Modeling the emergence of hysteresis in agri-food sectors

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Abstract

Most agri-food sectors are subject to important transport costs and food processing industries are likely to display increasing returns to scale. The economic geography literature highlights that in such frameworks, some external returns to scale are likely to occur and give rise to multiple equilibria, source of hysteresis in a system. However, classical trade models are unable to describe such characteristics and therefore fail to represent the irreversibility that can arise from hysteresis. This paper tackles this issue and proposes a model of a regional agri-food sector that allows the possibility of agglomeration economies and hysteretic behavior of the system. Built as a recursively dynamic partial equilibrium model, it describes the domestic agricultural production and a processing industry, as well as potential trade with a foreign region. It also offers the choice between a monopsonistic or an oligopsonistic representation of the structure of the local primary agricultural product market.

1 Introduction

Most agri-food sectors bear some characteristics that make them widely differ from their neo-classical representation in trade models. The productive capital is generally non-malleable, both in the primary production and at the processor's level, which generates some important sunk costs (Chavas, 2001). We observe economies of scale at the factory level in processing industries (Ollinger et al., 2005; Marchant, 2007; van den Heuvel et al., 2011; MacDonald and Ollinger, 2000). Finally, there are some important transport costs, in particular before processing, for fresh products like animal productions (Cohen and Paul, 2005).

It is well known in the economic geography literature that in presence of economies of scale and transport costs, some external economies of agglomeration can arise, due to the market size/market access marshallian externality (Fujita et al., 2001; Cohen and Paul, 2005). They are also known as pecuniary externalities (Fujita and Thisse, 2013). The same literature show as well that in presence of such externalities, multiple equilibria are possible, with various patterns of agglomeration, leading up to potential hysteretic behavior of the system. Such geographical concentrations of agri-food activities are well observed across the World (Camelia, 2010; Ben Arfa et al., 2009; Bagoulla et al., 2010; Cohen and Paul, 2005).

This potential for hysteresis and therefore irreversibility could be of crucial importance for the analysis and conception of policies that affect agri-food sectors. Indeed, hysteresis affects the resilience of a system as a shock may have permanent consequences (Perrings and Brock, 2009). In an uncertain environment, the potential benefits observed at a certain time may fade away in the future. The impossibility to back down after a decision has been made give rise to option value and information value (Pindyck, 2004). Therefore, it seems of first importance to account for long run consequences and risk exposure when taking decision about agri-food sectors. Hysteresis also makes the decisions not independent in time. If a decision A is taken before a decision B, the consequences may be different than when B is taken before A. This calls for taking into account the chronology of action when deciding about agri-food sectors.

Those seem especially true for trade policies. Indeed, trade agreements affect the risk exposure inherent to agri-food markets. We can also imagine that a trade policy decided to take advantage of certain market conditions may reveal to be an impediment if the conditions change. Moreover, the change in the production structure involved to fit a trade partner may not be fit for a different one. It would then appear important to be dynamically strategic in the set-up of the trade agenda. Finally, trade agreements are generally decided at the national or even supra-national level, whereas agglomeration economies rather occur at the regional level. This raise the case for potential political economy issues as interests in a trade agreement may differ and even oppose across the regions of a same country.

However, applied trade models of simulation used to analyze and support trade policies are unable to account for such agglomeration economies and hysteresis in agri-food sectors (Fujita and Thisse, 2013). This comes from the multiple modeling challenges that their representation would imply. First, internal economies of scale are susceptible to generate concentration and market power to the incumbent firms, in contradiction with the pure and perfect competition hypothesis. Second, most models are solely detailed at the country level, making impossible the representation of the regional effects occurring with agglomeration economies. Third, multiple-equilibria systems imply to be able to categorize each equilibrium and further specify the dynamic leading to one or another (Fujita et al., 2001). Fourth, sunk-cost cannot be accounted in a static equilibrium framework. They require making further assumption about the agents' intertemporal decisions and their anticipations.

Most models in the economic geography literature focus on the market size/market access effect on the demand side for firms that produce under increasing returns, leading to backward and forward linkage with labor availability and consumption (Fujita et al., 2001; Fujita and Thisse, 2013). In the case of agri-food sectors, as the transport cost are higher for the input processed (i.e. the primary product), the agglomeration economies are rather on the supply side. They arise from the proximity between farms producing a primary product used as an input in processing industry operating under increasing returns. Therefore, agglomeration economies affect both the primary production and the processers' localization. As far as we know, no model has been developed to explicitly represent such agglomeration dynamic across a value chain (Miron, 2010).

This paper fills into these gaps by proposing a model of agri-food sector that enable to represent agglomeration economies and therefore hysteretic behaviors. It is built as a recursively dynamic partial equilibrium model. It represents a region's agri-food sector composed of farms and processing factories. Farms can produce alternatively two homogeneous agricultural products. One of the agricultural outputs is a primary input and is processed by the factories into a final product. This primary product can also be traded with a foreign region, with import and export transport costs. It is also finally consumed inside and outside the domestic region. The final processed product and the other agricultural output are sold at a fixed price.

In the short run, corresponding to a one-year period, agricultural production and the number of factories are inelastic. Therefore, everything operates under constant or decreasing returns. We enable the representation of the market power of the processing industry with a monopsonistic or oligopsonistic structure of the primary product market. The short-run static partial equilibrium remains then fairly classical. In the long run, the model is recursively dynamic. We incorporate sunk costs for the set-up of processing factories. Those generate long-run economies of scale at the factory level and subsequently agglomeration economies in the agri-food sector. Factories have a definite durability and become obsolete after a certain number of periods. Farms can switch from one production to another following a long-run cost considerations, to represent for yearly land-use decisions. Intertemporal decisions in farms and in the processing sector are made under myopic assumptions, in a Cobbweb-like fashion.

This model is programmed under the software GAMS to be numerically resolved and enable simulation. We build reaction matrices of the dynamic system that prove the potential existence of multiple stable dynamic equilibria for this system. Those matrices enable us to analyze the stability and the characteristics of each equilibrium, depending on the parameters of the system. We also make dynamic simulations that display the potentially hysteretic behavior of the model. We show that an exogenous shock can make the system switch from one dynamic equilibrium to another. We also highlight the cyclical behavior that can arise in the system due to capital durability and imperfect coordination in the agents' anticipations.

This paper and the model it presents have been developed as a building block for the better representation of agri-food sectors in applied trade models, in particular recursively dynamic equilibrium models. This attempt at the representation of agglomeration externalities and hysteretic behavior enable to show the potential importance of such phenomena for trade policy analysis and offer a first path for their integration in applied model.

The following section will present the short run static partial equilibrium model. Some elements of analysis of the short-run equilibrium will be given. The long-run dynamic model will be presented in section 3. Finally, numerical resolutions will be presented to prove the existence of multiple equilibria and the possibility of hysteretic and cyclical behavior of the model, with the computation of the systems' reaction matrices and dynamic simulations.

2 Short run static model of an agri-food sector

The global model is built as a recursively dynamic partial equilibrium model. It represents a domestic region with local and external agents: agricultural primary producers, processing industries and final consumers. The short-run evolution of the system, equivalent to a one year period, is given by a static partial equilibrium model. The long-run dynamic evolution of the system will be built around the recursive resolution of this partial equilibrium for each year t. This section presents the structure of the short-run partial equilibrium model.

In the following presentation of the short-run model, all the variables and parameters will be presented without time indices for the sake of simplicity. However, keep in mind that they all describe the state of the system for a given year t. They will be likely to evolve across time in the dynamic long-run model in which further time dynamic specification will be added.

2.1 Local agricultural production

Farmers in the domestic region can produce alternatively two agricultural products. The first one is a primary product that can be traded, processed or finally consumed. Its domestic price P_{dom} is determined endogenously in the model. The total quantity produced in the region for the one year period studied is noted Q_p. The other agricultural product, considered as an alternative production can be sold at an exogenously fixed price P_{alt}. Its total quantity produced in the region at time t is noted Q_{alt}.

In the short-run, we assume that the local agricultural productions are inelastic. This represents the fact that farmers harvest and sell a production that has been planned previously. Therefore, the output quantities, Q_p and Q_{alt} are fixed exogenously in the short-run model. As the alternative agricultural production has exogenous price and quantity dynamics that are independent of the rest of the short-run model, it will be largely omitted in the rest of this description.

2.2 Local final demand for primary product

We model a local final demand for the primary agricultural product as issued from a Constant Elasticity of Substitution (CES) demand:

$$D_{dom} = a^{dom} \cdot \left(\frac{P_{dom}}{P_{dom}^{ref}} \right)^{\sigma_{dom}} \tag{1}$$

With $a^{dom} > 0$ a share parameter, $\sigma_{dom} < 0$ an elasticity parameter and $P_{dom}^{ref} > 0$ an exogenous price index.

2.3 Foreign market and trade

We add in this model a foreign region with an external supply and demand for the primary agricultural product. Trade with the domestic region is possible, under specified export and import costs. This gives rise to a "free-on-board" (FOB) price as regards to the domestic market price. No geographic scale is specified for this foreign region. It can represent the rest of the national market outside the domestic region studied, or the World market, or a different region inside or outside the same country.

2.3.1 Foreign supply in primary product

We model a primary agricultural product supply in the foreign region. In the same way as in the domestic region, it is inelastic in the short-run. The quantity produced for a year is exogenously fixed at Q_0. It is sold on the foreign market at an endogenous price P_{ext}, which represents the FOB price for the domestic market.

2.3.2 Foreign final demand in primary product

A demand for final consumption of primary agricultural product is also represented on the foreign market. It takes the same CES form as in the domestic region:

$$D_{ext} = a^{ext} \cdot \left(\frac{P_{ext}}{P_{ext}^{ref}} \right)^{\sigma_{ext}} \tag{2}$$

With $a^{ext} > 0$ a share parameter, $\sigma_{ext} < 0$ an elasticity parameter and $P_{ext}^{ref} > 0$ an exogenous price index.

