ASYMMETRIC ARMINGTON MODEL: METHOD AND APPLICATION

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## Abstract

To model two-way trade, most CGE models use the Armington assumption on the import side. In this paper, we extend the Armington assumption by introducing a distinction between responses to increases and decreases in the price of imports relative to domestic products; i.e., we assume that demand substitution possibilities are asymmetric. Specifically, for demanders, it is easier to substitute imports for domestic products than it is to substitute domestic products for imports. The paper presents the mathematical structure of the asymmetric Armington assumption and embeds it in a simple CGE model that is applied to a Mongolian dataset. A set of comparative-static simulations of terms of trade shocks with alternative assumptions for the elasticity of substitution between imports and domestic products are considered.

## 1. Introduction

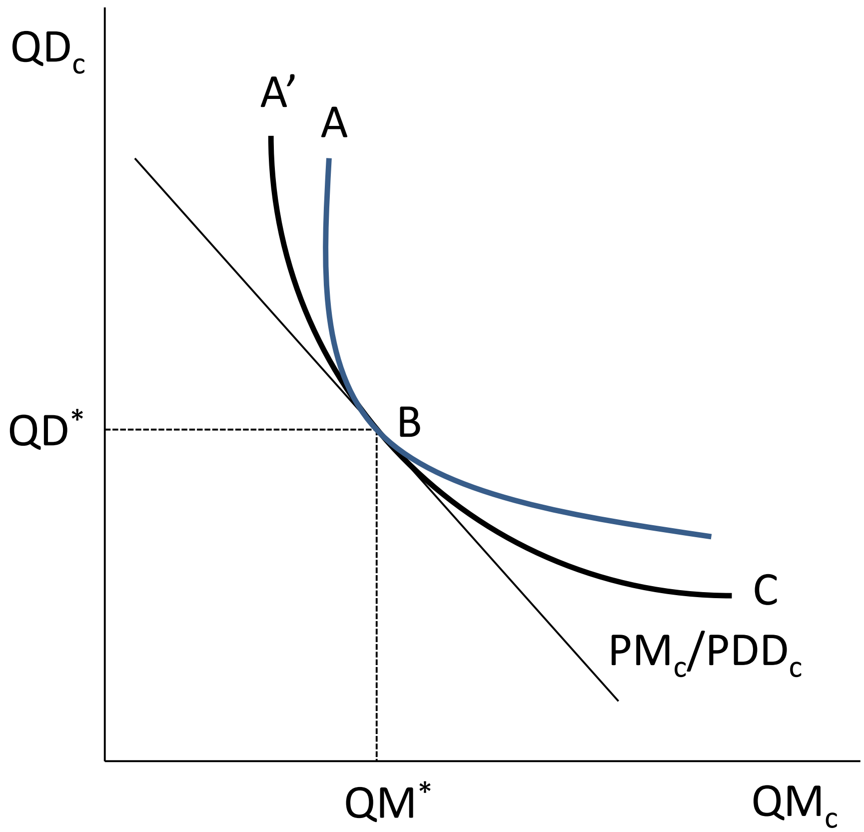
To model two-way trade (i.e., settings in which the same product is both exported and imported), most CGE models use the Armington assumption on the import side.[1] The Armington assumption distinguishes products by country of origin and assumes that a product produced in one country is an imperfect substitute for the same product produced in another country. Technically, it is assumed that domestic demanders minimize the cost of purchasing a composite of imports and domestic products subject to a constant-elasticity-of-substitution (CES) function. In their optimization decision, the domestic demanders take as given the prices they pay for these two products.

