Trade Conflicts Impact on Innovation

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Abstract

Rising popularity of isolationist policies and a more prominent role of geopolitical arguments in policy making have led political scientists to conjecture a possibility of globalization backsliding. We take that assessment as motivation to study the potential effects of increased and persistent trade conflicts on global economic growth and technological innovation. We do so by building a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion. Idea diffusion is mediated by the input-output structure of production, such that both sector cost shares and import trade shares characterize the source distribution of ideas. Using this framework, we explore the potential impact of a "decoupling of the global economy," a hypothetical scenario under which technology systems would diverge in the global economy. We divide the global economy in a U.S.-based bloc and a China-based bloc based on scores of geopolitical differences with the U.S. and China from the political science literature. We model decoupling through an increase in iceberg trade costs (full decoupling) or tariffs (tariff decoupling) between the U.S. bloc and the China bloc. Results yield three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, being as large as 15% in some regions. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

1 Introduction

A new wave of protectionism and trade conflicts loomed over international markets during the last decade. After decades of deepening in the international trade regime, diffused benefits and concentrated costs of globalization, mixed with sub-par policy response, may have prompted the beginning of a backlash. Recent empirical evidence highlights that local labor markets have adapted to economic shocks brought about by increased globalization

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very sluggishly and slow adaptation to trade shocks may have contributed to a rise in populist and isolationist parties to power[1].

Using these facts as motivation, this paper aims to estimate the potential effects of increased and persistent trade conflicts on economic growth and technological innovation. Some of the adverse effects of trade conflicts are well-known. Increased trade barriers decrease domestic welfare and gains from trade by shifting production away from the most cost-efficient producers and leaving households with a lower level of total consumption. Canonical trade models capture this *static result* through the fact that welfare is proportional to the degree of trade openness (Arkolakis, Costinot, and Rodríguez-Clare 2012)[2].

However, some of the main concerns of policymakers and practitioners regarding potentially detrimental effects of trade conflicts are abstracted away in standard models. For instance, these models typically assume a fixed technology distribution for domestic firms, thereby limiting gains from trade to static gains. This assumption renders it impossible to address some of the most important questions regarding the long-term consequences of continued trade conflict or receding globalization —namely, reduced technology and know-how spillovers that happen through trade.

A newer literature tries to overcome these limitations by incorporating knowledge diffusion that happens through trade over time. The earliest explorations of this topic go back to Eaton and S. Kortum (1999), who developed a multi-country dynamic model in which firms innovate by investing in research & development (R&D) and knowledge diffuses, after some lag, to other markets. In THEIR model, diffusion happened somewhat mechanically, was unrelated to trade and eventually reached all countries.

More recently, Alvarez et al. (2013) combined the Eaton and S. Kortum (2002) Ricardian model of trade with an idea diffusion process first presented by S. S. Kortum (1997). Importantly, the authors conjectured that the diffusion process is proportional to the quality of managers of firms whose product reach a given destination market. Ideas flow from one market to another in proportion to the trade linkages between them. Therefore, impediments to trade do not only have static, but also dynamic costs —as they decrease knowledge diffusion.

In order to realistically assess the impact of trade conflicts on global innovation, we build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion. As in Buera and Oberfield (2020), who generalized the aforementioned Alvarez et al. (2013) approach, we model the arrival of new ideas as a learning process from suppliers to a given country-sector. Through engaging in international markets, domestic innovators have access to new sources of ideas, whose quality depends on the productivity of the source country and sector.

In our model, idea diffusion is mediated by the input-output structure of production, such that both sectoral intermediate input cost shares and import trade shares characterize the source distribution of ideas. Innovation is summarized by describing productivity in different sectors as evolving according to a trade-share weighted-average of trade-partners productivities. This process is controlled by a parameter which determines the speed of

[1]For evidence of sluggish response of local labor markets to trade shocks, see analysis of the China Trade Shock in the United States by Autor, Dorn, and Hanson (2013) or trade liberalization in Brazil by Dix-Carneiro and Kovak (2017). For evidence of the impact of trade shocks on the rise of populist parties to power, see Colantone and Stanig (2018).

[2]This is true of all canonical trade models. As Arkolakis, Costinot, and Rodríguez-Clare (2012) show, Armington (1969), Krugman (1980), Eaton and S. Kortum (2002), and Melitz (2003), albeit different in motivation, are isomorphic and summarize gains from trade by some variation of the expression $\mathbb{G} \propto (\pi_{ii})^{\frac{1}{\varepsilon}}$, where π_{ii} is domestic trade share and ε is the elasticity of trade flows with respect to trade costs.

diffusion of ideas in that sector, which we calibrate using growth data.

Productivity thus evolves endogenously as a by-product to micro-founded market decisions —i.e., an externality that market agents affect with their behavior but do not take into account when making decisions. In this framework, a surge in global protectionism or bloc-specific trade conflicts will have spillover effects on the future path of sectoral productivities of all countries. Changes in trade costs induce trade diversion and creation, which, in turn, impact productivity dynamics in a way that is not anticipated or internalized by agents.

We contribute to the literature by incorporating the recent insights of the trade and innovation literature into a flexible multi-sector toolkit that permits assessing realistic trade policy experiments. Our model builds on the work that evaluates the impact of trade on innovation and shows that trade openness can increase the level of domestic innovation, building on the single-sector model of Buera and Oberfield (2020). Our model is also closely related to the one described by Santacreu, Li, and Cai (2017), who extended the original model by Eaton and S. Kortum (1999) incorporating lag-diffusion dynamics into a multi-sector framework.

After characterizing the model, we use it to perform policy experiments in the context of heightened global trade conflicts. We explore the potential impact of a "decoupling of the global economy," a hypothetical scenario under which technology systems would diverge in the global economy. We divide the global economy in a U.S.-based bloc and a China-based bloc based on scores of geopolitical differences with the U.S. and China from the political science literature.

We simulate increased trade costs arising from geopolitical circumstances, which increase frictions prohibitively if one country wants to trade with another one outside its bloc. Alternatively, we simulate an alternative scenario of a global increase in tariffs, in which all countries move from cooperative tariff setting in the context of the World Trade Organization (WTO) to non-cooperative tariff setting, which Nicita, Olarreaga, and Silva (2018) estimate to increase global tariffs, on average, by 32 percentage points. For simplicity, we use this average number as a reference and we assume that countries in different blocs raise tariffs against countries in the other bloc by such average amount.

This paper is organized as follows. First, we present the model, detailing production, demand, and consumption of the global economy. We also describe the dynamic evolution of productivities in different regions and sectors. Afterwards, we motivate our policy experiments and calibrations, and present the results of our main scenarios and also some alternative simulations. Finally, we conclude summarizing the key takeaways.

2 Environment

Time is discrete and indexed by $t \in \mathcal{T}$. There are $d \in \mathcal{D}$ regions in the global economy, which cover every part of the world economy, either as a stand-alone country, or a regional aggregate of countries. In each region, there are multiple industries $i \in \mathcal{I}$. Our model is characterized by a sequence of static equilibria. Given parameters and initial values for the endogenous variables, we solve for the changes in endogenous variables, and use them to compute end-of-period levels. .

2.1 Demand

In each region d and each period t a representative agent maximizes Cobb-Douglas preferences over private goods ($q^{pr}_{d,t}$) and government goods ($q^{go}_{d,t}$):

$$\begin{aligned}
\max_{\{q^m_{d,t}\}_{m\in\mathcal{M}}} \mathbb{U}_{d,t} &= \sum_{m\in\mathcal{M}} (q^m_{d,t})^{\kappa^m_d}, \\
s.t. \sum_{m\in\mathcal{M}} \kappa^m_d &= 1, \qquad \sum_{m\in\mathcal{M}} e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t}) \leq (1-s_{d,t})Y_{d,t} \\
Y_{d,t} &= \sum_{i\in\{h,l\}} w^i_{d,t}L^i_{d,t} + \rho_{d,t}k_{d,t} + F_{d,t} + T_{d,t} + \sum_{j\in\mathcal{I}} \Pi^j_{d,t}
\end{aligned}$$

where $\mathcal{M} = \{pr, go\}$ is the set of macro-sectors, $\mathbf{p}_{d,t} \equiv (p^{pr}_{d,t}, p^{go}_{d,t})$ is a vector of prices, and $e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t})$ are expenditure functions; $s_{d,t}$ the aggregate savings rate, which is assumed to be exogenous; $w^i_{d,t}, L^i_{d,t}$ are the wage and measure of workers of type $i \in \{h, u\}$, either high- or low-skilled; $\rho_{d,t}k_{d,t}$ is capital income; $F_{d,t}$ are other factor income; $T_{d,t}$ are transfers; and $\Pi^j_{d,t}$ are profits. Agents maximize utility subject an implicit budget constraint because, as it will become explicit below, we assume that preferences over private goods are non-homothetic.

Preferences over specific types of private, government, and savings goods follow a nested structure. Given optimal allocations and expenditure shares across the macro-sectors, agents allocate their expenditures following a sub-utility function that differs across macro-sectors.

Optimal expenditure functions satisfy:

$$e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t}) = \kappa^m_d Y_{d,t}\frac{\varphi^m_{d,t}}{\boldsymbol{\varphi}_{d,t}}$$

where $\varphi^m_{d,t} \equiv \frac{\partial q^m_{d,t}}{\partial e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t})}\frac{e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t})}{q^m_{d,t}}$ is the elasticity of consumption in macro-sector m with respect to expenditure on that sector; and $\boldsymbol{\varphi}_{d,t} \equiv \frac{\partial \mathbb{U}_{d,t}}{\partial Y_{d,t}}\frac{Y_{d,t}}{\mathbb{U}_{d,t}} = \sum_{n\in\mathcal{M}} \kappa^n_d \varphi^n_{d,t}$ is the elasticity of aggregate utility with respect with respect to total income.

For goods with homothetic preferences this elasticity is: $\varphi^{go}_{d,t} = 1$. If all goods had sub-utility functions with homothetic preferences, equation would generate the standard expression for Cobb-Douglas expenditure shares. With non-homothetic preferences for private goods, the share of spending on private goods is larger than the Cobb-Douglas parameter κ^{pr}_d if the elasticity of private quantity with respect to private expenditure ($\varphi^{pr}_{d,t}$) is larger than 1. This gives the consumer an incentive to spend a more than proportional amount on private goods.

We assume that the representative agent has non-homothetic preferences over different types of private goods. Following Comin, Lashkari, and Mestieri (2021), we adopt a non-homothetic CES preference structure. Let $i \in \mathcal{I}$ be industries of different private sector goods. Agents maximize the following preferences:

$$
\begin{aligned}
\max_{\{q_{d,t}^{pr,i}\}_{i\in\mathcal{I}}} \quad & q_{d,t}^{pr} \\
s.t. \quad & \sum_{i\in\mathcal{I}} [\Psi_i (q_{d,t}^{pr})^{\varepsilon_i}]^{\frac{1}{\sigma}} (q_{d,t}^{pr,i})^{\frac{\sigma-1}{\sigma}} = 1 \\
& \sum_{i\in\mathcal{I}} p_{d,t}^{pr,i} q_{d,t}^{pr,i} \leq e_{d,t}^{pr}
\end{aligned}
$$

where $q_{d,t}^{pr,i}$ is consumption of each private sector commodity indexed $i \in \mathcal{I}$; $q_{d,t}^{pr}$ is total aggregate private consumption; Ψ_i are preference shifters for each i; ε_i is a parameter that controls income elasticity of demand; and σ is a parameter that controls the elasticity of substitution. The following parametric restrictions are necessary: (1) $\sigma \in 0$ and $\sigma \neq 1$; (2) $\Psi_i > 0$ for all i; and (3) if $\sigma < 1$ ($\sigma > 1$), then $\varepsilon_i > 0$ ($\varepsilon_i < 0$) for all i.

As shown in the Appendix, the optimal demand function satisfies:

$$
q_{d,t}^{pr,i} = \Psi_i (q_{d,t}^{pr})^{\varepsilon_i} \left(\frac{p_{d,t}^{pr,i}}{e_{d,t}^{pr}(p_{d,t}^{pr,1}, \cdots, p_{d,t}^{pr,|\mathcal{I}|}; q_{d,t}^{pr})} \right)^{-\sigma}
$$

where $e_{d,t}^{pr}(p_{d,t}^{pr,1}, \cdots, p_{d,t}^{pr,|\mathcal{I}|}; q_{d,t}^{pr})$ is aggregate expenditure function in private goods, which depends on the total aggregate private consumption $q_{d,t}^{pr}$ and prices of each of $|\mathcal{I}|$ goods. As made explicit above, the parameter ε_i controls demand elasticity of good i with respect to total real expenditure.

The expenditure function $e_{d,t}^{pr}(p_{d,t}^{pr,1}, \cdots, p_{d,t}^{pr,|\mathcal{I}|}; q_{d,t}^{pr})$ can also be expressed as follows:

$$
e_{d,t}^{pr}(p_{d,t}^{pr,1}, \cdots, p_{d,t}^{pr,|\mathcal{I}|}; q_{d,t}^{pr}) \quad = \quad \left[\sum_{i\in\mathcal{I}} \Psi_i (q_{d,t}^{pr})^{\varepsilon_i} (p_{d,t}^{pr,i})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
$$

while expenditure for commodity i as a share of total private expenditures are:

$$
\eta_{d,t}^{pr,i} = \frac{\Psi_i (q_{d,t}^{pr})^{\varepsilon_i} (p_{d,t}^{pr,i})^{1-\sigma}}{\sum_{j\in\mathcal{I}} \Psi_j (q_{d,t}^{pr})^{\varepsilon_j} (p_{d,t}^{pr,j})^{1-\sigma}}
$$

Preferences for spending by the public sector across the different sectors are Cobb–Douglas, implying the following demand function for government goods in industry $i \in \mathcal{I}$:

$$
q_{d,t}^{go,i} = \kappa_d^{go,i} \Big(\frac{e_{d,t}^{go}}{p_{d,t}^{go}} \Big) \Big(\frac{p_{d,t}^{go,i}}{p_{d,t}^{go}} \Big)^{-1}
$$

where $\kappa_d^{go,i}$ is the Cobb-Douglas parameter in the sub-utility function over government goods $i \in \mathcal{I}$, $e_{d,t}^{go}/p_{d,t}^{go}$ is real expenditure in government goods and $p_{d,t}^{go,i}/p_{d,t}^{go}$ is the relative price of good i. The aggregate price for government goods satisfies:

$$
p_{d,t}^{go} = K^{go} \cdot \Pi_{i\in\mathcal{I}} (p_{d,t}^{go,i})^{\kappa_d^{go,i}}
$$

where $K^{go} = \Pi_{i\in\mathcal{I}} (\kappa_d^{go,i})^{-\kappa_d^{go,i}}$ is a constant.

2.2 Production

There are many producers of different varieties ω of each commodity i. Firms are endowed with an identical technology and combine factors of production $f^i_{d,t}$ and intermediate inputs $m^i_{d,t}$ to deliver varieties:

$$q^i_{d,t}(\omega) \quad = \quad z^i_{d,t}(\omega)\Big[(\Psi^{i,f}_{d,t})^{\frac{1}{\sigma_i}}(f^i_{d,t})^{\frac{\sigma_i-1}{\sigma_i}}+(\Psi^{i,m}_{d,t})^{\frac{1}{\sigma_i}}(m^i_{d,t})^{\frac{\sigma_i-1}{\sigma_i}}\Big]^{\frac{\sigma_i}{\sigma_i-1}} \tag{1}$$

where the cost of the unit input bundle is a function of the prices of factors of production and intermediate commodities:

$$c^i_{d,t}=\Big[\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\sigma_i}+\Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\sigma_i}\Big]^{\frac{1}{1-\sigma_i}}$$

Firms combine factors of production ($f^i_{d,t}$) and intermediate commodities $m^i_{d,t}$ are aggregated according to the following sub-production functions:

$$f^i_{d,t} \quad = \quad \Big[\sum_{\overline{f}\in\{k,h,u,t,n\}}(\Psi^{i,z}_{d,t})^{\frac{1}{\sigma_i}}\overline{f}_{d,t}^{\frac{\sigma_i-1}{\sigma_i}}\Big]^{\frac{\sigma_i}{\sigma_i-1}}$$

$$m^i_{d,t} \quad = \quad \Big[\sum_{j\in\mathcal{I}}\Psi^{i,j}_{d,t}{}^{\frac{1}{\sigma_i}}(q^j_{d,t})^{\frac{\sigma_i-1}{\sigma_i}}\Big]^{\frac{\sigma_i}{\sigma_i-1}}$$

The first aggregator combines capital ($k_{d,t}$), high-skilled labor ($h_{d,t}$), unskilled labor ($u_{d,t}$), land ($t_{d,t}$), and natural resources ($n_{d,t}$) as factors of production. The second aggregator one uses commodities $q^j_{d,t}$ as intermediate inputs.

In every sector $j\in\mathcal{I}$, a local producer of sectoral commodity $q^j_{d,t}$ sources the cheapest landed variety $q^i_{d,t}(\omega),\ \omega\in[0,1]$ from all countries $s\in\mathcal{D}$. It then aggregates it according to the following technology:

$$q^j_{d,t}=\Big[\int_{[0,1]}q^j_{d,t}(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\Big]^{\frac{\sigma}{\sigma-1}}$$

from where it follows that the price of commodity $j\in\mathcal{I}$ satisfies:

$$p^j_{d,t}=\Big[\int_{[0,1]}p^j_{d,t}(\omega)^{1-\sigma}d\omega\Big]^{\frac{1}{1-\sigma}}$$

2.3 Supply of Factors of Production

The supply of the five factors of production changes over time. Capital and labor are perfectly mobile between sectors and thus have a uniform price across sectors. Natural resources and land are sector specific and only serve as an input in the sector *primary resources* (`pri`).

For each country, there exists an exogenous path of labor endowments in both high-skilled and unskilled labor: $\{L^i_{d,t}\}_{t\in\mathcal{T},\ i\in\{h,u\}}\ \forall d\in\mathcal{D}$. As we will describe in the data section below, we use external projections by the United Nations and the International Monetary Fund to set forth labor paths.

