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# Empirical estimates of the elasticity of substitution of a KLEM production function without nesting constraints: The case of the Variable Output Elasticity-Cobb Douglas

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## Abstract

The outcome of Computable General Equilibrium models applied to climate crucially rely on the estimation of elasticities of substitution. We use a generalized production function that overcomes the restriction imposed by a nesting structure of the Constant Elasticity of Substitution (CES) production function assumed in most CGE models. Constructing a panel of 44 countries and 14 periods from the World Input-Output Database (WIOD) tables, we estimate the production functions for 54 sectors using a *Seemingly Unrelated Regression* model. We compare these results to two standard KLEM nesting structures used in CES specification and find direct implications on the estimation results, especially for Capital-Energy substitutability. The more general form of the CES production function on which we rely, the Variable Output Elasticity-Cobb Douglas (VOE-CB) supports substitution between these two inputs.

Keywords: KLEM production functions, Substitution elasticities, Energy and capital substitutability, Nested CES

# 1 Introduction

Computable General Equilibrium (CGE) models are widely used by different institutions (public administrations, academia, think tanks, international organizations) to support policy evaluations and prospective analysis. They rely on a complex representation of the economic system, which allows for quantitatively determining through a numerical resolution the *ex-ante* effects resulting from an exogenous shock (e.g. a technical shock) or the implementation of a given policy (e.g. a carbon tax). The first empirically estimated macro-econometric model was constructed for the Dutch economy by Tinbergen in 1936 (Dhaene and Barten, 1989) and had opened a field of research in applied macroeconomics. A CGE model that combines dynamic effects with a multi-sectoral representation of the economy was first proposed by Johansen (1960) following the strand of Input-Output analysis on inter-branch relations developed by Leontief. Their application has been revived by the climate change threat and the need for the evaluation of the economic impacts of sustainable long-term decarbonization strategies (Böhringer and Löschel, 2006).

However, CGE models have often been criticized for the fact that their results are highly sensitive to the value of exogenous parameters whose estimation is uncertain. In the case of energy transition scenarios analysis, their results are highly contingent on the level of substitutability of energy with other inputs (Németh et al., 2011). Due to the limited data availability, modelers frequently use either macroeconomic estimation of elasticity of substitution or econometric estimations on micro-data for specific sectors. Either way, it induces a bias because these estimations are not consistent with the set of data employed for the CGE model's calibration or because they are based on a different functional form than the model's equation.

Jacoby et al. (2006) demonstrate this impact in their MIT EPPA model<sup>1</sup>: changing value of the elasticity of substitution between energy and non-energy commodities would dramatically change the costs of a mitigation policy case, the Kyoto protocol. The conclusions were similar regarding the rebound effect: the value of the elasticity directly impacts its magnitude (Jaccard and Bataille, 2000). The values of the elasticities of substitution in the production function play a central role in the dynamic of CGE models, especially regarding price-based instruments such as implementing carbon or energy taxes. Okagawa and Ban (2008) for instance, found that conventional parameter distribution could overestimate the carbon price required for a given targeted level of emissions reduction by 44%. Landa Rivera et al. (2016) using a CGE analysis and simulating an energy

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<sup>1</sup>The Emissions Prediction and Policy Analysis (EPPA) model is a recursive-dynamic multi-regional general equilibrium model of the world economy that is part of the MIT Integrated Global Systems Model (IGSM) simulating the human systems.

transition scenario with a carbon tax policy in Mexico, show that the change of the elasticity between capital and energy (from 1.5 to 0) leads to a 20% difference in GHG emissions reduction by 2050<sup>2</sup>.

Modeling communities have attempted to tackle this issue using econometric estimation of these parameters. Although, due to the limited data availability, empirical estimations of the parameters of the production function at a sectoral level are rather limited<sup>3</sup>. Another point of debate remains in the choice of the production function specification to conduct the econometric estimations. Relying on a CES has the advantage to be consistent with the macroeconomic theory but imposes important constraints on the possibility of substitutions between inputs. The Translog specification<sup>4</sup> popularity in the 1980's comes from its higher flexibility but it relies on a approximation of the production function by a second-order Taylor-expansion and the well behaved properties of the production function prove difficult to impose (Diewert and Wales, 1987; Ryan and Wales, 2000). Despite continuous works to provide selection criteria on the form to adopt, there is still no consensus in the research community on which specification of the production function to favor. The same is true regarding the nested-CES structure that fits data the most accurately.

In this study, we perform empirical estimations of elasticities of substitution for a KLEM<sup>5</sup> production function using Seemingly Unrelated Model (SUR) estimation procedures. More specifically, we use the VOE-CD specification as the standard case and test two alternative nested structures. The originality of this approach is twofold. First, we rely on an original and consistent panel dataset from the *WIOD 2016 Release* and from which all the variables (prices and quantities) used in the estimation are derived. Secondly, we introduce a new specification of the production function, which has not yet been tested in an empirical analysis.

The remainder of the paper is organized as follows. We first introduce the VOE-CD specification of the production function and derive the estimated equations. We then describe the dataset construction in a third section and the econometric strategy we apply in Section 4. In Section 5, we present our estimation results, where we discuss which nesting structure fits the dataset the best. Section 6 concludes and discusses policy implications.

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<sup>2</sup>This difference can be interpreted as the contribution of energy efficiency measures to the total variation of GHG emissions.

<sup>3</sup>The first econometric estimation of these parameters from input-output data was done by Burniaux et al. (1992) for the CGE model GREEN, using OECD data on a sample of 12 countries and seven industries.

<sup>4</sup>The translog function is based on a second-order linear approximation of production function and is characterized by input symmetry and Hicks neutrality.

<sup>5</sup>This acronym stands for the inputs considered separately into the production function where the Value-Added is decomposed between Capital (K) and Labor (L) and the intermediate consumption between Energy (E) Materials (M).

## 2 The model specification

A production function describes a process of transforming a quantity of inputs into a quantity of output. In CGE models, the Cobb-Douglas, CES, and the Translog functions are the primary functional forms used. The modeling of the producer's behavior generally relies on three main assumptions:

- The firm produces only one output
- The production function is homogeneous of degree one meaning that the returns to scale are constant
- The substitutability between production inputs is limited

The CES production function introduced by Solow (1956) and formalized by Arrow et al. (1961) has become widely used in the CGE modeling community. It has the advantage to allow for representing a continuum of substitution possibilities between the inputs, from the Leontief production function where the Elasticity of Substitution (ES) is 0 (strict complementarity), to the linear production function where the ES is infinite (perfect substitution). The Cobb-Douglas function (unitary ES) is also a particular case of the CES function. However, the CES function limits the possibilities of substitution. As its name says, it imposes a constant ES along the isoquant. As shown by Uzawa (1962) and McFadden (1963), it constraints the elasticity to be equal across every pair of inputs, which may prove very limiting in the case of more than two inputs. To circumvent these limits, Sato (1967) proposed a nested form of the function. For instance, in a case with three inputs ( $X_1, X_2, X_3$ ), a system of nested CES function can be written:

$$Y = \left( \alpha X_1^{\frac{\eta_{X_1,Z}-1}{\eta_{X_1,Z}}} + (1-\alpha) Z^{\frac{\eta_{X_1,Z}-1}{\eta_{X_1,Z}}} \right)^{\frac{\eta_{X_1,Z}}{\eta_{X_1,Z}-1}} \quad (1)$$

$$Z = \left( \beta X_2^{\frac{\eta_{X_2,X_3}-1}{\eta_{X_2,X_3}}} + (1-\beta) X_3^{\frac{\eta_{X_2,X_3}-1}{\eta_{X_2,X_3}}} \right)^{\frac{\eta_{X_2,X_3}}{\eta_{X_2,X_3}-1}} \quad (2)$$

Equation (1) states that Input  $X_1$  is substitutable to the composite input  $Z$  in the production of output  $Y$  with an ES of  $\eta_{X_1,Z}$  whereas equation (2) states that  $X_1$  and  $X_2$  are two substitutable inputs in the production of the composite input  $Z$ .

Although this approach has been widely used in the literature, it suffers from some criticisms. As argued by van der Werf (2008), there is no theoretical reason to favor a nested structure upon another one. The choice of the nested structure is therefore left to the modelers' discretion. In one of the earliest work on this literature, Prywes (1986) on US manufacturing industries assumed a three-level-CES production function with a  $((KE)L)M$ <sup>6</sup> nested structure without providing theoretical nor empirical justifications.

Several studies attempt to provide approaches to determine the correct nested structure. In a study on Germany manufacturing sectors, Kemfert (1998) proposed a data-driven approach to discriminate between different nested structure. The strategy she uses is to estimate the different combinations of nested structure and select the model with the highest  $R^2$  statistics. However, using this criteria appears to be statistically inadequate to compare non-linear models since it assumes that the underlying model being fit is linear (Spiess and Neumeyer, 2010; Lagomarsino, 2020). Despite becoming popular in the CGE literature, it has also be questioned by some authors of this field who argue that in the case of an indirect method based on conditional factor demand is not recommended because the final comparison is made between models based on different dependent and explanatory variables (Baccianti, 2013; Dissou et al., 2015).

