Pricing Derivatives in the Agricultural Land Market

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1 Introduction

Agricultural land is an attractive investment for financial investors, not only since the financial crisis of 2008 (Deininger and Byerlee, 2011). Farmland generates comparatively secure returns in the form of lease payments and promises high increases in value, or at least compensation for inflation-related losses in purchasing power. In addition, income from land investments is only weakly correlated with income from other capital market investments (Sherrick and Mallory, 2013). Thus, the acquisition of agricultural land may not be exclusively for production, but often represents a financial investment with the aim of generating capital income.

The involvement of non-agricultural investors in land markets is regarded as part of a financialization process in the agricultural sector (cf. Odening and Hüttel, 2018). Discussions on their role, the consequences for farmers and agricultural production stimulate political debates on policy objectives. These arguments include the increased challenges for farmers to purchase land. One potential solution would be to develop a financial instrument that would allow financial investors to participate in the land market in a way that does not require the physical acquisition of land through land derivatives. Financial market instruments (such as futures, options, swaps) are used to transfer risks between land market participants, but also to speculate on price developments in the land market. With the introduction of these hypothetical financial market products, hedging against farmland price fluctuations, as the main factor of agricultural production, would become possible and further development of the financialization process might make these markets more efficient and transparent. With observed rising land prices, a scenario of declining land prices and need to hedge against them might be rather unlikely, but not impossible due to past developments. Rather, phases of stagnating land prices can be observed and even significant price declines (Falk and Lee, 1998; Olsen and Stokes, 2015). A producing farmer who does not intend to buy or sell land for the foreseeable future is not directly affected by changes in land prices. There is, however, an indirect concern. On the one hand, land is used to secure loans, and the loan-to-value ratio is derived from the market value. On the other hand, land represents a source of income in the form of lease payments, which are based on the value of the land, especially for farms that are being phased out. Other stakeholders are directly affected by changes in land prices. These include banks that have secured loans with agricultural land and other non-farmers who hold agricultural land in their financial portfolio. It does not necessarily include private individuals who have inherited land or acquired it to a small extent, but rather institutions such as real estate funds, insurance companies, public land privatization authorities or other state companies.

Despite sparse opportunities to trade country-based derivatives (call and put options on Gladstone land and Farmland Partners Inc. land fund), other possibilities of land derivatives have not yet been explored, especially those based on an index instead of funds with stocks possibly being more influenced by internal business decisions. This line of research has been previously limited by the unknown willingness to participate by relevant actors, the incompleteness of the market, and the lack of an adequate pricing framework, a prerequisite for their establishment.

Against this background, this paper adds to the sparse empirical literature on derivatives in land markets by identifying and applying a pricing framework. Potential market participants need to understand how the prices for these products are formed to then classify prices observed in the market as "fair" and to compare them to their own expectations. The proposed research enhances our understanding of opportunities and challenges related to development of land derivatives. This paper therefore contributes to pricing and potential implementation of land derivatives with the use of data from the National Council of Real Estate Fiduciaries (NCREIF) Farmland Index.

In the following, section 2 provides overview on the evolution of real estate derivatives and the current state of farmland derivatives, section 3 explains the construction of indices, followed by section 4 on pricing framework of derivatives and an empirical application to the NCREIF farmland index.

2 Evolution of real estate derivatives and current state of farmland derivatives

A motivation for exploring the potential of farmland derivatives originates in the development of the real estate derivatives market (property derivatives). Driven by the research agenda of Robert Shiller (1993), the possibility to acquire financial derivatives on commercial and residential real estate has existed for around 30 years. After a slow start, the real estate derivatives market took off in the mid-2000s, particularly in the U.S. and UK. Fabozzi et al. (2020) provide an overview of this market development and its drivers and obstacles. Irrespective of all differences between real estate and land markets in terms of size and supply and demand structures, both markets show structural similarities: Both (agricultural) land and real estate are immobile and characterized by heterogeneity of value-determining characteristics. Furthermore, both markets are imperfect in the sense that they have low liquidity, high transaction costs and short selling is not possible. Against this background, the question arises whether and under what conditions land derivatives could develop in a similar way to real estate derivatives.

The existence of isolated opportunities to trade land-based derivatives provides further reason for the elaboration of land derivatives. For example, call and put options on the Gladstone land fund are listed on Nasdaq and call and put options on the Farmland Partners Inc. land fund are listed on the NYSE. Gladstone is a listed real estate fund that, in addition to agricultural facilities (warehouses, cold stores), mainly holds agricultural land that is rented out. The value of agricultural land and real estate amounts to around US$1.6 billion at the end of 2022. The total of 46,944 hectares of agricultural land are spread over 164 farms in 15 U.S. states (Gladstone Land, 2023). Gladstone Land specializes in acquiring family farms on a sale-and-lease-back basis. The lease term varies between five and ten years. American Farmland constituted another agricultural real estate fund acquired by Farmland Partners Inc. in 2015. Farmland Partners Inc., like Gladstone Land, specializes in acquiring farmland for lease. The fund owns a total of 64,750 hectares in 17 states, which are spread over more than 100 companies. The market capitalization is US$ 755.82 million in April 2022. In addition to real estate funds for agriculture, there are also funds for wood: Rayonier, PotlatchDeltic, Weyerhaeuser and CatchMark are four U.S.-based public real estate funds owning forest land in 20 states with a market capitalization of US$41 billion (Baral and Mei, 2022; WRDS, 2021).

In principle, all types of financial market derivatives are available for use on agricultural land, i.e. forward contracts, options and swaps. The underlying for real estate derivatives in general and land derivatives in particular is a price index. Common and widely used indices for property markets are the IPD Index from the Investment Property Database in London, the National Property Index (NPI) from the National Council of Real Estate Fiduciaries (NCREIF) in the U.S. and the S&P Case-Shiller Index for house prices, also in the United States. An overview of indices used for real estate derivatives can be found in Tunaru (2017). There is currently no comparable variety of established indices for agricultural land. The NCREIF provides a farmland index for 12 regions in the USA. It is updated quarterly and shows the price development for arable land and permanent crops since 1991. The index covers fund assets of more than US$ 14 billion (as of the second quarter of 2022). The valuation is not based on transaction prices but is carried out by market experts (more details in chapter 3). It allows to capture total income from land ownership and split it into an income component and an appreciation component.[1] Figure 1 shows the development of the NCREIF Farmland Index (Figure 1a) and its return components (Figure 1b).

