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The GTAP Recursive Dynamic (GTAP-RD) Model: Version 1.0

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Abstract

First developed in 1992, the GTAP model has become the *de facto* standard and starting point for nearly all economy-wide analyses of global trade issues. The latest incarnation of the GTAP model is described in Corong et al. (2017), that also includes references to the numerous studies and extensions of the model—including a dynamic version, called GDyn, which is described in Ianchovichina and Walmsley (2012). This paper documents a newly developed GTAP recursive dynamic (GTAP-RD) model which includes a number of dynamic features that facilitate the creation of baselines and long term trade policy simulations. It replicates to some extent the description of the standard GTAP model (Corong et al., 2017), but also emphasizes new features added to the core comparative static model and the specification of model dynamics.

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Chapter 1

Introduction and overview

This paper documents the development of a new GTAP recursive dynamic (GTAP-RD) model, which is built on the latest version of the core GTAP trade model. First developed in 1992, the GTAP model has become the *de facto* standard and starting point for nearly all economy-wide analyses of global trade issues. The latest incarnation of the GTAP model is described in Corong et al. (2017), that also includes references to the numerous studies and extensions of the model— including a dynamic version, called GDyn, which is described in Ianchovichina and Walmsley (2012).

GTAP-RD was developed in response to a request by the World Trade Organization (WTO) for a version of the GTAP model which includes a number of dynamic features that facilitate the creation of baselines and long term trade policy simulations. It replicates to some extent the description of the standard GTAP model (Corong et al., 2017), but also emphasizes new features added to the core comparative static model and the specification of model dynamics.[1]

1.1   A brief history

The GTAP model builds on a rich tradition of general equilibrium models—both single-region and global in scope. It was immediately preceded by the Sectoral Analysis of Liberalising Trade in the East Asian Region (SALTER) model (Jomini et al., 1994), developed at the Australian Productivity Commission in the late 1980's by a team which included Rob McDougall, a stalwart of the GTAP Center. The SALTER model was, in turn, heavily influenced by the WALRAS model (Burniaux et al., 1990), developed at the OECD for analysis of agricultural trade policy in the industrialized economies of the world. WALRAS built on computable general equilibrium (CGE) modeling foundations established by John Whalley (Whalley, 1984), and John Shoven and John Whalley (Shoven and Whalley, 1992), as well as Victor Ginsburgh and Jean Waelbroeck (Ginsburgh and Waelbroeck, 1981). In addition, SALTER, and in turn, GTAP, were heavily influenced by the work of Peter Dixon and collaborators (Dixon et al., 1982). Dixon built on the work of Leif Johansen (Johansen, 1960) who developed the first computable general equilibrium model which involved totally differentiating the non-linear model and expressing the equations in terms of elasticities, cost shares and percentage change variables. This greatly facilitates analysis of the economic mechanisms at work in any given simulation.

Development of the GTAP model started in 1992, simultaneously with the development of the GTAP Data Base. The first full documentation of the GTAP model became available in Hertel

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[1]Though not described in this document, a variant of GTAP-RD (the so-called WTO Global Trade Model, incorporates imperfect competition, increasing returns to scale and Melitz specification based on the parsimonious GTAP-AEKM (Armington-Ethier-Krugman-Melitz) extension described in Bekkers and Francois (2018).

(1997), with model modifications and additions described in Hertel et al. (2000), Huff and Hertel (2001) and McDougall (2003). These core changes, along with some new additions and model streamlining were consolidated in a new model description documented in Corong et al. (2017), which is now referred to as the standard GTAP model. One of the major additions was the separation of activities from commodities using a so-called 'make' matrix. This had been an extension of many models that required either multi-product activities (such as ethanol) and/or commodities produced by more than one activities (such as electricity). Some of the streamlining elements included separating out investment expenditures from firm activities, changing the reference price for all taxes and subsidies to so-called basic prices, and more transparent use of indices.

1.2 Report structure

The core of the document are the model equations—laid out in a logical and modular fashion. It is not necessarily intended to be read in its entirety, i.e. interested readers are likely to jump around and read specific sections as needed. Each module includes easy-to-read algebraic equations with a description of the equations and includes as well the relevant TABLO code. The two may not necessarily coincide as the theoretical description may present equations in levels, where this seems more intuitive, whereas the TABLO code reflects the model in log-differentiated form. Typically, behavioral equations will be presented in log-differentiated form, but revenue and linear identities will often be presented in level form.

The description departs in some ways from that found in Corong et al. (2017). The use of signpost [*NEW*] in this document reflects changes compared to Corong et al. (2017).

The full model description appears in the next section. This is followed by a description of the dynamic elements of the model. The Appendices include additional information including a mathematical addendum for some of the model's key modules, the core accounting relations and how they relate to the GTAP Data Base.

Chapter 2

Model description

We now proceed to provide a detailed description of the GTAP-RD Model. The core code is derived from the standard GTAP model, version 7 (described in Corong et al. (2017)). The model includes some extensions to the standard model including more consistent definitions of price indices (e.g. GDP deflator, factor price index, etc.) and upward sloping supply curves for non-capital endowments. The following chapter describes extensions to the model to allow for dynamics.

We will describe the model using the circular flow logic of an economy outlined in Figure 2.1.[1] Here, production generates income accruing to endowments that is returned to the regional household and then spent on three sources of final demand: private expenditures, government spending and saving—which subsequently is translated into investment spending. Each source of spending, as well as purchases of intermediate goods comprise both domestic purchases and imported purchases, thereby generating both domestic and export sales by firms.

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[1]The figure is an adaptation from Brockmeier (2001).
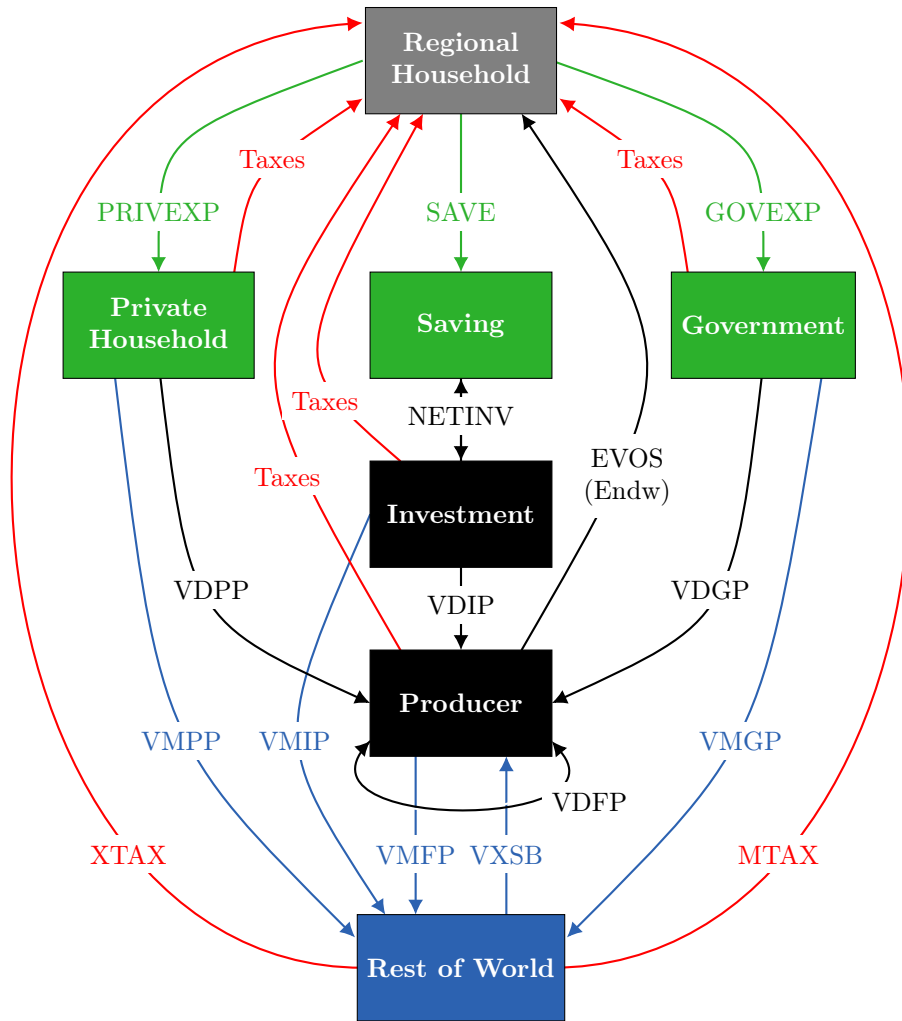
Figure 2.1: **Circular flows in a regional economy**

Notes: See Annex B for definition of value flows

Sets and subsets

The GTAP model's 'geometry' relies on a number of sets that are defined at model implementation. In other words, the model description is based on a generic definition of sets and its implementation will be based on a specific definition of the sets as defined by the user. Table 2.1 describes the most basic sets used in the GTAP model. For the regional indices, the model description will at times use s and d instead of r, particularly for bilateral trade variables. Their use is to clearly distinguish between source (s) and destination (d) regions. All bilateral variables assume that the source index appears before the destination index, i.e. exports are read along the row of the bilateral trade matrix and imports are read down a column.

The current version of the full GTAP database assumes a diagonal 'make' matrix, i.e. there is a one-to-one correspondence between an activity and a commodity. The aggregation facility allows for separate aggregation of activities and commodities and hence the model does not assume the

one-to-one correspondence.[2]

Table 2.1: **Basic sets in the model**

Name	Description
$REG(r,s,d)$	Regions (s for source, d for destination)
$ACTS(a)$	(Production) Activities
$COMM(c)$	Commodities
$ENDW(e)$	Endowments

Table 2.2 describes the only subsets for the commodity set. It represents a partition of the commodity space into margin and non-margin commodities. The former are specific to the international trade and transport module of the model.

Table 2.2: **Commodity subsets**

Commodities: $COMM(c)$	
$MARG(m)$	$NMRG(n)$
Margin commodities	Non-margin commodities

Table 2.3 describes the subsets used for endowments. There is an inititial partition of endowments between sector-specific factors and economy-wide factors. The former are specific to a single activity. These are typically the natural resource base of the activity such as oil reserves in the case of oil extraction.[3] The economy-wide endowments are partitioned into two separate sets. The first reflects the degree of mobility of the endowment—perfect mobility and partial (or sluggish) mobility. The user can decide the degree of mobility by defining the subsets $ENDWM$ and $ENDWS$, and in the case of sluggish factors, by also setting the transformation elasticity ($ETRAE$).[4] In a standard configuration, labor and capital endowments are mobile and land is sluggish. However, a simulation's time framework may warrant departures from the standard configuration. For example, a short-term time horizon may be more compatible with moving capital to the sluggish subset.

The second partition of the economy-wide endowments partitions these into the capital endowment and the non-capital endowments (most often labor and land). [NEW] This version of the model introduces upward sloping supply curves for the non-capital economy-wide endowments (the elasticity of which could be zero). The capital stock is typically fixed in both comparative static and recursive dynamics. In the case of the latter, the capital stock evolves according to the standard capital accumulation equation.[5]

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[2]Future releases of the GTAP database may depart from diagonality, for example a new version of the GTAP-Power database.

[3]In GTAP classic, these factors were part of the sluggish endowments, however with a very small transformation elasticity.

[4]Transformation elasticities are entered as negative values for the GTAP model.

[5]$K_{t+1} = (1-\delta)K_t + I_t$

Table 2.3: **Endowment subsets**

Endowments: $ENDW(e)$		
$ENDWF(e)$	$ENDWMS(e)$	
Sector-specific factors	Economy-wide factors	
	Mobility	
	$ENDWM(e)$	$ENDWS(e)$
	Mobile factors	Sluggish factors
	Supply assumption	
	$ENDWC(e)$	$ENDWMSXC(e)$
	Capital	Other non-capital

Depiction of commodity price linkages

Before going into the details of the model specification, it is useful to describe the main price linkages for commodities—though the individual price equations are introduced later. There is only one set of prices that truly determines all of the other prices. A natural way to think about these is as the prices which equilibrate supply and demand in this general equilibrium model. Thus we start with the market <u>P</u>rice for <u>D</u>omestically <u>S</u>upplied commodity c in region r, $PDS_{c,r}$. The top panel of Figure 2.2 depicts the various linkages. Working backwards to the suppliers of this commodity, we have the basic commodity- and activity-specific price, $PCA_{c,a,r}$. In the 'classic' GTAP model, where each activity produces a unique commodity, then $PCA_{c,a,r} = PDS_{c,r}$ for the appropriate correspondence between a and c. This is also the case when multiple activities produce a common commodity which is perfectly substitutable. However, in the new GTAP model there is scope for imperfect substitution at this stage, in which case these commodity prices are differentiated by activity, with the degree of differentiation governed by a substitution elasticity (discussed below).

The basic commodity- and activity-specific price, $PCA_{c,a,r}$, is equal to the supplier's price ($PS_{c,a,r}$) plus a commodity- and activity-specific tax/subsidy ($TO_{c,a,r}$)—in GTAP, all taxes are implemented as the power of the tax, i.e., $1 + $ tax rate (this has the advantage of allowing for additive price linkage equations when the model is totally differentiated). In order to allow for multi-product activities, a 'make' matrix is introduced. Therefore the supply prices must be aggregated before obtaining unit revenue associated with that activity. Due to zero profits, unit revenue must equal unit cost. Henceforth, we will refer to $PO_{a,r}$ as the *unit revenue of activity a in region r*. It is also useful to define an index for *unit revenue at basic prices* associated with sales by activity a in region r: $PB_{a,r}$. When employing the standard GTAP Data Base, the model will feature a diagonal 'make' matrix with a one-to-one correspondence between activity a and commodity c so that $PDS_{c,r} = PB_{a,r}$ for the appropriate correspondence between a and c.

From Figure 2.2, we see that domestic supplies are allocated across destination regions—the domestic market and all external destinations, i.e., bilateral exports. All of these sales of domestically supplied goods are priced at $PDS_{c,r}$. Export prices are obtained by multiplying $PDS_{c,r} \cdot TXS_{c,s,d}$ (where $TXS_{c,s,d} = 1 + $ export tax rate) and this converts the domestic supply price to the price of exports, $PFOB_{c,s,d}$, denoting the price before international freight and insurance are added. Given the presence of a (potentially) bilaterally varying export tax, this price is now destination-specific. In the figure, the indices for the export price reflect the demand side, and not the supply side. The first regional index, in this case s, reflects the source region, and the second regional index, in this case d, reflects the destination region. The top of the figure has the source region as s, so from the supply side, the FOB price should be written as $PFOB_{c,s,d}$, where d is the destination region. The

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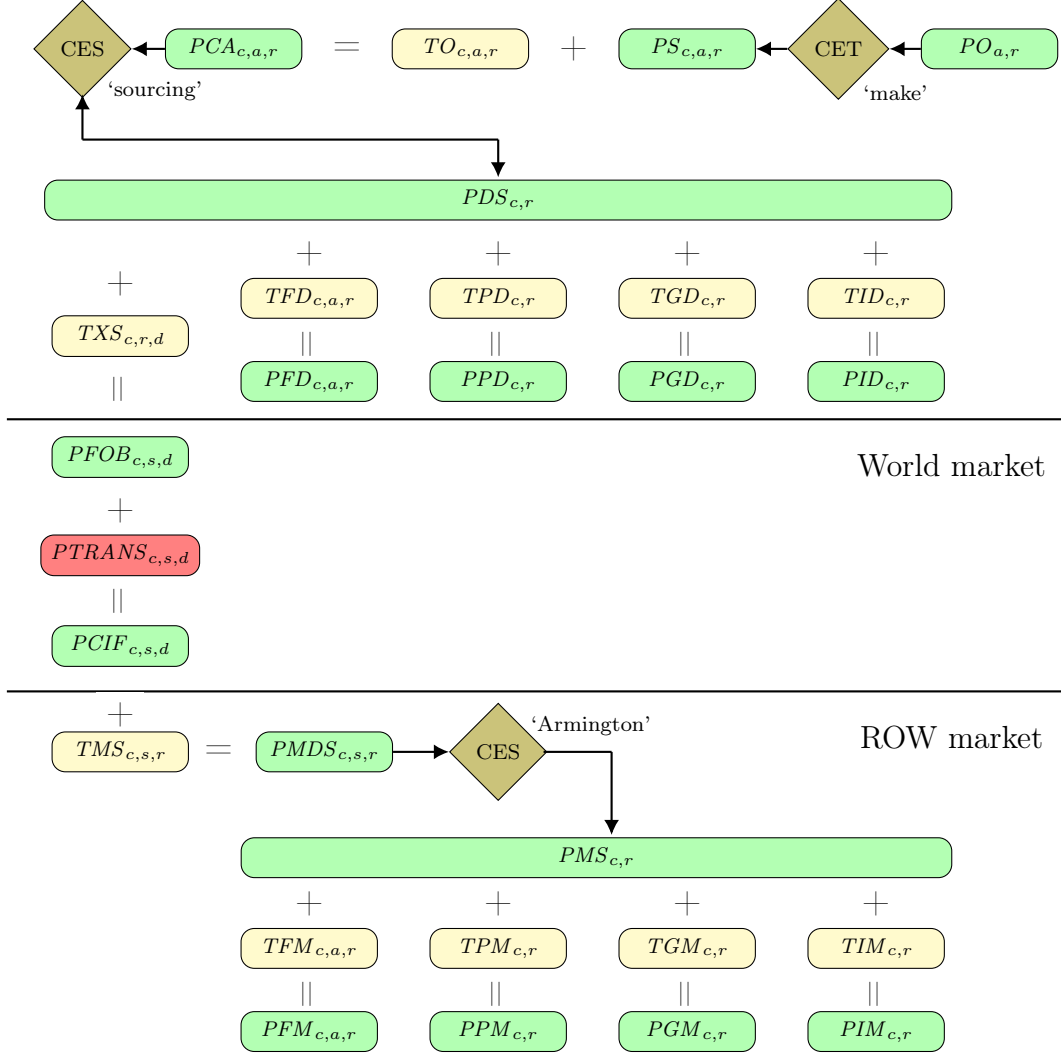
Figure 2.2: **Price linkages in the model**

FOB price undergoes two further transformations en route to its final destination. A transportation margin ($PTRANS_{c,s,d}$) is added to the FOB price to generate the CIF price of imports, $PCIF_{c,s,d}$. Then a bilateral tariff ($TMS_{c,s,d}$) is added to the latter to generate the $\underline{P}$rice of i$\underline{M}$ports in the $\underline{D}$omestic market by $\underline{S}$ource, $PMDS_{c,s,r}$. A 'national' importer aggregates bilateral imports from all sources to 'produce' an aggregate import bundle with a $\underline{P}$rice of i$\underline{M}$ported $\underline{S}$upplies, $PMS_{c,r}$.[6] Each agent in the economy—firms, households, government and investment—access this common import bundle market at the common price, $PMS_{c,r}$, and which competes with domestically supplied goods

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[6]In theory, it would be preferable to allow variation in the sourcing from individual exporters. However, there are two reasons for avoiding this. Firstly, the data are not there to support bilateral sourcing by agent. In practice, we are lucky if we can get the split of domestic and imported goods by sector/agent from database contributors (Aguiar et al., 2016). Secondly, if we were to introduce this bilateral sourcing by agent, we would have a large number of four-dimensioned arrays in the model (commodity × source × destination × sector) and this would create problems of model size for many users. For example, there are 140 regions and 57 sectors in the GTAP 9 Data Base. Full sourcing by agent would result in intermediate input arrays with more than 60 million elements! In light of the fact that the data to support such a model are not presently available, this seems like a poor choice.

which are priced at $PDS_{c,r}$. However, there are also agent-specific sales taxes which must be applied before reaching the prices actually paid by firms, private households, government and investors for the imported goods: $PFM_{c,a,r}$, $PPM_{c,r}$, $PGM_{c,r}$ and $PIM_{c,r}$, as well as for the domestic goods: $PFD_{c,a,r}$, $PPD_{c,r}$, $PGD_{c,r}$ and $PID_{c,r}$. The agents' price of the composite commodity obtained after aggregating the domestic and imported goods is represented by $PFA_{c,a,r}$, $PPA_{c,r}$, $PGA_{c,r}$ and $PIA_{c,r}$ respectively for firms, households, government and investment.

2.1 Firm behavior

Each producing activity, indexed by a, combines a set of intermediate goods and factors to produce output. Similar to many CGE models, the production structure is based on a sequence of nested Constant Elasticity of Substitution (CES) functions that aims to re-produce the substitution possibilities across the full set of inputs. The nested structure, or "technology tree" is depicted in Figure 2.3. The top level nest is composed of two aggregate composite bundles—intermediate demand and value added. The second level nests decompose each of the two aggregate nests into their components—on the one hand demand for individual intermediate goods (at the Armington level) and demand for individual factors.[7] A final nest decomposes demand for the composite good into domestic and imported components.
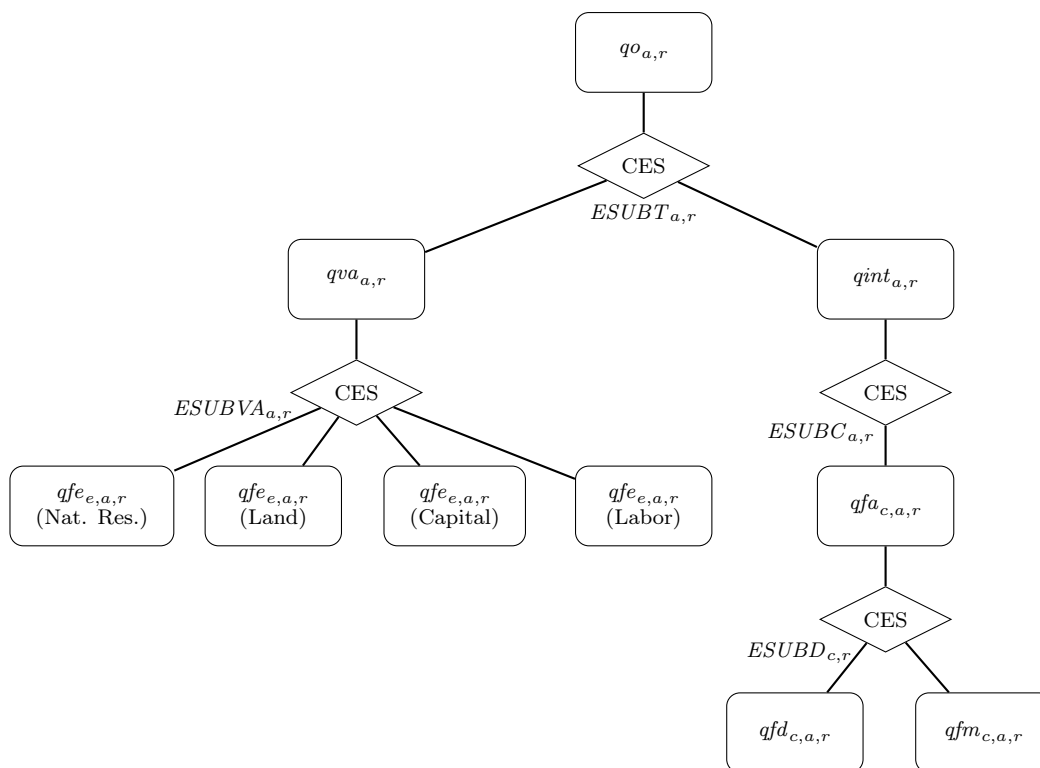
Figure 2.3: **Production structure**

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[7]Labor inputs are normally classified into skilled and unskilled categories; it is also possible to distinguish up to five labor types as per GTAP 9 Data Base.

2.1.1 Top production nest

The composite index of output from activity a, represented by $QO_{a,r}$ in levels, or $qo_{a,r}$ in percentage change form, is a combination of an intermediate demand bundle, $qint_{a,r}$, with the value added bundle, $qva_{a,r}$. Equations (1) and (2) define, respectively, the demand for the two top level bundles where the key substitution elasticity is $ESUBT_{a,r}$ (typically assumed to be zero thereby resulting in a Leontief specification). Equation (3) (written more conveniently as a levels equation) represents the zero-profit condition for activity a, i.e., the total revenue of this activity is equal to the sum of all the input costs, which can be totally differentiated and simplified using the envelope condition to give (3').

In this representation of production, we allow for technological change. All technical change variables are given the first letter a in place of the relevant quantity upon which they operate. Thus, for example, Hicks-neutral technical change is described by changes in $ao_{a,r}$ whereas factor-augmenting technical change would work through the variable $afe_{e,a,r}$. These technological change variables operate in three ways: (1) they reduce the input requirement for the augmented factor, (2) they modify the effective price of the input, and (3) they alter the unit cost of production, and hence, through the zero profits condition, output price. Henceforth, *we will list behavioral equations in percentage change form* and *accounting equations in levels form*. This eases the theoretical exposition. However, implementation in GEMPACK, as well as the code snippets provided in the text, will be solely in linearized form.

$$qint_{a,r} = qo_{a,r} - ao_{a,r} - aint_{a,r} - ESUBT_{a,r}\left(pint_{a,r} - aint_{a,r} - po_{a,r} - ao_{a,r}\right) \tag{1}$$

$$qva_{a,r} = qo_{a,r} - ao_{a,r} - ava_{a,r} - ESUBT_{a,r}\left(pva_{a,r} - ava_{a,r} - po_{a,r} - ao_{a,r}\right) \tag{2}$$

$$PO_{a,r}\,QO_{a,r} = PINT_{a,r}\,QINT_{a,r} + PVA_{a,r}\,QVA_{a,r} \tag{3}$$

$$
\begin{aligned}
po_{a,r} \;=\; & \sum_c STC_{c,a,r}\left(pfa_{c,a,r} - af_{c,a,r} - aint_{a,r}\right) \\
& + \sum_e STC_{e,a,r}\left(pfe_{e,a,r} - afe_{e,a,r} - ava_{e,r}\right) \\
& - ao_{a,r}
\end{aligned}
\tag{3'}
$$

This unit describes the variables $qint_{a,r}$, $qva_{a,r}$ and $po_{a,r}$, using respectively equations `E_qint`, `E_qva` and `E_qo`.

Listing 2.1: **GEMPACK equations for top level production nest**

```
1   Equation E_qint
2   # sector demands for composite intermediate commodity inputs by act. a in r #
3   (all,a,ACTS)(all,r,REG)
4       qint(a,r)
5           = - aint(a,r) + qo(a,r) - ao(a,r)
6           - ESUBT(a,r) * [pint(a,r) - aint(a,r) - po(a,r) - ao(a,r)];

8   Equation E_qva
9   # sector demands for primary factor composite #
10  (all,a,ACTS)(all,r,REG)
11      qva(a,r)
12          = -ava(a,r) + qo(a,r) - ao(a,r)
13          - ESUBT(a,r) * [pva(a,r) - ava(a,r) - po(a,r) - ao(a,r)];

15  Equation E_qo
16  # industry zero pure profits condition #
```

```
17    (all,a,ACTS)(all,r,REG)
18        po(a,r) + ao(a,r)
19            = sum{e,ENDW, STC(e,a,r) * [pfe(e,a,r) - afe(e,a,r) - ava(a,r)]}
20            + sum{c,COMM, STC(c,a,r) * [pfa(c,a,r) - afa(c,a,r) - aint(a,r)]}
21            + profitslack(a,r);
```

2.1.2 Second level nests

The two top level bundles in Figure 2.3, $qint_{a,r}$ and $qva_{a,r}$, are disaggregated into their components using additional CES nests. The intermediate demand bundle, $qint_{a,r}$, is a CES aggregation over commodities of $qfa_{c,a,r}$, which represent the intermediate demand for composite commodity c by activity a, see equation (4). The key substitution elasticity is $ESUBC_{a,r}$, whose default value is 0. There is a subsequent decomposition of composite intermediate input demand into demand for goods by source region described below. The price of the aggregate intermediate demand bundle, $pint_{a,r}$, is determined by the zero profit condition for this CES bundle, where $pfa_{c,a,r}$ represents the price of the intermediate components.

$$qfa_{c,a,r} = qint_{a,r} - afa_{c,a,r} - ESUBC_{a,r}\left(pfa_{c,a,r} - afa_{c,a,r} - pint_{a,r}\right) \tag{4}$$

$$PINT_{a,r}\,QINT_{a,r} = \sum_c PFA_{c,a,r}\,QFA_{c,a,r} \tag{5}$$

$$pint_{a,r} = \sum_c INTSHR_{c,a,r}\left(pfa_{c,a,r} - afa_{c,a,r}\right) \tag{5'}$$

In a similar fashion, the value added bundle, $qva_{a,r}$, is a CES aggregation of $qfe_{e,a,r}$, which represents demand for endowment (or primary factor) e by activity a, as given in equation (6). The key substitution elasticity is $ESUBVA_{a,r}$, which is differentiated by activity and region (although the default is that this is region-generic). The price of the value added bundle is given by equation (7), where $PFE_{e,a,r}$ is the sector and factor-specific price of endowment e.

$$qfe_{e,a,r} = qva_{a,r} - afe_{e,a,r} - ESUBVA_{a,r}\left(pfe_{e,a,r} - afe_{e,a,r} - pva_{a,r}\right) \tag{6}$$

$$PVA_{a,r}\,QVA_{a,r} = \sum_e PFE_{e,a,r}\,QFE_{e,a,r} \tag{7}$$

$$pva_{a,r} = \sum_e SVA_{e,a,r}\left(pfe_{e,a,r} - afe_{e,a,r}\right) \tag{7'}$$

This unit describes the variables $qfa_{c,a,r}$, $pint_{a,r}$, $qfe_{e,a,r}$ and $pva_{a,r}$, using respectively equations `E_qfa`, `E_pint`, `E_qfe` and `E_pva`.