Zha and Zhou (2014) insert a Translog specification into the two-level CES production function to select the most appropriate nested structure. Similarly, Lagomarsino (2020) proposed in a meta-analysis on the nested-CES production function to proceed through the use of a Translog specification of each nested structure. A Wald-test on the separability and homogeneity assumption for each Translog informs if the nested-model is rejected or not statistically.

In a recent study on CGE models in China, Feng and Zhang (2018) surveyed the nesting structure of their production function specification and find that the  $((KL)E)$  form has been mostly preferred in 75% of the cases, the  $((KE)L)$  nest being chosen three times and  $((EL)K)$  none. However, the choice is rarely motivated.

Some authors argued against taking the value-added variable as a composite variable from K and L to in the upper-level combine with E (referred to as a  $((KL)E)$  whereas others claimed to adopt a  $((KE)L)$  structure. This may reflect two visions of the functioning of the economic. The first one favors the income approach by combining Capital & Labor to form a added-value input in the production process. The second one put emphasis on the physical relation between Capital (equipments) & Energy in the production process <sup>7</sup>.

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<sup>6</sup>For this case and for the following of this paper, the brackets represent the organisation of the nest. In this case,  $K$  and  $E$  are combined to produce  $KE$ ,  $KE$  is combined with  $L$  to produce  $KEL$ , and  $KEL$  is combined with  $M$  to produce the output  $Y$ .

<sup>7</sup>At the microeconomic level, it is generally considered a Leontief production function between these two production

To overcome this limit, we take advantage of the VOE-CD specification of the production function (Reynès, 2019). It is a flexible form of the Cobb-Douglas production function, which provides a generalization of the CES functional form to the case where the Elasticity of Substitution (ES) between each pair of inputs is not equal. In this sense, it exhibits properties that are well-suited to the case of the multi-factors CES production function without assuming a specific nesting structure.

## 2.1 The VOE Cobb-Douglas function

Considering a general production function where output  $Y$  is produced from a combination of input  $X_i$  such as:

$$Y = Y(X_1, X_2, \dots, X_i, \dots, X_n) \quad (3)$$

and for which the standard assumptions apply: the production function is a continuous, twice differentiable function that is homogeneous of degree one; the output is increasing in inputs ( $Y'(X_i) = \frac{\partial Y}{\partial X_i} > 0$ ) and strictly concave ( $Y''(X_i) = \frac{\partial^2 Y}{\partial X_i^2} < 0$ )

Using the Euler theorem, Reynès (2019) shows that equation (3) can be written in growth rate<sup>8</sup> (or similarly in logarithm first difference).

$$\dot{Y} = \sum_i \varphi_i \dot{X}_i \leftrightarrow d \ln Y = \sum_i \varphi_i d \ln X_i \quad (4)$$

where  $\varphi_i$  is the output elasticity, which measures a relative change in output induced by a relative change in input  $i$ . It is defined according to the following equation:

$$\varphi_i = \left[ \sum_j \frac{Y'(X_j) X_j}{Y'(X_i) X_i} \right]^{-1} \quad (5)$$

The definition of the ES proposed by Hicks (1932) and Robinson (1933) measures the change in the ratio between two factors of production ( $i$  and  $j$ ) due to a change in their relative marginal productivity. Formally this writes:

$$-\eta_{ij} = \frac{d \ln(X_i/X_j)}{d \ln(Y'(X_i)/Y'(X_j))} \leftrightarrow \dot{X}_i - \dot{X}_j = -\eta_{ij} (\dot{Y}'(X_i) - \dot{Y}'(X_j)) \quad (6)$$

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factors.

<sup>8</sup>The first and second partial derivatives of the function  $Y$  with respect to  $X_i$  are respectively  $Y'(X_i) = \partial Y / \partial X_i$  and  $Y''(X_i) = \partial^2 Y / \partial X_i^2$ . Variables in growth rate are referred to as  $\dot{X} = dX/X = d(\ln X)/dX$ . All parameters written in Greek letter are positive.

Using the profit maximization behavior from the producer, we can derive the demand function by minimizing the production cost (7).

$$C = \sum_i P_i^X X_i \quad (7)$$

From the first-order conditions, the ratio between the marginal productivities of two inputs equals the ratio between prices ( $Y'(X_i)/Y'(X_j) = P_i^X/P_j^X$ ). Combining the first-order conditions with equation (5), the OE of input  $i$  corresponds to the cost share of input  $i$ :

$$\varphi_i = \frac{P_i^X X_i}{\sum_j P_j^X X_j} \quad (8)$$

Finally, combining the first-order conditions, the definition of the ES (6) and the production function (4) gives the demand function for each factor as a positive function of the output and a negative function of the relative prices between inputs:

$$\dot{X}_i = \dot{Y} - \sum_{j=1} \eta_{i,j} \varphi_j (\dot{P}_i^X - \dot{P}_j^X) \quad (9)$$

### 3 Data

Our econometric estimation is based on panel data that takes advantage of the combination of cross-section and time series data. This allows for considering a more apparent distinction between input substitution and technological change than time-series (Baccianti, 2013).

The ES estimation requires both having prices and quantities for all the different economic variables used in the economic regression. For the construction of the final database, we use the following data sources:

- WIOD Socio-Economic Account (WIOD SEA)
- WIOD National Supply-Use Tables (NIOT)
- WIOD World Input-Output Tables (WIOT)

These data sets belong to the World Input-Output Database Project (WIOD) (Timmer et al., 2015) which is a consistent regional input-output dataset with a detailed sectoral granularity of the world economy. In its latest version (2016 Release), the dataset covers the period from 2000 to 2014 and distinguishes 42 countries (plus the rest of the world) and 56 sectors (see Table 1 in Appendix A). Since the WIOD tables are both provided in current prices (*CP*) and previous year prices (*PYP*), we can distinguish, for each variable, value (in current price) and volume (nominal price) using the chained-price method<sup>9</sup>. The examples of other panel data sources employed in the literature include Eurostat's National Accounts and COMEXT (Németh et al., 2011), the IEA Energy Balances and the OECD International Sectoral Database (Saito, 2004; van der Werf, 2008) as well as the OECD International Trade by Commodities Statistics and the OECD Input-Output Database (Sato, 2014).

From the WIOT dataset, we extract for each sector their aggregate intermediate consumption of energy goods<sup>10</sup> and non-energy goods.

The price growth rate of the input  $X$  used in the sector  $i$  is computed according to the following equation:

$$\dot{P}_{i,t}^X = \frac{X_{i,t}^{CP}}{X_{i,t}^{PYP}} - 1 = \frac{X_{i,t} P_{i,t}^X}{X_{i,t} P_{i,t-1}^X} - 1 = \frac{P_{i,t}^X}{P_{i,t-1}^X} - 1 \quad (10)$$

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<sup>9</sup>In previous studies on the estimation of the ES of a KLEM production function, authors used the 2013 release of WIOD, which do not provide previous year prices national accounts. They, therefore, adopted an alternative source of data to construct the price series (see Baccianti (2013); Koesler and Schymura (2015); Antoszewski (2019)).

<sup>10</sup>The intermediate energy consumption aggregates the *Manufacture of coke and refined petroleum products* (C19) and *Electricity, gas, steam and air conditioning supply* (D35).

And taking a unitary value for the price at the base year ( $2000 = 1$ ) allows us to calculate the price index :

$$P_{i,t}^X = \prod_t (1 + \dot{P}_{i,t}^X) P_{i,0}^X \quad (11)$$

Finally, using it as a deflator on the input series expressed in current price allows for expressing these series in real terms:

$$X_{i,t} = \frac{X_{i,t}^{CP}}{P_{i,t}^X} \quad (12)$$

The capital and labor price and volume series are constructed from the WIOD SEA database following series: Total hours worked by employees (in millions) ( $H\_EMPE$ ), compensation of employees<sup>11</sup> ( $COMP$ ), capital compensation ( $CAP$ ) and nominal capital stock ( $K^{VAL}$ ). By default, the series are expressed in nominal value and in national currencies<sup>12</sup>.

Dividing  $COMP$  by  $H\_EMPE$  gives the hourly wage  $W$  for each period, country, and sector. We then compute the labor economic volume variable  $L$ , as the total work expressed in hours multiplied by the hourly wage base year value  $W_{i,0}$

$$L_{i,t} = W_{i,0} H\_EMPE_{i,t} \quad (13)$$

The price-variation of labor  $\dot{P}_{i,t}^L$  is directly derived from the wage growth rate ( $W_{i,t}/W_{i,t-1} - 1$ ), from which we directly derive the labor price index.

$$P_{i,t}^L = \prod_{t=1} (1 + \dot{P}_{i,t}^L) \quad (14)$$

Regarding the distinction between quantities and prices for capital, we can not use the same approach as for labor because there is no variable expressed in volume in the dataset that would allow for calculating a price deflator. To estimate the volume of capital stock we use the standard approach of the Perpetual Inventory Method (PIM) which consists in deriving the capital stock from data on investment flows<sup>13</sup> The capital accumulation equation can be written in value or volume metrics:

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<sup>11</sup>The WIOD SEA database provides an alternative metric for labor compensation ( $LAB$ ) that we did not consider because it includes self-employed workers.

<sup>12</sup>We convert the economic values in \$ currency using the same exchange rates table used in WIOD to construct the international Supply-Use Tables.