In this paper, we extend the Armington assumption by introducing a distinction between responses to increases and decreases in the price of imports relative to domestic products; i.e., we assume that demand substitution possibilities are asymmetric. Specifically, for demanders, it is easier to substitute imports for domestic products than it is to substitute domestic products for imports. This assumption is intuitively appealing: for any country it should be easier to find a close substitute for domestic products abroad (given the wide variety of products that are produced abroad) than it is to find a close substitute for imports among domestic products (given the relative narrow variety of products that are produced at home). Given that we work in a single country context, we focus on substitution between domestic products and imports and, not on substitution between imports from different countries. Empirically, this innovation may be important since the size and direction of real exchange rate adjustments and/or changes in balance of payments deficits caused by terms of trade shocks depend on these substitution elasticities. Certainly, whether imports (QM; priced at PM) and domestic products (QD; priced at PDD) are gross complements ($\sigma < 1$) or substitutes ($\sigma > 1$) in the context of a terms-of-trade shock is crucial for determining its impact. In Figure 1, we show the asymmetric Armington case as the indifference curve ABC while the symmetric case is represented by A'BC. In other words, the elasticity of substitution between domestic and imported products is higher

[1] It is named the Armington function after its originator (Armington 1969).

(lower) when the non-base PM-to-PDD ratio declines (increases) relative to the base PM-to-PDD.

*Figure 1.1. Asymmetric Armington indifference curve*




Source: Authors' elaboration.

## Literature Review

In the late 1960s and early 1970s, many theoretical articles proposed production functions with variable elasticities of substitution. Perhaps most importantly, Revankar (1971) and Lovell (1973) contributed to develop a generalization of the CES production function with the property of variable (non-constant) elasticity of substitution. Specifically, the VES (Variable Elasticity of Substitution) production function assumes that the elasticity of substitution is a function of the capital-to-labor ratio. More recently, production functions with a variable elasticity of substitution have been proposed in the context of growth models. For instance, Antony (2010) proposed a production function as an arbitrary spline of CES functions with different elasticities of substitution. In turn, Growiec and Mućk (2015) developed the Isoelastic Elasticity of Substitution production function by extending the normalized CES production function to allow for a varying elasticity of substitution as a function of four alternative determinants: relative factor shares, marginal rates of substitution, capital–labor ratios, or (iv) capital–output ratios. Overall, as opposed to our proposed asymmetric Armington model, the varying elasticity of substitution of the production functions developed in this literature is only indirectly linked to relative input prices.

It should be noted that the PM-PDD ratio may change for various reasons, including changes in the international prices of imports and the real exchange rate. In another strand of literature, authors such as Demian and di Mauro (2015) assess the impact of real exchange rate movements on exports; they find that exports are be sensitive to appreciation episodes but rather unaffected by depreciations.

[TBC]

## 2. Method

In summary, we extend an otherwise relatively simple single-country static CGE model by describing the (Armington) allocation of domestic demands between imports and domestic products with the equations stated in Table 2.1. To simplify, in the following mathematical presentation we omit the commodity index, which applies to all variables and parameters.

Together, equations (1)-(3) are used to impose $\sigma^{ql}$ or $\sigma^{qh}$ on $SIGMA^{Q}$. Figure 2.1 shows a graphical representation of equations (1)-(3). Equation (1) computes the positive (DEVP) or negative (DEVN) difference between the non-base (PM-PD ratio) and base (PM0-PD0 ratio) domestic price of imports. Equations (2a)-(2c) are used to impose the value of $\sigma^{ql}$ for $SIGMA^{Q}$.

If deviation between PM/PDD and PM0/PDD0 is positive (DEVP>0), then $SIGMA^{Q} = \sigma^{ql}$. Therefore, the elasticity of substitution is lower when there is an increase in the relative price of imports. Equations (3a)-(3c) are used to impose the value of $\sigma^{qh}$ for $SIGMA^{Q}$. If deviation between PM and PM0 is negative (DEVN>0), then $SIGMA^{Q} = \sigma^{qh}$. Therefore, the elasticity of substitution is higher when there is a decrease in the relative price of imports. Together, equations (4) and (5) determine the demand for domestic and imported commodities. Finally, equations (6)-(9) endogenously calibrate the parameters of the Armington CES function depending on the value of $SIGMA^{Q}$, the endogenous elasticity of substitution between domestic and imported commodities.

