Identifying the Natural Enemy-adjusted Economic Threshold (NEET) for Dynamically Optimal Pest

Management in High Tunnels[1]

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1 Introduction

Integrated pest management (IPM) is an ecosystem approach to crop production and protection that combines different management strategies and practices to grow healthy crops and minimize the use of pesticides (FAO, 2023)[9]. In practicing IPM, growers follow four steps: set action threshold, monitor and identify pests, prevention, and control (EPA, 2022)[6]. Among control strategies, pesticide application (chemical control) and the use of predators and parasitoids to suppress crop pest populations (biological control) are important. Unfortunately, the use of broad-spectrum insecticides can kill many common natural enemies along with the biological control services they can provide. When setting the action threshold for pesticide application, it is important to account for the detrimental effect on beneficial natural enemies. However, bioeconomic research on the natural enemy-adjusted economic threshold (NEET) of pesticide application is very limited and chemical manufacturers do not provide farmers with guidance on the threshold, which are major barriers to adopting IPM practices. One notable exception is the work by Zhang and Swinton (2009)[23] in the context of soybeans, aphids, and naturally occurring predators in an open-field

agriculture setting. We are not aware of similar work in protected agriculture and specialty crops that can inform the NEET for applying insecticide.

High tunnels (HTs) are one of the most accessible and affordable forms of protected agriculture and are used almost exclusively to produce specialty crops. Through capturing radiant heat and management strategies such as controlling the timing of ventilation and integrating row covers, farmers have the ability to alter the microclimate surrounding the crop. As a result, HTs increase farm viability through season extension, which provides a steady income stream during the traditional 'off-season' of farming (Bruce et al., 2019)[2]. Spinach is a cool-season crop that is frequently grown in high tunnels across the United States (Carey et al., 2009)[3]. According to survey data, 45% of farmers grow spinach in high tunnels in the Central U.S. (Knestson et al., 2010)[17].

Both ecological and economic factors indicate that control strategies used in open fields do not translate to the unique growing environment of HTs. Ecologically, HTs insect pest pressure is often greater than open-field production due to higher planting density and favorable conditions for pest growth (Ingwell et al., 2017, Ingewell et al., 2018)[14],[15]. On the other hand, HTs might be better suited for the adoption of natural enemies than open-field agriculture because their boundary can partially keep predators from flying away. Economically, Zhang and Swinton (2012)[24] show that NEET can vary with a wide range by the different market conditions such as crop prices and biological parameters such as reproducing rates. The unique growing environment in HTs leads to parameters differing a lot from open-field agriculture. Thus we expect NEET in HTs will also be different from open-field agriculture.

This work fits within a larger body of literature that considers optimal pest control decisions in either static or dynamic frameworks. Harper and Zilberman (1989)[13] and Harper (1991)[12] examine pesticide use decisions for static and dynamic models, respectively, that include a primary pest, a secondary pest, and a natural predator of the secondary pest which can be harmed by the pesticide used to control the primary pest. Through comparative static/dynamic analysis, they find that growers who ignore the effect of the pesticide on the predator will overuse the pes-

ticide. Zhang and Swinton (2009)[23] develop a dynamic bioeconomic model including one pest (soybean aphids) and one predator (lady beetles) of the pest. With deterministic and perfectly observable population dynamics, and given predator population densities, they solve the optimal spraying strategies under different combinations of initial pest and predator densities using numerical methods. Grogan (2014)[10] adopts a similar biological relation and deterministic setting as Zhang and Swinton but changes to continuous-time and infinite-horizon dynamic optimization. They explicitly include augmenting the predator as a second control strategy besides insecticide application. With linear functional forms and constant marginal control costs assumptions, they solve the closed-form solution that entails a "most rapid approach[1]" to the singular path.

The solution method of this work also fits in the literature on the management of invasive species with recently developed approaches. Marten and Moore (2011)[19] build an options based bioeconomic model for biological and chemical control of invasive species, without considering the detrimental effect of the pesticide on the predator. With fixed costs to exert the control strategies every time, they convert it to an impulse control problem with a continuation region (wait without control) and action regimes (exert at least one control). They include stochasticity in their model, such as weather affecting the occurrence of the pest and predator. They solve the optimal control strategies with numerical methods of solving optimal switching models developed by Balikcioglu (2008)[1] and Fackler (2018)[8]. The imperfectly observed population density, which is called state uncertainty, is another major barrier to resource management. Breaking up the state space into several categories and approximating the problem as a discrete-state Partially Observable Markov Decision Process (POMDP) has been applied to many resource management problems (Fackler and Haight, 2014; Haight and Polasky, 2010)[7],[11]. Recently, Kling et al. (2017)[16] adopt density projection as an approximating approach to solve a continuous-state POMDP problem, which is a breakthrough in solving state uncertainty problems in resource management.

The model that follows differs from previous work in several ways (Table 1). Compared to previous research on pest control and identifying the NEET (Zhang and Swinton, 2009; Grogan,

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[1]Most rapid approaches are characterized by a cornel solution for part of the trajectory, where the optimal control is constrained at an upper or lower bound until the steady-state optimum is achieved (Conroad and Rondeau, 2020)[5].

Table 1: Comparison of Pest/Invasive Species Control Models

Study	Control target	Control variable(s)	Pesticides harm predator	Stochasticity	State uncertainty
Zhang & Swinton (2009)	Pests	Chemical	Yes	No	No
Grogan (2014)	Pests	Chemical & Biological	Yes	No	No
Marten & Moore (2011)	Invasive species	Chemical & Biological	No	Yes	No
Haight & Polasky (2010)	Invasive species	Monitoring & Removal	Not applied	No	Discrete
Fackler & Haight (2014)	Invasive species	Monitoring & Removal	Not applied	No	Discrete
Kling et al. (2017)	Invasive species	Monitoring & Removal	Not applied	No	Continuous
This research	Pests	Chemical & Biological (& Monitoring)	Yes	Yes	(Discrete/Continuous)

Note: The parts within () are expected and we are still working on them.