2.3.3 Trade and trade costs between the domestic and foreign region

When no trade occur between the foreign and the domestic region, we have an autarky equilibrium on the foreign market where the final demand equals the supply: $D_{ext} = Q_0$. The price of autarkic equilibrium is then:

$$P_{ext}^{aut} = P_{ext}^{ref} \cdot \left(\frac{Q_0}{a_{ext}}\right)^{\frac{1}{\sigma_{ext}}} \tag{3}$$

We let the possibility for trade to occur between the domestic and foreign region. However, we represent transport costs under the form of an iceberg cost, at a rate τ_M for imports from the foreign region to the domestic region and a rate τ_X for exports from the domestic region to the foreign region.

The primary product is considered homogeneous. Therefore, we will observe imports to the domestic region only if the domestic price P_{dom} is greater than the foreign autarky market price plus the imports costs. In that case, the law-of-one-price will make the domestic price and the foreign price equals at the difference of the import costs. We will have:

$$P_{dom} - \tau_M = P_{ext} > P_{ext}^{aut} \tag{4}$$

Symmetrically, we will observe exports only if the domestic price P_{dom} is smaller than the foreign autarky market price minus the exports costs and we will have:

$$P_{dom} + \tau_X = P_{ext} < P_{ext}^{aut} \tag{5}$$

We notice that there can't be positive exports and imports simultaneously. Noting D_X the quantity of primary product demanded by the foreign region to be exported from the domestic region and Q_M the supply for imports to the domestic region, at all time we will have the following foreign market closure:

$$Q_0 + D_X = Q_M + D_{ext} \tag{6}$$

We can then infer the supply function for imports from the foreign to the domestic region:

$$\begin{aligned} Q_M &= \begin{cases} Q_0 - a_{ext} \cdot \left(\frac{P_{dom}-\tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}} & \text{if } P_{dom} > P_{ext}^{aut} + \tau_M \\ 0 \text{ if } P_{dom} \leq P_{ext}^{aut} + \tau_M \end{cases} \\ &= max\left(0; Q_0 - a^{ext} \cdot \left(\frac{P_{dom} - \tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right) \end{aligned} \tag{7}$$

For numerical modeling purposes and to ensure the derivability at every point, we use the approximation:

$$Q_M \approx \frac{1}{2} \cdot \left[Q_0 - a^{ext} \cdot \left(\frac{P_{dom} - \tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}} + \sqrt{\left(-Q_0 + a^{ext} \cdot \left(\frac{P_{dom} - \tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right)^2 + \Delta^2} \right] \tag{8}$$

With Δ close to 0.

Symmetrically, we have the export demand function:

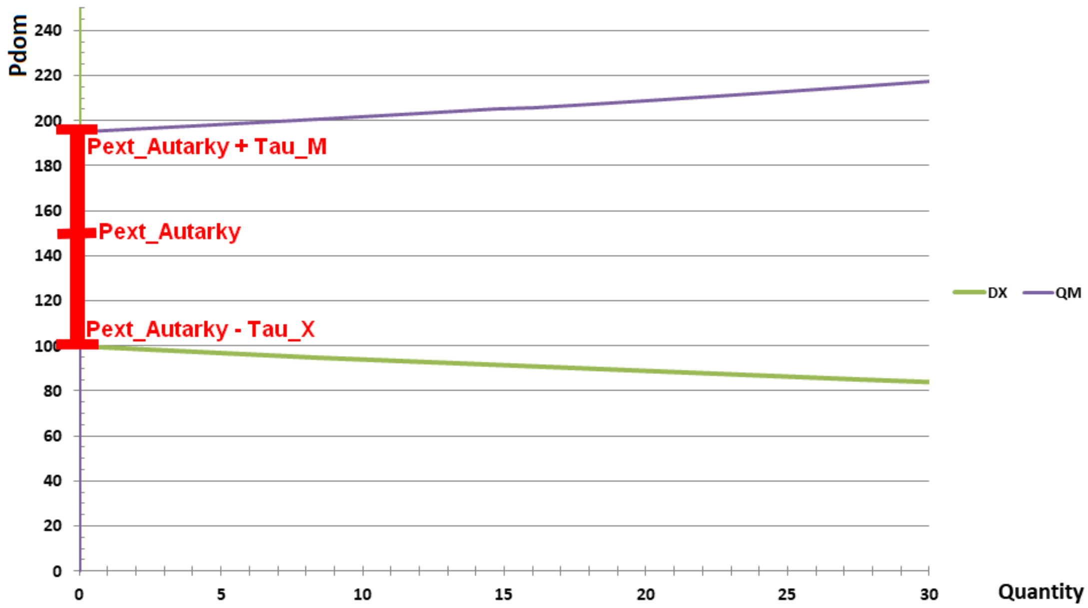
$$\begin{aligned} D_X &= \begin{cases} -Q_0 + a_{ext} \cdot \left(\frac{P_{dom}+\tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}} & \text{if } P_{dom} < P_{ext}^{aut} - \tau_X \\ 0 \text{ if } P_{dom} \geq P_{ext}^{aut} - \tau_X \end{cases} \\ &= max\left(0; -Q_0 + a^{ext} \cdot \left(\frac{P_{dom} + \tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right) \\ &\approx \frac{1}{2} \cdot \left[-Q_0 + a^{ext} \cdot \left(\frac{P_{dom} + \tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}} + \sqrt{\left(Q_0 - a^{ext} \cdot \left(\frac{P_{dom} + \tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right)^2 + \Delta^2} \right] \end{aligned} \tag{9}$$

Figure 1 displays the shape of import supply and export demand on the domestic primary product market.

2.4 Processing industry

We model a local industry which processes the primary agricultural product into a final product. We assume that this local industry is composed of n identical factories. Each factory is composed of a fixed indivisible amount of capital K. The number of factories is exogenously determined in the short run. Each factory has a CES production function

Figure 1: Import supply and export demand on the primary product domestic market



using capital and primary product in the following way:

$$Q_F = \alpha \left(\beta K^\theta + (1 - \beta) \cdot \left(\frac{D_F}{n} \right)^\theta \right)^{\frac{1}{\theta}} \tag{10}$$

With Q_F the quantity of final product produced by factory and D_F the quantity of primary product consumed by the whole processing industry, so $\frac{D_F}{n}$ is the quantity of primary product consumed by factory. $\alpha > 0$ is a scale parameter, $\beta \in]0; 1[$ a share parameter and $\theta < 1$ an elasticity of substitution parameter.

2.5 Market closure

On the foreign market for the primary product, we recall the following closure:

$$Q_0 + D_X = D_{ext} + Q_M \tag{11}$$

On the domestic market of the primary product we have:

$$Q_p + Q_M = D_F + D_{dom} + D_X \tag{12}$$

Finally, the alternative agricultural product and the final product are both sold at exogenously fixed prices.

2.6 Market structures options and demand from the processing firms

We assume that there is a multiplicity of farmers producing locally the primary product and that they are therefore price takers on this market. However, we propose two options to represent the market power of the processing industry on the primary product market. The first option corresponds to a monopsonistic structure, where all the local processing factories are owned by an unique company. It then maximizes its profit on the total quantity of primary product consumed D_F, considering the effect of its demand on the price P_{dom}. The second option describes an oligopsonistic structure where each factory is owned by a different company. Therefore, each factory/company optimize its profit on the quantity of primary product it consumed $\left(\frac{D_F}{n}\right)$, considering the effect of its demand and the other factories' demand on the price P_{dom}.

2.6.1 Option 1: Monopsonistic structure of the processing sector

The monopsonistic processing company optimizes its profit π_{mono} over its demand for primary product for all its factories D_F, taking into account the effect of this demand on the domestic price P_{dom}. To do so, it infers the input supply function from the primary product market closure, taking into account the demand and supply of the other agents in the market. The optimization program of the monopsonistic company is then:

$$\begin{aligned} \text{Maximize } \pi_{mono} &= n \cdot P_F \cdot Q_F(D_F) - D_F \cdot P_{dom} \\ &= n \cdot P_F \cdot \alpha \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta \right)^{\frac{1}{\theta}} - D_F \cdot P_{dom} \end{aligned} \tag{13}$$

Under the constraint:

$$Q_p + Q_M(P_{dom}) = D_F + D_{dom}(P_{dom}) + D_X(P_{dom}) \tag{14}$$

The Lagrangian of this system is:

$$\mathcal{L} = n \cdot P_F \cdot \alpha \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta \right)^{\frac{1-\theta}{\theta}} - D_F \cdot P_{dom}$$
$$-\lambda \left(D_F + D_{dom}(P_{dom}) + D_X(P_{dom}) - Q_p - Q_M(P_{dom})\right) \tag{15}$$

First order conditions :

The first orther conditions are:

$$\frac{\partial \mathcal{L}}{\partial D_F} = P_F \cdot \alpha \cdot (1-\beta) \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta \right)^{\frac{1-\theta}{\theta}} - P_{dom} - \lambda = 0 \tag{16}$$

And

$$\frac{\partial \mathcal{L}}{\partial P_{dom}} = -D_F - \lambda \left(\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}} \right) = 0 \tag{17}$$

So, identifying λ in 16 and 17, we have:

$$P_F \cdot \alpha \cdot (1-\beta) \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta \right)^{\frac{1-\theta}{\theta}} - P_{dom} = \frac{-D_F}{\left(\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}} \right)} \tag{18}$$

Finally, the demand for primary product by the monopsonistic processing company is given by:

$$P_F \cdot \alpha^\theta \cdot (1-\beta) \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot (QF)^{1-\theta} - P_{dom} = \frac{-D_F}{\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}}}$$
$$+P_F \cdot \alpha \cdot (1-\beta)^2 \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta \right)^{\frac{1-\theta}{\theta}} \tag{19}$$