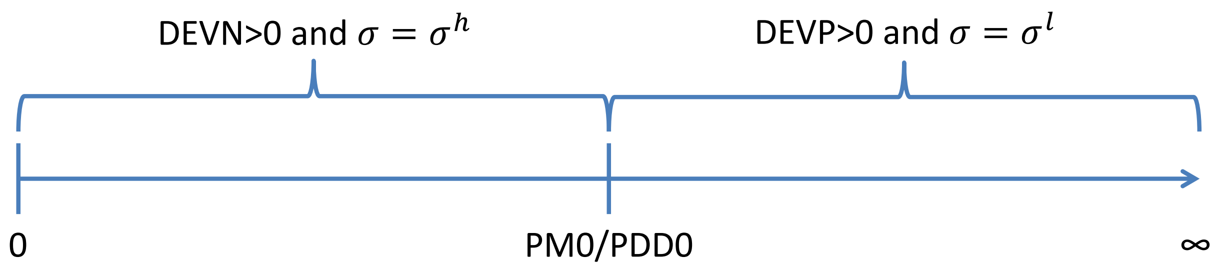
*Table 2.1. Equations for asymmetric Armington approach*

| | |
|---|---|
| (1) | $\frac{PM}{PDD} - DEVP + DEVN = \frac{PM^{0}}{PDD^{0}}$ |
| (2a) | $DEVP(SIGMA^{Q} - \sigma^{ql})$ |
| (2b) | $DEVP \geq 0$ |
| (2c) | $SIGMA^{Q} \geq \sigma^{ql}$ |
| (3a) | $DEVN(SIGMA^{Q} - \sigma^{qh})$ |
| (3b) | $DEVN \geq 0$ |
| (3c) | $SIGMA^{Q} \leq \sigma^{qh}$ |
| (4) | $QQ = PHI^{Q}\left(DELTA^{M} \cdot QM^{-RHO^{Q}} + DELTA^{DD} \cdot QD^{-RHO^{Q}}\right)^{\frac{-1}{RHO^{Q}}}$ |
| (5) | $\frac{QM}{QD} = \left(\frac{PDD}{PM} \cdot \frac{DELTA^{M}}{DELTA^{DD}}\right)^{SIGMA^{Q}}$ |
| (6) | $RHO^{Q} = \frac{1}{SIGMA^{Q}} - 1$ |
| (7) | $DELTA^{M} = \frac{PM \cdot QM^{\frac{1}{SIGMA^{Q}}}}{PM \cdot QM^{\frac{1}{SIGMA^{Q}}} + PDD \cdot QD^{\frac{1}{SIGMA^{Q}}}}$ |
| (8) | $DELTA^{DD} = \frac{PD \cdot QD^{\frac{1}{SIGMA^{Q}}}}{PM \cdot QM^{\frac{1}{SIGMA^{Q}}} + PDD \cdot QD^{\frac{1}{SIGMA^{Q}}}}$ |

| | |
|---|---|
| (9) | $PHI^{Q} = \frac{QQ}{\left(DELTA^{M} \cdot QM^{-RHO^{Q}} + DELTA^{DD} \cdot QD^{-RHO^{Q}}\right)^{\frac{-1}{RHO^{Q}}}}$ |

Source: Authors' elaboration.