2014)[23],[10], this model allows stochasticity (such as weather) to affect population dynamics of both the pest and predator. Our results confirm that adding stochasticity changes the NEET of pesticide application. We also plan to include monitoring as another control strategy to account for imperfect observed population densities, which would be another innovation of our work. Compared to previous research on invasive species control (Marten and Moore, 2011)[19], this paper explicitly considers the negative effect of chemical control on biological control, which leads to a trade-off between them.

This study fills the gap in the literature by deriving the natural enemy-adjusted economic threshold to support the adoption of integrated pest management in protected agriculture and specialty crops. We develop a dynamic bioeconomic optimization model to derive the optimal pest control strategies under different population densities of the pest and predator. The model explicitly takes into account the pest control service provided by natural enemies through a predator-prey submodel. Insecticide sprays targeted to the pest can impair the natural enemy and reduce the level of pest control services they provide, which adds an opportunity cost to insecticide use. By solving the dynamic optimization model, we provide an optimal solution to jointly adopting insecticides and commercial natural enemies, thereby providing farmers an economically viable option to adopt sustainable pest management practices that are not the extreme ends of the conventional-organic spectrum. We therefore expect this work to contribute more broadly to the literature on sustainable agriculture intensification (Pretty, 2018)[22]. By adjusting our parameter values based on the unique temperature conditions within HTs, we ensure that our recommendations are tailored to guide pest management in this type of protected agriculture.

2 Optimal Control Framework

We build a bioeconomic model to determine the dynamically optimal pest control strategies with and without uncertainty in this section. It is necessary to clarify the concept of uncertainty analyzed in our model. According to LaRiviere (2018)[18], there are four types of uncertainty in the con-

text of an optimal dynamic management framework: stochasticity, parametric uncertainty, model uncertainty, and state uncertainty. First, the dynamic processes can be influenced by stochasticity, such as intrinsic variability, shock, risk, or "noise". We include the stochasticity from weather conditions in our model. We model the stochasticity by adopting Brownian motions (Ito processes) in our equation of motions. Second, farmers do not know the accurate parameter values related to biology and ecology, such as the pest's intrinsic growth rate. We model the parameter value with the best knowledge that we can find through existing literature and field experiment by our research team. We will conduct a sensitivity analysis to account for parametric uncertainty. We are not interested in modeling farmers' learning in this research. Third, farmers do not know the accurate functional forms capturing the relationship between pest and predator growth. Although it is possible to model uncertainty by adopting functional forms that come from given distributions, such an approach can significantly complicate our model. Thus, we do not pursue to account for the model uncertainty. We will also conduct a sensitivity analysis to test how different functional forms can affect our results. Fourth, the state variables, such as population densities of pests and predators, are not accurately observable. We plan to use the Partially Observable Markov Decision Processes (POMDP) approach to account for the state uncertainty.

To control pests, we assume that a farmer has two control strategies, one is to apply pesticide (x_t) as a chemical control and the other is to release commercial predators at a certain proportion (A_t) of the current predator density as a biological control. Here, we assume x_t to be a binary variable ($x_1 = 0 \ or \ x_t = 1$) and A_t to be a continuous variable ($A_t \in [0, 1]$). Applying pesticide can kill both pests and predators. We assume k_S and k_P to be the pesticide's toxicity to the pests and predators, respectively, i.e. how much percentage of pests and predators being killed by one-time application of such pesticide. Thus, applying pesticides at t will cause the pest density (S) and predator density (P) changes to be

$$\dot{S}_t = -k_S \cdot x_t \cdot S_t \tag{1}$$

$$\dot{P}_t = -k_P \cdot x_t \cdot P_t \tag{2}$$

We assume the corresponding cost of applying pesticide to be

$$CX(x_t) = \theta(S_t) \cdot x_t \qquad (3)$$

where $\theta(S_t)$ is the marginal cost of applying pesticides at the pest density of S_t. The marginal cost is the money required to spray pesticide one more time when pest density is S_t, without accounting for any potential opportunity cost.

Augmentative strategy at t will cause the predator density changes to be

$$\dot{P}_t = A_t \cdot P_t \qquad (4)$$

The corresponding cost of augmenting the predator density is

$$CA(A_t) = \omega(P_t) \cdot A_t \qquad (5)$$

where $\omega(P_t)$ is the marginal cost of augmenting predators at the predator density of P_t. The marginal cost is the money required to release commercial predators at the amount equal to one more percent of the current predator density P_t, without accounting for any potential opportunity cost.

In the absence of controls the population dynamics of the two species are defined by a system of stochastic differential equations:

$$dS_t = a(S_t, P_t)dt + b(S_t)dW_t^S \qquad (6)$$

and

$$dP_t = c(S_t, P_t)dt + d(P_t)dW_t^P \qquad (7)$$

where W^S and W^P are standard Brownian motions and $E[dW^S dW^P] = \sigma_{SP}dt$.

We assume the pest-free potential harvest to be $\bar{y}$ and the pest density S_t, results in damages

$D(S_t)$. Although spinach is harvested (removed) at the end of a growing cycle, pests and predators can live to next growing cycle because farmers do polyculture inside HTs. The goal of the farmer is to maximize the expected discounted flow of revenues minus any cost associated with applying control strategies, with a discount factor ρ:

$$V(S_t, P_t) = \max_{x_t, A_t} E\left[\int_0^\infty e^{-\rho t}[p\bar{y}(1 - D(S_t)) - CX(x_t) - CA(A_t)]\right] dt \tag{8}$$