With by derivating equations 1, 8 and 9:

$$\frac{\partial D_{dom}}{\partial P_{dom}} = a^{dom} \cdot \frac{\sigma_{dom}}{(P_{dom}^{ref})^{\sigma_{dom}}} \cdot (P_{dom})^{\sigma_{dom}-1} < 0 \tag{20}$$

$$\frac{\partial D_X}{\partial P_{dom}} = \begin{cases} a^{ext} \cdot \frac{\sigma_{ext}}{(P_{ext}^{ref})^{\sigma_{ext}}} \cdot (P_{dom} + \tau_X)^{\sigma_{ext}-1} < 0 \text{ if } P_{dom} < P_{ext}^{aut} - \tau_X \\ 0 \text{ if } P_{dom} \geq P_{ext}^{aut} - \tau_X \end{cases}$$
$$\approx \frac{1}{2} \cdot \frac{a^{ext} \cdot \sigma_{ext} \cdot \left(\frac{P_{dom}+\tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}}}{P_{dom} + \tau_X} \cdot \left[\frac{-Q_0 + a^{ext} \cdot \left(\frac{P_{dom}+\tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}}}{\sqrt{\left(-Q_0 + a^{ext} \cdot \left(\frac{P_{dom}+\tau_X}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right)^2 + \Delta^2}} + 1 \right] \tag{21}$$

$$\frac{\partial Q_M}{\partial P_{dom}} = \begin{cases} -a^{ext} \cdot \frac{\sigma_{ext}}{(P_{ext}^{ref})^{\sigma_{ext}}} \cdot (P_{dom} - \tau_M)^{\sigma_{ext}-1} > 0 \text{ if } P_{dom} > P_{ext}^{aut} + \tau_M \\ 0 \text{ if } P_{dom} \leq P_{ext}^{aut} + \tau_M \end{cases}$$
$$\approx -\frac{1}{2} \cdot \frac{a^{ext} \cdot \sigma_{ext} \cdot \left(\frac{P_{dom}-\tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}}}{P_{dom} - \tau_M} \cdot \left[\frac{Q_0 - a^{ext} \cdot \left(\frac{P_{dom}-\tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}}}{\sqrt{\left(Q_0 - a^{ext} \cdot \left(\frac{P_{dom}-\tau_M}{P_{ext}^{ref}}\right)^{\sigma_{ext}}\right)^2 + \Delta^2}} + 1 \right] \tag{22}$$

Second-order conditions :

We have:

$$\frac{\partial^2 \mathcal{L}}{\partial D_F^2} = P_F \cdot n \cdot \frac{\partial^2 QF(\frac{DF}{n})}{\partial D_F^2} \tag{23}$$

QF is a Constant Elasticity of Substitution function and $\theta < 1$, therefore QF is concave and $\frac{\partial^2 QF(\frac{DF}{n})}{\partial D_F^2} < 0$. So: $\frac{\partial^2 \mathcal{L}}{\partial D_F^2} < 0$.

On the other hand, we have:

$$\frac{\partial^2 \mathcal{L}}{\partial P_{dom}^2} = -\lambda \left(\frac{\partial^2 D_{dom}}{\partial P_{dom}^2} + \frac{\partial^2 D_X}{\partial P_{dom}^2} - \frac{\partial^2 Q_M}{\partial P_{dom}^2} \right) \tag{24}$$

With:

$$\lambda = \frac{-D_F}{\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}}} > 0 \tag{25}$$

$$\frac{\partial^2 D_{dom}}{\partial P_{dom}^2} > 0 \tag{26}$$

$$\frac{\partial^2 D_X}{\partial P_{dom}^2}\begin{cases} > 0 \text{ if } P_{dom} < P_{ext}^{aut} - \tau_X \\ = 0 \text{ if } P_{dom} \geq P_{ext}^{aut} - \tau_X \end{cases} \tag{27}$$

$$\frac{\partial^2 Q_M}{\partial P_{dom}^2}\begin{cases} < 0 \text{ if } P_{dom} > P_{ext}^{aut} + \tau_M \\ = 0 \text{ if } P_{dom} \leq P_{ext}^{aut} + \tau_M \end{cases} \tag{28}$$

Therefore, $\frac{\partial^2 \mathcal{L}}{\partial P_{dom}^2} < 0$ and $\frac{\partial \mathcal{L}}{\partial P_{dom}}$ is decreasing on $]-\infty; P_{ext}^{aut} - \tau_X[$, $[P_{ext}^{aut} - \tau_X; P_{ext}^{aut} + \tau_M]$ and $]P_{ext}^{aut} + \tau_M; +\infty[$. So the profit Π_{mono} has a local maximum on each of those intervals.

We have reduced to three the number of potential solutions of the system. In order to see if we can reduce it further, we look at the points where the Lagrangian is not derivable without approximation. First, in $P_{ext}^{aut} - \tau_X$:

$$\lim_{P_{dom}\to(P_{ext}^{aut}-\tau_X)^-} \frac{\partial \mathcal{L}}{\partial P_{dom}} = -D_F - \lambda\left(\frac{a^{dom}\cdot\sigma_{dom}}{(P_{dom}^{ref})^{\sigma_{dom}}}\cdot(P_{ext}-\tau_X)^{\sigma_{dom}-1} + \frac{a^{ext}\cdot\sigma_{ext}}{(P_{ext}^{ref})^{\sigma_{ext}}}\cdot(P_{ext}^{aut})^{\sigma_{ext}-1}\right) \tag{29}$$

$$\lim_{P_{dom}\to(P_{ext}^{aut}-\tau_X)^+} \frac{\partial \mathcal{L}}{\partial P_{dom}} = -D_F - \lambda\left(a^{dom}\cdot\frac{\sigma_{dom}}{(P_{dom}^{ref})^{\sigma_{dom}}}\cdot(P_{ext}-\tau_X)^{\sigma_{dom}-1}\right) \tag{30}$$

Therefore,

$$\lim_{P_{dom}\to(P_{ext}^{aut}-\tau_X)^-} \frac{\partial \mathcal{L}}{\partial P_{dom}} > \lim_{P_{dom}\to(P_{ext}^{aut}-\tau_X)^+} \frac{\partial \mathcal{L}}{\partial P_{dom}} \tag{31}$$

So, $\frac{\partial \mathcal{L}}{\partial P_{dom}}$ is decreasing around $P_{ext}^{aut} - \tau_X$, therefore it is decreasing on $]-\infty; P_{ext}^{aut} + \tau_M[$. We ensure the continuity and derivability of $\frac{\partial \mathcal{L}}{\partial P_{dom}}$ on $]-\infty; P_{ext}^{aut} + \tau_M[$ using the approximation presented in 21. So the profit π_{mono} has a unique local maximum on the whole interval $]-\infty; P_{ext}^{aut} + \tau_M[$.

Looking now at the point $P_{ext}^{aut} + \tau_M$, we have:

$$\lim_{P_{dom}\to(P_{ext}^{aut}+\tau_M)^-} \frac{\partial \mathcal{L}}{\partial P_{dom}} = -D_F - \lambda\left(a^{dom}\cdot\frac{\sigma_{dom}}{(P_{dom}^{ref})^{\sigma_{dom}}}\cdot(P_{ext}^{aut}+\tau_M)^{\sigma_{dom}-1}\right) \tag{32}$$

$$\lim_{P_{dom}\to(P_{ext}^{aut}+\tau_M)^+} \frac{\partial \mathcal{L}}{\partial P_{dom}} = -D_F - \lambda\left(\frac{a^{dom}\cdot\sigma_{dom}}{(P_{dom}^{ref})^{\sigma_{dom}}}\cdot(P_{ext}^{aut}+\tau_M)^{\sigma_{dom}-1} + \frac{a^{ext}\cdot\sigma_{ext}}{(P_{ext}^{ref})^{\sigma_{ext}}}\cdot(P_{ext}^{aut})^{\sigma_{ext}-1}\right) \tag{33}$$

Therefore,

$$\lim_{P_{dom} \to (P_{ext}^{aut} - \tau_X)^-} \frac{\partial \mathcal{L}}{\partial P_{dom}} < \lim_{P_{dom} \to (P_{ext}^{aut} - \tau_X)^+} \frac{\partial \mathcal{L}}{\partial P_{dom}} \tag{34}$$

So, $\frac{\partial \mathcal{L}}{\partial P_{dom}}$ is increasing around $P_{ext}^{aut} - \tau_X$. Therefore, we can only tell that $\frac{\partial \mathcal{L}}{\partial P_{dom}}$ is decreasing respectively on $]-\infty; P_{ext}^{aut} + \tau_M]$ and $]P_{ext}^{aut} + \tau_M; +\infty[$. We conclude that π_{mono} has a unique local maximum on each of those segments. Our short-run model therefore could still have two potential solutions.

The local maximum on $]-\infty; P_{ext}^{aut} + \tau_M]$ corresponds to the case when the monopsonistic company consumes less primary product than what is locally available, therefore paying a smaller price. The local maximum on $]P_{ext}^{aut} + \tau_M; +\infty[$ corresponds to the case when the company chooses to process more primary product and therefore has to import and pay a higher price.

So in order to find the global maximum, we solve the complete model on each of these price intervals and the equilibrium is the solution that ensures the greatest profit to the monopsonistic processing company.