The supply of natural resources ($r^i_{d,t}$) and land ($t^i_{d,t}$) are determined by an isoelastic supply function with $pt^i_{d,t}$ and $pr^i_{d,t}$ respectively the supply of land and natural resources:

$$\begin{aligned} r_{d,t} &= \overline{r_d}\,(pr_{d,t})^{\xi_r} \\ t_{d,t} &= \overline{t_d}\,(pt_{d,t})^{\xi_t} \end{aligned}$$

with $\xi_r, \xi_t \geq 0$. Aggregate capital is a function of capital in the previous period, depreciation, and investment, evolving according to the following law of motion:

$$k_{d,t} = (1 - \delta_d)k_{d,t-1} + q^{in}_{d,t} \tag{2}$$

There is an investment sector that combines different sectoral commodities $q^j_{d,t}$ with Leontief technology under perfect competition:

$$q^{in}_{d,t} = \min\left\{\frac{q^1_{d,t}}{\overline{\chi}^1_{d,t}}, \cdots, \frac{q^{|\mathcal{I}|}_{d,t}}{\overline{\chi}^{|\mathcal{I}|}_{d,t}}\right\}$$

Hence, the demand for any particular input i and the price of the aggregate investment good are, respectively:

$$\begin{aligned} q^i_{d,t} &= \overline{\chi}^i_{d,t} q^{in}_{d,t} \\ p^{in}_{d,t} &= \sum_{i \in \mathcal{I}} \overline{\chi}^i_{d,t} p^i_{d,t} \end{aligned}$$

We assume that the ratio of a region's trade balance to its total income is fixed. Abstracting from other components of the current account, the capital account is equal to the trade balance. Assuming a fixed trade balance ratio (relative to income) thus implies that the investment rate is equal to the savings rate.

We abstract from modelling intertemporal optimization and assume a constant savings rate. In equilibrium, it must be the case that:

$$p^{in}_{d,t} q^{in}_{d,t} = s_{d,t} Y_{d,t} \tag{3}$$

2.4 International trade

International trade happens both at the final goods market and through intermediate goods, as variety producers use the final good as intermediate inputs in the production process. Our model is inclusive of production, exports, import, and origin taxes, as well as transportation and trade costs. Let $x^i_{sd,t}(\omega)$ be the landed unit cost of supplying variety ω of commodity $i \in \mathcal{I}$ produced in source region $s \in \mathcal{D}$ and delivered to region $d \in \mathcal{D}$:

$$x^i_{sd,t}(\omega) = \frac{tm^i_{sd,t} \cdot \tau^i_{sd,t} \cdot c^i_{sd,t}}{z^i_{s,t}(\omega)} = \frac{\tilde{x}^i_{sd,t}}{z^i_{s,t}(\omega)}$$

where $tm^i_{sd,t}$ are gross import taxes, which can be source and destination specific; $\tau^i_{sd,t}$ are bilateral trade costs; and $z^i_{s,t}(\omega)$ is the firm's productivity. The last equality follows from defining $\tilde{x}^{m,i}_{sd,t}$ as the landed input bundle costs of final goods.

Since varieties can be sourced from every region $s \in \mathcal{D}$ consumers in destination region $d \in \mathcal{D}$ will only buy variety ω from the source with the lowest landed price. Following Bernard et al. (2003), producers engage in Bertrand competition. For each country, order firms $k = [1, 2, \cdots]$ such that $z^i_{1s,t}(\omega) > z^i_{1s,t}(\omega), \cdots$. If the lowest-cost provider of the variety ω to country $d \in \mathcal{D}$ is a producer from country $s \in \mathcal{D}$, the price in d satisfies:

$$p^i_{d,t}(\omega) = \min\left\{ \underbrace{\frac{\sigma}{\sigma-1}\frac{\tilde{x}^i_{sd,t}}{z^i_{1s,t}(\omega)}}_{\text{optimal monopolist price}}, \underbrace{\frac{\tilde{x}^i_{sd,t}}{z^i_{2s,t}(\omega)}}_{\substack{\text{MC of 2nd most}\\ \text{productive firm from } s}}, \min_{n\neq s} \underbrace{\frac{\tilde{x}^i_{nd,t}}{z^i_{1n,t}(\omega)}}_{\substack{\text{MC of most productive}\\ \text{firm from other countries}}} \right\} \tag{4}$$

Assumption 1 (Productivity draws)**.** *We follow the canonical Eaton and S. Kortum (2002) assumption that and take* $z^i_{s,t}(\omega) : \mathcal{A} \times \mathcal{D} \times \mathcal{I} \times \Omega \to \mathbb{R}_+$ *to be the realization of an i.i.d. random variable, where* $\mathcal{A}$ *is the set of states of the world. Productivity is distributed according to a Type II Extreme Value Distribution (Fréchet).*

$$F^i_{s,t}(z) = \exp\{-\lambda^i_{s,t} z^{-\theta_i}\} \tag{5}$$

The country-specific Fréchet distribution has a region-commodity-specific location parameter $\lambda^i_{s,t}$, which denotes absolute advantage (better draws for all varieties), and a common scale parameter θ_i, which governs comparative advantage (higher θ_i implies less variability in productivity and lower potential for diversification according to comparative advantage).

We show in the Appendix that prices in the destination region $d \in \mathcal{D}$ will be, respectively:

$$p^i_{d,t} \quad = \quad \mathbf{\Gamma}_1 (\Phi^i_{d,t})^{-\frac{1}{\theta_i}} \tag{6}$$

where $\mathbf{\Gamma}_1$ is a constant[3]; $\Phi^i_{d,t} \equiv \sum_{s\in\mathcal{D}} \lambda^i_{s,t}(\tilde{x}^i_{sd,t})^{-\theta_i}$; $\tilde{x}^i_{sd,t} \equiv [x^i_{s,t} \cdot tp^i_{s,t} \cdot tx^i_{sd,t} + \gamma^i_{sd,t} \cdot ps^i_{sd,t}] \cdot tm^i_{sd,t} \cdot \tau^i_{sd,t}$ is the landed input bundle costs of final goods.

As there are infinitely many varieties in the unit interval, by the law of large numbers, the expenditure share of destination region $d \in \mathcal{D}$ on goods coming from source country $s \in \mathcal{D}$ converges to their expected values. Let $\pi^i_{sd,t}$ denote the share of final goods expenditures of consumers in region $d \in \mathcal{D}$ on commodity $i \in \mathcal{I}$ coming from region $s \in \mathcal{D}$.

$$\pi^i_{sd,t} \equiv \frac{\lambda^i_{s,t}(\tilde{x}^i_{sd,t})^{-\theta_i}}{\Phi^i_{d,t}} \tag{7}$$

In the presence of Bertrand Competition, we show in the Appendix that source firms realize a profit which is proportional to the total expenditure of destination countries. In particular, profits are:

$$\Pi^i_{s,t} = \frac{1}{1+\theta_i} \sum_{d\in\mathcal{D}} \pi^i_{sd,t} e^i_{d,t} \tag{8}$$

[3]Specifically, $\mathbf{\Gamma}_1 \equiv \left[1 - \frac{\sigma-1}{\theta_i} + \frac{\sigma-1}{\theta_i}\left(\frac{\sigma}{\sigma-1}\right)^{-\theta_i}\right]\Gamma\left(\frac{1-\sigma+\theta_i}{\theta_i}\right)$, where $\Gamma(\cdot)$ is the Gamma function

2.5 Market Clearing

In ever region, factor markets must clear, such that the use of each factor of production in all sectors region $s \in \mathcal{D}$ by producers of varieties must equal its supply:

$$\sum_{i\in\mathcal{I}} k^i_{s,t} = k_{s,t}, \quad \sum_{i\in\mathcal{I}} u^i_{s,t} = L^u_{s,t}, \quad \sum_{i\in\mathcal{I}} h^i_{s,t} = L^h_{s,t}, \quad \sum_{i\in\mathcal{I}} t^i_{s,t} = t_{s,t}, \quad \sum_{i\in\mathcal{I}} r^i_{s,t} = r_{s,t}$$

Trade happens through demand for varieties used as inputs in the production of sectoral goods $q^j_{s,t}$. These goods, in turn, are used in two different ways: as intermediate inputs in the production of varieties and investment goods; and in final consumption in both the private and public sectors. Expenditure on goods coming from industry j of region s satisfies:

$$\begin{aligned} e^j_{s,t} \quad = \quad & \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}} \pi^j_{sd,t} \times \eta^{i,j}_{d,t} \times \eta^{i,m}_{d,t} \times \frac{\theta_i}{1+\theta_i} e^i_{d,t} \\ + \quad & \sum_{d\in\mathcal{D}} \pi^j_{sd,t} \times \eta^{j,in}_{d,t} \times p^{in}_{d,t} q^{in}_{d,t} \\ + \quad & \sum_{d\in\mathcal{D}} \pi^j_{sd,t} \times \eta^{pr,j}_{d,t} \times e^{pr}_{d,t} \\ + \quad & \sum_{d\in\mathcal{D}} \pi^j_{sd,t} \times \kappa^{go,j}_{d,t} \times e^{go}_{d,t} \end{aligned} \tag{9}$$

The first and second lines denote the expenditure regarding from the use of varieties in the production of intermediate inputs in the production of varieties and investment goods, respectively. $\pi^j_{sd,t}$ is the trade share of s in varieties demanded by the producer of sectoral good j in region d; $\eta^{i,j}_{d,t} = \Psi^{i,j}_{d,t}(p^j_{d,t})^{1-\sigma}/(\sum_{k\in\mathcal{I}} \Psi^{i,k}_{d,t}(p^k_{d,t})^{1-\sigma})$ is the cost share of sector j in the total intermediate expenditure use in sector i; $\eta^{i,m}_{d,t} = \Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\sigma}/(\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\sigma} + \Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\sigma})$ is the cost share of intermediates in total cost in sector i; and $\frac{\theta_i}{1+\theta_i} e^i_{d,t}$ is the total cost payments. $\eta^{i,in}_{d,t} = p^i_{d,t}\overline{\chi}^i_{d,t}/\sum_{k\in I} p^k_{d,t}\overline{\chi}^k_{d,t}$ stands for the cost share of sector i in the production of the investment good.

The third and fourth lines represent expenditure related to the use of varieties in the production of sectoral goods for final consumption in the private and public macro-sectors. $\eta^{pr,i}_{d,t} = \Psi_i(q^{pr}_{d,t})^{\varepsilon_i}(p^{pr,i}_{d,t})^{1-\sigma}(\sum_{j\in\mathcal{I}} \Psi_j(q^{pr}_{d,t})^{\varepsilon_j}(p^{pr,j}_{d,t})^{1-\sigma})$ is the expenditure share of sector i as a fraction of total private expenditures and $\kappa^{go,i}_{d,t}$ is the Cobb-Douglas parameter that denotes expenditure share in sector i as a fraction of total government expenditures.

Prices of factors of production is proportional to their use and total cost. Since factors are used in every sector, we aggregate over sectors to calculate aggregate payments to each factor of production:

$$
\begin{aligned}
w^n_{s,t} L^n_{s,t} &= \sum_{i\in\mathcal{I}} \eta^{i,f}_{d,t} \cdot \eta^{i,n}_{d,t} \cdot \frac{\theta_i}{1+\theta_i} e^i_{d,t} \qquad \text{for } n \in \{h,u\} \\
\rho_{s,t} k_{s,t} &= \sum_{i\in\mathcal{I}} \eta^{i,f}_{d,t} \cdot \eta^{i,k}_{d,t} \cdot \frac{\theta_i}{1+\theta_i} e^i_{d,t} \\
pr_{d,t} r_{d,t} &= \sum_{i\in\mathcal{I}} \eta^{i,f}_{d,t} \cdot \eta^{i,k}_{r,t} \cdot \frac{\theta_i}{1+\theta_i} e^i_{d,t} \\
pt_{d,t} t_{d,t} &= \sum_{i\in\mathcal{I}} \eta^{i,f}_{d,t} \cdot \eta^{i,t}_{s,t} \cdot \frac{\theta_i}{1+\theta_i} e^i_{d,t}
\end{aligned}
$$

where $\eta^{i,f}_{d,t} = \Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\sigma}/(\Psi^{i,f}_{d,t}(pf^i_{d,t})^{1-\sigma} + \Psi^{i,m}_{d,t}(pm^i_{d,t})^{1-\sigma})$ is the cost share of value added in total cost; and, for each factor of production $n \in \{h,u,k,n,t\}$, $\eta^{i,n}_{d,t} = \Psi^{i,n}_{d,t}(pn^i_{d,t})^{1-\sigma}/\sum_{q\in\{h,u,k,n,t\}} \Psi^{i,q}_{d,t}(pq^i_{d,t})^{1-\sigma}$ is the cost share of factor n in total expenditure on factors of production, with $pn^i_{d,t}$ standing for the price of factor n.

The government collects tariffs and directs them to the representative household as lump-sum transfers:

$$
T_{s,t} = \sum_{n\in\mathcal{D}} \sum_{i\in\mathcal{I}} \frac{tm_{ns,t}-1}{tm_{ns,t}} \pi^i_{ns,t} e^i_{n,t}
$$

Recalling that profits are $\Pi^i_{s,t} = \frac{1}{1+\theta_i}\sum_{d\in\mathcal{D}} \pi^i_{sd,t} e^i_{d,t}$ and stating other factor income $F_{d,t} \equiv pr_{d,t} r_{d,t} + pt_{d,t} t_{d,t}$ complete all elements necessary to characterize domestic income:

$$
Y_{s,t} = \sum_{i\in\{h,l\}} w^i_{s,t} L^i_{s,t} + \rho_{s,t} k_{s,t} + F_{s,t} + T_{s,t} + \sum_{j\in\mathcal{I}} \Pi^j_{s,t}
$$

To close the model, we fix the trade balance as a share of income to a value observed in the data, which is country specific ($b_s \in \mathbb{R}$). In equilibrium, expenditure in every sector of the economy must equal national income plus the trade balance, assumed to be fixed and exogenous as share of income:

$$
Y_{s,t} = \sum_{i\in\mathcal{I}} e^i_{s,t} + b_s Y_{s,t} \tag{10}
$$

2.6 Dynamic innovation

Unlike in the standard Eaton and S. Kortum (2002) model or in the Bertrand-competition version developed in Bernard et al. (2003), we assume that each countries region's location parameter evolves over time. Each commodity $i \in \mathcal{I}$ and each country $d \in \mathcal{D}$ has a different period-specific productivity distribution $F^i_{d,t}(z)$.

Our model follows a strand of the literature which models ideas diffusion through random matches between domestic and foreign managers[4]. Seminal examples of this work include Jovanovic and Rob (1989) and S. S. Kortum (1997). More recently, Alvarez et al. (2013) and Buera and Oberfield (2020) explored how idea diffusion is intertwined with trade linkages.

[4]For a detailed review of this literature, see the comprehensive review chapter published by Buera and Lucas (2018).

Like Buera and Oberfield (2020), we assume that a manager draws new insights as a by-product of sourcing a basket inputs.

We extend this framework to a model diffusion of ideas in a multi-sector context and solve it in a recursive fashion that permits forward-looking assessment of policy experiments. The idea diffusion mechanism is mediated by the input-output structure of production, such that both sector cost shares and import trade shares characterize the source distribution of ideas.[5]

Assumption 2 (Idea formation)**.** *New ideas are the transformation of two random variables, namely: (i) original insights o, which arrive according to a power law: $O_t(o) = Pr(O < o) = 1 - \alpha_t o^{-\theta}$; (ii) derived insights z', drawn from a source distribution $G^i_{d,t}(z)$. After the realization of those two random variables, the new idea has productivity $z = o(z')^{\beta}$, where o is the original component of the new idea, z' is the derived insight, and $\beta \in [0, 1)$ captures the contribution of the derived insights to new ideas. Local producers only adopt new ideas if their quality dominates the quality of local varieties. Therefore, for any period, domestic technological frontier evolves according to*[6]*:*

$$F^i_{d,t+\Delta}(z) = \underbrace{F^i_{d,t}(z)}_{Pr\{productivity<z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_\tau z^{-\theta_i}(z')^{\beta\theta} dG^i_{d,\tau}(z') d\tau\right)}_{Pr\{\text{no better draws in } (t,t+\Delta)\}}$$

Lemma (Generic Law of Motion, Buera and Oberfield 2020)**.** *Given Assumption 2, if, for any t, $F^i_{d,t}(z)$ is Fréchet with location parameter $\lambda^i_{d,t} = \int_{-\infty}^{t} \alpha_\tau \int (z')^{\beta\theta_i} dG^i_{d,\tau}(z') d\tau$ and scale parameter θ, the former evolving according to the following law of motion:*

$$\Delta\lambda^i_{d,t} = \alpha_t \int (z')^{\beta\theta_i} dG^i_{d,t}(z') \tag{11}$$

where α_t is a parameter that controls the arrival rate of ideas and β is the sensitivity of current productivity to derived insights. The integral on the right hand side of the equation denotes the average productivity of ideas drawn from source distribution $G^i_{d,t}(z')$[7]*.*

Proof. Appendix. □

To fully characterize (11), we need to define the source distribution. We assume that managers learn from their suppliers, such that $G^i_{d,t}(z')$ is proportional to the sourcing decisions in production of commodity i in country d. Productivity thus evolves endogenously as a by-product of sourcing decisions. Additionally, we assume that insights take time to come to fruition. Rather than drawing insights from interactions with suppliers in the current period, we assume that insights take one period to materialize. Intuitively, we are assuming that entrepreneurs have to study their purchases for one period and only then draw insights. This assumption will be convenient because it will allow us to compute the law of motion

[5] As mentioned earlier, our work is closely related to Santacreu, Li, and Cai (2017), who extend S. S. Kortum (1997) to a multi-sector framework. We differ in that they model diffusion as happening separately from trade, rather than a trade-externality.