This model represents a two dimensional singular control problem with two controls for which the solution is defined by a variational inequality (VI), representing the Hamilton-Jacobi-Bellman equation and the necessary smooth pasting conditions (Øksendal and Sulem, 2007)[21]. Following Balikcioglu (2008)[1], the optimality conditions of the VI are

$$\rho V(S_t, P_t) \geq p\bar{y}(1 - D(S_t)) + a(S_t, P_t)V_S + c(S_t, P_t)V_P$$
$$+ \frac{1}{2}b^2(S_t)V_{SS} + \frac{1}{2}d^2(P_t)V_{PP} + \sigma_{SP}b(S_t)d(P_t)V_{SP} \tag{9}$$

$$\theta(S_t) \geq -k_S \cdot S_t \cdot V_S - k_P \cdot P_t \cdot V_P \tag{10}$$

and

$$\omega(A_t) \geq P_t \cdot V_P \tag{11}$$

where V_s and V_p are the partial derivatives of the value function $V(\cdot)$ with respect to pests and predators, respectively. One of the conditions must hold with equality. Which condition holds with equality will determine the optimal control strategy. If conditions 10 and 11 hold with strict inequality, it is implied that the value at the current state of nature, $V(S_t, P_t)$, is greater than any other which could be obtained when taking into account the cost of exerting the controls. Thus, condition 9 must hold with equality, which means that it is optimal for the farmer to wait before applying any of the controls. The economic interpretation is that the rate of return obtainable by investing $V(S_t, P_t)$ must be equal to the flow of income plus the expected change in $V(S_t, P_t)$. The corresponding certain regions of the state space is called continuation region. On the other hand,

if conditions 10 or/and 11 hold with equality, a higher value may be obtained if at least one control is used, then it is optimal to exert the corresponding control strategy. Thus, condition 9 must hold with strict inequality, which means that the flow of income plus the expected change in $V(S_t, P_t)$ is smaller than the rate of return obtainable by investing $V(S_t, P_t)$ if the farmer does not adopt any control strategies. The corresponding certain regions of the state space are action regions. It is worth noting that in condition 10, the marginal cost of applying pesticide also includes the opportunity cost of the loss of predators caused by pesticides.

3 Green Peach Aphids Control in High Tunnels

3.1 Stochastic Model

In order to apply the optimal control framework presented in the previous section, we specify functional forms for the evolution equations and cost functions for control. We adopt similar functional forms to Grogan (2014)[10] but we change the deterministic model to a stochastic model with geometric Brownian processes (GBM). GBM reflects the fact that green peach aphids (GPA) have the potential to increase exponentially in the field with enough food but no appropriate control. By not reaching 0 in any finite time, GBM also accounts for that full eradication is unlikely is impossible given the potential for reintroductions. We adopt a logistic growth form for the pest and a dynamic Lotka-Volterra predator-pery model to describe the population density changes in the predator:

$$dS_t = \left[\alpha S_t (1 - \frac{S_t}{K}) - \beta S_t \cdot P_t \right] dt + \sigma_S S_t dW_t^S \tag{12}$$

and

$$dP_t = [\gamma \eta P_t \cdot S_t - \tau P_t] dt + \sigma_P P_t dW_t^P \tag{13}$$

where α is the intrinsic growth rate of GPA. K is the carrying capacity of GPA. Both β and η are the predation rate ($\beta = \eta$). We use different letters (β and η) to account for the fact that the predation may have different effects on the growth of the pest and predator in future. γ is a

parameter related to the growth of the predator. τ is the death and run off rate of the predator. σ_S and σ_P are volatility parameters. We assume $E[dW^S dW^P] = \sigma_{XP} dt$ to be the covariance between the natural shocks to aphids and predators. If $\sigma_{XP} dt = 0$ then the shocks have independent effects on aphids and predators. However, given that the weather condition, which affects both aphids and predators, is one important source of the natural shocks. We expect that $\sigma_{XP} dt \neq 0$. We will explore the influence of the correlation factor on optimal control strategies more in the sensitivity analysis.

Previous literature argue that while many non-linear specifications are possible for $D(S_t)$, for agricultural insect pests linear specifications are appropriate and have in some cases been found to outperform non-linear ones (Ceddia et al., 2009)[4]. Thus, we adopt a linear damage functional form that the pest damages a proportion of the crop, S_t/K

$$D(S_t,t) = \frac{S_t}{K} \tag{14}$$

We assume that the farmer takes a calendar-based spraying (no scouting). Thus the marginal cost of spraying is constant.

$$\theta(S_t) = \theta \tag{15}$$

and

$$\omega(P_t) = \omega \tag{16}$$

$$V(S,P) = \max_{x_t,A_t} \left[\int_0^\infty e^{-\rho t} [p\bar{y}(1 - \frac{S_t}{K}) - \theta x_t - \omega A_t] \right] dt \tag{17}$$

The optimality conditions for the singular control problem:

$$\rho V(S_t,P_t) \geq p\bar{y}(1 - \frac{S_t}{K}) + \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t \right] V_S + [\gamma\eta P_t \cdot S_t - \tau P_t] V_P$$
$$+ \frac{1}{2}\sigma_S^2 S_t^2 V_{SS} + \frac{1}{2}\sigma_P^2 P_t^2 V_{PP} + \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t V_{SP} \tag{18}$$

$$\theta \geq -V_S \cdot k_S \cdot S_t - V_P \cdot k_P \cdot P_t \tag{19}$$

11

and

$$\omega \geq V_P \cdot P_t \tag{20}$$

3.2 Deterministic Model

For stochastic models, the state and action generally will not converge to specific values, and the long-run behavior of the model can only be described probabilistically. Nonetheless, in these cases, it is often useful to derive the steady state of the deterministic "certainty-equivalent" problem obtained by fixing all exogenous random shocks at their respective means (Miranda and Fackler, 2002)[20]. We are interested in comparing the optimal control strategies, values, and shadow prices under deterministic and stochastic cases.