The NCREIF farmland index value has experienced a tenfold growth over the past 30 years. The value has increased continuously, but not at a steady proportional rate. The total return lies between 5 and 9% in the 1990s, climbing temporarily to over 30% in the mid-2000s, then falling to 5% afterwards. The total return varies due to fluctuation in the appreciation return, whereas the income return (from renting out) remains relatively stable.

[1] There have been attempts by private providers, such as PeakSoil, to establish other agricultural land indices as underlyings for land derivatives, but these have not been successful so far.

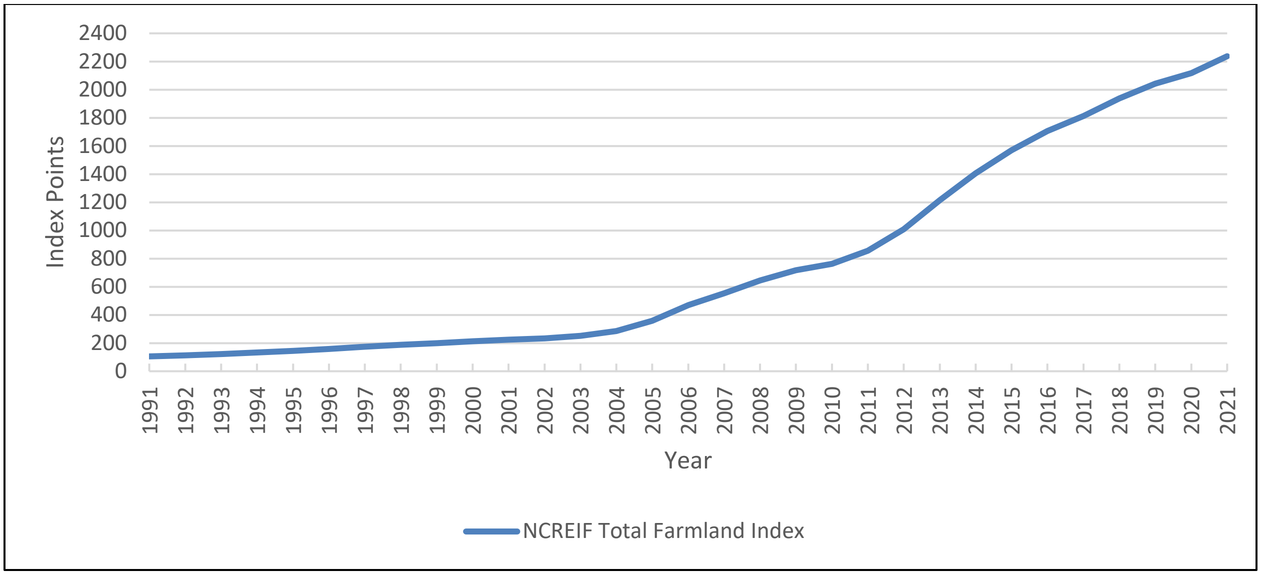

Figure 1a: NCREIF Farmland Index

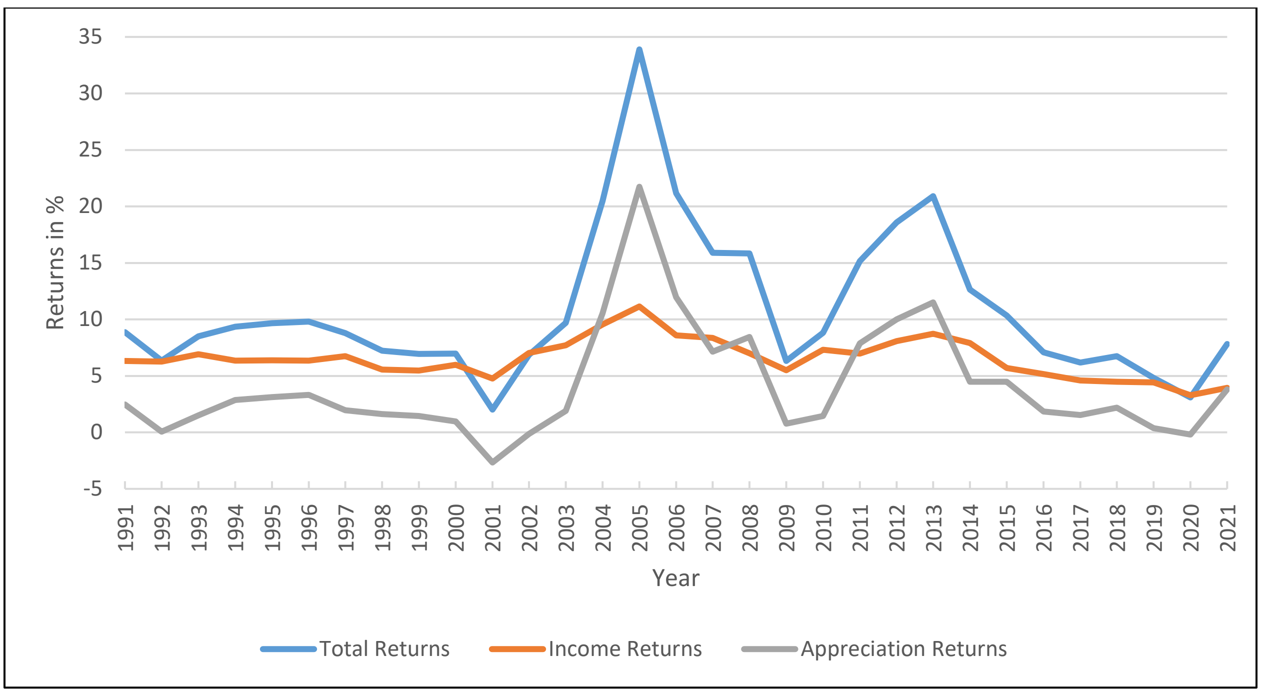

Figure 1b: Returns of the NCREIF Index

Compared to other index derivatives, land indices are not updated daily, but quarterly in the case of the NCREIF Farmland Index, and the update is delayed by a few weeks. Thus, cash settlement of derivatives must be settled retrospectively after the index value has been published. Further, the establishment of land derivatives, such as forwards contracts, necessitates a specified duration. The duration of forward contracts for real estate ranges from one to 30 years, corresponding to the long holding period for this asset class (Tunaru, 2017). In contrast, little is known about how long agricultural land is held in the portfolios of financial investors.