[6] Here we simply use the fact that $o = z(z')^{-\beta}$ and note that, given an insight z', at any moment t the arrival rate of ideas of quality better than z is $Pr(O > o) = Pr(O > z(z')^{-\beta}) = \alpha_t z^{-\theta}(z')^{\beta\theta}$. We then integrate over all possible values of z'.

[7] Equation (11) is a discrete-time approximation of the continuous-time law of motion derived in the Appendix.

for technology without relying on present period trade shares. Therefore, we will be able to solve the model recursively and use it for forward-looking counterfactual analysis.

Assumption 3 (Source Distribution from Intermediates)**.** *The source distribution* $G^i_{d,t}(z') \equiv \sum_{j\in\mathcal{I}} \eta^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} H^{i,j}_{sd,t-1}(z')$*, where* $\eta^{i,j}_{d,t} = \Psi^{i,j}_{d,t}(p^j_{d,t})^{1-\sigma}/(\sum_{k\in\mathcal{I}} \Psi^{i,k}_{d,t}(p^k_{d,t})^{1-\sigma})$ *is the intermediate cost share of sector* j *when producing good* i *in region* d*; and* $H^{i,j}_{sd,t-1}(z')$ *is the fraction of commodities for which the lowest cost supplier in period* $t-1$ *is a firm located in* $s \in \mathcal{D}$ *with productivity weakly less than* z'*.*

Proposition 1 (Law of Motion in a Multi-Sector Framework)**.** *Given Assumptions 1-3, in the multi-sector multi-region economy described in the previous section, the country-sector-specific technology parameter evolves according to the following process:*

$$\Delta\lambda^i_{d,t} = \alpha_t \sum_{j\in\mathcal{I}} \Gamma(1-\beta)\eta^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} (\pi^{i,j}_{sd,t-1})^{1-\beta}(\lambda^j_{s,t-1})^{\beta} \tag{12}$$

where $\Gamma(\cdot)$ *is the gamma function,* $\eta^{i,j}_{d,t-1}$ *are cost shares, and* $\pi^{i,j}_{sd,t-1}$ *are intermediate input trade shares.*

Proof. Combining the result of the Lemma stated above and Assumption 2, we can express the law of motion as:

$$\begin{aligned} \Delta\lambda^i_{d,t} &= \alpha_t \int z^{\beta\theta} dG^i_{d,t}(z) \\ &= \alpha_t \sum_{j\in\mathcal{I}} \eta^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \int z^{\beta\theta} dH^{i,j}_{sd,t-1}(z) \end{aligned}$$

The strategy of the proof relies on Bertrand competition and Assumption 1, regarding productivity draws. The joint distribution of the two most productive firms in a given industry i in country s is given by $F^i_{s,t}(z_1, z_2) = (1 + \lambda^i_{s,t}[z_2^{-\theta} - z_1^{-\theta}]) \exp\{-\lambda^i_{s,t} z_2^{-\theta}\}$.

Incorporating landed input bundle costs $\tilde{x}^i_{sd,t}$, we calculate the probability that the lowest cost producer at destination d is from s and has productivity lower than z_2 or in the range $[z_2, z_1)$. In the Appendix, we use the work the each integral $\int z^{\beta\theta} dH^{i,j}_{sd,t-1}(z)$ and derive the result stated in the Proposition. □

This result extends Buera and Oberfield (2020) to a multi-sector framework. We will use equation (12) and a calibrated path for α_t to solve for an endogenous path for $\lambda^i_{d,t}$.

3 Discussion and Intuition of Ideas Diffusion in a Multi-sector Framework

In this section, we provide some intuition regarding how the idea diffusion mechanism operates in the multi-sector framework. We will use a simplified two-country, two-region economy to show how free-trade allocations exhibit both within- and between sector deviations from the first best allocations that maximize idea diffusion.

Below we denote industries as $i, -i$ and label regions are home (h) and foreign (f). η^i denotes own-cost share of industry i, assumed to be identical in both countries; λ^i_h is the productivity in sector i at home; and $\pi_h^{i,-i}$ stands for the domestic trade share of h in the total intermediate cost of inputs from industry $-i$ in the production of industry i. We drop time subscripts for simplicity. In a two-by-two symmetric economy, equation (12) for industry i at home is proportional to a weighted average of the two sector input shares:

$$\begin{aligned}\Delta\lambda^i &\propto \eta^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta] \\ &+ (1-\eta_d^i)[(\pi_h^{i,-i})^{1-\beta}(\lambda_h^{-i})^\beta + (1-\pi_h^{i,-i})^{1-\beta}(\lambda_f^{-i})^\beta]\end{aligned}$$

What would the optimal total domestic trade shares in in sectors $i, -i$ be? If a planner were to optimize $\pi_h^{i,i}, \pi_h^{i,-i}$ in order to maximize idea diffusion[8], the ratio of optimal total expenditure shares within a given sector satisfies:

$$\left(\frac{\eta^i\pi_h^{i,i}}{\eta^i(1-\pi_h^{i,i})}\right)^{\text{Planner}} = \frac{\lambda_h^i}{\lambda_f^i}$$

How does this compare with the total domestic trade shares that results from free-trade optimization? Free-trade allocations incorporate unit costs x_h^i and trade costs $\tau \geq 1$ (assumed to be symmetric):

$$\left(\frac{\eta^i\pi_h^{i,i}}{\eta^i(1-\pi_h^{i,i})}\right)^{\text{Free Trade}} = \frac{\lambda_h^i(x_h^i)^{-\theta}}{\lambda_f^i(\tau\cdot x_f^i)^{-\theta}}$$

In general, the free trade allocation will be different from the planner one, except if differences in trade and unit costs exactly cancel out, i.e.: $\tau = x_h^i/x_f^i$. This within sector distortion mimics the single-sector results of Buera and Oberfield (2020). Below, we show that in a multi-sector framework not only there are deviations within each sector, but also that in general they are not proportional across sectors: i.e., there are cross-sector distortions. Consider first the ratio of domestic shares in total trade expenditures in sectors $i, -i$ that induce optimal idea diffusion:

$$\left(\frac{\eta^i\pi_h^{i,i}}{(1-\eta^i)\pi_h^{i,-i}}\right)^{\text{Planner}} = \underbrace{\frac{\eta^i}{1-\eta^i}}_{\text{cost share}} \times \underbrace{\frac{\lambda_h^i}{\lambda_h^{-i}}}_{\text{own-productivity}} \times \underbrace{\left(\frac{\lambda_h^i+\lambda_f^i}{\lambda_h^{-i}+\lambda_f^{-i}}\right)^{-1}}_{\text{industry-wise productivity}} \quad (13)$$

The ratio can be decomposed into a cost share component, a relative own-productivity component; and a industry-wise relative productivity component. Intuitively, optimal domestic trade allocation in industry i will increase relative to industry $-i$ if intermediate cost share of industry i increases and if the relative domestic productivity of industry i goes up. The ratio is decreasing in industry-wise relative productivity: if the productivity gap between foreign and home is larger in industry i relative to industry $-i$, optimal domestic trade share of industry i will decrease relative to industry $-i$.

How does this compare with the total domestic trade shares that results from free-trade optimization? Industry-wise productivity ratio is adjusted by unit and trade costs:

[8]Specifically, a planner is maximizing $\max_{\{\pi_h^{i,i},\pi_h^{i,-i}\}} \eta^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta] + (1-\eta_d^i)[(\pi_h^{i,-i})^{1-\beta}(\lambda_h^{-i})^\beta + (1-\pi_h^{i,-i})^{1-\beta}(\lambda_f^{-i})^\beta]$, which is a separable and strictly concave programming problem in $\pi_h^{i,i}, \pi_h^{i,-i}$.

$$\left(\frac{\eta^i \pi_h^{i,i}}{(1-\eta^i)\pi_h^{i,-i}}\right)^{\text{Free Trade}} = \underbrace{\frac{\eta^i}{1-\eta^i}}_{\text{cost share}} \times \underbrace{\frac{\lambda_h^i (x_h^i)^{-\theta}}{\lambda_h^{-i}(x_h^{-i})^{-\theta}}}_{\text{own cost-adj. productivity}} \times \underbrace{\left(\frac{\lambda_h^i (x_h^i)^{-\theta} + \lambda_f^i (\tau \cdot x_f^i)^{-\theta}}{\lambda_h^{-i}(x_h^{-i})^{-\theta} + \lambda_f^{-i}(\tau \cdot x_f^{-i})^{-\theta}}\right)^{-1}}_{\text{industry-wise cost-adj. productivity}} \tag{14}$$

These differences induce a gap between the planner's allocation and the free-trade allocation. Define $\aleph$ as as the ratio of equations (14) for (13):

$$\aleph = \underbrace{\left(\frac{x_h^i}{x_h^{-i}}\right)^{-\theta}}_{\text{domestic cost gap}} \times \underbrace{\left(\frac{\lambda_h^i (x_h^i)^{-\theta} + \lambda_f^i (\tau \cdot x_f^i)^{-\theta}}{\lambda_h^i + \lambda_f^i}\right)^{-1}}_{\text{industry-wise cost-induced deviation in } i} \times \underbrace{\left(\frac{\lambda_h^{-i}(x_h^{-i})^{-\theta} + \lambda_f^{-i}(\tau \cdot x_f^{-i})^{-\theta}}{\lambda_h^{-i} + \lambda_f^{-i}}\right)}_{\text{industry-wise cost-induced deviation in } -i} \tag{15}$$

Whenever $\aleph \neq 1$, there is a sectoral distortion in domestic trade expenditure shares. If $\aleph > 1$ (< 1), domestic trade expenditure share on sector i relative to sector $-i$ is above (below) the planner's ratio. Even if $\aleph = 1$, that does not guarantee absence of deviations from the optimal diffusion point. Rather, it means that deviations (or absence thereof) are proportional in both sectors, such that domestic trade share in one sector is not disproportionately higher (lower) in sector i relative to sector $-i$.

In general, deviations need not be proportional. In fact, only in knife edge cases $\aleph = 1$: if countries have identical input costs across industries $x_h^i = x_h^{-i}, x_f^i = x_f^{-i}$; and either industries in each country have identical productivity ($\lambda_h^i = \lambda_h^{-i}$, $\lambda_f^i = \lambda_f^{-i}$); or sector-specific productivities at home are a linear transformation of the sector-specific productivities at foreign ($\lambda_h^i = \kappa \cdot \lambda_f^i$, $\lambda_f^{-i} = \kappa \cdot \lambda_f^{-i}$, $\kappa \in \mathbb{R}_{++}$).

This underscores that, in a multi-sector framework, there will not only be *within sector distortions*, but also *between sector distortions*. Domestic sourcing will be biased towards the industry with lowest relative cost, even if that industry is not very productive. For instance, if costs are disproportionately low in one industry i relative to industry $-i$, either domestically or industry-wise, domestic trade share will be disproportionately high in industry i under free-trade relative to the optimal allocation and $\aleph > 1$.

There are additional complexities that arise in a multi-sector framework, which we illustrate geometrically. First, consider what happens *within* sector i in a fully symmetric two-by-two economy. Even with identical countries, the strict concavity of diffusion equation implies that idea diffusion is not uniform as $\pi_h^{i,i}$ varies[9]. The optimal diffusion point is $(\pi_h^{i,i})^{\text{Planner}} = \lambda_h^i/(\lambda_h^i + \lambda_f^i) = 1/2$. Under free trade, trade costs induce home bias such that domestic share is $(\pi_h^{i,i})^{\text{Free Trade}} = 1/(1+\tau^{-\theta}) > 1/2$ and ideas diffusion is below the optimal point. If trade costs increase and $\tau \to \infty$, the home country moves to autarky and deviations from the optimal idea diffusion reach a maximum. We plot the optimal, free trade, and autarky points along the ideas diffusion function for sector i on the left hand side panel of Figure 1.

The right panel illustrates what happens when the home country has a lower productivity in sector i. The curve shifts down at the autarky point and the planner's solution moves to the left (smaller domestic trade share)[10]. When $\lambda_h^i < \lambda_f^i$, diffusion losses from high trade

[9]Add intuition for the concavity.

[10]Formally, once countries are no longer symmetric, we need to make the following regularity condition

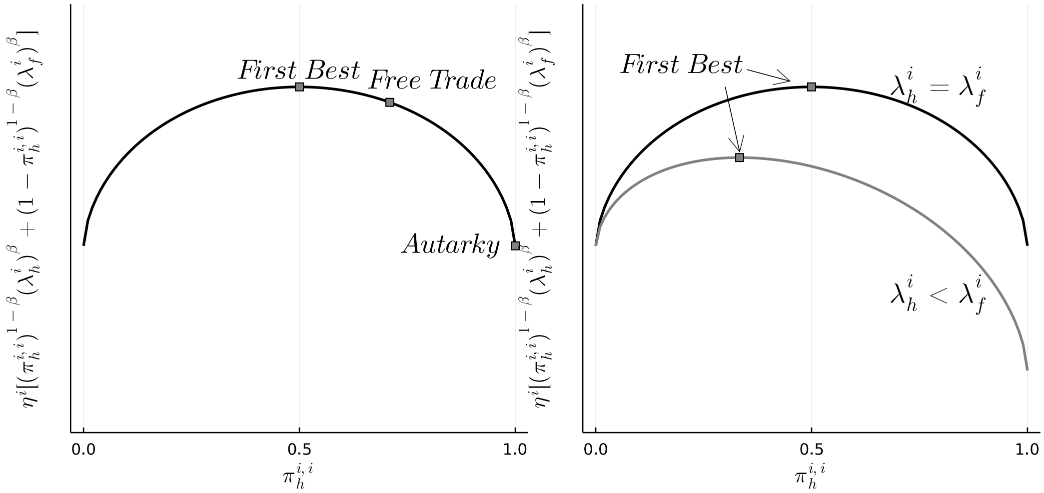

Figure 1: **Within sector idea diffusion functions in a two-by-two economy.** Both panels plot the idea diffusion functions for the home country in a two-by-two model within sector i: $\eta^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta]$. The left panel shows the optimal, free trade, and autarky points along the ideas diffusion function when countries are fully symmetric ($\lambda_h^i = \lambda_f^i$). The righ panel plots the functions and planner's solutions for the cases when countries have identical productivities $\lambda_h^i = \lambda_f^i$ and the home country is less productive $\lambda_h^i < \lambda_f^i$.

costs are higher. This highlights a key characteristic of this class of models: countries that are *less productive* in a given sector have *higher dynamic gains from trade*[11].

When considering a multi-sector framework, within sector inefficiencies accumulate. For instance, suppose that in sector i domestic trade share $\pi_h^i = 0$ while, in sector $-i$, $\pi_h^i = 1$. If $\eta^i = 1/2$, deviations from optimal idea diffusion will be at a maximum even though total trade share will be at the optimal point (1/2). In a multi-sector framework, the fact that *total domestic trade share* is at its optimal point is a necessary but *no longer sufficient for optimal diffusion.* To maximize total diffusion, trade shares must be at their optimal point *in every sector.*

Figure 2 underlines this fact. It shows that there is a unique point in the $[0,1]^2 \times [0,\infty)$ space that maximizes idea diffusion in a two-by-two symmetric model as $\pi_h^{i,i}, \pi_h^{i,-i}$ vary. With $\eta^i = 1/2$, every point in the diagonal $\pi_h^{i,i} = 1 - \pi_h^{i,-i}$ will have total trade share at its optimal point 1/2. Additionally, every point in the counterdiagonal $\pi_h^{i,i} = \pi_h^{i,-i}$ has absence of between sector distortion ($\aleph = 1$). However, neither fact is sufficient to guarantee optimal diffusion. Only if trade shares are optimal in both sectors (i.e., $\pi_h^{i,i} = \pi_h^{i,-i} = 1/2$) diffusion is maximized in this simplified economy. Any other point will have some degree of inefficiency.

---

to guarantee convergence to the autarky equilibrium: $\lim_{\tau\to\infty} (\tau x_f^i)/x_h^i = +\infty$. Most models make this assumption either explicitly or implicitly.

[11]In fact, for any $\pi_h^i \in (0,1]$, the marginal change in diffusion as π_h^i increases will be increasing in a country's productivity. To see that, take $\frac{\partial \Delta\lambda_h^i}{\partial \pi_h^i} = \alpha \cdot \Gamma(1-\beta) \cdot \eta^i(1-\beta)[(\pi_h^{i,i})^{-\beta}(\lambda_h^i)^\beta - (1-\pi_h^{i,i})^{-\beta}(\lambda_f^i)^\beta]$, which is increasing in λ_h^i.

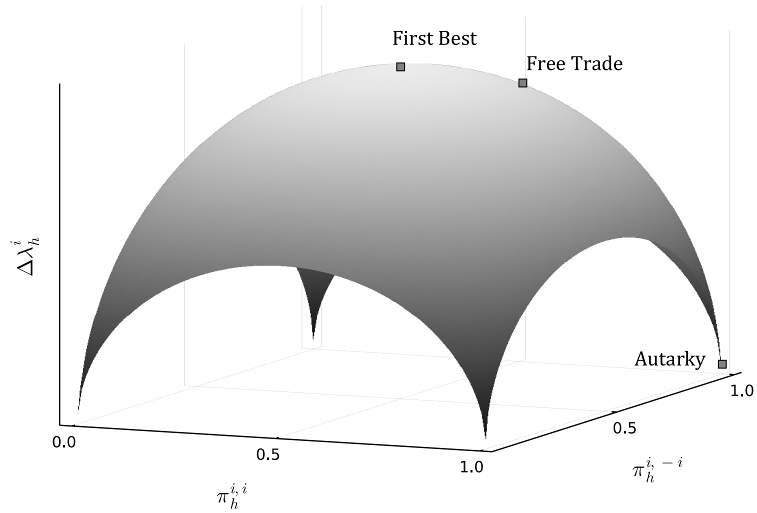

Figure 2: **Idea diffusion function in a two-by-two economy.** The graph shows $\Delta\lambda^i \propto \eta^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta] + (1-\eta_d^i)[(\pi_h^{i,-i})^{1-\beta}(\lambda_h^{-i})^\beta + (1-\pi_h^{i,-i})^{1-\beta}(\lambda_f^{-i})^\beta]$ as a function of domestic trade share in sectors $i, -i$: π_h^i, π_h^{-i}. If countries and sectors are identical and $\eta^i = 1/2$, Planner's, Free Trade, and Autarky allocations are as represented in this figure. The marginal contribution of each sector to total diffusion are as shown in the left panel of Figure 1

It is easy to see how the problem increases in complexity and inefficiencies accumulate as the number of countries and sectors increase. With N countries and J sectors, there are $(N-1)J$ free parameters that would need to be at their optimal points if diffusion were to be maximized. In a free trade equilibrium, those parameters will not be at their optimal points and each one of them will contribute to some deviation from optimal diffusion. Therefore, a multi-sector framework is important to have a more realistic assessment of diffusion in policy experiments. We derive multi-sector, multi-region versions of equations 13, 14, and 15 in Appendix C, but most of the intuition can be represented with the simplified version presented in this section.