The certainty-equivalent optimization problem:

$$\max_{x_t, A_t} \int_0^\infty e^{-\rho t} [p\bar{y}(1 - \frac{S_t}{K}) - \theta x_t - \omega A_t] dt \tag{21}$$

Subject to:

$$dS_t = \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t - k_S \cdot x_t \cdot S_t \right] dt \tag{22}$$

$$dP_t = [\gamma \eta P_t \cdot S_t - k_P \cdot x_t \cdot P_t + (A_t - \tau) \cdot P_t] dt \tag{23}$$

The current-value Hamiltonian:

$$H_t = p\bar{y}(1 - \frac{S_t}{K}) - \theta x_t - \omega A_t + \lambda_{S_t}(\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t - k_S \cdot x_t \cdot S_t)$$
$$+ \lambda_{P_t}(\gamma \eta P_t \cdot S_t - k_P \cdot x_t \cdot P_t + (A_t - \tau) \cdot P_t) \tag{24}$$

The first order conditions:

$$
\frac{\partial H}{\partial x_t} =
\begin{cases}
0 & \text{if } \theta + \lambda_{P_t} k_P P_t > -\lambda_{S_t} k_S S_t \\[2mm]
x^{SS} & \text{if } \theta + \lambda_{P_t} k_P P_t = -\lambda_{S_t} k_S S_t \\[2mm]
x_{max} & \text{if } \theta + \lambda_{P_t} k_P P_t < -\lambda_{S_t} k_S S_t
\end{cases}
\tag{25}
$$

$$
\frac{\partial H}{\partial A_t} =
\begin{cases}
0 & \text{if } \omega > \lambda_{P_t} \cdot P_t \\[2mm]
A^{SS} & \text{if } \omega = \lambda_{P_t} \cdot P_t \\[2mm]
A_{max} & \text{if } \omega < \lambda_{P_t} \cdot P_t
\end{cases}
\tag{26}
$$

$$
\dot{\lambda}_{S_t} - \lambda_{S_t} = -\frac{\partial H}{\partial S_t} = \frac{p\bar{y}}{K} - \lambda_{S_t}\left(\alpha\left(1 - \frac{2S_t}{K}\right) - \beta \cdot P_t - k_S \cdot x_t\right) - \lambda_{P_t} \gamma\eta P_t
\tag{27}
$$

$$
\dot{\lambda}_{P_t} - \lambda_{P_t} = -\frac{\partial H}{\partial P_t} = \lambda_{S_t}(\beta S_t) - \lambda_{P_t}(\gamma\eta S_t - k_P \cdot x_t + A_t - \tau)
\tag{28}
$$

where x^{SS} and A^{SS} are the singular solutions defined in Eqs. 31 and 32 below, x_{max} is the maximum allowed application rate for the particular pesticide, and A_{max} is the maximum possible augmentation rate that can be achieved. Because of our constant marginal cost and other linearity assumptions, the optimal control strategies are in the "most rapid[2]" forms. In Eq. 25, $-\lambda_{S_t} k_S S_t$ represents the value of crop damages avoided by one more unit of pesticide application. The farmer will apply no control if the cost of control (both the marginal cost of spraying and the opportunity cost of losing predators) exceeds the value of crop losses prevented by the pesticide. On the other hand, the farmer applies the maximum chemical control when the cost of doing so is less than the value of damages prevented. When the cost and damage prevented are equal, the farmer applies the singular level of control. The cost and benefit of augmentative strategies similarly determine the level of biological control.

Eq. 25 has an important implication for pest eradication. As S_t goes to zero, $-\lambda_{S_t} k_S S_t$ ap-

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[2]Most rapid approaches are characterized by a cornel solution for part of the trajectory, where the optimal control is constrained at an upper or lower bound until the steady-state optimum is achieved (Conroad and Rondeau, 2020)[5].

proaches zero. Thus, $\theta + \lambda_{P_t} k_P P_t > -\lambda_{S_t} k_S S_t$ always holds, which implies that when pest density is low enough, it is optimal to stop spraying pesticides.

The steady-state solution to this problem can be derived from the first-order conditions (see Appendix A for derivations):

$$S^{SS} = \frac{\theta + \omega k_P}{(\frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S K} - \omega \gamma \eta) k_S} \tag{29}$$

$$P^{SS} = \frac{\omega k_S}{\beta(\theta + \omega k_P)} \tag{30}$$

$$x^{SS} = \alpha(1 - \frac{\theta + \omega k_P}{(\frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S K} - \omega \gamma \eta) k_S K}) - \frac{\omega}{\theta + \omega k_P} \tag{31}$$

$$A^{SS} = \tau + k_P \left[\alpha(1 - \frac{\theta + \omega k_P}{(\frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S K} - \omega \gamma \eta) k_S K}) - \frac{\omega}{\theta + \omega k_P} \right] - \gamma \eta \frac{\theta + \omega k_P}{(\frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S K} - \omega \gamma \eta) k_S} \tag{32}$$

The steady state solution shows the optimal combination of chemical and biological control strategies, which depends on the parameter values, to keep the system at the steady state. At the steady state, there exist certain amounts of both pests and predators, which implies that eradication of pests is not optimal.

4   Numerical Results

(This part is still in progress. We are waiting for the results from field experiments and farmer's survey from our research team to improve our parameter value. We attach the preliminary result for expositional purposes.)

Given the complex nature of this continuous time, two states, two singular control stochastic dynamic programming problem, closed form solutions for the value function, and optimal control policy (do not exist). Therefore we rely on numerical methods to derive a solution to the problem. Since the form of the value function is unknown, we define a relatively flexible approximation for $V(S_t, P_t)$ over the relevant portion of the state space using piecewise linear functions (and/or some

other functional forms). The goal is then to solve for the set of coefficients of the approximating functions.

Following Fackler (2018)[8], Balikcioglu (2008)[1], and Marten and Moore (2011)[19], Suppose $V(S_t, P_t) \approx \phi(S_t, P_t)\theta$, where ϕ represents a set of n basis functions for a family of approximating functions and θ is an n-vector of coefficients for the value function.