<sup>13</sup>Some authors, such as Lee (2005) and (Soytas and Sari, 2007) use directly investment data as a proxy for capital stock. This approach underestimates the capital stock since it does not take into account the lifespan of capital.

$$K_{i,t}^{VAL} = K_{i,t-1}^{VAL} (1 - \delta_{i,t}) + P_{i,t}^I I_{i,t} \quad (15)$$

$$K_{i,t} = K_{i,t-1} (1 - \delta_{i,t}) + I_{i,t} \quad (16)$$

Inverting equation (15) and using the definition of the growth rate of capital,  $\dot{K}_{i,t}^{VAL} = \frac{K_{i,t}^{VAL}}{K_{i,t-1}^{VAL}} - 1$ , allows for deriving a relation for the depreciation rate:

$$\delta_{i,t} = \frac{P_{i,t}^I I_{i,t}}{K_{i,t-1}^{VAL}} - \dot{K}_{i,t}^{VAL} \quad (17)$$

This equation is used to derive the depreciation ratio from the WIOD database which contains time series for the capital stock and investment in value. The depreciation rate is then used to estimate the capital stock in volume thanks to equation (16). Finally taking the nominal to real capital stock ratio provides capital price index.

$$P_{i,t}^K = \frac{K_{i,t}^{VAL}}{K_{i,t}} \quad (18)$$

As in Antoszewski (2019), this price index will be used as a proxy for the cost of the capital input. This specification has the advantage of simplicity. Its main drawback is that it does not account for the opportunity cost related to investment. The specification of the cost of capital remains however controversial (Jorgenson and Griliches, 1967; Hall and Jorgenson, 1969; Hudson and Jorgenson, 1974; Levinsohn and Petrin, 2003; Collard-Wexler and Loecker, 2016) in the literature for several reasons among which the difficulties to distinguish between physical and financial capital or between the user cost, opportunity cost or desired rate of return. This issue goes largely beyond the scope of this paper which is to investigate the impact of the nest structure on the estimation of the ES. Therefore we keep the impact of alternative specifications of the cost of capital for further research.

The final panel dataset gathers the following variables in volume ( $Y, K, L, E, M$ ) and prices ( $pY, pK, pL, pE, pM$ ). We also compute their respective growth rates ( $\dot{Y}, \dot{K}, \dot{L}, \dot{E}, \dot{M}, p\dot{Y}, p\dot{K}, p\dot{L}, p\dot{E}, p\dot{M}$ ) from which we perform the econometric estimations presented in the next section.

## 4 Econometric strategy

Our empirical analysis considers a four inputs production function, often known as KLEM : Capital (K), Labor (L), Energy (E) and non-energy intermediate inputs (M). The parameters of the function to estimate are determined for each sector  $s$  specified in the WIOD database. In the panel, we distinguish 13 periods  $t$  and 44 countries or regions  $r$ . Three approaches have been proposed in the literature to estimate a nested CES production function. The direct approach based on its non-linear estimation, the indirect approach based on a cost minimization program, and the approximation based on its Kmenta linearization.

The direct approach consists in using non-linear least squares estimation based on *ad-hoc* non-linear optimization algorithms<sup>14</sup>. However, their use is intricate because of the need to find a proper starting value to achieve a numerical convergence<sup>15</sup>. Since the CES production function is not-linear in its parameters, it implies that their values cannot be directly estimated with standard OLS estimator.

The indirect approach has been often used to estimate nested CES production function (Prywes, 1986; Okagawa and Ban, 2008; Antoszewski, 2019). It relies on the assumption of the maximizing behavior of the supply-side (either through a cost-minimization or a profit-maximization problem), and therefore involves collection of data on prices, besides quantities data.

An alternative approach to non-linear estimation is the one proposed by Kmenta (1967). It uses a linear approximation of the CES function to estimate its parameters. More specifically, this approximation is a linear Taylor series expansion when the ES is around 1. The outcome is a restricted form of the general Translog function. This method has been criticized by Thursby and Lovell (1978) arguing that Kmenta approximation only converges to the underlying CES function in region of convergence which is determined by the true parameters of the CES function. For these reasons, the linearization method proposed by Kmenta was rarely chosen<sup>16</sup>.

The specification we are going to test is derived from the demand function for factor determined by the VOE-Cobb Douglas production function as stated in 9. Since the economic framework assumes constant return to scale, and to avoid endogeneity in the estimation, we are going to take as explained variable the difference between the growth rates of input  $j$  and output  $Y$ . We also

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<sup>14</sup>A version of this algorithm developed by Henningsen and Henningsen (2012) has been made available for empirical applications in the package *micEconCES*.

<sup>15</sup>According to Henningsen and Henningsen (2012), results obtained through this method should be taken with caution, since they were not able to replicate the original findings from the (Kempfert, 1998) article adopting this approach.

<sup>16</sup>Koesler and Schymura (2015) compare the estimates obtained from the Kmenta approximation with a non-linear estimation and conclude that the former performs less well in terms of statistics fit.

consider time and country fixed effects. We regress our model on a sub-panel independently defined for each sector. Regarding our strategy, we want to take into account the advantage of the general form of the VOE-CD to perform a regression on the system of equations that defines the production process.

We adopt a Seemingly Unrelated Regression (SUR) approach originally developed by Zellner (1962) and extended to panel data analysis by Avery (1977) and Baltagi (1980). This allows for accounting for potential correlation between the errors from the different equations of the system. Moreover, in order to take into account the assumption of symmetry of the ES between inputs ( $\eta_{ij} = \eta_{ji}$ ), we have to impose cross-constraints restriction of the system of equations (19). Having derived the inputs demand (9) for a system of four inputs, the estimated system is:

$$\left\{ \begin{array}{l} \dot{K}_{r,t} - \dot{Y}_{r,t} = \alpha^K + \eta^{K,L} \varphi_{r,t-1}^L (\dot{P}_{r,t}^K - \dot{P}_{r,t}^L) + \eta^{K,E} \varphi_{r,t-1}^E (\dot{P}_{r,t}^K - \dot{P}_{r,t}^E) + \\ \quad \eta^{K,M} \varphi_{r,t-1}^M (\dot{P}_{r,t}^K - \dot{P}_{r,t}^M) + \mu_t^K + \mu_r^K + \epsilon_{r,t}^K \\ \dot{L}_{r,t} - \dot{Y}_{r,t} = \alpha^L + \eta^{L,K} \varphi_{r,t-1}^K (\dot{P}_{r,t}^L - \dot{P}_{r,t}^K) + \eta^{L,E} \varphi_{r,t-1}^E (\dot{P}_{r,t}^L - \dot{P}_{r,t}^E) + \\ \quad \eta^{L,M} \varphi_{r,t-1}^M (\dot{P}_{r,t}^L - \dot{P}_{r,t}^M) + \mu_t^L + \mu_r^L + \epsilon_{r,t}^L \\ \dot{E}_{r,t} - \dot{Y}_{r,t} = \alpha^E + \eta^{E,K} \varphi_{r,t-1}^K (\dot{P}_{r,t}^E - \dot{P}_{r,t}^K) + \eta^{E,L} \varphi_{r,t-1}^L (\dot{P}_{r,t}^E - \dot{P}_{r,t}^L) + \\ \quad \eta^{E,M} \varphi_{r,t-1}^M (\dot{P}_{r,t}^E - \dot{P}_{r,t}^M) + \mu_t^E + \mu_r^E + \epsilon_{r,t}^E \\ \dot{M}_{r,t} - \dot{Y}_{r,t} = \alpha^M + \eta^{M,K} \varphi_{r,t-1}^K (\dot{P}_{r,t}^M - \dot{P}_{r,t}^K) + \eta^{M,L} \varphi_{r,t-1}^L (\dot{P}_{r,t}^M - \dot{P}_{r,t}^L) + \\ \quad \eta^{M,E} \varphi_{r,t-1}^E (\dot{P}_{r,t}^M - \dot{P}_{r,t}^E) + \mu_t^M + \mu_r^M + \epsilon_{r,t}^M \end{array} \right. \quad (19)$$

The system of equation (19) is solved for each sector  $s$  based on 2408 observations. The parameter  $\alpha^i$  is the constant,  $\eta_{ij}$  are the elasticities of substitution between input  $i$  and  $j$ .  $\mu_r^M$  and  $\mu_t^M$  are respectively the country and the time fixed-effect terms,  $\epsilon_{r,t}$  is the error term. The input shares that intervene with a time lag in system (19) to avoid endogeneity bias are computed according to equation (8):