Listing 2.2: **GEMPACK equations for second level production nests**

```
1   Equation E_qfa
2   # industry demands for intermediate inputs c by act. a in region r #
3   (all,c,COMM)(all,a,ACTS)(all,r,REG)
4       qfa(c,a,r)
5           = - afa(c,a,r) + qint(a,r)
6           - ESUBC(a,r) * [pfa(c,a,r) - afa(c,a,r) - pint(a,r)];

8   Equation E_pint
9   # price of composite intermediate commodity inputs by act. a in r #
10  (all,a,ACTS)(all,r,REG)
```

```
11        pint(a,r) = sum{c,COMM, INTSHR(c,a,r) * [pfa(c,a,r) — afa(c,a,r)]};

13    Equation E_qfe
14    # demands for endowment commodities #
15    (all,e,ENDW)(all,a,ACTS)(all,r,REG)
16        qfe(e,a,r)
17            = — afe(e,a,r) + qva(a,r)
18            — ESUBVA(a,r) * [pfe(e,a,r) — afe(e,a,r) — pva(a,r)];

20    Equation E_pva
21    # effective price of primary factor composite in each sector/region #
22    (all,a,ACTS)(all,r,REG)
23        pva(a,r) = sum{e,ENDW, VASHR(e,a,r) * [pfe(e,a,r) — afe(e,a,r)]};
```

2.1.3 Sourcing of commodities by firms

The final nest in production describes the composition of the commodity bundle, $qfa_{c,a,r}$, by source—domestic vs. imported. At this level, the demand for imports represents the demand for a composite bundle, i.e., a bundle of imports from all (external) source regions. Equations (8) and (9) determine, respectively, firms' demand for domestically produced goods ($qfd_{c,a,r}$) and the composite import good ($qfm_{c,a,r}$). The key substitution elasticity is $ESUBD_{c,r}$, the so-called (top-level) Armington[8] elasticity that determines the degree of substitutability between domestic and imported goods. Equation (10) defines the price of the composite (Armington) bundle and (10') gives the percentage change form of $pfa_{c,a,r}$.

$$qfd_{c,a,r} = qfa_{c,a,r} - ESUBD_{c,r}\left(pfd_{c,a,r} - pfa_{c,a,r}\right) \tag{8}$$

$$qfm_{c,a,r} = qfa_{c,a,r} - ESUBD_{c,r}\left(pfm_{c,a,r} - pfa_{c,a,r}\right) \tag{9}$$

$$PFA_{c,a,r}\,QFA_{c,a,r} = PFD_{c,a,r}\,QFD_{c,a,r} + PFM_{c,a,r}\,QFM_{c,a,r} \tag{10}$$

$$pfa_{c,a,r} = \left(1 - FMSHR_{c,a,r}\right)pfd_{c,a,r} + FMSHR_{c,a,r}\,pfm_{c,a,r} \tag{10'}$$

This unit describes $qfd_{c,a,r}$, $qfm_{c,a,r}$ and $pfa_{c,a,r}$, respectively equations `E_qfd`, `E_qfm` and `E_pfa`.

Listing 2.3: **GEMPACK equations for sourcing of commodities by firms**

```
1    Equation E_qfd
2    # act. a demands for domestic good c #
3    (all,c,COMM)(all,a,ACTS)(all,s,REG)
4        qfd(c,a,s) = qfa(c,a,s) — ESUBD(c,s) * [pfd(c,a,s) — pfa(c,a,s)];

6    Equation E_qfm
7    # act. a demands for composite import c #
8    (all,c,COMM)(all,a,ACTS)(all,s,REG)
9        qfm(c,a,s) = qfa(c,a,s) — ESUBD(c,s) * [pfm(c,a,s) — pfa(c,a,s)];

11   Equation E_pfa
12   # industry price for composite commodities #
13   (all,c,COMM)(all,a,ACTS)(all,r,REG)
14       pfa(c,a,r) = [1 — FMSHR(c,a,r)] * pfd(c,a,r) + FMSHR(c,a,r) * pfm(c,a,r);
```

---

[8]Armington (1969) in a seminal paper described import demand using a differentiated goods model.

2.2 Commodity supply

The new standard GTAP model introduces one major innovation, which is the possibility of a non-diagonal 'make' matrix. In GTAP 'classic', each production activity was associated with one, and only one commodity. The new version allows for activities to produce more than one good, for example a biofuels sector that produces both ethanol and distiller's dried grains with solubles (DDGS). This also allows for the supply of a single commodity (e.g., electricity) to be composed of output from multiple activities, for example nuclear and coal-fired generation. The 'make' matrix is an extension which many users have had to introduce themselves over the past two decades. Including this as an option in the standard model enhances its utility as a basis for new extensions and applications.

On the supply side, activities with multiple outputs are given a Constant Elasticity of Transformation (CET) specification wherein they maximize their total revenue stream subject to being on the constant elasticity of transformation frontier. On the demand side, buyers of a commodity produced by multiple activities wish to minimize the total cost of supply subject to a CES preference function. However, the latter is written in such a way as to permit users to eliminate this feature, rendering the goods perfect substitutes, as might be the case, for example with irrigated and rain-fed wheat.

Equation (11) describes changes in the supply of commodity c produced by activity a, $qca_{c,a,r}$. This depends on the overall level of activity in the sector, $qo_{a,r}$, as well as any shift in the mix of commodities supplied by that sector. The latter will depend on changes in the price received by the firm for this commodity, $ps_{c,a,r}$, relative to the firm's unit revenue of activity, $po_{a,r}$, as discussed previously. The key transformation elasticity is given by $ETRAQ_{a,r} < 0$. In the case of a diagonal 'make' matrix, this equation is harmless and simply transforms the output of activity a into the supply of commodity c.[9] Equation (12) computes unit revenue under the zero-profit assumption. Equation (13) links the basic price of commodity by activity, $pca_{c,a,r}$, to the commodity- and activity-specific unit cost, $ps_{c,a,r}$, via the power of the output tax, $to_{c,a,r}$. Equation (14) calculates the average basic (tax-inclusive) price of activity, $pb_{a,r}$, as a share-weighted sum of basic commodity- and activity-specific prices, $pca_{c,a,r}$. Note that if the 'make' matrix is diagonal, then, $pb_{a,r} = pds_{c,r}$ for the appropriate correspondence between a and c.

$$qca_{c,a,r} = qo_{a,r} - ETRAQ_{a,r}\left(ps_{c,a,r} - po_{a,r}\right) \tag{11}$$

$$PO_{a,r}QO_{a,r} = \sum_c PS_{c,a,r}QCA_{c,a,r} \tag{12}$$

$$po_{a,r} = \sum_c MAKESACTSHR_{c,a,r}ps_{c,a,r} \tag{12'}$$

$$pca_{c,a,r} = ps_{c,a,r} + to_{c,a,r} \tag{13}$$

$$PB_{a,r}QO_{a,r} = \sum_c PCA_{c,a,r}QCA_{c,a,r} \tag{14}$$

$$pb_{a,r} = \sum_c MAKEBACTSHR_{c,a,r}pca_{c,a,r} \tag{14'}$$

---

[9]In the case of a diagonal matrix, $ps_{c,a,r} = po_{a,r}$, and the elasticity becomes redundant.

Analogously, a 'national supplier' of composite commodity c purchases its inputs from all activities a producing c using a CES preference function (for example a national electricity supplier). Equation (15) reflects a CES price expression where the key parameter $ESUBQ_{c,r}$ represents the inverse of the CES substitution elasticity[10]—i.e., $ESUBQ = 1/\sigma$ with a default value of 0.[11] Thus Equation (15) simplifies to $pca_{c,a,r} = pds_{c,r}$ when $ESUBQ = 0$, suggesting that the law of one price holds and that the 'national supplier' can perfectly substitute among the same commodities produced by various activities. The variable $qca_{c,a,r}$ represents the desired demand for commodity c produced by activity a. Equation (16) represents the zero-profit condition and in essence determines the domestic supply of good c, $qc_{c,r}$.[12] These equations determine respectively $pca_{c,a,r}$ and $qc_{c,r}$. The variable qca from the CET-side is the supply, while the variable qca from the CES-side is the demand for these commodities and we could identify them separately and then include an equilibrium condition that determines pca. We skip this step and substitute out the equilibrium condition.

$$pca_{c,a,r} = pds_{c,r} - ESUBQ_{c,r}\left(qca_{c,a,r} - qc_{c,r}\right) \tag{15}$$

$$PDS_{c,r}QC_{c,r} = \sum_a PCA_{c,a,r}QCA_{c,a,r} \tag{16}$$

$$qc_{c,r} = \sum_a MAKEBCOMSHR_{c,a,r}\,qca_{c,a,r} \tag{16'}$$

This unit determines $qca_{c,a,r}$, $qo_{a,r}$, $ps_{c,a,r}$, $pb_{a,r}$, $pca_{c,a,r}$ and $qc_{c,r}$ using equations `E_qca`, `E_po`, `E_ps`, `E_pb`, `E_pca` and `E_qc`.

Listing 2.4: **GEMPACK equations for commodity supply**

```
1   Equation E_qca
2   # supply of commodities by act. a #
3   (all,c,COMM)(all,a,ACTS)(all,r,REG)
4       qca(c,a,r) = IF[MAKES(c,a,r) gt 0,
5                    qo(a,r) - ETRAQ(a,r) * [ps(c,a,r) - po(a,r)]];

7   Equation E_po
8   # average unit (tax-exclusive) cost of output of act. a #
9   (all,a,ACTS)(all,r,REG)
10      po(a,r) = sum{c,COMM, MAKESACTSHR(c,a,r) * ps(c,a,r)};

12  Equation E_ps
13  # links basic and supply price of commodity c produced by activity a in r #
14  (all,c,COMM)(all,a,ACTS)(all,r,REG)
15      pca(c,a,r) = ps(c,a,r) + to(c,a,r);

17  Equation E_pb
18  # price index: basic (tax-inclusive) price of output of act. a #
19  (all,a,ACTS)(all,r,REG)
20      pb(a,r) = sum{c,COMM, MAKEBACTSHR(c,a,r) * pca(c,a,r)};

22  Equation E_pca
```

---

[10]Horridge (2014) notes that where high or infinite elasticity values are permissible, it is necessary to write the CES specification in GEMPACK as $p_i = p_{ave} - \tau(x_i - x_{tot})$ and $x_{tot} = \sum_i S_i x_i$ where $\tau = 1/\sigma$, instead of the primal form $x_i = x_{tot} - \sigma(p_i - p_{ave})$ and $p_{ave} = \sum_i S_i p_i$.

[11]By default, $ESUBQ_{c,r} = 0$ is hard coded in the model code, i.e., perfect substitution is the default specification. Users could change this default value in the parameter data file. A value of 0 implies perfect substitution, while higher (infinite) $ESUBQ$ values imply imperfect substitution.

[12]Equation (16) could be replaced with the CES primal function, explicitly defining $QC_{c,r}$. The price $PDS_{c,r}$ is determined by the goods market equilibrium condition described below.

```
23  # CES allocation of commodity output by activity (ESUBQ(c,r) is 1/CES elast.) #
24  (all,c,COMM)(all,a,ACTS)(all,r,REG)
25     pca(c,a,r) = IF[MAKEB(c,a,r) gt 0, pds(c,r)
26              — ESUBQ(c,r) * [qca(c,a,r) — qc(a,r)]];          ! Inverse CES !

28  Equation E_qc
29  # market clearing condition for total commodity supply #
30  (all,c,COMM)(all,r,REG)
31     qc(c,r) = sum{a,ACTS, MAKEBCOMSHR(c,a,r) * qca(c,a,r)};     ! Inverse CES !
```

2.3 Income distribution

In keeping with its primary role as a trade model, focusing on the international incidence of policies, the model has a single representative household for each region. The household receives all gross factor payments net of the capital depreciation allowance, plus the receipts from all indirect taxes. Equation (17) represents gross factor payments equal to total factor remuneration—summed across all activities and factors—evaluated at market prices, less the depreciation allowance. Equation (18) represents total regional income—the sum of factor income and the fiscal revenues from all indirect taxes (sales tax on domestic and imported goods, taxes on factor use, output tax, and import and export taxes).[13] Section 2.9 will derive the indirect tax revenue flows and total tax revenues.[14]

$$FINCOME_r = \sum_a \sum_e PEB_{e,a,r} QES_{e,a,r} - \delta_r PINV_r KB_r \tag{17}$$

$$Y_r = FINCOME_r + INDTAX_r \tag{18}$$

This unit determines $FINCOME_r$ and Y_r using equations E_fincome and E_y.

Listing 2.5: **GEMPACK equations for regional income equations**

```
1  Equation E_fincome
2  # factor income at basic prices net of depreciation #
3  (all,r,REG)
4  FY(r) * fincome(r)
5     = sum{e,ENDW, sum{a,ACTS, EVFB(e,a,r)*[peb(e,a,r) + qes(e,a,r)]}}
6     — VDEP(r)  * [pinv(r) + kb(r)];

8  Equation E_y
9  # regional income = sum of primary factor income and indirect tax receipts #
10 (all,r,REG)
11 INCOME(r) * y(r)
12    = FY(r) * fincome(r)
13    + 100.0 * INCOME(r) * del_indtaxr(r)
14    + INDTAX(r) * y(r)
15    + INCOME(r) * incomeslack(r);
```

2.4 Allocation of income across expenditure categories

Regional income is distributed across three broad categories—private consumption, public expenditures and saving. The idea of treating saving as a commodity in a static utility function derives

---

[13]There are also income taxes on total factor remuneration. These are incorporated in the *FINCOME* variable.

[14]The GTAP model evaluates tax revenues relative to regional income and this will be reflected in the TABLO implementation of the relevant equations, if not in the mathematical description. For the interested reader, Section A.0.2 in the Mathematical Appendix describes this with additional detail.

from Lluch (1973) and Howe (1975). The saving good proxies demand for future consumption in this comparative static model. Similarly, the demand for government spending is treated as a proxy for the welfare obtained from public goods provided by such spending. This idea is motivated by Keller (1980, chapter 8), who demonstrates that if: (1) preferences for public goods are separable from preferences for private goods, and (2) the utility function for public goods is identical across households within the regional economy, then we can derive a public utility function. The aggregation of this index with private utility in order to make inferences about regional welfare requires the further assumption that (3) the level of public goods provided in the initial equilibrium is optimal. Users who do not wish to invoke this assumption can employ an alternative closure, such as fixing the level of aggregate government utility while letting private consumption adjust to exhaust regional income on expenditures. However, doing so destroys the appealing welfare properties of the regional household utility function.

A top-level utility function, using a Cobb-Douglas specification, governs the allocation of aggregate expenditure across these three broad categories.[15] More specifically, regional households act so as to maximize utility:

$$U = A \sum_f U_f^{B_f}$$

subject to the budget constraint:

$$\sum_f E_f (U_f, P_f)$$

where U denotes overall regional utility, U_f is sub-utility from source f, $E_f (U_f, P_f)$ the expenditure required to achieve sub-utility U_f at price vector P_f. and B_f are the Cobb-Douglas distribution parameters; and the index f ranges over the three broad categories of private consumption, government consumption, and saving.

Saving is a unitary good, but for government and private consumption, aggregator functions relate overall sub-utility to consumption of individual commodities. For government consumption, the function is of the CES form (Cobb-Douglas by default); for private consumption, it is the Constant-Differences-of-Elasticities (CDE) system (Hanoch, 1975). This system affords no closed-form solution for expenditure, but that is not a problem in a numerical model where it can be numerically computed.

A key point is that the budget equation involves, in general, not prices of sub-utility but price vectors P_f. For saving, indeed, there is a single saving price P_s, and for government consumption, we can derive a constant marginal cost P_g. But for private consumption, CDE preferences being non-homothetic, we cannot determine the price of private utility independently of the level of private consumption (McDougall, 2003). With the marginal cost of utility from private consumption endogenous, the top-level expenditure shares, as it turns out, depend on the elasticity of expenditure with respect to utility, both in aggregate and for private consumption individually (for saving and government consumption, the elasticity is identically one). Accommodating this feature required a major theory extension which is fully developed in McDougall (2003), to which the technically-oriented reader is referred and an abbreviated version is developed in Section A.0.3 in the Mathematical Appendix.

Besides the variables U and U_f, the utility function involves parameters A and B_f. We treat these as variables in the model. Changes in the distribution parameters B_f represent changes in regional household preferences; users may endogenize these, in effect overruling the regional

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[15]This specification was introduced by McDougall (2003).

household demand system so as to meet targets represented by other, exogenized variables. Of course, as noted above, this destroys the welfare properties of the model.

"Expenditure" on the single saving good is the product of its price $PSAVE_r$ and quantity $QSAVE_r$, i.e., the value of saving in percentage change form, $psave_r + qsave_r$. The share of saving in regional income is then $psave_r + qsave_r - y_r$. With fixed prices and preferences, this would be a constant—in percentage change form, zero—but in the more complex situation obtaining, we have

$$psave_r + qsave_r - y_r = uelas_r + dpsave_r \tag{19}$$

where $uelas_r$ denotes (percentage change in) the elasticity of expenditure with respect to utility and $dpsave_r$ the Cobb-Douglas distribution parameter B_S. Government consumption expenditure yg_r likewise is given by

$$yg_r - y_r = uelas_r + dpgov_r \tag{20}$$

where $dpgov_r$ is the Cobb-Douglas distribution parameter B_G. Private consumption expenditure is given by

$$yp_r - y_r = uelas_r - uepriv_r + dppriv_r \tag{21}$$

where $dppriv_r$ is the Cobb-Douglas distribution parameter B_P and $uepriv_r$ represents the elasticity of expenditure on private consumption with respect to utility therefrom. This elasticity, unlike its counterparts for government consumption and saving, is variable in levels and non-zero in percentage changes. The overall elasticity of expenditure $uelas_r$ is an index of the elasticities for the three broad categories, but, since two of the three are fixed, its percentage change equation involves only the remaining one:[16]

$$uelas_r = XSHRPRIV_r uepriv_r + dpav_r \tag{22}$$

where $XSHRPRIV_r$ represents the share of private consumption expenditure in regional income, and $dpav_r$ an index of the change in the distribution parameters:

$$dpav_r = XSHRPRIV_r dppriv_r + XSHRGOV_r dpgov_r + XSHRSAV_r dpsave_r \tag{23}$$

where $XSHRGOV_r$ and $XSHRSAV_r$ represent respectively the share of public expenditure and saving in regional income. This suffices to determine the top-level demands; we also calculate two descriptive variables, an overall price index for disposition of income:

$$p_r = XSHRPRIV_r priv_r + XSHRGOV_r pgov_r + XSHRSAV_r psave_r \tag{24}$$

and top-level utility,

$$\begin{aligned} u_r &= au_r + DPARPRIV_r \log{(UTILPRIV_r)}\, dppriv_r \\ &+ DPARGOV_r \log{(UTILGOV_r)}\, dpgov_r \\ &+ DPARSAV_r \log{(UTILSAVE_r)}\, dpsave_r \\ &+ UTILELAS_r^{-1}\,(y_r - pop_r - p_r) \end{aligned} \tag{25}$$

Here au_r represents the percentage change in the parameter A in the overall utility function, $DPARPRIV_r$ the levels value of the distributional parameter B_P for private consumption, the levels value of utility U_P from private consumption is given by $UTILPRIV_r$, and so on. In most

---

[16]This formula is developed in Section A.0.3 in the Mathematical Appendix.

simulations, the change in the distributional terms are zero (and even when we let the distributional parameters vary, we arrange things so their contributions are zero to first order), and the equation reduces to the simpler form:

$$u_r = UTILELAS_r^{-1} \left(y_r - pop_r - p_r \right)$$

that is, utility u_r depends on real *per capita* income $y_r - pop_r - p_r$, with a sensitivity given by the inverse of the elasticity $UTILELAS_r$ of expenditure with respect to utility; this inverse being just the elasticity of utility with respect to income.

This unit determines $qsave_r$, yg_r, yp_r, $uelas_r$, $dpav_r$, p_r and u_r using equations E_qsave, E_yg, E_yp, E_uelas, E_dpav, E_p and E_u.

Listing 2.6: **GEMPACK equations for top level utility equations**

```
1  Equation E_qsave
2  # saving #
3  (all,r,REG)
4     psave(r) + qsave(r) − y(r) = uelas(r) + dpsave(r);

6  Equation E_yg
7  # government consumption expenditure #
8  (all,r,REG)
9     yg(r) − y(r) = uelas(r) + dpgov(r);

11 Equation E_yp
12 # private consumption expenditure #
13 (all,r,REG)
14    yp(r) − y(r) = −[uepriv(r) − uelas(r)] + dppriv(r);

16 Equation E_uelas
17 # elasticity of cost of utility wrt utility #
18 (all,r,REG)
19    uelas(r) = XSHRPRIV(r) * uepriv(r) − dpav(r);

21 Equation E_dpav
22 # average distribution parameter shift #
23 (all,r,REG)
24    dpav(r)
25       = XSHRPRIV(r)  * dppriv(r)
26       + XSHRGOV(r)   * dpgov(r)
27       + XSHRSAVE(r)  * dpsave(r);

29 Equation E_p
30 # price index for disposition of income by regional household #
31 (all,r,REG)
32    p(r)
33       = XSHRPRIV(r) * ppriv(r)
34       + XSHRGOV(r)  * pgov(r)
35       + XSHRSAVE(r) * psave(r);

37 Equation E_u
38 # regional household utility #
39 (all,r,REG)
40    u(r)
41       = au(r)
42       + DPARPRIV(r) * loge(UTILPRIV(r)) * dppriv(r)
43       + DPARGOV(r)  * loge(UTILGOV(r))  * dpgov(r)
44       + DPARSAVE(r) * loge(UTILSAVE(r)) * dpsave(r)
45       + [1.0 / UTILELAS(r)] * [y(r) − pop(r) − p(r)];
```

17

2.5 Domestic final demand

The top level distribution of regional income is disbursed for private and public expenditures. Domestic saving is used to purchase capital goods, i.e., investment. The supply of domestic saving may be adjusted by net foreign saving flow. A positive capital account leads to investment higher than domestic saving, and the reverse for a negative capital account balance. The allocation of global investment and capital account closure is described below.

2.5.1 Private expenditures

Equation (26) defines demand for the composite commodity c for private expenditures. In per capita terms, the percent change in demand for good c is the inner-product of the percent change in composite consumer prices with the appropriate row of the matrix of own- and cross-price elasticities ($EP_{c,r}$), plus the percent change in per capita income adjusted by the income elasticity ($EY_{c,r}$).

$$qpa_{c,r} - pop_r = \sum_k EP_{c,k,r} ppa_{k,r} + EY_{c,r} (yr_r - pop_r) \tag{26}$$

Equation (26) is just a generic Marshallian demand equation; what makes our private demand system an economically coherent demand system is the fact that these elasticities depend on prices and quantities (or budget shares) and are derived from an underlying sub-utility function for private expenditure. The particular functional form which is chosen here to represent preferences for private spending is based on the Constant Differences of Elasticities implicitly additive expenditure function (CDE) by Hanoch (1975). The CDE has been shown to be well-suited to CGE applications (Hertel et al., 1991), as it allows more flexibility than the CES or LES functional forms, since it has $2n$ behavioral parameters (where n is the number of commodities). Half of these parameters relate to the compensated price responsiveness and the remainder relate to the response of commodity demands to income. This contrasts with the LES, for example, where just one parameter governs the price responsiveness of all n demands. Another option, short of the parameter-hungry, fully flexible functional forms (e.g., translog) which may not be globally well-behaved, is the 'An Implicitly Directly Additive Demand System' (AIDADS) by Rimmer and Powell (1992). AIDADS is a generalization of the LES which allows for additional Engel flexibility by including two marginal budget shares for each commodity—one governing expenditure patterns at low income levels and one ruling the day at very high income levels. However, as with LES, the price responsiveness of AIDADS is still very limited, and, as income grows and subsistence quantities become relatively small, the uncompensated price elasticities of demand converge to one.[17] This is unattractive in comparative static simulations where income changes are small, relative to price changes.