3 Construction of land price indices

An essential prerequisite for the establishment of a functioning market for land derivatives is the construction of a land index corresponding to the expectations and requirements of market participants. First, an important characteristic is transparency, i.e. the calculation of the index needs to be understandable, traceable and free from manipulation. Second, the index should reflect the "actual" value of the land as precisely as possible. Third, the index should capture and reflect all market information relevant to the valuation as quickly as possible. A distinction in the index construction is made between transaction-based and valuation-based indices.

Transaction-based indices are derived from the prices of properties sold within a specified period of time. They have the advantage of capturing buyers' actual willingness to pay and sellers' payment requirements. However, transaction-based indices display some challenges in their construction: Only a fraction of the real estate portfolio is sold within the selected observation period and the properties are very heterogeneous, i.e. they differ in their value-determining characteristics and thus, in their prices. Thus, price changes of the index might not be caused by changes in valuation, but by structural changes in the recorded transactions. In the literature, two approaches are discussed to address this problem, which are also applied in practice. They encompass the analysis of repeated sales and hedonic price regressions (Geltner and Fisher, 2007). The repeated sales analysis takes as its data basis properties that were sold at least twice during the observation period. It considers the change in value of identical objects over time. Technically, the price index is determined as a regression coefficient of a dummy variable for the year of sale (Case and Shiller, 1989). The Case-Shiller index for residential real estate underlies this principle, for example. The advantage of this approach are no temporal changes in the composition of the objects underlying the index. A disadvantage is the potential small number of observations, which can be attributed to the low liquidity or the long holding period of real estate. The problem is exacerbated with a regional disaggregation of the index (Clapp and Giacotto, 1992). This disadvantage also applies to agricultural land. Aggregated at the federal state level of Lower Saxony, only 3% of land sold several times over a period of 14 years (2005-2019).

An alternative to the analysis of repeated sales constitutes a hedonic regression, accounting for and partially eliminating the changing quality composition of the objects over time. Hedonic models are used extensively for empirical analysis of price determinants of agricultural land (e.g. Featherstone et al., 1993; Huang et al., 2006; Ritter et al., 2020; Tsoodle et al., 2006; Nickerson and Zhang, 2014). Yang et al. (2017) use this approach to adjust price time series for quality differences. The adjusted prices then hold for areas of average quality. Clapp and Giacotto (1992) derive the price index directly from the regression coefficients for time dummies being included in the regression model, in addition to price-determining attributes. However, the use of hedonic regression models in the context of calculating land price indices is not unproblematic because, like any regression model, the results are subject to a potential estimation and specification error (c.f. Nickerson and Zhang, 2014). In fact, there is no generally accepted hedonic land price model that could be used to determine land price indices. Ritter et al. (2020) apply a meta-analysis to demonstrate the variation of hedonic model results across regions and over time and the sensitivity of the results on the model specification.

Valuation-based indices are based on estimates by appraisers and real estate experts. Typically, valuation-based price indices show less volatility than transaction-based ones. In Germany, for instance, expert committees for land values are state institutions that deal with the valuation of developed and undeveloped land, including agricultural land. They are responsible for the collection of transaction data and the determination of standard land values for agricultural and non-agricultural land (Helbing et al., 2017). However, the standard land values are only updated every two years and are not aggregated into a price index for agricultural land. Hence, they are not suitable as reference objects for land derivatives in their present form.

The NCREIF Farmland Index is a valuation-based price index. The valuations made by members of the NCREIF relate to developed or undeveloped agricultural land that is owned by financial investors. The farms involved can be self-managed or leased. As family farms are not included in the reporting, the database is not a representative sample of all forms of ownership, however relevant from the perspective of potential financial investors. The basis of the index changes over time due to new reporting members and (sales) purchases of land. The participating investment managers estimate the market value of their managed land on a quarterly basis, i.e. the Farmland Index is based on a subjective evaluation. To limit subjective evaluation errors, an evaluation is carried out by external experts at least every two years. In addition to the market value estimates, information on the current income from land ownership (lease payments or income from agricultural production) is reported. Values of individual agricultural properties are then aggregated by region and combined into a land index or yield index with reference to 1990.

The peculiarities of agricultural land markets and the construction of land price indices are reflected in the statistical properties of the price time series. The NCREIF Farmland indices are affected by random estimation error and smoothing error, which can lead to obsolete values, seasonality and lags in the data (Webb, 1994; Fisher et al., 1999; Cannon and Cole, 2011). (Partial) autocorrelation tests are displayed in Figure 2 for the returns of the NCREIF Farmland index from 1991 to 2021. The positive autocorrelation of the previous year (level 1) exceeds the statistical significance band and an AR (1) process can be determined. The NCREIF sub-indices show a similar picture. It implies time series to be more than white noise and with some predictability. The autocorrelation raises some theoretical problems when trying to develop a suitable pricing model for derivatives, as this dynamic can lead to incorrect pricing of the derivatives. The assumption of a geometric Brownian process for a pricing model is not compatible with an autocorrelated index.

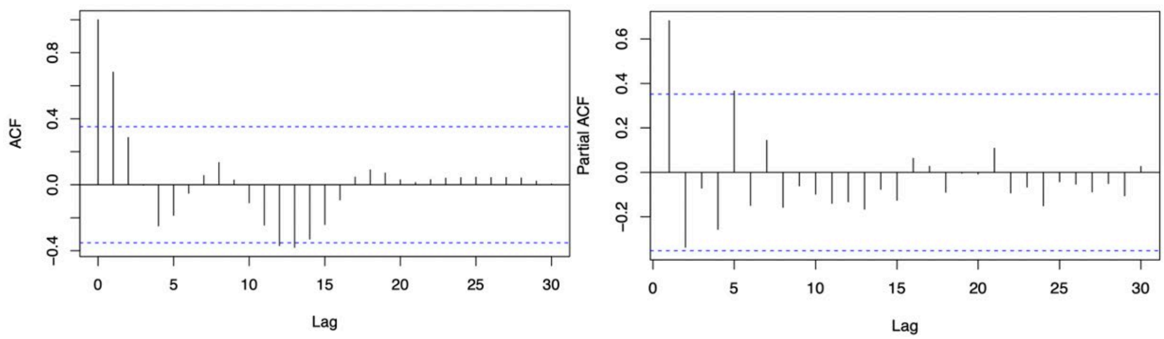

Figure 2: Autocorrelation and partial autocorrelation of NCREIF Index.