2.6.2 Option 2: Oligopsonistic structure of the processing sector

Under the oligopsonistic option, each company/factory optimizes its own profit π_{oli} on its own demand for primary product $\frac{D_F}{n}$. It takes into account the effect of its demand on price in the same way as in the monopsonistic case, by inferring the inverse demand function from the market closure. However, this time it has to account for the demand from the other factories. As all the firms are identical, we note this remaining demand D_F^{rem} and we have: $D_F^{rem} = \frac{(n-1) \cdot D_F}{n}$. We can see this framework as the oligopsonistic symmetric of a Cournot oligopoly framework. The optimization program of an oligopsonistic factory is then:

$$\text{Maximize } \pi_{oli} = P_F \cdot \alpha \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n} \right)^\theta \right)^{\frac{1}{\theta}} - \frac{D_F}{n} \cdot P_{dom} \tag{35}$$

Under the constraint:

$$Q_p + Q_M(P_{dom}) = \frac{D_F}{n} + D_F^{rem} + D_{dom}(P_{dom}) + D_X(P_{dom}) \tag{36}$$

The Lagrangian of this system is:

$$\begin{aligned}\mathcal{L} = P_F \cdot \alpha \left(\beta K^\theta + (1-\beta)\cdot\left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1}{\theta}} - \frac{D_F}{n}\cdot P_{dom} \\ -\lambda\left(\frac{D_F}{n} + D_F^{rem} + D_{dom}(P_{dom}) + D_X(P_{dom}) - Q_p - Q_M(P_{dom})\right)\end{aligned} \tag{37}$$

First-order conditions :

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial D_F} = P_F \cdot \alpha \cdot \frac{1-\beta}{n^\theta} \cdot (D_F)^{\theta-1} \cdot \left(\beta K^\theta + (1-\beta)\cdot\left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1-\theta}{\theta}} - \frac{P_{dom}}{n} - \frac{\lambda}{n} = 0 \tag{38}$$

And

$$\frac{\partial \mathcal{L}}{\partial P_{dom}} = -\frac{D_F}{n} - \lambda\left(\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}}\right) = 0 \tag{39}$$

So, identifying λ in 38 and 39, we have:

$$P_F\cdot\alpha\cdot(1-\beta)\cdot\left(\frac{D_F}{n}\right)^{\theta-1}\cdot\left(\beta K^\theta + (1-\beta)\cdot\left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1-\theta}{\theta}} - P_{dom} = \frac{-D_F}{n\cdot\left(\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}}\right)} \tag{40}$$

Subsequently, the demand for primary product by the processing industry in the oligopsonistic case is given by:

$$P_F \cdot \alpha^\theta \cdot (1-\beta)\cdot\left(\frac{D_F}{n}\right)^{\theta-1}\cdot (Q_F)^{1-\theta} - P_{dom} = \frac{-D_F}{n\cdot\left(\frac{\partial D_{dom}}{\partial P_{dom}} + \frac{\partial D_X}{\partial P_{dom}} - \frac{\partial Q_M}{\partial P_{dom}}\right)} \tag{41}$$

We notice that when $n = 1$, we find back the result of the monopsonistic case. When the number of factories increase and $n \mapsto \infty$, we find back the result that we would get in a perfectly competitive framework[1].

Second-order conditions :

The second-order conditions are similar to the ones in the monopsonistic case (subsection 2.6.2), therefore leading to a unique local maximum on each of the intervals $]-\infty; P_{ext}^{aut} + \tau_M]$ and $]P_{ext}^{aut} + \tau_M; +\infty[$. However, contrary to the monopsonistic case,

[1] The perfectly competitive framework is solved in Appendix A

in the oligopsonistic case the quantity of primary product demanded by a processing company depends on the expected demand of the other companies $\left(D_F^{rem} = \frac{(n-1)\cdot D_F}{n}\right)$.

The solution of the system on $]-\infty; P_{ext}^{aut} + \tau_M]$ would correspond to a situation when all the firms would choose to consume only part of the local primary production, therefore at a reduced price. The solution on $]P_{ext}^{aut} + \tau_M; +\infty[$ would correspond to the case when all the firms would demand more, therefore importing primary product and pay a higher price. In each case, it would likely not be profitable for a single firm to deviate alone . Therefore, each of the local maximum could correspond to an acceptable Nash-equilibrium.

We solve this issue of potential multiplicity of equilibria in the short-term by assuming that local processing firms manage to coordinate sufficiently to choose the pay-off dominant equilibrium. So in the same way as in the monopsonistic case, we find the short-run equilibrium of the system for a given year by solving the complete model on each of the price intervals and we choose the solution that ensures the greatest profit to the monopsonistic processing companies.

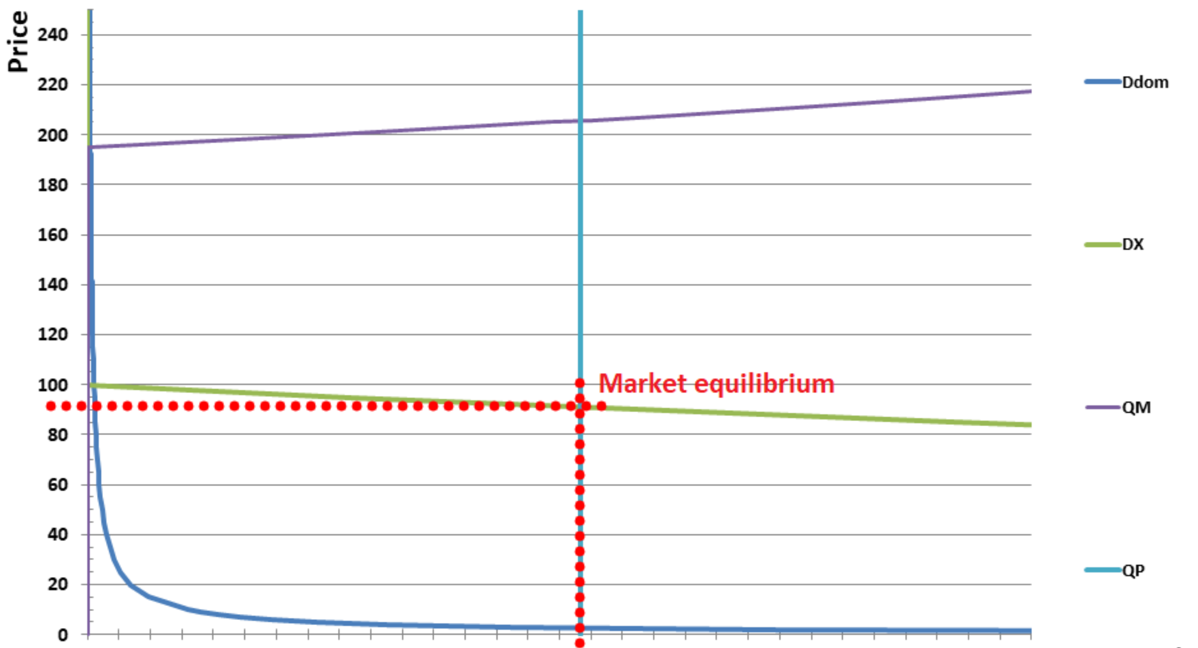
2.7 Analysis of the short-run partial equilibrium model

Figures 2, 3, 4 and 5 present the short-run equilibrium under various configurations of the model, with or without domestic agricultural production, domestic processing industry, transport costs or under the monopsonistic or oligopsonistic option.

We can see on Figure 2 that when there are transport costs with the foreign region and a domestic agricultural production but no local processing industry, the domestic price of the primary product will necessarily be under the limit export price. Indeed, the farmers will quickly saturate the domestic final demand and will have to export the rest of their production.

Conversely, we can see on Figure 3 that when a domestic industry is present but there are no farmers producing domestically the primary product, final consumers and the processing industry will have to import it. The domestic price of the primary product will consequently be above the limit import price. We can notice that the short-run equilibrium price and quantities consumed will be very close under the monopsonistic and oligopsonistic structures. Indeed, as the import supply is very elastic, even a monopsonistic company

Figure 2: Short-run equilibrium with no local processing industry

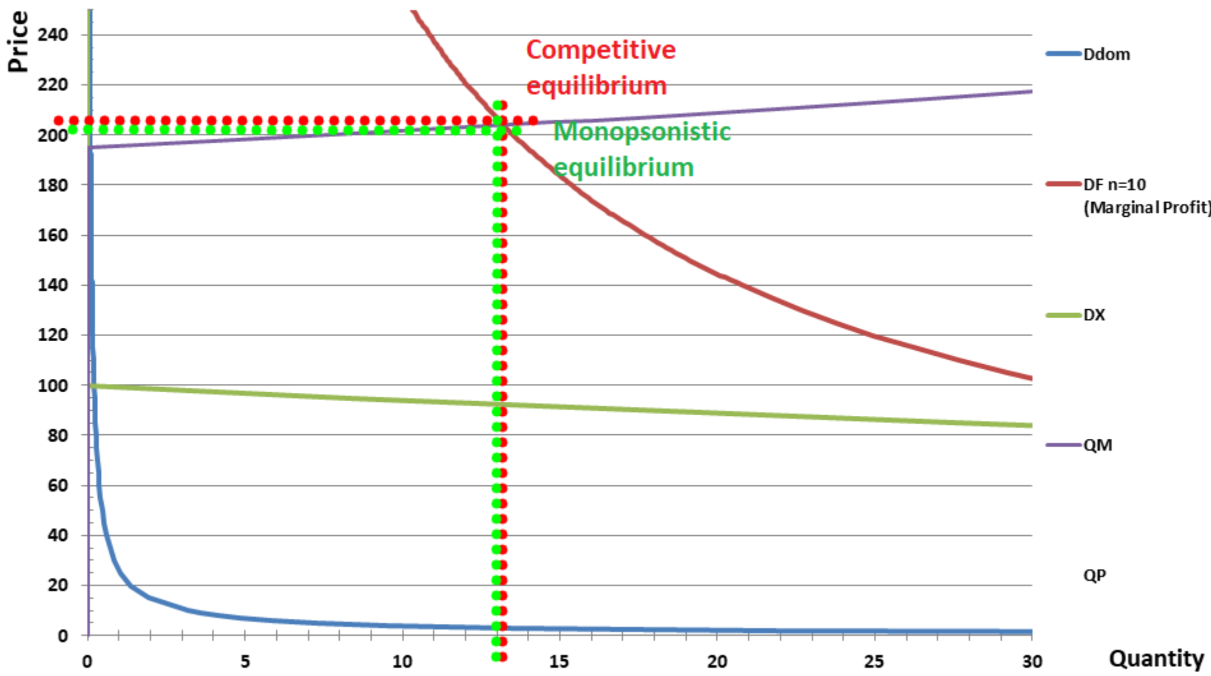


will have a much reduced market power when it will have to import.