The CDE demand system has the following generic formulation:

$$\max U : \sum_c a_c U^{e_c b_c} \left(\frac{P_c}{Y} \right)^{b_c} \equiv 1 \quad \text{subject to } Y = \sum_c P_c X_c$$

The parameter e is referred to as the expansion parameter and is linked to the income elasticity and b is the substitution parameter (respectively $INCPAR_{c,r}$ and $SUBPAR_{c,r}$ in the model). Using Roy's identity and the implicit function theorem, the budget shares are given by the following:

---

[17]A further extension of AIDADS involves allowing subsistence quantities to vary as a function of per capita income. In this way, price responses at high income levels can be made more realistic. This Modified AIDADS (MAIDADS) demand system has been proposed by Preckel et al. (2010) and may hold promise for future CGE modeling—particularly when income growth plays an important role.

$$s_c = \frac{Z_c}{\sum_k Z_k} \quad \text{where } Z_c = a_c b_c U^{e_c b_c} \left(\frac{P_c}{Y}\right)^{b_c}$$

Note that with this definition for Z, the utility expression simplifies to:

$$\sum_c \frac{Z_c}{b_c} \equiv 1$$

The CDE system allows us to write out explicit formulae describing how the price and expenditure elasticities of demand, $EPC_{c,k,r}$ and $EY_{c,r}$, vary with changing budget shares and these are given by equations (27–29), that thus feed into equation (26).[18] Equation (27) simplifies the resulting expressions for the price elasticities. It defines the Allen partial elasticity for the CDE function. The parameter $ALPHA_c = 1 - SUBPAR_c$ and the parameter δ is the Kronecker δ that takes the value 1 when the indices are identical (i.e., for diagonal elements), otherwise the value 0. The coefficient $CONSHR$ represents the relevant budget share for commodity c. Equation (28) defines the income elasticities. Equation (29) defines the uncompensated price elasticities, which are a simple function of the Allen partial and income elasticities. Since the parameters of the CDE function are invariant, the only things that change in these elasticity expressions are the continuously updated budget shares.

$$
\begin{aligned}
APE_{c,k,r} &= ALPHA_{k,r} + ALPHA_{c,r}\left(1 - \frac{\delta_{c,k}}{CONSHR_{k,r}}\right) \\
&\quad - \sum_{c'} CONSHR_{c',r} ALPHA_{c',r}
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
EY_{c,r} &= \Big[INCPAR_{c,r}\left(1 - ALPHA_{c,r}\right) \\
&\quad + \quad \sum_k CONSHR_{k,r} INCPAR_{k,r} ALPHA_{k,r}\Big] \\
&\quad / \quad \big[\sum_k CONSHR_{k,r} INCPAR_{k,r}\big] \\
&\quad + \quad \big[ALPHA_{c,r} - \sum_k CONSHR_{k,r} INCPAR_{k,r}\big]
\end{aligned}
\tag{28}
$$

$$EP_{c,k,r} = CONSHR_{c,r}\left(APE_{c,k,r} - EY_{c,r}\right) \tag{29}$$

For the top level of the demand system, we need the elasticity of private consumption expenditure with respect to utility from private consumption; this in levels, as Hanoch (1975) shows, is the expenditure-share-weighted average of the CDE expansion parameters $INCPAR_{c,r}$, in percentage changes, the $INCPAR$-times-expenditure weighted average of the budget shares:

$$upriv_r = \sum_c \frac{CONSHR_{c,r} INCPAR_{c,r}}{\sum_k CONSHR_{k,r} INCPAR_{k,r}} \left(ppa_{c,r} + qpa_{c,r} - yp_r\right) \tag{30}$$

We also calculate two descriptive variables. The private consumption price index $ppriv_r$ is just a weighted average of prices of the composite goods:

$$ppriv_r = \sum_c CONSHR_{c,r} ppa_{c,r} \tag{31}$$

utility from and per capita expenditure on private consumption are related by the percentage-change form of the expenditure function:

---

[18]These expressions are derived in Section A.0.4 in the Mathematical Appendix.

$$yp_r - pop_r = ppriv_r + UELASPRIV_r\, up_r \tag{32}$$

Private expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (33), (34) and (35) determine private demand for domestic goods ($qpd_{c,r}$), imported goods ($qpm_{c,r}$) and the consumer price of the composite good ($ppa_{c,r}$).

$$qpd_{c,r} = qpa_{c,r} - ESUBD_{c,r}\left(ppd_{c,r} - ppa_{c,r}\right) \tag{33}$$

$$qpm_{c,r} = qpa_{c,r} - ESUBD_{c,r}\left(ppm_{c,r} - ppa_{c,r}\right) \tag{34}$$

$$PPA_{c,r}\,QPA_{c,r} = PPD_{c,r}\,QPD_{c,r} + PPM_{c,r}\,QPM_{c,r} \tag{35}$$

$$ppa_{c,r} = \left(1 - PMSHR_{c,r}\right)ppd_{c,r} + PMSHR_{c,r}\,ppm_{c,r} \tag{35'}$$

This unit determines $qpa_{c,r}$, $uepriv_r$, $ppriv_r$, up_r, $qpd_{c,r}$, $qpm_{c,r}$ and $ppa_{c,r}$ using equations `E_qpa`, `E_uepriv`, `E_ppriv`, `E_up`, `E_qpd`, `E_qpm` and `E_ppa`. In addition, consumer demand requires updating of the income and price elasticity expressions, $APE_{c,k,r}$, $EY_{c,r}$ and $EP_{c,k,r}$ using update formulas for $ALPHA_{c,r}$ and $CONSHR_{c,r}$.

Listing 2.7: **GEMPACK equations for consumer demand equations**

```
1   Formula (all,c,COMM)(all,r,REG)
2      ALPHA(c,r) = 1 — SUBPAR(c,r);

4   Formula (all,c,COMM)(all,r,REG)
5      CONSHR(c,r) = VPP(c,r) / PRIVEXP(r);

7   Formula (all,c,COMM)(all,k,COMM)(all,r,REG)
8      APE(c,k,r)
9         = ALPHA(c,r) + ALPHA(k,r) — sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)};

11  Formula (all,c,COMM)(all,r,REG)
12     APE(c,c,r)
13        = 2.0 * ALPHA(c,r)
14        — sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)}
15        — ALPHA(c,r) / CONSHR(c,r);

17  Formula (all,c,COMM)(all,r,REG)
18     EY(c,r)
19        = [1.0 / sum{n,COMM, CONSHR(n,r) * INCPAR(n,r)}]
20        * [INCPAR(c,r) * [1.0 — ALPHA(c,r)]
21        + sum{n,COMM, CONSHR(n,r) * INCPAR(n,r) * ALPHA(n,r)}]
22        + [ALPHA(c,r) — sum{n,COMM, CONSHR(n,r) * ALPHA(n,r)}];

24  Formula (all,c,COMM)(all,k,COMM)(all,r,REG)
25     EP(c,k,r) = [APE(c,k,r) — EY(c,r)] * CONSHR(k,r);

27  Equation E_qpa
28  # private consumption demands for composite commodities #
29  (all,c,COMM)(all,r,REG)
30     qpa(c,r) — pop(r)
31        = sum{k,COMM, EP(c,k,r) * ppa(k,r)} + EY(c,r) * [yp(r) — pop(r)];

33  Equation  E_uepriv
34  # elasticity of expenditure wrt utility from private consumption #
35  (all,r,REG)
36     uepriv(r) = sum{c,COMM, XWCONSHR(c,r) * [ppa(c,r) + qpa(c,r) — yp(r)]};
```

```
38  Equation E_ppriv
39  # price index for private consumption expenditure #
40  (all,r,REG)
41      ppriv(r) = sum{c,COMM, CONSHR(c,r) * ppa(c,r)};

43  Equation E_up
44  # computation of utility from private consumption in r #
45  (all,r,REG)
46      UELASPRIV(r) * up(r) = yp(r) — ppriv(r) — pop(r) ;

48  Equation E_qpd
49  # private consumption demand for domestic goods #
50  (all,c,COMM)(all,r,REG)
51      qpd(c,r) = qpa(c,r) — ESUBD(c,r) * [ppd(c,r) — ppa(c,r)];

53  Equation E_qpm
54  # private consumption demand for aggregate imports #
55  (all,c,COMM)(all,r,REG)
56      qpm(c,r) = qpa(c,r) — ESUBD(c,r) * [ppm(c,r) — ppa(c,r)];

58  Equation E_ppa
59  # private consumption price for composite commodities #
60  (all,c,COMM)(all,r,REG)
61      ppa(c,r) = [1 — PMSHR(c,r)] * ppd(c,r) + PMSHR(c,r) * ppm(c,r);
```

2.5.2 Public expenditures

The sub-utility function for public expenditure is based on a CES utility function. Equation (36) determines composite commodity demand by the government for commodity c where $ESUBG_r$ is the substitution elasticity across commodities.[19] The government expenditure price index is provided in equation (37), where the ratio term expresses the budget shares.

$$qga_{c,r} = yg_r - pgov_r - ESUBG_r \left(pga_{c,r} - pgov_r \right) \tag{36}$$

$$pgov_r = \sum_c \frac{VGP_{c,r}}{GOVEXP_r} pga_{c,r} \tag{37}$$

Utility from and expenditure on government consumption are related by the percentage change form of a linearly homogeneous expenditure function:

$$yg_r - pop_r = pgov_r + ug_r \tag{38}$$

Public expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (39), (40) and (41) determine public demand for domestic goods ($qgd_{c,r}$), imported goods ($qgm_{c,r}$) and the government price of the composite good ($pga_{c,r}$).

$$qgd_{c,r} = qga_{c,r} - ESUBD_{c,r} \left(pgd_{c,r} - pga_{c,r} \right) \tag{39}$$

$$qgm_{c,r} = qga_{c,r} - ESUBD_{c,r} \left(pgm_{c,r} - pga_{c,r} \right) \tag{40}$$

---

[19]There is no aggregate volume of government expenditure. It could be convenient to hold this fixed in some simulations and endogenize some instrument, such as the top-level utility share parameter for government expenditures. The equation $xg = yg - pgov$ would define the real volume of government expenditure (this is equivalent to ug).

$$PGA_{c,r} QGA_{c,r} = PGD_{c,r} QGD_{c,r} + PGM_{c,r} QGM_{c,r} \qquad (41)$$

$$pga_{c,r} = (1 - GMSHR_{c,r}) \, pgd_{c,r} + GMSHR_{c,r} pgm_{c,r} \qquad (41')$$

This unit determines $qga_{c,r}$, $pgov_r$, ug_r, $qgd_{c,r}$, $qgm_{c,r}$ and $pga_{c,r}$ using equations E_qga, E_pgov, E_ug, E_qgd, E_qgm and E_pga.

Listing 2.8: **GEMPACK equations for government demand**

```
1   Equation E_qga
2   # government consumption demands for composite commodities #
3   (all,c,COMM)(all,r,REG)
4       qga(c,r) = yg(r) - pgov(r) - ESUBG(r) * [pga(c,r) - pgov(r)];
5
6   Equation E_pgov
7   # price index for aggregate gov't purchases #
8   (all,r,REG)
9       pgov(r) = sum{c,COMM, [VGP(c,r) / GOVEXP(r)] * pga(c,r)};
10
11  Equation E_ug
12  # utility from government consumption in r #
13  (all,r,REG)
14      ug(r) = yg(r) - pgov(r) - pop(r);
15
16  Equation E_qgd
17  # government consumption demand for domestic goods #
18  (all,c,COMM)(all,r,REG)
19      qgd(c,r) = qga(c,r) - ESUBD(c,r) * [pgd(c,r) - pga(c,r)];
20
21  Equation E_qgm
22  # government consumption demand for aggregate imports #
23  (all,c,COMM)(all,r,REG)
24      qgm(c,r) = qga(c,r) - ESUBD(c,r) * [pgm(c,r) - pga(c,r)];
25
26  Equation E_pga
27  # government consumption price for composite commodities #
28  (all,c,COMM)(all,r,REG)
29      pga(c,r) = [1 - GMSHR(c,r)] * pgd(c,r) + GMSHR(c,r) * pgm(c,r);
```

2.5.3 Investment expenditures

The sub-utility function for investment expenditure, i.e., gross investment, is based on a Leontief utility function. Equation (42) determines composite commodity demand by the capital goods sector for commodity c. The aggregate volume of investment, $qinv_r$, will be defined below, but essentially it comes from the nominal investment equals saving identity, where saving is the sum of domestic saving and net capital inflows from foreign economies. The investment expenditure price index is provided in equation (43) and equation (43') in percentage terms where the ratio term expresses the expenditure shares.

$$qia_{c,r} = qinv_r \qquad (42)$$

$$PINV_r QINV_r = \sum_c QIA_{c,r} PIA_{c,r} \qquad (43)$$

$$pinv_r = \sum_c \frac{VIP_{c,r}}{REGINV_r} pia_{c,r} \qquad (43')$$

Investment expenditures on the composite goods are subsequently decomposed into demand for domestic and imported goods using a CES sub-utility preference function. Equations (44), (45) and (46) determine investement demand for domestic goods ($qid_{c,r}$), imported goods ($qim_{c,r}$) and the price of the composite investment good ($pia_{c,r}$).

$$qid_{c,r} = qia_{c,r} - ESUBD_{c,r} \left(pid_{c,r} - pia_{c,r} \right) \tag{44}$$

$$qim_{c,r} = qia_{c,r} - ESUBD_{c,r} \left(pim_{c,r} - pia_{c,r} \right) \tag{45}$$

$$PIA_{c,r} QIA_{c,r} = PID_{c,r} QID_{c,r} + PIM_{c,r} QIM_{c,r} \tag{46}$$

$$pia_{c,r} = (1 - IMSHR_{c,r}) \, pid_{c,r} + IMSHR_{c,r} \, pim_{c,r} \tag{46'}$$

This unit determines $qia_{c,r}$, $pinv_r$, $qid_{c,r}$, $qim_{c,r}$ and $pia_{c,r}$ using equations `E_qia`, `E_pinv`, `E_qid`, `E_qim` and `E_pia`.

Listing 2.9: **GEMPACK equations for investment demand**

```
1   Equation E_qia
2   # Top level (Leontief) demand for investment goods #
3   (all,c,COMM)(all,r,REG)
4       qia(c,r) = qinv(r);
5
6   Equation E_pinv
7   # defines the price of investment #
8   (all,r,REG)
9       pinv(r) = sum{c,COMM, [VIP(c,r) / REGINV(r)] * pia(c,r)};
10
11  Equation E_qid
12  # demand for domestic investment commodity c #
13  (all,c,COMM)(all,r,REG)
14      qid(c,r) = qia(c,r) - ESUBD(c,r) * [pid(c,r) - pia(c,r)];
15
16  Equation E_qim
17  # demand for imported investment commodity c #
18  (all,c,COMM)(all,r,REG)
19      qim(c,r) = qia(c,r) - ESUBD(c,r) * [pim(c,r) - pia(c,r)];
20
21  Equation E_pia
22  # invesment price for composite commodities #
23  (all,c,COMM)(all,r,REG)
24      pia(c,r) = [1 - IMSHR(c,r)] * pid(c,r) + IMSHR(c,r) * pim(c,r);
```

2.6 Trade, goods market equilibrium and prices

2.6.1 Sourcing of imports

At this juncture, all agents in the economy have a well-specified commodity-specific demand for domestic goods and composite imported goods.[20] The sourcing of imports by region of origin is done at the regional level in the destination country.[21] Equation (47) aggregates all of the agent-based demand for the import composite good. With a CES preference function for the sourcing of

---

[20]The domestic supplier of international trade and transport margins is explicitly assumed to only directly purchase domestic goods and services.

[21]This implies an assumption that the preferences for imports by source region are uniform across all domestic agents. A MRIO-based model assumes that sourcing by region is done at the agent level.

imports, the demand for each good by region of origin is given by equation (48), where $ESUBM_{c,d}$ is the substitution elasticity for imports by destination region and the price $pmds_{c,s,d}$ is the basic price of commodity c produced in region s augmented by a bilateral export tax, the node-specific trade and transport margin and the relevant bilateral tariff.[22] (Recall the top panel of Figure 2.2.) The aggregate import price, $PMS_{c,d}$, is defined in equation (49).[23]

$$QMS_{c,r} = \sum_a QFM_{c,a,r} + QPM_{c,r} + QGM_{c,r} + QIM_{c,r} \tag{47}$$

$$qxs_{c,s,d} = qms_{c,d} - ESUBM_{c,d}\left(pmds_{c,s,d} - pms_{c,d}\right) \tag{48}$$

$$PMS_{c,d}QMS_{c,d} = \sum_s PMDS_{c,s,d}QXS_{c,s,d} \tag{49}$$

This unit determines $qms_{c,d}$, $qxs_{c,s,d}$ and $PMS_{c,d}$ using equations `E_qms`, `E_qxs` and `E_pms`.

Listing 2.10: **GEMPACK equations for sourcing of imports by region**

```
1  Equation E_qms
2  # assures mkt clearing for imported goods entering each region #
3  (all,c,COMM)(all,r,REG)
4     qms(c,r)
5         = sum{a,ACTS, FMCSHR(c,a,r) * qfm(c,a,r)}
6         + PMCSHR(c,r) * qpm(c,r)
7         + GMCSHR(c,r) * qgm(c,r)
8         + IMCSHR(c,r) * qim(c,r);
9
10 Equation E_qxs
11 # regional demand for disaggregated imported commodities by source #
12 (all,c,COMM)(all,s,REG)(all,d,REG)
13    qxs(c,s,d)
14        = -ams(c,s,d) + qms(c,d)
15        - ESUBM(c,d) * [pmds(c,s,d) - ams(c,s,d) - pms(c,d)];
16
17 Equation E_pms
18 # price for aggregate imports #
19 (all,c,COMM)(all,d,REG)
20    pms(c,d) = sum{s,REG, MSHRS(c,s,d) * [pmds(c,s,d) - ams(c,s,d)]};
```

2.6.2 International trade and transport margins

Trade flows from region s to region d generate demand for trade and transport services. Equation (50) describes the demand for trade and transport service m, to deliver good c from region s to region d. Demand is in fixed proportion to the quantity being delivered, with the possibility of improvements in transport efficiency as captured by the technical coefficient *atmfsd* giving the per unit efficiency of Transportation by Mode of Freight c from Source to Destination. Given the lack of bilateral supplies of shipping services, each mode of transport, m, is supplied at a uniform price PT_m across the world. This global transport price is a composite based on the price of national

---

[22]All bilateral variables have two regional indices. The first is always the source region and the second is the destination region. Thus total exports from region s is written as follows where d is the importing (i.e., destination) region:

$$TEXP_{c,s} = \sum_d QXS_{c,s,d}$$

[23]The CES sourcing specification allows for a shift in preferences using the variable *ams*.

margin services exports, as given by equation (54). To compute the composite FOB-CIF margin, it is necessary to aggregate these modal-specific prices over all relevant modes of transport for that particular commodity. Any transport efficiency changes enter into this calculation as well, giving equation (51).

The global demand for margin service m is the sum of demand across all commodities and across all bilateral trade nodes, equation (52). There is a fictional 'global' transport sector which purchases the services m from each region. The global purchaser wishes to minimize the cost of purchasing the services across regions subject to a CES preference function. Optimal demand is given by equation (53), which determines $QST_{m,r}$, i.e., the regional supply of trade service m.

$$qtmfsd_{m,c,s,d} = qxs_{c,s,d} - atmfsd_{m,c,s,d} \tag{50}$$

$$ptrans_{c,s,d} = \sum_m VTFSD\_MSH_{m,c,s,d} \left(PT_m - atmfsd_{m,c,s,d} \right) \tag{51}$$

$$QTM_m = \sum_c \sum_s \sum_d QTMFSD_{m,c,s,d} \tag{52}$$

$$qst_{m,r} = qtm_m - ESUBS_m \left(pds_{m,r} - pt_m \right) \tag{53}$$

$$PT_m QTM_m = \sum_r PDS_{m,r} QST_{m,r} \tag{54}$$

This unit determines $qtmfsd_{m,c,s,d}$, $ptrans_{c,s,d}$, QTM_m, $qst_{m,r}$ and PT_m using, respectively, equations E_qtmfsd, E_ptrans, E_qtm, E_qst and E_pt.

Listing 2.11: **GEMPACK equations for international trade margins**

```
1   Equation E_qtmfsd
2   # bilateral demand for transport services #
3   (all,m,MARG)(all,c,COMM)(all,s,REG)(all,d,REG)
4      qtmfsd(m,c,s,d) = qxs(c,s,d) - atmfsd(m,c,s,d);

6   Equation E_ptrans
7   #  generates flow-specific modal average cost of transport index (cf. HT7) #
8   (all,c,COMM)(all,s,REG)(all,d,REG)
9      ptrans(c,s,d) = sum{m,MARG, VTFSD_MSH(m,c,s,d)
10                     * [pt(m) - atmfsd(m,c,s,d)]};

12  Equation E_qtm
13  # global demand for margin m #
14  (all,m,MARG)
15     qtm(m) = sum{c,COMM, sum{s,REG, sum{d,REG,VTMUSESHR(m,c,s,d)
16             * qtmfsd(m,c,s,d)}}};

18  Equation E_qst
19  # generate demand for regional supply of global transportation service #
20  (all,m,MARG)(all,r,REG)
21     qst(m,r) = qtm(m) - ESUBS(m) * [pds(m,r) - pt(m)];

23  Equation E_pt
24  # generate price index for composite transportation services #
25  (all,m,MARG)
26     pt(m) = sum{r,REG, VTSUPPSHR(m,r) * pds(m,r)};
```

2.6.3 Trade prices

Each bilateral trade flow is associated with four prices (recall Price Linkages in Figure 2.2). Domestic supplies are made available at the price $PDS_{c,r}$ which is the same across all regions of destination, i.e., from the supplier's perspective, there is no differentiation across destination markets (including the domestic market).[24] The uniform supply price is subject to a (potentially) bilateral export tax/subsidy between the supplier and the border. The border price, known as the *free on board price* or the FOB export price, is described in levels in equation (55). The variable $TXS_{c,s,d}$ represents the power of export tax/subsidy.[25] The first regional index is the region imposing the tax and the second regional index refers to the destination region of the exports.

Between the port of origin and the port of destination, an additional wedge is added to the price of exports that represents the international trade and transport margins, described above. This converts the FOB price to the import border price, also known as the *cost, insurance and freight*, or CIF, price. This is represented in levels in equation (56), where the parameter ζ represents an index of efficiency of margin services used per unit of export and $PTRANS_{c,s,d}$ is the average price of the margin services. The final transformation of the export price is the adjustment for bilateral import tariffs. This generates the price $PMDS_{c,s,d}$ in levels in equation (57) where $TMS_{c,s,d}$ represents the power of the tariff. It should be noted that, while the export and import taxes are multiplicative wedges, the transport margin is an additive wedge. The latter implies that if the FOB export price doubles, the CIF import price will not double unless the price of the margin services also doubles.

$$PFOB_{c,s,d} = PDS_{c,s} TXS_{c,s,d} \tag{55}$$

$$PCIF_{c,s,d} = PFOB_{c,s,d} + \zeta_{c,s,d} PTRANS_{c,s,d} \tag{56}$$

$$PMDS_{c,s,d} = PCIF_{c,s,d} TMS_{c,s,d} \tag{57}$$

This unit determines the variables $pfob_{c,s,d}$, $pcif_{c,s,d}$ and $pmds_{c,s,d}$ using equations `E_pfob`, `E_pcif` and `E_pmds`.

Listing 2.12: **GEMPACK equations for trade prices**

```
1   Equation E_pfob
2   # links basic and FOB exports prices #
3   (all,c,COMM)(all,s,REG)(all,d,REG)
4       pfob(c,s,d) = pds(c,s) + tx(c,s) + txs(c,s,d);

6   Equation E_pcif
7   # links FOB and CIF prices for good c shipped from region s to d #
8   (all,c,COMM)(all,s,REG)(all,d,REG)
9       pcif(c,s,d)
10          = FOBSHR(c,s,d) * pfob(c,s,d)
11          + TRNSHR(c,s,d) * ptrans(c,s,d);

13  Equation E_pmds
14  # links basic domestic import prices and CIF import prices #
15  (all,c,COMM)(all,s,REG)(all,d,REG)
16      pmds(c,s,d) = pcif(c,s,d) + tm(c,d) + tms(c,s,d);
```

---

[24]Some multi-regional trade models introduce imperfect transformation across destination markets—using for example a nested CET structure (see for example van der Mensbrugghe (2013)). This results in differentiated supply prices across destination markets.

[25]The FOB export price equation includes an additional, region-generic, tax/subsidy shifter that is only source specific and is a handy way to increase the tax wedge uniformly across all destination regions.

2.6.4 Goods market equilibrium

There is a single equilibrium condition for the goods market that determines the domestic market price $pds_{c,r}$. Domestic supply of good c must equal demand for good c, and demand is the sum of domestic demand plus the sum of exports to all export destinations. Equation (58) represents the sum of domestic demands for domestic goods across domestic agents—firms, private households, government and investment—excluding margin services exporters.[26] The equilibrium condition is represented by equation (59), which adds in merchandise exports as well as sales to the global trade and transport sector and this determines the market price of commodity c in region r, where $QC_{c,r}$ represents commodity supply.

$$QDS_{c,r} = \sum_a QFD_{c,a,r} + QPD_{c,r} + QGD_{c,r} + QID_{c,r} \tag{58}$$

$$QC_{c,r} = QDS_{c,r} + QST_{c,r} + \sum_d QXS_{c,r,d} \tag{59}$$

This unit determines $qds_{c,r}$ and $pds_{c,r}$ using equations `E_qds` and `E_pds`.

Listing 2.13: **GEMPACK equations for goods market equilibrium**

```
1  Equation E_qds
2  # assures market clearing for domestic sales #
3  (all,c,COMM)(all,r,REG)
4      qds(c,r) = sum{a,ACTS, FDCSHR(c,a,r) * qfd(c,a,r)}
5                  + PDCSHR(c,r) * qpd(c,r) + GDCSHR(c,r) * qgd(c,r)
6                  + IDCSHR(c,r) * qid(c,r);
7
8  Equation E_pds
9  # assures market clearing for commodities #
10 (all,c,COMM)(all,r,REG)
11     qc(c,r) = DSSHR(c,r) * qds(c,r) + sum(d,REG, XSSHR(c,r,d) * qxs(c,r,d))
12                 + IF[c in MARG, STSHR(c,r) * qst(c,r)]
13                 + tradslack(c,r);
```

2.6.5 Agents' prices for goods

Each domestic agent faces two market prices—$PDS_{c,r}$ and $PMS_{c,r}$—which are uniform across all agents. $PDS_{c,r}$ represents the price of domestic goods sold to the domestic market and $PMS_{c,r}$ is the CES aggregate price of imports, the latter based on the price $PDS_{c,r}$ of these goods in the source regions, but is also inclusive of the aforementioned bilateral trade wedges. The price actually paid depends on agent-specific sales taxes, which are also differentiated between domestic and import goods. The following set of equations determines the agents' price of demand for respectively domestic goods and the import bundle.

$$PFD_{c,a,r} = PDS_{c,r}\,TFD_{c,a,r} \tag{60}$$

$$PFM_{c,a,r} = PMS_{c,r}\,TFM_{c,a,r} \tag{61}$$

$$PPD_{c,r} = PDS_{c,r}\,TPD_{c,r} \tag{62}$$

---

[26]It is the domestic goods counterpart to equation (47).

$$PPM_{c,r} = PMS_{c,r}\, TPM_{c,r} \tag{63}$$

$$PGD_{c,r} = PDS_{c,r}\, TGD_{c,r} \tag{64}$$

$$PGM_{c,r} = PMS_{c,r}\, TGM_{c,r} \tag{65}$$

$$PID_{c,r} = PDS_{c,r}\, TID_{c,r} \tag{66}$$

$$PIM_{c,r} = PMS_{c,r}\, TIM_{c,r} \tag{67}$$

This unit determines $pfd_{c,a,r}$, $pfm_{c,a,r}$, $ppd_{c,r}$, $ppm_{c,r}$, $pgd_{c,r}$, $pgm_{c,r}$r, $pid_{c,r}$ and $pim_{c,r}$ using equations E_pfd, E_pfm, E_ppd, E_ppm, E_pgd, E_pgm, E_pid and E_pim.

Listing 2.14: **GEMPACK equations for agent's prices of goods**

```
1  Equation E_pfd
2  # links domestic basic and firm prices #
3  (all,c,COMM)(all,a,ACTS)(all,r,REG)
4     pfd(c,a,r) = pds(c,r) + tfd(c,a,r);
5
6  Equation E_pfm
7  # links domestic basic and firm prices #
8  (all,c,COMM)(all,a,ACTS)(all,r,REG)
9     pfm(c,a,r) = pms(c,r) + tfm(c,a,r);
10
11 Equation E_ppd
12 # links basic and private consumption prices for domestic c #
13 (all,c,COMM)(all,r,REG)
14    ppd(c,r) = pds(c,r) + tpd(c,r);
15
16 Equation E_ppm
17 # links domestic basic and private consumption prices #
18 (all,c,COMM)(all,r,REG)
19    ppm(c,r) = pms(c,r) + tpm(c,r);
20
21 Equation E_pgd
22 # links domestic basic and government consumption prices #
23 (all,c,COMM)(all,r,REG)
24    pgd(c,r) = pds(c,r) + tgd(c,r);
25
26 Equation E_pgm
27 # links imported basic and government consumption prices #
28 (all,c,COMM)(all,r,REG)
29    pgm(c,r) = pms(c,r) + tgm(c,r);
30
31 Equation E_pid
32 # links basic and investment prices for domestic commodity c #
33 (all,c,COMM)(all,r,REG)
34    pid(c,r) = pds(c,r) + tid(c,r);
35
36 Equation E_pim
37 # links basic and investment prices for imported commodity c #
38 (all,c,COMM)(all,r,REG)
39    pim(c,r) = pms(c,r) + tim(c,r);
```

2.7 Factor market equilibrium

Factors of production, or endowments, are of three types: perfectly mobile (e.g., labor and capital), partially mobile or sluggish (e.g., land) and sector-specific factors (natural resources). It should be noted that the model user has full flexibility in designating the degree of mobility for all factors—this is determined in the GTAP aggregation facilities.[27] [NEW] The model user could also has flexibility to implement upward sloping supply curves for the non-capital endowments.

