Fiscal sustainability, public expenditure and economic growth

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Abstract

There are many investment opportunities in a society that are not taken advantage because their benefits cannot be appropriated by the private sector, but which could generate positive externalities of undoubted benefit to society. Therefore, it is a matter of creating public-private schemes that guarantee the appropriateness of these externalities by private agents, and which therefore trigger virtuous circles of economic growth that are beneficial to society.

In this sense, this work evaluates an infrastructure construction program through public-private partnerships, using a Recursive Computable General Equilibrium Dynamic Model, which explains in detail the functioning of the Colombian economy. The model considers formal and informal markets, monopolistic competition, and includes heterogeneous companies, using a Melitz type model.

The emphasis of the modeling is on 3 aspects, which correspond to the trilemma of public finances: 1) short-term effects, assessed through an "investment-led" macroeconomic closing, which allows capturing the impact of investment in roads on the level of economic activity; 2) the inclusion of externalities in productive functions, to capture supply and productivity effects; and 3) the careful and detailed modeling of public debt, including endogenous revenues that reflect productive activity, the financial costs of borrowing, that are sensitive to the level of that borrowing, and the payment of investment through "future funds"provisions.

Keywords: Private-public partnerships, externalities, public finance trilemma, G-RDEM recursive-dynamic Computable General Equilibrium model, construction of infrastructure and economic growth.
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1. Introduction

After the Great Recession, the world seems to have recognized that government budget constraints are much more binding than was previously thought. This occurs at a time when public spending is a key tool for driving economic dynamism, even as the international context becomes more complex owing to risks such as those associated with trade wars and the symptoms of secular stagnation that some agents observe.This has led to complex discussions between those who emphasize the sustainability risks that would arise from not addressing binding restrictions, and therefore recommend fiscal austerity policies, and those who insist on the need to find space for public spending, in order to face not only the problems of growth dynamics, but also the distributive problems, which have gained prominence in recent years.

The discussion is, however, meaningless: both sides are right in their considerations. The issue of sustainability is not an issue that has to do with the subjective opinion of analysts, but rather an objective issue of access to capital markets: governments with a poor management of public finances, will lose much more in the long-term, as the costs associated with the excess spending would outweigh the benefits. But this, of course, should not make us forget the urgent need for public policies that encourage growth, improve the reallocation of resources, boost innovation and productivity, and confront the problems of legitimacy that derive from the generalized perceptions about the distributive issues that affect the world today.

Consequently, the solution must be in the public policies careful designed, while maintaining strict compliance with budget restrictions, make room for spending tools with a long-term impact on economic activity. The problem is that these policies do not always seem to be within the reach of governments, they imply important risks that must be anticipated, faced and mitigated, and often require sophisticated models for their evaluation, which not only incorporate short-term effects, but also account the sustainability problems that capital markets incorporate in their decisions.

The thesis of this paper is that there is a wide field for these policies, in the productive relationship of governments with private capital. There are many investment opportunities in a society, which are not exploited because its benefits are not privately appropriable, but which generate positive externalities of undoubted benefit to society. It is then a matter of creating public-private schemes that convert externalities into positive benefits to society, guarantee their appropriability by private agents, at least to the point where they justify the investment, and thus unleash virtuous circles of economic growth which benefit society.

A typical example is the Colombian transportation infrastructure investment program, through Public-Private Partnerships (PPP), and which is generally known as the 4G (fourth generation) highways program. The program involves the construction of nearly 8,000 km of roads, through concession programs, and manages to solve what we could call the fundamental trilemma of public finances: mobilize public resources efficiently, without sacrificing the financial sustainability of public entities, and generating welfare benefits for the entire society.

The program invites the private sector to make investments in infrastructure, guaranteeing an adequate financial return, through “future budgetary commitments”, and therefore has three types of effects: short-term Keynesian effects, due to the role that public investment it has in the short term dynamics of the economy (demand effects); positive externalities on production and competitiveness, allowing the country’s producers to connect with international markets (supply effects); and positive effects on public finances because, when the programs are well structured, they generate public revenues in advance, which are then used to pay the disbursements contemplated in the “future funds” (sustainability effect).

This paper evaluates the 4G Road Program using a Recursive Computable General Equilibrium Dynamic Model, which explains in detail the operation of the Colombian economy, incorporating 12 productive branches, 16 basic types of products, 16 trade partners in the world and three types of productive resources: skilled labor, unskilled labor and capital. The model considers formal and informal markets, attending to a fundamental characteristic of the Colombian economy (its degree of informality, close to 50 % if measured through informal work), incorporates monopolistic competition, and includes heterogeneous companies, using a Melitz's standard scheme, which tries to account for the way the country is inserted in international markets.

The emphasis of the modeling is on three main aspects, which correspond to the aforementioned public finance trilemma: 1) the short-term effects, evaluated through a macroeconomic closure "guided by investment", which allows capturing the impact of investment in roads above the level of economic activity; 2) the inclusion of externalities in the production functions, to capture the supply and productivity effects; and 3) the careful and detailed modeling of public debt, including endogenous income that reflects productive activity, the financial costs of indebtedness (sensitive to the level of that indebtedness), and the payment of the investment through provisions of "future funds", which are incorporated into the budget.

The model is calibrated for 2017, and projects 20 years, in which the investment is carried out and its payment is assumed, through the mechanism of "future budgetary commmitments". The results are suggestive: an investment program of 73.5 trillion pesos in 2017 over an 8-year period (which represents 0.8 % of GDP in that period) generates an increase in GDP of 167 trillion in the considered timeframe, which increases the growth rate of the economy by 4 basis points long term growth rate. But most importantly: it slightly improves the trajectory of the public debt of the Central National Government (CNG): in the basic scenario, that debt evolves from 43.2 % of GDP to 34.1 %. With the 4G Program, the resulting final debt is 33.5 % of GDP.

Thus, the program illustrates what should be the priority of public policies at the present time: expansionary demand policies, which generate productivity effects in the medium and long term, without sacrificing the sustainability of public finances.

The article consists of five sections in addition to this introduction: the second section is the literature review; the third section describes in detail the model and the operating mechanisms it incorporates; the fourth describes the calibration of the model; the fifth presents the results; and the sixth outlines the conclusions.

2. Literature review

The role of infrastructure in economic growth and welfare economics has been studied extensively in the literature over the past three decades. The reconstruction after World War II presented a model in which governments invested in economies to create an enabling environment for the private sector (Ahmed et al. 2013), so the public-private partnerships is a subject well known in the literature.