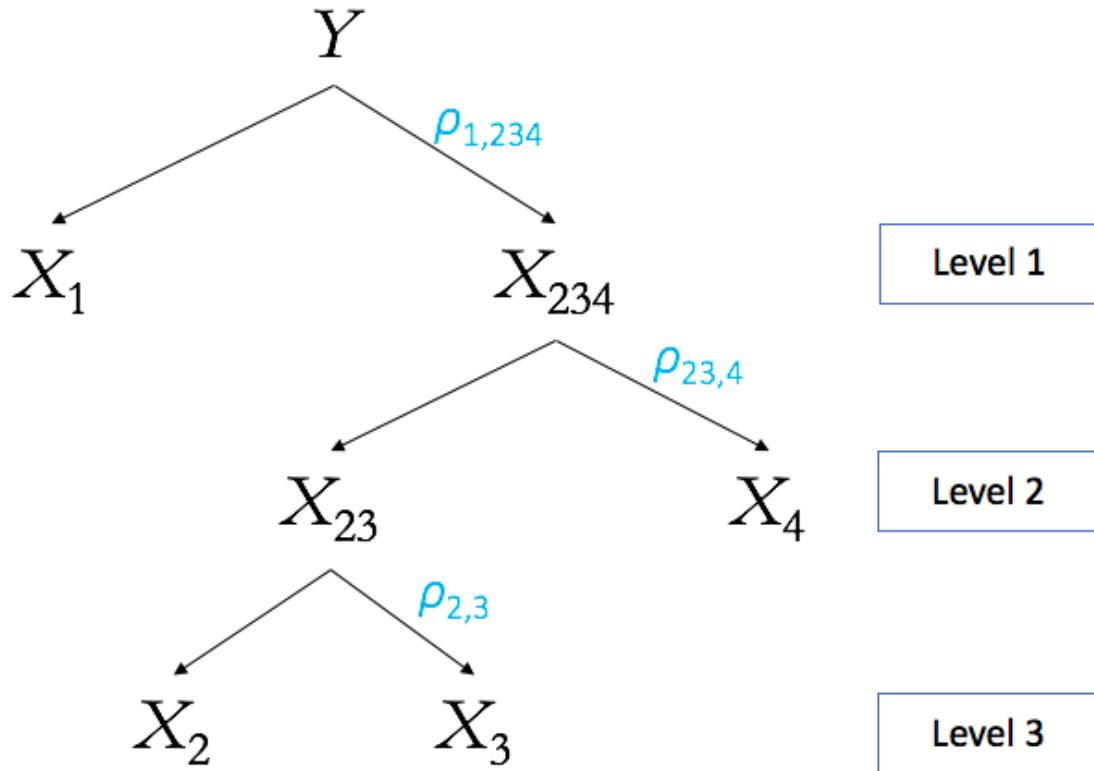
$$\varphi_{r,t}^X = \frac{P_{r,t}^X X_{r,t}}{\sum_j P_{r,t}^X X_{r,t}} \quad (20)$$

#### 4.1 The nesting structure

The VOE-CD being a generalization of the CES function, it also encompasses nested CES structures. We can therefore use to test the ES estimation is consistent with a nested CES structure. To do so, we adopt a standard three level nested structure of the type  $(X_1((X_2 X_3) X_4))$  as shown on Figure

1)

Figure 1: Nesting structure of a four inputs CES production function



Writing  $\varphi_{k'}^k$  the share of the input  $k$  into the output  $k'$ , ( $k'$  being either the final output  $Y$  at the first level of the nest or a composite input of production for lower levels)<sup>17</sup> , the model can be reformulated as follows :

<sup>17</sup>For the second level of the nested production function, the share of the composite good  $X_{23}$  into to the output  $X_{234}$  is  $\varphi_{234}^{23} = (1 - \varphi_{234}^4)$ .

$$\left\{ \begin{array}{l} \dot{X}_1 = \dot{Y} + \rho^{1,234} \varphi_Y^{234} (\dot{P}^{X_1} - \dot{P}^{X_{234}}) \\ \dot{X}_{234} = \dot{Y} + \rho^{234,1} \varphi_Y^1 (\dot{P}^{X_{234}} - \dot{P}^{X_1}) \\ \dot{X}_2 = \dot{X}_{23} + \rho^{2,3} \varphi_{23}^3 (\dot{P}^{X_2} - \dot{P}^{X_3}) \\ \dot{X}_3 = \dot{X}_{23} + \rho^{3,2} \varphi_{23}^2 (\dot{P}^{X_3} - \dot{P}^{X_2}) \\ \dot{X}_4 = \dot{X}_{234} + \rho^{4,234} \varphi_{234}^{23} (\dot{P}^{X_4} - \dot{P}^{X_{23}}) \\ \dot{X}_{23} = \dot{X}_{234} + \rho^{4,234} \varphi_{234}^4 (\dot{P}^{X_{23}} - \dot{P}^{X_4}) \end{array} \right. \quad (21)$$

We confront the two most used nesting structure of the production function in the literature. The first nesting is of the form  $((KL)E)M$ , which considers the value-added as a meaningful economic variable in relation with the intermediate inputs. The alternative case  $((KE)L)M$  sees the Capital-Energy relation as essential since it is based on engineering observations of a productive capital functioning. The  $((KL)E)M$  nesting has been adopted in several articles (Okagawa and Ban, 2008; Koesler and Schymura, 2015; Antoszewski, 2019)) whereas the  $((KE)L)M$  nesting has been preferred by others<sup>18</sup> (Prywes, 1986; Chang, 1994).

The system of equations we estimate is defined as follow:

$$\left\{ \begin{array}{l} \dot{X}_1 - \dot{Y} = \alpha^{X_1} + \rho^{X_1, X_{234}} \varphi_Y^{X_{234}} (\dot{P}^{X_1} - \dot{P}^{X_{234}}) + \mu_t^{X_1} + \mu_r^{X_1} + \epsilon_{r,t}^{X_1} \\ \dot{X}_{234} - \dot{Y} = \alpha^{X_{234}} + \rho^{X_1, X_{234}} \varphi_Y^{X_1} (\dot{P}^{X_{234}} - \dot{P}^{X_1}) + \mu_t^{X_{234}} + \mu_r^{X_{234}} + \epsilon_{r,t}^{X_{234}} \\ \dot{X}_2 - \dot{X}_{234} = \alpha^{X_2} + \rho^{X_2, X_{34}} \varphi_{X_{234}}^{X_{34}} (\dot{P}^{X_2} - \dot{P}^{X_{34}}) + \mu_t^{X_2} + \mu_r^{X_2} + \epsilon_{r,t}^{X_2} \\ \dot{X}_{34} - \dot{X}_{234} = \alpha^{X_{34}} + \rho^{X_{34}, X_2} \varphi_{X_{234}}^{X_2} (\dot{P}^{X_{34}} - \dot{P}^{X_2}) + \mu_t^{X_{34}} + \mu_r^{X_{34}} + \epsilon_{r,t}^{X_{34}} \\ \dot{X}_3 - \dot{X}_{34} = \alpha^{X_3} + \rho^{X_3, X_4} \varphi_{X_{34}}^{X_4} (\dot{P}^{X_3} - \dot{P}^{X_4}) + \mu_t^{X_3} + \mu_r^{X_3} + \epsilon_{r,t}^{X_3} \\ \dot{X}_4 - \dot{X}_{34} = \alpha^{X_4} + \rho^{X_4, X_3} \varphi_{X_{34}}^{X_3} (\dot{P}^{X_4} - \dot{P}^{X_3}) + \mu_t^{X_4} + \mu_r^{X_4} + \epsilon_{r,t}^{X_4} \end{array} \right. \quad (22)$$

Regarding the system of equations (22), this leads to  $(X_1 = M; X_2 = E; X_3 = L; X_4 = K)$  in the first case  $((KL)E)M$  and to  $(X_1 = M; X_2 = L; X_3 = E; X_4 = K)$  in the second one  $((KE)L)M$ .

By developing the system (21), we can derive the explicit production factors demand as in the system (19). We can also write the ES between each pair of inputs implicitly defined by the system (21),  $\eta$  being a function of the ES  $\rho$  estimated in the nested specification of the production function. Extending the system of equation (21) by replacing the composite inputs, it leads to the explicit

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<sup>18</sup>In a three inputs-case  $(K; L; E)$ , we notice that the preferences are more oriented towards the  $((KE)L)$  form (Feng and Zhang, 2018; Kemfert, 1998).

formulation of each input as in (9). This yields after simplifying the equations<sup>19</sup> the relation between the ES of the form:

$$\left\{ \begin{array}{l} \eta_{1,2} = \eta_{1,3} = \eta_{1,4} = \rho_{1,234} \\ \eta_{2,3} = \frac{\rho_{2,3}}{1 - \varphi_1 - \varphi_4} - \frac{\rho_{1,234} \varphi_1}{1 - \varphi_1} - \frac{\rho_{23,4} \varphi_4}{(1 - \varphi_1)(1 - \varphi_1 - \varphi_4)} \\ \eta_{2,4} = \eta_{3,4} = \frac{\rho_{23,4} - \rho_{1,234} \varphi_1}{1 - \varphi_1} \end{array} \right. \quad (23)$$

To be noted that in this case, we still have three constrained values ( $\eta_{1,3}$ ,  $\eta_{1,4}$  and  $\eta_{3,4}$ ) with respect to the VOE-CD general case. We also consider for their computation the average shares  $\varphi$  on the whole period covered by the data panel.

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<sup>19</sup>For a full demonstration see Reynès (2019).

## 5 Results

In this section, we expose the econometric results for three cases of structure of the production function: two constraints cases and the unconstrained case. Then we use them to calibrate a CGE model and simulate the impact of a carbon tax policy depending on the estimated production structure.

### 5.1 Estimation results

In order to facilitate the interpretation considering the number of sectors, we present the results graphically (see Figure 2). The detailed estimation tables are provided in Appendix B for the three cases.