2.7.1 Mobile endowments

Perfect mobility implies that prices should be equated across all uses. Therefore, the percent change in endowment returns is uniform across activities and market equilibrium is determined by setting aggregate demand equal to (exogenous) supply. Equation (68) represents the equilibrium condition for mobile endowments where $QE_{e,r}$ represents the (fixed) aggregate endowment and $QFE_{e,a,r}$ is demand for endowment e by activity a.[28] Equation (68) thus determines $PE_{e,r}$, which is the economy-wide return to perfectly mobile factors. To simplify specification of activities' unit cost definitions and accounting identities, the model includes equation (69) that produces the activity-specific factor cost, which is uniform across activities.

$$QE_{e,r} = \sum_e QFE_{e,a,r} \qquad \text{for } e \in \{ENDWM\} \tag{68}$$

$$pes_{e,a,r} = pe_{e,r} \qquad \text{for } e \in \{ENDWM\} \tag{69}$$

2.7.2 Sluggish endowments

For each sluggish endowment, there is an aggregate quantity, typically in fixed supply, for example total agricultural land. The supply of the aggregate factor to individual activities is less than perfectly elastic, as there is a transformation frontier that moderates the movement of the factor across activities. A CET specification is used as the transformation frontier. Equation (70) determines the supply of the sluggish factor for use in activity a, $qes_{e,a,r}$, where the key transformation elasticity is $ETRAE_{e,r}$ and $pes_{e,a,r}$ represents the (after tax) activity-specific return to the sluggish factor.[29] Equation (71) defines the aggregate return to the sluggish factor, $pe_{e,r}$.[30] Equation (72) represents the equilibrium condition that determines the after tax-market equilibrium price, $pes_{e,a,r}$, for the use of the sluggish factor in activity a.

$$qes_{e,a,r} = qe_{e,r} - ETRAE_{e,r} \left(pes_{e,a,r} - pe_{e,r} \right) \qquad \text{for } e \in \{ENDWS\} \tag{70}$$

$$PE_{e,r} QE_{e,r} = \sum_a PES_{e,a,r} QES_{e,a,r} \qquad \text{for } e \in \{ENDWS\} \tag{71}$$

$$QES_{e,a,r} = QFE_{e,a,r} \qquad \text{for } e \in \{ENDWS\} \tag{72}$$

---

[27]i.e., GTAPAgg2 or FlexAgg

[28]The endowment subset $ENDWM$ covers mobile endowments.

[29]The endowment subset $ENDWS$ covers sluggish endowments.

[30]There is a subtle difference in the interpretation of $PE_{e,r}$ between mobile and sluggish factors. In the case of mobile factors, $PE_{e,r}$, is the economy-wide pre-(income) tax return to the endowment. In the case of sluggish factors, $PE_{e,r}$ represents the CET aggregate price of the post-(income) tax returns to the endowment.

2.7.3 Sector-specific endowments

The final endowment category is the set of sector-specific factors, typically these are natural resources—such as mineral deposits, fossil fuel reserves, forestry stocks, etc. The sector-specific supply is exogenous, i.e., $QES_{e,a,r}$ is fixed. Only one equation is thus needed, which is the equilibrium condition[31], identical to that for sluggish endowments, thus equation (73) replicates equation (72).[32]

$$QES_{e,a,r} = QFE_{e,a,r} \qquad \text{for } e \in \{ENDWF\} \tag{73}$$

Equation (74) links the equilibrium market price of endowments to the producer price that includes an endowment and activity-specific tax, where the power of the tax is identified with $TFE_{e,a,r}$. Equation (75) links the pre- and post- (income) tax price of endowments (at the activity level—though in most cases the income tax is independent of which activity the endowment is used for). In summary, each factor is associated with three prices and two wedges. $PFE_{e,a,r}$ represents the activities' price of the endowment use, inclusive of the tax $TFE_{e,a,r}$ that is imposed on the price $PEB_{e,a,r}$, which represents the equilibrium price-derived—through economy-wide arbitration, or activity-specific arbitration. $PES_{e,a,r}$ represents the take-home, or after tax remuneration, where the $TINC_{e,a,r}$ represents the (power of the) tax on income derived from endowment use.[33]

$$PFE_{e,a,r} = PEB_{e,a,r} \, TFE_{e,a,r} \tag{74}$$

$$PEB_{e,a,r} = PES_{e,a,r} \, TINC_{e,a,r} \tag{75}$$

In summary, the supply of the aggregate endowment, $QE_{e,r}$, is determined by an iso-elastic supply curve for mobile and sluggish endowments (see below)[34], whereas the sector-specific endowment, $QES_{e,a,r}$, is associated with a sector-specific supply curve.[35] Activity-level equilibrium conditions determine activity-specific returns for sluggish and fixed endowments. For accounting purposes and model simplification, $PEB_{e,a,r}$ for mobile endowments is included and is equal to the economy-wide return, $PE_{e,r}$, determined by market equilibrium.

This unit determines $PE_{e,r}$ for mobile and sluggish endowments, $PEB_{e,a,r}$ and $PES_{e,a,r}$ for all endowments, and $QES_{e,a,r}$ for sluggish endowments. Equation `E_pe2` implements equation (68), `E_qes1` implements equation (69), `E_qes2` implements equation (70), `E_pe1` implements equation (71), `E_peb` implements equations (72) and (73) which also holds for mobile endowments and simply sets supply equal to demand. Equation `E_pfe` implements equation (74) and equation `E_pes` implements equation (75).

Listing 2.15: **GEMPACK equations for equilibrium conditions for factor markets**

```
1  Equation E_pe1
2  # mkt clearing for perfectly mobile endowments in each r #
3  (all,e,ENDWM)(all,r,REG)
4     qe(e,r) = sum{a,ACTS, ENDWMSHR(e,a,r) * qfe(e,a,r)} + endwslack(e,r);
5
6  Equation E_qes1
```

---

[31]The endowment subset $ENDWF$ covers sector-specific endowments.

[32]In the $TABLO$ code, equations (72) and (73) are merged in `E_peb` and the equilibrium equation is conditioned on the set $ENDW$. For mobile endowments, this trivially sets activity supply to activity demand.

[33]In the $TABLO$ code, the power of income tax is $TINC_{e,a,r} = EVFB_{e,a,r}/EVOS_{e,a,r}$.

[34]The explicit assumption in the standard GTAP model is that the supply curves—either aggregate or activity-specific are vertical

[35]The model does not include a definition for $QE_{e,r}$ for the sector-specific endowments (nor for the variable $PE_{e,r}$) as the interpretation of the aggregate is not very intuitive. This requires changes to the income identities—factor income (FY_r) and revenues generated by taxes on factor use (see below).

```
7   # basic price of mobile endowments in a in r #
8   (all,e,ENDWM)(all,a,ACTS)(all,r,REG)
9      pes(e,a,r) = pe(e,r);

11  Equation E_qes2
12  # allocation of sluggish endowments across sectors #
13  (all,e,ENDWS)(all,a,ACTS)(all,r,REG)
14     qes(e,a,r) = qe(e,r) − ETRAE(e,r) * [pes(e,a,r) − pe(e,r)]
15              − endwslack(e,r);

17  Equation E_pe2
18  # composite price for sluggish endowments #
19  (all,e,ENDWS)(all,r,REG)
20     pe(e,r) = sum{a,ACTS, REVSHR(e,a,r) * pes(e,a,r)};

22  Equation E_peb
23  # mkt clearing for endowments in each r #
24  (all,e,ENDW)(all,a,ACTS)(all,r,REG)
25     qfe(e,a,r) = qes(e,a,r);

27  Equation E_pfe
28  # links basic and firm demand prices for mobile endowments #
29  (all,e,ENDW)(all,a,ACTS)(all,r,REG)
30     pfe(e,a,r) = peb(e,a,r) + tfe(e,a,r);

32  Equation E_pes
33  # links supply (pre−) and basic (post−tax) endowment prices #
34  (all,e,ENDW)(all,a,ACTS)(all,r,REG)
35     peb(e,a,r) = pes(e,a,r) + tinc(e,a,r);
```

2.7.4 Upward sloping factor supply curves

[NEW] The model allows for upward sloping supply specification for the non-capital endowments—i.e., land, labor and natural resources—albeit default supply elasticities are zero. Equation (76), which is conditioned on the set $ENDWMSXC$[36], specifies an upward sloping supply curve for aggregate land and labor endowments as a function of the endowment- and region-specific supply elasticity, $ESUPMS_{e,r}$, and the endowment's real return. The latter is derived as a ratio of an endowment's aggregate return, $pe_{e,r}$, to the private consumption price index, $ppriv_r$.

Similarly, Equation (77) implements upward sloping supply for the sector-specific endowment as a function of its supply elasticity $ESUPF_{e,a,r}$ and real return (i.e., sector-specific return, $pes_{e,a,r}$, relative to the private consumption price index, $ppriv_r$.

$$qe_{e,r} = ESUPMS_{e,r}[pe_{e,r} - ppriv_r] \qquad \text{for } e \in \{ENDWMSXC\} \tag{76}$$

$$qes_{e,a,r} = ESUPF_{e,a,r}[pes_{e,a,r} - ppriv_r] \qquad \text{for } e \in \{ENDWF\} \tag{77}$$

This unit determines $QE_{e,r}$ for land and labor endowments, and $QES_{e,a,r}$ for the sector-specific endowment when the upward sloping supply specification for these factors are active. In the model code, equation E_qelsupply implements equation (76), while E_qesfsupply implements equation (77).[37]

The TABLO codes in Listing 2.16 include two shift variables that facilitate upward sloping supply specification. To specify upward sloping supply for land and/or labor, the shift variable

---

[36]MSXC is a shorthand for Mobile and Sluggish endowments eXcept Capital. In the model code, $ENDWMSXC$ is derived as the complement of mobile/sluggish ($ENDWMS$) and capital ($ENDWC$) sets.

[37]Equations E_qelsupply and E_qesfsupply simplify to $qe_{e,r} = qelsupply_{e,r}$ and $qes_{e,a,r} = qesfsupply_{e,a,r}$ when their associated supply elasticities are zero.

qelsupply$_{e,r}$ must be swapped with *qe*$_{e,r}$ in the closure file.[38] Similarly, swapping the shift variable *qesfsupply*$_{e,a,r}$ with *qesf*$_{e,a,r}$ in the closure file activates the upward sloping supply specification for the sector-specific factor.

Listing 2.16: **GEMPACK equations for upward sloping supply**

```
1  Equation E_qelsupply
2  # upward sloping aggregate supply specification for land (sluggish) and labor #
3  (all,e,ENDWMSXC)(all,r,REG)
4  qe(e,r) = ESUPMS(e,r) * [pe(e,r) − ppriv(r)] + qelsupply(e,r);
5
6  Equation E_qesfsupply
7  # upward sloping supply specification for sector−specific (natres) factor #
8  (all,e,ENDWF)(all,a,ACTS)(all,r,REG)
9  qes(e,a,r) = ESUPF(e,a,r) * [pes(e,a,r) − ppriv(r)] + qesfsupply(e,a,r);
```

2.8 Allocation of global savings

Savings of each regional household are aggregated across all regions and given to a global investor, also known as the 'global bank'. The disbursement of global saving across regions to finance investment is specified using one of four mechanisms.[39] Under one mechanism the 'global bank' maximizes its returns by an investment allocation which achieves *equiproportionate* changes in the expected regional rates of return to investment.[40] The second mechanism ignores any expectations about region specific capital markets and allocates investment across regions according to the initial regional investment shares. [NEW] A third mechanism fixes net foreign capital flows in real terms. Given the simplified version of the balance of payments, this in essence fixes the net trade balance. An *ex ante* change to the trade balance, for example a tariff reduction, typically requires a change to the real exchange rate. In this case the global bank ignores changes in the relative rates of return, as in the case of a fixed investment allocation, and it is the net saving flow that is fixed, not the regional investment shares. [NEW] A fourth mechanism fixes net capital flows relative to regional income. Both the third and fourth mechanisms can only be implemented for $n-1$ regions and thus there is a residual region that becomes the lender/borrower of last resort.

2.8.1 Investment preliminaries

Given that GTAP is a static model, the beginning of period capital stock, KB_r, is fixed and not normally impacted by a simulation shock. Changes to the investment level only affect the composition of demand, but have no impact on the beginning of period capital stock. The end of period capital stock, KE_r, is equal to the initial capital stock, less depreciation, plus new investment. Equation (78) determines the end of period capital stock, where $QINV_r$, is the volume of investment.

$$KE_r = (1 - \delta_r) KB_r + QINV_r \tag{78}$$

---

[38]The user could selectively implement this specification by conditioning the swap statements with the relevant factor sets—e.g., *ENDWS* if limited to the sluggish land factor, *ENDWL* if limited to labor, or *ENDWMXSC* for both land and labor.

[39][NEW] This version of the model includes two additional closures for the capital account.

[40]This approach in theory is a simple application of the theory of investment allocation across industries in the single-country ORANI model of the Australian economy (Dixon et al., 1982).

The rate of return on capital, $RENTAL$, is defined as the after-tax return to capital.[41] In levels, it is given by equation (79). $PES_{endwc,a,r}$ is the after tax return to capital in sector a.

$$RENTAL_r = \sum_a PES_{endwc,a,r} \, QES_{endwc,a,r} \Big/ KB_r \qquad (79)$$

Equation (80) defines the net current rate of return, the rental rate adjusted for the price of replacing capital, less the depreciation rate. Therefore, a tariff reduction, that would lower the price of investment goods, would reduce the replacement price of capital and increase the rate of return (all else equal).[42]

$$RORC_r = \frac{RENTAL_r}{PINV_r} - \delta_r \qquad (80)$$

2.8.2 Rate-of-return sensitive investment allocation

The first specification for allocating investment is that the global investor is sensitive to changes in relative rates of return. Equation (81) defines the expected rate of return—it represents an adjustment in expectations relative to the net rate of return, where the adjustment depends on the rate of growth of the capital stock. A large increase in the capital stock, all else equal, will tend to dampen the expected rate of return relative to the current net rate of return, where the elasticity $RORFLEX_r$ determines the level of the dampening. Obviously, at one extreme, a value of 0 means that there is no change in expectations. High values of the elasticity reflect high changes in the expected rate of return.

$$RORE_r = RORC_r \left(\frac{KE_r}{KB_r} \right)^{-RORFLEX_r} \qquad (81)$$

The equilibrating mechanism is provided in equation (82) that equates the expected rate of return across countries. In effect, equation (82) determines $QINV_r$, i.e., global investment will be allocated across regions so as to insure that the expected rate of return is equalized across regions.

$$RORE_r = RORG \qquad (82)$$

The equation that determines $RORG$ is the global saving equals global investment identity. However, Walras' Law is invoked and that equation is dropped. It is checked after the fact to verify that the model has been properly implemented.

One can also show that the following accounting identity must hold:

$$PINV_r \, QINV_r = SAVE_r + FSAVE_r + \delta_r PINV_r KB_r \qquad (83)$$

where $FSAVE_r$ is the region's net nominal foreign capital flow. In this case, equation (82) could be seen as determining a region's net foreign capital flow and the equation above determines domestic

---

[41]The rental equation in GTAP.TAB is based on the ratio of $VES_{endwc,r}$ to $GROSSCAP_r$ (i.e., gross returns on capital) where $endwc$ is a single element set containing capital endowment. This ratio is defined on an after-income taxes basis.

[42]When log-differentiated, the equation takes the form:

$$\dot{rorc}_r RORC_r = (RENTAL_r/PINV_r) \left(\dot{rental}_r - \dot{pinv}_r \right)$$

Replacing the ratio of $(RENTAL_r/PINV_r)$ with $RORC + \delta_r$, leads to the version in the TABLO code, where $GRNETRATIO$ is equal to $(RORC + \delta_r)/RORC$.

investment. Summing the equation above across all regions, we have gross investment on the left-hand side and saving plus depreciation on the right-hand side. The sum of $FSAVE_r$ across all regions will be zero. An alternative, therefore could be to keep the equation above, but define it only for $(R-1)$ regions.

2.8.3   Investment allocation based on initial capital shares

The second specification for allocating investment assumes that the regional composition of capital stocks is invariant and does not respond to changes in expected relative rates of return. Equation (84) defines global net investment, i.e., the sum across all regions of regional net investment.

$$GLOBALCGDS = \sum_r (QINV_r - \delta_r KB_r) \tag{84}$$

Regional investment is assumed to be a constant share of global investment, where the share is calculated using base investment shares, equation (85).

$$QINV_r - \delta_r KB_r = \chi_r^I GLOBALCGDS \tag{85}$$

Note that this equation only needs to be defined for $(R-1)$ regions as the global saving/investment balance will determine the residual regional investment level. In the case of this 'insensitive' allocation rule, the model still produces the aggregate rate of return variable, $RORG$, which becomes the weighted sum of the regional expected rate of return variable, equation (86). The weights are provided by the regional share of global net investment.

$$RORG = \sum_r \varphi_r RORE_r \qquad \text{where } \varphi_r = \frac{PINV_r (QINV_r - \delta_r KB_r)}{\sum_s PINV_s (QINV_s - \delta_s KB_s)} \tag{86}$$

2.8.4   Fixed real foreign savings

[NEW] The third specification fixes net real foreign savings—typically to base year levels, but not required, i.e. one can shock net foreign savings. Foreign savings are fixed for $n-1$ regions and are given by the exogenous parameter $FSAVEX$, equation (87). These are multiplied by a global price, in this case $GLOBALCGDS$, which is the average price of investment defined in equation (88). Equation (89) is implemented to ensure that global net foreign savings sum to zero, in essence evaluating foreign savings for the residual region.

$$FSAVE_r = PGLOBALCGDS \times FSAVEX_r \tag{87}$$

$$PGLOBALCGDS = \sum_r PINV_r (QINV_r - \delta_r KB_r) \tag{88}$$

$$\sum_r FSAVE_r \equiv 0 \tag{89}$$

2.8.5 Fixed relative foreign savings

[NEW] The fourth specification fixes net foreign savings relative to regional income. Relative foreign savings are fixed for $n-1$ regions and are set equal to the exogenous parameter χ^f, equation (90). The parameter is calibrated to the base year ratio of foreign savings to income, but can be changed in a simulation shock. Equation (89) determines net foreign savings for the residual region, i.e. the borrower/lender of last resort.

$$FSAVE_r = \chi_r^f Y_r \qquad (90)$$

In summary we have four foreign savings closure. Common to all closures are the following equations: equation (78), equation (79), equation (80), equation (81), equation (84), equation (88), (83) and (89). Common to the last three closures, i.e. all except the rate-of-return sensitive closure is also equation (86).

The four cases involve specifying the following to essentially determine *FSAVE*:

- *RoR sensitive* Equation (82)

- *Fixed investment shares* Equation (85)

- *Fixed capital account* Equation (87)

- *Fixed capital account ratio* Equation (90)

This unit determines $GLOBALCGDS$, KE_r, $RENTAL_r$, $RORC_r$, $RORE_r$, $QINV_r$ and $RORG$ using equations E_ke, E_rental, E_rorc, E_rore, E_qinv that implements equations (80) and (85), E_globalcgds that implements equations (84) and (86), E_del_rfsave that implements $FSAVEX$ in equation (87) and E_del_fsavery that implements equation (90).[43]

Listing 2.17: **GEMPACK equations for investment allocation**

```
1   Equation E_ke
2   # ending capital stock equals beginning stock plus net investment.  #
3   (all,r,REG)
4     ke(r) = INVKERATIO(r) * qinv(r) + [1.0 — INVKERATIO(r)] * kb(r);

6   Equation E_rental
7   # defines a variable for capital rental rate #
8   (all,r,REG)
9     rental(r) = sum{e,ENDWC, [VES(e,r) / GROSSCAP(r)] * pe(e,r)};

11  Equation E_rorc
12  # current rate of return on capital in region r #
13  (all,r,REG)
14    rorc(r) = GRNETRATIO(r) * [rental(r) — pinv(r)];

16  Equation E_rore
17  # expected rate of return depends on the current return and investment #
18  (all,r,REG)
19    rore(r) = rorc(r) — RORFLEX(r) * [ke(r) — kb(r)];

21  Equation E_qinv
```

---

[43]The parameter *RORDELTA* determines global investment closure. A value of 1 uses the rate-of-return sensitive closure. A value of 0 uses the fixed investment allocation closure. A value of 1 coupled with the appropriate swap statement facilitate either fixed real or fixed relative foreign savings closure. For fixed real foreign savings closure, the appropriate swap statement is: `Swap cgdslack(REGN-1) = del_rfsave(REGN-1)`, while the appropriate swap statement for fixed relative foreign savings closure is: `Swap cgdslack(REGN-1) = del_fsavery(REGN-1)`. In both cases, `REGN-1` is the set of all regions except a residual region.

```
22   # either gross investment or expected rate of return in region r #
23   (all,r,REG)
24      RORDELTA * rore(r)
25         + [1 — RORDELTA]
26         * [[REGINV(r) / NETINV(r)] * qinv(r) — [VDEP(r) / NETINV(r)] * kb(r)]
27         = RORDELTA * rorg + [1 — RORDELTA] * globalcgds + cgdslack(r);

29   Equation E_globalcgds
30   # either expected global rate of return or global net investment #
31      RORDELTA * globalcgds + [1 — RORDELTA] * rorg
32         = RORDELTA
33         * sum{r,REG,
34            [REGINV(r) / GLOBINV] * qinv(r) — [VDEP(r) / GLOBINV] * kb(r)}
35         + [1 — RORDELTA] * sum{r,REG, [NETINV(r) / GLOBINV] * rore(r)};

37   Equation E_del_rfsave
38   # computes ordinary change in real foreign saving by region, $US million #
39   (all,r,REG)
40   del_rfsave(r) = 0.01 * [NETINV(r) * qnetinv(r) — SAVE(r) * qsave(r)];

42   Equation E_del_fsavery
43   # computes change in foreign saving as a percentage of regional income #
44   (all,r,REG)
45   INCOME(r) * 100 * del_fsavery(r)
46   = NETINV(r) * vnetinv(r) — SAVE(r) * vsave(r) — FSAVE(r) * y(r);
```

2.8.6 Price of saving

This final section of the investment module defines the price of saving, $PSAVE_r$, needed among other things to complete the utility module, i.e., to define the utility of saving. In the absence of cross-border saving flows, the price of saving would be equal to the price of capital goods, i.e., $PINV_r$. In the presence of capital flows, regional investment is assumed to occur at the cost of purchasing capital goods regionally ($PINV_r$), but the counterbalancing saving flow is a mix of both domestic and international saving. We have adopted a somewhat *ad hoc* rule but that has the advantage of holding exactly at the global level, i.e., the global price of saving is equal to the global price of investment. Equation (91) defines the regional price of saving and it is set to $PINV_r$ plus an adjustment factor that is common across all regions. The adjustment factor is a weighted share of the regional investment price indices. The share parameter is equal to the regional share of the difference between net investment and saving in global net investment. Adding these up globally gives us the desired outcome that at the global level the price of saving is equal to the price of investment as the adjustment factor sums to 0 when aggregated across regions.

$$psave_r = pinv_r + \sum_s \varphi_s pinv_s \qquad \text{where } \varphi_s = \frac{NETINV_s - SAVE_s}{GLOBINV} \tag{91}$$

This unit determines $PSAVE_r$ using equation `E_psave`.

Listing 2.18: **GEMPACK equations for price of saving**

```
1   Equation E_psave
2   # price of saving #
3   (all,r,REG)
4      psave(r)
5         = pinv(r)
6         + sum{s,REG, [[NETINV(s) — SAVE(s)] / GLOBINV] * pinv(s)}
7         + psaveslack(r);
```

2.9   Tax revenue streams

The aggregate indirect tax revenue stream, $INDTAX_r$, was referenced above when describing income distribution. We have collected them in this section as now all of the relevant variables have been described that fully define these tax revenue streams.

2.9.1   Tax revenues generated in production

There are three separate taxes paid by firms: tax on output, taxes on factor use and taxes on purchases of domestic and imported goods. Equation (92) defines the tax revenues derived from the production tax, where $TO_{c,a,r} - 1$ represents the *ad valorem* tax rate, $PS_{c,a,r}$ is the pre-tax supply price (equal to unit cost in the case of perfectly competitive firms with constant returns to scale) and $QCA_{c,a,r}$ is output by activity a of commodity c.[44] The revenues from factor use are described in equation (93), where $TFA_{e,a,r} - 1$ represents the relevant tax rate. Equation (94) describes the sales tax revenues from intermediate consumption of goods and services, where $TFD_{c,a,r} - 1$ and $TFM_{c,a,r} - 1$ represent respectively the commodity- and activity-specific tax rates on domestic and imported goods.

$$TAXROUT_r = \sum_c \sum_a \left(TO_{c,a,r} - 1 \right) PS_{c,a,r} QCA_{c,a,r} \tag{92}$$

$$TAXRFU_r = \sum_e \sum_a \left(TFE_{e,a,r} - 1 \right) PEB_{e,a,r} QFE_{e,a,r} \tag{93}$$

$$
\begin{aligned}
TAXRIU_r \; = \; & \sum_c \sum_a \left(TFD_{c,a,r} - 1 \right) PDS_{c,r} QFD_{c,a,r} \\
+ \; & \sum_c \sum_a \left(TFM_{c,a,r} - 1 \right) PMS_{c,r} QFM_{c,a,r}
\end{aligned}
\tag{94}
$$

This unit determines $TAXROUT_r$, $TAXRFU_r$ and $TAXRIU_r$ using GEMPACK equations `E_del_taxrout`, `E_del_taxrfu` and `E_del_taxriu`.

Listing 2.19: **GEMPACK equations for tax revenues from domestic production**

```
1   Equation E_del_taxrout
2   # change in ratio of output tax payments to regional income #
3       100.0 * INCOME(r) * del_taxrout(r) + TAXROUT(r) * y(r)
4               = sum{c,COMM, sum{a,ACTS,
5                   MAKEB(c,a,r) * to(c,a,r) + PTAX(c,a,r) * [ps(c,a,r) + qca(c,a,r)]}};

7   Equation E_del_taxrfu
8   # change in ratio of tax payments on factor usage to regional income #
9   (all,r,REG)
10      100.0 * INCOME(r) * del_taxrfu(r) + TAXRFU(r) * y(r)
11        = sum{e,ENDW, sum{a,ACTS,
12          VFP(e,a,r) * tfe(e,a,r) + ETAX(e,a,r) * [peb(e,a,r) + qfe(e,a,r)]}};

14  Equation E_del_taxriu
15  # change in ratio of tax payments on intermediate goods to regional income #
16  (all,r,REG)
17      100.0 * INCOME(r) * del_taxriu(r) + TAXRIU(r) * y(r)
18        = sum{c,COMM, sum{a,ACTS,
19          VDFP(c,a,r) * tfd(c,a,r) + DFTAX(c,a,r) * [pds(c,r) + qfd(c,a,r)]}}
20      + sum{c,COMM, sum{a,ACTS,
21          VMFP(c,a,r) * tfm(c,a,r) + MFTAX(c,a,r) * [pms(c,r) + qfm(c,a,r)]}};
```

---

[44]Note that the implemented equations are somewhat more complex. First, they are defined as a percent of regional income, Y_r. And second, since 0 is a legitimate value, they are defined as an ordinary change and not a percent change. We are ignoring these complications in the write-up, but Section A.0.2 in the Mathematical Appendix provides additional description of the implementation of these equations.

2.9.2 Tax revenues generated in domestic final demand

Equations (95), (96) and (97) represent respectively the revenues from sales taxes on domestic and imported purchases by the private agent, the government and investment. Note that the pre-tax supply prices are identical across all agents.

$$TAXRPC_r = \sum_c \left[(TPD_{c,r} - 1)\, PDS_{c,r}\, QPD_{c,r} + (TPM_{c,r} - 1)\, PMS_{c,r}\, QPM_{c,r} \right] \qquad (95)$$

$$TAXRGC_r = \sum_c \left[(TGD_{c,r} - 1)\, PDS_{c,r}\, QGD_{c,r} + (TGM_{c,r} - 1)\, PMS_{c,r}\, QGM_{c,r} \right] \qquad (96)$$

$$TAXRIC_r = \sum_c \left[(TID_{c,r} - 1)\, PDS_{c,r}\, QID_{c,r} + (TIM_{c,r} - 1)\, PMS_{c,r}\, QIM_{c,r} \right] \qquad (97)$$

This unit determines $TAXRPC_r$, $TAXRGC_r$ and $TAXRIC_r$ using equations `E_del_taxrpc`, `E_del_taxrgc` and `E_del_taxric`.