Figure 4 gives a proof for the possibility of agglomeration economies across the local agri-food value chain in this model. We can see that when there are both primary producers and a processing industry in the domestic region, the domestic price can be higher for the farmers that won't have to export and suffer the export cost. At the same time, the processing industry will also benefit of a price lower than the import price. This highlights the local synergy that can exist between the agricultural primary production and the processing sector.

We notice that if the quantities purchased will be quite similar under the monopsonistic and the oligopsonistic option, the equilibrium price will be much different. Indeed, in both case the industry can purchase most of the domestic production. However, as in the oligopsonistic framework, the more companies there will be, the closer the equilibrium price will be to the marginal profit they gain from another unit of input and thus of the result in a competitive structure. On the contrary, as long as a monopsonistic will be satisfied with the quantity of input locally supplied, it won't pay much more than the export price.

Figure 3: Short-run equilibrium with no local agricultural production



Indeed, as the domestic supply is inelastic and the local final demand reduced, the market power of a monopsonistic firm will be very big until the price is as low as the limit export price, in the absence of other competitive agents.

Finally, we can highlight the role of transport costs in the existence of local agglomeration economies. Indeed, when there are no transport costs the domestic and the foreign will form one unique market with one unique price. However, this big market will be much more elastic. We can then see on 5 that in the absence of transport costs, the price will be stabilized around the foreign autarky equilibrium price. There will be little difference whether there is a domestic agricultural production or not or if there is a domestic processing sector or not. The price will be very similar under the monopsonistic or the oligopsonistic framework as in any case; the market power of the processing industry will be reduced.

Figure 4: Short-run equilibrium with local agricultural production and processing industry

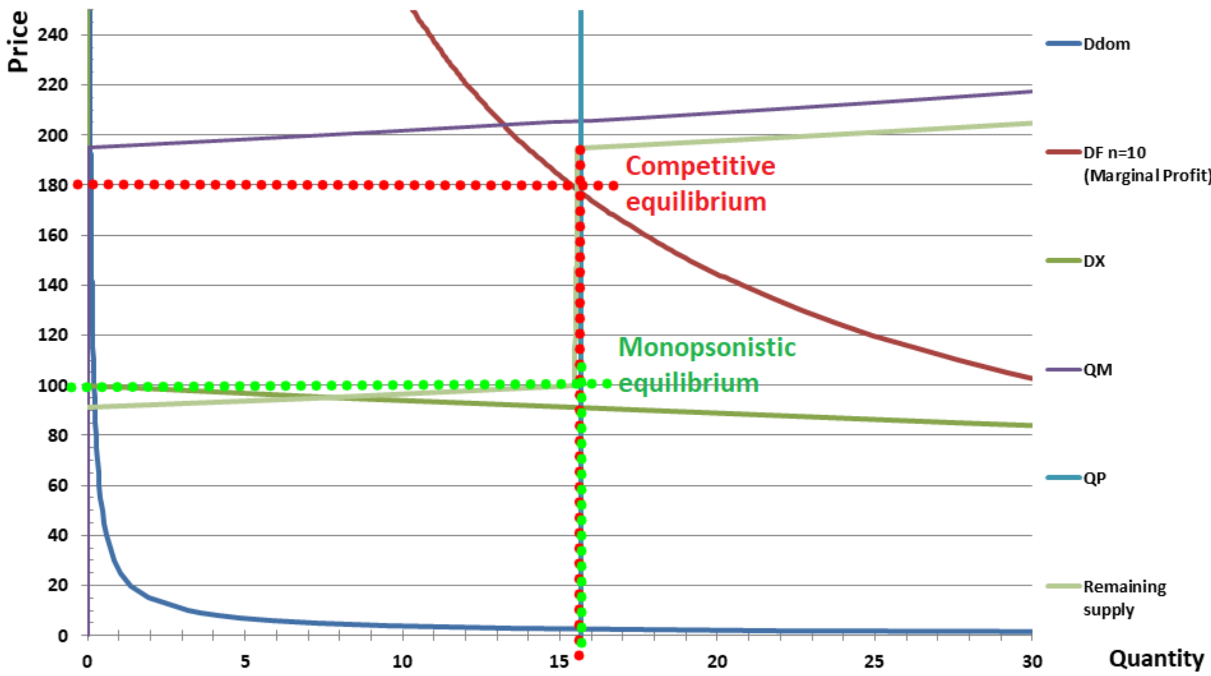
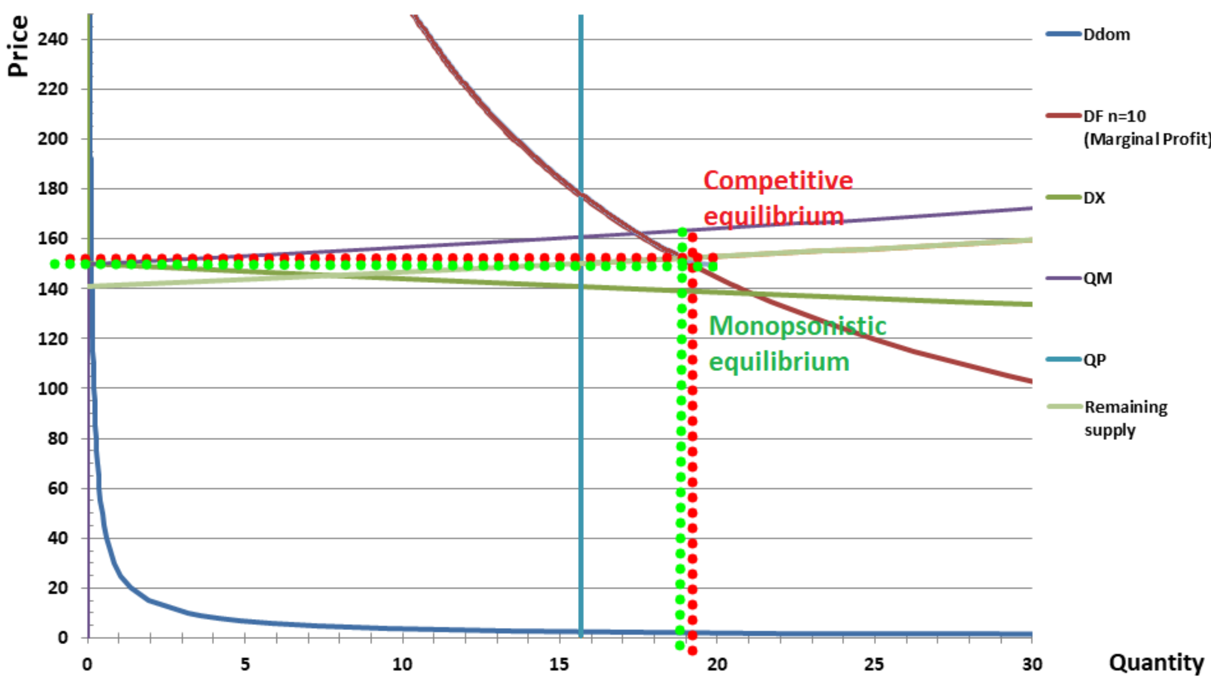


Figure 5: Short-run equilibrium with no transport costs



3 Long run recursively dynamic model of an agri-food sector

The long-run model enables to represent the long-run evolution of the local agricultural production and the number of processing factories in the agri-food sector. It is constructed as the recursive resolution of the static short-run partial equilibrium model, which describes the state of the system for each one-year period t. Between each period, the long-run model describes the reaction of the farmers and the processing industry to the previous equilibria in terms of production choices and investments for the future periods, according to their anticipations and long-run cost considerations.

3.1 Further description of the local agricultural sector and long-run production dynamic

We consider that the local agricultural sector is composed of a fixed number of farms $nF = card(F)$, with $F = \{1, ..., f..., nF\}$ the set of all local farms.

In order to produce the primary product, a farm has to make a fixed capital investment, for a cost $KM \cdot PK$. KM the quantity of capital necessary and PK the price of capital are exogenous to the model. This sunk cost can be seen as the required indivisible investment in capital to produce a certain product. For example, in dairy production this could be a milking parlor. It then enables the farms f that has made this investment to produce an homogeneous quantity Q_P^{farm} of primary product for T_{farm} years. During those T_{farm} years, those farms are stuck in this production, they can't stop production, switch to another or make further investments. After the T_{farm} years, the production unit is considered obsolete and the farm can choose to build a new one or not.

We consider that farms that have not made such investments are engaged in the alternative agricultural production. We can imagine this alternative production to be cereal production or any other that requires little specific capital. Farms engaged in the alternative production will produce each year a fixed quantity $Q_{alt}^{farm}(f)$. However, as no specific capital investments are made, we assume that those farms can each year choose to switch and invest in the primary production or to remain in the alternative production. In order to represent an heterogeneous productivity of land across the farms of the domestic region, we consider that the farm production $Q_{alt}^{farm}(f)$ is heterogeneous and that it follows a linear distribution so that:

$$Q_{farm}^{alt}(f) = \frac{f}{nF} \cdot Q_{farm}^{\bar{alt}} \tag{42}$$

Before each year t, farms that are not stuck in the primary production can choose to invest to produce the primary product or to stay in the alternative production which doesn't require further investments. To take this decision, they compute their anticipated net present value for both productions on the period of the longest investment durability (T_{farm}). We assume they do so under the myopic anticipations that the prices of both products remains stable at $P_{dom,t-1}$ and $P_{alt,t-1}$, in a Cobbweb like fashion (Gouel, 2012).

Each farm f then have for year t:

$$NPV_{P,t} = -KM \cdot PK + \sum_{t1=t}^{t+T_{farm}-1} \frac{Q_P^{farm} \cdot P_{dom,t-1}}{(1+r)^{t1}} \tag{43}$$

$$NPV_{alt,f,t} = \sum_{t1=t}^{t+T_{farm}-1} \frac{Q_{farm}^{alt}(f) \cdot P_{alt,t-1}}{(1+r)^{t1}} \tag{44}$$

With r, the discount rate. A farm f that can switch production before year t will then choose to produce the primary product for the next T_{farm} if: $NPV_{P,t} > NPV_{alt,f,t}$.