The nesting structure has important implications since it leads to two opposite diagnostics regarding the substitution between capital and energy. In the ( $KL(E(M))$ ) case where Energy is a direct substitute to the Value-added component in the production function, the econometric estimation finds that the ES between capital and energy is positive in a majority of sectors (thirty out of fifty-four). This indicates a strong complementary between these two inputs (see Prywes (1986)). In the ( $KE(L(M))$ ) case, capital and energy are on the contrary diagnosed as strong substitute: forty-three sectoral estimations out of fifty-four have a negative ES, and among them, thirty-three with an absolute value greater than 1. Regarding at the average of sectors, the ES between Capital and Energy is 0,20 in the ( $KL(E(M))$ ) case against -1,75 in the ( $KE(L(M))$ ) case. This is a clear contrast with the Labor-Capital ES estimation where the results are consistent across the specifications (the average Labor-Capital ES is -0.31 for the case ( $KL(E(M))$ ) and - 0,27 for the case ( $KE(L(M))$ )). This indicates a specificity of the capital-energy relationship. When considered as direct substitutes, they are strongly substitutable and when they are integrated into a composite input, they are strongly complementary.

The more general unconstrained VOE case rather indicates substitutability between capital and energy. Out of the 31 sectors providing significant results, 29 sectors have a negative elasticity<sup>20</sup>. For results significant at a 99% level<sup>21</sup>, we find an average ES between Capital and Energy of -0,76 (resp. -0.5 for the median ES), of -0,83 (resp. -0,85) between Capital and Materials of -0,48

<sup>20</sup>Out of the three sectors showing complementarity J61 = Telecommunications; C27 = Manufacture of electrical equipment and R\_S = Other services activities), two of them are related to electrical equipment, suggesting a sectoral feature in terms of capital and energy use.

<sup>21</sup>After excluding an outlier: the values estimated for the sector E37-E39 (Sewerage; waste collection, treatment; materials recovery and other waste management services) are in absolute terms higher than 10 for two ES, suggesting misspecification of a data issue.

(resp. -0,42) between Labor and Energy, of -1,65 (resp. -1,31) between Labor and Materials of -0,80 (resp. -0,65) and of -2,37 (resp. -1,72) between Energy and Materials. The results confirm recent findings in the literature. Based on an empirical analysis of the production function using a Translog specification as a benchmark, Lagomarsino and Turner (2017) conclude that a (KE(L(M))) nested structure is the most appropriate form.

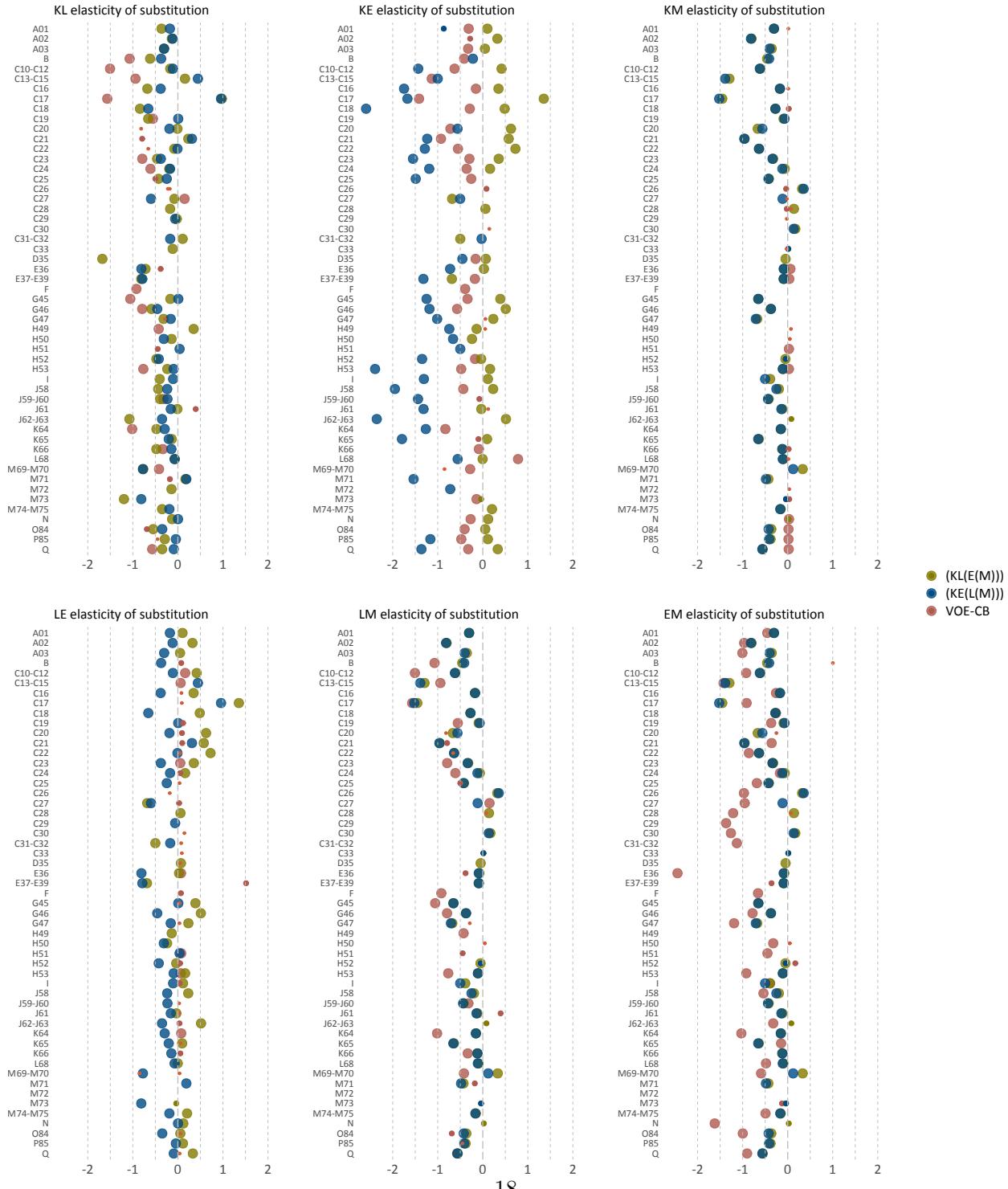
Another point to raise is the differences in estimation that brings the VOE specification concerning the nested specifications. Indeed due to the restrictions imposed by the constraints on the ES estimations (see equation 23), non energy-inputs (M) are considered less substitutable with the other inputs than in the VOE case<sup>22</sup>

Without further statistical tests, it remains tedious to assess the superiority of a specification to another from an empirical point of view. However, imposing a nesting structure necessarily induces more constraints on the estimation. In the case of the *KM*, *LM*, and *EM* ES, it seems that these restrictions can even be misleading. If it matches pretty well the estimations from the VOE for the *KM* substitutability, the findings suggesting a complementarity between capital and energy for most of the sectors remains questionable.

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<sup>22</sup>In the alternative cases, Materials are substitute to the composite input (KL)E) or (KE)L)).

Figure 2: Estimation of the elasticities



## 5.2 Simulations

As stated in the introduction, results from simulations conducted on CGE are sensitive to the distribution of the exogenous parameters, including the elasticities of substitution. In this part, we will mobilize the CGE model ThreeME to conduct a sensitivity analysis regarding the distribution of these parameters on the aggregate and sectoral results. The model ThreeME is a dynamic CGE model characterized by neo-keynesian features. It allows for sub-optimal equilibria and therefore transition phases before reaching a long-term steady state equilibrium (see details in Appendix C). We take as the baseline a 17 sectors version of the model<sup>23</sup>, calibrated on the NAF nomenclature, compatible with the NACE Rev2.1 EU nomenclature (and therefore WIOD), which has been used to estimate the impact of the COVID restrictions on the french economy (Malliet et al., 2020).

Our ES estimations is based on the WIOD sectoral disaggregation. They need to be adjusted to match the NAF nomenclature sector disaggregation used in ThreeME. For each sector of the NAF nomenclature, the ES are calculated as the weighted average of the estimated ES from WIOD data using the production weight from WIOD. The distribution is provided in the Figure 3

Following the Quinet commission report (Quinet, 2019), we assume a carbon tax trajectory increasing by 250 EUR in 2030, by 500 EUR in 2040, and 775 EUR in 2050. We consider a neutral carbon tax scenario with no monetary transfers between households and firms: proceeds of the carbon tax paid by households are redistributed to them, while each sector receives a share of the carbon tax paid by the private sector proportional to its share of total employment. This mode of allocation is favorable to labor-intensive sectors.