*Figure 2.1. Segments for PM/PDD and their relation with SIGMA_Q*

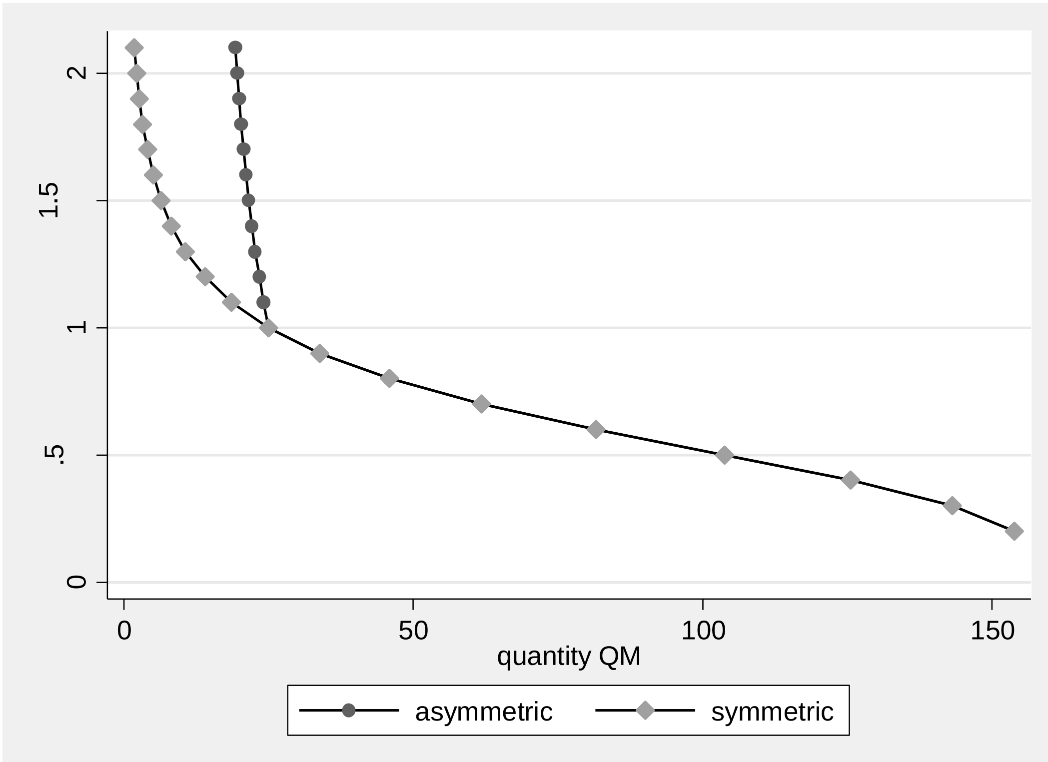



Source: Authors' elaboration.

## Partial Equilibrium Example

To test how this works, we first developed a simple partial equilibrium model to simulate changes in the world price of imports, from -80% to +110%. As shown in Figure 2.2, we introduce two demand curves, one for the symmetric (standard) case and one for the asymmetric case. The two curves are the same when PM decreases from its initial level of one. However, when PM increases, the curve for the asymmetric case is steeper than for the symmetric case: for the same increase in price, there is less import substitution (lower sigma) for the asymmetric case.

*Figure 2.2. Import demand curves with symmetric and asymmetric Armington approaches derived from simulations with partial equilibrium model*




Source: Authors' elaboration.

## 3. Illustrative Simulations

In this section, we develop a simple static CGE model extended with the same asymmetric Armington approach as we used for the partial equilibrium model. Our CGE model is typical of models of small open economies with optimizing behavior for households and producers, domestic markets for commodities and factors cleared by flexible prices and wages, respectively, and a government that consumes, saves, taxes, and both receives and pays transfers.[2] Below, we present the assumptions that are used in the simulations. The CGE model is applied to a 2015 dataset for Mongolia, including an aggregated version of the SAM described

[2] For a related model with a more detailed documentation, see Lofgren et al. (2002).