3 Pricing models for Property Derivatives

3.1 Overview about pricing models

An essential prerequisite for the establishment of land derivatives is the understanding of potential market participants how the prices for these products are (theoretically) formed. It allows the participants to then classify prices observed in the market as "fair" and to compare them to their own expectations and, if necessary, to discover arbitrage opportunities. Generally accepted valuation models for classic financial market products cannot be easily transferred to real estate derivatives in general and land derivatives in particular. A fundamental difficulty results from the non-tradability of the underlying index, i.e. unlike classic financial derivatives, the index itself cannot be bought or sold. As a result, markets for real estate and land derivatives are imperfect and risk-free hedge portfolios cannot be easily built. Similar theoretical problems arise when pricing weather derivatives (Xu et al., 2008). Irrespective of this, a number of assessment approaches have been proposed in the real estate literature, which are briefly presented below. An overview can be found in Tunaru (2017, Chapter 7).

An early contribution to the pricing of real estate derivatives is made by Shiller and Weiss (1999). Shiller and Weiss start by stating that real estate prices, in contrast to assets (e.g. stocks), do not follow a purely random process, but rather an autoregressive process. It results in a partial predictability of relative price changes. They propose to calculate the price of a real estate derivative as the expected value of the discounted returns. For European options, they derive a pricing formula with the same structure as the

Black-Scholes model, except that the risk-free rate is replaced by the empirically estimated expected value of the price index return. However, this ad hoc specification does not ensure the option price to be arbitrage-free.

Syz (2008) bases his valuation model on spot-forward parity $F_t(T) = S_t \cdot e^{r(T-t)}$, which ensures freedom from arbitrage on frictionless financial markets. F indicates the forward price and S is the spot price, r represents the risk-free interest rate and T is the maturity of the contract. In light of frictions in real estate and land markets, Syz modifies this relationship to $F_t(T) = S_t \cdot e^{r+\rho(T-t)}$. The property spread ρ represents a difference between actual returns on real estate swaps and a risk-free interest rate. This difference can be positive or negative and conceptually corresponds to a convenience yield in commodity futures. This model then results in a modified version of the Black-Scholes model for the pricing of options on real estate indices, as in Shiller and Weiss (1999).

Fabozzi et al. (2012) further develop the model of Shiller and Weiss (1999) by applying the principle of risk-neutral valuation. The implementation of this pricing approach requires the estimation of the market price for risk. Fabozzi et al. (2012) derive this parameter from the quotations of real estate futures contracts with different maturities. This approach is described in greater detail in section 3.2, as we apply this model developed to cope with the particularities of the broader real estate market to the agricultural land market. It accounts for the autoregression of the NCREIF index and provides an approach to complete the market.

Cao and Wei (2010) propose another valuation approach built on Lucas ' (1978) equilibrium model. An optimal (utility-maximizing) portfolio is determined for a representative market participant, which contains fixed-income securities and shares. The prices for the traded financial instruments are stochastic and are determined endogenously, under the requirement of market clearance. Cao and Wei (2010) extend the Lucas model by adding real estate to the portfolio. The challenge of its practical application lies in the assumption of a risk-utility function for the representative market participant and the estimation of the risk aversion parameter.

3.2 The Fabozzi-Shiller-Tunaru (FST) Model

The main components of the FST-Model are a stochastic process $Y_t = ln\{X_t\}_{t\geq 0}$ for the farmland index X_t, a risk-free rate r, a liquid bond market with zero-bonds for any maturity and price $p(t,T)$ for time t and maturity T, as well as a money market account. To account for the autocorrelation of returns and the predictability of real estate indices, FST assume a mean reverting process for Y_t with a long-run mean (LRM) Ψ_t. It then holds that

$$dY_t = \left[\frac{d\Psi_t}{dt} - \theta(Y_t - \Psi_t)\right] dt + \sigma dW_t \qquad (1)$$

Herein W_t is a Wiener process, θ denotes a mean reversion parameter, and σ is a volatility measure. Using the definition

$$\hat{Y} = Y_t - \Psi_t$$

(1) can be rewritten as an Orstein-Uhlenbeck process:

$$dY_t = \left[-\theta\hat{Y}_t\right]dt + \sigma dW_t \qquad (2)$$

Arbitrage free derivative prices can be derived by replacing the actual (physical) stochastic process (2) by a risk-neutral one. This is attained by adjusting the drift rate with the market price of risk parameter λ (e.g. Hull, 2006, p. 590):

$$dY_t = \left[-\theta\hat{Y}_t - \lambda\sigma\right]dt + \sigma dW_t^Q \qquad (3)$$

where the Wiener process is under an equivalent martingale measure Q. Fabozzi et al. (2012) show that under the previous assumptions, the land price index X_t follows a log-normal distribution, i.e.:

$$Y_T|Y_t \sim N\left(m_{y;t,T}; \sigma^2_{y;t,T}\right) \tag{4}$$

with an expected value

$$m_{y;t,T} = \Psi_T - \frac{\lambda\sigma}{\theta} + \left(Y_t - \Psi_t + \frac{\lambda\sigma}{\theta}\right) e^{\theta(t-T)} \tag{5}$$

and variance

$$\sigma^2_{y;t,T} = \frac{\sigma^2}{2\theta}\left[1 - e^{2\theta(t-T)}\right]. \tag{6}$$

Based on this distribution for the log price index, Fabozzi et al. (2012) derive closed form equations for various types of property derivatives. The no-arbitrage price for a futures contract with maturity T is:

$$F(t,T) = E_t^Q[X_T] = \exp\left\{m_{y;t,T} + \frac{\sigma^2_{y;t,T}}{2}\right\} \tag{7}$$

Since the land price index is assumed to be log-normal, prices for European call options c based on a Futures contract $F\,(0,T)$ with strike price K follow a Black-Scholes-type formula:

$$c = p(0,T)F(0,T)\Phi(d_+) - Kp(0,T)\Phi(d_-) \tag{8}$$

with $d_+ = \frac{\ln\left(\frac{F(0,T)}{K}\right) + \frac{\sigma^2_{y;0,T}}{2}}{\sigma_{y;0,T}}$, $d_- = d_+ - \sigma_{y;0,T}$

