Stochastic simulation with informed rotations of Gaussian quadratures

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Background

In the past decade, the increase of available computational power and speed has led simulation models, especial the ones addressing agricultural and environmental issues, to grow in levels of detail and complexity. Complexity can be represented by the number of parameters and variables that a model accounts for. By increasing the number of parameters and variables, however, we also increase the degree of uncertainty of the model associated with each parameter and variable. In order to account for this uncertainty, the application of uncertainty and sensitivity analysis techniques has become a standard modeling practice. According to the study by Douglas-Smith et al. (2020), the number of published journal articles in the field of environmental sciences that apply uncertainty or sensitivity analysis methods has increased seven times in the last two decades. Despite the increasing application of different uncertainty analysis tools, there is still a necessity of developing more efficient and accurate methods of uncertainty quantification in simulation models according to the same study.

The standard method of uncertainty/sensitivity analysis in the GTAP model is Gaussian Quadratures (GQ) (Arndt 1996). This method, compared to the other widely used methods such as Monte Carlo or LHS, requires very little computational resources. However, recently several studies (Artavia, Grethe, and Zimmermann 2015; Villoria and Preckel 2017; Stepanyan et al. 2018) have pointed out potential inaccuracies of approximations by this method. Stepanyan et al. (2019) presented a novel approach to uncertainty analysis called MRGQ that compared to the GQ method uses a higher number of iterations at the same time improving the quality of results. The MRGQ method was motivated by Artavia, Grethe, and Zimmermann (2015) showing that depending on the rotation of Stroud's octahedron the quality of the results produced by the GQ method often deviated largely compared to the benchmark. It was not clear, however, which factors were responsible for those deviations. Therefore, Stepanyan et al. (2019) suggested randomly rotating Stroud's octahedron several times and using the generated families of GQ points simultaneously which showed great improvement in the quality of approximations.

In this study, we have identified the factors affecting the quality of GQ results allowing to gain high-quality results using minimal computational capacity (2N points where N is the number of stochastic variables or parameters). We have built an LP model that given the base values of the stochastic parameters and the covariance matrix is able to identify the exact rotations of Stroud's octahedron that produce GQ points yielding to high-quality stochastic results. To avoid biases created by different models and covariance structures we have tested the proposed approach in one CGE and two PE models, namely a recursive-dynamic CGE model (IFPRI model) and two global PE models (GLOBIOM and ESIM).

Methods

Simulation Models and Data

The proposed efficient method of uncertainty analysis has been tested in 3 well-established economic simulation models addressing agricultural markets. Namely, we have used one CGE model: a recursive-dynamic CGE model (Diao and Thurlow 2012) and two global PE models: the GLOBIOM (Petr Havlik et al. 2011; P. Havlik et al. 2014), and the ESIM (Grethe et al. 2012). In all models, we have tested the uncertainty of agricultural yield[1]. For each model, we have generated a reliable benchmark using the LHS method with converged sample size, although this required enormous computational resources. In the case of the ESIM, we have taken the study by Artavia, Grethe, and Zimmermann (2015) who found that different rotations of Stroud's octahedron generated GQ points with heterogeneous quality but were unable to explain the exact factors affecting this quality. Using the same data and the model we demonstrate those factors and show that our method is able to identify the rotations that yield high-quality results, thus, solving the problem using minimal computational resources. The covariance matrices have been created using historical data from FAOSTAT.

Informed Rotations of Gaussian Quadratures

Each rotation of Stroud's octahedron generates GQ points with different dispositions. Our conjecture was that if the rotation generates points closely located to each other, the quality of approximations will be low. However, if the generated points lie far from each other but still within the region of integration, the quality will be much better. This could be explained by the fact that points lying further out are able to capture the region of integration much better which is in line with the finding by Preckel et al. (2011). To test our hypothesis we designed a linear programming model that given the base values of the stochastic parameters and the covariance matrices performs all the possible rotations (the number of all the possible rotations is equal to the factorial of the number of stochastic parameters, i.e. N!) and out of those rotations selects 10 good rotations and 10 bad rotations. In this way, we generated GQ points for each of our models and tested our hypothesis.

Results

The results from all 3 models demonstrated that our hypothesis is indeed correct and the rotations generating GQ points that lie close to each other yield to results with a large deviation from the benchmark. In contrary, the rotations generating GQ points lying further out but still within the integration region yield to fairly good quality results.

Conclusions

This study demonstrates the importance of informed selection of GQ points since in certain cases they might lead to approximations with low quality. To avoid this and further decrease the computational requirement of stochastic simulation modeling we designed a linear programming model that given the necessary data generates GQ points that produce high-quality results. The LP

[1] In the case of CGE models the total factor productivity parameters have been made stochastic.

is tested in 3 different simulation models with different covariance structures with respect to yield uncertainty. The model is able to identify those rotations that produce high quality as well as those producing low-quality results.

Keywords

Model validation, uncertainty analysis, systematic sensitivity analysis, CGE, PE, yield uncertainty.

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