in Cicowiez and Lofgren (2018). It has 4 factors (labor, capital, land, and an extractive natural resource), 7 sectors (activities and commodities), and a single representative household. On the basis of SAM data, Table 3.1 summarizes the sectoral structure of the Mongolian economy in 2015: it shows sectoral shares in value-added (column VAshr), production (column PRDshr), employment (column EMPshr), exports and imports (columns EXPshr and IMPshr, respectively), as well as the split of domestic sectoral supplies between exports and domestic sales (column EXP-OUTshr), and domestic sectoral demands between imports and domestic output (column IMP-DEMshr). For instance, while (primary) agriculture represents a significant share of employment (around 27.8 percent), its shares of value added, production, and exports are much smaller (in the range of 4.8-14.5 percent). For mining, the output share that is exported amounts to 95.4 percent. The share of exports due to mining (around 67.6 percent) is far above its share in total value added (15.6 percent). On the imports side, machinery and equipment stands out in terms of its 91.9 percent ratio between imports and total demand. Besides, its share on total imports is 14.8 percent. Table 3.2 shows the elasticity values that are used. To model household consumption, we use a Cobb-Douglas utility function.

*Table 3.1: sectoral structure, Mongolia in 2015*

*(percent)*

| | VAshr | PRDshr | EMPshr | EXPshr | EXP-OUTshr | IMPshr | IMP-DEMshr |
|---|---|---|---|---|---|---|---|
| Agriculture | 14.51 | 10.52 | 27.84 | 4.83 | 10.27 | 0.81 | 2.31 |
| Mining | 15.58 | 16.86 | 3.44 | 67.56 | 95.39 | 0.11 | 3.64 |
| Food and beverages | 4.52 | 7.80 | 1.92 | 0.73 | 1.97 | 8.22 | 20.52 |
| Machinery and equipment | 0.11 | 0.13 | 0.05 | 0.06 | 9.66 | 14.78 | 91.90 |
| Other manufacturing | 5.54 | 6.70 | 5.47 | 13.07 | 40.19 | 50.82 | 70.65 |
| Other industry | 6.75 | 12.35 | 8.87 | 0.64 | 1.32 | 4.40 | 8.38 |
| Services | 52.99 | 45.63 | 52.41 | 13.11 | 7.28 | 20.84 | 10.84 |
| total | 100.00 | 100.00 | 100.00 | 100.00 | 23.51 | 100.00 | 26.71 |

Source: Authors' elaboration.

*Table 3.2: value added, trade, and consumption elasticities*

| | VA | Armignton low | Armington high | CET | LES* |
|---|---|---|---|---|---|
| Agriculture | 0.25 | 0.90 | 4.00 | 4.00 | 1.00 |
| Mining | 0.20 | 0.90 | 4.00 | 4.00 | 1.00 |
| Food and beverages | 0.95 | 0.90 | 4.00 | 4.00 | 1.00 |
| Machinery and equipment | 0.95 | 0.90 | 4.00 | 4.00 | 1.00 |
| Other manufacturing | 0.95 | 0.90 | 4.00 | 4.00 | 1.00 |
| Other industry | 0.95 | 0.90 | 4.00 | 4.00 | 1.00 |
| Services | 0.95 | 0.90 | 4.00 | 4.00 | 1.00 |
| *Utility function is Cobb-Douglas. | | | | | |

Source: Authors' elaboration.

## Scenarios

To test how the model works, we simulate a 25 percent increase and a 25 percent decrease in the world import price of machinery and equipment, one of Mongolia's main imports (see Table 2). We run these two scenarios under alternative assumptions for the Armington treatment – asymmetric, symmetric low, and symmetric high –and for the closure rules for the current account of the balance of payments. Table 3.3 summarizes the assumptions made in each of the 12 scenarios we consider.