4 Policy Experiments

Our main motivation for simulating large-scale trade conflicts is the possibility of receding globalization due to a political backlash. Decades of continuous deepening of the international trade regime were characterized by a large consensus regarding the need to reduce trade costs and prioritize gains from trade.

Even though there is large evidence of gains from openness, which can be as large as of 50% of national income (Ossa 2015), empirical findings about frictions in local labor markets (Autor, Dorn, and Hanson 2013; Dix-Carneiro and Kovak 2017) highlight distributional concerns that translate into political grievances and may have led to an increase in the number of populist and isolationist parties in Western countries (Colantone and Stanig 2018).

The clearest example of the shift in political consensus is perhaps the trade war waged by the Trump Administration (2017-20) against China and other countries. The economic discourse shifted away from emphasis on gains from trade to a framing of trade as a zero-sum game and to the unorthodox use of national-security provisions of the international trade regime to engage in protectionist policy-making[12].

Challenges to the international trade regime might seem like some circumstantial discontinuity in a long-run trend towards increasing openness. However, political scientists in the West and in China argue that there is reason to believe that strategic geopolitical rivalries could trump economic gains —at least partially. These disputes are exemplified not only in the trade war between the U.S. and China but also in sectoral clashes, such as the U.S. government pressuring allies against allowing participation of Chinese telecommunications companies in new infrastructure developments.

Some Chinese analysts see an escalating and continuous conflict between China and the U.S. as a natural and "structural" development of a shifting international system that is moving from unipolar (the U.S. being the only superpower) to bipolar (China becoming a superpower on an equal footing to the U.S.)[13]. In the West, political scientist Joseph S. Nye Jr. highlights that, while an abrupt decoupling between the U.S. and China is unlikely, both parties will try to decrease their (inter-)dependence with respect to each other's actions, except where the costs of disengagement are too high to bear (Nye Jr. 2020)[14].

We are agnostic about the future degree of decoupling between the U.S. and China in the future. Nonetheless, the fact that international relations scholars envisage disengagement as a real possibility underscores that estimating the potential consequences of decoupling to trade, growth, and innovation is an important exercise. As our model highlights, changes in trade patterns and sourcing decisions have not only static effects, but also dynamic effects with respect to potential growth and innovation. Our policy experiments try to disentangle the static and dynamic costs of two different decoupling scenarios.

In order to do so, we classify different regions as belonging to a U.S. or a Chinese bloc, based on the Foreign Policy Similarity Database, which uses UN General Assembly voting for a large set of countries in order to calculate foreign policy similarity of country pairs (Häge 2011). Intuitively, the index takes countries who vote similarly in the United Nations (compared to the expected level of similarity in a random event) as being similar in their foreign policy.

We ordered country groups in terms of their similarity compared to China and the United States and created ten different regions based on quantiles. The results shown in Figure 3 are in line with conventional wisdom: Europe, Canada, Australia, Japan, South Korea fall in the U.S. bloc. Latin America and and Sub-Saharan Africa fall somewhere in between, with the former being closer to the U.S. than the latter. India, Russia, and most of North Africa and Southeast Asia fall closer to China.

After classifying the regions into the Chinese or American influence blocs, we conducted two different policy experiments. We first increase iceberg trade costs $\tau^i_{sd,t}$ to a point where

[12]For a contemporaneous review of the policies implemented, see Bown and Irwin (2019).

[13]Wei (2019) provides review of debates among Chinese scholars. In the context above, the balance of power between functionally equivalent states (the "international structure") provides incentives for strategic behavior by governments that try to maximize their power. This is known as the "structural realism" theory of international politics, developed by Kenneth Waltz (2010).

[14]Nye Jr. is mostly known for his joint work with Robert Koehane on "complex interdependence" during the post-World War II era (Keohane and Nye Jr 2011). The authors focus their analysis on the creation of international rules and practices in a world in which the use of military force is very costly due to interdependence between multiple agents that engage both internationally and domestically. For instance, a great degree of trade integration increases the costs (and decreases the probability) of outright military conflict.

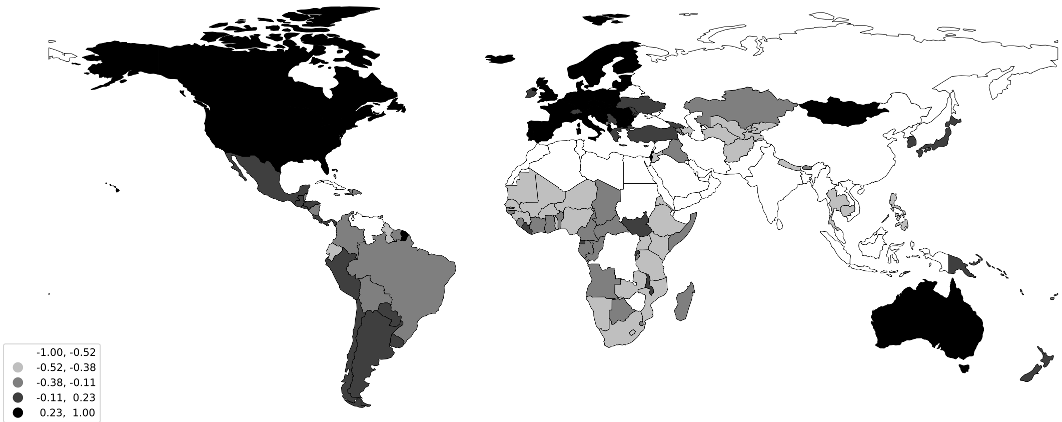

Figure 3: Differential Foreign Policy Similarity Index. Values are normalized such that 1 represents maximum relative similarity with the U.S. and −1 represents maximum relative similarity with China. The map shows the difference between pairwise similarity indices $\kappa_{i,US} - \kappa_{i,China}$. The parameter $\kappa_{i,j}$ represents the foreign policy similarity of countries i, j, based on vote similarity in the United Nations General Assembly. Given vote possibilities $n, m \in \{1, \cdots, k\}$, one can calculate a matrix $P = [p_{nm}]$, where entry p_{nm} represents the share of votes in which country i took position n and country j took position m. Given matrix P, $\kappa_{i,j} = 1 - \sum_{m \neq n} p_{mn} / \sum_{m \neq n} p_m p_n$, where p_m, p_n are expected marginal propensity of any country to take position m, n at a random vote. For more details, see Häge (2011).

virtually all of the trade happens exclusively within each bloc. In total, we increase bilateral trade costs by ~ 160 percentage points. We label this scenario **full decouple**. This provides an important limiting case that can be useful for putting bounds on potential effects.

The second scenario relies on work by Nicita, Olarreaga, and Silva (2018), who estimate that a move from cooperative to non-cooperative tariff setting would increase average tariffs by 32 percentage points globally. We simulate what would happen if countries kept cooperative tariff setting within their trade blocs but moved to non-cooperative tariff setting across trade blocs. For simplicity, we assume that means that regions in different blocs increase bilateral tariffs $tm^i_{sd,t}$ by 32 percentage points against regions outside the bloc. These numbers might sound unrealistically high, but the average tariff increase in the China-U.S. trade war reached North of 21pp. We call this scenario **tariff decouple**.

4.1 Data and Behavioral Parameters

4.1.1 Baseline Data

The model is calibrated to trade and production data from the 2014 version of the GTAP Data Base, Version GTAP10A. This means that all spending and cost shares are set equal to the shares in the 2014 database, following the same calibration procedure as in models employing exact hat algebra (Dekle, Eaton, and S. Kortum 2007). The data are aggregated to 10 regions, 6 sectors, and 5 factors of production as specified in Table 1. The model is solved until 2040 in a sequence of recursive dynamic simulations, thus solving the model period per period, using the model solution in the previous period as the starting point for the next period. The savings rate is kept constant, ideas diffuse as specified in the previous section and capital is accumulated according to equation (2). Population grows based on UN population projections and labor supply grows based on International Monetary Fund projections for employment (until 2025) and United Nations projections regarding working age population (from 2026 until 2040).

Table 1: Overview of regions, sectors, and endowments

Region		Sector		Endowment
Code	Description	Code	Description	
`chn`	China	`pri`	Primary (agri & natres)	Land
`e27`	European Union 27	`lmn`	Light manufacturing	Unskilled labor
`jpn`	Japan	`hmn`	Heavy manufacturing	High-skilled labor
`ind`	India	`elm`	Electronic Equipment	Capital
`lac`	Latin America	`tas`	Business services	Nat. Ressources
`ode`	Other developed	`ots`	Other Services	
`rwc`	ROW - China bloc			
`rwu`	ROW - USA bloc			
`rus`	Russia			
`usa`	USA			

The data in the GTAP Data Base do not include profit income as in our model with Bertrand competition. Therefore, we have to modify the baseline data employed, considering that profit income $\Pi^i_{s,t}$ is a share $\frac{1}{1+\theta_i}$ of the total value of sales in sector i in region s. We have done this following two approaches.

In the first approach, we proceed in two steps. First, we reduced the value of payments to the production factor capital (capital income) by 50% and reallocated it to profit income. With this step the share of profit income in the value of sales is not yet equal to $\frac{1}{1+\theta_i}$. Therefore, in a second step we employ our model to modify the base data to target the share of profit income in the value of sales for each country and sector. The reason to proceed in two steps is that capital income in some cases is smaller than profit income required by the model. This is especially the case in sectors with large intermediate linkages and a small trade elasticity, because profit income is a share of gross output in the Bertrand model, whereas capital income is part of net output.

In the second approach, we reduce the value of payments to factors of production by an identical share for all production factors and reallocate this to profit income. The reallocation is set such that profit income $\Pi^i_{s,t}$ becomes a share $\frac{1}{1+\theta_i}$ of the value of sales.

4.1.2 Behavioral Parameters

The parameters of the non-homothetic CES, the substitution elasticity σ and ε_i are based on the sectoral estimates in Comin, Lashkari, and Mestieri (2021). The dispersion parameters of the Fréchet distribution, θ_i, equal to the trade elasticities are estimates from Hertel et al. (2007). The substitution elasticity between value added and intermediates, between intermediates, and between investment goods are all set equal to zero, thus implying a Leontief structure. The substitution elasticities between production factors are also based on the values in the GTAP Data Base[15].

Table 2 displays the values of the dispersion parameter of the Fréchet distribution, θ_i, the parameter controlling the income elasticity of private household demand, ε_i, and the substitution elasticity between production factors (endowments).

[15]The supply elasticity of land and natural resources is set conservatively equal to 0.5, but will not have a large impact on the simulation results.

Table 2: Behavioral parameters

	θ_i	ε_i	σ_i
Primary (agriculture & natres)	10.09	0.146	0.27
Light manufacturing	4.60	-0.2	1.20
Heavy manufacturing	5.99	-0.26	1.26
Electronic Equipment	7.80	-0.26	1.26
Business services	2.80	-0.416	1.26
Other Services	2.90	-0.672	1.42
Source	Hertel et al. 2007	Comin, Lashkari, and Mestieri 2021	Hertel et al. 2007

Following Buera and Oberfield (2020), we calibrate the initial value of $\lambda^i_{s,t}$ as equal to GDP per capita and set the growth rate of the autonomous arrival rate of ideas α_t at 1% per year. The ideas diffusion parameter β is uniform across sectors and determined based on model validation, using simulated methods of moments, as described below.

We simulate the model starting from the base year 2014 with no policy changes for one period. We employ different different values of β and evaluate for which values of β the variance of the growth rates of GDP are closest to the variance in the data. More specifically, starting at $\beta = 0$, β is raised in steps of 0.05 up to 0.6[16].

The variance of the growth rates of GDP rises as β increases, because at higher levels of β there is more diffusion of ideas and the divergence between growth rates of poor and rich countries increases. As discussed in Section 3, countries starting with a lower productivity parameters have larger dynamic gains from trade. That effect increases in the value of β[17]

Table 3 displays the summary statistics for different values of β, in the data (IMF) and also in the OECD Shared Socioeconomic Pathways Middle of the Road (SSP2) projections.

Table 3: Growth Rate of Real GDP using Different Values of β

β	Mean	St.Dev.	max	min
0.05	1.72	1.13	4.50	0.35
0.10	1.75	1.14	4.54	0.35
0.15	1.80	1.18	4.60	0.36
0.20	1.90	1.22	4.71	0.37
0.25	2.07	1.32	4.90	0.40
0.30	2.39	1.55	5.26	0.46
0.35	3.00	2.06	6.52	0.57
0.40	4.20	3.19	10.62	0.78
0.45	6.61	5.64	18.90	1.20
0.50	11.63	10.89	36.23	2.05
0.55	22.34	22.37	73.63	3.81
0.60	45.89	48.13	157.19	7.57
IMF past data	1.79	1.81	5.67	0.01
OECD SSP2 projections	3.28	2.08	8.14	1.36
N	10			

[16]For values larger than 0.6 the variance in the simulations becomes unrealistically high, so these are disregarded.

[17]In the limiting limiting point $\beta \to 1$, ideas diffuse instantly and every country that is not in autarky experience equal productivity gains in absolute terms. With equal gains in absolute terms, those countries with lower productivity experience larger growth in relative terms as β increases.

The targeted moments rare the mean (μ) the standard deviation (σ) of real GDP per capita growth in both the IMF data and the OECD SSP2 forecasts. Formally, we are minimizing the following loss function:

$$\min_{\beta} \sum_{m \in \{\mu,\sigma\}} w^{OECD}(m(\beta) - m^{OECD})^2 + (1 - w^{OECD})(m(\beta) - m^{IMF})^2$$

The solution β will depend on choices of the weights w^{OECD}. Regardless of the choice of weight, the solution converges to $\beta \in [0.3, 0.35]$. In the simulations we work with a β equal to 0.35. Furthermore, the average growth rates in the model are close to the average projected growth rates in the OECD SSP2 growth rates, but somewhat below the actual growth rates from the IMF. However, for projection work it can be argued that the projected growth rates from the OECD SSP scenarios are more relevant.

4.2 Main Results

We have four main scenarios. We simulate full decouple and tariff decouple, defined as explained above. Either scenario is simulated both with and without diffusion of ideas, in order to assess the differential impact of this particular mechanism. In practice, we simulate the dynamic world economy with no policy change, then do the same with the policy change, and report the long run cumulative percentage difference: $\hat{x} = \sum_{t=p}^{T}(x'_t - x_t)/\sum_{t=p}^{T} x_t$, where p is the date of the first policy change, x'_t, x_t are the values of variable x with and without the policy change, respectively. Later in this section, we also report experiments that restrict decoupling to the electronics and equipment sector or change Latin America from the U.S.-centric to the Chinese bloc.

As expected, all scenarios show large negative impacts on cross-bloc trade after the introduction of the policy intervention. In the **full decouple** scenario, trade between the countries in the U.S. bloc and China is virtually shut down, with imports and exports dropping by 98%. Those countries also shift a substantial part of their trade to the U.S., with trade flows increasing anywhere between $20-39\%$ depending on the region and scenario. The domestic spending share in the U.S. increases between $6-11\%$. The converse happens in the Chinese bloc but with larger dispersion across regions. Trade with the U.S. drops by $65-90\%$ while trade with China increases by $9-71\%$. The domestic trade share in China increases by 5%. The **tariff decouple** scenario yields qualitatively similar results but with smaller magnitudes. We show the results by region and scenario in Figure 4.

One of the reasons behind the asymmetry in trade decreases between blocs is the assumption of a fixed trade-balance-to-income ratio in all regions but one. This implies that regions with large trade surpluses will shift proportionately less of their trade flows away from regions in other trade blocs in order to satisfy the fixed trade balance assumption.

Every country in the Chinese bloc, except for Russia, shifts its trade flows much less towards China whenever the diffusion of ideas mechanism is on. [There are two reasons for Russia diverging. First, Russia starts from a higher income level than other countries in its bloc, which renders the catch-up effect induced by trading with more productive countries less effective. Second, Russia currently imports a smaller share of its intermediate goods from the West, with a larger share of its supply chain already concentrated in China.]