The relationship between public expenditure and aggregate productivity has been explored for more than two decades as it is made clear in Aschauer (1989) where it is argued that critical infrastructure, such as roads, highways, airports, public transportation, sewers and water systems, plays an important role in promoting economic growth and improving productivity. The positive impact of infrastructure systems in the economy has been widely confirmed in works such as: Okiyama & Tokunaga (2017) which measures the economic impact of the rebuilding budget to achieve sustained economic growth for the Japanese economy, the authors find the need to take other measures such as productivity improvements in each industry located within the region affected by the disaster, using fiscal transfers; Mbekeani (2013) which analyzes the African continent; the study of Asturias et al. (2019) which studies the effects of a road infrastructure project in India called the Golden Quadrilateral and finds that the construction of this project increases real income in the domestic manufacturing sector in 2.72 %.

PPP (Public Private Partnerships) are "working arrangements based on mutual commitment (over and above that implied in any contract) between a public sector organization and any organization outside of the public sector" (Bovaird, 2004, p. 200). Those commitments may be related to policy design, coordination, monitoring, evaluation, review, implementation including the provision of public services.

There are some advantages associated with PPP due to the economic incentives and productivity nature of the private and public sectors. The provision of public services by public institutions has not allocated incentives effectively to promote efficiency. In this sense, the private sector is more effective due to competition (Saussier & Breux, 2018). Thus, there are some technical advantages in outsourcing the provision of public services: private experience, economies of scale, range of experience. Furthermore, there are management advantages: competition (i.e., between firms to obtain a concession), well defined incentives in management, and risk-sharing between the private and public sectors. But there are costs associated with PPP which can reduce its effectiveness. Those are related to the contractual design (specificity and flexibility), and asymmetries of information, uncertainty and institutional environment.

The link between PPP and economic growth is multi-dimensional. PPP can promote an increase in R&D on industries (Scott & Scott, 2015), or an increase in education funds (Issayeva et al., 2018). However, some evidence suggests harmful results for social welfare such as the increase of prices in water supply (Chong, Huet, Saussier, Steiner, 2006).

Infrastructure has received some attention in PPP's literature due to its relevance for economic growth. Baker et al (2019) study the relation of political stability on the private commitment to infrastructure investments. There is large evidence of the impact of infrastructure on economic growth through competitiveness and lower transaction costs, thus, PPP play a huge role especially for developing countries. However, this participation is affected in a negative way by political instability due to uncertainty and the use of political power to renegotiate. Using a panel data model for 14 developing countries, the authors find positive evidence between the participation of the private sector on infrastructure investments and political stability.

In the same line, Li et al (2005) study the most relevant factors that explain the success of the PPP on the construction of public facilities and on the provision of public services in the United Kingdom. Factor Analysis with a survey research with organizations involved in PFI (Private Financing Initiative) shows that the most relevant CSF (Critical Success Factors) for PPP/PFI are strong and good private consortium, appropriate risk allocation and available financial market.

Galilea & Medda (2010), investigate how the political and economic characteristics affect the success of a PPP in transport projects. They find evidence that factors such as PPP experience in the past, levels of GDP, worldwide perception of corruption, and political instability affect considerably infrastructure projects financed through PPP. Another finding shows Latin America as an interesting region for PPP projects mainly due to its experience.

Economic Research has shown strong evidence of the impact of infrastructure on economic growth (Estache & Fay, 2010). In fact, Démurger (2001) provides empirical evidence on the relationship between infrastructure investment and economic growth in China using a panel data of 24 Chinese provinces. Results show that transport facilities explain the growth gap of Chinese regions. Melo, Graham & Brage-Ardao (2013) conduct a meta-analysis of estimations from 33 studies related to the research of the relationship between infrastructure and economic growth. They found that the impact of infrastructure is higher in the US than Europe and is higher in roads than other kinds of infrastructure. The impacts on productivity vary among industries.

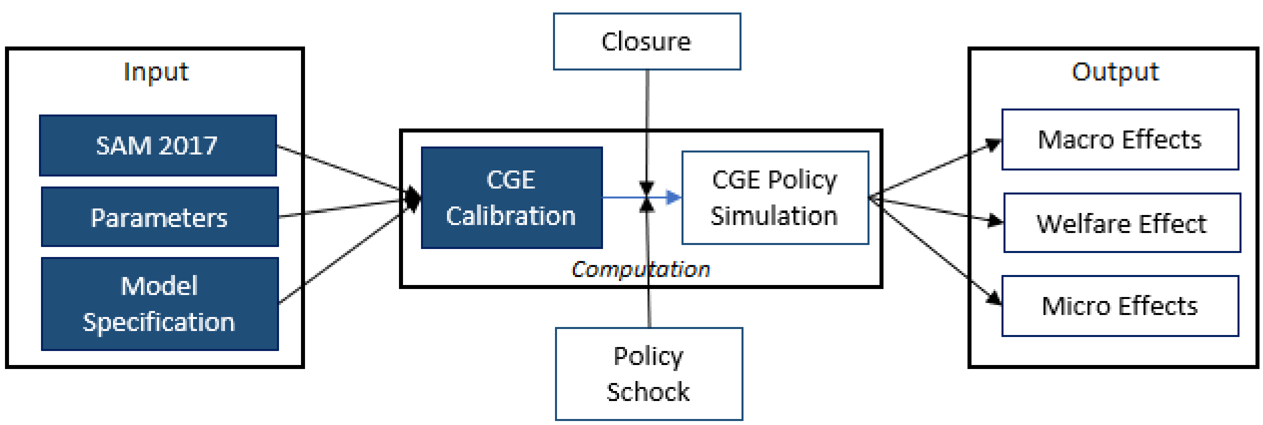
The methodology is another key factor affecting the understanding of the contribution of a system of infrastructure spending and fiscal sustainability in the economy. As mentioned by Chen & Haynes (2017), several pioneering studies assessed the economic contribution of infrastructure to economic growth and improved productivity following a neoclassical approach by measuring the elasticity of the economic output of infrastructure through some forms of aggregate production function using a format regression (Duffy-Deno & Eberts, 1991; Gramlich 1994, 2001; Harmatuck 1996; Nadiri & Mamuneas 1996; Fernald, 1999; Boarnet 1997; Boarnet & Haughwout, 2000; Matton, 2002; Bhatta & Drennan, 2003). Elasticities are estimated and results vary substantially with a range between 0.15 and 0.56, due to differences in the data and forms specific modeling (Melo et al. 2013), however, it was considered a partial equilibrium. Given the interdependent nature between the APP investment, government spending, fiscal sustainability and economic growth, the analysis need to be addressed under a general equilibrium framework.

With the advancement of computer technology and modeling platforms applied, the model of computable general equilibrium (CGE) has been widely adopted. The current CGE modeling techniques enable analysts to achieve a robust impact simulation by capturing regional economic structures heterogeneous and dynamic interactions of economic activities.