Real estate swaps are more complex than futures and options, because they involve payments at time points t_1, t_2, …t_k. A common maturity is five years with one payment per year. A variable payment depending on the return on the index is exchanged against a fixed payment in the form of a floating rate which consists of a risk-free rate plus a spread δ. No principal is exchanged. Under the assumption of a constant (risk-free) interest rate r and no income return (only appreciation), Tunaru (2017) derives the following expression for the spread of a swap:

$$\delta = \frac{\sum_{j=1}^{N} e^{-r_{t_j}} (E_t^Q \left[X_{t_j}\right] - e^{r_{t_j}} E_t^Q [X_{t_{j-1}}])}{\sum_{j=1}^{N} e^{-r_{t_j}} E_t^Q [X_{t_{j-1}}]} \tag{9}$$

The spread can be expressed in terms of futures prices with different maturities (c.f. eq. 7). Bjork and Clapman (2002) claim that the theoretical arbitrage-free price of a swap should be zero, which is actually not the case for existing real estate linked swaps. Fabozzi (2009) explain this finding by low fungibility and short sale constraints of real estate markets.

Estimation of the pricing model proceeds in several steps. First, a functional form for the long run mean of the index Ψ_t has to specified and estimated. In the subsequent application we choose a linear trend, i.e. $\Psi_t = \alpha + \beta t$ and estimate the parameters α and β with OLS. We then estimate the mean-reverting parameter and the volatility.

$$\hat{\theta}^{(N)} = \ln\left(\frac{\sum_{k=1}^{N} \hat{Y}_{k-1}^2}{\sum_{k=1}^{N} \hat{Y}_k \hat{Y}_{k-1}}\right) \tag{10}$$

The variance σ^2 can be estimated by a quadratic variation estimator

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{k=1}^{N} [Y_k - Y_{k-1}]^2 \tag{11}$$

Finally, the market price of risk, λ, is estimated by "reverse engineering", which means that we use observed land derivative prices to calibrate the equilibrium pricing equations (7), (8) or (9) with respect to this parameter.

4 Pricing derivatives on the NCREIF Farmland Index

4.1 Checking sensitivities on the market price of risk

The pricing of land derivatives on the NCREIF Farmland Index centers around the determination of the market price of risk, because its retrieval completes the market. It is calculated through the combination of the pricing model by FST built on an equivalent martingale probability measure and available data of a derivative market, i.e. futures prices. This methodology respects the empirical characteristics of the land market, i.e. the autocorrelation of the NCREIF Farmland index.

The empirical application starts with the specification of the functional form of the long run mean by choosing a linear trend (table 1).

	Coefficients	Standard error
α	4.3940	0.0247
β (years)	0.0283	0.0003
Adj. R^2	0.9819	

Table 1: Regression results with log index values as dependent variable.

This allows to estimate the mean reversion parameter (10) and volatility measure (11) (table 2).

θ	0.0262
σ^2	0.0013
σ	0.0366

Table 2: Estimation of mean reversion and volatility parameter.

Before calibrating the market price of risk via reverse engineering from land market data, we turn to the market prices of risk retrieved by Fabozzi et al. (2012) from market futures prices on the IPD UK Annual Property Total Returns index (Table 3). Irrespective of the maturity of IPD futures, they are regarded as hypothetical values of λ to check the sensitivity of derivative prices on different market prices of risk. For different maturities, values for λ range from 0.7062 to 2.5862. We use these market prices of risk to calculate futures and options (eq. 7,8).[2]

[2] Due to the swap price (eq. 9) being a combination of futures prices (eq. 7), we focus on the determination of the futures price.

Date	Maturity	Dec 2008	Dec 2009	Dec 2010	Dec 2011	Dec 2012
Mar 25, 2009	Eurex Futures price	77.9	80.75	102.25	110	112.75
	Market price of risk λ	1.4693	2.5409	0.7354	0.7062	0.8193
Date		Dec 2009	Dec 2010	Dec 2011	Dec 2012	Dec 2013
Apr 1, 2009	Eurex Futures price	80.5	102.25	110	112.75	110.75
	Market price of risk λ	2.5862	0.7393	0.7084	0.8213	0.9919

Table 3: λ values as calibrated in Fabozzi et al. (2012)

A varying market price of risk influences the level of futures prices, as displayed in Figure 3. Futures on NCREIF Farmland Index with a shorter maturity (1 year) have a lower price level than those with a longer maturity (5 years) for the same value of λ. Regardless of the price level, futures prices decline with an increasing λ. However, this effect is less pronounced for futures with shorter maturity, as the time for changing market expectation is shorter.

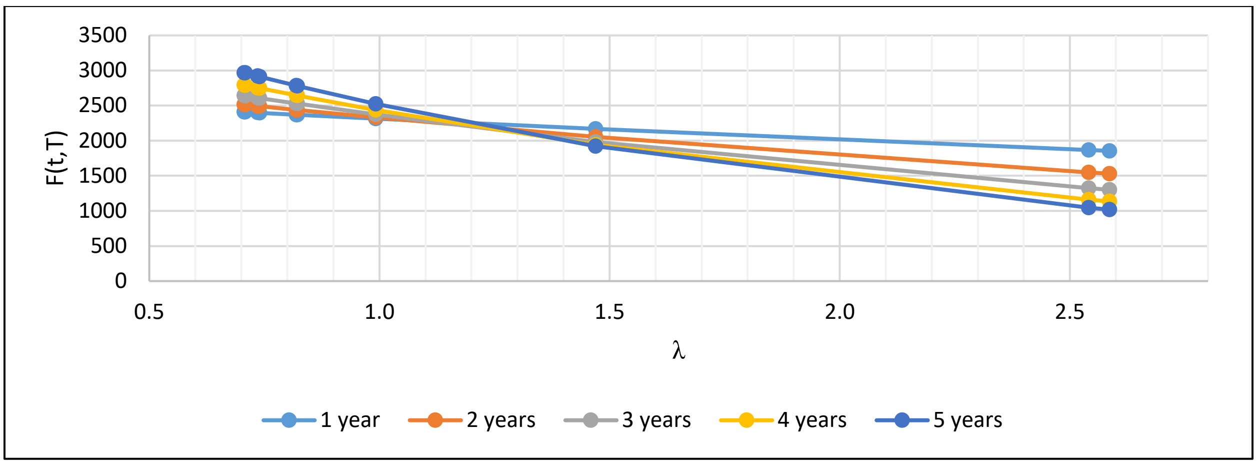

Figure 3: Futures on NCREIF Farmland Index with different maturities, determined with market price of risk λ from Fabozzi et al. (2012).