Recall the optimality conditions of the VI are

$$\rho V(S,P) \geq p\bar{y}(1 - \frac{S_t}{K}) + \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t\right] V_S + [\gamma \eta P_t \cdot S_t - \tau P_t] V_P$$
$$+ \frac{1}{2}\sigma_S^2 S_t^2 V_{SS} + \frac{1}{2}\sigma_P^2 P_t^2 V_{PP} + \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t V_{SP} \quad (33)$$

$$\theta \geq -V_S \cdot k_S \cdot S_t - V_P \cdot k_P \cdot P_t \quad (34)$$

and

$$\omega \geq V_P \cdot P_t \quad (35)$$

At least one the of above expressions must hold with equality. If the first expression holds with equality, the farmer is in the continuation regime. If either/both of the last two expressions hold with equality, the farmer is in the corresponding action regime(s).

The optimality conditions can be restated in the form:

$$0 = \min\left[\rho V(S_t, P_t) - p\bar{y}(1 - \frac{S_t}{K}) - LV(S_t, P_t), \theta + V_S \cdot k_S \cdot S_t + V_P \cdot k_P \cdot P_t, \omega - V_P \cdot P_t\right] \quad (36)$$

where

$$LV(S_t, P_t) = \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t\right] V_S + [\gamma \eta P_t \cdot S_t - \tau P_t] V_P$$
$$+ \frac{1}{2}\sigma_S^2 S_t^2 V_{SS} + \frac{1}{2}\sigma_P^2 P_t^2 V_{PP} + \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t V_{SP} \quad (37)$$

The full details of the numerical method are presented in Appendix B.

Table 2: Parameterization of the bioeconomic model

Parameter	Definition	Value	Units	Comments
p	Output price	5	\$/lb	4.5-5.5
$\bar{y}$	Max potential yield	1.5	Lbs/sq ft	
K	Carrying capacity	500	Pests/sq ft	
α	Intrinsic growth rate of GPA	0.156	GPA	
β	Predation rate	0.15	Pests/predator	
γ	Intrinsic growth rate of predators	0.05		
η	Predation rate	0.15	Pests/predator	same as β
τ	Death & Run off rate of predators	0.2	Predators	
θ	Marginal cost parameter for chemical control	0.014	\$/sq ft	
ω	Marginal cost parameter for biological control	0.02	\$/sq ft	
σ_i	Volatility parameters	0.25		Assumptions tested
σ_{XP}	Brownian motion correlation	0.5		in sensitivity analysis.

Our current parameter values are listed in Table 2. To see how well the biological model describes the ladybeetle-GPA system, we simulate model predictions of untreated predator-free GPA population density and untreated ladybeetle-GPA population dynamics. Figure 1 shows the untreated and predator-free GPA population density when we set the GPA carrying capacity to 500 and the initial GPA density to 10. Figure 2 shows the untreated ladybeetle-GPA population densities when we set the initial GPA density to 10. We notice that under our current parameter values, the initial ladybeetle density of 6 has a significant effect in GPA control before they run off, which necessitates the addition of commercial predators. Figure 3 shows the untreated ladybeetle-GPA population densities when we set the initial GPA density to be 300. Under high initial GPA density, the initial ladybeetle density of 6 still has a significant effect on GPA control before they run off. The simulation results validate our biological model. We decide to set the GPA density range to 0-500 and the ladybeetle density range to 0-8 to solve for our optimal control strategies.

To numerically solve the optimality conditions of the dynamic model, we use a piecewise linear family of basis functions and 21*21 breakpoints spread evenly over the two-dimensional state space to approximate the value function. Our preliminary results are listed as figure 4 and figure 5.

Based on the preliminary results, we find that including stochasticity indeed changes the opti-
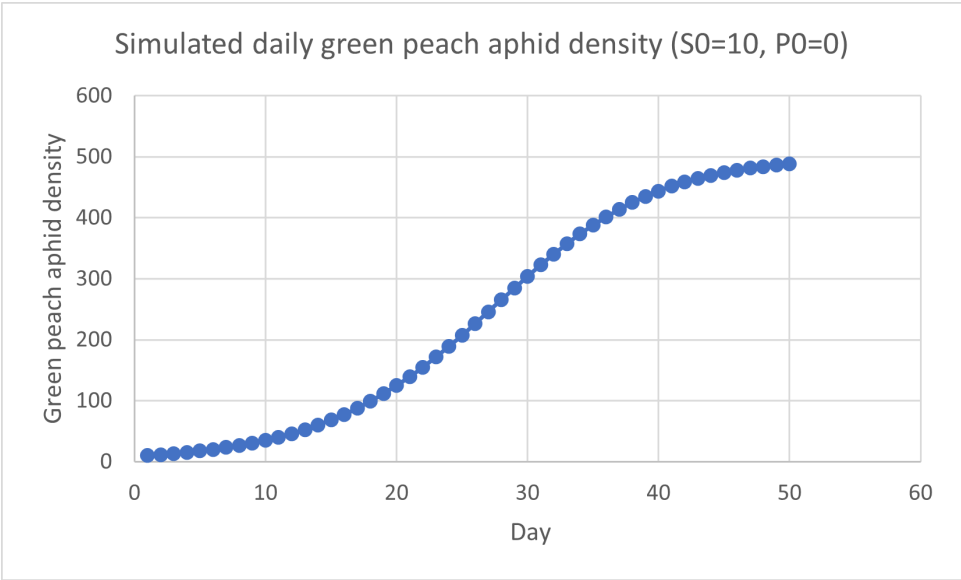
Figure 1: Simulated untreated and predator-free GPA density
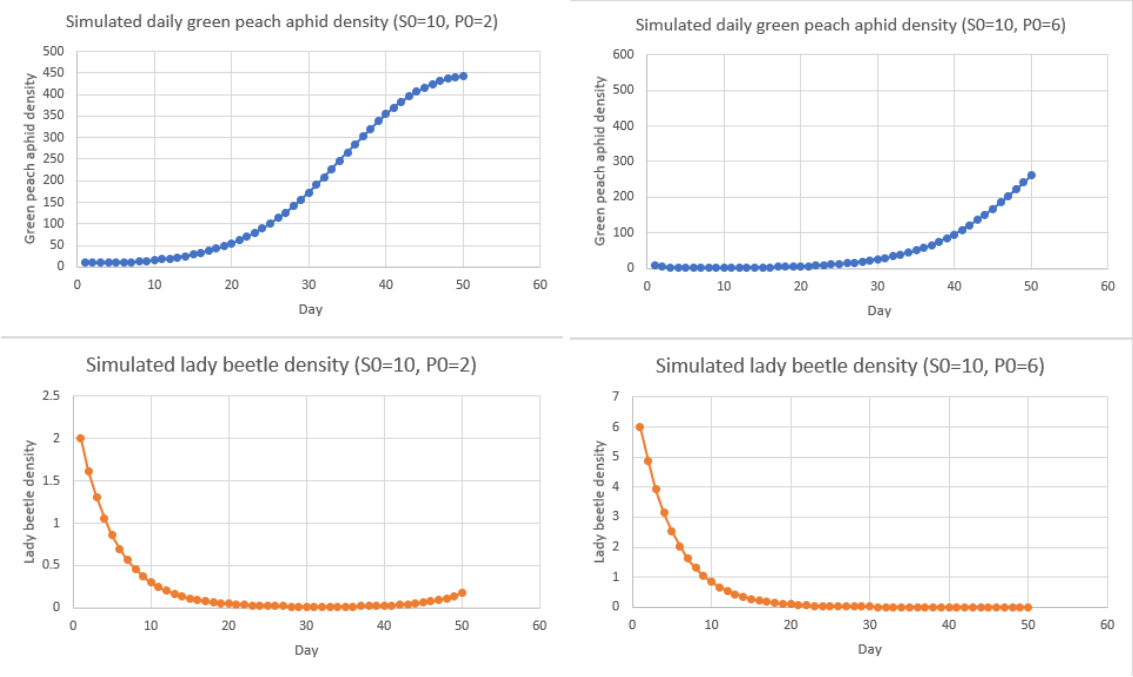