Listing 2.20: **GEMPACK equations for tax revenues from domestic final demand**

```
1  Equation E_del_taxrpc
2  # change in ratio of private consumption tax payments to regional income #
3  (all,r,REG)
4     100.0 * INCOME(r) * del_taxrpc(r) + TAXRPC(r) * y(r)
5        = sum{c,COMM,
6           VDPP(c,r) * tpd(c,r) + DPTAX(c,r) * [pds(c,r) + qpd(c,r)]}
7        + sum{c,COMM,
8           VMPP(c,r) * tpm(c,r) + MPTAX(c,r) * [pms(c,r) + qpm(c,r)]};
9
10 Equation E_del_taxrgc
11 # change in ratio of government consumption tax payments to regional income #
12 (all,r,REG)
13    100.0 * INCOME(r) * del_taxrgc(r) + TAXRGC(r) * y(r)
14       = sum{c,COMM,
15          VDGP(c,r) * tgd(c,r) + DGTAX(c,r) * [pds(c,r) + qgd(c,r)]}
16       + sum{c,COMM,
17          VMGP(c,r) * tgm(c,r) + MGTAX(c,r) * [pms(c,r) + qgm(c,r)]};
18
19 Equation E_del_taxric
20 # change in ratio of investment tax payments to regional income #
21 (all,r,REG)
22    100.0 * INCOME(r) * del_taxric(r) + TAXRIC(r) * y(r)
23       = sum{c,COMM,
24          VDIP(c,r) * tid(c,r) + DITAX(c,r) * [pds(c,r) + qid(c,r)]}
25       + sum{c,COMM,
26          VMIP(c,r) * tim(c,r) + MITAX(c,r) * [pms(c,r) + qim(c,r)]};
```

2.9.3 Tax revenues generated in international trade

Equation (98) describes revenues generated by import tariffs where $TMS_{c,s,d} - 1$ represents the *ad valorem* tariff rate imposed in region d for exports sourced in region s for commodity c. The relevant base price is the border price of imports, i.e., $PCIF_{c,s,d}$. Equation (99) represents the revenues generated by export taxes/subsidies, where $TXS_{c,s,d} - 1$ represents the export tax imposed by region s on exports towards region d for commodity c. A careful look at the indices will show that import tariff revenues are collected by the destination country, d, and that export tax revenues are collected by the source country, s. These differences are also reflected in the different indices used for the regional sums.

$$TAXRIMP_d = \sum_s \sum_c (TMS_{c,s,d} - 1) PCIF_{c,s,d} QXS_{c,s,d} \tag{98}$$

$$TAXREXP_s = \sum_d \sum_c (TXS_{c,s,d} - 1) PDS_{c,s} QXS_{c,s,d} \tag{99}$$

This unit determines variables $TAXRIMP_d$ and $TAXREXP_s$ using equations `E_del_taxrimp` and `E_del_taxrexp`.

Listing 2.21: **GEMPACK equations for tax revenues from international trade**

```
1  Equation E_del_taxrimp
2  # change in ratio of import tax payments to regional income #
3  (all,d,REG)
4     100.0 * INCOME(d) * del_taxrimp(d) + TAXRIMP(d) * y(d)
5        = sum{c,COMM, sum{s,REG, VMSB(c,s,d) * [tm(c,d) + tms(c,s,d)]
6        + MTAX(c,s,d) * [pcif(c,s,d) + qxs(c,s,d)]}};

8  Equation E_del_taxrexp
9  # change in ratio of export tax payments to regional income #
10 (all,s,REG)
11    100.0 * INCOME(s) * del_taxrexp(s) + TAXREXP(s) * y(s)
12       = sum{c,COMM, sum{d,REG, VFOB(c,s,d) * [tx(c,s) + txs(c,s,d)]
13       + XTAXD(c,s,d) * [pds(c,s) + qxs(c,s,d)]}};
```

2.9.4 Income taxes and other tax identities

Equation (100) describes revenues generated by income taxes where $TINC_{e,a,r} - 1$ represents the income tax rate on endowment e used in activity a. The base price of the tax is $PES_{e,a,r}$. Thus if the power of tax, $TINC_{e,a,r}$, is equal to 1.2, the tax rate is 20%. Equation (101) describes total revenues from all indirect taxes, $INDTAX_r$. This is part of household income. The other part of household income is total factor remuneration that is evaluated at the equilibrium price for factors and is thus inclusive of income taxes. Equation (102) defines total tax revenues—indirect and direct. This variable is purely definitional and does not appear anywhere else in the model (though presumably could be endogenized to target a total tax to income ratio level).

$$TAXRINC_r = \sum_e \sum_a (TINC_{e,a,r} - 1) PES_{e,a,r} QFE_{e,a,r} \tag{100}$$

$$
\begin{aligned}
INDTAX_r &= TAXROUT_r + TAXRFU_r \\
&+ TAXRIU_r + TAXRPC_r + TAXRGC_r + TAXRIC_r \\
&+ TAXRIMP_r + TAXREXP_r
\end{aligned} \tag{101}
$$

$$TTAXR_r = INDTAX_r + TAXRINC_r \tag{102}$$

This unit determines variables $TAXRINC_r$, $TAXRINC_r$ and $TTAXR_r$ using GEMPACK equations `E_del_taxrinc`, `E_del_indtaxr` and `E_del_ttaxr`.

Listing 2.22: **GEMPACK equations for income tax revenues and tax identities**

```
1  Equation E_del_taxrinc
2  # change in ratio of income tax payments to regional income #
3  (all,r,REG)
4     100.0 * INCOME(r) * del_taxrinc(r) + TAXRINC(r) * y(r)
5        = sum{e,ENDW, sum{a,ACTS, EVFB(e,a,r) * [tinc(e,a,r)]
6        + INCTAX(e,a,r) * [pes(e,a,r) + qfe(e,a,r)]}};
```

```
 8   Equation E_del_indtaxr
 9   # change in ratio of indirect taxes to INCOME in r #
10   (all,r,REG)
11      del_indtaxr(r)
12         = del_taxrout(r) + del_taxrfu(r) + del_taxriu(r)
13         + del_taxrpc(r) + del_taxrgc(r) + del_taxric(r)
14         + del_taxrimp(r)+ del_taxrexp(r);

16   Equation E_del_ttaxr
17   # change in ratio of taxes to INCOME in r #
18   (all,r,REG)
19      del_ttaxr(r) = del_indtaxr(r) + del_taxrinc(r);
```

2.10 Numéraire and closure

2.10.1 A note on price indices

The latest version of the model has moved to Fisher price indices for both the factor price and GDP indices. The Fisher price index, sometimes also referred to as the "ideal" price index (see for example Eurostat (2008)), is the geometric mean of the Laspeyres and Paasches indices.

We outline the specification for a generic formulation before depicting the specific formulations used in the GTAP model. For a price index over set i, the Fisher *price* index is equal to the following:[45]

$$F^p = \sqrt{\frac{\sum_i P_{i,t}Q_{i,t-1}}{\sum_i P_{i,t-1}Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t}}} = \sqrt{L^p \cdot P^p}$$

The Fisher price index represents the geometric mean of the Laspeyres (L^p) and Paasche price indices (P^p), with the former using lagged volume weights and the latter using current volume weights.

$$L^p = \frac{\sum_i P_{i,t}Q_{i,t-1}}{\sum_i P_{i,t-1}Q_{i,t-1}} \qquad P^p = \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t}}$$

These indices represent percent change relative to the previous period and thus the price indices are chain weighted. The price index itself is thus defined as:

$$PI_t = PI_{t-1}F_t^p$$

where typically the price index will be set to 1 or 100 in some base period.

The Fisher *volume* index has a similar definition using different price weights:

$$F^q = \sqrt{\frac{\sum_i P_{i,t-1}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t-1}} \cdot \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t}Q_{i,t-1}}} = \sqrt{L^q \cdot P^q}$$

The Fisher volume index represents the geometric mean of the Laspeyres (L^q) and Paasche volume indices (P^q), with the former using lagged price weights and the latter using current price weights.

$$L^q = \frac{\sum_i P_{i,t-1}Q_{i,t}}{\sum_i P_{i,t-1}Q_{i,t-1}} \qquad P^q = \frac{\sum_i P_{i,t}Q_{i,t}}{\sum_i P_{i,t}Q_{i,t-1}}$$

---

[45]It is natural to think of the index 't' as time. However, in a comparative static framework it represents the pre- and post-shock values.

It is easy to show that the aggregate nominal value is equal to the product of the price and volume indices:

$$P_t \cdot Q_t = \sum_i P_{i,t} \cdot Q_{i,t} = F_t^p \cdot F_t^q \cdot P_{t-1} \cdot Q_{t-1}$$

These formulas can be log-differentiated for use in GEMPACK. We start with the following expressions:

$$L_t^p = \frac{\sum_i Q_{i,t-1} P_{i,t}}{\sum_i Q_{i,t-1} P_{i,t-1}} = \frac{LQ1P_t}{LQ1P1_t} \qquad P_t^p = \frac{\sum_i Q_{i,t} P_{i,t}}{\sum_i Q_{i,t} P_{i,t-1}} = \frac{LQP_t}{LQP1_t}$$

Since $LQ1P1_t$ only has lags, it is a constant and does not require differentiation. The other three formulas in log-differentiated form are:

$$lq1p_t = \sum_i \varphi_{i,t}^p p_{i,t} \qquad \text{where } \varphi_{i,t}^p = \frac{Q_{i,t-1} P_{i,t}}{\sum_i Q_{i,t-1} P_{i,t}}$$

$$lqp_t = \sum_i \varphi_{i,t}^v (q_{i,t} + p_{i,t}) \qquad \text{where } \varphi_{i,t}^v = \frac{Q_{i,t} P_{i,t}}{\sum_i Q_{i,t} P_{i,t}}$$

$$lqp1_t = \sum_i \varphi_{i,t}^q q_{i,t} \qquad \text{where } \varphi_{i,t}^q = \frac{Q_{i,t} P_{i,t-1}}{\sum_i Q_{i,t} P_{i,t-1}}$$

We can insert these in the expression for L^p and P^p:

$$l_t^p = lq1p_t \qquad p_t^p = lqp_t - lqp1_t$$

The final step is to insert these expressions into the price index:

$$p_t = 0.5[l_t^p + p_t^p]$$

Thus we can proceed in four steps:

1. Calculate the share vectors, of which there are three: φ^p, φ^v and φ^q

2. Calculate the percentage change in the numerators and denominators of the Laspeyres and Paasche indices—using the weights

3. Calculate the percentage change in the Laspeyres and Paasche indices

4. Calculate the percentage change in the Fisher price index

2.10.2 Factor price index

We defined two indices for factor prices—a regional and global factor price index. Equation (103) defines the regional factor price index, $PFACTOR_r$, using the Fisher price index expression. The relevant factor prices are represented by the variable PEB, i.e. the market price, or basic price, of factors.

Any single price, or price index, could be chosen as the model numéraire, or price anchor. The default numéraire is the global price index of factor remuneration using the Fisher price index expression, $PFACTWLD$, which is aggregated over all endowments, activities and regions, i.e., it represents the average global return to endowments, equation (104). The left-out equation,

or Walras' Law was described in the investment section and represents the global saving=global investment identity.[46]

$$
\begin{aligned}
PFACTOR_r &= PFACTOR_{r,-1} \\
&\times \sqrt{\frac{\sum_e \sum_a PEB_{e,a,r} QFE_{e,a,r,-1}}{\sum_e \sum_a PEB_{e,a,r,-1} QFE_{e,a,r,-1}} \cdot \frac{\sum_e \sum_a PEB_{e,a,r} QFE_{e,a,r}}{\sum_e \sum_a PEB_{e,a,r,-1} QFE_{e,a,r}}}
\end{aligned}
\tag{103}
$$

$$
\begin{aligned}
PFACTWLD &= PFACTWLD_{-1} \\
&\times \sqrt{\frac{\sum_r \sum_e \sum_a PEB_{e,a,r} QFE_{e,a,r,-1}}{\sum_r \sum_e \sum_a PEB_{e,a,r,-1} QFE_{e,a,r,-1}} \cdot \frac{\sum_r \sum_e \sum_a PEB_{e,a,r} QFE_{e,a,r}}{\sum_r \sum_e \sum_a PEB_{e,a,r,-1} QFE_{e,a,r}}}
\end{aligned}
\tag{104}
$$

This unit determines the regional and global factor price indices in the model. Equation `E_pfactor` implements equation (103) while `E_pfactwld` implements equation (104).

Listing 2.23: **GEMPACK equations for numéraire definition and Walras' Law**

```
1  Equation E_pfactor
2  # computes % change in factor (Fisher) price index, by region #
3  (all,r,REG)
4  pfactor(r) = 0.5 * [pfacLaspeyres(r) + pfacPaasche(r)];

6  Equation E_rorg
7  # computes % change in world factor (Fisher) price index #
8  pfactwld = 0.5 * [pfacwLaspeyres + pfacwPaasche];

10  Equation E_walras_sup
11  # extra equation: computes change in supply in the omitted market #
12     walras_sup = pcgdswld + globalcgds;

14  Equation E_walras_dem
15  # extra equation: computes change in demand in the omitted market #
16     GLOBINV * walras_dem = sum{r,REG, SAVE(r) * [psave(r) + qsave(r)]};

18  Equation E_walraslack
19  # Check Walras' Law.  Value of "walraslack" should be zero #
20     walras_sup = walras_dem + walraslack;
```

2.10.3 Price of regional Absorption and MUV price index

[NEW] The model also includes two additional price indices—a regional absorption price index and a global index of so-called manufactured exports from high-income countries[47]

Equation (105) defines the absorption price index for each region as a weighted average price index of the final demand subset $DOMABS$ which includes private, public and investment demand. Equation (106) then defines the regional absorption price using the Fisher index.

Equation (107), defines a weighted average of FOB export prices for a subset of commodities and regions. The commodity index covers manufactured commodities defined in the subset $CMUV$. The first regional sum covers all regions defined in the subset $RMUV$, which consists of high-income countries. The second regional sum is over all regions. Equation (108) defines the associated MUV Fisher price index.

This unit describes the variables $PABS_r$, $PABSFISHER_r$, $PMUV$ and $PMUVFISHER$ using equations `E_pabs`, `E_pabsFisher`, `E_pmuv` and `E_pmuvFisher`.

---

[46]The equation in the TABLO code is labeled `E_rorg`. This is because $PFACTWLD$ is exogenous as the model's price anchor and thus this equation 'explains' $RORG$ which has no separate equation.

[47]Intended to be closely related to the World Bank's Manufactured Unit Value (MUV) index.

$$PABS_r = \sum_{d \in DOMABS} \varphi_{r,d} PGDPEXP_{r,d} \tag{105}$$

$$
\begin{aligned}
PABSFISHER_r &= PABSFISHER_{r,-1} \\
&\times \sqrt{\frac{PABS_r QABS_{r,-1}}{PABS_{r,-1} QABS_{r,-1}} \cdot \frac{PABS_r QABS_r}{PABS_{r,-1} QABS_r}}
\end{aligned}
\tag{106}
$$

$$PMUV = \sum_{c \in CMUV} \sum_{s \in RMUV} \sum_d \alpha_{c,s,d} PFOB_{c,s,d} \tag{107}$$

$$
\begin{aligned}
PMUVFISHER &= PMUVFISHER_{-1} \\
&\times \sqrt{\frac{\sum_c \sum_s \sum_d PFOB_{c,s,d} QXS_{c,s,d,-1}}{\sum_c \sum_s \sum_d PFOB_{c,s,d,-1} QXS_{c,s,d,-1}} \cdot \frac{\sum_c \sum_s \sum_d PFOB_{c,s,d} QXS_{c,s,d}}{\sum_c \sum_s \sum_d PFOB_{c,s,d,-1} QXS_{c,s,d}}}
\end{aligned}
\tag{108}
$$

Listing 2.24: **GEMPACK equations for regional absorption and MUV price indices**

```
1  Equation E_pabs
2  # computes % change in the price of aggregate domestic absorption in region r #
3  (all,r,REG)
4  pabs(r) = sum{d,DOMABS, ABSSHR(d,r) * pgdpexp(r,d)};
5
6  Equation E_pabsFisher
7  # computes % change in in aggreg. absorption: Fisher price index in r #
8  (all,r,REG)
9  pabsFisher(r) = 0.5 * [pabsLaspeyres(r) + pabsPaasche(r)];
10
11 Equation E_pmuv
12 # computes % change in manufactures unit value (MUV) price index #
13 pmuv = sum{c,CMUV, sum{s,RMUV, sum{d,REG, MUVSHR(c,s,d) * pfob(c,s,d)}}};
14
15 Equation E_pmuvFisher
16 # computes % change in MUV Fisher price index #
17 pmuvFisher = 0.5 * [pmuvLaspeyres + pmuvPaasche];
```

2.10.4 Definition of GDP

Equation (109) defines nominal GDP at market prices (in levels). It is the sum of the value of private, public and investment expenditures, exports of international trade and transport services, and net trade at world prices. Equation (110) defines the Fisher *volume* index for real GDP, where the formula for *VGDP* follows. The expression for *VGDP* evaluates the generic GDP expression at prices *tp* and volumes *tq*.[48] Equation (111) rather trivially determines the GDP at market price deflator.[49]

$$
\begin{aligned}
GDP_r &= \sum_c \left[PPA_{c,r} QPA_{c,r} + PGA_{c,r} QGA_{c,r} + PIA_{c,r} QIA_{c,r} \right] \\
&+ \sum_m PDS_{m,r} QST_{m,r} \\
&+ \sum_c \left[\sum_d PFOB_{c,r,d} QXS_{c,r,d} - \sum_s PCIF_{c,s,r} QXS_{c,s,r} \right]
\end{aligned}
\tag{109}
$$

---

[48]In a dynamic context these will represent t and $t - 1$. For comparative static simulations, they represent pre- and post-shock values.

[49]N.B. $GDP_t = VGDP_{r,t,t}$.

$$QGDP_{r,t} = QGDP_{r,t-1}\sqrt{\frac{VGDP_{r,t-1,t}}{VGDP_{r,t-1,t-1}} \cdot \frac{VGDP_{r,t,t}}{VGDP_{r,t,t-1}}}$$

$$= QGDP_{r,t-1}\sqrt{\frac{GDP_{r,t}}{GDP_{r,t-1}} \cdot \frac{VGDP_{r,t-1,t}}{VGDP_{r,t,t-1}}} \qquad (110)$$

$$VGDP_{r,tp,tq} = \sum_c \left[PPA_{c,r,tp}QPA_{c,r,tq} + PGA_{c,r,tp}QGA_{c,r,tq} + PIA_{c,r,tp}QIA_{c,r,tq} \right]$$

$$+ \sum_m PDS_{m,r,tp}QST_{m,r,tq}$$

$$+ \sum_c \left[\sum_d PFOB_{c,r,d,tp}QXS_{c,r,d,tq} - \sum_s PCIF_{c,s,r,tp}QXS_{c,s,r,tq} \right]$$

$$PGDP_r = GDP_r/QGDP_r \qquad (111)$$

[NEW] This unit describes the variables GDP_r, $QGDP_r$, $VGDP_{r,tp,tq}$ and $PGDP_r$ using equations `E_vgdp`, `E_qgdpFisher`, `E_vgdpFisher` and `E_pgdpFisher`.

Listing 2.25: **GEMPACK equations for nominal GDP**

```
1  Equation E_vgdp
2  # change in value of GDP #
3  (all,r,REG)
4  GDP(r) * vgdp(r)
5  = sum{c,COMM, VGP(c,r) * [qga(c,r) + pga(c,r)]}
6  + sum{c,COMM, VPP(c,r) * [qpa(c,r) + ppa(c,r)]}
7  + sum{c,COMM, VIP(c,r) * [qinv(r) + pinv(r)]}
8  + sum{c,COMM, sum{d,REG, VFOB(c,r,d) * [qxs(c,r,d) + pfob(c,r,d)]}}
9  + sum{m,MARG, VST(m,r) * [qst(m,r) + pds(m,r)]}
10 — sum{c,COMM, sum{s,REG, VCIF(c,s,r) * [qxs(c,s,r) + pcif(c,s,r)]}};

12 pgdpexp(r,g) # GDP—expenditure component price indices #;
13 Equation E_pgdpexp
14 # GDP—expenditure component price indices #
15 (all,r,REG)(all,g,GDPEX)
16 pgdpexp(r,g) = IF[g="household", ppriv(r)] + IF[g="Investment", pinv(r)]
17 + IF[g="Government", pgov(r)] + IF[g="Exports", pfobxw(r)]
18 + IF[g="IntnlMargins", pstxw(r)] + IF[g="Imports", pmwreg(r)];

20 Equation E_qgdpexp
21 # GDP—expenditure side quantity indices #
22 (all,r,REG)(all,g,GDPEX)
23 qgdpexp(r,g) = IF[g="household", qp(r)]  + IF[g="Investment", qinv(r)]
24 + IF[g="Government", qg(r)] + IF[g="Exports", qfobxw(r)]
25 + IF[g="IntnlMargins", qstxw(r)] + IF[g="Imports", qmwreg(r)];

27 Equation E_vgdpexp
28 # value index for each GDP—expenditure component #
29 (all,r,REG)(all,g,GDPEX)
30 vgdpexp(r,g) = pgdpexp(r,g) + qgdpexp(r,g);

32 Equation E_pgdpLaspeyres
33 # computes % change in GDP Laspeyres price index in r #
34 (all,r,REG)
35 0 = sum{g, GDPEX, GDPEXP_FPIQ(r,g) * [pgdpexp(r,g) — pgdpLaspeyres(r)]};

37 Equation E_qgdpLaspeyres
38 # computes % change in GDP Laspeyres quantity index in r #
39 (all,r,REG)
40 0 = sum{g, GDPEX, GDPEXP_IPFQ(r,g) * [qgdpexp(r,g) — qgdpLaspeyres(r)]};
```

```
42  Equation E_pgdpPaasche
43  # computes % change in GDP Paasche price index in r #
44  (all,r,REG)
45  pgdpPaasche(r) = vgdp(r) − qgdpLaspeyres(r);

47  Equation E_qgdpPaasche
48  # computes % change in GDP Paasche quantity index in r #
49  (all,r,REG)
50  qgdpPaasche(r) = vgdp(r) − pgdpLaspeyres(r);

52  Equation E_pgdpFisher
53  # computes % change in GDP Fisher price index in r #
54  (all,r,REG)
55  pgdpFisher(r) = 0.5 * [pgdpLaspeyres(r) + pgdpPaasche(r)];

57  Equation E_qgdpFisher
58  # computes % change in GDP Fisher quantity index in r #
59  (all,r,REG)
60  qgdpFisher(r) = 0.5 * [qgdpLaspeyres(r) + qgdpPaasche(r)];

62  Equation E_vgdpFisher
63  # computes % change in GDP Fisher value in r #
64  (all,r,REG)
65  vgdpFisher(r) = pgdpFisher(r) + qgdpFisher(r);
```

2.11   Measurement and decomposition of welfare[50]

Regional welfare in the standard GTAP model is reported as the percentage change in regional utility, or, alternatively, as the associated *Equivalent Variation* (EV). Most policy-oriented studies report the latter, as policy makers prefer to think about the value-based welfare change associated with a given policy. However, in a model with vastly different sized regional economies, expressing EV as a percentage of initial period expenditure, or equivalently, reporting the percentage change in utility, is preferred for inter-regional comparisons. Small percentage changes in welfare in large regional economies can dwarf proportionately more important changes in the welfare of smaller economies.

The most difficult aspect of general equilibrium policy analysis is that of explaining the results—in particular the welfare results. In the standard GTAP model, these are a function of terms of trade changes (inter-regional shifting of welfare) and allocative efficiency changes (i.e., changes in production or consumption efficiency due to the presence of distortions). In many simulations, authors also vary technology, population, and possibly endowments as well. Some of these components may vary endogenously via closure changes (e.g., unemployment, technological spillovers, etc.), or they may be determined exogenously (they may simply assume that 'something good happens' due to a policy reform—e.g., improved productivity). Disentangling all of these factors affecting regional welfare is a very difficult task indeed. Fortunately, an analytical welfare decomposition has been developed which permits a break-down of the sources of welfare gain to be undertaken. The decomposition was originally developed by Huff and Hertel in the early 1990's and subsequently revised (Huff and Hertel, 2001) in light of the work of McDougall (2003), and involves a rather lengthy set of algebraic substitutions and simplifications, resulting in an expression for regional equivalent variation which, instead of being based on the regional household's expenditure function, is based instead on the various sources of efficiency changes as well as changes in endowments, technology and the region's

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[50]This section draws heavily on Hertel (2013).

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terms of trade.[51]

Huff and Hertel (2001) begin this decomposition with the model equation which expresses the change in regional income as a function of payments to endowments (net of depreciation), plus tax revenue, less subsidies paid. Into this income change equation they substitute the linearized zero profit conditions for each sector, the linearized market clearing conditions for traded goods and endowments, and the price linkage equations. The change in income on the left hand side of this expression is next deflated by the change in the regional household price index, and this is also subtracted from the right hand side of the expression. Through a series of algebraic simplifications, an expression is obtained which gives the change in real per capita expenditure as a function of changes in endowments and taxes, interacting with quantity changes. Appropriate scaling converts the real income change into the regional equivalent variation. Due to the non-homotheticity in final demand, we must also factor in population in this welfare decomposition. Due to its complexity, we do not seek to translate this from the model code, but rather present the code in Listing 2.26, along with some edits to facilitate discussion and interpretation.

Listing 2.26: **GEMPACK equations for welfare decomposition in the GTAP model**

```
1   Equation E_EV_ALT
2   # decomposition of Equivalent Variation #
3   (all,r,REG)
4       EV_ALT(r) =

6       : Preference shifts
7           −[0.01 * UTILELASEV(r) * INCOMEEV(r)]
8           * [DPARPRIV(r) * loge(UTILPRIVEV(r) / UTILPRIV(r)) * dppriv(r)
9           + DPARGOV(r) * loge(UTILGOVEV(r) / UTILGOV(r)) * dpgov(r)
10          + DPARSAVE(r) * loge(UTILSAVEEV(r) / UTILSAVE(r)) * dpsave(r)]

12      : Scaling factor
13          + [0.01 * EVSCALFACT(r)]

15      : Changes in allocative efficiency
16          * [sum{c,COMM, sum{a,ACTS, PTAX(c,a,r) * [qca(c,a,r) − pop(r)]}}
17          + sum{e,ENDW, sum{a,ACTS, INCTAX(e,a,r) * [qes(e,a,r) − pop(r)]}}
18          + sum{e,ENDW, sum{a,ACTS, ETAX(e,a,r) * [qfe(e,a,r) − pop(r)]}}
19          + sum{c,COMM, sum{a,ACTS, MFTAX(c,a,r) * [qfm(c,a,r) − pop(r)]}}
20          + sum{c,COMM, sum{a,ACTS, DFTAX(c,a,r) * [qfd(c,a,r) − pop(r)]}}
21          + sum{c,COMM, MPTAX(c,r) * [qpm(c,r) − pop(r)]}
22          + sum{c,COMM, DPTAX(c,r) * [qpd(c,r) − pop(r)]}
23          + sum{c,COMM, MGTAX(c,r) * [qgm(c,r) − pop(r)]}
24          + sum{c,COMM, DGTAX(c,r) * [qgd(c,r) − pop(r)]}
25          + sum{c,COMM, MITAX(c,r) * [qim(c,r) − pop(r)]}
26          + sum{c,COMM, DITAX(c,r) * [qid(c,r) − pop(r)]}
27          + sum{c,COMM, sum{d,REG, XTAXD(c,r,d) * [qxs(c,r,d) − pop(r)]}}
28          + sum{c,COMM, sum{s,REG, MTAX(c,s,r) * [qxs(c,s,r) − pop(r)]}}

30      : Changes in endowments
31          + sum{e,ENDW, sum{a,ACTS, EVOS(e,a,r) * [qes(e,a,r) − pop(r)]}}

33      : Depreciation
34          − VDEP(r) * [kb(r) − pop(r)]

36      : Changes in technology
37          + sum{a,ACTS, VOS(a,r) * ao(a,r)}
38          + sum{a,ACTS, VVA(a,r) * ava(a,r)}
39          + sum{c,COMM, sum{a,ACTS, VFP(c,a,r) * aint(a,r)}}
40          + sum{a,ACTS, sum{e,ENDW, VFP(e,a,r) * afe(e,a,r)}}
41          + sum{a,ACTS, sum{c,COMM, VFP(c,a,r) * afa(c,a,r)}}
42          + sum{m,MARG, sum{c,COMM, sum{s,REG, VTMFSD(m,c,s,r) * atmfsd(m,c,s,r)}}}
```

---

[51]Baldwin and Venables (1995) independently developed a theoretical decomposition in the context of the gains from regional trade agreements which is similar in spirit.

```
43                + sum{c,COMM, sum{s,REG, VMSB(c,s,r) * ams(c,s,r)}}

45        : Changes in terms of trade
46                + sum{c,COMM, sum{s,REG, VFOB(c,r,s) * pfob(c,r,s)}}
47                + sum{m,MARG, VST(m,r) * pds(m,r)}
48                + NETINV(r) * pinv(r)
49                − sum{c,COMM, sum{s,REG, VFOB(c,s,r) * pfob(c,s,r)}}
50                − sum{m,MARG, VTMD(m,r) * pt(m)}
51                − SAVE(r) * psave(r)]

53        : Changes in population
54                + 0.01 * INCOMEEV(r) * pop(r);
```

The left hand side of this expression is given the name EV_ALT. This is because it is a distinct variable from EV. The latter is computed from the utility function, whereas EV_ALT is computed indirectly. However, if the theory is correctly implemented, these two variables should be the same—subject to computational accuracy. So this is another check on model implementation (along with Walras' Law).