We note $F_{P,t}$ the set of farms engaged in the primary production at year t and $F_{alt,t}$ the set of farms engaged in the alternative production at year t. The total domestic agricultural productions of the primary product for year t is then :

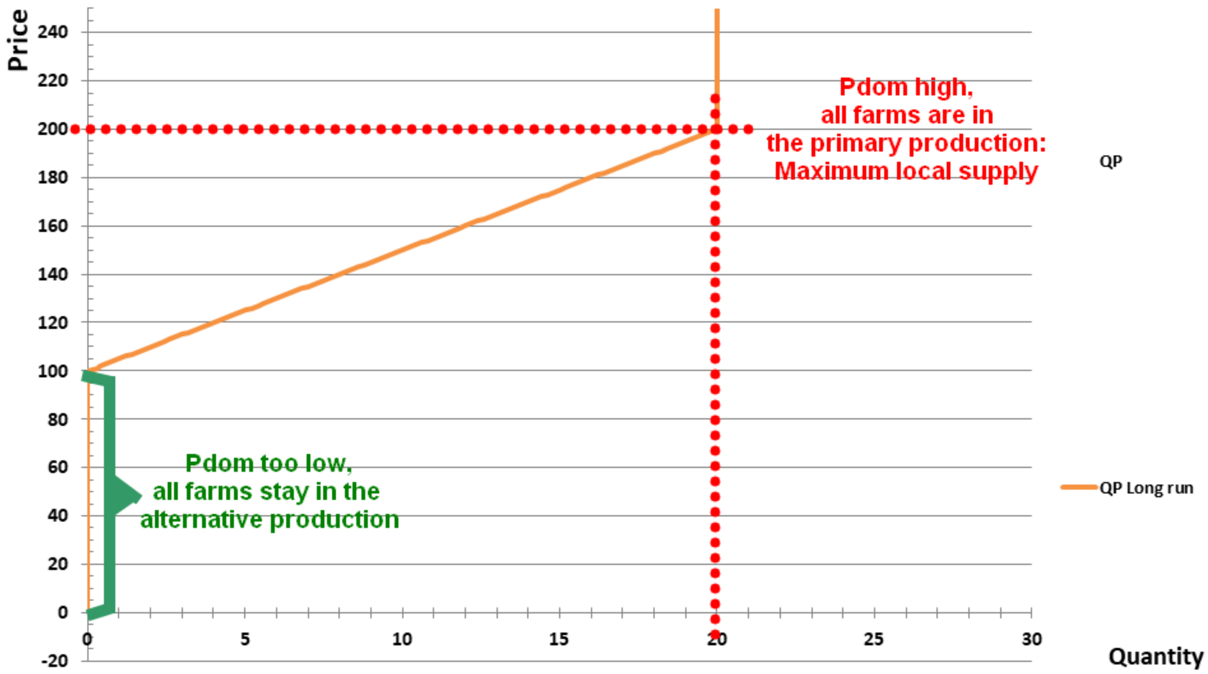
$$Q_{P,t} = \sum_{f \in F_{P,t}} Q_P^{farm} \tag{45}$$

And for the alternative product:

$$Q_{alt,t} = \sum_{f \in F_{alt,t}} Q_{farm}^{alt}(f) \tag{46}$$

Therefore, the long-run domestic primary product supply will have the shape presented on Figure 6. When the domestic price of the primary product will be too low, no farms will invest in this production and the regional production will become null. Conversely, past a certain price, all the local farms turn to this production and the regional production reach a plateau.

Figure 6: Long-run domestic supply of the primary product



3.2 Long run evolution of the number of processing factories

The long-run model describes the dynamic of the number of domestic processing factories n_t. Each year t, this number reacts to the state of the system in the previous years equilibria, following anticipations on the future state of the system and long-run cost considerations. The set-up of a new factory correspond to an investments in an indivisible amount K of capital units. For the owner, this represents a sunk costs $K \cdot PK$. Processing factories are productive for a number T_F of one year periods once they are set-up. After this time, they are considered obsolete and disappear. The next two sub-sections presents the process following which this number of factories evolve, under the monopsonistic and oligopsonistic

3.2.1 Investment decisions of the local monopsonistic processing company in new factories (Option 1)

In the monopsonistic case, the company that owns all the local processing factories take before every year t the decision to invest or not in new processing factories. Therefore, the monopsonistic firm seek the number of new processing unit $newD_t$ to set up at time t to maximize its anticipated net present value $NPV_{F,mono,t}$. The effect on the price of the primary product through the additional demand generated by new factories are not independent across time, as they are not additive if $T_F > 1$. The monopsonistic company therefore maximize each year t its net present value over an infinite period for all $t1 \geq t$, by scheduling all its future investments $newD_{t1}^{ant}$ and primary product demand $D_{F,t1}^{ant}$, considering the anticipated reaction of the primary product market to this demand [2]. We assume that this optimization is made under the myopic anticipation that the primary production in the region remains stable at the level $Q_{P,t-1}$ and the price of the final product remains at $P_{F,t-1}$. The optimization program is then:

$$\underset{\substack{(newD_{t1}^{ant}, D_{F,t1}^{ant}) \\ \forall t1 \geq t}}{\text{maximize}} NPV_{F,mono,t} = \sum_{t1=t}^{\infty} \frac{n_{t1}^{ant} \cdot Q_{F,t1}^{ant}(n_{t1}^{ant}, D_{F,t1}^{ant}) \cdot P_{F,t-1} - D_{F,t1}^{ant} \cdot P_{dom,t1}^{ant}(D_{F,t1}^{ant}) - newD_{t1}^{ant} \cdot KF \cdot PK}{(1+r)^{t1-t}} \tag{47}$$

Under the constraints:

$$n_{t1}^{ant} = n_{t1-1}^{ant} + newD_{t1}^{ant} - newD_{t1-T_{Proc}}^{ant} \tag{48}$$

$$Q_{p,t-1} + Q_{M,t-1}(P_{dom,t1}^{ant}) = D_{F,t1}^{ant} + D_{dom,t-1}(P_{dom,t1}^{ant}) + D_{X,t-1}(P_{dom,t1}^{ant}) \tag{49}$$

All the variables with an "*ant*" exponents correspond to anticipated variables.

Once this system is solved, the number of new factories is determined as $newD_t = newD_t^{ant}$. The number of processing factories is then fixed for year t as:

$$n_t = n_{t-1} + newD_{t1} - newD_{t1-T_F} \tag{50}$$

[2] For computation purposes in the numerical model, we approximate this infinite period by defining a large time horizon, so that the optimization program is solved on a finite period. We consider that the time discount make negligible the future anticipated profits and investments past this horizon.

And the new equilibrium for year t can then be computed.

3.2.2 Company entry/factory set-up under Zero-Profit Condition in an oligopsonistic processing sector (Option 2)

In the oligopsonistic case, each processing factory corresponds to a different company. Therefore, the set-up of new factories corresponds to the entry of new companies on the market. We assume that new companies will enter every year t until the anticipated net present value of a new factory $NPV_{F,oli}$ is equal to 0. Contrary to the monopsonistic case, as the net present value is computed for a unique factory/company, with a definite time of existence T_F, it is computed on a horizon equal to the firm durability D_F. The computation of the anticipated net present value for a new factory is made by anticipating the future demand for primary product by factory as it is made in the short-run model. This is made under the myopic anticipations that the number of active factories besides the potential new entrants remains stable at its previous level excluding the factories that just get out at this period, so it is at $nD_{t-1} - newD_{t-T_F}$ for all the investment period T_F. The local primary production is also assumed to remain at $Q_{P,t-1}$ and the price of the final product at $P_{F,t-1}$.

Therefore, to determine the number of new factories set-up or companies entering the market at time t, we increase this number $newD_t \in \mathbb{N}$ until the net present value of a new entrant, with one more factory, becomes negative. So that:

$$\begin{aligned} & NPV_{F,oli,t}(newD_t + 1) < 0 \\ \iff & -KF \cdot PK + \sum_{t1=t}^{t+T_{proc}-1} Q_{F,t1}^{ant}\left(\frac{D_{F,t1}^{ant}}{n_{t1}^{ant}}\right) \cdot P_{F,t-1} - \frac{D_{F,t1}^{ant}}{n_{t1}^{ant}} \cdot Pdom_{t1}^{ant}(D_{F,t1}^{ant})(1+r)^{t1-t} < 0 \end{aligned} \tag{51}$$

With:

$$n_{t1}^{ant} = n_{t-1} - newD_{t-T_F} + newD_t + 1 \tag{52}$$

$$Q_{p,t-1} + Q_{M,t-1}(P_{dom,t1}^{ant}) = \frac{D_{F,t1}^{ant}}{n_{t1}^{ant}} + D_{F,t1}^{rem,ant} + D_{dom,t-1}(P_{dom,t1}^{ant}) + D_{X,t-1}(P_{dom,t1}^{ant}) \tag{53}$$

The number of active processing units is then fixed for year t at:

$$n_t = n_{t-1} + newD_{t1} - newD_{t1-T_F} \quad (54)$$

3.3 Analysis of the long-run dynamic system

In the long-run model, the processing industry takes an investment decision by comparing the input price and the quantity available that would result from the short-run equilibrium following this investment, to its long-run average profit function. Figure 7 display the average profit function for the set-up of one factory. We can see that with the anticipated primary agricultural production Q_p, the input price that would result both under monopsonistic and oligopsonistic structure would be around the limit export price. For the quantity that would be consumed, the resulting input price is below the average profit function so this investment would be profitable. At least one factory would be built for this level of anticipated input production, whether under oligopsonistic or monopsonistic structure. We can notice on this Figure that because of the integration of sunk costs in the long-run average profit function, it is not monotonous and display increasing returns to scale for small quantities of input processed.