The underlying factor driving the divergence in results between the two blocs is a difference in the evolution of productivity, represented by the scale parameter of the Fréchet distribution of different sectors. Sourcing goods from high productive countries puts do-

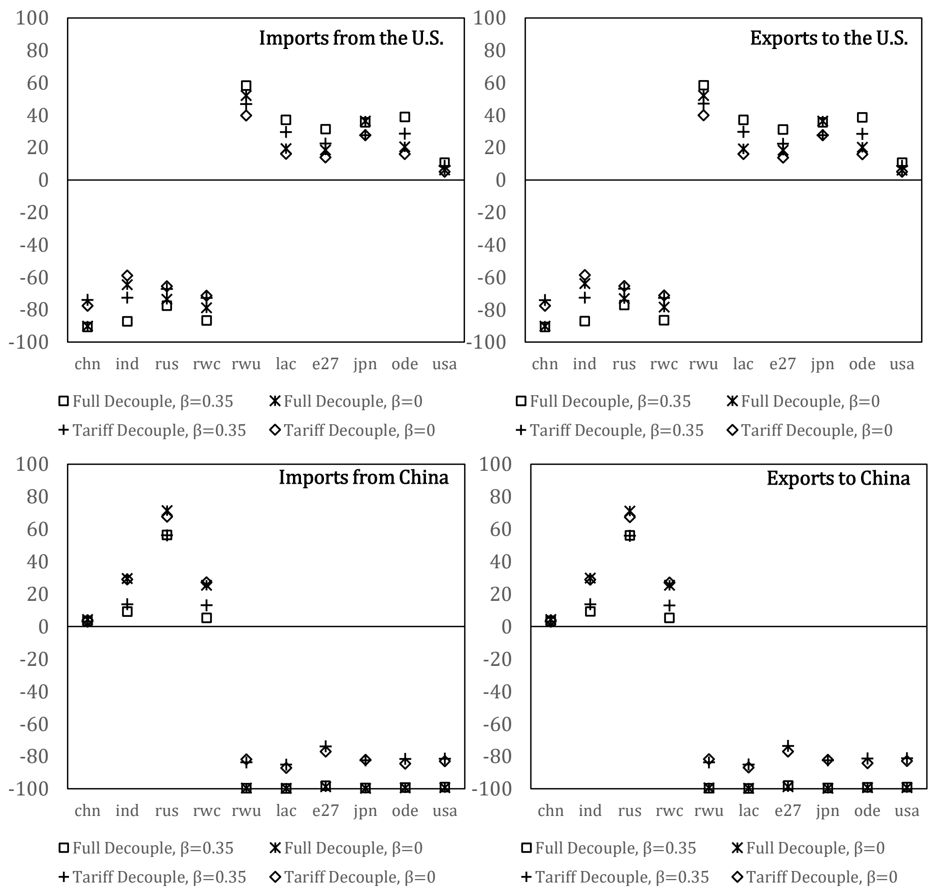

Figure 4: Cumulative Percentage Change in Trade Flows with China and the United States, respectively, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^{i}_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^{i}_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

mestic managers in contact with better quality designs that inspire better ideas through innovation or imitation.

Importantly, the dynamics governed by equation (12) incorporate the input-output structure of production, such that domestic managers in each sector innovate in proportion to the quality and share of their inputs. Losing access to high quality designs does not only lead to static losses, but also to a lower level of future innovation, which implies larger dynamic losses. Additionally, the input-output structure of the model implies that cutting ties to innovative regions is particularly costly if that country is a supplier of many intermediate inputs.

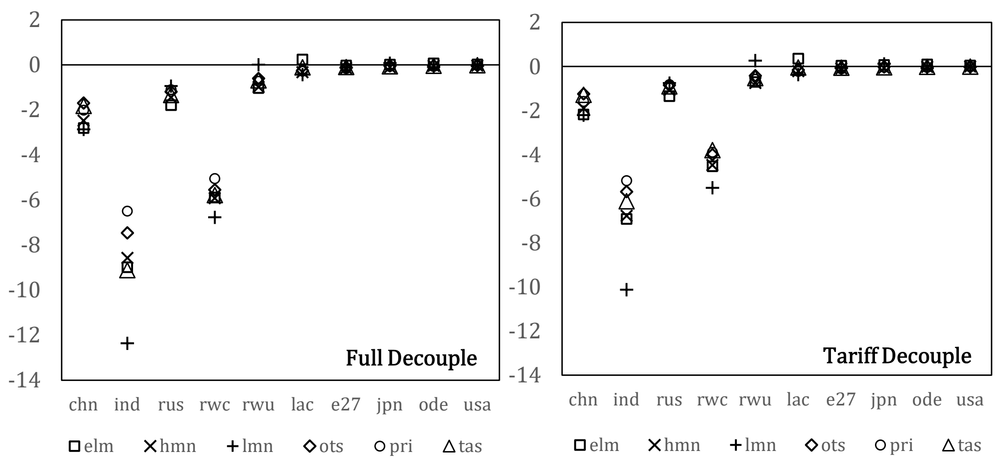

Figure 5: Cumulative Percentage Change in the Fréchet Distribution location parameter $\lambda^i_{d,t}$, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

For those reasons, in our policy experiments, countries that currently have a lower level of productivity and have larger ties with innovative countries have larger losses. By looking at results in Figure 5, one can see the stark contrast between the differential evolution of $\lambda^i_{d,t}$ for those countries in the U.S. bloc and those in the Chinese bloc.

The reasoning is the same as above: by cutting ties with richer and innovative markets such as OECD countries, China, India, Asia, and Africa shift their supply chains towards lower quality inputs, which, in turn, induce less innovation. By contrast, while countries in the U.S. bloc do suffer welfare losses, their innovation paths are virtually unchanged after decoupling, suggesting that nearly all of their losses are static, rather than dynamic.

There is large dispersion across both sectors and countries in differential losses. India and the region that aggregates all of Africa and Asia in the China-bloc (excluding China, India, Russia, and Japan) suffers large losses. Starting from a lower income level than China and Russia, those regions have a much slower productivity catch-up after severing trade ties with the West. [Additionally, include numbers for intermediate linkages].

Among those regions in the China bloc, differential losses are larger in the manufacturing sectors (-5% and -4% with full decoupling and tariff decoupling, respectively; this includes `elm`, `lmn`, and `hmn`) than in the services (-4.2% and -3%, respectively; `ots tas`) or primary (-3.6% and -2.8%, respectively; `pri`) sectors.

Both large increases in iceberg trade costs and retaliatory tariff hikes induce substantial welfare decreases for all countries. The effects, however, are asymmetric. While welfare losses in the U.S. bloc range anywhere between -1% and -6% (median: -2%) in the Chinese bloc it falls in the -1% to -15% range (median: -6%). Welfare losses are larger for those countries that have larger differential losses in the Fréchet Distribution location parameter, namely India and Rest-of-the-World China bloc.

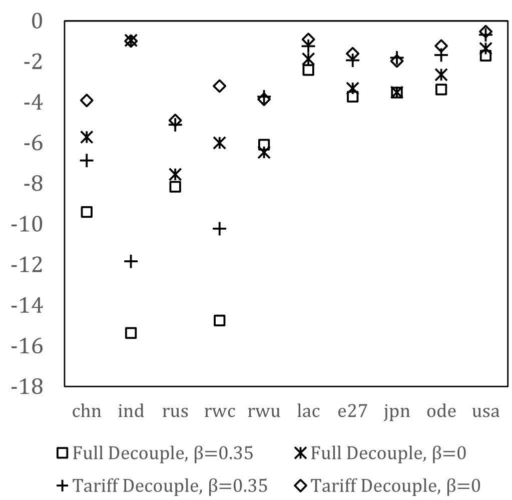

Figure 6: Cumulative Percentage Change in Real Income, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. β is a parameter that controls the the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

Perhaps the most interesting feature of reported welfare losses, however, is the contrast between the static effect (when the diffusion of ideas mechanism is shut down) and the dynamic effect. For most regions of the Chinese bloc, dynamic losses far outsize static losses, which can be explained through the loss of access to higher quality inputs. However, the magnitude of this effect is very heterogeneous across the regions of the bloc.

In India, static welfare losses amount to 1% while total losses range from $12-15\%$, depending on the decoupling scenario. Static losses to real income are small because India is a relatively large country, which implies a larger domestic share in steady state with the negative effects of distortions of tariffs partially compensated for by gains in terms of trade. However, because it is relatively poor, its losses in the diffusion of ideas version of the model are much larger. By severing ties with the U.S. bloc, it limits the role of trade-induced innovation that is a by-product of having access to high quality suppliers.

By contrast, in Russia including dynamics leads only to small additional effects: welfare losses are very similar with or without the ideas diffusion mechanism. As explained above, this stems both from a higher income starting point and limited input-output linkages with the West.

4.3 Diffusion Inefficiencies Multi-sector vs. Single-sector Frameworks

In Section 3, we stressed that, except in knife-edge cases, within- and between-sector inefficiencies accumulate as the number of countries and sectors increase. The concavity of the diffusion process implies that *total* trade shares being at their optimal points is no longer sufficient for optimal diffusion. Optimal diffusion requires trade shares to be at their optimal points *at every sector*. This suggests that, in most cases, diffusion inefficiencies should increase with the number of sectors.

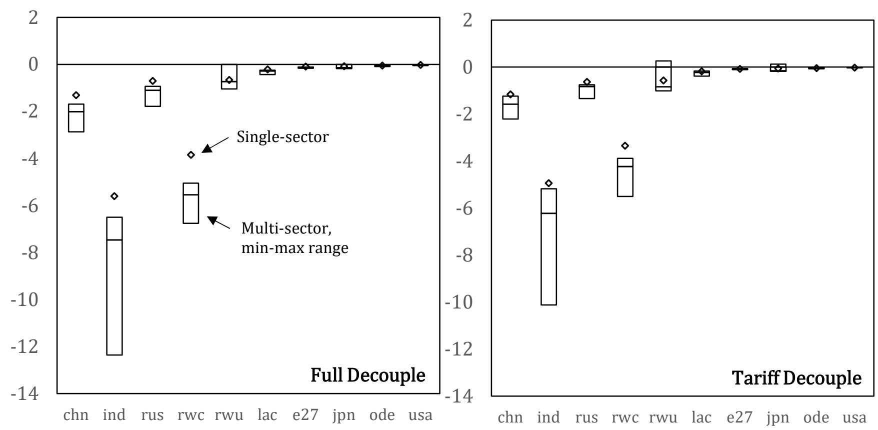

Figure 7: Multi-sector vs. Single-sector: Cumulative Percentage Change in the Fréchet Distribution location parameter $\lambda^i_{d,t}$, after policy change, by 2040. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

Our empirical results confirm that theoretical intuition. Figure 7 contrasts the results of either the **full decouple** or the **tariff decouple** scenarios under the baseline specification presented in the previous section and an alternative simulation in which we collapse the model to a single-sector framework.

In both scenarios, every country whose firms lose access to the most productive suppliers face higher cumulative diffusion inefficiencies in a multi-sector framework after trade costs go up. These results underscore one important takeaway of this paper: modeling trade diffusion in a simplified single-sector framework can underestimate the level of dynamic gains (losses) induced by a decrease (increase) in trade costs.

4.4 Consequences of bloc membership

In this section, we consider the consequences of moving one of the regions —Latin America and the Caribbean (LAC) —from the U.S. bloc to the Chinese bloc. Intuitively, we expect that, by losing access to the highest productivity suppliers, LAC will experience less productivity growth. Nonetheless, the quantitative exercise allows U.S. to have a sense of the magnitude induced by the change in group membership.

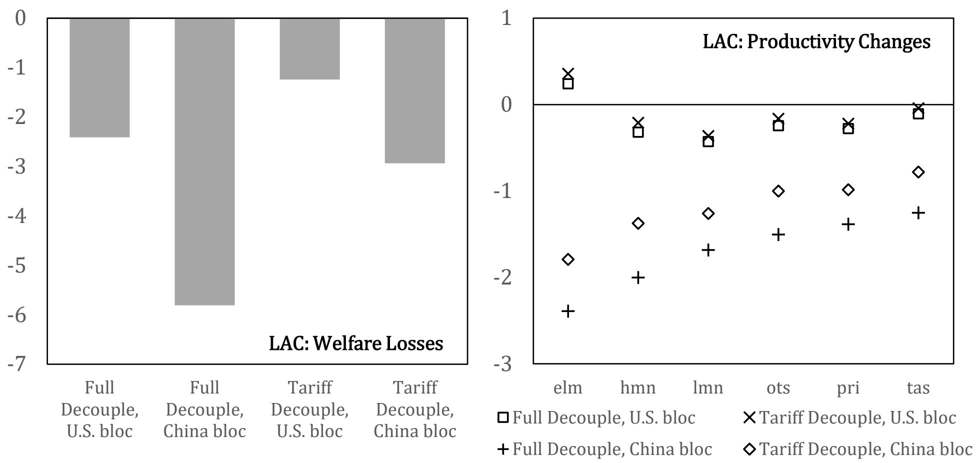

Figure 8: Left Panel: Cumulative Percentage Change in Real Income in LAC Region, by scenario. Right Panel: Cumulative Percentage Change of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$ in LAC Region, by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points. *Tariff decouple* increases bilateral tariffs $tm^i_{sd,t}$, across groups, by 32 percentage points. In all cases, we set parameter that controls the the diffusion of ideas according $\beta = 0.35$. Sector codes: `elm`, Electronic Equipment; `hmn`, Heavy manufacturing; `lmn`, Light manufacturing; `ots`, Other Services; `pri`, Primary Sector; `tas`, Business services. Tables with the values for these charts can be found in the Appendix.

Figure 8 compares the results of identical decoupling scenarios: we set $\beta = 0.35$ and simulate either *full decouple* or *tariff decouple*. The only difference is LAC bloc membership. Moving LAC to the China bloc reduces welfare losses in India and China by about $1 p.p.$ (10%) and $2 p.p.$ (15%), respectively. LAC has higher income than India and the Rest of the China bloc. All else equal, on average, its inclusion in the bloc raises average productivity and decreases dynamic losses. Additionally, lower tariff or iceberg trade costs between the China bloc and LAC induce lower static losses for those countries.

As expected, most of the changes are concentrated in the LAC region. The left panel of Figure 8 shows that welfare losses in LAC are about $2.4x$ larger when it is included in the China bloc, for both scenarios. Increased losses stem almost entirely from dynamic losses, as the overall level of domestic trade share remains virtually unchanged in LAC for either scenario, which implies similar static welfare losses.

The right hand side panel of Figure 8 shows the differential productivity changes in the LAC region for different sectors. When LAC is included in the U.S. bloc, there are essentially no dynamic productivity losses in any sector: the evolution of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$ in the LAC Region behaves very similarly to a scenario with no policy changes.

By contrast, all sectors have dynamic productivity losses weakly greater than 1% when we simulate decoupling with LAC as part of the China bloc. There is large sectoral heterogeneity. Under full decoupling, productivity losses range from 2.4% in Electronic Equipment (`elm`) to 1.3% in Business Services (`tas`). [These differences are induced by input-output linkages. While LAC sources X percent of `elm` intermediates from the U.S., it only sources

Y percent of `tas`].

This experiment underscores that costs of decoupling might be unbearably high to low and middle income countries who are excluded from the U.S. bloc. Many countries in Latin America and Africa benefit from increasingly large trade ties to China through both having larger market access and access to lower input costs. However, as the dynamic costs of severing ties with the West are very high, political leaders in those countries might have an incentive to keep an equidistant relationship between the U.S. and China, by preserving both mid-term gains from the relationship with China and longer term dynamic gains from having access to Western supply chains.

4.5 Electronic Equipment Decoupling

The final quantitative exercise that we perform is a full decoupling between the original China and U.S. blocs (with LAC in the U.S. bloc) but restricting the increase in iceberg trade costs $\tau^i_{sd,t}$ only to the electronic equipment (`elm`) sector. This scenario is motivated by U.S. and Chinese authorities being increasingly at loggerheads with each other in the technological arena. One important example of this process has been the conflict involving Chinese telecom giant Huawei Technologies. Since 2019, American corporations have been banned from doing business with Huawei. In a similar move, the New York Exchange delisted China Unicom, China Mobile, and China Telecom. Despite legal challenges and a new administration, as of April 2021 neither decision has been reversed.

Additionally, the U.S. has been using its foreign policy arsenal to pressure allies to join them in limiting Chinese telecom companies reach. In particular, there is a desire to limit Chinese participation in 5G technology auctions, citing national security and privacy concerns[18]. So far, Australia, the United Kingdom and some European allies have chosen to ban or limit Chinese participation in technological auctions.

This conflict suggests that a large increase in trade costs between the U.S. allies and Chinese allies regarding technological equipment is a positive probability scenario in the future. In this case, decoupling would mean a near total separation of electronic equipment sectors of the two blocs.

Huawei and Google breaking their business connections after the U.S. government sanctions against the Chinese corporation is a good illustration of what this separation could look like in real life. Huawei used Google's *Android* ecosystem in their smartphones, which gave their users access to Google-approved updates and apps. After the ban issued by the Trump administration, however, Google announced it would comply with the U.S. government directives and Huawei was forced to shift away from Google software and design their own operating system *HarmonyOS*.

Since this separation is driven primarily by regulation rather than tariffs, it is appropriate to think of it as an increase in iceberg trade costs $\tau^i_{sd,t}$ between blocs in the electronic equipment sector. In what follows, we compare our baseline scenario of *full decouple* in **all sectors** with a *full decouple* **restricted to the electronics equipment sector**. In both scenarios, we assume that the ideas diffusion mechanism works as described by equation (12) and we set $\beta = 0.35$, according to the calibration described before.

[18] North American Treaty Organization (NATO) researchers Kaska, Beckvard, and Minárik (2019) review the arguments put forth from a Western national security perspective. This topic is extremely contentious and some Chinese commentators argue that the U.S. is using national security concerns as excuses to implement industrial policy.