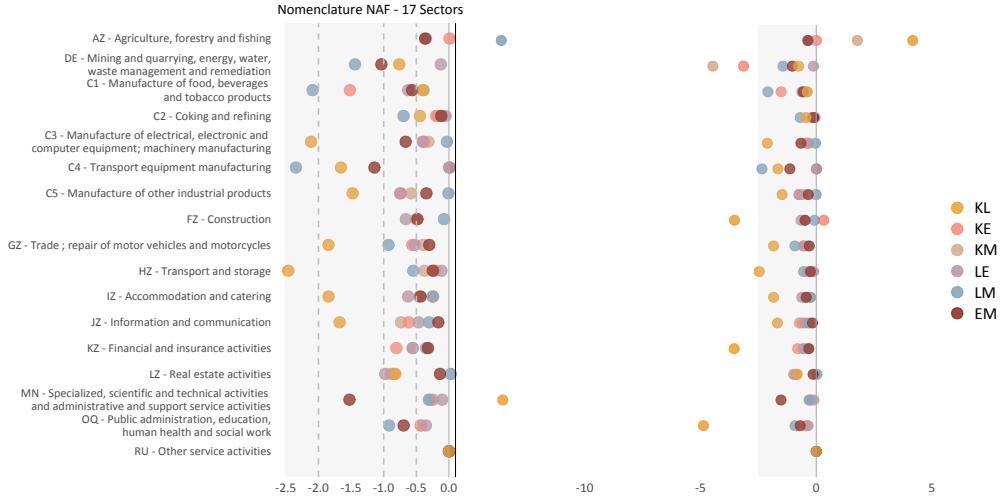
We compare four simulations of this scenario where only the value of the elasticities of substitution is altered. The first one with an aggregate elasticity of substitution calibrated to  $-0.5$  represents a relative inelastic case, a second one to  $-1$  corresponds to a Cobb-Douglas specification of the production function. The third one with  $-2$  states an elastic version of the production function, and in the last one, we report the results obtained from the econometric regression and calibrated on the 17 sectors. The results are reported in the Figure 4 in relative deviation to the baseline scenario (where no carbon tax policy is implemented).

From a macroeconomic point of view, the scenario with the VOE-CD estimation does not appear as an outlier. It evolves in the same range as the ad-hoc elasticities scenarios with a long-term effect between the the  $ES : -2$  and  $ES : -0.5$  scenarios. Regarding the GDP, we observe a positive increase by 2050 of 0.24% (the amplitude of the deviation is slightly under the Cobb-Douglas scenario for which the impact is of 0.47%). From a general point of view, we can see that the results

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<sup>23</sup>The source code can be retrieved from the github repository: [https://github.com/fosem/ThreeME\\_V3-open](https://github.com/fosem/ThreeME_V3-open).

Figure 3: Elasticities for the NAF nomenclature



Note: The gray area corresponds to a zoom of the distribution of the elasticities between  $-2.5$  and  $0$  values/ The dashed lines corresponds to the assumptions made for the alternative scenarios for the values of the elasticities of substitution.

are strongly related to ES assumption since the results cover a broad amplitude. At the end year of the simulation, 2050, we find a range from  $-0.20\%$  for the scenario  $ES : -0.5$ , to  $+1.15\%$  for the scenario  $E : -2$ ).

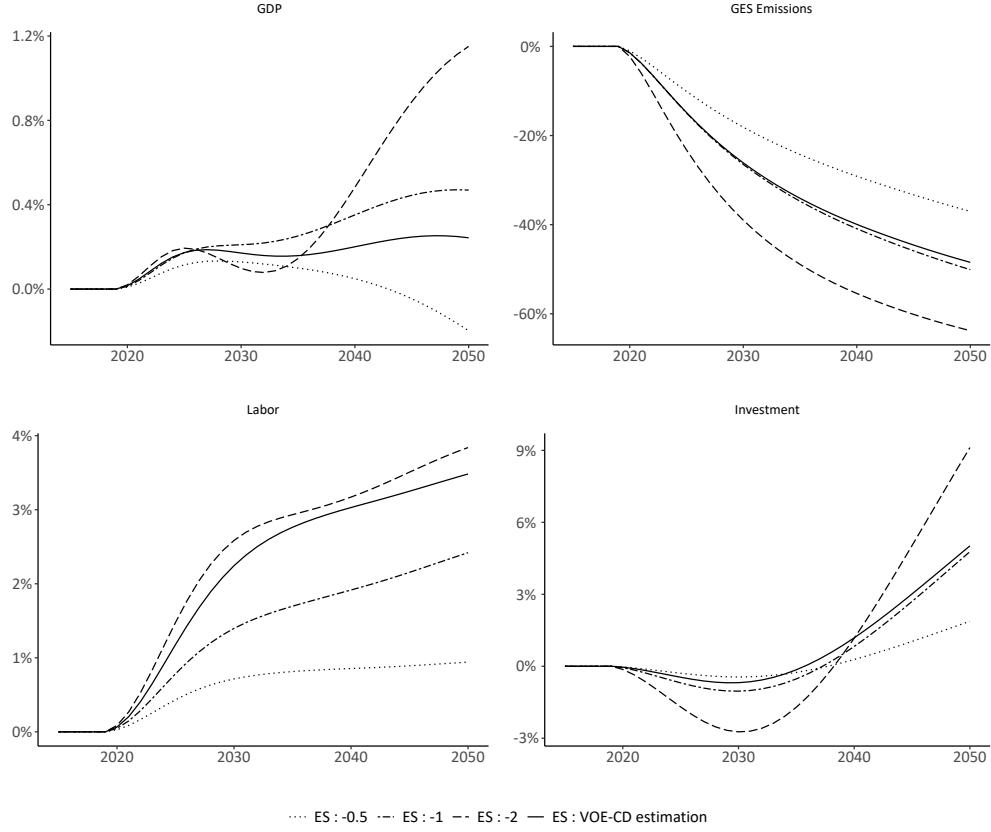
For the more elastic case, we can see a downturn on the GDP trajectory from 2025 to 2032, which corresponds to a similar shrinking of investments onto the same period before catching up and reaching by 2050 a  $1.15\%$  increase with respect to the baseline. In the most inelastic case (ES:  $-0.5$ ), the GDP % deviation remains small compared to the others and leads to a long-run negative impact with a  $0.2\%$  deviation by 2050.

The dynamics induced on the labor market are pretty straightforwards as well. The more substitutable the inputs, the larger the impact on employment<sup>24</sup>. It reaches by 2050 a positive deviation of  $0.9\%$  for the scenario  $ES : -0.5$ ,  $2.4\%$  for the intermediate case  $ES : -1$ , and  $3.8\%$  for the scenario  $ES : -2$ . The estimated elasticities scenario appears to follow the same dynamic as the latest with a long-term impact of  $3.5\%$ .

Finally, the overall impact on emissions ranges from  $-63\%$  to  $-37\%$  by 2050, the lowest reduction being associated with the scenario  $ES : -0.5$  and the highest to the scenario  $ES : -2$ . The more

<sup>24</sup>To be noted that the recycling scheme plays a central role in the direction of the results. Other recycling scheme would not necessarily lead to a positive effect on employment.

Figure 4: Simulations of a carbon tax policy for each distribution of parameters



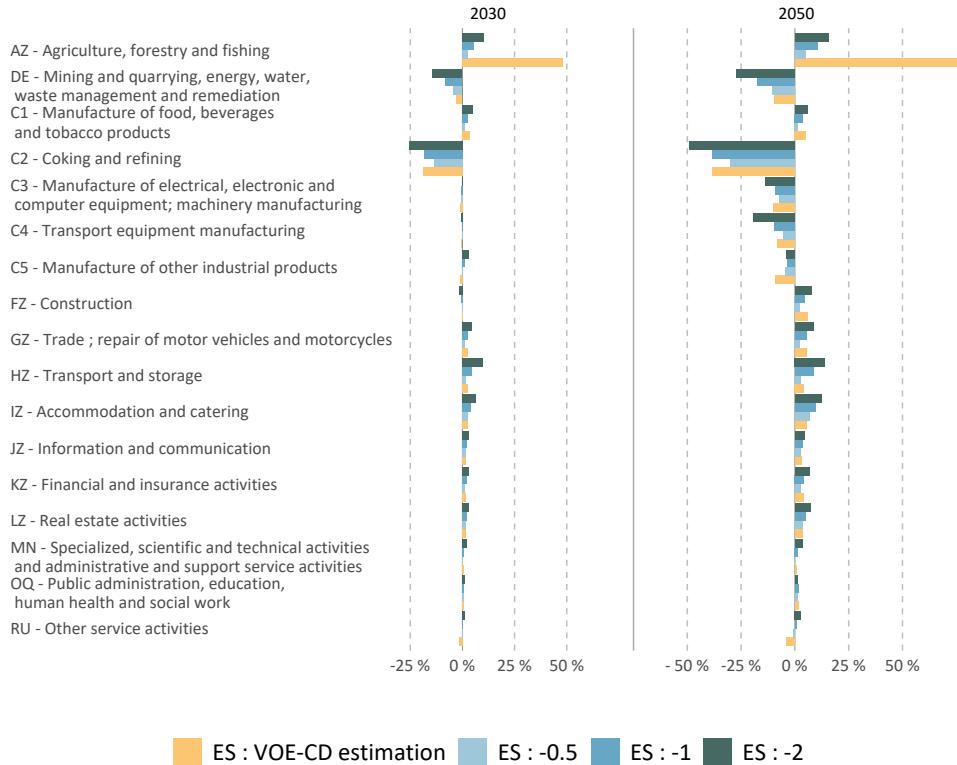
Note: Simulations conducted with the model ThreeME

substitutable the inputs, the lower the fossil fuel energy demand. The scenario with the estimated ES leads to a similar reduction of emissions than in the  $ES : -1$  case, with a relative deviation by 2050 equals to  $-48.5\%$  (resp.  $-50.1\%$ ). It can be seen as the direct consequence of the values for ES between Capital and Energy, which for some sectors are closer to  $-2$  than to  $-1$ <sup>25</sup>

Breaking down at the sectoral level (see Figure 5 and looking at the value added variable, the results from the estimated elasticities deliver the same conclusions as for the aggregate indicators. The effects dwell within the same range as the macroeconomic indicators, except for the agricultural sector where the variation of labor use is much larger. This is the direct consequence of the estimation of the elasticity between materials and labor, which is equal to  $-15.2$  for this sector. This

<sup>25</sup>The estimations for the ES between Energy and Labor though are not as much elastic since the results are distributed between 0 and  $-1$ .