*Table 3.3: definition of scenarios*

| Name | price shock | sigma low | sigma high | RoW closure* |
|---|---|---|---|---|
| asym+** | +25% | 0.90 | 3.00 | endog REXR |
| asym-** | -25% | 0.90 | 3.00 | endog REXR |
| syml+ | +25% | 0.90 | n.a. | endog REXR |
| syml- | -25% | 0.90 | n.a. | endog REXR |
| symh+ | +25% | n.a. | 3.00 | endog REXR |
| symh- | -25% | n.a. | 3.00 | endog REXR |
| asym2+** | +25% | 0.90 | 3.00 | endog SAVF |
| asym2-** | -25% | 0.90 | 3.00 | endog SAVF |
| syml2+ | +25% | 0.90 | n.a. | endog SAVF |
| syml2- | -25% | 0.90 | n.a. | endog SAVF |
| symh2+ | +25% | n.a. | 3.00 | endog SAVF |
| symh2- | -25% | n.a. | 3.00 | endog SAVF |

*REXR is real exchange rate and SAVF is foreign savings.

**Asymmetric Armington: endogenous selection of low or high sigma.

Source: Authors' elaboration.

## Results and Analysis

Table 3.4 shows the Armington elasticities (exogenously or endogenously) selected for each non-base scenario. Tables 3.5-3.7 and Figures 3.1 and 3.2 summarize the results for the simulations, covering both the macro and sector levels.

Notes:

- Overall, and independently of the Armington assumption made (i.e., symmetric vs. asymmetric), macro impacts are as expected; an increase (decrease) in the world price of imports has a negative (positive) effect on macro indicators such as private consumption.
- The elasticity selection for the targeted commodity is as expected: sigma_ql and sigma_qh are selected for machinery and equipment in the scenarios with increases (+) and decreases (-) in the PM-PDD ratios, respectively.
- Given GE effects, we do not see that all elasticities are low or high simultaneously. In other words, results for asym+ (asym-) and syml+ (symh-) are different.

*Table 3.4. Elasticities of substitution SIGMAQ*

| | asym+ | asym- | syml+ | syml- | symh+ | symh- |
|---|---|---|---|---|---|---|
| c-agr | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-min | 3.00 | 0.90 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-foodbevtob | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-macheq | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-othmnfc | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-othind | 0.90 | 0.90 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-svc | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |

*Table 3.4, cont. Elasticities of substitution SIGMAQ*

| | asym2+ | asym2- | syml2+ | syml2- | symh2+ | symh2- |
|---|---|---|---|---|---|---|
| c-agr | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-min | 3.00 | 0.90 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-foodbevtob | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-macheq | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-othmnfc | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-othind | 3.00 | 0.90 | 0.90 | 0.90 | 3.00 | 3.00 |
| c-svc | 0.90 | 3.00 | 0.90 | 0.90 | 3.00 | 3.00 |

*Table 3.5. Change in real macro aggregates (percent)*

| Item | base-year | asym+ | asym- | syml+ | syml- | symh+ | symh- |
|---|---|---|---|---|---|---|---|
| Absorption | 22,917,915 | -1.50 | 1.57 | -1.50 | 1.53 | -1.52 | 1.57 |
| PrvCon | 13,671,168 | -2.51 | 2.63 | -2.51 | 2.57 | -2.55 | 2.63 |
| Exports | 10,451,638 | 1.67 | -1.08 | 1.67 | -1.67 | 1.10 | -1.08 |
| Imports | -10,219,167 | -1.40 | 2.21 | -1.40 | 1.54 | -2.04 | 2.21 |
| GDPMP | 23,150,386 | -0.10 | 0.09 | -0.10 | 0.08 | -0.11 | 0.09 |
| NetIndTax | 1,979,032 | -1.00 | 1.15 | -1.00 | 1.05 | -1.08 | 1.15 |
| GDPFC | 21,171,354 | -0.01 | 0.00 | -0.01 | 0.00 | -0.01 | 0.00 |
| REXR | 1 | 1.56 | -1.05 | 1.56 | -1.58 | 1.05 | -1.05 |