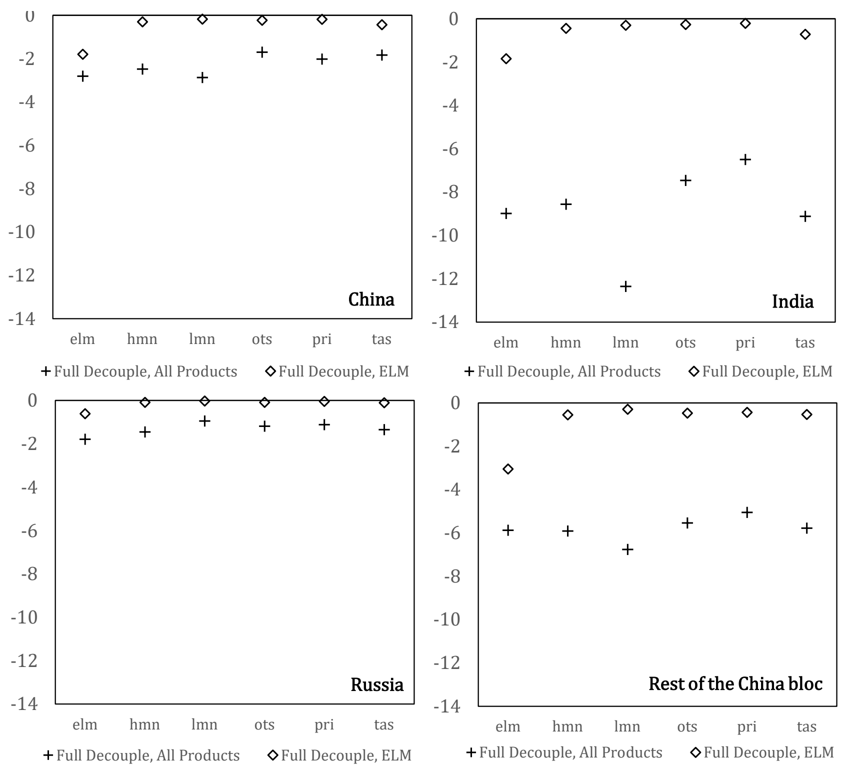

Figure 9: Cumulative Percentage Change of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$, by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points in either all sectors or only in the Electronic Equipment (`elm`) sector. In both cases, we set parameter that controls the the diffusion of ideas according $\beta = 0.35$. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

Note that, due to the multi-sector structure of the model, an increase in iceberg trade costs in one particular sector potentially has an indirect effect in all sectors of the economy. The magnitude of such impact in a given sector can be split between a direct effect (proportional to input use from the `elm` sector as intermediates) and an indirect effect (proportional to the use of the `elm` sector in the production of intermediates inputs).

Results in Figure 9 show the productivity losses induced by policy changes represented by the evolution of the Fréchet Distribution scale parameter $\lambda^i_{d,t}$ for those regions in the China bloc. Contrasting the full decoupling in all sectors and one restricted to electronic equipment shows that, across all regions, productivity losses are substantially reduced and mostly restricted to the `elm` sector.

While there is some negative spillover effect to other sectors due to input-output linkages, particularly to business services (`tas`), these are very small for most regions. Regions such as Russia, which already had limited exposure to Western intermediate sourcing in the main scenario, see productivity losses go down to nearly zero across all sectors under the scenario that limits decoupling to the `elm` sector. China's losses in the `elm` sector are roughly similar to losses when decoupling happens in all products; other sectors are not substantially affected.

All other regions have non-negligible losses in the `elm` sector. The largest changes happen for India and the Rest of the China bloc. Those regions have a lower productivity starting point and benefited proportionately more from exposure to higher quality intermediate inputs. For that reason, full decoupling in all products lead to large differential losses in productivity in those regions. The more restricted full decoupling in `elm` scenario limits losses, since those are proportional to the use of Western electronic equipment as inputs in the production of other sectors.

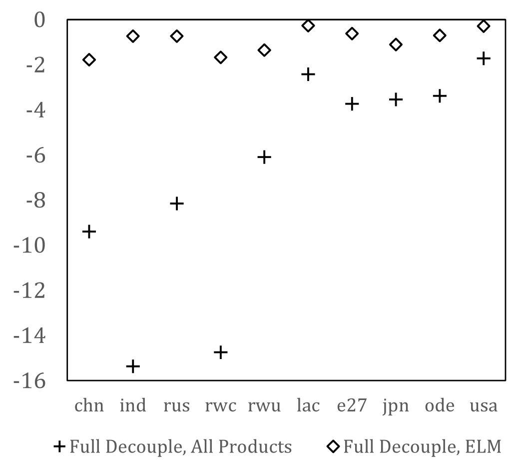

Figure 10: Cumulative Percentage Change in Welfare (Real Income), by scenario. *Full Decouple* increases iceberg trade costs $\tau^i_{sd,t}$ by 160 percentage points in either all sectors or only in the Electronic Equipment (`elm`) sector. In both cases, we set parameter that controls the the diffusion of ideas according $\beta = 0.35$. Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of China bloc; `rwu`, Rest of U.S. bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

Changes in productivity map onto changes in welfare, pictured in Figure 10. While welfare losses are non-zero, ranging from $0.7 - 1.8\%$, they are very different in magnitude to the devastating results of a full decoupling in all products, in which losses range between $8.5 - 15.4\%$.

These results underline two important facts. First, the costs of sector-specific decoupling might be limited enough for this scenario to be feasible. Second, input-output structures play an important role in magnifying dynamic losses. Limiting decoupling to one specific sector tapers down indirect magnification effects that happen through the input-output network.

5 Conclusion

We built a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion in order to realistically investigate the impact of large and persistent trade conflicts on global patterns of economic growth and innovation. Canonical trade models typically start from a fixed technology assumption, which misses many aspects of gains from trade that work from channels other than changes in terms of trade.

In our theoretical contribution, we show that large trade costs can lead to dynamic inefficiencies in knowledge diffusion. Furthermore, we show that in a multi-sector framework, deviations from optimal knowledge diffusion happen both within- and between-sectors. Additionally, in a multi-sector model, sectoral deviations accumulate, such that trade shares being close to their aggregate optimal diffusion points is no longer sufficient to guarantee optimal diffusion. A takeaway of our theoretical discussion is that, as the number of sector increases, so do the number of deviations from optimality and diffusion losses tend to be higher with multiple sectors.

We then use this toolkit to to simulate some a receding globalization characterized by economic decouple between the U.S. and China. Results yield three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, being as large as 15% in some regions. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

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A Tables With Detailed Results

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B Mathematical Derivation of Dynamic Innovation

B.1 Evolution of the Productivity Frontier

In this section, we largely follow the steps of the mathematical appendix to **bueraoberfield2020** to the particularities of our model. For any period, domestic technological frontier evolves according to:

$$F^{m,i}_{d,t+\Delta}(z) = \underbrace{F^{m,i}_{d,t}(z)}_{Pr\{\text{productivity}<z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_\tau z^{-\theta} (z')^{\beta\theta} dG^{m,i}_{d,\tau}(z') d\tau \right)}_{Pr\{\text{no better draws in } (t,t+\Delta)\}}$$

Rearranging and using the definition of the derivative:

$$\frac{d}{dt} \ln F^{m,i}_{s,t}(z) = \lim_{\Delta \to 0} \frac{F^{m,i}_{s,t+\Delta}(z) - F^{m,i}_{s,t}(z)}{F^{m,i}_{s,t}(z)} = -\int \alpha_t z^{-\theta} (z')^{\beta\theta} dG^{m,i}_{d,t}(z')$$

Define $\lambda^{m,i}_{s,t} = \int_{-\infty}^{t} \alpha_\tau \int (z')^{\beta\theta} dG^{m,i}_{s,\tau}(z') d\tau$ and integrate both sides wrt to time:

$$\begin{aligned}
\int_0^t \frac{d}{d\tau} \ln F^{m,i}_{s,\tau}(z) d\tau &= -z^{-\theta} \int_0^t \int \alpha_\tau (z')^{\beta\theta} dG^{m,i}_{d,\tau}(z') d\tau \\
\ln \left(\frac{F^{m,i}_{s,\tau}(z)}{F^{m,i}_{s,0}(z)} \right) &= -z^{-\theta} (\lambda^{m,i}_{s,t} - \lambda^{m,i}_{s,0}) \\
F^{m,i}_{s,t}(z) &= F^{m,i}_{s,0}(z) \exp\{-z^{-\theta} (\lambda^{m,i}_{s,t} - \lambda^{m,i}_{s,0})\}
\end{aligned}$$

Assuming that the initial distribution is Fréchet $F^{m,i}_{s,0}(z) = \exp\{-\lambda^{m,i}_{s,0} z^{-\theta}\}$ guarantees that the distribution will be Fréchet at any point in time:

$$F^{m,i}_{s,t}(z) = \exp\{-\lambda^{m,i}_{s,t} z^{-\theta}\} \tag{A-1}$$

B.2 Law of Motion of Productivity

As seen above, we have defined:

$$\lambda^{m,i}_{s,t} = \int_{-\infty}^{t} \alpha_\tau \int (z')^{\beta\theta} dG^{m,i}_{s,\tau}(z') d\tau$$

Differentiating this definition with respect to time and applying Leibnitz's Lemma yields:

$$\dot{\lambda}^{m,i}_{s,t} = \alpha_t \int (z')^{\beta\theta} dG^{m,i}_{s,t}(z')$$

We use these results and work with a discrete approximation of the law of motion for productivity:

$$\Delta \lambda^{m,i}_{s,t} = \alpha_t \int (z')^{\beta\theta} dG^{m,i}_{s,t}(z') \tag{A-2}$$

The source distribution $G^{m,i}_{d,t}(z') \equiv \sum_{j \in \mathcal{I}} \eta^{i,j}_{d,t-1} \sum_{s \in \mathcal{D}} H^{i,j}_{sd,t-1}(z')$, where $\eta^{i,j}_{d,t}$ is the expenditure share of sector j in the cost of intermediates when producing good i in region d;

and $H^{i,j}_{sd,t-1}(z')$ is the fraction of commodities for which the lowest cost supplier in period $t-1$ is a firm located in $s \in \mathcal{D}$ with productivity weakly less than z'. Finally, we assume that the diffusion parameter can differ by sector-pair. Then:

$$\begin{aligned} \Delta\lambda^i_{d,t} &= \alpha_t \int z^{\beta\theta} dG^{m,i}_{d,t-1}(z) \\ &= \alpha_t \sum_{j\in\mathcal{I}} \eta^{i,j}_{d,t-1} \sum_{s\in\mathcal{D}} \int z^{\beta^{i,j}\theta} dH^{i,j}_{sd,t-1}(z) \end{aligned}$$

We focus our attention on the integral $\int z^{\beta^{i,j}\theta} dH^{i,j}_{sd,t}(z)$. Note that $F^{m,i}_{s,t}(z_2, z_2) = \exp\{-\lambda^{m,i}_{s,t} z_2^{-\theta}\}$ and $F^{m,i}_{s,t}(z_1, z_2) = (1 + \lambda^{m,i}_{s,t}[z_2^{-\theta} - z_1^{-\theta}]) \exp\{-\lambda^{m,i}_{s,t} z_2^{-\theta}\}$ are, respectively, the probability that a productivity draw is below z_2, and that the maximum productivity is z_1 and the second highest productivity is z_2[19]. Let for each n, $A_{n,t} \equiv \tilde{x}^{m,i}_{nd,t}/\tilde{x}^{m,i}_{sd,t}$, such that s will have a lower cost than d iff $A_{n,t} z^{m,i}_{n,t}(\omega) < z^{m,i}_{s,t}(\omega)$. Region s with highest productivity producers z_1, z_2 will supply the commodity $i \in \mathcal{I}$ in region d with the following probability:

$$\begin{aligned} \mathcal{F}^{i,j}_{sd,t-1}(z_1, z_2) &= \int_0^{z_2} \Pi_{n\neq s} F^{m,j}_{n,t-1}\Big(A_{n,t}y, A_{n,t}y\Big) dF^{m,j}_{s,t-1}(y, y) \\ &+ \int_{z_2}^{z_1} \Pi_{n\neq s} F^{m,i}_{n,t-1}\Big(A_{n,t}z_2, A_{n,t}z_2\Big) \frac{d}{dz_1} F^{m,j}_{s,t-1}(z_1, z_2) \end{aligned}$$

The first term in the right hand side denotes the probability that the lowest cost producer at destination d is from s and has productivity lower than z_2, while the second term accounts for the probability that the lowest cost producer at destination d is from s and has productivity in the range $[z_2, z_1)$. We will evaluate each integral separately. First, take the first term:

[19] To see the latter, note that:

$$\begin{aligned} Prob(z_1 \le Z_1, z_2 \le Z_2) &= F^{m,i}_{s,t}(Z_2) + \int_0^{Z_2} \int_{Z_2}^{Z_1} f^{m,i}_{s,t}(y) f^{m,i}_{s,t}(y') dy' dy \\ &= F^{m,i}_{s,t}(Z_2) + F^{m,i}_{s,t}(Z_2)(F^{m,i}_{s,t}(Z_1) - F^{m,i}_{s,t}(Z_2)) \\ &= (1 + \lambda^{m,i}_{s,t}[Z_2^{-\theta} - Z_1^{-\theta}]) \exp\{-\lambda^{m,i}_{s,t} Z_2^{-\theta}\} \end{aligned}$$

$$\int_0^{z_2} \Pi_{n\neq s} F^{m,j}_{n,t-1}\Big(A_{n,t-1}y, A_{n,t-1}y\Big) dF^{m,j}_{s,t-1}(y,y)$$
$$= \int_0^{z_2} \exp\left\{ -\sum_{n\neq s} \lambda^{m,j}_{n,t-1}\Big(A_{n,t-1}y\Big)^{-\theta}\right\} \theta\lambda^{m,j}_{s,t} y^{-\theta-1} \exp\{-\lambda^{m,j}_{s,t-1}y^{-\theta}\}dy$$
$$= \lambda^{m,j}_{s,t-1} \int_0^{z_2} \theta y^{-\theta-1} \exp\left\{ -\sum_n \lambda^{m,j}_{n,t-1}\Big(A_{n,t-1}\Big)^{-\theta} y^{-\theta}\right\}dy$$
$$= \lambda^{m,j}_{s,t-1}\left[\frac{1}{\sum_n \lambda^{m,j}_{n,t-1}\Big(A_{n,t-1}\Big)^{-\theta}} \exp\left\{ -\sum_n \lambda^{m,j}_{n,t-1}\Big(A_{n,t-1}\Big)^{-\theta} y^{-\theta}\right\}\right]_{y=0}^{y=z_2}$$
$$= \pi^{i,j}_{sd,t-1} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\}$$

Now consider the second term.

$$\int_{z_2}^{z_1} \Pi_{n\neq s} F^{m,j}_{n,t-1}\Big(A_{n,t}z_2, A_{n,t}z_2\Big) dF^{m,j}_{s,t-1}(dy, q_2)$$
$$= \int_{z_2}^{z_1} \exp\left\{ -\sum_{n\neq s} \lambda^{m,i}_{n,t-1}\Big(A_{n,t-1}z_2\Big)^{-\theta}\right\} \theta\lambda^{m,j}_{s,t} z_1^{-\theta-1} \exp\{-\lambda^{m,j}_{s,t-1}z_2^{-\theta}\}dz_1$$
$$= \exp\left\{ -\sum_n \lambda^{m,j}_{n,t-1}\Big(A_{n,t-1}z_2\Big)^{-\theta}\right\} \lambda^{m,j}_{s,t-1} \int_{z_2}^{z_1} \theta z_1^{-\theta-1} dz_1$$
$$= \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\} \lambda^{m,j}_{s,t-1}(z_2^{-\theta} - z_1^{-\theta})$$

Therefore:

$$\mathcal{F}^{i,j}_{sd,t-1}(z_1,z_2) = \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\}\Big(\pi^{m,j}_{sd,t-1} + \lambda^{m,j}_{s,t-1}(z_2^{-\theta} - z_1^{-\theta})\Big) \tag{A-3}$$

Note that:

$$\int z^{\beta^{i,j}\theta} dH^{i,j}_{sd,t}(z) = \int_0^\infty \int_{z_2}^\infty z_1^{\beta^{i,j}\theta} \frac{\partial^2 \mathcal{F}^{i,j}_{sd,t-1}(z_1,z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \tag{A-4}$$

and that we can calculate the joint density explicitly:

$$\begin{aligned}
\frac{\partial^2 \mathcal{F}^{m,i}_{sd,t-1}(z_1,z_2)}{\partial z_1 \partial z_2} &= \frac{\partial}{\partial z_2} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\} \theta\lambda^{m,j}_{s,t-1} z_1^{-\theta-1} \\
&= \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\} (\theta\lambda^{m,j}_{s,t-1} z_1^{-\theta-1})(\theta\lambda^{m,j}_{t-1,t} z_2^{-\theta-1})
\end{aligned}$$

Plugging this into (A-4):

$$\int_0^\infty \int_{z_2}^\infty z_1^{\beta^{i,j}\theta} \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{i,j}_{sd,t-1}} z_2^{-\theta}\Big\}(\theta\lambda^{m,j}_{s,t-1} z_1^{-\theta-1})(\theta\lambda^{m,j}_{t-1,t} z_2^{-\theta-1}) dz_1 dz_2$$
$$= \int_0^\infty \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{m,j}_{sd,t-1}} z_2^{-\theta}\Big\}(\theta\lambda^{m,j}_{s,t-1} z_2^{-\theta-1})\lambda^{m,j}_{s,t-1}\int_{z_2}^\infty (\theta z_1^{-\theta(1-\beta^{i,j})-1}) dz_1 dz_2$$
$$= \int_0^\infty \frac{1}{\pi^{i,j}_{sd,t-1}} \exp\Big\{ -\frac{\lambda^{m,j}_{s,t-1}}{\pi^{m,j}_{sd,t-1}} z_2^{-\theta}\Big\}(\theta\lambda^{m,j}_{s,t-1} z_2^{-\theta-1})\lambda^{m,j}_{s,t-1}\frac{1}{1-\beta} z_2^{-\theta(1-\beta)} dz_2$$

Using a change of variables, let $\gamma \equiv \frac{\lambda^{m,i}_{s,t-1}}{\pi^{m,i}_{sd,t-1}} z_2^{-\theta}$, which implies that $d\gamma = -\theta\frac{\lambda^{m,i}_{s,t-1}}{\pi^{m,i}_{sd,t-1}} z_2^{-\theta-1} dz$ Replacing above:

$$(\lambda^{m,j}_{s,t-1})^{\beta^{i,j}}(\pi^{m,j}_{sd,t-1})^{1-\beta^{i,j}}\frac{1}{1-\beta^{i,j}}\int_0^\infty \exp\Big\{-\gamma\Big\}\eta^{(1-\beta^{i,j})} d\gamma$$
$$= (\lambda^{m,j}_{s,t-1})^{\beta}(\pi^{m,j}_{sd,t-1})^{1-\beta^{i,j}}\frac{1}{1-\beta^{i,j}}\Gamma(2-\beta)$$
$$= (\lambda^{m,j}_{s,t-1})^{\beta^{i,j}}(\pi^{m,j}_{sd,t-1})^{1-\beta^{i,j}}\Gamma(1-\beta^{i,j}) \qquad (\because \Gamma(y+1) = y\Gamma(y))$$

Therefore, replacing into the law of motion for the location parameter of the Fréchet distribution:

$$\begin{aligned}
\Delta\lambda^i_{d,t} &= \alpha_t \int z^{\beta\theta} dG^{m,i}_{d,t}(z) \\
&= \alpha_t \sum_{j\in\mathcal{I}} \eta^{i,m,j}_{d,t-1} \sum_{s\in\mathcal{D}} \int z^{\beta^{i,j}\theta} dH^{i,j}_{sd,t-1}(z) \\
&= \alpha_t \sum_{j\in\mathcal{I}} \eta^{i,j}_{d,t-1}\Gamma(1-\beta^{i,j}) \sum_{s\in\mathcal{D}} (\lambda^{m,j}_{s,t-1})^{\beta^{i,j}}(\pi^{m,j}_{sd,t-1})^{1-\beta^{i,j}}
\end{aligned}$$

which is the same expression as in equation (12).

