each member's cost of increasing his output by f_i fraction of a unit equals fraction f_i of member's total cost of increasing output by one unit. Then

(22) $\Delta MTPC/\Delta q_x = (1/f_i)\ (\Delta M_i C/\Delta q_{xi})$ for all i.

The cost to all members of increasing output by Δq_x equals $(1/f_i)$ times the cost to member i of increasing his output by $f_i \ \Delta q_x$. If $f_i = .05$, e.g., the cost to all members of increasing output by Δq_x equals 20 times the cost to member i of increasing his output by $.05 \ \Delta q_i$. Now (20) is equivalent to Phillips' equilibrium condition but is inconsistent with Aresvik's conclusion. Aresvik argued that a member firm maximizes its profits by equating the sum of its marginal cost and the cooperative's average cost to the cooperative's average revenue.

Nevertheless, Phillips' condition faces an operational problem: Who makes and carries out the decisions that cause the member's equilibrium conditions to be satisified? An individual member's marginal return is price plus refund and it maximizes its profit by equating its marginal return to its marginal cost, i.e., by operating at a level of output that satisfies

(23) $\partial M_iC/\partial q_{xi} = p_x + r_x.$

If the member firm does this, it cannot be sure of following the Phillips' rule unless it can assure that $p_x + r_x = \partial TCR/\partial q_z - \partial TCC/\partial q_z$. Or, the member firm can follow the Phillips' rule: Adjust its level of output to equate its marginal cost to the excess of the cooperative's marginal revenue over its marginal cost. Then the member cannot be sure of maximizing its own profit unless it can assure that price plus refund equals this excess. Thus, to follow the Phillips' rule to maximize its own profit, the member must control price and refund paid by the cooperative to assure that their sum equals the cooperative's marginal revenue minus marginal cost. But cooperatives do not allow individual members the authority to determine prices and refunds. These determinations are collective decisions. Thus, although the Phillips' model provides the existence of a profit maximization rule for an individual member, the individual member lacks the authority to assure conformance to this rule.

Helmberger and Hoos

In the Helmberger and Hoos model, the cooperative is a decisionmaking unit that attempts to maximize the surplus available for payment to its members for a raw product (x). The cooperative processes this product and markets the finished product (z). The production function can be expressed

(24) $q_z = q_z\ (q_x, Q_V, F).$

Helmberger and Hoos assumed that all of the cooperative surplus is distributed to members in the form of the price paid for the raw product. Thus net savings (or profit as Helmberger and Hoos termed it) equals zero

$$(25)\ NS = p_z q_z - p_x q_x - \sum_{i \varepsilon V} p_i q_i - FCC = 0$$

and surplus can be expressed

$$(26)\ CS = p_x q_x = p_z q_z\ (q_x,\ Q_V,\ F) - \sum_{i \varepsilon V} p_i q_i - FCC.$$

The cooperative maximizes this surplus, and thus the price p_x for the quantity of raw product supplied by members, by producing the finished product at the level at which price equals marginal cost. According to Helmberger and Hoos, there exists a unique functional relationship, which they called the short-run net returns function, between the maximum price the cooperative can offer and the quantity of raw product supplied by members

$$(27)\ p_{xd} = p_{xd}(x)$$

The intersection of this short-run net returns function and the aggregate supply function of the members

$$(28)\ p_{xs} = p_{xs}(x)$$

determines the equilibrium price and quantity of the raw product. At the equilibrium, the price received by members can be expressed

$$(29)\ p_x = \frac{(p_z - ATC_z)\ q_z}{q_x}.$$

The model presented here differs from the Helmberger and Hoos model in two respects: (1) in the model presented here, price is not the only means by which the cooperative can distribute surplus, and (2) in this model, the cooperative maximizes total member profits instead of cooperative surplus.