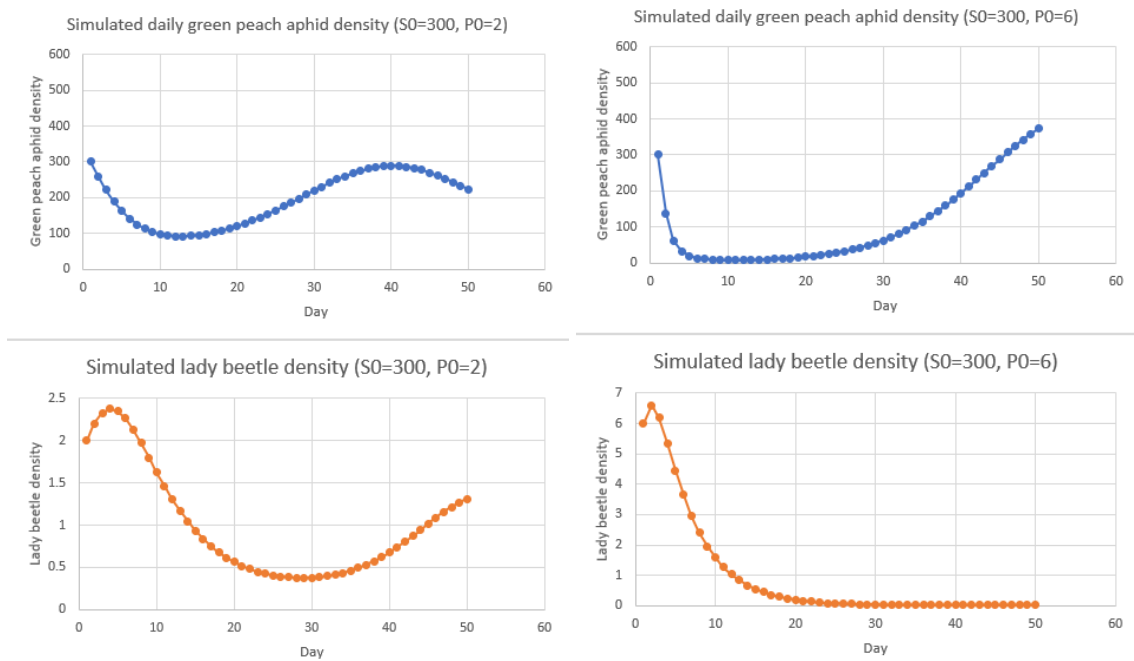Figure 2: Simulated ladybeetle-GPA population densities with low initial GPA density

Figure 3: Simulated ladybeetle-GPA population densities with high initial GPA density
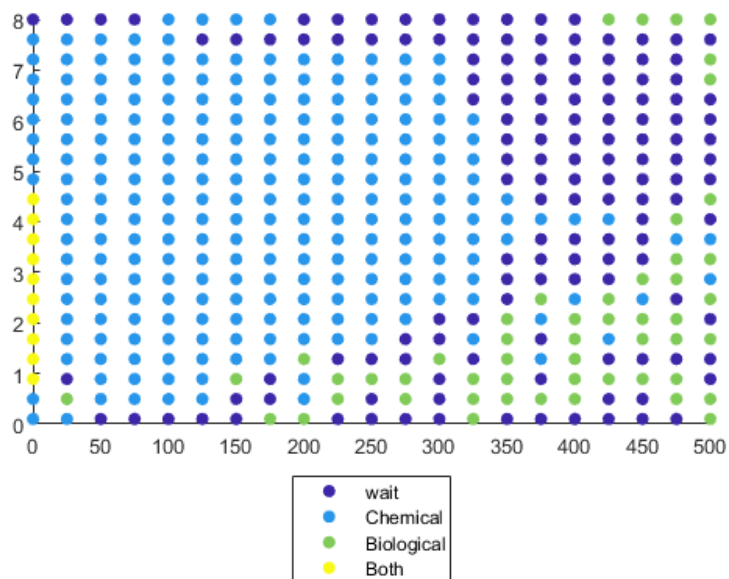