The CGE model has become an important tool used by global economic organizations such as the World Bank, World Trade Organization (WTO), the Organization for Economic Co-operation and Development (OECD) to measure the impact of shocks or policy changes (Meng & Siriwardana, 2017).

The term "computable" in CGE modeling indicates that it can quantify the effect of a shock (Hosoe, Gasawa & Hashimoto, 2010), there lies the importance of its use in our analysis. Since there are numerous markets in an economy, to build a general equilibrium involves a lot of data, so it is common that a computer is used in CGE modeling (Dixon & Jorgenson, 2013). The term "computable" also implies that a computer is involved in CGE modeling, in this case we will make use of the GAMS (General Algebraic Modeling System) program as is used in the empirical literature. A typical mechanism CGE modeling shown in Figure 1 is explained extensively in Hosoe, Gasawa & Hashimoto (2010) and to see a greater level of detail in a compendium you can see Dixon & Jorgenson (2013).

Figure 1: **Process Modeling CGE**



Source: Compiled by the authors adopted the work of Chen & Haynes (2017).

There are a lot of CGE models developed by some international organizations, World Bank, OECD, WTO for Latin America. There are too those developed by the IDB Giordano, Watanuki & Gavagnin, (2013), not counting the myriad of practical applications in different regions of the world. To mention a few important applications: models of fiscal policy as Lora (2015) and Botero (2011) for Colombia; international trade Dixon, Jerie, & Rimmer (2016) and also integrating and modeling heterogenous firms Jafari & Britz (2018) and improving calibration Dixon, Jerie, & Rimmer (2018) ; agricultural economy Perfetti & Botero (2018), Foreign Direct Investment of Latorre et al. (2019); health and pollution to the Nam et al., (2019); among others; but in general, they show the complexity and calibration have been improved substantially, allowing better analysis of the real economy and its impact with measurable results increasingly reliable.

With respect to our analysis combining modeling CGE, we found the work of Chen, Daito & Gifford (2017) entitled "Socioeconomic impacts of transportation public-private partnerships: A dynamic CGE assessment"that evaluates the socio-economic impacts of infrastructure projects; on the same line and using modeling CGE there is the study of Zugasti et al. (2001) for Spain economy; traffic, transportation, infrastructure and externalities of Conrad (1997); financing road infrastructure Conrad & Heng (2002); transport of Kim et al. (2011); Ahmed, Abbas & Ahmed (2013) study the impact of infrastructure investment in GDP growth using a dynamic CGE model linked with a microsimulation model in the Pakistan context. An analysis of this relationship is made by two approaches: infrastructure project funding by international borrowing or by productive taxes. Results show that both approaches have a positive effect, taxation reduces the impact on the short- term because of strain on economic output, but international borrowing indicates certain Dutch disease-like effects showed by a decline in exports. Rioja (2001) also studies the influence of infrastructure investment on economic growth, investment, and welfare for Peru, Mexico, and Brazil using a general equilibrium model of a small economy. His findings show that public infrastructure investment has a positive impact on economic growth.

Other research papers are those of transport infrastructure of Chen & Haynes (2014) that add spatial econometrics (SECGE); on fiscal measures to rebuild the Tohoku region in Japan of Okiyama & Tokunaga (2017); measuring the impact of infrastructure systems Chen Haynes (2017); infrastructure in Peru of Montaud et al. (2019); Investments in transport infrastructure Rokicki et al. (2020) for Poland; among others.

Today, it has emerged more strongly what has been called the new generation of CGE models worldwide taking into account the enormous growth, enabling the transition to more modern that are considerably higher compared to ORANI traditional, involving calibration Bayes (Go et al., 2015), behavior economics (Ahmed et al., 2018), advanced calibration Melitz (Dixon, Jerie & Rimmer, 2018), which integrate micro simulations as presented in several works of the recent book published at the end of the 2018 entitled "The New Generation of Computable General Equilibrium Models: Modeling the Economy" edited by Perali & Scandizzo. In brief, CGE models have multiple advantages when compared to other existing economic models. They are flexible, practical and easy to understand.

In short, due to the advantages and flexibility of modeling CGE, we will try to understand the phenomenon of infrastructure spending, PPP, "future budgetary commitments", fiscal sustainability and its impact on the growth of GDP in an emerging economy as Colombia. There are large investments in infrastructure projects such as 4G, that have been used and will be used as one of the main tools for structural policies at country level.

3. Detailed description of the model

3.1. General Structure

The model considers twelve sectors or productive branches, in which sixteen types of products are manufactured (see Table 1). In each sector there are two types of technologies: formal technology (which generates 85.3 % of the added value of the sector, 54.9 % of total employment, and 38.6 % of unskilled employment) and informal technology (which participates with 14.6 % of value added and generates 45.1 % of total employment and 61.4 % of unskilled workers).

Table 1: **Product and sectors in CGE modeling**

SECTORS AND PRODUCTS.	
Agricultura activities	Agricultural products
Mining	Oil
	Other products of mining
Manufacturing	Foods
	Other consumption products
	Intermediate inputs
	Capital goods
Electricity and gas	Electricity and gas
Building	Housing and infrastructure
Commerce	Wholesale, Retail and hotels
	Transport
TIC	Information and telecommunication
Finance	Financial activities and insurance
Real estate	Real estate activities
Profesional services	Profesional services
Public administration	Publica administration, Health and education services
Other services	Other services activities

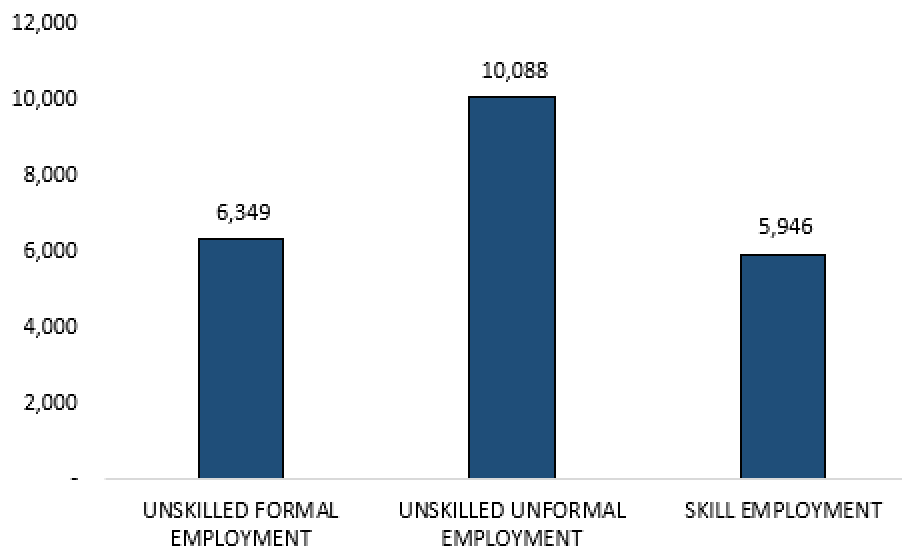
Source: Own elaboration based on the model.