The first term on the right hand side of this equation captures the impact of changing preferences on welfare. Normally this should be zero, as it doesn't really make sense to measure welfare using a utility function which is changing over the course of the simulation. However, as noted above, there are some instances where policy analysts wish to alter the rate of saving or government spending, and this is done by endogenizing these shift parameters. By isolating the effect of these utility function changes, we can still use the other parts of the welfare decomposition.

The next term in Listing 2.26 is a scaling factor. Since all the remaining variables are expressed in percentage change terms (i.e., proportion change * 100%), we must divide them all by 100 (or multiply by 0.01). Furthermore, due to the non-homotheticity in final demand, the value of additional income falls as the income level rises. For this reason, we must also apply a scale factor, $EVSCALFACT_r$, which depends on changes in the marginal utility of income (see McDougall (2003) for more details).

The next block of terms in the welfare decomposition measure changes in allocative efficiency in the economy. These arise from the interplay between economic distortions and the reallocation of resources in the economy. To gain insight into the nature of these expressions, turn to Figure 2.4 which depicts what happens when we eliminate the bilateral tariff on imports into region d of commodity c from trading partner s. In the initial equilibrium, the *ad valorem* power of the tariff is given by τ so that the size of the distortion is $\tau \cdot PCIF$. This explains the difference between the initial CIF price, $PCIF_0$, and the initial domestic price for this imported good, $PMDS_0$. The resulting cost to the economy is captured by the shaded triangle representing the area between consumer's willingness to pay for this imported good and the marginal cost of supply. When the tariff is eliminated, we move to a new equilibrium wherein $PCIF_0 = PMDS_0$. So, absent shifts in the supply and demand curves, the allocative efficiency effect represents the increased welfare associated with this tariff reform.

We capture this allocative efficiency gain from tariff reform in the final term of the allocative efficiency block in Listing 2.26. Picking out only the term associated with this particular tariff on commodity c shipped from source s to destination d, we can write the allocative efficiency expression as follows:

$$EV_d\left(\tau_{Mc,s,d}\right) = \psi_d\left(\tau_{Mc,s,d}PCIF_{c,s,d}dQXS_{c,s,d}\right) \tag{112}$$

As in Figure 2.4, $(\tau_{Mc,s,d}PCIF_{c,s,d})$ is the per unit tariff revenue on imports of good c from s into d, associated with the *ad valorem* tariff rate $\tau_{Mc,s,d}d$. This is multiplied by the change in the volume of imports of c from s into d: $dQXS_{c,s,d}$. In order to evaluate the area of this

"Harberger triangle" as the tariff is eliminated, we must consider both the "base" ($\tau_{Mc,s,d}PCIF_{c,s,d}$) and the "height" ($dQXS_{c,s,d}$).[52] By continually reevaluating the base of this triangle as the tariff is eliminated, we track the diminishing gap between $PCIF_{c,s,d}$ and $PMDS_{c,s,d}$. In this way, we are able to accurately measure its area, which is then added to the aggregate welfare measure, subject to application of the appropriate scaling factor, ψ_d. In order to properly perform this numerical integration, the welfare decomposition equations must be solved in conjunction with the CGE model, using appropriate solution procedures. We use the *GEMPACK* software suite developed by Harrison and Pearson (1996) which is ideally suited to this problem, as it solves the non-linear CGE model using a linearized version of the behavioral equations, coupled with updating equations that link the change, in this case $dQXS_{c,s,d}$, with the levels variables, $QXS_{c,s,d}$. Standard extrapolation techniques can be used to obtain arbitrarily accurate solutions to any well-posed non-linear problem (Harrison and Pearson, 1996). To relate these terms back to the expression in Listing 2.26, note that $MTAX_{c,s,d} = \tau_{Mc,s,d}PCIF_{c,s,d}QXS_{c,s,d}$ and $100 \times dQXS_{c,s,d}/QXS_{c,s,d} = qxs_{c,s,d}$. Note that the allocative efficiency effects also pick up the interplay between partial policy reforms and pre-existing distortions which are unchanging. This is important and highlights the value of CGE analysis in evaluating the welfare impacts of potentially second-best reforms.

The next two terms in Listing 2.26 capture the impact of changing endowments on regional welfare. Obviously any exogenous increase in labor or land will improve regional welfare—at least assuming no change in the regional terms of trade. Note that any shock to capital stock will increase not only capital services, but also depreciation and this is factored into the welfare calculation.
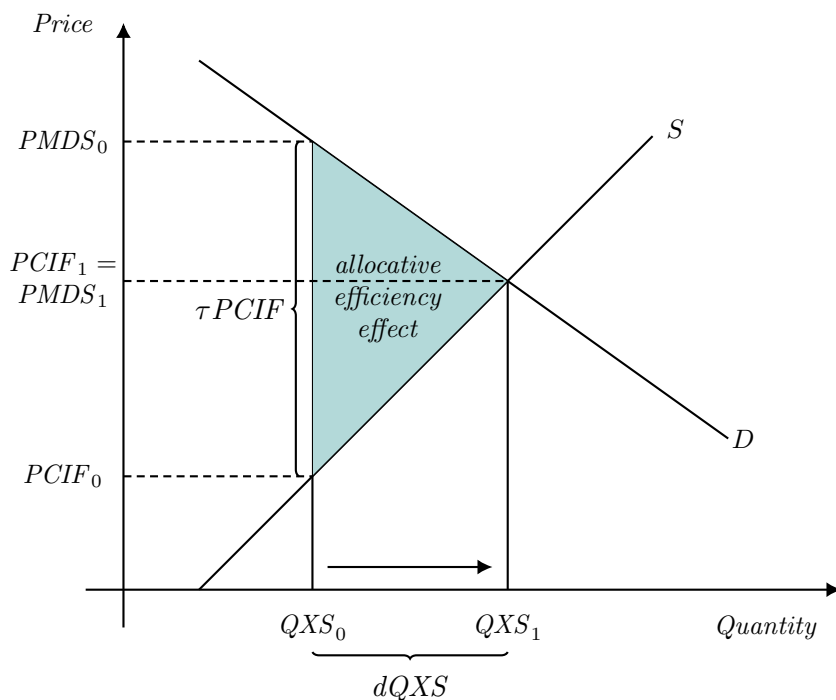
Figure 2.4: **Computing allocative efficiency effects**

The next block of terms in the welfare decomposition relates to changes in technology. This is

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[52]For those accustomed to computing "Harberger triangles" as $\frac{1}{2}$ *base* $\times$ *height*, it may appear that we need a $\frac{1}{2}$ pre-multiplying the right hand side of equation (112). However, this is not required. The numerical integration procedure facilitated by GEMPACK continually re-evaluates the base of the "triangle" Harrison and Pearson (2002).

straightforward. Enhanced efficiency in the economy will boost welfare, again holding the terms of trade constant.

And finally, turning to the terms of trade—these are captured in six terms at the bottom of Listing 2.26. The first of these captures the fact that higher export prices boost regional welfare, with the impact depending on the amount of exports sold into world markets. However, it is not just normal goods and services which are exported by a given region in the model. There are also international trade and transport services sales to the global transport sector which must be accounted for. Hence the term involving $VST_{m,r}$. Finally, regions also 'export' the sale of investment services to satisfy the demands of savers worldwide. This, too, has the potential to influence a region's terms of trade.

On the import side of the terms of trade, higher prices diminish welfare, as do higher prices for the margins services implicitly imported along with merchandise goods. For this reason, imports are valued at their FOB prices, and the component of CIF prices reflected by the international trade and transportation services is separately modeled. Since the transactions with the global bank must be squared as well, purchases of saving by the regional household show up as a potential source of terms of trade changes. However, the price of saving is defined in a way which tends to minimize these effects.

The final term in Listing 2.26 relates to the change in population. Since EV refers to total regional welfare, and up to this point we have been working on a per capita basis, we must now bring in population as an explanatory variable. If population doubles, we would roughly expect that regional welfare would double.

In summary, it is little surprise that the welfare decomposition tool in GTAP is among the most widely used analysis tools in trade policy analysis. By providing a comprehensive decomposition of sources of welfare change from a given policy, it prevents confabulation and enriches the presentation of findings to policy makers, thereby enhancing the credibility of the policy analyst.

Chapter 3

Model dynamics

3.1 Introduction

From a specification point of view, dynamics broadly involves four areas:

1. Update of exogenous endowments

2. Changes to technologies

3. Changes to preferences

4. Changes to policies

In all other ways, the dynamic model is solved as a sequence of time-based comparative static equilibria with updates in the aforementioned four areas occurring between periods. A dynamic solution can be done in the absence of any external information with informed guesses about the evolution of endowments, technology and preferences. However, a dynamic baseline typically is coupled to external information about certain future trends, for example population, labor force growth, per capita incomes and the baseline scenario is then used to calibrate some key parameters based on the external information.

3.2 Endowments

3.2.1 Population and labor force

In most baselines, population and labor force growth are taken from external demographic projections, for example the United Nation's Population Division typically biennial projections (http://www.un.org/en/development/desa/population/).[1] Data sources for baselines are discussed further below.

Demographic projections typically include various demographic slices—for example by age group (0-4, 5-9,...,95-99,100+ or some aggregation thereof), by gender, by country-origin and/or by education level. The standard GTAP-based baseline mostly uses total population as well as some indicator for the labor force. The simplest for the latter is to equate labor force growth to the growth of the so-called 'working age' population—often defined as the 15-64 age group. A somewhat more sophisticated version could use age- and gender-specific labor force participation rates

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[1]Their latest projection, entitled *2017 Revision of World Population Prospects* was issued in June, 2017.

and create an aggregate labor force growth rate combining the labor force participation rates with the demographic projections. Assuming the demographic projections and the assumptions on labor force participation are exogenous, these calculations can be done separately and the inputs to the baseline are simply the growth of total population and the labor force. Due to the lack of additional information, we are currently assuming no differentiation in the growth of labor endowments across skill types. We could use the education information included with the International Institute for Applied Systems Analysis (IIASA) projections to guide the relative growth of skilled versus unskilled workers, further discussed below.

3.2.2 Capital

The model uses the standard capital accumulation expression to drive the growth in the capital stock. This is essentially already included in the standard model—see equation (78). Note that if we move to multi-year steps, the capital accumulation equation needs to include investment for the skipped years. If we assume that investment growth is constant in the inter-period step, the capital accumulation formula is:

$$K_t = (1 - \delta)^n K_{t-n} + \frac{(1 + g)^n - (1 - \delta)^n}{g + \delta} I_{t-n}$$

where g is the inter-period growth rate of investment:

$$g = \left(\frac{I_t}{I_{t-n}}\right)^{1/n} - 1$$

Note that in this case the capital stock is no longer pre-determined but depends on the endogenous level of investment in the current period. Capital growth will be determined by savings behavior (possibly adjusted by public and foreign savings) and the rate of depreciation. It is often the case that adjustments will be required for both the rate of saving and depreciation to develop a baseline with long-run steady state properties, which could be defined for example, by a (near) constant rate of return. In this case, we may have an exogenous target for the investment to GDP ratio in which case we would override the savings level determined via the top level regional household utility function.

Listing 3.1 implements the year-on-year link between investment and capital growth in the model's TABLO code. In dynamic simulations, this link is activated in equation E_capadd by swapping the variable $capadd_r$ with $qe_{'capital',r}$ in the closure file[2] and by shocking the homotopy variable, del_unity, with a value of 1. With $capadd_r$ exogenous, equation E_capadd then associates net investment changes to aggregate capital growth in each region, $qecap_r$, based on changes in the ratio of two coefficients—i.e., the value of investment generated within the year adjusted for depreciation ($VKADD_r$) relative to the initial value of beginning period capital stock ($VKBINI_r$). The next equation, E_qecap, is simply a bridging equation that relates aggregate capital growth, $qecap_r$, to aggregate capital endowment, $qe_{e,r}$.[3] Equation E_kb—which is already included in the standard model—associates changes in aggregate capital endowment, $qe_{e,r}$, at the end of the current period to next period's beginning capital stock, kb_r.

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[2]This swap exogenizes $capadd_r$ and endogenizes $qe_{'capital',r}$. The appropriate statement in the closure file is: Swap qe("capital",REG) = capadd(REG)

[3]Equation E_qecap simplifies to qecap(r) = qe("capital",r) since the capital endowment set, $ENDWC$, contains just one element named capital. This equation along with the variable $qecap_r$ can be deleted from the model code. However, we retain them for flexibility and to avoid hard-coding $qe_{'capital',r}$ in place of $qecap_r$ in equation E_capadd.

Listing 3.1: **GEMPACK equations for year-on-year capital accumulation function**

```
1  Equation E_capadd
2  # asscociates net investment changes to aggregate capital endowment in r #
3  (all,r,REG)
4  VKADD(r) * del_unity + capadd(r) = 0.01 * VKBINI(r) * qecap(r);
5
6  Equation E_qecap
7  # computes % change in aggregate capital endowment in reqion r #
8  (all,r,REG)
9  qecap(r) = sum{e,ENDWC, [VES(e,r) / GROSSCAP(r)] * qe(e,r)};
10
11 Equation E_kb
12 # associates change in cap. services w/ change in cap. stock #
13 (all,r,REG)
14 kb(r) = sum{e,ENDWC, [VES(e,r) / GROSSCAP(r)] * qe(e,r)};
```

3.2.3 Natural resources

Allowing for a plausible aggregate land supply function, i.e. one with lower and upper limits such as a logistic function, as baseline would imply movements up and down the supply curve.[4] Eventually, one could also adjust the shift parameters in the land supply curve to handle possible events such as sea-level rise.

Sector-specific natural resources—such as fossil fuel reserves—should also have their own sector-specific supply functions. We have most often used iso-elastic supply functions—with the possibility of kinks in the supply elasticities. The kinks allow for differential response based on market conditions as it is typically easier to contract supply than to expand it. More sophisticated models might have resource depletion modules for natural resources that puts an overall cap on supply—this is typically the case for energy-based models.

3.3 Technology

The baseline normally involves calibrating one economy-wide variable to hit a given target for GDP (per capita) growth. A simplified model can be specified as:

$$Y = A \cdot F(\lambda^l L, \lambda^k K)$$

where Y is output that is a function of labor and capital in efficiency units. The variables λ^l and λ^k represent the growth of labor and capital in efficiency units, normally set to 1 in the base year. The variable A is a total factor productivity (TFP) shifter, whereas the other two variables induce biased technical change. The discussion above described the dynamics of labor and capital. If the growth of output, Y, is given, then we have three parameters that enable the model to target output growth—TFP or one of the two biased technical change parameters, or some combination of the three. In principle the choice should be based on historical observations and linked to other variables such as the evolution of factor prices. History provides only partial guidance—particularly at the global level. The most frequent assumption used in dynamic baselines, particularly those associated with integrated assessment, is to assume that residual growth is embedded in labor-biased technical change. Hence given the output equation above, F is inverted to evaluate the value of λ^l given L, K and Y.[5] One alternative, used in the GREEN model (van der Mensbrugghe, 1994), is to implement

---

[4]The model currently uses an iso-elastic supply curve for aggregate land. For relatively short-term horizons, this is probably not a significant issue, particularly if the supply elasticity is relatively low for land-scarce regions.

[5]Note that exogenous changes to A and λ^k are also possible.

so-called balanced growth. The interpretations is that the long-term capital/labor ratio, expressed in efficiency units is constant:

$$\frac{\lambda^k K}{\lambda^l L} = \chi$$

where χ could be set to the base year capital to labor ratio, or some anticipated long-term trend. With Y and χ given, we have two equations and two unknowns and thus we can calibrate λ^l and λ^k simultaneously. A third strategy, pioneered by Dixon and Rimmer (2002) is to use the historical evolution of the capital/labor ratio, subject to a cost-neutral constraint on the cost function, (to be described below) and TFP to target Y, i.e. calibrate A to the growth target. This is the so-called 'twist' strategy.

In its simplest form, the 'twist' strategy is applied to a CES bundle. The idea is to move the capital/labor ratio from some initial value, to some terminal value:

$$\frac{K}{L} = (1 + tw)\frac{K_0}{L_0}$$

where tw represents the percent change in the ratio. The 'twist' is done in such a way maintain the same cost level (with unchanged factor prices). In the case of the CES function, this implies:

$$P = \left[\alpha^l \left(\frac{W}{\lambda^l}\right)^{1-\sigma} + \alpha^k \left(\frac{R}{\lambda^k}\right)^{1-\sigma}\right]^{1/(1-\sigma)} = P_0 = \left[\alpha^l W^{1-\sigma} + \alpha^k R^{1-\sigma}\right]^{1/(1-\sigma)}$$

We now have a system of three equations—the output equation, the twist equation and the constant cost equation—and these can be solved simultaneously to determine A, λ^l and λ^k. With a little bit of algebra it is possible to provide expressions for λ^l and λ^k with respect to the desired twist, tw, and the value shares:

$$\lambda^l = \left[1 + tw \cdot s^k\right]^{1/(1-\sigma)}$$

$$\lambda^k = \left[\frac{1 + tw \cdot s^k}{1 + tw}\right]^{1/(1-\sigma)}$$

where tw represents the percent change in the capital to labor ratio and s^k is the base year value share of capital.

This unit implements baseline calibration of alternative economy-wide factor productivity variables to hit a given target for GDP. As shown in Listing 3.2, Equation `E_afe` allows 3 possible alternatives:

- Hicks-neutral TFP shifter ($afereg_r$)

- Labor-biased technology shifter ($afelabreg_r$)

- Non-capital-biased technology shifter ($afendwxcreg_r$)

This unit also implements labor-capital twist mechanisms by modifying equation `E_qfe`, initially presented in Listing 2.2. In the revised code shown in Listing 3.2, equation `E_qfe` has been partitioned into two parts, i.e., the labor and capital endowment set, `ENDWLC`, and non labor-capital endowments set, `ENDWXLC`. The former implements labor-capital twists via the variable $afetwist_{e,a,r}$. The next equation, `E_afetwistave`, imposes a cost-neutral constraint on the labor-capital bundle while the last equation, `E_afetwist`, provides user flexibility with the inclusion of various labor-capital twist shifters—e.g., activity-specific and all region twist shifter ($afetwistact_{a,r}$), all activity and region-specific shifter ($afetwistreg_{e,r}$), and activity- and region-specific shifter ($afetwistall_{e,a,r}$).

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Listing 3.2: **GEMPACK equations for factor demands with labor-capital 'twist'**

```
1   Equation E_afe
2   # sector/region specific average rate of prim. factor e augmenting tech change #
3   (all,e,ENDW)(all,a,ACTS)(all,r,REG)
4   afe(e,a,r)
5   = afecom(e) + afesec(a) + afereg(r) + afeall(e,a,r) + afecomreg(e,r)
6   !< endowment—biased technical change and shifters >!
7   + IF[e in ENDWL,  afelabreg(r) + afelabact(a,r) + afelab(e,a,r)]
8   + IF[e in ENDWXC, afendwxcreg(r) + afendwxcact(a,r) + afendwxc(e,a,r)];

10  Equation E_qfe
11  # demands for endowment commodities #
12  (all,e,ENDW)(all,a,ACTS)(all,r,REG)
13  qfe(e,a,r)
14  = IF[e in ENDWLC,
15        — afe(e,a,r) + [afetwist(e,a,r) — afetwistave(a,r)] + qva(a,r)
16        — ESUBVA(a,r) * [pfe(e,a,r) — afe(e,a,r) — pva(a,r)]]
17  + IF[e in ENDWXLC,
18        — afe(e,a,r) + qva(a,r)
19        — ESUBVA(a,r) * [pfe(e,a,r) — afe(e,a,r) — pva(a,r)]];

21  Equation E_afetwistave
22  # average endowment twist shifter in activity a in reg r #
23  (all,a,ACTS)(all,r,REG)
24  0 = sum{e,ENDWLC, VFP(e,a,r) * [afetwist(e,a,r)  — afetwistave(a,r)]};

26  Equation E_afetwist
27  # sector and region specific twist towards use of endowment e #
28  (all,e,ENDWLC)(all,a,ACTS)(all,r,REG)
29  afetwist(e,a,r) = afetwistact(e,a) + afetwistreg(e,r) + afetwistall(e,a,r);
```

3.3.1   Multi-sector models

The description above is straight-forward in the case of a single sector macro model, but multi-sector models need additional assumptions. In the case of the labor-biased technical change, there are as many potential labor productivity factors as there are activities and labor types. The simplest assumption imposes uniformity across activities and skill types. Somewhat more generically, we can assume that labor productivity takes the following form:

$$\pi^l_{l,a} = \alpha^l_{l,a} + \beta^{l,a}\gamma^l$$

where γ^l is an economy-wide parameter that can be calibrated to target GDP. Uniformity implies α^l is zero and β^l is one for all skill types and activities (See equation listing below for corresponding TABLO implementation). While the econometric evidence is relatively scant, it nonetheless suggests that there are differences across activities (see Jorgenson et al. (2013) and Roson and van der Mensbrugghe (2017)) that can be exploited. Our standard starting point is to assume that α^l is between 1 and 2 percent in agriculture and 2 percent in manufacturing[6]. This implies that the calibrated γ^l represents labor productivity in services and that there is a constant (positive) wedge in agriculture and manufacturing. These assumptions obviously have structural assumptions beyond those generated by shifting patterns of demand.

If we use the Dixon and Rimmer (2002) approach, one would have assumptions about the 'twist' between capital and labor, i.e. some assumptions regarding the capital/labor ratio, at the activity level. Dixon and Rimmer (2002) are able to use the richness of historical data for Australia and the United States (Dixon and Rimmer, 2005) to measure historical 'twists' that can then inform

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[6]with β^l set to 1

the projected 'twist'. The approach above, for labor-biased technical change can then be used to determine TFP with a calibrated economy-wide productivity measure and linear transformations to allocate the economy-wide measure across activities.

In the balanced growth scenarios, there has been no explicit assumption about productivity across activities/labor types. Instead the following expression is assumed to hold, which is a generalization of the balanced growth equation above:

$$\frac{\sum_a \lambda_a^k K_a}{\sum_a \sum_l \lambda_{l,a}^l L_{l,a}} = \chi$$

Typical baselines may also include other exogenous assumptions on technological progress such as yield changes in crops, energy efficiency improvement (identified by end-user, energy carrier, etc.) and autonomous improvements in trade and transport margins. The use of expert models could be used to drive some of these assumptions such as crop and energy models.

This unit implements labor-biased technical change, with many potential labor productivity factors as there are activities and labor types, using Equation `E_afelab`. The variables $afelabreg_r$ and $afelabadd_{e,a,r}$ correspond to γ^l and α^l respectively, while the coefficient ($LABMULT_{e,a,r}$) corresponds to the β^l parameter.

Listing 3.3: **GEMPACK equations for labor productivity changes**

```
1  Equation E_afelab
2  # labor productivity shifters #
3  (all,e,ENDWL)(all,a,ACTS)(all,r,REG)
4  afelab(e,a,r) = afelabadd(e,a,r) + LABMULT(e,a,r) * afelabreg(r);
```

3.4   Preferences

Trade preferences are governed by a two-level nested CES (Armington) structure. Similar to Dixon and Rimmer (2002), we have introduced a twist specification in the top-level Armington nest that allows for shifts in preferences for domestic vs. imported commodities for all agents—i.e., firms, private households, investment and government. We have also introduced a twist specification in the second-level nest that allows for shifts in import preferences across trading partners.

This unit implements the two-level nested CES (Armington) structure with twist preferences by modifying equations `E_qfd`, `E_qfm`, `E_qpd`, `E_qpm`, `E_qid`, `E_qim`, `E_qgd` and `E_qgm` previously presented in Listing 2.3 and `E_qxs` previously shown in Listing 2.10. The agent-specific domestic and imported twist variables —$afd_{c,a,r}$, $apd_{c,r}$, $aid_{c,r}$, $agd_{c,r}$, $afm_{c,a,r}$, $apm_{c,r}$, $aim_{c,r}$ and $agm_{c,r}$— facilitate cost-neutral preference shifts towards domestic and imported commodities in each agent's top level Armington demand. The second level Armington twist variable, $amdstwist_{c,s,d}$, in equation `E_qxs` facilitates preference shifts towards a specific trading partner or a group of trading partners. The last equation, `E_amdstwistave`, imposes a cost-neutral constraint on the second level import cost function.