Given the sensitivity of futures prices on a range of λ, changes in the market price for risk also influences option prices due to their relation as displayed in formula (8). Figure 4 displays the reaction of the option prices to different λ. The call option price decreases with an increasing market price of risk. Holding the market price of risk constant, a higher price level is noted for options with a lower strike level. Moreover, the closer the maturity, the higher the option price given the same market price of risk. Thus, the level of the market price of risk influences the pricing of options.

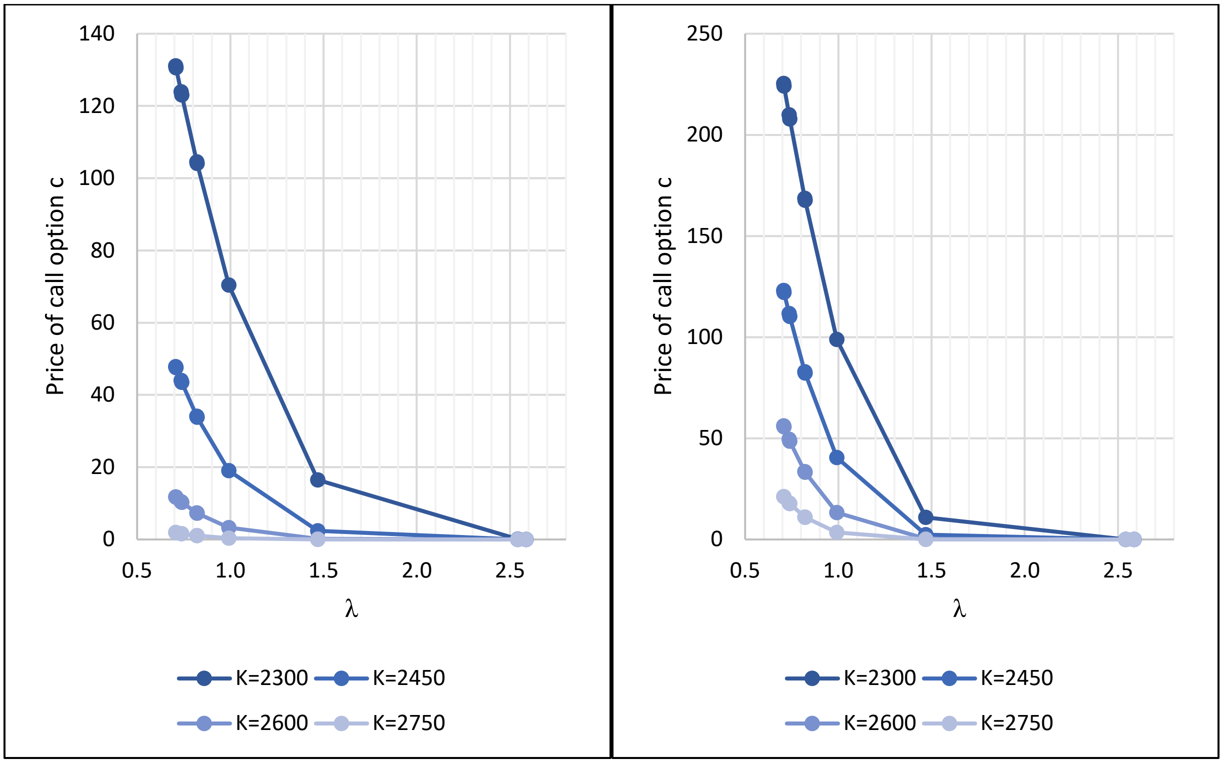

Figure 4: Options on NCREI Farmland Index (one-year call option on the left, two-year call option on the right) with different strikes, determined with market price of risk λ from Fabozzi et al. (2012).

Given the sensitivity of different derivative prices on the market price of risk calibrated independently from the NCREIF farmland index, we now turn to the calibration of λ in relation to this index.

4.2 Calibrating the market price of risk

The calibration of the market price of risk for the NCREIF Farmland index is restricted to the extent that no derivatives are traded on this index that would allow to reversely engineer λ. For this reason, we turn to one of the appraising actors of the NCREIF Farmland index, namely Gladstone Land. Option prices for different maturities are available. It is the closest available match to the NCREIF Farmland index, because of its status as a US-based land-owning company and data-providing member to the index. We use weekly closing stock prices from 2013, the company's IPO in January 2013, until August 2022.[3]. Options with four different maturities are focused on: September 21, 2022, October 21, 2022, November 18, 2022 and February 17, 2023.

The calibration of the market price of risk retrieved from the option prices (data retrieved in August, 2022) requests the specification of the functional form for the long run mean (LRM) and the estimation of the mean reversion and volatility parameter. A linear model for the LRM results in a R^2 of 0.312 and negative λ for most of maturity-strike combinations used in the calibration. Thus, we turn to a quadratic model with $LRM = \alpha + \beta t + \rho t^2$ (R^2 of 0.82) and estimate the parameters to then calibrate the market price of risk. To check for the robustness of the retrieved market price of risk, we re-estimate the market price of risk with data from December, 2022.

	Date 1: August 30, 2022	Date 2: December 9, 2022
α	2.8304	2.7923
β (week)	-0.0048	-0.0042
ρ (week^2)	0.0000121	0.0000107
Adj. R^2	0.8182	0.7686
θ	0.0455	0.0284
σ^2	0.0017	0.0017
σ	0.0413	0.0416

Table 4: Estimation results of long run mean, mean reversion and volatility parameter of Gladstone land stock prices (log).

Taken the results of table 4, we calibrate the market price of risk on both dates (table 5). The values derived for λ in the quadratic model are positive and their mean ranges from 1.046 to 1.165 for options over different strike level on August 30, 2022 and from 0.865 to 0.908 on December 9, 2022. It is notable that the mean market price of risk decreases for the option with maturity in February 2023 on observation date in December 2022 (λ = 0.908) compared to August 2022 (λ = 1.165). Moreover, it holds for both

[3] Stock price data on Gladstone Land Corporation was downloaded from Yahoo Finance (August 30, 2022), leading to 500 observations.

dates that the market price of risk decreases with an increasing strike price. Holding the strike level constant, the market price of risk does not display a clear trend of increase or decrease over the different strike levels. When comparing the first and last maturity on both observation dates, the market price of risk increases on average.