Formal technologies use capital, skilled labor, and unskilled labor, and their value added is modeled by a CES production function at two levels: in the first, skilled and unskilled labor are added, while in the second, they are aggregated work and capital. Entrepreneurs minimize costs, given the capital stock they have, and demand both types of work, equating wages with the value of their marginal product.

Informal technology is a fixed coefficient technology, in which the added value depends only on the amount of work done.

Both technologies use intermediate inputs in fixed proportions to produce the final good or services, but there are indirect taxes on formal technologies.

Figure 2: **Total Employment in Colombia 2017 (thousands)**



Source: GEIH and DANE.

For the formal technologies, we assume monopolistic competition in all sectors but agriculture, which operated under perfect competition. Given all this, the price is set using the mark-up rule, as described below.

Let n be the number of companies that participate in a market, each producing a differentiated good. The consumer demands an added good in the form:

$$y = \left(\sum_i y_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

The consumer minimizes spending:

$$Min_{y_i} p_i y_i$$

The demand function derived from each good is:

$$y_i = \left(\frac{p_i}{p} \right)^{-\sigma} \frac{y}{p}$$

Where:

$$p = \left(\sum_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Each company maximizes its profit:

$$Max_{p_i} \left(p_i - \lambda_i \right) y_i$$

Where: λ_i is the unitary cost for each company i

The first order condition is:

$$-\sigma \left(p_i - \lambda_i \right) \frac{p_i^{-\sigma-1}}{p_i^{-\sigma+1}} y + \frac{p_i^{-\sigma}}{p_i^{-\sigma+1}} y$$

Solving for p_i we have

$$p_i = \frac{\sigma}{\sigma - 1} \lambda_i$$

Informal technology, meanwhile, operates under perfect competition, because there is no differentiation in its services and products, and there is no market power either.

The aggregation of formal and informal products and services is modelled using a CES function, in such a way that it is assumed that the productions of both technologies are imperfect substitutes.

The total production resulting from this aggregation is distributed among the types of products through a production matrix, which details the composition of the sector's production. The types of products obtained serve as basic inputs to value-generating entrepreneurs, who satisfy the demands of domestic and external consumers, and who are heterogeneous in terms of their productivity and differentiated in their ability to satisfy consumer needs with its generated economic value. The narrative underlying this modeling scheme is as follows: there are productive, formal and informal entrepreneurs, who generate goods and services, which are used by value entrepreneurs to satisfy the demand of different types of consumers. Thus, the model distinguishes the task of producing objects or services (touch screens, housings, microchips, for example) from the task of assembling them to turn them into valuable products or services for consumers (cell phones, in the example). The underlying logic for productive entrepreneurs is to manufacture objects or to produce services efficiently, that is, at the least cost possible, and the concept of productivity as efficiency operates for them. The underlying logic of value entrepreneurs, on the other hand, is to convert those objects and services into well-being for consumers and the concept of productivity that apply to them is improvement of their products through the innovation. The underlying logic of value entrepreneurs, on the other hand, is to convert those objects and services into well-being for consumers and apply the concept of productivity as innovation to them. Companies generally result from the mixture of both types of entrepreneurs (the former corresponding to the areas of production and operations, and the latter to the areas of marketing that create added value), but the distinction is important, because value chains are built by entrepreneurs of the second type, who hire entrepreneurs of the first type to produce the elements that compose a suitable emsamble that will satisfy the consumers' needs.

These value entrepreneurs (heterogeneous, due to their nature as innovators) are modeled following Melitz (2003), in the way described below.

Let r, s, and i be the company, market and sector. The production of sector i to market s is:

$$Q_{i,s} = \left[\sum_{r \epsilon S(i,s)} Q_{r,i,s}^{(\sigma_i - 1/\sigma_i)} \right]^{\sigma_i/(\sigma_i - 1)}$$

The dual function cost is

$$P_{i,s} = \left[\sum_{r \epsilon S(i,s)} P_{r,i,s}^{(1-\sigma_i)} \right]^{1/(1-\sigma_i)}$$

The derived demand function is:

$$Q_{r,i,s} = Q_{i,s} \left(\frac{P_{i,s}}{P_{r,i,d}} \right)^{\sigma_i}$$

The optimal price of the company that maximizes its profit is:

$$P_{r,i,s} = \frac{\tau_{r,i,s} PY_i}{(1 - 1/\sigma_i)\, \phi_{r,i,s}}$$

Where τ corresponds to what is called in the literature "iceberg cost" and $\phi_{r,i,s}$ is the firm's productivity, which is assumed to be the realization of a random variable, which is modeled as a Pareto distribution, whose density function is:

$$g_i(\phi) = \frac{a_i}{\phi} \left(\frac{b_i}{\phi} \right)^{a_i}$$

And whose cumulative distribution function is:

$$G_i(\phi) = 1 - \left(\frac{b_i}{\phi}\right)^{a_i}$$

The proportion of productivity values that are above a minimum value is defined by:

$$\int_{\phi_{min}}^{\infty} g(\phi_{i,j,s}) = \phi_{min}^{-\alpha_j}$$

Since we assume fixed costs to operate in a market (other than installation costs, to enter the sector), only companies that exceed a minimum level of productivity will be active.

The minimum productivity is:

$$fk_{i,s} PY_I = \frac{r\left(\phi_{i,s}^*\right)}{\sigma_i}$$

where $fk_{i,s}$ are the fixed costs for a company that belongs to sector i and operates in the market s.

Note that $r(\phi_{r,i,s}) = Q_{i,r,s} P_{i,r,s}$ and the profit on variable cost in the monopolistic competition model is $\pi_{r,i,s} = \frac{r_{r,i,s}}{\sigma_i}$.