Figure 8 displays average profit curves for several numbers of factories (1, 10 and 20) in the processing industry and the subsequent equilibria for each situation. The input quantity available without importing (which enables a monopsonistic to pay the limit export price) is limited by the anticipated domestic production. We can see that building too few (one factory) doesn't enable the company to take advantage of all the available input. On the contrary, building too many (twenty factories here) would be inefficient. Therefore, the monopsonistic will choose to set up the number of factories that maximizes its anticipated surplus. Among the three possibilities offered here, it would choose to build ten factories.

Figure 9 present the same curves but under the competitive option that we will associate to the oligopsonistic case. We notice that the anticipated input prices for the subsequent equilibria are higher than in the monopsonistic situation as the market power is reduced. Here, some factories will be built, some companies will enter the market until the anticipated profits are exhausted. Therefore, the positive surplus anticipated for one factory means that more factories will be set-up. The surplus becomes null for ten factories. This will then be number of factories set-up under this structure for this anticipated

Figure 7: Investment decision in the processing industry

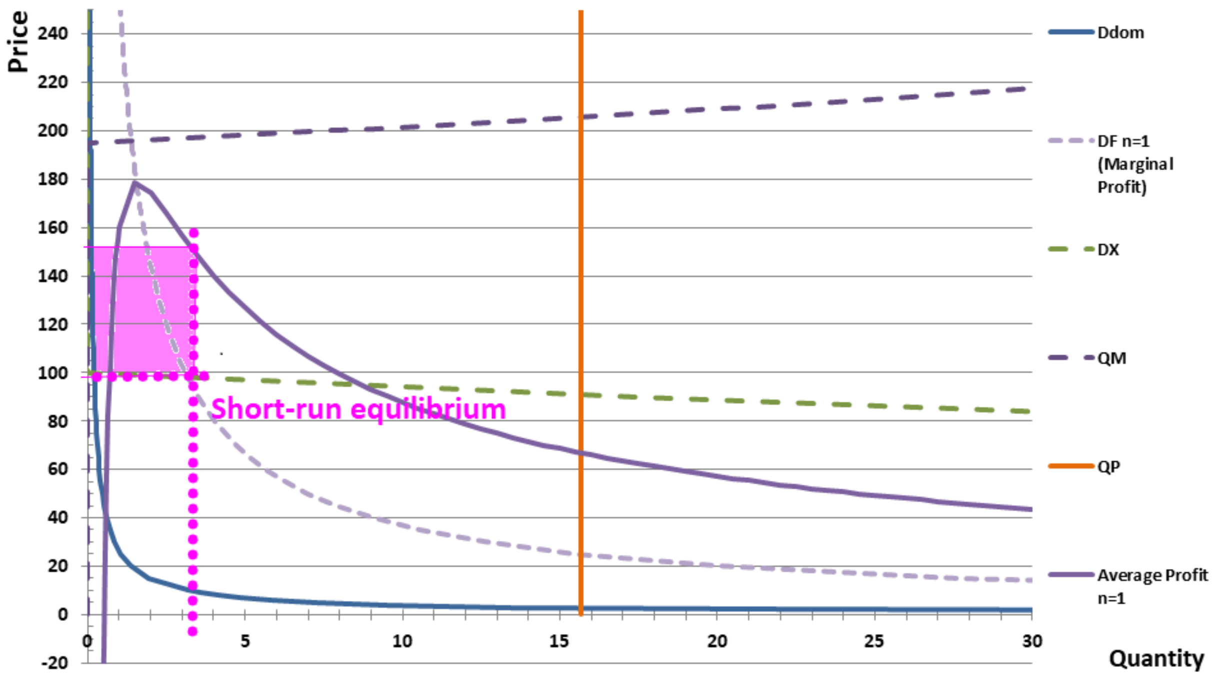
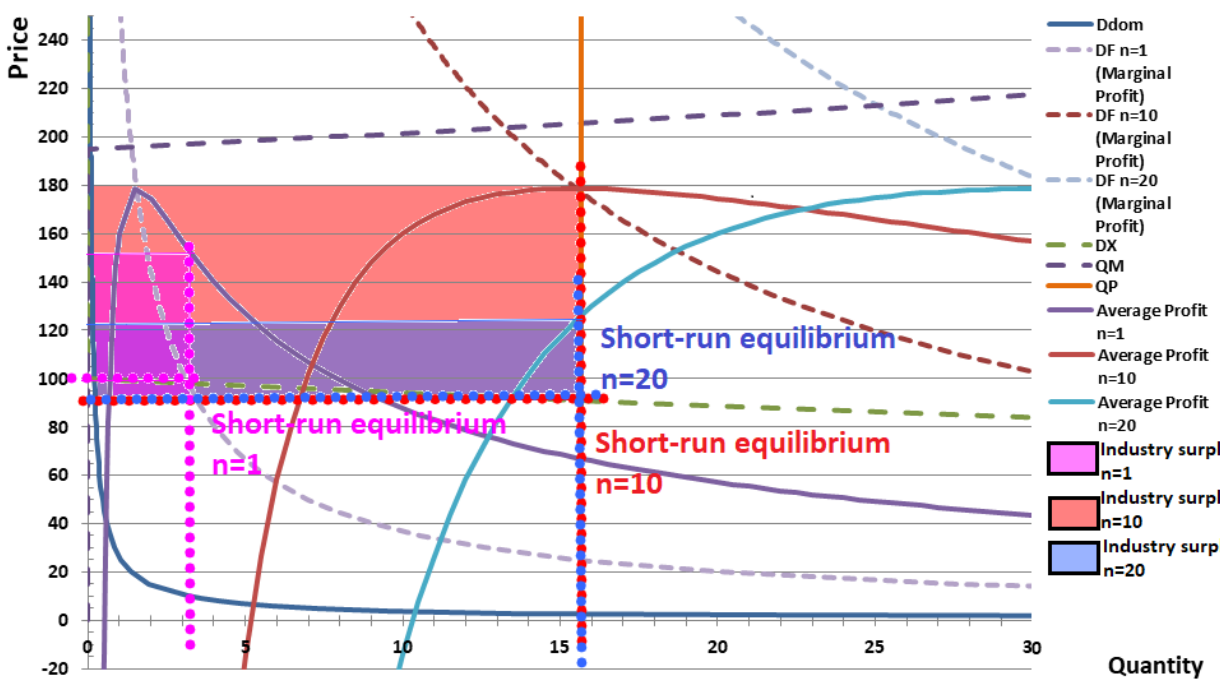
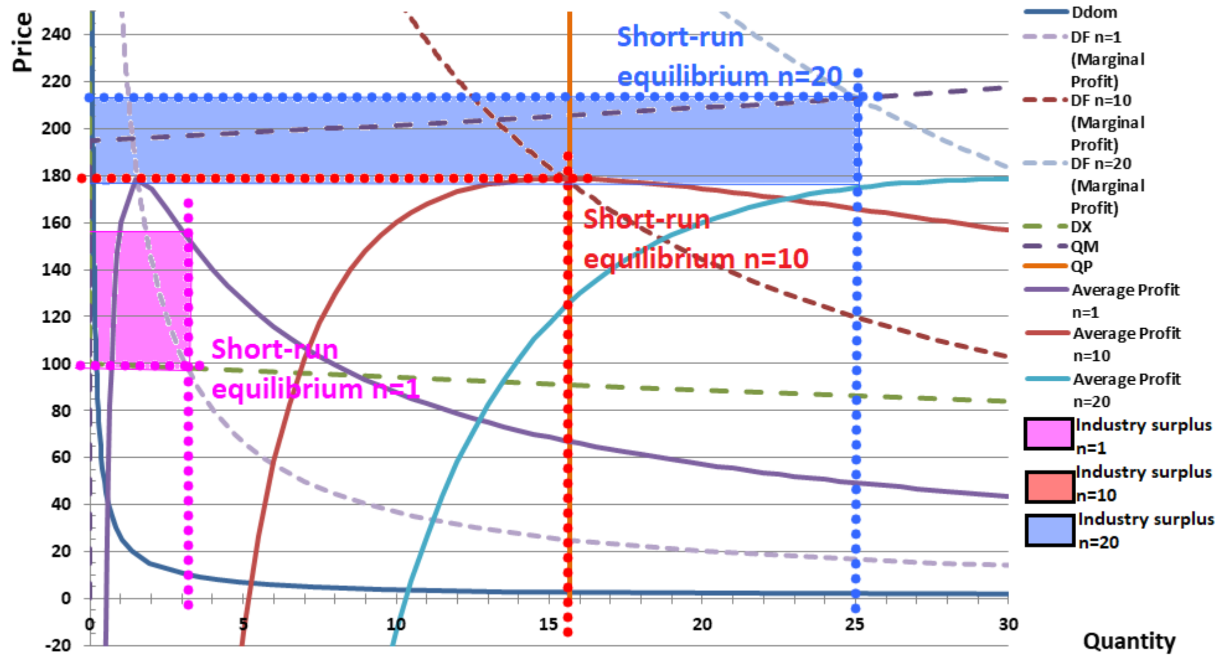


Figure 8: Number of factory decision under the monopsonistic structure



agricultural production. If more factories are built, as in the twenty factories' case, the surplus becomes negative so the number of factories will decrease.