Date 1: August, 30, 2022					Date 2: December 9, 2022				
	Maturity					Maturity			
Strike	Sep 2022	Oct 2022	Nov 2022	Feb 2023	Strike	Dec 2022	Jan 2023	Feb 2023	May 2023
	λ					λ			
\$20	1.019	-	1.205	1.337	\$15	0.894	-	-	0.970
\$22.5	1.047	1.088	1.146	1.316	\$20	0.932	-	0.944	0.927
\$25	1.186	1.004	1.135	1.207	\$22.5	-	0.917	0.976	0.894
\$30	1.165	-	0.978	1.093	\$25	-	0.931	0.899	0.791
\$35	-	-	0.861	0.872	\$30	0.769	-	0.811	-
Mean	1.104	1.046	1.065	1.165	Mean	0.865	0.924	0.908	0.896

Table 5: Reverse engineering of market price of risk λ from Gladstone options for two dates of observation. “- “denotes a maturity-strike combination without an available option

This relation (increasing strike price and decreasing market price of risk) is to be found in related studies. Buckle (2002) develops a model for isolating the sharpe ratio (i.e. market price of risk) in an option’s environment and demonstrates a negative relation between sharpe ration and strike size. Härdle et al. (2021) find a positive relationship between the time to maturity and the market price of risk for their application on winter power futures. Fabozzi et al. (2012) recover curves for the market price of risk for different maturities for the IPD real estate index. They find external factors (e.g. new taxation regime) influencing market expectations to affect the relation between market price of risk and maturity. After such external events, the market price of risk for shorter maturity is lower compared to the one for a longer maturity.

4.3. Calculating derivative prices

The calibration of the market price of risk completes the market and allows to calculate land derivative prices (equation 7 and 8).[4] For this example, the market prices for the different maturities retrieved from Gladstone option in August 2022 were used. Figure 5 displays the calculated futures prices on the underlying NCREIF Farmland Index.[5]

Maturity	September 21, 2022	October 21, 2022	November 18, 2022	February 17, 2023
λ	1.104	1.046	1.065	1.165
$F(t,T)$	2190.2	2220.4	2239.3	2288.6
$I_t - F(t,T)$	231.4	201.2	182.3	133

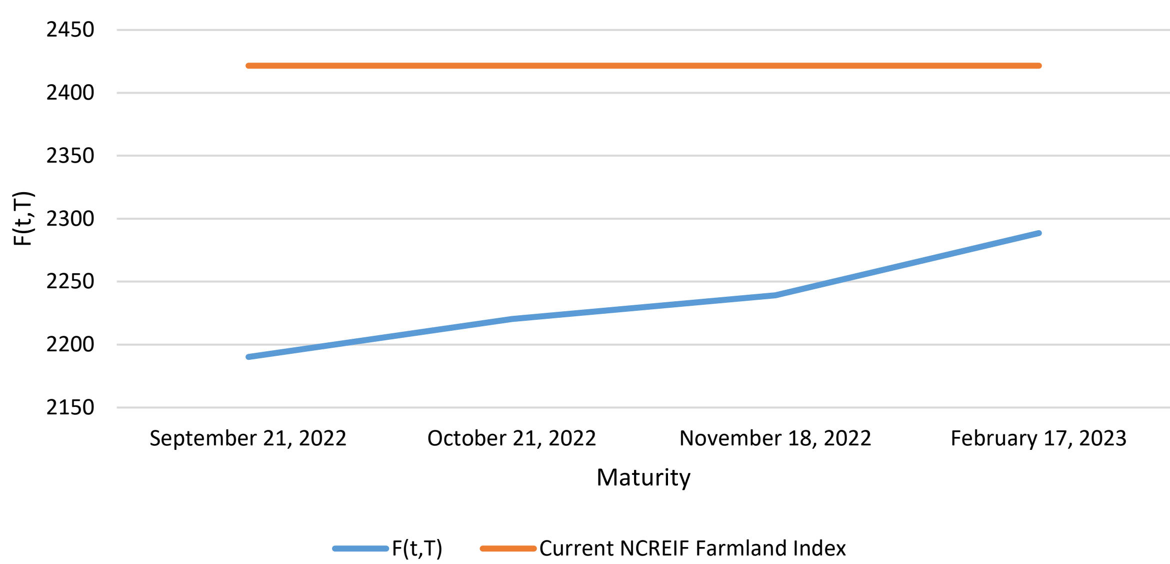

Figure 5: Futures prices on the NCREIF with current index value $\boldsymbol{I_t} = 2421.6$ (August 26, 2022)

The futures prices with the maturities are lower than the current index value, indicating negative short-term expectations for the NCREIF Farmland Index. It is unlikely that agricultural land values will decline in

[4] A risk-free rate of $r = 2\%$ was assumed.
[5] The derivative prices are not expressed in a specific currency, as the framework uses index points as a basis.

the long term. However, short-term price reductions can occur and are mainly caused by expected macroeconomic changes, such as increases in key interest rates, which occurred repeatedly in the course of 2022.

Call options with five different strike levels ranging from 2200 to 2600 (current index value $I_{t=}$ 2421.6) were calculated for each of the four available maturities. Table 6 and figure 6 display that the calculated call option prices are more expensive, the lower the strike price (13.85 with K =2200 and 0.15 with K = 2300). The same holds for the relation of price and maturity, as the price increases with an increasing maturity (for K =2200, the option prices lie at 13.85 versus 101.61 for maturities in September 2022 and February 2023, respectively. Reasons are the higher probability of an option's profitability with lower strike prices (higher intrinsic values) and longer maturity (higher time value), in line with economic theory.