The productivity of the representative company is:

$$\phi_{i,s} = \left[\frac{1}{1 - G_{i,s}\left(\phi_{i,s}^*\right)} \int_{\phi_{i,s}^*}^{\infty} \phi^{\sigma_i - 1} g(\phi)\, d\phi\right]^{1/(\sigma_i - 1)}$$

In the case of the Pareto's distribution:

$$\phi_{i,s} = \left[\frac{a_i}{a_i + 1 - \sigma_i}\right]^{1/(\sigma_i - 1)} \phi_{i,s}^*$$

Using the relationship between incomes and the profit function, we have[6]

$$\phi_{i,s} = b_i \left(\frac{a_i}{a_i + 1 - \sigma_i}\right)^{1/(\sigma_i - 1)} \left(\frac{N_{i,s}}{M_i}\right)^{-1/a_i}, \tag{1}$$

Now, we must re-express production and prices in terms of the representative company:

$$pf_{i,s}(\phi_{i,s}) = \frac{\tau_{r,i,s} PY_i}{(1 - 1/\sigma_i)\, \phi_{i,s}}, \tag{2}$$

$$qf_{i,s}(\phi_{i,s}) = Q_{i,s} \left(\frac{P_{i,s}}{pf_{k,i,d}}\right)^{\sigma_i}, \tag{3}$$

And

$$Q_{i,s} = [N_{i,s}]^{\sigma_i/(\sigma_i - 1)} qf_{i,s}$$

$$P_{i,s} = [N_{i,s}]^{1/(1 - \sigma_i)} pf_{i,s}, \tag{4}$$

[6] See Balistreri and Rutherford (2013), p, 14-15

And the short-term and long-term conditions of free entry are:

$$PY_i\delta_{f,i} = \sum_s \left(\frac{N_{i,s}}{M_i}\right) pf_{i,s}qf_{i,s}\frac{\sigma_i - 1}{a_i\sigma_i}, \tag{5}$$

$$PY_iFC_{i,s} = \left(pf_{i,s}qf_{i,s}\left(\frac{a_i + 1 - \sigma_i}{a_i\sigma_i}\right)\right), \tag{6}$$

And the total demand for inputs is:

$$Y_i = \delta_{f,i}MT_i + \sum_s N_{i,s}\left(FC_{i,s} + \tau_{i,s}qf_{i,s}/\phi_{i,s}\right), \tag{7}$$

The set of markets served (the 's' set) is made up of the domestic market and the country's 16 export destinations. External demand is modeled using the function of compound good (the traditional Armington function of the general equilibrium literature), while in the domestic markets, there are trade entrepreneurs, who buy domestic goods and imported goods to minimize the cost to consumers of the purchases made. This is also expressed in the well-known CES function system, in the Armington way.

The model considers five agents: households, companies, central government, decentralized government and the rest of the world. It generates the corresponding factor incomes, and includes inter-institutional transactions, in four main categories: payment of taxes; social security contributions and benefits; property income represented in dividends, interest, royalties or other payments; and transfers. The resulting disposable income is dedicated to saving, or consumption. Government demand is distributed among product types, using fixed coefficients. Household demand, on the other hand, is modeled on two levels: at the first level, the representative household chooses a given category of goods: agricultural, industrial, or services, which is modeled using the Almost Ideal Demand System, which allows to discriminate price and income elasticities between the categories considered. At the second level, consumption is distributed among the types of goods, in fixed proportions.

The modeling of the ideal demand system is described below:

$$lnc = \sum_k \alpha_k lnp_k + \frac{1}{2}\sum_k\sum_j \gamma_{k,j}lnp_k lnp_j + u\prod_k p_k^{\beta_k} \tag{8}$$

The representative consumer preferences of each of the categories considered in the model are represented by an expense function of the PIGLOG type:

By Shepards lemma

$$\frac{\partial lnc}{\partial lnp_i} = w_i = \alpha_i + \sum_j \gamma_{i,j}lnp_j + \beta_i ln\left(\frac{c}{p}\right)$$

Where

$$lnp = \sum_k \alpha_k lnp_k + \frac{1}{2}\sum_k\sum_j \gamma_{k,j}lnp_k lnp_j \tag{9}$$

For the calculation of the welfare impacts, we use the indirect utility function, inverting the spending function: the impact that a given change in public policy has on household welfare is measured by the equivalent variation:

$$VE_{IHOG} = c\left(u_1, p_0\right) - c\left(u_0, p_0\right) \tag{10}$$

Given that:

$$u_1 = V\left(c_1, p_1\right))$$

Where c is the expense, u the utility, V the indirect utility function, and the subscript 0 refers to the initial situation and 1 to the final situation.

Business and government savings are determined on a residual basis. The external market is adjusted by exchange rate, given an exogenously determined capital flows. And household savings are adjusted to balance the model, given an exogenously determined investment, which includes investment in infrastructure. It is therefore an ïnvestment-driven"model.

4. Model Calibration

The model has been calibrated with information corresponding to the year 2017, and its equilibrium replicates the functioning of the Colombian economy in that period. For this purpose, we begin by constructing a Social Accounting Matrix (SAM) that represents the circular flow of economic activity and serves as a basis for calibrating the initial values of the variables and parameters included in the model. The SAM's structure that fits the CGE model described in this paper will be the following one:

Figure 3: **SAM Scheme**

		Expenses					
		Productive sectors (12)	**Products (16)**	**Production factors (3)**	**Institutions (5)**	**Transactions**	**Savings-Investment**
Income	**Productive sectors (12)**		Production matrix				
	Products (16)	Input - Output			Consumption expenditure - Exports		GFCF - Inventories
	Production factors (3)	Added value	Taxes - Tariffs				
	Institutions (5)		Imports	Payment factors assignment - Indirect taxes		Tax income received	
	Transactions				Transfers		
	Savings-Investment				Savings - Debts		

Source: Own Elaboration.

This matrix clearly identifies the income and expenditure flows associated with each of the 12 production sectors, the 16 basic types of products, the five agents (households, enterprises, central government, decentralized government and the rest of the world) and the three production factors (skilled labor, unskilled labor and capital). In addition, the SAM is complemented by two annexed modules. The first one gathers information about the 16 export destinations, which, as mentioned in the previous section, are modelled following Melitz (2003). The second one characterizes the evolution of the future funds over the projection period (2017-2037). The information sources that were consulted for the construction of the SAM and the modules that support the calibration of the model are shown in the next table.

Table 2: **Information sources**

Source	Information Data Base
Colombian National Administrative Department of Statistics (DANE*)	Gross Domestic Product (GDP)
	Integrated Economic Accounts (IEA)
	Offer – Use Table
	Labor Matrix
	Large Integrated Household Survey (GEIH*)
	Gross Fixed Capital Formation (GFCF) by Product
Ministry of Finance and Public Credit	Central National Government Fiscal Balance
Colombian Central Bank	Balance of Payments
	Exchange rate (TRM*)
Trade Map	Exports by Trading Partner
National Infrastructure Agency (ANI*)	Future-funded investment projects
* Represents the acronym in its original language (Spanish)	

Source: Own Elaboration.