C Optimal Diffusion Levels

C.1 Two-by-Two Economy

If a Benevolent Planner were to chose domestic trade shares to maximize idea diffusion to a given sector at the home economy, she would solve the following concave programming problem:

$$\max_{\{\pi_h^{i,i},\pi_h^{i,-i}\}} \eta^i[(\pi_h^{i,i})^{1-\beta}(\lambda_h^i)^\beta + (1-\pi_h^{i,i})^{1-\beta}(\lambda_f^i)^\beta] + (1-\eta_d^i)[(\pi_h^{i,-i})^{1-\beta}(\lambda_h^{-i})^\beta + (1-\pi_h^{i,-i})^{1-\beta}(\lambda_f^{-i})^\beta] \tag{A-5}$$

For $\pi_h^{i,i}$, the first order condition satisfies:

$$\begin{aligned} \eta^i(1-\beta)[(\pi_h^{i,i})^{-\beta}(\lambda_h^i)^\beta - (1-\pi_h^{i,i})^{-\beta}(\lambda_f^i)^\beta] &= 0 \\ (\pi_h^{i,i})^{-\beta}(\lambda_h^i)^\beta &= (1-\pi_h^{i,i})^{-\beta}(\lambda_f^i)^\beta \\ (\pi_h^{i,i})^{Planner} &= \frac{\lambda_h^i}{\lambda_f^i+\lambda_h^i} \end{aligned}$$

This result is the building block of the ratios that we express in Section 3. If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

$$\left(\frac{\eta^i\pi_h^{i,i}}{\eta^i(1-\pi_h^{i,i})}\right)^{\text{Planner}} = \frac{\lambda_h^i}{\lambda_f^i+\lambda_h^i} \times \left(\frac{\lambda_f^i}{\lambda_f^i+\lambda_h^i}\right)^{-1} = \frac{\lambda_h^i}{\lambda_f^i}$$

Similarly, if we want to write a cross-sector ratio of total domestic trade expenditure shares, we can write:

$$\left(\frac{\eta^i\pi_h^{i,i}}{(1-\eta^i)\pi_h^{i,-i}}\right)^{\text{Planner}} = \underbrace{\frac{\eta^i}{1-\eta^i}}_{\text{cost share}} \times \underbrace{\frac{\lambda_h^i}{\lambda_h^{-i}}}_{\text{own-productivity}} \times \underbrace{\left(\frac{\lambda_h^i+\lambda_f^i}{\lambda_h^{-i}+\lambda_f^{-i}}\right)^{-1}}_{\text{industry-wise productivity}}$$

which is the same as equation (13).

C.2 Multi-Sector, Multi-Region Economy

For each commodity i in macrosector j, the Benevolent Planner maximizes:

$$\max_{\{\pi_{sd,t-1}^{i,j}\}_{j,i\in\mathcal{I},s\in\mathcal{D},m\in\mathcal{M}}} \sum_{j\in\mathcal{I}} \eta_{d,t-1}^{i,j} \sum_{s\in\mathcal{D}} (\pi_{sd,t-1}^{i,j})^{1-\beta}(\lambda_{s,t-1}^{m,j})^\beta \tag{A-6}$$

$$s.t. \quad \forall(m,i,j)\in\mathcal{M}\times\mathcal{I}\times\mathcal{I} \qquad \sum_{s\in\mathcal{D}}\pi_{sd,t-1}^{i,j} = 1$$

Let φ be the Lagrange multiplier. Then, for each (s,m,i,j) first order conditions satisfy:

$$\begin{aligned} (1-\beta)\eta_{d,t-1}^{i,j}(\pi_{sd,t-1}^{i,j})^\beta(\lambda_{s,t-1}^{m,j})^\beta &= \varphi \\ (\pi_{sd,t-1}^{i,j})^{Planner} &= \varphi^{-\frac{1}{\beta}}[(1-\beta)\eta_{d,t-1}^{i,j}]^{\frac{1}{\beta}}\lambda_{s,t-1}^{m,j} \end{aligned}$$

using the constraint:

$$\sum_{s\in\mathcal{D}}(\varphi^{-\frac{1}{\beta}}[(1-\beta)\eta^{i,j}_{d,t-1}]^{\frac{1}{\beta}}\lambda^{m,j}_{s,t-1})=1 \iff \varphi^{-\frac{1}{\beta}}=[(1-\beta)\eta^{i,j}_{d,t-1}]^{-\frac{1}{\beta}}(\sum_{s\in\mathcal{D}}\lambda^{m,j}_{s,t-1})^{-1}$$

Therefore:

$$(\pi^{i,j}_{sd,t-1})^{Planner}=\frac{\lambda^{m,j}_{s,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,j}_{k,t-1}} \tag{A-7}$$

If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

$$\left(\frac{\eta^{i,j}_{d,t-1}\pi^{i,j}_{sd,t-1}}{\eta^{i,j}_{d,t-1}\pi^{i,j}_{nd,t-1}}\right)^{\text{Planner}}=\frac{\lambda^{m,j}_{s,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,j}_{k,t-1}}\times\left(\frac{\lambda^{m,j}_{n,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,j}_{k,t-1}}\right)^{-1}=\frac{\lambda^{m,j}_{s,t-1}}{\lambda^{m,j}_{n,t-1}}$$

Similarly, if we want to write a cross-sector ratio of total domestic trade expenditure shares, we can write:

$$\left(\frac{\eta^{i,j}_{d,t-1}\pi^{i,j}_{sd,t-1}}{\eta^{m,i,p}_{d,t-1}\pi^{m,i,p}_{nd,t-1}}\right)^{\text{Planner}}=\frac{\eta^{i,j}_{d,t-1}}{\eta^{m,i,p}_{d,t-1}}\times\frac{\lambda^{m,j}_{s,t-1}}{\lambda^{m,p}_{n,t-1}}\times\left(\frac{\sum_{k\in\mathcal{D}}\lambda^{m,j}_{k,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,p}_{k,t-1}}\right)^{-1}$$

which is analogous to equation 13. The free trade allocation under the multi-country, multi-sector framework satisfies:

$$\left(\frac{\eta^{i,j}_{d,t-1}\pi^{i,j}_{sd,t-1}}{\eta^{m,i,p}_{d,t-1}\pi^{m,i,p}_{nd,t-1}}\right)^{\text{Free Trade}}=\frac{\eta^{i,j}_{d,t-1}}{\eta^{m,i,p}_{d,t-1}}\times\frac{\lambda^{m,j}_{s,t-1}(\tilde{x}^{m,i}_{sd,t-1})^{-\theta}}{\lambda^{m,p}_{n,t-1}(\tilde{x}^{m,p}_{nd,t-1})^{-\theta}}\times\left(\frac{\sum_{k\in\mathcal{D}}\lambda^{m,i}_{k,t}(\tilde{x}^{m,i}_{kd,t})^{-\theta}}{\sum_{k\in\mathcal{D}}\lambda^{m,p}_{k,t}(\tilde{x}^{m,p}_{kd,t})^{-\theta}}\right)^{-1}$$

which is analogous to equation 14. The ratio the free trade allocation for the planner's allocation satisfies:

$$\aleph=\frac{(\tilde{x}^{m,i}_{sd,t-1})^{-\theta}}{(\tilde{x}^{m,p}_{nd,t-1})^{-\theta}}\times\left(\frac{\sum_{k\in\mathcal{D}}\lambda^{m,j}_{k,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,i}_{k,t}(\tilde{x}^{m,i}_{kd,t})^{-\theta}}\right)\times\left(\frac{\sum_{k\in\mathcal{D}}\lambda^{m,p}_{k,t-1}}{\sum_{k\in\mathcal{D}}\lambda^{m,p}_{k,t}(\tilde{x}^{m,p}_{kd,t})^{-\theta}}\right)^{-1}$$

D Other Mathematical Derivations

D.1 Demand and expenditure functions of the upper nest Cobb-Douglas

First order conditions satisfy:

$$e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t}) = \frac{\mathbb{U}_{d,t}}{\lambda}\kappa^m_d\varphi^m_{d,t} \tag{A-8}$$

where $\varphi^m_{d,t} \equiv \frac{\partial q^m_{d,t}}{\partial e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t})}\frac{e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t})}{q^m_{d,t}}$ is the elasticity of consumption in macro-sector m with respect to expenditure on that sector. By summing up over macro-sectors $m \in \mathcal{M}$ we can solve for λ:

$$\sum_{m\in\mathcal{M}} e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t}) = Y_{d,t} = \frac{\mathbb{U}_{d,t}}{\lambda}\sum_{m\in\mathcal{M}}\kappa^m_d\varphi^m_{d,t} \implies \lambda = \frac{\mathbb{U}_{d,t}}{Y_{d,t}}\sum_{m\in\mathcal{M}}\kappa^m_d\varphi^m_{d,t} \tag{A-9}$$

Replacing this on equation $(A-8)$ delivers the expenditure function:

$$e^m_{d,t}(q^m_{d,t};\mathbf{p}_{d,t},Y_{d,t}) = \kappa^m_d Y_{d,t}\varphi^m_{d,t}\frac{1}{\sum_{n\in\mathcal{M}}\kappa^n_d\varphi^n_{d,t}} \equiv \kappa^m_d Y_{d,t}\frac{\varphi^m_{d,t}}{\boldsymbol{\varphi}_{d,t}} \tag{A-10}$$

where $\boldsymbol{\varphi}_{d,t} = \frac{\partial\mathbb{U}_{d,t}}{\partial Y_{d,t}}\frac{Y_{d,t}}{\mathbb{U}_{d,t}}$ is the elasticity of aggregate utility with respect with respect to total income. Following McDougall (2003), we can show that $\boldsymbol{\varphi}^m_{d,t} = \sum_{m\in\mathcal{M}}\kappa^m_d\varphi^m_{d,t}$. First recall that, by the envelope theorem, once optimized, $\frac{\partial\mathbb{U}_{d,t}}{\partial Y_{d,t}} = \lambda$. Then, the derivation follows smoothly:

$$\begin{aligned}
\boldsymbol{\varphi}_{d,t} &= \frac{\partial\mathbb{U}_{d,t}}{\partial Y_{d,t}}\frac{Y_{d,t}}{\mathbb{U}_{d,t}} \\
&= \frac{Y_{d,t}}{\mathbb{U}_{d,t}}\lambda \\
&= \frac{Y_{d,t}}{\mathbb{U}_{d,t}}\frac{\mathbb{U}_{d,t}}{Y_{d,t}}\sum_{m\in\mathcal{M}}\kappa^m_d\varphi^m_{d,t} \\
&= \sum_{m\in\mathcal{M}}\kappa^m_d\varphi^m_{d,t}
\end{aligned}$$

as required.

D.2 Demand functions and price levels of non-homothetic CES

The Lagrangian of this maximization problem can be set up as follows, with ρ and λ as Lagrange multipliers:

$$\max_{\max_{\{q^{pr,i}_{d,t}\}_{i\in\mathcal{I}}}} \mathcal{L} = g(q^{pr}_{d,t}) + \rho\left[1 - \sum_{i\in\mathcal{I}}[\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}]^{\frac{1}{\sigma}}(q^{pr,i}_{d,t})^{\frac{\sigma-1}{\sigma}}\right] + \lambda\left[e^{pr}_{d,t} - \sum_{i\in\mathcal{I}}p^{pr,i}_{d,t}q^{pr,i}_{d,t}\right]$$

The $|\mathcal{I}|$ first-order conditions satisfy:

$$\rho\frac{\sigma-1}{\sigma}[\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}]^{\frac{1}{\sigma}}(q^{pr,i}_{d,t})^{-\frac{1}{\sigma}} = \lambda p^{pr,i}_{d,t} \tag{A-11}$$

Multiplying both sides by C_i and then summing over i results in:

$$\begin{aligned}
\rho\frac{\sigma-1}{\sigma}\underbrace{\sum_{i\in\mathcal{I}}[\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}]^{\frac{1}{\sigma}}(q^{pr,i}_{d,t})^{\frac{\sigma-1}{\sigma}}}_{=1} &= \lambda\underbrace{\sum_{i\in\mathcal{I}}\sum_{i\in\mathcal{I}}p^{pr,i}_{d,t}q^{pr,i}_{d,t}}_{=e^{pr}_{d,t}} \\
\therefore \qquad e^{pr}_{d,t} &= \frac{\rho}{\lambda}\frac{\sigma-1}{\sigma}
\end{aligned}$$

Replacing this result into (A-11) and solving for $q^{pr,i}_{d,t}$:

$$q^{pr,i}_{d,t} = \Psi_i(q^{pr}_{d,t})^{\varepsilon_i}\Big(\frac{p^{pr,i}_{d,t}}{e^{pr}_{d,t}}\Big)^{-\sigma}$$

which is a familiar expression for CES preferences. As made explicit above, the parameter ε_i controls demand elasticity of good i with respect to total real expenditure.

Finally, we can obtain an expression for the ideal price index $e^{pr}_{d,t}$ through the following steps. First, multiplying both sides in (2.1) by $p^{pr,i}_{d,t}$ and sum over i:

$$\sum_{i\in\mathcal{I}}p^{pr,i}_{d,t}q^{pr,i}_{d,t} = e^{pr}_{d,t} = \sum_{i\in\mathcal{I}}\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}\Big(\frac{p^{pr,i}_{d,t}}{e^{pr}_{d,t}}\Big)^{-\sigma}p^{pr,i}_{d,t}$$

Then, solve for expenditure in private goods $p^{pr}_{d,t}$:

$$e^{pr}_{d,t}(p^{pr,1}_{d,t},\cdots,p^{pr,|\mathcal{I}|}_{d,t};q^{pr}_{d,t}) = \left[\sum_{i\in\mathcal{I}}\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}(p^{pr,i}_{d,t})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A-12}$$

which gives an implicit expression for $p^{pr}_{d,t}$.

Expenditure shares can be derived in the following fashion:

$$\eta^{pr,i}_{d,t} = \frac{p^{pr,i}_{d,t}q^{pr,i}_{d,t}}{\sum_{j\in\mathcal{I}}p^{pr,j}_{d,t}q^{pr,j}_{d,t}} = \frac{\Psi_i(q^{pr}_{d,t})^{\varepsilon_i}(p^{pr,i}_{d,t})^{1-\sigma}}{\sum_{j\in\mathcal{I}}\Psi_j(q^{pr}_{d,t})^{\varepsilon_j}(p^{pr,j}_{d,t})^{1-\sigma}}$$

D.3 Trade shares

In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region $d\in\mathcal{D}$ on goods coming from source country $s\in\mathcal{D}$ converge to their expected values. Let $\pi^{m,i}_{sd,t}$ denote the share of expenditures of consumers in region $d\in\mathcal{D}$ on commodity $i\in\mathcal{I}^m$ coming from region $s\in\mathcal{D}$ and, let for each n, $A^{-1}_{n,t}\equiv\tilde{x}^{m,i}_{sd,t}/\tilde{x}^{m,i}_{nd,t}$. This share will satisfy:

$$
\begin{aligned}
\pi^{m,i}_{sd,t} &= Pr\Big(\frac{\tilde{x}^{m,i}_{sd,t}}{z^{m,i}_{s,t}(\omega)} < \min_{(n\neq s)}\Big\{\frac{\tilde{x}^{m,i}_{nd,t}}{z^{m,i}_{n,t}(\omega)}\Big\}\Big) \\
&= \int_0^\infty Pr(z^{m,i}_{s,t}(\omega)=z)Pr(z^{m,i}_{n,t}(\omega)<zA_n)dz \\
&= \int_0^\infty f^{m,i}_{s,t}(z)\Pi_{(n\neq s)}F_{n,t}(A_n z)dz \\
&= \int_0^\infty \theta\lambda^{m,i}_{s,t}z^{-(1+\theta)}e^{-(\sum_{n\in\mathcal{D}}\lambda^{m,i}_{n,t}A_n^{-\theta})z^{-\theta}}dz \\
&= \frac{\lambda^{m,i}_{s,t}(\tilde{x}^{m,i}_{sd,t})^{-\theta}}{\sum_{n\in\mathcal{D}}\lambda^{m,i}_{n,t}(\tilde{x}^{m,i}_{md,t})^{-\theta}} \\
&= \frac{\lambda^{m,i}_{s,t}(\tilde{x}^{m,i}_{sd,t})^{-\theta}}{\Phi^{m,i}_{d,t}} \qquad \text{(A-13)}
\end{aligned}
$$

Similarly, since countries use the same aggregate final goods as intermediate inputs, cost shares in intermediates for each supplying sector j and region s used in the production of good i in region d satisfies:

$$
\pi^{i,j}_{sd,t} = \frac{\lambda^{m,i}_{s,t}(\tilde{x}^{m,i}_{sd,t})^{-\theta}}{\Phi^{m,i}_{d,t}} \qquad \text{(A-14)}
$$

which are the same as expressed in (7) and (**??**).