Maturity	September 21, 2022	October 21, 2022	November 18, 2022	February 17, 2023
λ	1.104	1.046	1.065	1.165
c (K =2200)	13.85	37.36	55.14	101.61
c(K =2300)	0.15	3.77	10.88	40.28
c(K =2400)	0	0.09	0.88	10.86
c(K =2500)	0	0	0.03	1.92
c(K =2600)	0	0	0	0.22

Table 6: Example options' prices (c) with different strike level (K) on the NCREIF Farmland Index

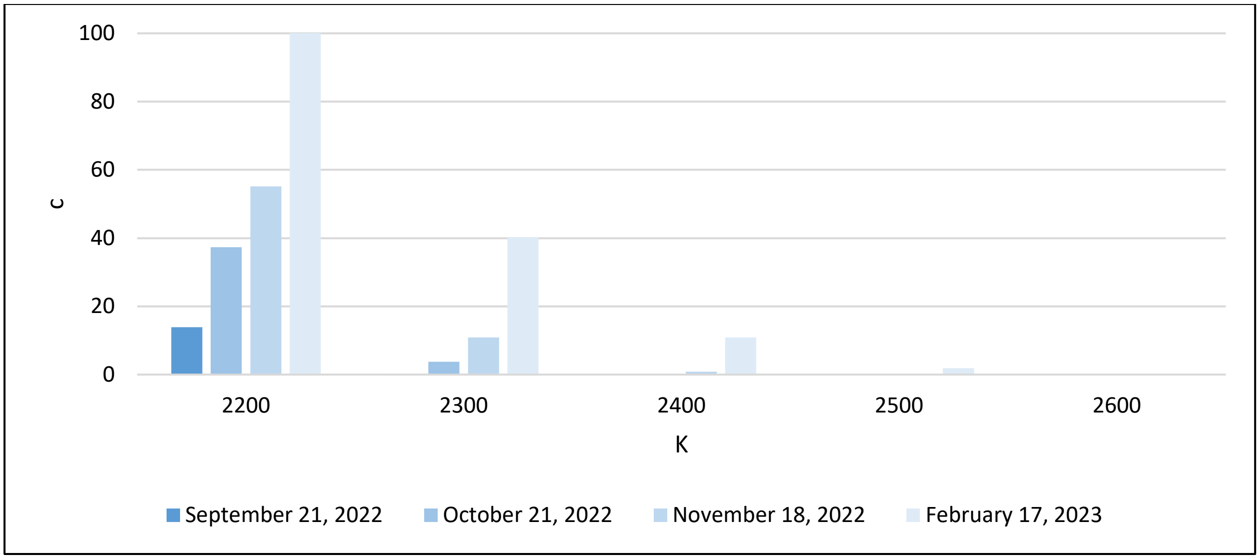

Figure 6: Example options' prices (c) with different strike level (K) on the NCREIF Farmland Index

Furthermore, the bound of values for option prices is checked. The lower bound condition, i.e. the call option price is higher or equal to a call option discounted expected return, is fulfilled for all of the presented strike-maturity combinations. Table 7 compares the presented option prices with their discounted intrinsic values (DIV).

	September 21, 2022		October 21, 2022		November 18, 2022		February 17, 2023	
	c	DIV | 0	c	DIV | 0	c	DIV | 0	c	DIV | 0
$K = 2200$	13.85	0	37.36	20.37	55.14	39.13	101.62	87.75
$K = 2300$	0.15	0	3.77	0	10.88	0	40.28	0
$K = 2400$	0	0	0.09	0	0.88	0	10.86	0
$K = 2500$	0	0	0	0	0.03	0	1.92	0
$K = 2600$	0	0	0	0	0	0	0.22	0

Table 7: Verification of option price conditions (lower bound). All negative DIV are replaced by 0.

The upper bound implying that call option price is lower than the underlying price is also fulfilled. Taken this condition to the extreme, the price of a call option with $K = 1$ and maturity in September, 2022 amounts to $c = 2187.33$ (which is lower than $I_{t=}$ 2421.6).

5 Conclusions

The aim of this paper is to contribute to the analysis of the potential of land derivatives. Discussions on their establishment relates to the increased engagement of non-agricultural investors in the land market, as part of its financialization process. Consequences for farmers and thus, agricultural production include the increased challenge to purchase land. Land derivatives provide an instrument that permit non-agricultural investors to participate in the market without physically acquiring land. These financial instruments allow risks resulting from uncertain value development of agricultural land to be passed on to market participants, in return for a risk premium. Unknown willingness to participate by relevant actors, the incompleteness of the market and lacking an adequate pricing framework have prevented an in-depth exploration of the establishment of land derivatives. Against this background, this paper tackles one of the obstacles by focusing on the pricing of derivatives. Transparency of the price formation process for land derivatives is crucial for potential market participants.

This paper therefore contributes to the pricing and potential implementation of land derivatives with the use of data from the National Council of Real Estate Fiduciaries (NCREIF) Farmland Index. In this regard, reference can be made to the historical development of the market for real estate derivatives. Despite the differences between real estate market and land markets in term of size, supply and demand structures, both markets show structural similarities. We therefore apply a model developed to cope with the particularities of the broader real estate market to the agricultural land market. The model by Fabozzi et al. (2012) accounts for autoregressive characteristics of the underlying index, derives exact formulae for the derivative prices, and facilitates pricing by retrieving an exogenously applicable market price of risk and thereby completes the market. The model holds that the expected value of future returns of the derivative is not calculated on the basis of the actual process of the underlying index, but with the help of a corrected process to complete the market. This correction is made by the market price of risk, and thus, the implementation of this risk-neutral valuation approach requires its empirical estimation.

We apply data of the NCREIF Farmland Index to determine prices of futures and options. The empirical application centers around the determination of the market price of risk. It is calibrated using existing prices for one derivative and is then applied to price further derivatives on the same index. The main challenge is the lack of derivative prices on the NCREIF farmland index. We use data from one of its appraising actors (i.e. options on Gladstone land) to derive the market price of risk via reverse-engineering assuming that the market participants follow the suggested pricing framework.

The calibrated market price of risk of real estate and land markets lie within the same range. We find the market price of risk depending on the strike level as well as on the maturity of the option. Derivative prices are sensitive on the market price of risk. The calculated derivative prices obey usual no-arbitrage conditions. The option prices are not above the index value and not less than the intrinsic value. However, the results highlight the obstacles for the establishment of land derivatives: the pricing for the traded products is inhibited by missing information (the existence of prices of one derivative on the same underlying) to determine the market price of risk. Thus, the retrieval of land derivative prices is necessary, but their introduction becomes challenging because the data needed for the pricing model is not yet available. The direction of future research is twofold. Applying an alternative pricing model to the land market could provide a solution to the problem of missing market information. Further, the willingness to participate of potential market participants needs to be empirically inquired to assess whether trading will be realized.

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