Subsequently, the equilibrium values for the set of variables included in the model are established and the parameterization that will govern its successful performance is also determined. In this way, it is possible to carefully replicate the functioning of the Colombian economy in a general equilibrium context. The essential parameters that characterize the adequate behavior of the markets are presented below:

Table 3: **Key parameters**

Parameter	Sector	Value
Aggregation elasticity of unskilled work	All	0.9
Aggregation elasticity of skilled and unskilled work	All	0.9
Elasticity of the frontier of production possibilities CET	All	-1.5
Export demand elasticity	All	1.5
Composite good elasticity	Goods	1.48
Elasticity of the product to public capital	Central National Government	0.06
Added value elasticity	All	0.9

Source: Own Elaboration.

With all the above, it is possible to establish that the computable general balance model replicates the Colombian economy in its base year (2017); thereby, its calculations are satisfactorily adjusted to the macroeconomic performance which is reflected in the main interest variables such as the GDP, the unemployment rate, the current account balance and the fiscal deficit of the government. In conclusion, the model is ready to carry out the simulations under the PPP investment scenario that this article aims to evaluate.

5. Results

This article evaluates the Colombian program of construction of infrastructure through public-private partnerships. This program includes the so-called Fourth Generation of toll roads (4G), which seeks to build 8,000 km of national roads, interconnecting productive centers between them and with the country's ports. The government tenders the construction of strategically selected roads among private agents, granting the right to collect tolls on the road, and guaranteeing, through future budget allocations, the financial closure of the project for the private agent, under the conditions offered in the tender.

The exercise compares the results of a "base scenario" which projects the evolution of the Colombian economy over the 20-year horizon, under reasonable assumptions of job supply evolution, total factorial productivity, oil price, government income and expenditure structure, with an "alternative scenario" simulating the effect of investing 73.5 trillion pesos in 9 years (81 % of the 2020-2048 total estimated investment , detailed in Annex 1, to 2019 pesos).

To facilitate the analysis in the "alternative scenario" the program of execution of the works is adjusted, so that they are carried out between 2020 and 2028, and paid in during the projection horizon, which in this case is 20 years (2018-2037). Thus, all possible effects associated with the infrastructure program are incorporated in the model, extinguishing the commitments acquired by the government with the program, and thus closing the investment-payment cycle on the horizon used.

Although in practice the projects are structured by "functional units" that can start to operate once completed, the model assumes that the infrastructure projects are completed in two stages: in the fourth year of the program the first half of the project is completed, while the second half is completed at the end of the investment horizon. This implies that the externalities associated with the project, incorporated into the production function of the formal[7] sector through a component of sectorial productivity, that depends on the cumulative stock of public infrastructure, occur only in those two moments.

Payments of "future validities" are incorporated into the government budget and disbursed once each stage has been completed. It is assumed that these disbursements allow the financial closure of projects, with a nominal rate of return of 6.5 %, and that they are completed on the projection horizon.

[7]This component reflects the positive impact that works have on economic activity, through the additional productivity that entrepreneurs achieve as an effect of greater integration with domestic and global markets

Table 4: **Structure of simulated projects**

Investment projection, activation of works and future values. Millions of pesos 2019				
Year	Investment	Activation of works	Future budgetary commitments	Contingent Liability Balance with agents
2020	8,168			8,168
2021	8,168			16,867
2022	8,168			26,131
2023	8,168	32,671		35,997
2024	8,168			46,505
2025	8,168			57,696
2026	8,168			69,614
2027	8,168			82,307
2028	8,168	40,840		95,825
2029			14,396	87,657
2030			14,396	78,958
2031			14,396	69,694
2032			14,396	59,827
2033			14,396	49,320
2034			14,396	38,129
2035			14,396	26,211
2036			14,396	13,518
2037			14,396	-
TOTAL	**73,511**	**73,511**	**129,568**	

Source: Own calculations.

The effect of the infrastructure investment is threefold: first, it generates an important source of demand growth, associated with the investment made; second, it generates productive externalities, which improve the overall productivity of the economy as a whole; and third, the fiscal effort is spread over the investment period, generating additional payment sources, associated with the incremental tax collections that are generated.

Table 5: **Results of simulation**

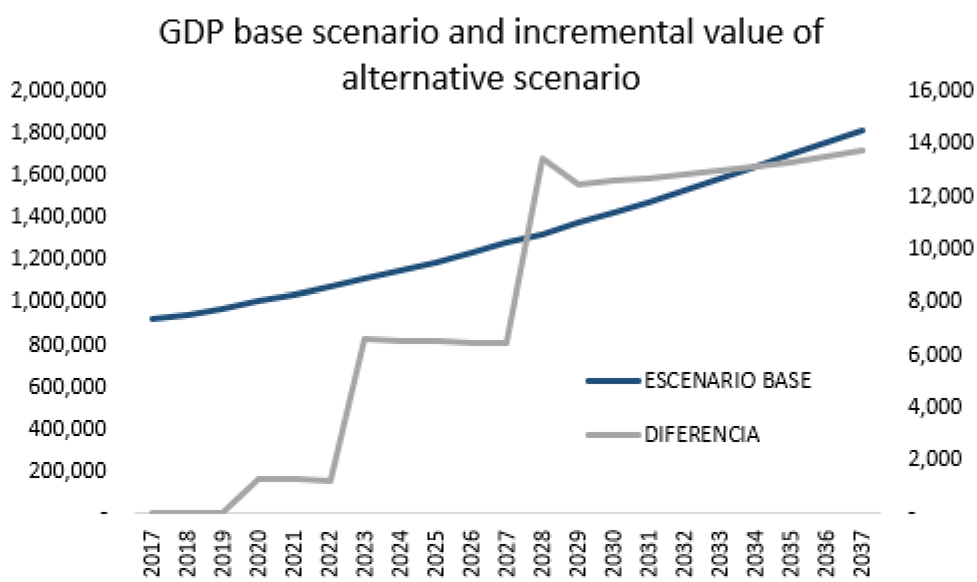
Simulation results. Alternative scenario final values			
Variable	Initial year. Billions of pesos	Final year. Base scenario. Billions of pesos of 2017	Final year. Alternative scenario. Billions of pesos of 2018
GDP	920,195	1,818,606	1,832,352
Unemployment rate	9.37%	8.40%	8.30%
Public susplus or deficit	-3.70%	-2.70%	-2.70%

Source: Model CGE.

The table may prove misleading, because final GDP shows relatively little difference with base scenario, but it highlights three things: first, it is a public effort carried out through public-private partnerships that permanently raises GDP; second, improves the overall employment in the economy; and third, does it without adversely affecting public finances.

Figure 4, which shows the evolution of GDP in the base scenario and its variation due to the alternative scenario, allows for a better analysis of the effect of public policy over time.

Figure 4: **Simulation results in billions**



Source: Model CGE.