Listing 3.4: **GEMPACK equations for Armington demands with 'twist' specification**

```
1  Equation E_qfd
2  # act. a demands for domestic good c #
3  (all,c,COMM)(all,a,ACTS)(all,r,REG)
4  qfd(c,a,r) = qfa(c,a,r) — ESUBD(c,r) * [pfd(c,a,r) — pfa(c,a,r)]
5  + afd(c,a,r);
6
7  Equation E_qfm
```

```
 8   # act. a demands for composite import c #
 9   (all,c,COMM)(all,a,ACTS)(all,r,REG)
10   qfm(c,a,r) = qfa(c,a,r) — ESUBD(c,r) * [pfm(c,a,r) — pfa(c,a,r)]
11   + afm(c,a,r);

13   Coefficient (all,c,COMM)(all,a,ACTS)(all,r,REG)
14   FMSHR(c,a,r) # share of firms' imports in dom. composite, purch. prices #;
15   Zerodivide default 0.5;
16   Formula (all,c,COMM)(all,a,ACTS)(all,r,REG)
17   FMSHR(c,a,r) = VMFP(c,a,r) / VFP(c,a,r);
18   Zerodivide off;

20   Equation E_afd
21   # equation to facilitate shift towards domestic input c in act. a in region r #
22   (all,c,COMM)(all,a,ACTS)(all,r,REG)
23   afd(c,a,r) = FMSHR(c,a,r) * afdmtwist(c,a,r);

25   Equation E_afm
26   # equation to facilitate shift towards imported input c in act. a in region r #
27   (all,c,COMM)(all,a,ACTS)(all,r,REG)
28   afm(c,a,r) = [FMSHR(c,a,r) — 1] * afdmtwist(c,a,r);

30   Equation E_qpd
31   # private consumption demand for domestic goods #
32   (all,c,COMM)(all,r,REG)
33   qpd(c,r) = qpa(c,r) — ESUBD(c,r) * [ppd(c,r) — ppa(c,r)] + apd(c,r);

35   Equation E_qpm
36   # private consumption demand for aggregate imports #
37   (all,c,COMM)(all,r,REG)
38   qpm(c,r) = qpa(c,r) — ESUBD(c,r) * [ppm(c,r) — ppa(c,r)] + apm(c,r);

40   !<  Composite Tradeables >!
41   Coefficient (all,c,COMM)(all,r,REG)
42   PMSHR(c,r) # share of imports in private hhld cons. at producer prices #;
43   Formula (all,c,COMM)(all,r,REG)
44   PMSHR(c,r) = VMPP(c,r) / VPP(c,r);

46   Equation E_apd
47   # equation to facilitate shift towards cons of dom. c by priv hhld in region r #
48   (all,c,COMM)(all,r,REG)
49   apd(c,r) = PMSHR(c,r) * apdmtwist(c,r);

51   Equation E_apm
52   # equation to facilitate shift towards cons of imp. c by priv hhld in region r #
53   (all,c,COMM)(all,r,REG)
54   apm(c,r) = [PMSHR(c,r) — 1] * apdmtwist(c,r);

56   Equation E_qid
57   # demand for domestic investment commodity c #
58   (all,c,COMM)(all,r,REG)
59   qid(c,r) = qia(c,r) — ESUBD(c,r) * [pid(c,r) — pia(c,r)] + aid(c,r);

61   Equation E_qim
62   # demand for imported investment commodity c #
63   (all,c,COMM)(all,r,REG)
64   qim(c,r) = qia(c,r) — ESUBD(c,r) * [pim(c,r) — pia(c,r)] + aim(c,r);

66   !<  Composite tradeables >!
67   Coefficient (all,c,COMM)(all,r,REG)
68   IMSHR(c,r) # share of imports for investment at producer prices #;
69   Formula (all,c,COMM)(all,r,REG)
70   IMSHR(c,r) = VMIP(c,r) / VIP(c,r);

72   Equation E_aid
73   # equation to facilitate shift towards use of dom. investment commodity c in r #
74   (all,c,COMM)(all,r,REG)
75   aid(c,r) = IMSHR(c,r) * aidmtwist(c,r);
```

```
77   Equation E_aim
78   # equation to facilitate shift towards use of imp. investment commodity c in r #
79   (all,c,COMM)(all,r,REG)
80   aim(c,r) = [IMSHR(c,r) — 1] * aidmtwist(c,r);

82   Equation E_qgd
83   # government consumption demand for domestic goods #
84   (all,c,COMM)(all,r,REG)
85   qgd(c,r) = qga(c,r) — ESUBD(c,r) * [pgd(c,r) — pga(c,r)] + agd(c,r);

87   Equation E_qgm
88   # government consumption demand for aggregate imports #
89   (all,c,COMM)(all,r,REG)
90   qgm(c,r) = qga(c,r) — ESUBD(c,r) * [pgm(c,r) — pga(c,r)] + agm(c,r);

92   !<  Composite tradeables >!
93   Coefficient (all,c,COMM)(all,r,REG)
94   GMSHR(c,r) # share of imports for gov't hhld at producer prices #;
95   Formula (all,c,COMM)(all,r,REG)
96   GMSHR(c,r) = VMGP(c,r) / VGP(c,r);

98   Equation E_agd
99   # equation to facilitate shift towards cons. of domestic c by gov in region r #
100  (all,c,COMM)(all,r,REG)
101  agd(c,r) = GMSHR(c,r) * agdmtwist(c,r);

103  Equation E_agm
104  # equation to facilitate shift towards cons. of imported c by gov in region r #
105  (all,c,COMM)(all,r,REG)
106  agm(c,r) = [GMSHR(c,r) — 1] * agdmtwist(c,r);

108  Equation E_qxs
109  # regional demand for disaggregated imported commodities by source #
110  (all,c,COMM)(all,s,REG)(all,d,REG)
111  qxs(c,s,d)
112  = —ams(c,s,d) + [amdstwist(c,s,d) — amdstwistave(c,d)]
113  + qms(c,d) — ESUBM(c,d) * [pmds(c,s,d) — ams(c,s,d) — pms(c,d)];

115  Equation E_amstwistave
116  # average top—level import twist shifter for com c by importing reg d #
117  (all,c,COMM)(all,d,REG)
118  0 = sum{s,REG, VMSB(c,s,d) * [amdstwist(c,s,d) — amdstwistave(c,d)]};

120  Equation E_amstwist
121  # commodity and region specific twist towards imports of com c from source s #
122  (all,c,COMM)(all,s,REG)(all,d,REG)
123  amdstwist(c,s,d) = amdstwistall(c,s,d) + amdstwistsrc(c,s) + amdstwistreg(s,d);
```

3.5   Policies

A dynamic baseline starts in a given year—for example 2014—and with policies calibrated to the base year. A number of policy changes may already be in the process of being implemented—for example the commitments made under the 2015 Paris Agreement, or bilateral or regional trade agreements, or may already have been negotiated with a future phase-in period, such as the Trans-Pacific Partnership Agreement. The baseline scenario should include these policy changes, to the extent possible. In some cases, the changes are relatively straightforward, such as a percentage cut in a tariff rate. In other cases it may require interpretation. For example, is a commitment to an x-percent improvement in energy efficiency a binding commitment, or one that might have been anticipated in a baseline.

3.6 Data sources for exogenous baseline assumptions

GDP or per capita GDP projections are typically sourced from international agencies such as the International Monetary Fund (IMF), the Organisation for Economic Cooperation and Development (OECD) and the World Bank—though often these are short- and medium-term projections. Global population and labor force growth are available from demographic projections by the United Nations Population Division[7] or IIASA[8].

An alternative source of exogenous baseline assumptions is IIASA's Shared Socioeconomic Pathways (SSPs) dataset, which allows users to choose among five potential socioeconomic development pathways.[9] The SSPs include projections on GDP, population and educational attainment—the latter could be used to differentiate the relative growth of skilled versus unskilled workers. Three economic modeling teams developed an independent set of projections for the five SSPs—the OECD, IIASA and PIK. All harmonized to the same set of demographic projections for the 5 SSPs undertaken by the demographers at IIASA. The GDP projections of the OECD and IIASA are based on country-level assumptions (some 150+ countries covering most of world GDP and population). The PIK GDP projections are available for a fixed set of 32 aggregate regions—some of which are individual countries. GTAP Center's staff has prepared a GTAP-friendly version of the SSPs (including the UN's population projections). It has extended the SSP dataset: 1) gap filled all of the missing countries to match the UN's population dimensionality; and 2) annualized the projections that were initially available in 5-year time steps.

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[7] http://www.un.org/en/development/desa/population/

[8] http://www.iiasa.ac.at/web/home/research/researchPrograms/WorldPopulation/Introduction.html

[9] For further information on SSPs, see https://tntcat.iiasa.ac.at/SspDb or the special issue of the Journal of Global Environmental Change devoted to SSP http://www.iiasa.ac.at/web/home/research/researchPrograms/Energy/news/161005-SSP.html

Chapter 4

Further reading

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Appendix A

Mathematical appendix

We derive here some of the key mathematical features of the GTAP model. The first section introduces some of the key concepts underpinning the implementation of the model in the *GEMPACK* software, i.e., the log-linearization of the model equations. The second section expands on the implementation of the tax revenue streams in the GTAP model and highlights additional details regarding the log-linearization of the model. There are two key features of the GTAP model that differentiate it with respect to many other CGE models—the top-level utility function of the representative household and the Constant-Differences-in-Elasticity (CDE) sub-utility function for the allocation of private expenditures. Few models have any explicit top-level utility function for the allocation of regional income to private, public and saving expenditures. And many models use some variant of the Linear Expenditure System (LES) for allocating private expenditures. These special features of GTAP are developed in Sections A.0.3 and A.0.4.

A.0.1 Log linearization

The GTAP model derives from the Johansen/ORANI/MONASH tradition (Johansen (1960), Dixon et al. (1982), Dixon and Rimmer (2002)). As such, all of its equations in levels are converted to percent change form using log-linearization.[1]

The conversion of a model in levels into a model in percentage change form relies on three rules described in Table A.1.[2] To illustrate the mechanics of the conversion, and its usefulness for providing economic insights, we will derive the log-linearization of a generic CES function.

The most generic form of the CES function, for example in terms of a single nested production function, can be written as follows in levels form:

$$\min C = \sum_i P_i X_i$$

$$\text{subject to:}$$

$$Q = A \left[\sum_i a_i \left(\lambda_i X_i \right)^\rho \right]^{1/\rho}$$

where C is total cost $(P \cdot Q)$, X_i are the input components with a cost of P_i, Q is output, with a price

---

[1]There are some rare exceptions to deal with variables that could take zero as a value in levels (for example tax revenues) in which case the equations are specified as ordinary changes.

[2]Reprised from Dixon and Rimmer (2002).

| | Representation in: | | |
	levels		percentage changes
Multiplication rule	$X = YZ$	$\Rightarrow$	$x = y + z$
Power rule	$X = Y^\alpha$	$\Rightarrow$	$x = \alpha y$
Addition rules	$X = Y + Z$	$\Rightarrow$	$Xx = Yy + Zz$
			or $x = S_y y + S_z z$

X, Y and Z are levels of variables, x, y and z are percentage changes, α is a parameter and S_y and S_z are shares evaluated at the current solution. In the first step of a Johansen/Euler computation, the current solution is the initial solution. Hence $S_y = Y_0/X_0$ and $S_z = Z_0/X_0$. In subsequent steps, S_y and S_z are recomputed as X, Y and Z move away from their initial values.

Table A.1: **Rules for deriving a percentage-change version of a model**

of P, a_i are the so-called primal share parameters, ρ is the CES exponent, A is a neutral technology shifter, and the λ_i parameters are input-specific technology shifters (or biased technology shifters).

It is easy to show that the first order conditions of the CES leads to the following set of equations in levels:

$$X_i = \alpha_i \left(\lambda_i A\right)^{\sigma-1} \left(\frac{P}{P_i}\right)^\sigma Q$$

$$P = \frac{1}{A} \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \iff P \cdot Q = \sum_i P_i X_i$$

where the primal and dual parameters are linked by $\sigma = 1/(1-\rho)$ and $\alpha_i = a_i^\sigma$. The parameter $\sigma > 0$ is the so-called CES substitution elasticity. The implementation of the CES can typically dispense with the primal parameters. The levels form requires calibrating the α parameters using base year data.

Using the formulas from Table A.1, the CES demand function and its accompanying price expression are readily converted to percent change form:

$$\dot{x}_i = \dot{q} - \dot{a} - \dot{\lambda}_i - \sigma \left(\dot{p}_i - \dot{\lambda}_i - \dot{p} - \dot{a}\right)$$
$$\dot{p} = -\dot{a} + \sum_i S_i \left(\dot{p}_i - \dot{\lambda}_i\right) \quad \text{and } S_i = \frac{P_i X_i}{P \cdot Q}$$

where we are using a dot over the lower case variable to designate the variable in percentage change form. The first equation is intuitively appealing (ignoring the technical change coefficents for a moment). The percentage change in demand for component i changes in proportion with overall output, $\dot{q}$, adjusted for any change in the price P_i relative to the aggregate price, P, where σ determines the level of adjustment. At one extreme, when σ is zero, demand changes in exact proportion with output—the so-called Leontief technology. As σ increases, the price adjustments become more prominent. The price formula is also intuitive. A first order approximation of an increase in the price of P_i on aggregate cost is the percentage change in P_i adjusted by its cost share.

As we can see from this example, the implementation of the CES in percentage change form does not require calibrating the CES share parameters, though it does require specifying an updating formula for the cost shares in the price expression.

In the model write-up, we have used the percentage change formulation for the behavioral equations where there appeal to intuition is most clear. However, many of the accounting identities and equilibrium conditions have been left in level form to improve exposition.

A.0.2 Revenue tax streams in the GTAP model

A generic form of a tax stream in levels form can be expressed as:

$$REV = \sum_i (T_i - 1) PB_i X_i$$

where T is the power of the tax, PB is the base price of a good (or factor), and X is the quantity being taxed. In fact, in the implementation of the GTAP model, the equations are formulated as the ratio of the tax revenue and income, and thus the relevant variable is $R = REV/Y$.

Since taxes and thus tax revenues, can be zero, positive or negative, there is a danger to using the standard percentage change form for the equation, and instead, the tax revenue equations are expressed as ordinary changes. Thus we have:

$$Y \cdot \Delta R + R \cdot \Delta Y = \sum_i \left[\Delta T_i PB_i X_i + (T_i - 1) \Delta PB_i X_i + (T_i - 1) PB_i \Delta X_i \right]$$

The relevant variables can be converted to percentage change form:

$$Y \cdot \Delta R + R \cdot Y \cdot \dot{y} = \sum_i \left[T_i PB_i X_i \dot{t}_i + (T_i - 1) PB_i X_i (\dot{p}_i + \dot{x}_i) \right]$$

In the final step, we replace the variables in levels with GTAP coefficients that are updated at each iteration (replacing $R \cdot Y$ with REV) and multiplying ΔR by 100 to convert it to a change in percent[3]:

$$100 \cdot Y \cdot \Delta R + REV \cdot \dot{y} = \sum_i \left[VXP_i \dot{t}_i + VTAX_i \left(\dot{pb}_i + \dot{x}_i \right) \right]$$

where VXP is the value of X at producer prices (PP) and $VTAX$ is the value of the tax revenue (for each commodity i). Thus we have the following equations and updating formulas:

$$\dot{pp}_i = \dot{pb}_i + \dot{t}_i$$
$$VXP_i = \dot{pp}_i \dot{x}_i$$
$$VXB_i = \dot{pb}_i \dot{x}_i$$
$$VTAX_i = VXP_i - VXB_i$$
$$REV = \sum_i VTAX_i$$

where VXB is the value of X at basic prices. The first expression is a model equation. The remaining four represent model coefficients that are updated at each iteration. The updating expressions for VXP and VXB are interpreted by GEMPACK to mean to apply the percentage change of the relevant variables to their product:

$$VXP_{i,n} = PP_{i,n} X_{i,n} = PP_{i,n-1}(1 + 0.01\dot{pp}_{i,n}) X_{i,n-1}(1 + 0.01\dot{x}_{i,n})$$
$$VXB_{i,n} = PB_{i,n} X_{i,n} = PB_{i,n-1}(1 + 0.01\dot{pb}_{i,n}) X_{i,n-1}(1 + 0.01\dot{x}_{i,n})$$

---

[3]All the indirect tax ratios are evaluated as decimal changes, thus there use in percentage change formulas requires multiplying by 100

where n represents an iteration counter. For the first iteration, the relevant base is the initial solution.

The listing below shows the relevant equations and formulas for the import tax revenues and they can be readily identified with the equations and formulas above. The two key differences are that the sums are over commodities and regions of origin (s) and that the import tariff is composed of two components—a bilateral tariff, initially calibrated to base year levels, and a shifter, uniform across source regions, that is set to 0 in the initial database.

Listing A.1: **GEMPACK equations for Import tax revenues and related formulas**

```
1   Update (all,c,COMM)(all,s,REG)(all,d,REG)
2       VMSB(c,s,d) = pmds(c,s,d) * qxs(c,s,d);
3   Update (all,c,COMM)(all,s,REG)(all,d,REG)
4       VCIF(c,s,d) = pcif(c,s,d) * qxs(c,s,d);

6   Formula (all,c,COMM)(all,s,REG)(all,d,REG)
7       MTAX(c,s,d) = VMSB(c,s,d) − VCIF(c,s,d);
8   Formula (all,d,REG)
9       TMTAX(d) = sum{c,COMM, sum{s,REG, MTAX(c,s,d)}};

11  Equation E_pmds
12  # links basic domestic import prices and CIF import prices #
13  (all,c,COMM)(all,s,REG)(all,d,REG)
14      pmds(c,s,d) = pcif(c,s,d) + tm(c,d) + tms(c,s,d);

16  Equation E_del_taxrimp
17  # change in ratio of import tax payments to regional income #
18  (all,d,REG)
19      100.0 * INCOME(d) * del_taxrimp(d) + TMTAX(d) * y(d)
20          = sum{c,COMM, sum{s,REG, VMSB(c,s,d) * [tm(c,d) + tms(c,s,d)]
21          + MTAX(c,s,d) * [pcif(c,s,d) + qxs(c,s,d)]}};
```

The equation for regional income levels is given by:

$$Y = FY + RT \cdot Y$$

where RT is the share of all indirect taxes in income and FY is aggregate factor income (including income taxes). Converting this into ordinary change, we have:

$$\Delta Y = \Delta FY + Y \cdot \Delta RT + RT \cdot \Delta Y$$

which can be converted to variables in percent differences:

$$Y \cdot \dot{y} = FY \cdot \dot{fy} + 100 \cdot Y \cdot \Delta RT + REVT \cdot \dot{y}$$

where $REVT = RT \cdot Y$ is total revenues from indirect taxes. Note again that the variable ΔRT is multiplied by 100 to convert to a percent. See equation `E_y` in the TABLO code.

A.0.3 Top level utility function of the representative household

Regional income is given by Y in nominal terms.[4] Income is allocated across i expenditure components such as to maximize total utility:

$$\max_{U_i} U = A \prod_i U_i^{\beta_i} \text{ subject to } \sum_i Y_i(U_i) = Y \tag{1}$$

---

[4]This section has been adapted from McDougall (2003).

where U_i represents the sub-utility derived from expenditure Y_i on expenditure component i. The β_i parameters are the Cobb-Douglas preference parameters and A is a scale parameter. Solving this for Y_i, yields the following equation:

$$Y_i = \frac{\beta_i}{\varphi_i} \frac{U}{\lambda} \tag{2}$$

where λ is the Lagrangian multiplier from the constrained optimization and φ_i is the elasticity of expenditure on commodity i with respect to sub-utility i:

$$\varphi_i = \frac{\partial Y_i}{\partial U_i} \frac{U_i}{Y_i} \tag{3}$$

Taking the sum of equation (2) and isolating λ, we have:

$$\lambda = \frac{U}{Y} \sum_j \frac{\beta_j}{\varphi_j}$$

and re-inserting back into equation (2) we derive:

$$Y_i = \frac{\beta_i}{\varphi_i} \frac{Y}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{4}$$

or

$$S_i = \frac{Y_i}{Y} = \frac{\beta_i}{\varphi_i} \frac{1}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{5}$$

where S_i is the expenditure share of component i.

We can also derive a relation between the elasticity of total expenditure to total utility, as a function of the sub-utility expenditure elasticities.

$$\Phi = \frac{\partial Y}{\partial U} \frac{U}{Y} = \frac{U}{Y} \sum_i \frac{\partial Y_i}{\partial U_i} \frac{\partial U_i}{\partial U} \tag{6}$$

This can be shown to be:[5]

$$\Phi = \frac{1}{\sum_j \frac{\beta_j}{\varphi_j}} \tag{7}$$

Inserting this expression back into equation (5) we get another expression for the budget shares:

$$S_i = \frac{Y_i}{Y} = \frac{\beta_i}{\varphi_i} \Phi \tag{8}$$

In log-differentiated form, this yields:

$$\dot{y}_i - \dot{y} = \dot{\Phi} + \dot{\beta}_i - \dot{\varphi}_i \tag{9}$$

Equation (7) can also be log-differentiated to yield:

$$\dot{\Phi} = \sum_i S_i \dot{\varphi}_i - \sum_i S_i \dot{\beta}_i \tag{10}$$

---

[5]See McDougall (2003).

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The second term on the right-hand side is the weighted sum of the percent change in the preference parameters. In most simulations preferences will be unchanged and thus this expression is typically zero. In the model implementation it is replaced by a variable, say χ, and thus we write:

$$\dot{\Phi} = \sum_i S_i \dot{\varphi}_i - \chi \qquad (11)$$

and add an equation for χ:[6]

$$\chi = \sum_i S_i \dot{\beta}_i \qquad (12)$$

In the GTAP model, there are three broad expenditure components at the top-level—private (P), public (or government, G) and saving (S). Public expenditures and saving are associated with a generic CES sub-utility function.[7] We know that for a CES utility function, the expenditure function can be written as $Y = U \times P$, where P is the aggregate price index. Thus clearly the expenditure elasticity is equal to 1, and in percentage change terms it is 0. We thus need to derive an expression for the elasticity of private expenditure with respect to private sub-utility. Hanoch shows that in the case of the CDE, the expression has the following form:

$$\varphi_p = \sum_c S_c^p e_c$$

where S_c^p represents the budget share of commodity c and e_c is the so-called expansion parameter of the CDE utility function (see following section). Log-differentiating this expression leads to:

$$\dot{\varphi}_p = \frac{\sum_c \dot{s}_c^p S_c^p e_c}{\varphi_p}$$

The differentiated share variable can be replaced by its components to yield:

$$\dot{\varphi}_p = \sum_c S_c^e (\dot{q}_c + \dot{p}_c - \dot{y}) \qquad (13)$$

where q_c is private purchases of commodity c and p_c is its price. The 'share' parameter s^e is given by the following expression:

$$S_c^e = \frac{S_c^p e_c}{\varphi_p}$$

We finally add two more results that are required for implementation. By log-differentiating the top level utility expression, equation (1), we derive the following expression:

$$\dot{u} = \dot{a} + \sum_i \beta_i \dot{u}_i + \sum_i \beta_i \ln(U_i) \dot{\beta}_i \qquad (14)$$

It can be shown that the following holds:

$$\sum_i \beta_i \dot{u}_i = \frac{\dot{y} - \dot{p}}{\Phi}$$

---

[6]Note that χ is not a percent change in a specific variable, it simply represents the weighted average of the percent change in the preference parameters.

[7]Saving almost trivially since it is a scalar.

We start with the following formula that expresses the change in sub-utility as a function in the change in expenditure:[8]

$$\dot{u}_i = \frac{\dot{y}_i - \dot{p}_i}{\varphi_i}$$

in which we replace $\dot{y}_i$ with expression (9), thus:

$$
\begin{aligned}
\sum_i \beta_i \dot{u}_i &= \sum_i \frac{\beta_i}{\varphi_i} \left(\dot{y} + \dot{\Phi} + \dot{\beta}_i - \dot{\varphi}_i - \dot{p}_i \right) \\
&= \frac{\dot{y}}{\Phi} + \frac{\dot{\Phi}}{\Phi} + \sum_i \frac{\beta_i}{\varphi_i} \left(\dot{\beta}_i - \dot{\varphi}_i - \dot{p}_i \right) \\
&= \frac{\dot{y}}{\Phi} + \frac{\dot{\Phi}}{\Phi} - \sum_i \frac{S_i}{\Phi} \left(\dot{\varphi}_i - \dot{\beta}_i \right) - \sum_i \frac{S_i}{\Phi} \dot{p}_i \\
&= \frac{\dot{y} - \dot{p}}{\Phi}
\end{aligned}
$$

The second line uses the expression for Φ, the third line replaces the ratio of β/φ with expression (8) and the fourth line uses the definition of $\dot{\Phi}$ to cancel out the middle terms.

With this substitution, we get the final expression for the top-level utility function:

$$\dot{u} = \dot{a} + \frac{\dot{y} - \dot{p}}{\Phi} + \sum_i \beta_i \ln\left(U_i\right) \dot{\beta}_i \tag{15}$$

where we define the aggregate price index—in percent change form—using the following expression.

$$\dot{p} = \sum_i S_i \dot{p}_i \tag{16}$$

Thus implementation of the top-level utility function relies on equations (9), (11), (11), (12), (15) and (16). Listing these out, we have the following set of equations, where we use the fact that $\dot{\varphi}_G = \dot{\varphi}_S = 0$:[9]

$$
\begin{aligned}
\dot{y}_P &= \dot{y} + \dot{\Phi} + \dot{\beta}_P - \dot{\varphi}_P \\
\dot{y}_G &= \dot{y} + \dot{\Phi} + \dot{\beta}_G \\
\dot{y}_S &= \dot{y} + \dot{\Phi} + \dot{\beta}_S \\
\dot{\Phi} &= S_P \dot{\varphi}_P - \chi \\
\chi &= S_P \dot{\beta}_P + S_G \dot{\beta}_G + S_S \dot{\beta}_S \\
\dot{\varphi}_P &= \sum_c S_c^e \left(\dot{q}_c + \dot{p}_c - \dot{y}_P \right) \\
\dot{u} &= \dot{a} + \frac{\dot{y} - \dot{p}}{\Phi} + \sum_i \beta_i \ln\left(U_i\right) \dot{\beta}_i \\
\dot{p} &= S_P \dot{P}_P + S_G \dot{P}_G + S_S \dot{P}_S
\end{aligned}
$$

In terms of the model implementation, Table A.2 provides a link between the generic variables and coefficients described above and those implemented in the model. The remaining details of the implementation in the model, such as the updating formulas for the coefficients are available in the TABLO code.[10]

---

[8]See equations (32) and (38) and a similar expression can be formulated for the utility of saving.

[9]The formulas simplify considerably in the case of fixed preference parameters.

[10]This refers specifically to updating of the share coefficients, the sub-utilities in levels and the Φ coefficient.

Table A.2: **Main variables and coefficients in utility module**

Generic	Model	Description
$\dot{y}_P$	yp	Aggregate private expenditures
$\dot{y}_G$	yg	Aggregate government expenditures
$\dot{y}_S$	qsave, psave	Aggregate savings
$\dot{\Phi}$	uelas	Elasticity of income wrt to utility
χ	dpav	Average shift in preference parameters
$\dot{\varphi}_P$	uepriv	Elasticity of private expenditures wrt to private utility
$\dot{u}$	u	Aggregate utility
$\dot{p}$	p	Aggregate price index
$\dot{a}$	au	Uniform shifter in utility function
$\dot{\beta}_P$	dppriv	Private consumption distribution parameter
$\dot{\beta}_G$	dpgov	Public consumption distribution parameter
$\dot{\beta}_S$	dpsave	Savings distribution parameter
Φ	UTILELAS	Elasticity of income wrt to utility
β_P	DPARPRIV	Private consumption distribution parameter
β_G	DPARGOV	Public consumption distribution parameter
β_S	DPARSAVE	Saving distribution parameter
S_P	XSHRPRIV	Private consumption budget share
S_G	XSHRGOV	Public consumption budget share
S_S	XSHRSAVE	Saving budget share

A.0.4 Constant Difference of Elasticities

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods and for non-homotheticity.[11] The starting point is an implicitly additive indirect utility function (see Hanoch (1975)) from which we can derive demand using Roy's identity (and the implicit function theorem).

General form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$V(P, U, Y) = \sum_{i=1}^{n} \alpha_i U^{e_i b_i} \left(\frac{P_i}{Y} \right)^{b_i} \equiv 1 \tag{17}$$

where P is the vector of commodity prices, U is (per capita) utility and Y is (per capita) income. Using Roy's identity and the implicit function theorem[12] we can derive demand, X, where v is the indirect utility function (defined implicitly):

$$X_i = -\frac{\partial v}{\partial P_i} \Big/ \frac{\partial v}{\partial Y} = -\left(\frac{\partial V}{\partial P_i} \Big/ \frac{\partial V}{\partial U} \right) \Big/ \left(\frac{\partial V}{\partial Y} \Big/ \frac{\partial V}{\partial U} \right) = -\left(\frac{\partial V}{\partial P_i} \Big/ \frac{\partial V}{\partial Y} \right) \tag{18}$$

This then leads to the following demand function:

---

[11] More detailed descriptions of the CDE can be found in Hertel et al. (1991), Surry (1993) and Hertel (1997).

[12] See Varian (1992), p. 106.

$$X_i = \frac{\alpha_i b_i U^{e_i b_i} \left(\dfrac{P_i}{Y}\right)^{b_i - 1}}{\sum\limits_j \alpha_j b_j U^{e_j b_j} \left(\dfrac{P_j}{Y}\right)^{b_j}} \tag{19}$$

Implementation is easier if we define the following variable:

$$Z_i = \alpha_i b_i U^{e_i b_i} \left(\frac{P_i}{Y}\right)^{b_i} \tag{20}$$

Then the budget shares can be expressed as:

$$S_i = \frac{P_i X_i}{Y} = \frac{Z_i}{\sum_j Z_j} \tag{21}$$

and the demand expression is:

$$X_i = \frac{S_i}{P_i} Y \tag{22}$$

Implementation also requires evaluating U. This can be done by implementing equation (17) and inserting the expression for Z:

$$\sum_{i=1}^{n} \frac{Z_i}{b_i} \equiv 1 \tag{23}$$

Elasticities

In this section we derive the price and income elasticities for the CDE function. These formulas will be needed to implement the CDE in percentage change form.[13]

The own-price elasticity is given by the following:

$$\varepsilon_i = \frac{\partial X_i}{\partial P_i} \frac{P_i}{X_i} = b_i \left(1 - S_i\right) - 1 - S_i \left[e_i b_i - \sum_j S_j e_j b_j\right] \bigg/ \sum_j S_j b_j \tag{24}$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

$$\frac{\partial U}{\partial P_i} \frac{P_i}{U} = -\frac{P_i}{U} \left(\frac{\partial V}{\partial P_i}\right) \bigg/ \left(\frac{\partial V}{\partial U}\right) = -\frac{S_i}{\sum_j S_j e_j} \tag{25}$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters, e, is close to unity.