*Table 3.5, cont. Change in real macro aggregates (percent)*

| Item | base-year | asym2+ | asym2- | syml2+ | syml2- | symh2+ | symh2- |
|---|---|---|---|---|---|---|---|
| Absorption | 22,917,915 | -0.11 | 0.26 | -0.14 | 0.18 | -0.19 | 0.23 |
| PrvCon | 13,671,168 | -0.19 | 0.43 | -0.24 | 0.30 | -0.32 | 0.38 |
| Exports | 10,451,638 | -0.08 | 0.07 | -0.08 | 0.07 | -0.07 | 0.07 |
| Imports | -10,219,167 | -0.23 | 0.55 | -0.28 | 0.38 | -0.39 | 0.48 |
| GDPMP | 23,150,386 | -0.05 | 0.05 | -0.05 | 0.05 | -0.05 | 0.04 |
| NetIndTax | 1,979,032 | -0.15 | 0.27 | -0.16 | 0.21 | -0.21 | 0.25 |
| GDPFC | 21,171,354 | -0.03 | 0.03 | -0.04 | 0.04 | -0.03 | 0.03 |
| REXR | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

*Table 3.6. Change in imports of machinery and equipment (QMXP) (percent)*

| | asym+ | asym- | syml+ | syml- | symh+ | symh- |
|---|---|---|---|---|---|---|
| c-agr | -2.40 | 5.00 | -2.40 | 2.45 | -3.80 | 3.94 |
| c-min | 0.26 | -0.06 | 0.15 | -0.14 | 0.16 | -0.15 |
| c-foodbevtob | -3.07 | 5.29 | -3.07 | 3.11 | -4.29 | 4.42 |
| c-macheq | -1.50 | 2.53 | -1.50 | 2.23 | -1.66 | 2.43 |
| c-othmnfc | -0.93 | 1.03 | -0.93 | 0.94 | -1.00 | 1.01 |
| c-othind | -0.26 | 0.04 | -0.26 | 0.30 | -0.17 | 0.22 |
| c-svc | -2.04 | 4.02 | -2.04 | 2.11 | -3.13 | 3.28 |

*Table 3.6, cont. Change in imports of machinery and equipment (QMXP) (percent)*

| | asym2+ | asym2- | syml2+ | syml2- | symh2+ | symh2- |
|---|---|---|---|---|---|---|
| c-agr | -0.08 | 0.52 | -0.12 | 0.10 | -0.30 | 0.28 |
| c-min | 0.01 | 0.05 | -0.04 | 0.03 | -0.02 | 0.02 |
| c-foodbevtob | -0.10 | 0.64 | -0.15 | 0.14 | -0.39 | 0.36 |
| c-macheq | -1.41 | 2.34 | -1.40 | 2.05 | -1.56 | 2.24 |
| c-othmnfc | -0.10 | 0.12 | -0.09 | 0.09 | -0.11 | 0.10 |
| c-othind | 1.68 | -0.48 | 0.49 | -0.49 | 1.13 | -1.12 |
| c-svc | -0.16 | 0.50 | -0.18 | 0.18 | -0.33 | 0.34 |

[TBC]

## References


Antony, J. 2010. A Class of Changing Elasticity of Substitution Production Functions. Journal of Economics 100 (2): 165–183.

Armington, P. S. 1969. A Theory of Demand for Products Distinguished by Place of Production. Staff Papers (International Monetary Fund) 16 (1): 159-178.

Cicowiez, M. and Lofgren H. 2018. Technical Note on the Construction of a Social Accounting Matrix for Mongolia 2015. Unpublished.

Growiec, J. and Mućk, J. 2015. Isoelastic Elasticity of Substitution Production Functions. Discussion Papers Department of Economics University of Copenhagen No. 15-13.

Lovell, C. A. K. 1973. CES and VES Production Functions in a Cross-Section Context. Journal of Political Economy 81 (3): 705-720.

Revankar, N. S. 1971. A Class of Variable Elasticity of Substitution Production Functions. Econometrica 39 (1): 61-71.