The increase in GDP begins in 2020, because of the Keynesian effect of demand, associated with the investment. In the years between 2023 and 2037, in which the investment is made, the supply effect occurs: program is completed, having a positive impact on factorial productivity and the output. The final differential be a permanent level change, which must of course have additional effects, but even, without considering these permanent effects, the net present value of the difference between the GDP in the alternative scenario and the base scenario is 105 trillion pesos or 1.44 times the investment made. An outcome that is achieved without increasing the taxes that society must pay!

6. Conclusions

This article argues the following: there are investment opportunities in a country that are not adequately exploited, because they produce external effects that cannot be captured by investors. These opportunities correspond to investments that increase total productivity, allowing for higher levels of production without necessarily increasing the productive resources used.

If there are no adequate institutional conditions, such investments will not be made. The role of the government should be to design markets suitable for these externalities, making investment possible. In carefully chosen investments, which have a clear social and productive benefit, projects can generate the future income that pays for them, thus producing a positive welfare effect on society.

Of course, this makes the "expansive policies-sustainability" dilemma perhaps a poorly posed dilemma, because there is a wide range of opportunities in which it would not operate. It is up to the government to identify "worthwhile investment opportunities" that increase overall well-being, without overwhelming impact on the balance of public finances. With a suitable mix of market design and financial engineering, it is possible to realize these opportunities. Some of them, those associated with human capital for example, which are limited not only by general externalities that are not fully privately[8] appropriated, but also by liquidity constraints, are opportunities for state intervention. But there are certainly many others, associated with productive infrastructure or disruptive transformations, for which the right markets and instruments will have to be designed. Of course, there must be a threshold from which an investment can be classified as "meritorious" in the suggested sense, that is, that they generate a sum of private benefits and public benefits greater than its cost, and for which the role of the state is to design markets, mechanisms and efficient financial instruments. But identifying those opportunities, designing markets and the mechanisms to implement them, becomes perhaps one of the most important functions of the state.

[8]We refer, of course, to the additional benefits that education brings to society, and which are not reflected exclusively in the individual income received: benefits of dissemination of knowledge, of attracting new technologies, innovation and productive and social systems improved by virtue of education.

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Mathematical Appendix: The Structure of the Model

Formal sector block

Adding formal sector Jobs:

$$TTA_{II} = TNCA_{II} + TC_{II} \tag{11}$$

Expansion path formal Jobs:

$$\frac{TNCA_{II}}{TC_{II}} = K_{TT} * \left(\frac{WC_{II} * \delta_{AII}}{WNCA_{II} * (1 - \delta_{AII})} \right)^{\sigma_{AII}} \tag{12}$$

Average price of formal jobs:

$$TTA_{II} * WA_{II} = TNCA_{II} * WNCA_{II} + TC_{II} * WC_{II} \tag{13}$$

Labor and capital aggregation:

$$VA_{II} = \left(\frac{k_{tp}}{k_{po}} \right)^{\varepsilon_{kp}} BVA_{II} * \left[\left(\delta_{VA(II)} * TTA_{II} \right)^{\frac{\delta_{VAII}-1}{\delta_{VAII}}} + (1 - \delta_{VAII}) * K_{II}{}^{\frac{\delta_{VAII}-1}{\delta_{VAII}}} \right]^{\frac{\delta_{VAII}}{\delta_{VAII}-1}} \tag{14}$$

Value-added expansion path:

$$\frac{TT_{II}}{K_{II}} = KCT_{II} * \left(\frac{R_{II} * \delta_{VAII}}{W_{AII} * (1 - \delta_{VAII})} \right)^{\sigma_{VAII}} \tag{15}$$

Identity book value added:

$$PVAA_{II} * VAA_{II} = TTA_{II} * WA_{II} + K_{II} * R_{II} \tag{16}$$

Informal sector block

Informal sector:

$$VAI_{II} = KVAI_{II} * TNCI_{II} \tag{17}$$

Accounting identity value added:

$$PVAA_{II} * VAA_{II} = TTA_{II} * WA_{II} + K_{II} * R_{II} \tag{18}$$

Modeling monopolistic competition:

$$\lambda * A_{ICM} * Y_{ICM} = RPUA_{ICM} * K_{ICM} + WA_{ICM} * TTA_{ICM} + YA_{ICM} * \sum_{i=1}^{n} IO_{ICM} * PXC_I \tag{19}$$

Total production independent technology:

$$PYA_{ICM} = (1 + MARKUPA_{ICM})\lambda A_{ICM} \tag{20}$$

Profit per unit of capital:

$$GANA_{ICM} = PYA_{ICM} - \lambda_{ICM} * YA_{ICM} \tag{21}$$

Basic remuneration borderless capital:

$$RCMA_{ICM} = \frac{GANA_{ICM}}{K_{ICM}} \tag{22}$$

Including capital gain margins:

$$R_{ICM} = RCMA_{ICM} + RPUA_{ICM} \tag{23}$$

Block perfectly competitive sectors

Value of production sectors without monopolistic competition:

$$PYA_{INCM} * YA_{INCM} = PVAA_{INCM} * VAA_{INCM} + \sum_{i=1}^{n} PXC * YA_{INCM} * IO_{INCM,I} \tag{24}$$

Demand for intermediate inputs:

$$VAA_{II} = YA_{II} * (1 - \sum_{i=1}^{n} IO_{II}) \tag{25}$$

Demand for intermediate inputs:

$$VAI_{II} = YI_{II} * (1 - \sum_{i=1}^{n} IO_{II}) \tag{26}$$

Production value:

$$PYI_{INCM} * YI_{INCM} = PVAI_{INCM} * VAI_{INCM} + \sum_{i=1}^{n} PXC * YI_{INCM} * IO_{INCM,I} \tag{27}$$

Formal and informal production aggregation:

$$YIMP_{II} = YIMPA_{II} + YI_{II} \tag{28}$$

Expansion path:

$$\frac{YI_{II}}{YIMPA_{II}} = \left(\frac{PYIMPA_{II} * \delta_{DII}}{PYI_{II} * (1 - \delta_{DII})}\right)^{\sigma_{DII}} \tag{29}$$

Identity book value added:

$$PYPIMP_{II} * YIMP_{II} = PYI_{II} * YI_{II} + PYIMPA_{II} * YIMPA_{II} \tag{30}$$

Total demand and import demand:

$$C_I + G_I + INV_I + V_I + ZINV_I = (M_1 + D_1) * (1 + IVA_1) \tag{31}$$

Expansion path domestic production and imports:

$$\frac{M_{IM}}{D_{IM}} = FEM * \left(PD_{IM} * \frac{\delta_{IM}}{(PM_{IM} * (1 + AVE_{IM}))^{(1-\delta_{IM})}}\right)^{\sigma_{MMI}} \tag{32}$$