Letting $\sigma_i = 1 - b_i$ (or $b_i = 1 - \sigma_i$), we can also write:

---

[13]The formulas can also be used to calibrate the CDE function in levels given a set of initial budget shares and income and price elasticities.

$$\varepsilon_i = S_i \left[\sigma_i - \frac{e_i \left(1 - \sigma_i\right)}{\sum\limits_j S_j e_j} - \frac{\sum\limits_j S_j e_j \sigma_j}{\sum\limits_j S_j e_j} \right] - \sigma_i \tag{26}$$

The derivation of the cross elasticities is almost identical and is not carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes the following form where we use the Kronecker product, δ:[14]

$$\varepsilon_{ij} = S_j \left[-b_j - \frac{e_i b_i}{\sum\limits_k S_k e_k} + \frac{\sum\limits_k S_k e_k b_k}{\sum\limits_k S_k e_k} \right] + \delta_{ij} \left(b_i - 1\right) \tag{27}$$

Again, we replace b by $1 - \sigma$, to get:

$$\varepsilon_{ij} = S_j \left[\sigma_j - \frac{e_i \left(1 - \sigma_i\right)}{\sum\limits_k S_k e_k} - \frac{\sum\limits_k S_k e_k \sigma_k}{\sum\limits_k S_k e_k} \right] - \delta_{ij} \sigma_i \tag{28}$$

The income elasticities are derived in a similar fashion:

$$\eta_i = \frac{\partial X_i}{\partial Y} \frac{Y}{X_i} = \frac{1}{\sum\limits_k S_k e_k} \left[e_i b_i - \sum_k S_k e_k b_k \right] - \left(b_i - 1\right) + \sum_k S_k b_k \tag{29}$$

For this, we need the elasticity of utility with respect to income:

$$\frac{\partial U}{\partial U} \frac{Y}{U} = -\frac{Y}{U} \left(\frac{\partial V}{\partial Y} \right) \bigg/ \left(\frac{\partial V}{\partial U} \right) = \frac{1}{\sum\limits_k S_k e_k} \tag{30}$$

Replacing b with $1 - \sigma$, equation (29) can be re-written to be:

$$\eta_i = \frac{1}{\sum\limits_k S_k e_k} \left[e_i \left(1 - \sigma_i\right) + \sum_k S_k e_k b_k \right] + \sigma_i - \sum_k S_k \sigma_k \tag{31}$$

From the Slutsky equation, we can calculate the compensated demand elasticities:

$$\xi_{ij} = \varepsilon_{ij} + S_j \eta_i = -\delta_{ij} \sigma_i + S_j \left[\sigma_j + \sigma_i - \sum_k S_k \sigma_k \right] \tag{32}$$

The cross-Allen partial elasticities are equal to the compensated demand elasticities divided by the share:

$$\sigma_{ij}^a = \sigma_j + \sigma_i - \sum_k S_k \sigma_k - \delta_{ij} \sigma_i / S_j \tag{33}$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities*.

$$\sigma_{ij}^a - \sigma_{il}^a = \sigma_j - \sigma_l \tag{34}$$

---

[14]δ takes the value of 1 along the diagonal (i.e., when $i = j$) and the value 0 off-diagonal (i.e., when $i \neq j$).

CDE in first differences

It is useful to decompose changes in demand using a linearized version of the demand function, and that which is used in the standard GEMPACK version of the CDE function. We begin with differentiating the expression for Z_i above, this will prove useful below:

$$\dot{z}_i = e_i b_i \dot{u} + b_i \dot{p}_i - b_i \dot{y} \iff \sum_k \frac{S_k}{b_k} \dot{z}_k = \dot{u} \sum_k S_k e_k + \sum_k S_k \dot{p}_k - \dot{y}$$

We can also differentiate the implicit utility function defined in terms of Z_i:

$$\sum_j \Delta Z_j / b_j = \sum_j Z_j \dot{z}_j / b_j = \sum_j \left[\sum_k Z_k \right] S_j \dot{z}_j / b_j \equiv 0 \iff \sum_j S_j \dot{z}_j / b_j \equiv 0$$

Merging the two expressions above we can derive an expression for $\dot{u}$ in terms of $\dot{y}$ and $\dot{p}_i$:

$$\dot{u} = \left[\dot{y} - \sum_k S_k \dot{p}_k \right] / \sum_k S_k e_k$$

We now proceed to insert the expression for $\dot{z}$ from above into the percent change for the budget shares and finally into the percent change for the demand function:

$$\dot{s}_i = \dot{z}_i - \sum_j S_j \dot{z}_j \Rightarrow \dot{x}_i = \dot{z}_i - \sum_j S_j \dot{z}_j + \dot{y} - \dot{p}_i$$

After substitution we derive:

$$\dot{x}_i = e_i b_i \dot{u} + b_i \dot{p}_i - b_i \dot{y} - \sum_j [S_j e_j b_j \dot{u} + b_j \dot{p}_j - b_j \dot{y}] + \dot{y} - \dot{p}_i$$

We then insert the expression for $\dot{u}$ from above and collect terms to get:

$$
\begin{aligned}
\dot{x}_i &= \dot{y} \left[1 - b_i + \sum_k S_k b_k + \frac{e_i b_i - \sum_k S_k e_k b_k}{\sum_k S_k e_k b_k} \right] \\
&+ \sum_j \left\{ S_j \left[-b_j - \frac{e_i b_i}{\sum_k S_k e_k} + \frac{\sum_k S_k e_k b_k}{\sum_k S_k e_k} \right] \dot{p}_j \right\} + (b_i - 1) \dot{p}_i
\end{aligned}
$$

We can identity the terms for $\dot{y}$ and $\dot{p}$ as the income and price elasticities respectively and thus we have:

$$\dot{x}_i = \eta_i \dot{y} + \sum_j \varepsilon_{ij} \dot{p}_j \tag{35}$$

Implementation in GTAP

Equation 35 is implemented in the GEMPACK version of GTAP as the following:

$$qpa_{c,r} - pop_r = \sum_k EP_{c,k,r} ppa_{k,r} + EY_{c,r} (yp_r - pop_r)$$

where qpa is aggregate private demand (hence the population correction), $ppa_{k,r}$ is the vector of consumer prices and $yp_r - pop_r$ represents per capita total expenditure. The coefficients EP and EY represent respectively the price and income elasticities. The latter are updated at each iteration using the formulas from above and the intermediate values of the budget shares. Instead of using directly the price elasticity formula, the implementation uses the simpler expression for the Allen partial elasticity that is held in the coefficent APE. The implementation then uses the coefficients EY and APE to calculate the matrix of price elasticities using $\varepsilon_{ij} = S_j(\sigma_{ij}^a - \eta_i)$. The current implementation does not use Kronecker's δ and instead the formula for APE is specified twice— once for all (c, k) combinations and a second time for (c, c). In the formulas, the b parameters are represented by $SUBPAR$ and the e parameters are represented by the $INCPAR$. Finally, the elasticity formulas are expressed in terms of the σ parameter from above that is represented by the coefficient $ALPHA$, where $ALPHA = 1 - SUBPAR$.

The model implementation also requires an expression for utility. Above we derived a key relation between utility, income and prices:

$$\dot{u} = \left[\dot{y} - \sum_k S_k \dot{p}_k \right] / \sum_k S_k e_k$$

This is implemented in the model where the expression $\sum_k S_k \dot{p}_k$ is replaced by the variable $ppriv$ that represents the percent change in the consumer price index, and $\dot{y}$ is replaced with per capita expenditure, i.e., $yp_r - pop_r$. Finally, the denominator, that represents the share weighted sum of the expansion parameters, is updated using the coefficient $UELASPRIV$.

Appendix B

Accounting relations

B.0.1   Introduction

This appendix outlines the key accounting identities in the GTAP database and its correspondence in the GTAP model. The organizing framework is a Social Accounting Matrix—an international standard for preparing and presenting national accounts.[1] The following list describes some of the key accounting extensions relative to the 'classic' version of the GTAP model:

- The new standard version of the model has the capability of having a non-diagonal 'make' matrix. Among other things, this implies that there is no longer a one-to-one correspondence between activities and commodities.

- The production tax vector has been converted to an $a \times c$ matrix and the tax is applied at the individual product level—not the aggregate output level. Note that the data is still only being compiled at the activity level and thus the rates will be uniform across commodities supplied by the same activity, though not necessarily after an aggregation of the database.

- Output and export taxes are now evaluated relative to basic prices and not so-called market prices. Thus tax rates will be positive and subsidy rates are negative.

- The income tax vector has been converted to an $a \times e$ matrix, thus taxes on factor remuneration are activity as well as endowment specific. For the moment, the tax rates are uniform across activities, though not necessarily after an aggregation of the database.

- Investment expenditures (or the formation of capital) has been extracted from 'production' activities. Thus variables such as *QFD* no longer have a 'CGDS' activity, which is now included as a separate final demand account with names such as *QID*.

- Separate variable names have been introduced for endowments (or factors of production). In GTAP 'classic', variables such as *PM* and *QO* were defined for both produced commodities (including 'CGDS') and endowments.

- All endowments in the 'classic' version had an aggregate level and price. Only sluggish variables had an activity specific return (*PMES*). To facilitate a number of new expressions, all endowments have an activity specific return—in the case of mobile factors, percent changes in the activity specific return will equal the percent change in the aggregate return. The new

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[1]See Pyatt and Round (1985) and Reinert and Roland-Holst (1997). McDonald and Thierfelder (2004) provided an earlier exposition of the GTAP accounting framework in a SAM format.

model also introduces an activity-specific endowment (natural resources). This implies that we no longer necessarily have aggregate endowment levels and prices—an additional reason to treat endowments as activity specific in most expressions.

B.0.2 Accounting relations in new Standard Model

This section describes the accounting framework for the new standard version of the GTAP database and model. Table B.1 shows the entire SAM using the header arrays from the GTAP database and thus provides an overall view of the accounts for each region/country. Below are some additional notes on the SAM accounting framework:

- The SAM deviates somewhat from a standard SAM. The number of columns is not equal to the number of rows. The indirect and direct tax columns have been collapsed into a single column, labeled $TTAX$, that represents all of the tax revenues collected (and provided to the regional household).

- The database hews to the previous practice of providing all value flows as pre- and post-tax matrices, i.e. the revenues derived from taxes (and subsidies) are derived via residual. The new standard database also introduces a new harmonization rule and that is that all taxes are assessed relative to what is now called 'basic' prices and thus tax revenues will be evaluated as positive flows (and subsidies as negative flows). This affects only two of the 'classic' taxes—taxes on domestic output and taxes on exports.

- The value of output at suppliers' prices is given by *(VOS)*, formerly *VOA*.

- The domestic supply column ('DOM') has the possibility of having a non-diagonal 'make' matrix—the cell identified as `MAKES`, the 'make' matrix at suppliers' prices. In fact, the 'make' matrix is no longer required to be square, it has the dimensions of $a \times c$.

- The `PTAX` matrix has the same dimensionality as the `MAKE` matrix, i.e. taxes on domestic supply are both activity- and commodity-specific. As well, since all taxes are evaluated relative to basic prices, the `PTAX` matrix will be positive for taxes. In the 'classic' SAM, the `PTAX` matrix was represented by the vector `OSEP`, which would enter as `-OSEP` instead of the current `PTAX` matrix.

- Export taxes (`XTRV`), derived from the difference between exports at border prices and basic prices is now a positive matrix. Note that the relevant basic price is PDS, which by assumption is uniform across destination markets. The export tax is bilateral and hence the border price, $PFOB$, is bilateral.

- The parameter `VOSB` corresponds to the parameter `VOM` from GTAP 'classic', i.e. the value of output gross of the output tax (i.e. `VOM = VOA - OSEP`).

- Income taxes are now both activity- and endowment-specific. In the SAM they are captured as a single row with one column for each of the endowments. The formula for each cell of the row vector is therefore:

$$\sum_a \left(TINC_{e,a} - 1 \right) PES_{e,a} QFE_{e,a}$$

and the term `EVFB-EVOS` is short-hand for the sum over activities, as the individual matrices are indexed by a and e.

- **TTAX** represents the sum of taxes on domestic sales (**DTAX**), taxes on import sales (**ITAX**), taxes on factor-use (**ETAX**), taxes on domestic output (**PTAX**), import tariffs (**MTAX**), export taxes (**XTAX**) and income taxes (**YTAX**).

- Household income is derived from the sum of after tax factor remuneration (**EVOS**), net of depreciation, and total taxes (**TTAX**). It is allocated across three categories of expenditures: private expenditures (**PRIVEXP**), public expenditures (**GOVEXP**) and **SAVE**. The latter is part of the database. The former are derived from the expenditure vectors **VDPP**, **VMPP**, **VDGP** and **VMGP**. Thus one test of data consistency is to assess the consistency of regional household income with its disposition.

- The investment expenditure vectors **VDIB**, **VMIB**, **VDIP**, **VMIP** are new to the database. Investment was derived from the 'CGDS' column of firms' expenditures in GTAP 'classic'.

- The balance of payments account is another measure of the consistency of the database. The scalar **SAVF** is not part of the database, but can be caclulated by residual, i.e. the difference between total exports, including margin exports, and total imports. At the global level, the sum of **SAVF** across regions should be zero.

Table B.1: **Social accounting scheme for GTAP Data Base**

	ACT	DOM	IMP	VA	TTAX	REGH	PRIV	GOV	INV	DEPR	ITT	TRADE	ROW	Total
Activities		MAKES												VOS
Domestic commodities	VDFB						VDPB	VDGB	VDIB		VST	VFOB		VOSB+XTAX
Imported commodities	VMFB						VMPB	VMGB	VMIB					VMSB
Value added	EVFB													EVFB
Tax on dom. commodities	VDFP-VDFB						VDPP-VDPB	VDGP-VDGB	VDIP-VDIB					DTAX
Tax on imp. commodities	VMFP-VMFB						VMPP-VMPB	VMGP-VMGB	VMIP-VMIB					ITAX
Tax on factor use	EVFP-EVFB													ETAX
Production tax		MAKEB-MAKES												PTAX
Import tax			VMSB-VCIF											MTAX
Export tax		VFOB-VXSB												XTAX
Income tax				EVFB-EVOS										YTAX
Regional household				EVOS	TTAX									REGY
Private expenditures						PRIVE.								PRIVEXP
Public expenditures						GOVEX								GOVEXP
Investment expenditures						SAVE				VDEP			SAVF	EINV
Depreciation allowance						VDEP								VDEP
Margin exports												VST		VST
Trade			VCIF									VFOB-VCIF		VFOB
Balance of payments														0
Total	VOS	VOSB+XT	VMSB	EVFB	TTAX	REGY	PRIVE.	GOVEX	EINV	VDEP	VST	VFOB	0	

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Tables B.2 and B.3 provide a concordance between the matrices and vectors in the database (the so-called header arrays) and the corresponding expressions in levels using the model variables. They can also be linked to the entries in the SAM, thus there is a three-way correspondence between the header arrays in the database, the entries in the SAM, and the model variables. It would be possible to re-construct the SAM post-simulation using the formulas in these tables.[2]

We conclude this section with additional figures highlighting the linkages in the model. The price linkages were described in section 2 and depicted in Figure 2.2. Below are two additional figures, one for the quantity linkages (Figure B.1) and the other for the value linkages (Figure B.2).
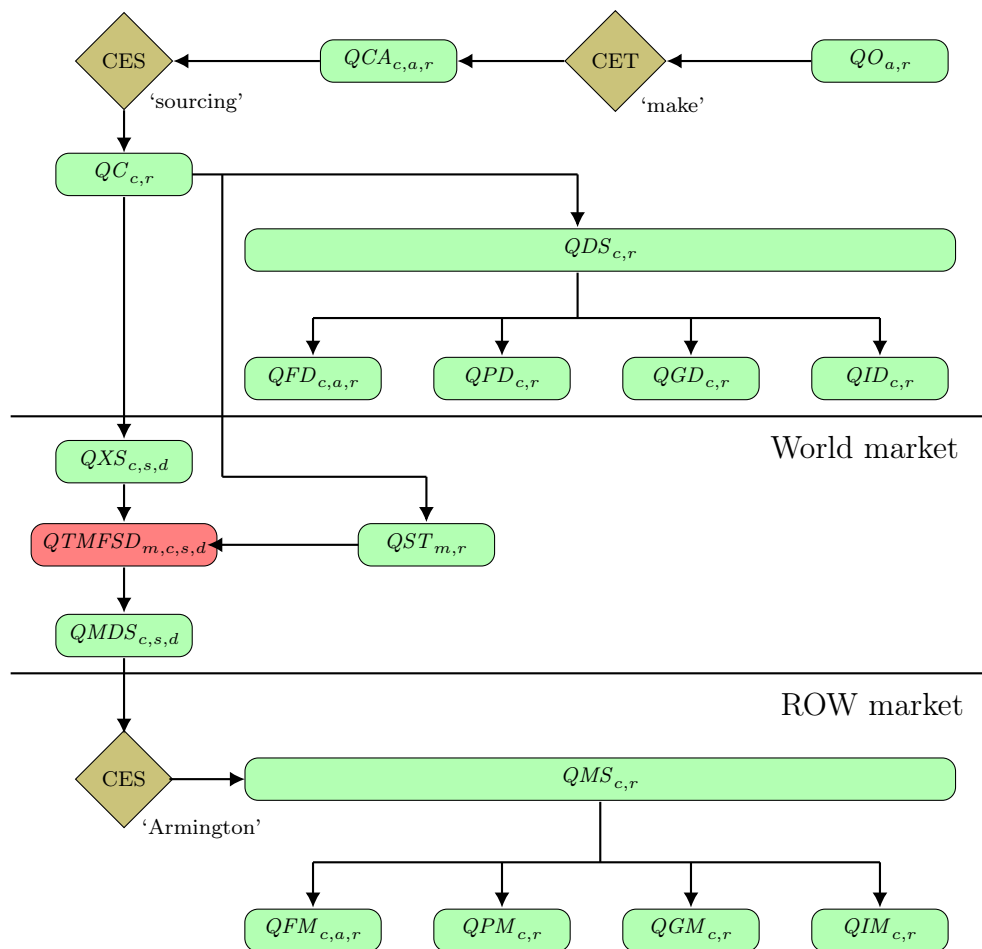

Figure B.1: **Quantity linkages in the model**

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[2]The foreign saving component can be derived from the `TBAL` coefficient.

Table B.2: **Supply relationships**

Row description	Data table name	Expression
Firm demand for dom. commodities at basic prices	`VDFB(COMM,ACTS)`	$PDS_{c,r}\,QFD_{c,a,r}$
Firm demand for imp. commodities at basic prices	`VMFB(COMM,ACTS)`	$PMS_{c,r}\,QFM_{c,a,r}$
Factor payments at basic prices	`EVFB(ENDW,ACTS)`	$PEB_{e,a,r}\,QFE_{e,a,r}$
Firm demand for dom. commodities at producer prices	`VDFP(COMM,ACTS)`	$TFD_{c,a,r}PDS_{c,r}\,QFD_{c,a,r} = PFD_{c,a,r}\,QFD_{c,a,r}$
Firm demand for imp. commodities at producer prices	`VMFP(COMM,ACTS)`	$TFM_{c,a,r}PMS_{c,r}\,QFM_{c,a,r} = PFM_{c,a,r}\,QFM_{c,a,r}$
Factor payments at producer prices	`EVFP(ENDW,ACTS)`	$TFE_{e,a,r}PEB_{e,a,r}\,QFE_{e,a,r} = PFE_{e,a,r}\,QFE_{e,a,r}$
Value of output at suppliers' prices	`VOS(ACTS) =` `sum(COMM,VDFP+VMFP)` `+ sum(ENDW,EVFP)`	$\begin{aligned} PO_{a,r}\,QO_{a,r} \;=\;& \sum_{c} PFD_{c,a,r}\,QFD_{c,a,r} \\ +\;& \sum_{c} PFM_{c,a,r}\,QFM_{c,a,r} \\ +\;& \sum_{e} PFE_{e,a,r}\,QFE_{e,a,r} \end{aligned}$
Value of domestic supply at suppliers' prices	`MAKES(ACTS,COMM)`	$PS_{c,a,r}\,QCA_{c,a,r}$
Value of domestic supply at basic prices	`MAKEB(ACTS,COMM)=` `MAKES(ACTS,COMM) +` `PTAX(ACTS,COMM)`	$PCA_{c,a,r}\,QCA_{c,a,r} = TO_{c,a,r}PS_{c,a,r}\,QCA_{c,a,r}$
Value of exports at basic prices	`VXSB(COMM,DEST)`	$PDS_{c,s}\,QXS_{c,s,d}$
Value of exports at world prices	`VFOB(COMM,DEST) =` `VXSB(COMM,DEST) +` `XTRV(COMM,DEST)`	$TXS_{c,s,d}PDS_{c,s}\,QXS_{c,s,d} = PFOB_{c,s,d}\,QXS_{c,s,d}$
Value of imports at world prices	`VCIF(COMM,SOURCE)`	$PCIF_{c,s,d}\,QXS_{c,s,d}$
Value of imports at domestic basic prices	`VMSB(COMM,SOURCE)=` `VCIF(COMM,SOURCE) +` `TFRV(COMM,SOURCE)`	$PMDS_{c,s,d}\,QXS_{c,s,d} = TMS_{c,s,d}PCIF_{c,s,d}\,QXS_{c,s,d}$

Table B.3: **Income and demand relations**

| Row description	Data table name	Expression
Household factor income	EVOS(ENDW,ACTS)	$PES_{e,a,r} QFD_{e,a,r}$
Household expenditures	PRIVEXP	YP_r
Government expenditures	GOVEXP	YG_r
Domestic savings	SAVE	$SAVE_r = PSAVE_r QSAVE_r$
Depreciation allowance	VDEP	$\delta_r PINV_r KB_r$
Household demand for dom. commodities at basic prices	VDPB(COMM)	$PDS_{c,r} QPD_{c,r}$
Household demand for imp. commodities at basic prices	VMPB(COMM)	$PMS_{c,r} QPM_{c,r}$
Household demand for dom. commodities at producer prices	VDPP(COMM)	$TPD_{c,r} PDS_{c,r} QPD_{c,r} = PPD_{c,r} QPD_{c,r}$
Household demand for imp. commodities at producer prices	VMPP(COMM)	$TPM_{c,r} PMS_{c,r} QPM_{c,r} = PPM_{c,r} QPM_{c,r}$
Gov. demand for dom. commodities at basic prices	VDGB(COMM)	$PDS_{c,r} QGD_{c,r}$
Gov. demand for imp. commodities at basic prices	VMGB(COMM)	$PMS_{c,r} QGM_{c,r}$
Gov. demand for dom. commodities at producer prices	VDGP(COMM)	$TGD_{c,r} PDS_{c,r} QGD_{c,r} = PGD_{c,r} QGD_{c,r}$
Gov. demand for imp. commodities at producer prices	VMGP(COMM)	$TGM_{c,r} PMS_{c,r} QGM_{c,r} = PGM_{c,r} QGM_{c,r}$
Inv. demand for dom. commodities at basic prices	VDIB(COMM)	$PDS_{c,r} QID_{c,r}$
Inv. demand for imp. commodities at basic prices	VMIB(COMM)	$PMS_{c,r} QIM_{c,r}$
Inv. demand for dom. commodities at producer prices	VDIP(COMM)	$TID_{c,r} PDS_{c,r} QID_{c,r} = PID_{c,r} QID_{c,r}$
Inv. demand for imp. commodities at producer prices	VMIP(COMM)	$TIM_{c,r} PMS_{c,r} QIM_{c,r} = PIM_{c,r} QIM_{c,r}$
Demand for margins exports	VST(MARG)	$PDS_{m,r} QST_{m,r}$
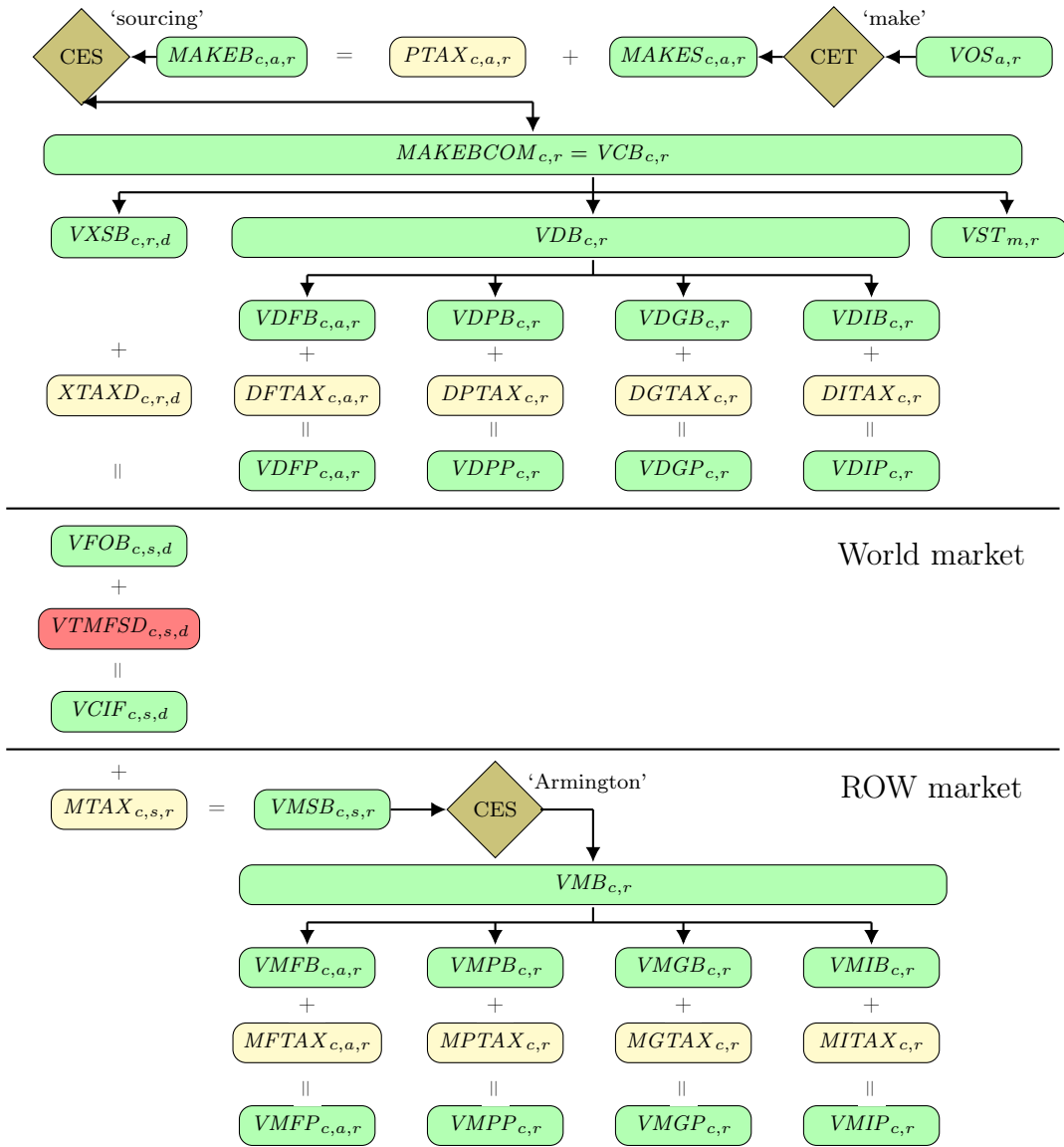
Figure B.2: **Value linkages in the model**