Figure 5: Value added in % deviation wrt baseline for two selected years



Note: Simulations conducted with the model ThreeME

raises questions on the data quality for this sector since such a value is an outlier. It leads to a long-term variation of value added and employment of more than 60%, which is much greater than for other scenarios.

## 6 Conclusion

We contribute to the empirical literature on substitutions between production factors by proposing the first econometric estimation of the VOE-CD specification. Moreover, we construct an original panel dataset derived from the WIOD database, one of the most used sources for CGE analysis. We then proceed to estimate of the ES between KLEM inputs for 54 economic sectors. We evaluate and compare three specifications of the production function, among which two main forms of nested CES production function, namely  $((KL)E)M$  and  $((KE)L)M$ . We obtain highly significant estimation results for most of the sectors. A comparison of the different specifications allows for deriving three main conclusions:

- By imposing constraints on the estimations, the form of the nest has important implications on the estimated results.
- The Capital-Energy substitution behavior especially is highly dependent on the nest structure since it leads to opposing conclusions: either substitution or complementarity depending on the nest structure's choice.
- The VOE specification supports substitutability between these two factors of production, suggesting that the  $((KE)L)M$  nest may be closer to the reality.

These results shed some light on the Capital-Energy controversy initiated by the opposite results on the value of the ES between these two inputs (on the one hand Berndt and Christensen (1973) finding complementarity and on the other hand Griffin and Gregory (1976) finding substitutability) and which has not been thoroughly answered yet. The VOE specification appears as a relevant, flexible, functional form of the production function. It has the advantage of linear tractability while relaxing the constraint imposed by the CES production function. It is, therefore, a relevant alternative for CGE models. When applied to energy and carbon policy evaluations, the VOE CD function shows that the nest's choice appears to impact the econometric results critically.

To investigate the implications of these estimations on the simulation results of a CGE model, we perform a sensitivity analysis regarding the level of the ES. Including the values estimated econometrically, we compare them to 3 standard cases of ES. Our results confirm the crucial role of the distribution of the ES parameters on the results of CGE conducted simulations. Regarding the estimated values we obtained from the econometric regressions and compared with a Cobb-Douglas specification of the production function (i.e., with an ES equal to -1), they indicate relatively lower substitutability between energy and capital, leading to fewer emissions reduction. On the other

hand, the elasticities between labor and energy in the majority of the sectors are lower than -1. The implications on the simulation results are paramount. It sketches a more labor-intensive substitution effect from the carbon tax policy than what could be expected in the Cobb-Douglas case but associated with an equivalent reduction in the emissions.

These results could be further investigated in several directions. A first lead would be to compare them with those estimated from another flexible production function such as the Translog. This would allow for disentangling the respective role of the data and of the estimated specification on the results. Another possible investigation is the impact of the indicators used on the estimated elasticity level. For instance, the definition of the capital stock use may have an impact on the results. Investigation on the original dataset should also be carried on since they result from a necessary transformation process of raw data from statistical institutes that can be a source of estimation bias, especially at the sector level where the observations. Nonetheless, the implications of the calibration of the elasticities are critical in terms of effect and cannot be ignored or neglected. The data stringency argument that has been raised in the past to justify the use of ad-hoc values appears no longer valid, and the development of econometric studies for CGE modeling should be more systematized.

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## 6.1 Appendix A

Table 1: Estimation results-VOE production function

Sectors	$\eta^{KL}$	$\eta^{KE}$	$\eta^{KM}$	$\eta^{LE}$	$\eta^{LM}$	$\eta^{EM}$
A01	-0.389*** (0.053)	-0.283*** (0.464)	-0.294*** (0.035)	0.908 (4.47)	-0.165*** (0.133)	-2.875** (2.063)
A02	-0.563** (0.081)	-3.32*** (1.961)	-1.053*** (0.158)	7.606 (9.618)	-0.71*** (0.129)	12.087 (6.17)
A03	-0.635*** (0.092)	-1.407 (0.748)	-0.475*** (0.092)	3.548 (4.065)	-0.6*** (0.196)	2.904** (2.569)
B	-0.428*** (0.067)	-0.465 (0.067)	-0.668*** (0.083)	-0.492*** (0.16)	-0.216*** (0.274)	-0.911*** (0.172)
C10-C12	-0.459*** (0.053)	-0.287*** (0.423)	-0.572*** (0.064)	-5.753 (2.076)	-0.808*** (0.101)	-0.456** (1.201)
C13-C15	-0.433*** (0.064)	-0.769*** (0.162)	-0.61*** (0.091)	-0.724*** (0.714)	-0.793*** (0.126)	-1.074 (0.352)
C16	-0.712*** (0.098)	-0.497*** (0.535)	-0.331*** (0.048)	1.08 (2.516)	-0.772*** (0.129)	-0.844** (0.681)
C17	-0.474*** (0.075)	0.165*** (0.189)	-0.572*** (0.053)	-1.466 (1.468)	-1.084*** (0.113)	-2.654 (0.361)
C18	-0.733*** (0.088)	-0.125*** (0.366)	-0.52*** (0.067)	-0.046 (1.717)	-1.078*** (0.121)	-1.047* (1.041)
C19	-1.022*** (0.135)	0.224* (0.207)	-0.234*** (0.076)	0.103 (0.457)	-0.108** (0.159)	0.163*** (0.074)
C20	-1.488** (0.209)	-0.478** (0.255)	-0.348*** (0.07)	0.554 (1.662)	-0.119*** (0.299)	-1.146 (0.432)
C21	-0.393*** (0.092)	-0.21* (0.406)	-0.142*** (0.129)	-1.4 (1.45)	-1.161 (0.136)	-2.47 (0.548)
C22	-0.392*** (0.098)	-0.875*** (0.254)	-0.588*** (0.056)	-0.903*** (1.043)	-1.027*** (0.106)	-1.621 (0.415)
C23	-0.457*** (0.087)	-1.062*** (0.309)	-0.453*** (0.048)	1.514*** (1.602)	-0.796*** (0.127)	-1.524 (0.552)
C24	-0.483*** (0.102)	-0.313*** (0.243)	-0.465*** (0.05)	-6.222 (1.914)	-0.579*** (0.092)	-1.085 (0.387)
C25	-0.692*** (0.077)	-0.321 (0.354)	-0.569*** (0.054)	1.159 (1.237)	-0.831*** (0.099)	-1.576* (0.336)
C26	0.129** (0.073)	0.008 (0.034)	0.356 (0.21)	-0.535 (0.607)	-1.102 (0.148)	-0.876 (0.486)
C27	1.023* (0.134)	0.203 (0.123)	4.612*** (0.45)	-0.043 (0.743)	-1.447*** (0.178)	-0.09 (0.488)
C28	0* (0.072)	-0.008 (0.03)	-0.001 (0.067)	-0.751 (0.638)	-1.033 (0.144)	-2.257 (0.672)
C29	1.798* (0.203)	0.455 (0.155)	5.127*** (0.499)	-1.88** (0.916)	-2.732*** (0.286)	-0.657 (0.953)
C30	4.666*** (0.469)	-0.062 (0.393)	5.551*** (0.55)	-5.147 (3.096)	-3.528*** (0.755)	2.161 (4.053)
C31-C32	0.019 (0.049)	-0.108 (0.05)	0.032 (0.072)	-5.21* (0.464)	-1.39 (0.164)	-0.081 (0.398)
C33	-0.569* (0.049)	-0.718* (0.05)	-0.636** (0.072)	1.67 (0.464)	-0.565*** (0.164)	-1.563 (0.398)