Because patronage refunds can be used to distribute surplus in excess of that distributed by price, (29) can be relaxed and p_x can be used as an instrument instead of as a maximand. Differentiating

$$(30)\quad CS = p_z q_z (q_x, Q_V, F) - \sum_{i \varepsilon V} p_i q_i - FCC$$

with respect to q_x,

$$(31)\quad \partial CS/\partial q_x = [p_z + q_z (\partial p_z/\partial q_z)] (\partial q_z/\partial q_x) = 0$$

the cooperative maximizes surplus by using the raw product x at the level at which the marginal revenue product of x is zero.

Assume all patronage refunds are paid in cash. By simplifying (11) or by maximizing total member profits in the Helmberger and Hoos model,

$$(32)\quad \Pi = p_x q_x + \sum_{i \varepsilon X_0} p_i q_i - \sum_{i \varepsilon Y_0} p_i q_i + p_z q_z (q_x, Q_V, F) - p_x q_x$$

$$- \sum_{i \varepsilon V} p_i q_i - FCC$$

we find that the following condition must hold for a maximum

$$(33)\quad \partial \Pi/\partial q_x = \sum_{i \varepsilon X_0} p_i (\partial q_i/\partial q_x) - \sum_{i \varepsilon Y_0} p_i (\partial q_i/\partial q_x)$$

$$+ [p_z + q_z (\partial p_z/\partial q_z)] (\partial q_z/\partial q_x) = 0.$$

Condition (33) differs from condition (31) in that it contains the first two terms which are included because of consideration of the effect production of x has on the private profits of members. If x is the only product produced by members, $\sum_{i \varepsilon X_0} p_i (\partial q_i/\partial q_x)$ can be interpreted as the marginal increase in the cost of member patrons from producing x. Thus, to maximize member total profits, the cooperative should use x up to the point at which its marginal revenue product equals the marginal increase in the cost of member patrons from producing x. To maximize cooperative surplus only, the cooperative should use x up to the point at which its marginal revenue product is zero.

To further compare these optima with the equilibrium suggested by the Helmberger and Hoos model, assume that production of each unit of z requires one unit of x. Then condition (33) is equivalent to (19).

If member patrons do not expect to receive patronage refunds, the Helmberger and Hoos equilibrium quantity (q_x) and price (p_x), as shown in figure 3, are determined by the intersection of the short-run net returns function (SRNR) and the aggregate supply function S_x. The SRNR function is determined by the difference between average revenue (AR_z) and average total cost (ATC_z). Net savings is zero; all of the cooperative surplus is distributed by price p_x.

Maximization of cooperative surplus (31) is acheived when the condition

$$(34)\quad \partial CS/\partial q_z = p_z + q_z\,(\partial p_z/\partial q_z) - \sum_{i\varepsilon V} p_i\,(\partial q_i/\partial q_z) = 0$$

holds: z should be produced at the level at which marginal revenue equals marginal cost. Optimal output in figure 4 is q_x and the corresponding price is p_x. The cooperative surplus (SRNR) is distributed to patrons through price p_x and per-unit patronage refund r_x.

Maximization of total member profits occurs when condition (19) holds, as in figure 2: z should be produced at the level at which marginal revenue equals the marginal cost to member patrons from supplying the raw product x plus the marginal cost of processing z. Of course, profit of the typical member patron is larger under this objective than under either maximization of cooperative surplus or price.