D.4 Price levels

Recall from equations (2.2) and (**??**) that the prices of commodities and intermediate goods can be expressed, respectively, as:

$$
p^{m,i}_{d,t} = \Big[\int_{[0,1]} p^{m,i}_{d,t}(\omega)^{1-\sigma}d\omega\Big]^{\frac{1}{1-\sigma}}
$$

Let $\Omega^{m,i}_{sd,t}$ and $\Omega^{i,j}_{sd,t}$ denote the subsets of $\Omega = [0,1]$ for which the region $s \in \mathcal{D}$ is a supplier in destination region $d \in \mathcal{D}$. We can then rewrite price levels above as:

$$
p^{m,i}_{d,t} = \Big[\sum_{s\in\mathcal{D}}\int_{\Omega^{m,i}_{sd,t}} p^{m,i}_{d,t}(\omega)^{1-\sigma}d\omega\Big]^{\frac{1}{1-\sigma}}
$$

Similarly, we restate $\mathcal{F}^{m,i}_{sd,t}(z_1,z_2)$ and the analogous measure $\mathcal{F}^{i,j}_{sd,t}(z_1,z_2)$:

$$
\mathcal{F}^{m,i}_{sd,t}(z_1,z_2) = \exp\Big\{-\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}z_2^{-\theta}\Big\}\Big(\pi^{m,i}_{sd,t}+\lambda^{m,i}_{s,t}(z_2^{-\theta}-z_1^{-\theta})\Big) \qquad \text{(A-15)}
$$

which denote the fraction of varieties that d purchases from s with productivity up to z_1 and whose second best producer is not more efficient than than z_2. Recall that, from the Bertrand competition assumption, we can write, for each variety ω:

$$p_{d,t}^{m,i}(\omega) = \min\left\{\frac{\sigma}{\sigma-1}\frac{\tilde{x}_{sd,t}^{m,i}}{z_{1s,t}^{m,i}(\omega)}, \frac{\tilde{x}_{sd,t}^{m,i}}{z_{2s,t}^{m,i}(\omega)}\right\}$$

So we can rewrite the equation $\int_{\Omega_{sd,t}^{m,i}} p_{d,t}^{m,i}(\omega)^{1-\sigma}d\omega$ in the following fashion:

$$\begin{aligned}
&\int_{\Omega_{sd,t}^{m,i}} p_{d,t}^{m,i}(\omega)^{1-\sigma}d\omega \\
= &\int_0^\infty \int_{z_2}^\infty (p_{d,t}^{m,i})^{1-\sigma}\frac{\partial^2 \mathcal{F}_{sd,t}^{m,i}(z_1,z_2)}{\partial z_1 \partial z_2}dz_1 dz_2 \\
= &\int_0^\infty \int_{z_2}^\infty \min\left\{\frac{\sigma}{\sigma-1}\frac{\tilde{x}_{sd,t}^{m,i}}{z_1}, \frac{\tilde{x}_{sd,t}^{m,i}}{z_2}\right\}^{1-\sigma}\frac{1}{\pi_{sd,t}^{m,i}}\exp\Big\{-\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}z_2^{-\theta}\Big\}(\theta\lambda_{s,t}^{m,i}z_1^{-\theta-1})(\theta\lambda_{s,t}^{m,i}z_2^{-\theta-1})dz_1dz_2
\end{aligned}$$

With a change of variables, denote $\eta_1 \equiv \frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}z_1^{-\theta}$ and $\eta_2 \equiv \frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}z_2^{-\theta}$ and $d\eta_1 = -\frac{\theta\lambda_{s,t}^{m,i}z_1^{-\theta-1}}{\pi_{sd,t-1}^{m,i}}dz_1$, $d\eta_2 = -\frac{\theta\lambda_{s,t}^{m,i}z_2^{-\theta-1}}{\pi_{sd,t-1}^{m,i}}dz_2$, which allows U.S. to rewrite the equation above as:

$$\begin{aligned}
&\int_{\Omega_{sd,t}^{m,i}} p_{d,t}^{m,i}(\omega)^{1-\sigma}d\omega \\
= &\pi_{sd,t}^{m,i}\int_0^\infty\int_0^{\eta_2}\min\left\{\frac{\sigma}{\sigma-1}\frac{\tilde{x}_{sd,t}^{m,i}}{z_1}, \frac{\tilde{x}_{sd,t}^{m,i}}{z_2}\right\}^{1-\sigma}\exp\Big\{-\eta_2\Big\}d\eta_1 d\eta_2 \\
= &\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}_{sd,t}^{m,i})^{1-\sigma}\int_0^\infty\int_0^{\eta_2}\min\left\{\Big(\frac{\sigma}{\sigma-1}\Big)^{\theta}\eta_1,\eta_2\right\}^{\frac{1-\sigma}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_1 d\eta_2 \\
= &\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}_{sd,t}^{m,i})^{1-\sigma}\Bigg[\int_0^\infty\int_{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\eta_2}^{\eta_2}\eta_2^{\frac{1-\sigma}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_1 d\eta_2 \\
+ &\int_0^\infty\int_0^{\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\eta_2}\Big(\frac{\sigma}{\sigma-1}\Big)^{1-\sigma}\eta_1^{\frac{1-\sigma}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_1 d\eta_2\Bigg] \\
= &\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}_{sd,t}^{m,i})^{1-\sigma}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}+\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\cdot\int_0^\infty\eta_2^{\frac{1-\sigma}{\theta}+1}\exp\Big\{-\eta_2\Big\}d\eta_2 \\
= &\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}_{sd,t}^{m,i})^{1-\sigma}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}+\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\Gamma\Big(\frac{1-\sigma}{\theta}+2\Big) \\
= &\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}_{sd,t}^{m,i})^{1-\sigma}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}+\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\frac{1-\sigma+\theta}{\theta}\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big) \\
= &\Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\cdot\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\cdot\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}(\tilde{x}_{sd,t}^{m,i})^{-\theta}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}} \\
= &\Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\cdot\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\cdot\pi_{sd,t}^{m,i}\Big(\frac{\lambda_{s,t}^{m,i}(\tilde{x}_{sd,t}^{m,i})^{-\theta}}{\pi_{sd,t}^{m,i}}\Big)^{-\frac{1-\sigma}{\theta}} \\
= &\Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\cdot\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\cdot\pi_{sd,t}^{m,i}\Big(\sum_{n\in\mathcal{D}}\lambda_{n,t}^{m,i}(\tilde{x}_{nd,t}^{m,i})^{-\theta}\Big)^{-\frac{1-\sigma}{\theta}}
\end{aligned}$$

Therefore:

$$
\begin{aligned}
p^{m,i}_{d,t} &= \Big[\sum_{s\in\mathcal{D}}\int_{\Omega^{m,i}_{sd,t}} p^{m,i}_{d,t}(\omega)^{1-\sigma}d\omega\Big]^{\frac{1}{1-\sigma}} \\
p^{m,i}_{d,t} &= \Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]^{\frac{1}{1-\sigma}}\cdot\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)^{\frac{1}{1-\sigma}}\cdot\Big(\sum_{n\in\mathcal{D}}\lambda^{m,i}_{n,t}(\tilde{x}^{m,i}_{nd,t})^{-\theta}\Big)^{-\frac{1}{\theta}}\cdot\Big[\sum_{s\in\mathcal{D}}\pi^{m,i}_{sd,t}\Big]^{\frac{1}{1-\sigma}} \\
p^{m,i}_{d,t} &= \Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]^{\frac{1}{1-\sigma}}\cdot\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)^{\frac{1}{1-\sigma}}\cdot\Big(\sum_{n\in\mathcal{D}}\lambda^{m,i}_{n,t}(\tilde{x}^{m,i}_{nd,t})^{-\theta}\Big)^{-\frac{1}{\theta}} \qquad \text{(A-16)}
\end{aligned}
$$

Which is the same as (6).

D.5 Marginal costs and profits

From equation (2.2) we can derive standard CES demand functions as:

$$
\begin{aligned}
q^{m,i}_{d,t}(\omega) &= \Big(\frac{p^{m,i}_{d,t}(\omega)}{p^{m,i}_{d,t}}\Big)^{-\sigma}\frac{e^{m,i}_{d,t}}{p^{m,i}_{d,t}} \qquad \text{(A-17)} \\
c^{i,j}_{d,t}(\omega) &= \Big(\frac{p^{m,j}_{d,t}(\omega)}{p^{m,j}_{d,t}}\Big)^{-\sigma}\frac{e^{i,j}_{d,t}}{pc^{m,j}_{d,t}} \qquad \text{(A-18)}
\end{aligned}
$$

where $p^{m,i}_{d,t}$ satisfies equations (6); $e^{m,i}_{d,t}$ denotes expenditure on commodity i of macro-sector m in country d; and $e^{i,j}_{d,t}$ denotes expenditure on intermediate input j used in the production of commodity i of macro-sector m in country d.

As in previous subsections of the Appendix, we will derive the expression for the marginal cost and mark-up for the production of variety $q^{m,i}_{d,t}(\omega)$ and state a corresponding expression for $c^{i,j}_{d,t}(\omega)$. The marginal cost of producing variety ω sourced in country s and consumed in country s is:

$$
\frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}q^{m,i}_{d,t}(\omega)
$$

and total cost of varieties sourced in country s and consumed in country s can be expressed as:

$$
\int_{\Omega^{m,i}_{sd,t}}\frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}q^{m,i}_{d,t}(\omega)d\omega = \int_{\Omega^{m,i}_{sd,t}}\frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}\Big(\frac{p^{m,i}_{d,t}(\omega)}{p^{m,i}_{d,t}}\Big)^{-\sigma}\frac{e^{m,i}_{d,t}}{p^{m,i}_{d,t}}d\omega
$$

As in the previous section of the Appendix, we let $\Omega^{m,i}_{sd,t}$ and $\Omega^{i,j}_{sd,t}$ denote the subsets of $\Omega=[0,1]$ for which the region $s\in\mathcal{D}$ is a supplier in destination region $d\in\mathcal{D}$. We can then rewrite the integral above as:

$$
\begin{aligned}
&\int_{\Omega^{m,i}_{sd,t}} \frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}\Big(\frac{p^{m,i}_{d,t}(\omega)}{p^{m,i}_{d,t}}\Big)^{-\sigma}\frac{e^{m,i}_{d,t}}{p^{m,i}_{d,t}}d\omega \\
= \quad & \tilde{x}^{m,i}_{d,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\int_0^\infty\int_{z_2}^\infty (z_1)^{-1}(p^{m,i}_{d,t})^{-\sigma}\frac{\partial^2\mathcal{F}^{m,i}_{sd,t}(z_1,z_2)}{\partial z_1\partial z_2}dz_1dz_2 \\
= \quad & \tilde{x}^{m,i}_{d,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\int_0^\infty\int_{z_2}^\infty \frac{1}{z_1}\min\Big\{\frac{\sigma}{\sigma-1}\frac{\tilde{x}^{m,i}_{sd,t}}{z_1},\frac{\tilde{x}^{m,i}_{sd,t}}{z_2}\Big\}^{-\sigma}\frac{1}{\pi^{m,i}_{sd,t}}\exp\Big\{-\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}z_2^{-\theta}\Big\}(\theta\lambda^{m,i}_{s,t}z_1^{-\theta-1})(\theta\lambda^{m,i}_{s,t}z_2^{-\theta-1})dz_1dz_2
\end{aligned}
$$

Once again, use a change of variables, denote $\eta_1 \equiv \frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}z_1^{-\theta}$ and $\eta_2 \equiv \frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}z_2^{-\theta}$ and $d\eta_1 = -\frac{\theta\lambda^{m,i}_{s,t}z_1^{-\theta-1}}{\pi^{m,i}_{sd,t-1}}dz_1$, $d\eta_2 = -\frac{\theta\lambda^{m,i}_{s,t}z_2^{-\theta-1}}{\pi^{m,i}_{sd,t-1}}dz_2$, which allows U.S. to rewrite the equation above as:

$$
\begin{aligned}
&\int_{\Omega^{m,i}_{sd,t}} \frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}\Big(\frac{p^{m,i}_{d,t}(\omega)}{p^{m,i}_{d,t}}\Big)^{-\sigma}\frac{e^{m,i}_{d,t}}{p^{m,i}_{d,t}}d\omega \\
= \quad & \pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1-\sigma}\int_0^\infty\int_0^{\eta_2}\eta_1^{\frac{1}{\theta}}\min\Big\{\Big(\frac{\sigma}{\sigma-1}\Big)^{\theta}\eta_1,\eta_2\Big\}^{-\frac{\sigma}{\theta}}d\eta_1d\eta_2 \\
= \quad & \pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1-\sigma}\Bigg[\int_0^\infty\int_{\big(\frac{\sigma}{\sigma-1}\big)^{-\theta}\eta_2}^{\eta_2}\eta_1^{\frac{1}{\theta}}\eta_2^{\frac{-\sigma}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_1d\eta_2 \\
+ \quad & \int_0^\infty\int_0^{\big(\frac{\sigma}{\sigma-1}\big)^{-\theta}\eta_2}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\sigma}\eta_1^{\frac{1-\sigma}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_1d\eta_2\Bigg] \\
= \quad & \pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1-\sigma}\Bigg[\int_0^\infty\frac{\theta}{1+\theta}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]\eta_2^{\frac{1-\sigma+\theta}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_2 \\
+ \quad & \int_0^\infty\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\eta_1^{\frac{1-\sigma+\theta}{\theta}}\exp\Big\{-\eta_2\Big\}d\eta_2\Bigg] \\
= \quad & \pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1-\sigma}\Bigg[\frac{\theta}{1+\theta}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]+\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Bigg]\Gamma\Big(\frac{1-\sigma}{\theta}+2\Big) \\
= \quad & \pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1}-\sigma\Bigg[\frac{\theta}{1+\theta}\Big[1-\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]+\frac{\theta}{1-\sigma+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Bigg]\frac{1-\sigma+\theta}{\theta}\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big) \\
= \quad & \Big[1-\frac{\sigma}{1+\theta}+\frac{\sigma}{1+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1-\sigma}{\theta}}(\tilde{x}^{m,i}_{sd,t})^{1-\sigma} \\
= \quad & \Big[1-\frac{\sigma}{1+\theta}+\frac{\sigma}{1+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\frac{(\tilde{x}^{m,i}_{sd,t})^{-\theta}\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{\frac{\sigma}{\theta}}\Big(\frac{(\tilde{x}^{m,i}_{sd,t})^{-\theta}\lambda^{m,i}_{s,t}}{\pi^{m,i}_{sd,t}}\Big)^{-\frac{1}{\theta}} \\
= \quad & \Big[1-\frac{\sigma}{1+\theta}+\frac{\sigma}{1+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\pi^{m,i}_{sd,t}\frac{e^{m,i}_{d,t}}{(p^{m,i}_{d,t})^{1-\sigma}}\Big(\sum_{n\in\mathcal{D}}(\tilde{x}^{m,i}_{nd,t})^{-\theta}\lambda^{m,i}_{n,t}\Big)^{-\frac{1-\sigma}{\theta}}
\end{aligned}
$$

Using the expression for $(p^{m,i}_{d,t})^{1-\sigma}$:

$$
\begin{aligned}
&= \frac{\Big[1-\frac{\sigma}{1+\theta}+\frac{\sigma}{1+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\pi^{m,i}_{sd,t}e^{m,i}_{d,t}\Big(\sum_{n\in\mathcal{D}}(\tilde{x}^{m,i}_{nd,t})^{-\theta}\lambda^{m,i}_{n,t}\Big)^{-\frac{1-\sigma}{\theta}}}{\Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]\Gamma\Big(\frac{1-\sigma+\theta}{\theta}\Big)\Big(\sum_{n\in\mathcal{D}}\lambda^{m,i}_{n,t}(\tilde{x}^{m,i}_{nd,t})^{-\theta}\Big)^{-\frac{1-\sigma}{\theta}}} \\
&= \frac{\Big[1-\frac{\sigma}{1+\theta}+\frac{\sigma}{1+\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-1-\theta}\Big]}{\Big[1-\frac{\sigma-1}{\theta}+\frac{\sigma-1}{\theta}\Big(\frac{\sigma}{\sigma-1}\Big)^{-\theta}\Big]}\pi^{m,i}_{sd,t}e^{m,i}_{d,t} \\
&= \frac{\theta}{1+\theta}\frac{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}}{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}}\pi^{m,i}_{sd,t}e^{m,i}_{d,t}
\end{aligned}
$$

Therefore, total cost equals:

$$
C^{m,i}_{s,t} = \sum_{d\in\mathcal{D}}\int_{\Omega^{m,i}_{sd,t}}\frac{\tilde{x}^{m,i}_{d,t}}{z_1(\omega)}\Big(\frac{p^{m,i}_{d,t}(\omega)}{p^{m,i}_{d,t}}\Big)^{-\sigma}\frac{e^{m,i}_{d,t}}{p^{m,i}_{d,t}}d\omega = \frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi^{m,i}_{sd,t}e^{m,i}_{d,t} \tag{A-19}
$$

Profits can be expressed compactly as total revenue minus total cost:

$$
\Pi^{m,i}_{s,t} = \sum_{d\in\mathcal{D}}\pi^{m,i}_{sd,t}e^{m,i}_{d,t} - \frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi^{m,i}_{sd,t}e^{m,i}_{d,t} = \frac{1}{1+\theta}\sum_{d\in\mathcal{D}}\pi^{m,i}_{sd,t}e^{m,i}_{d,t} \tag{A-20}
$$

Analogously, total costs and profits of intermediary producers are, respectively:

$$
C^{m,i}_{s,t} = \frac{\theta}{1+\theta}\sum_{d\in\mathcal{D}}\pi^{i,j}_{sd,t}e^{i,j}_{d,t}, \qquad \Pi^{m,i}_{s,t} = \frac{1}{1+\theta}\sum_{d\in\mathcal{D}}\pi^{i,j}_{sd,t}e^{i,j}_{d,t} \tag{A-21}
$$