Expansion path:

$$\frac{M_{INM}}{D_{INM}} = {}_{.FEM*MDEXO_{INM}} \tag{33}$$

Well composed accounting identity:

$$PXC_I * (C_I + G_I + INV_I + V_I + ZINV_I) = (1 + IVA_I) * (PM_I * M_I + PD_I * D_I) \tag{34}$$

Aggregation of domestic production and exports CET:

$$YP_{INE} = \sum_{i=1}^{n} SE, X_{INE,SE} + D_{INE} \tag{35}$$

Export prices:

$$PWX_{INEX,SE} = PWXEXO_{INEX,SE} \tag{36}$$

Production possibilities frontier:

$$\frac{X_{INEE,SE}}{D_{INEE}} = \delta_{CINEE} * \left(\frac{PD_{INEE}}{PX_{INEE,SE}}\right)^{\sigma_{CINEE}} \tag{37}$$

Production value:

$$PP_{INE} * YP_{INE} = \sum_{i=1}^{n} SE, PX * X_{INE,SE} + PD_{INE} * D_{INE} \tag{38}$$

Block foreign markets

Global export demand:

$$XXT_{INE,SE} = X_{INE,SE} + XT_{INE,SE} \tag{39}$$

Quantities exported:

$$X_{INEX,SE} = XEXO_{INEX,SE} \tag{40}$$

Global export demand:

$$XXT_{IE,SE} = X_{IE,SE} + XT_{IE,SE} \tag{41}$$

Export demand (expansion path):

$$\frac{X_{IE,SE}}{XT_{IE,SE}} = \left(\frac{PW_{IE,SE} * \delta_{E(IE,SE)}}{PWX_{IE,SE} * \left(1 - \delta_{E(IE,SE)}\right)} \right)^{\sigma_{E(IE,SE)}} \tag{42}$$

Aggregation Jobs:

$$TNC_{II} = TNCA_{II} * TNCI_{II} \tag{43}$$

Accounting identity aggregation of jobs:

$$TNC_{II} * WNC_{II} = TNCA_{II} * WNCA_{II} + TNCI_{II} * WNCI_{II} \tag{44}$$

Block factor income

Factorial corporate income:

$$YFAC_{EMP} = SHARE_{KI} * \sum_{i=1}^{n} K_{II} * R_{II} \tag{45}$$

Factorial government income:

$$YFAC_{gob} = SHARE_{gI} * \sum_{i=1}^{n} K_{II} * R_{II} \tag{46}$$

Income factorial rest of the world:

$$YFAC_{rm} = SHARE_{rm} * (\sum_{i=1}^{n} K_{II} * R_{II})/ER \tag{47}$$

Factorial income of the National Central Government:

$$YFAC_{gnc} = SHARE_{gnc} * (\sum_{i=1}^{n} K_{II} * R_{II}) \tag{48}$$

Determining tax companies:

$$IMPUES_{EMP} = TIMP_{EMP} * (YFAC_{EMP} * (1 - TINT_{EMP} - TINT_{RM}) \tag{49}$$

Saving capital:

$$SAVE_{RM} = FK * ER \tag{50}$$

Consumption by sector:

$$CS_{IS} = \propto si_i + \sum_{j=1}^{N} \gamma si_{ij} lnp_{ij} + \beta si_i ln \left(\frac{Xsi_t}{Psi_t} \right) * (1 - MSAVE/PXCS)\, ; Para\ i = 1, 2, ..N \tag{51}$$

Public expenditure for sector:

$$G_I = SHARE_{GI} * GPX_{GNC} + GPX_{GOB} \tag{52}$$

Investment by sector:

$$INV_I = SHARE_{II} * INVT_I + INVGNC + INVAPP \tag{53}$$

Other sectoral demands:

$$ZINV_I = TZINV_I * (C_I + G_I + INV_I + V_I) \tag{54}$$

Intermediate purchases:

$$V_I = \sum_{i=\mathbf{1}}^{n} (YA_{II} + YI_{II}) * IO_{II,I} \tag{55}$$

External prices in domestic currency:

$$PM_I = PWM_I * (1 + ARAN_I) * ER \tag{56}$$

Export prices in domestic currency:

$$PX_{I,SE} = PWX_{I,SE} * ER \tag{57}$$

Equilibrium in the labor market:

$$TSC * (\mathbf{1} - DESEMC) = \sum_{i=\mathbf{1}}^{n} TC_{II,I} \tag{58}$$

Equilibrium in the labor market:

$$DESEMA = (TSNCA - \sum_{i=\mathbf{1}}^{n} TNCA_{II,I})/TSNCA \tag{59}$$

Equilibrium unskilled job:

$$TSNCI = \sum_{i=\mathbf{1}}^{n} TNCI_{II,I} \tag{60}$$

Salary scale unskilled market wage:

$$WNCA_{II} = WDISTNCA_{II} * WMNCAX_{II} \tag{61}$$

Scale compensation not qualified independent:

$$WNCI_{II} = WDISTNCI_{II} * WMNCI_{II} \tag{62}$$

Scale compensation qualified independent:

$$WC_{II} = WDISTC_{II} * WMC_{II} \tag{63}$$

Savings investment balance:

$$\sum_{i=\mathbf{1}}^{n} INST, SAVE_{INST} = \sum_{i=\mathbf{1}}^{n} INV_I + ZINV_I * PXC_I \tag{64}$$

Price Index:

$$P = IPC \tag{65}$$

Total product Branches:

$$YP_I = \sum_{i=1}^{n} MP_{I,II} * YIMP_{II} \tag{66}$$

Walras Law: Product total branches to n-1 products:

$$YP_{I,j} = \sum_{i=1}^{n} MP_{II,IJ} * YIMP_{II} \tag{67}$$

Production branch tax:

$$YIMPA_{II} = YA_{II} * (1 + IMPRA_{II}) \tag{68}$$

Branch price with taxes:

$$YIMPA_{II} * PYIMPA_{II} = YA_{II} * PYA_{II} * (1 + IMPRA_{II}) \tag{69}$$

Index installed capacity:

$$IUSO = KIUSO \tag{70}$$

Investments:

$$INVT = K * INVT \tag{71}$$

Interest on the public debt:

$$INTEE = RME * DEUDAEXT * ER \tag{72}$$

Interest on the public debt:

$$INTEI = RMI * DEUDAINT \tag{73}$$

Interest rate public debt:

$$RME = RMFR + *(K_{deuda} * DPIB)^{\propto_{deuda}} \tag{74}$$

Interest rate public debt:

$$RMI = RME * RMEI \tag{75}$$

Objective Function:

$$VAT = \sum_{i=1}^{n} VAA_{II} * VAI_{II} \tag{76}$$