Figure 4: Optimal control strategies under different population densities with stochasticity (x-axis: GPA, y-axis: ladybeetle)
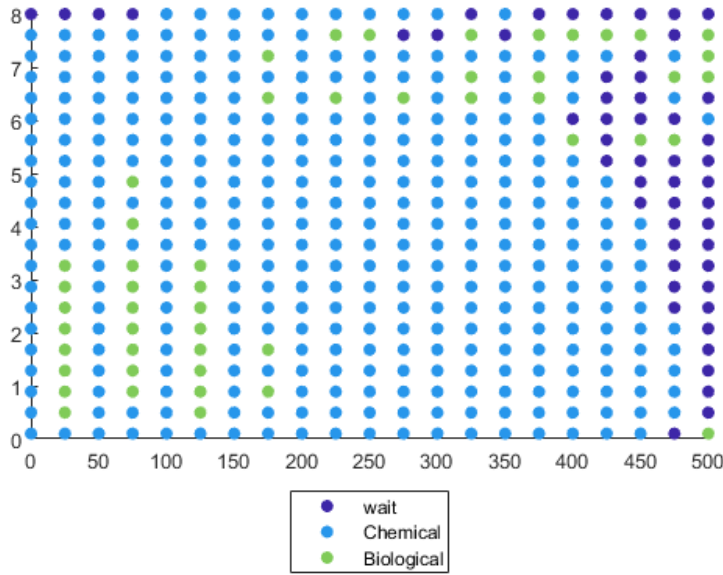
Figure 5: Optimal control strategies under different population densities without stochasticity (x-axis: GPA, y-axis: ladybeetle)

mal control strategies, which confirms our contribution to the existing literature that derives NEET without considering stochasticity. Figure 4 (with stochasticity) shows that under low pest density, farmers should rely more on chemical control, while the optimal control strategies would change to biological control or waiting without any control depending on the predator density under high pest density. Figure 5 (without stochasticity) shows that chemical control is the optimal control strategy under almost all the scenarios.

Since our parameter values are not precise enough at this moment (we are still waiting for the results from field experiments from our research team), our preliminary results are just for expositional purposes. We expect to update the parameter values and get more precise results later. We will also conduct a sensitivity analysis based on that.

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A Mathematical Solutions to the Deterministic Case

$$H_t = p\bar{y}(1 - \frac{S_t}{K}) - \theta x_t - \omega A_t + \lambda_{S_t}(\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t - k_S \cdot x_t \cdot S_t)$$
$$+ \lambda_{P_t}(\gamma \eta P_t \cdot S_t - k_P \cdot x_t \cdot P_t + (A_t - \tau) \cdot P_t) \quad (38)$$

The first order conditions:

$$\frac{\partial H}{\partial x_t} = -\theta - \lambda_{S_t} k_S S_t - \lambda_{P_t} k_P P_t \quad (39)$$

$$\frac{\partial H}{\partial A_t} = -\omega + \lambda_{P_t} P_t \quad (40)$$

$$\dot{\lambda}_{S_t} - \lambda_{S_t} = -\frac{\partial H}{\partial S_t} = \frac{p\bar{y}}{K} - \lambda_{S_t}(\alpha(1 - \frac{2S_t}{K}) - \beta \cdot P_t - k_S \cdot x_t) - \lambda_{P_t}\gamma \eta P_t \quad (41)$$

$$\dot{\lambda}_{P_t} - \lambda_{P_t} = -\frac{\partial H}{\partial P_t} = \lambda_{S_t}(\beta S_t) - \lambda_{P_t}(\gamma \eta S_t - k_P \cdot x_t + A_t - \tau) \quad (42)$$

At the steady state, from Eq. 40,

$$\lambda_{P_t} = \frac{\omega}{P_t} \quad (43)$$

Differentiate both sides with respect to time yields:

$$\dot{\lambda}_{P_t} = \frac{-\omega \dot{P}_t}{P_t^2} \quad (44)$$

Plug Eq. 43 into 39 yields

$$\lambda_{S_t} = -\frac{\theta + \omega k_P}{k_S S_t} \quad (45)$$

Differentiate both sides with respect to time yields:

$$\dot{\lambda}_{S_t} = \frac{(\theta + \omega k_P)\dot{S}_t}{k_S(S_t)^2} \quad (46)$$

23

Plug Eq. 45 and Eq. 43 into Eq. 41, solving for $\dot{\lambda}_{S_t}$ yields

$$\dot{\lambda}_{S_t} = -\frac{\theta + \omega k_P}{k_S S_t} + \frac{p\bar{y}}{K} + \frac{\theta + \omega k_P}{k_S S_t}(\alpha(1 - \frac{2S_t}{K}) - \beta \cdot P_t - k_S \cdot x_t) - \omega \gamma \eta \tag{47}$$

Combining Eq. 46 and Eq. 47 yields

$$\frac{(\theta + \omega k_P)\dot{S}_t}{k_S(S_t)^2} = -\frac{\theta + \omega k_P}{k_S S_t} + \frac{p\bar{y}}{K} + \frac{\theta + \omega k_P}{k_S S_t}(\alpha(1 - \frac{2S_t}{K}) - \beta \cdot P_t - k_S \cdot x_t) - \omega \gamma \eta \tag{48}$$

Which can be simplified as

$$0 = -\frac{\theta + \omega k_P}{k_S S_t} + \frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S S_t}\frac{S_t}{K} - \omega \gamma \eta \tag{49}$$

Where we use the equation of motion Eq. 22

Solving Eq. 49 for S_t yields the singular path for the pest population:

$$S^{SS} = \frac{\theta + \omega k_P}{(\frac{p\bar{y}}{K} - \frac{\theta + \omega k_P}{k_S K} - \omega \gamma \eta)k_S} \tag{50}$$