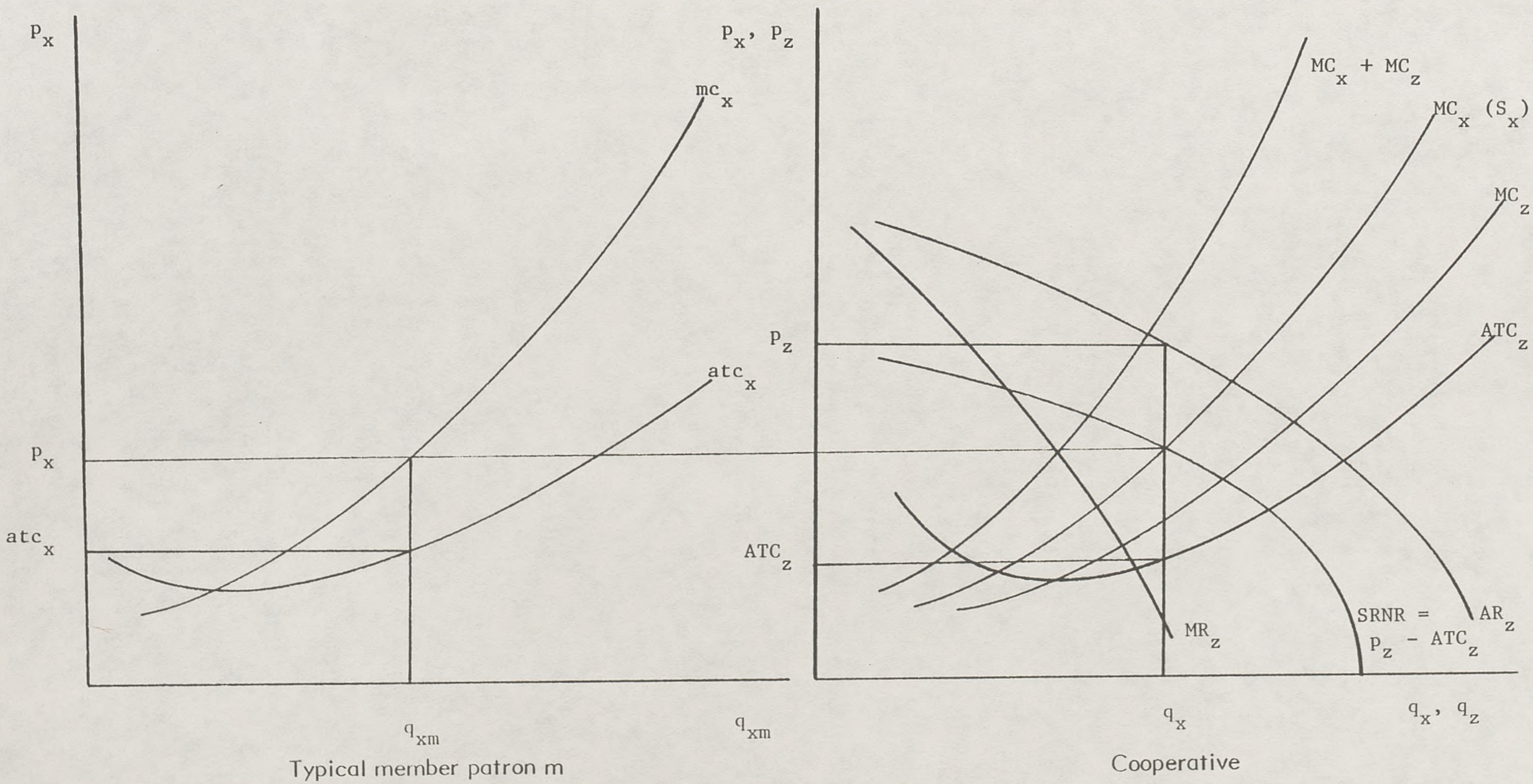


Figure 3. Helmberger and Hoos model of a marketing cooperative

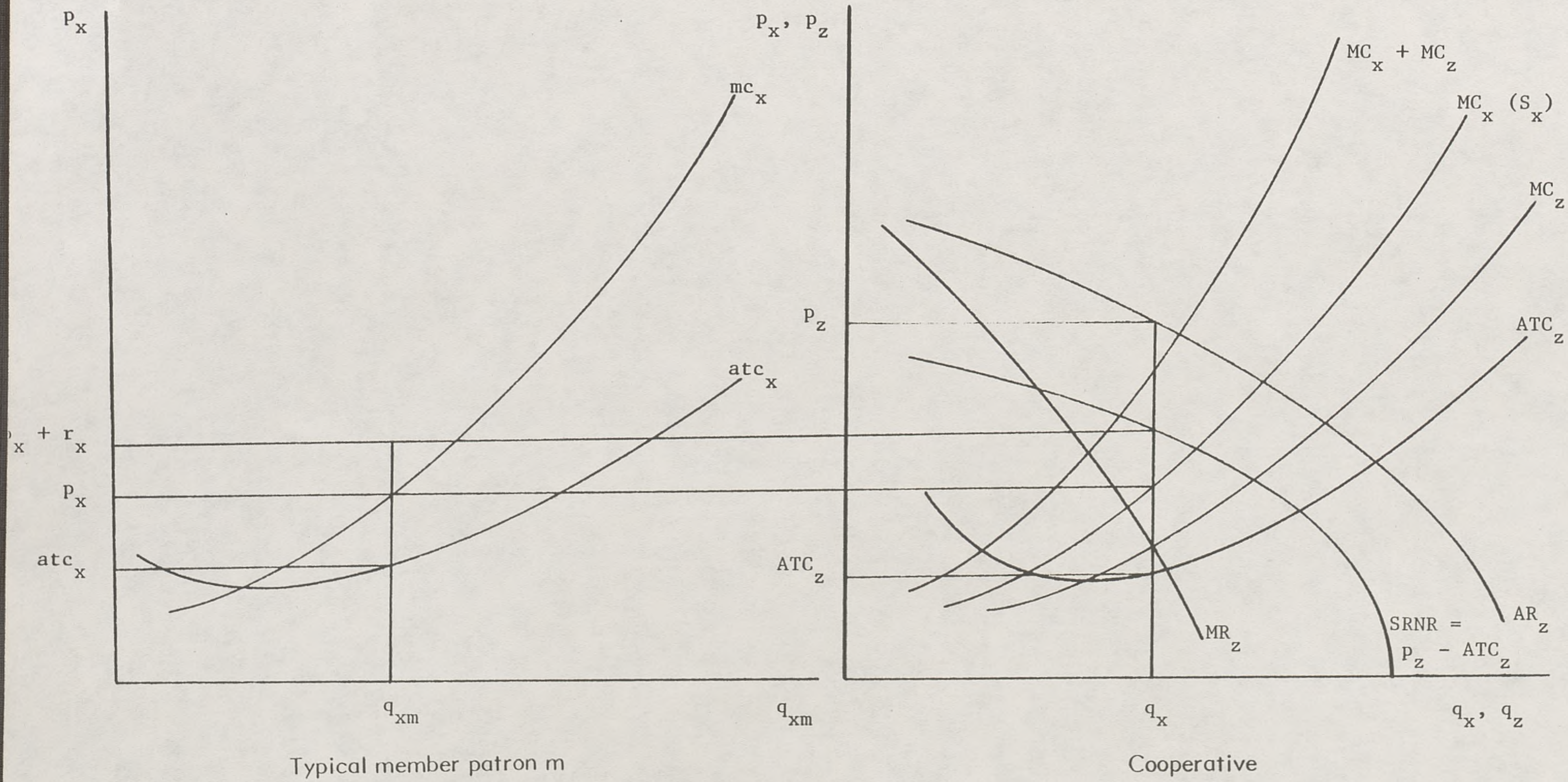


Figure 4. Maximization of cooperative surplus

SUMMARY

This paper presents a model of a producers' cooperative association in which the use of patronage refunds has been incorporated. The objective of the cooperative is maximization of total member profits. In maximizing member profits, the cooperative must recognize the impact of its pricing decisions on both private and collective costs and revenues. As a consequence, maximization of the total profits of member patrons is not the same as maximization of net savings.

The optimality condition presented here for a single-product marketing cooperative is similar to that in the Phillips model. However, the condition presented here is for the cooperative association whereas Phillips' condition is for an individual member, and the individual member lacks the authority to assure conformance to this rule. This model differs from the Helmberger and Hoos model in that price is not the only means by which the cooperative can distribute surplus and the cooperative maximizes total member profits instead of cooperative surplus. Because of these differences, the optimality conditions derived here differ from the equilibrium condition of Helmberger and Hoos.

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