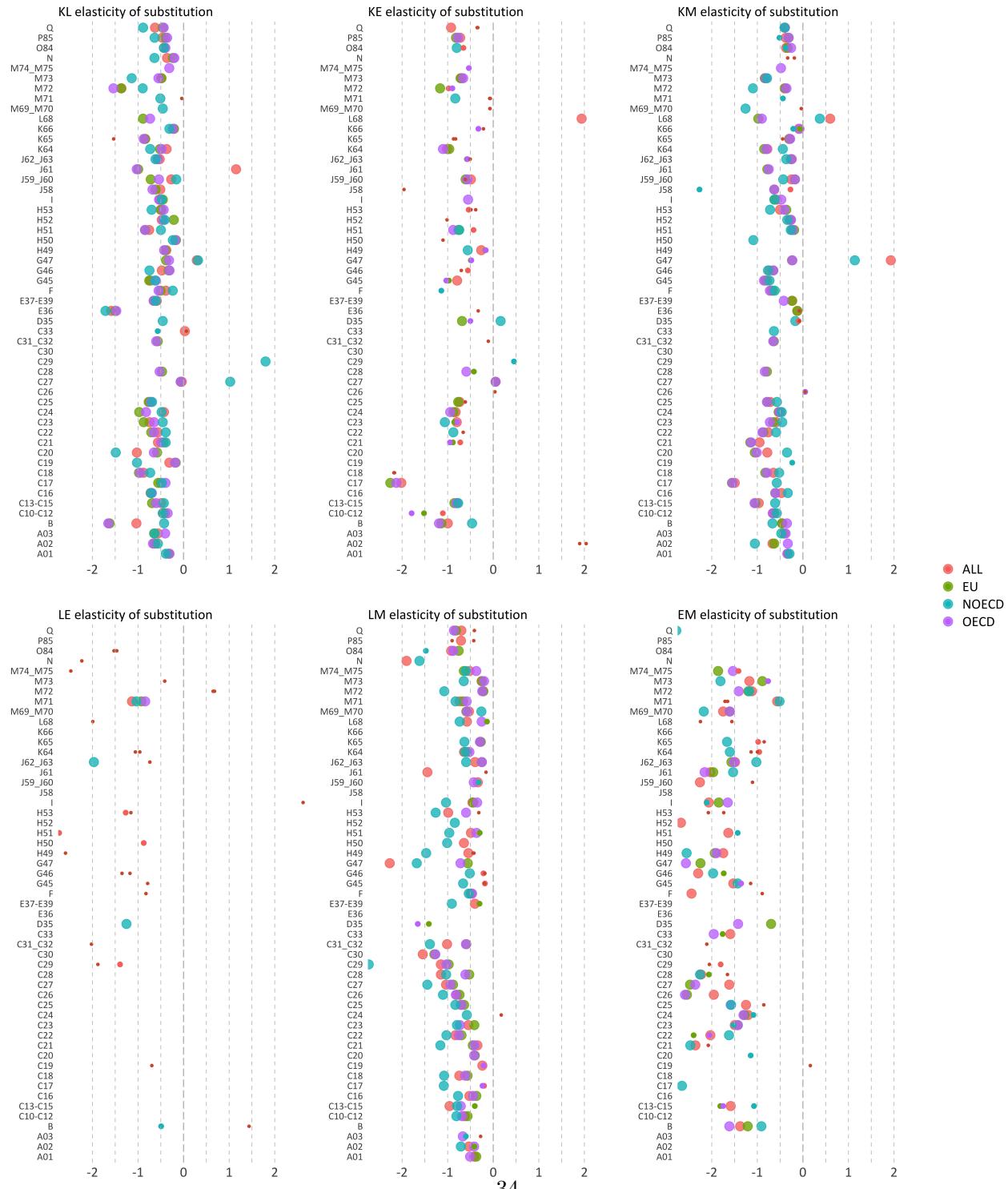
	(0.197)	(1.067)	(0.144)	(2.663)	(0.319)	(0.959)
D35	-0.457*** (0.058)	0.162*** (0.027)	-0.166*** (0.037)	-1.253*** (0.165)	-0.551*** (0.349)	0.128*** (0.066)
E36	-1.714*** (0.254)	-0.238 (0.423)	-0.09*** (0.102)	3.514 (5.291)	-1.913 (1.083)	-5.929*** (2.982)
E37-E39	-0.608*** (0.094)	0.204 (0.35)	-0.161*** (0.108)	-0.721 (1.009)	-0.909 (0.146)	-4.024*** (1.314)
F	-0.238*** (0.066)	-1.139*** (0.366)	-0.618*** (0.087)	-0.825** (0.408)	-0.537*** (0.082)	-0.892** (0.381)
G45	-0.627*** (0.067)	-0.395*** (0.295)	-0.733*** (0.059)	-0.788 (0.39)	-0.661*** (0.081)	-1.431 (0.388)
G46	-0.748*** (0.057)	-0.361*** (0.311)	-0.766*** (0.062)	-0.175 (0.441)	-0.514*** (0.086)	-1.971* (0.44)
G47	0.317*** (0.026)	0.047 (0.062)	1.139*** (0.061)	-1.717 (0.906)	-1.678*** (0.156)	-3.518 (0.588)
H49	-0.165*** (0.104)	-0.559 (0.158)	0.006 (0.048)	-2.591*** (1.288)	-1.472 (0.235)	-2.551*** (0.53)
H50	-0.236*** (0.065)	-1.102*** (0.451)	-1.086*** (0.198)	-0.017* (1.231)	-1.011*** (0.172)	-0.052* (1.206)
H51	-0.497*** (0.056)	-0.745*** (0.211)	-0.266*** (0.053)	2.122*** (1.267)	-0.964*** (0.143)	-1.431** (0.524)
H52	-0.416*** (0.101)	-1.014*** (0.477)	-0.343*** (0.101)	-1.263* (0.837)	-0.843*** (0.112)	-0.076 (0.531)
H53	-0.702*** (0.062)	-0.385*** (0.184)	-0.721*** (0.082)	-1.156* (0.495)	-1.263*** (0.193)	-1.737*** (0.882)
I	-0.478*** (0.072)	-0.315*** (0.479)	-0.623*** (0.09)	2.623 (1.066)	-1.036*** (0.122)	-2.109* (0.691)
J58	-0.316 (0.449)	-42.303 (76.828)	-2.269 (0.689)	67.411 (116.917)	-1.287** (0.929)	226.966 (369.362)
J59-J60	-0.161*** (0.038)	-0.613*** (0.258)	-0.432*** (0.055)	-0.209* (0.665)	-0.323*** (0.112)	-2.956 (0.819)
J61	5.31*** (0.371)	0.108** (0.186)	13.352*** (0.802)	-0.509 (0.61)	-3.814*** (0.334)	-1.532 (0.372)
J62-J63	-0.617*** (0.107)	-0.505** (0.204)	-0.366*** (0.071)	-1.968* (0.569)	-0.594*** (0.086)	-1.022 (0.202)
K64	-0.733*** (0.136)	0.31* (0.27)	-0.441*** (0.113)	-1.197 (0.628)	-0.619*** (0.172)	-1.602** (0.325)
K65	-1.534*** (0.669)	-2.938 (1.494)	-0.444* (0.188)	1.697 (1.051)	-0.633* (0.14)	-1.665 (0.426)
K66	-0.311*** (0.07)	-3.623** (2.207)	-0.212*** (0.067)	2.293 (53.546)	-2.847** (1.399)	6.318 (11.484)
L68	-0.027 (0.025)	0.221 (0.203)	0.371 (0.111)	-0.987 (0.816)	-0.736*** (0.066)	-2.249*** (1.123)
M69-M70	-0.457 (0.084)	-0.592*** (0.518)	-1.259*** (0.168)	0.088 (0.2)	-0.26*** (0.078)	-2.175** (0.612)
M71	-0.51 (0.041)	-0.833*** (0.189)	-0.438*** (0.145)	-1.034*** (0.295)	-0.826** (0.116)	-0.512** (0.134)
M72	-0.897*** (0.265)	0.717* (1.044)	-1.095*** (0.141)	1.414 (1.143)	-1.075*** (0.16)	-1.177** (0.263)
M73	-1.138*** (0.138)	0.25*** (0.25)	-0.786*** (0.444)	0.444 (0.444)	-0.646*** (0.444)	-1.809*** (0.444)

	(0.168)	(0.824)	(0.109)	(0.65)	(0.08)	(0.252)
M74-M75	-0.002	-0.027	-0.002	-2.183	-0.604	-2.37**
	(0.009)	(0.398)	(0.031)	(3.69)	(0.084)	(1.331)
N	-0.639***	0.097	-0.334***	-2.232	-1.619*	-4.493***
	(0.041)	(0.619)	(0.151)	(0.998)	(0.448)	(17.978)
O84	-0.432***	-0.8	-0.384***	-1.178***	-1.478**	7.859***
	(0.044)	(0.201)	(0.142)	(0.724)	(0.461)	(5.661)
P85	-0.635***	-0.461	-0.514***	-0.625	-0.902**	-4.957***
	(0.046)	(0.336)	(0.163)	(1.076)	(0.375)	(6.271)
Q	-0.888***	-0.353***	-0.405***	0.633*	-0.414***	-2.779***
	(0.061)	(0.164)	(0.082)	(0.602)	(0.205)	(0.683)

## 6.2 Appendix B

We perform the SUR regression on 4 different panels, each one composed of a different subset of countries to test the effect of the level of development on the results. The 27 member states of the European Union composed the panel *EU*, OECD countries are gather in the *OECD* group, and non-OECD countries into the *NOECD* group. *ALL* stands for the whole sample used in the main analysis. We do not observe stark differences on the general levels between the different panels on the estimation of the ES. Some sectoral specificity yields opposite results though, but that cannot be generalized. We notice however that Labor is relatively more substitutable to Energy in the non-OECD countries than in the OECD members countries. The level of development might be explaining this difference, since it would be easier to replace energy to manpower (or inversely) for some tasks (field labor, crafting, transportation...).

Figure 6: Estimation of the elasticities for different panels



Note: The size of the points indicate the level of significance of the estimations (big = 1%; medium = 5%; small = 10%).

### 6.3 Appendix C

The model ThreeME