Plugging Eqs. 43 44 and 45 into Eq. 42 yields

$$-\frac{\omega \dot{P}_t}{P_t^2} - \frac{\omega}{P_t} = -\frac{\theta + \omega k_P}{k_S S_t}(\beta S_t) - \frac{\omega}{P_t}(\gamma \eta S_t - k_P \cdot x_t + A_t - \tau) \tag{51}$$

Solving Eq. 51 for P_t using euqaiton of motion Eq. 23 yields the singular path for the predator population:

$$P^{SS} = \frac{\omega k_S}{\beta(\theta + \omega k_P)} \tag{52}$$

Since this is an autonomous linear control optimization problem, the singular chemical control occurs when:

$$\dot{S}_t = \alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t - k_S \cdot x_t \cdot S_t = 0 \tag{53}$$

Plugging Eqs. 50 and 52 into Eq. 53 and solving for x_t, yields the singular chemical control:

$$\alpha \frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S}(1-\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S K})$$
$$-\beta\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S}\frac{\omega k_S}{\beta(\theta+\omega k_P)}-\frac{x_t(\theta+\omega k_P)}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)}=0 \quad (54)$$

$$\alpha\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S}(1-\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S K})-\frac{\omega}{\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta}$$
$$-\frac{x_t(\theta+\omega k_P)}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)}=0 \quad (55)$$

$$x^{SS}=\alpha(1-\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S K})-\frac{\omega}{\theta+\omega k_P} \quad (56)$$

Similarly, in steady state,

$$\dot{P}_t=\gamma\eta P_t\cdot S_t-k_P\cdot x_t\cdot P_t+(A_t-\tau)\cdot P_t=0 \quad (57)$$

Plugging Eqs. 50, 52 and 56 into Eq. 57, yields the singular biocontrol:

$$\gamma\eta\frac{\omega k_S}{\beta(\theta+\omega k_P)}\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S}$$
$$-k_P\left[\alpha(1-\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S K})-\frac{\omega}{\theta+\omega k_P}\right]\frac{\omega k_S}{\beta(\theta+\omega k_P)}+(A_t-\tau)\frac{\omega k_S}{\beta(\theta+\omega k_P)}=0$$
$$(58)$$

$$A^{SS}=\tau+k_P\left[\alpha(1-\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S K})-\frac{\omega}{\theta+\omega k_P}\right]-\gamma\eta\frac{\theta+\omega k_P}{(\frac{p\bar{y}}{K}-\frac{\theta+\omega k_P}{k_S K}-\omega\gamma\eta)k_S} \quad (59)$$

B Numerical Method Solution

Recall the optimality conditions of the VI are

$$\rho V(S,P) \geq p\bar{y}(1 - \frac{S_t}{K}) + \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t\right] V_S + [\gamma \eta P_t \cdot S_t - \tau P_t] V_P$$

$$+ \frac{1}{2}\sigma_S^2 S_t^2 V_{SS} + \frac{1}{2}\sigma_P^2 P_t^2 V_{PP} + \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t V_{SP} \quad (60)$$

$$\theta \geq -V_S \cdot k_S \cdot S_t - V_P \cdot k_P \cdot P_t \quad (61)$$

and

$$\omega \geq V_P \cdot P_t \quad (62)$$

At least one the of above expressions must hold with equality. If the first expression holds with equality, the farmer is in the continuation regime. If either/both of the last two expressions hold with equality, the farmer is in the corresponding action regime(s).

The optimality conditions can be restated in the form:

$$0 = \min\left[\rho V(S_t, P_t) - p\bar{y}(1 - \frac{S_t}{K}) - LV(S_t, P_t), \theta + V_S \cdot k_S \cdot S_t + V_P \cdot k_P \cdot P_t, \omega - V_P \cdot P_t\right] \quad (63)$$

where

$$LV(S_t, P_t) = \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t\right] V_S + [\gamma \eta P_t \cdot S_t - \tau P_t] V_P$$

$$+ \frac{1}{2}\sigma_S^2 S_t^2 V_{SS} + \frac{1}{2}\sigma_P^2 P_t^2 V_{PP} + \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t V_{SP} \quad (64)$$

Since we don't know $V(\cdot)$, following Fackler (2018)[8] and Balicioglu (2008)[1], suppose $V(S_t, P_t) \approx \phi(S_t, P_t)\theta$, where ϕ represents a set of n basis functions for a family of approximating functions and θ is an n-vector of coefficients for the value function. Then the complementarity

problems in Eq. 63 - evaluated at the n nodal points in the state space, (S,P), can be rewritten as

$$0 = \min(B_1\theta + q_1, B_2\theta + q_2, B_3\theta + q_3) \tag{65}$$

where

$$B_1 = \rho\phi(S_t, P_t) - \left[\alpha S_t(1 - \frac{S_t}{K}) - \beta S_t \cdot P_t\right]\phi_S - [\gamma\eta P_t \cdot S_t - \tau P_t]\phi_P$$

$$- \frac{1}{2}\sigma_S^2 S_t^2 \phi_{SS} - \frac{1}{2}\sigma_P^2 P_t^2 \phi_{PP} - \sigma_{SP}\sigma_S\sigma_P S_t \cdot P_t \phi_{SP} \tag{66}$$

$$B_2 = \phi_S \cdot k_S \cdot S_t + \phi_P \cdot k_P \cdot P_t \tag{67}$$

$$B_3 = -\phi_P \tag{68}$$

$$q_1 = -p\bar{y}(1 - \frac{S_t}{K}) \tag{69}$$

$$q_2 = \theta \tag{70}$$

and

$$q_3 = \omega \tag{71}$$

Then use Fischer-Burmeister function

$$\Phi^-(x,y) = x + y - \sqrt{x^2 + y^2}$$

to set

$$F_1(\theta) = \Phi^-(B_1\theta + q_1, F_2(\theta))$$

$$F_2(\theta) = \Phi^-(B_2\theta + q_2, F_3(\theta))$$

and

$$F_3(\theta) = B_3\theta + q_3$$

Then Eq. 65 is equivalent to solving

$$F_1(\theta) = 0$$