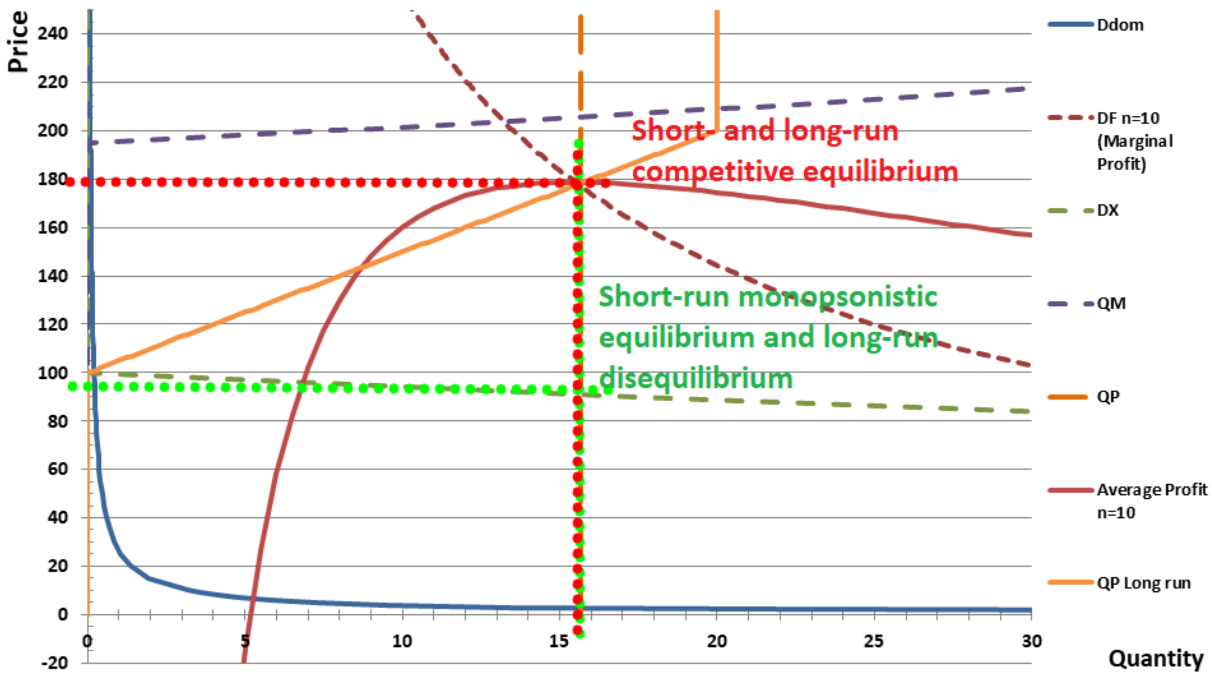
Figure 9: Number of factory decision under the oligopsonistic structure



In order to get a steady state dynamic equilibrium, the short-run equilibrium should coincide with the long-run equilibrium, in other words, the long-run and short-run demand and supply function should cross at the same point. Figure 10 shows such a situation in the competitive situation. However, we notice that the same example would not provide a dynamic equilibrium under the monopsonistic structure. Indeed, the resulting price would be too low to sustain the primary agricultural production at this level.

We can see on Figure 11 that with the same calibration, another long-run dynamic equilibrium is possible. Indeed, when the domestic primary product price is at the limit export price, the domestic agricultural production is very low. For such a level of domestically available input, increasing returns to scale make the set-up of a processing factory unprofitable, whether the industry is under a monopsonistic or oligopsonistic structure. Therefore, there exists a stable state of the system with a reduced agricultural production and no processing industry.

Figure 10: Long-run equilibrium and disequilibrium

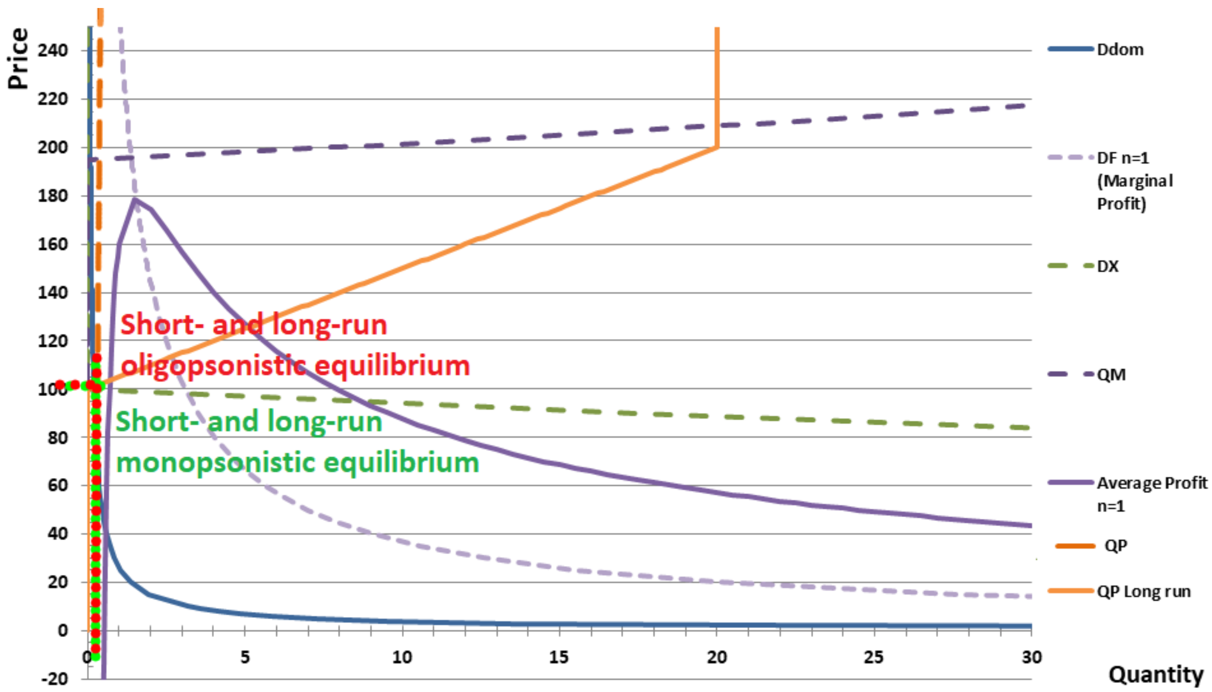


This proves the existence of multiple dynamic equilibria in the model that could result in hysteresis. In the dynamic equilibrium that we observe, an important production in the primary agricultural product goes in pair an important production in the processed product. This highlights the existence of agglomeration economies in this model.

4 Computation of reaction matrices and analysis of the dynamic system

In order to analyze the behavior of the model, we compute numerically the short-run partial equilibrium for a whole set of domestic primary agricultural production quantities and number of factories in the processing industry for an hypothetical case. This enables us to get the resulting state of the system for each of those combinations. Subsequently, we compute the next period long-run dynamic reaction of those variables to each state. This enables us to construct dynamic reaction matrices of the number of factories and of the quantity of local agricultural production for the next period following every possible state of the system. Tables 1a and 1b presents an example of those results under the

Figure 11: Unproductive long-run equilibrium



oligopsonistic option, Tables 2a and 2b presents the results with a similar calibration but under the monopsonistic option. The parameters used for those numerical computations are displayed in appendix B.

Matrices 1a and 2a give us the reaction of the primary agricultural production $Q_{P,t}$ for year t following the anticipations that the farmers make based on the resulting state of the system at year $t-1$ for all the set of couple of values $(Q_{P,t-1}, n_{t-1})$. It enables us in particular to see around which states this production remains at a steady dynamic equilibrium, when $Q_{P,t} = Q_{P,t-1}$. Those steady production states are highlighted by the red-line on Tables 1a and 2a.

We notice as could have been suspected that the higher the number of factories and the smaller the primary agricultural production are at $t-1$, the bigger this production will be at time t. This can be understood as the price $P_{dom,t-1}$ of the primary product at time $t-1$, on which the farmers base their anticipations to invest or not in the primary product, will be higher with less local supply and more demand from the processing factories. We can also notice that under the monopsonistic option, the steady primary

Table 1: Reaction matrices of the primary agricultural production $Q_{P,t}$ and the number of new factories set-up $newD_t$ to a set of possible previous state of the system $(Q_{P,t-1}, n_{t-1})$ under the oligopsonistic option

(a) Reaction of the primary agricultural production $Q_{P,t}$

n_{t-1} \ $Q_{p,t-1}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	20	20	20	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	20	20	20	20	16	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	20	20	20	20	20	20	13	6	1	0	0	0	0	0	0	0	0	0	0	0	0
4	20	20	20	20	20	20	20	18	12	6	2	0	0	0	0	0	0		0	0	0
5	20	20	20	20	20	20	20	20	20	15	11	7	3	0	0	0	0	0	0	0	0
6	20	20	20	20	20	20	20	20	20	20	18	14	10	7	4	1	0	0	0	0	0
7	20	20	20	20	20	20	20	20	20	20	20	20	17	13	10	7	4	2	0	0	0
8	20	20	20	20	20	20	20	20	20	20	20	20	20	19	15	12	9	7	4	2	0
9	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	17	14	12	9	7	5
10	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	19	16	14	11	9
11	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	18	15	13
12	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	19	17
13	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
14	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
15	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

The red line correspond to the states where the primary agricultural production remains steady: $Q_{P,t} = Q_{P,t-1}$

(b) Reaction of the number of new factories set-up $newD_t$

n_{t-1} \ $Q_{p,t-1}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	0	1	1	2	3	4	4	5	6	7	8	8	9	10	11	11	12	13	14	14
1	0	0	0	0	1	2	3	3	4	5	6	7	7	8	9	10	10	11	12	13	13
2	0	0	0	0	0	1	2	2	3	4	5	2	6	7	8	9	9	10	11	12	12
3	0	0	0	0	0	0	1	1	2	1	1	2	5	6	7	8	8	9	10	11	11
4	0	0	0	0	0	0	0	0	1	2	3	1	1	1	5	7	4		9	10	4
5	0	0	0	0	0	0	0	0	0	1	2	3	3	4	1	1	8	9	8	9	4
6	0	0	0	0	0	0	0	0	0	0	1	2	2	3	4	5	5	1	1	8	8
7	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	4	5	6	1	1
8	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	3	4	5	6	6
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	3	4	5	5
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	3	4	4
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	3
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The squares highlighted in green correspond to the states of the system where the number of factory remains steady, even if some factories arrive at their obsolescence limit: $newD(Q_{P,t-1}, n_{t-1} = 0)_t = n_{t-1}$.

production line is at a lower level than in the oligopsonistic case for a given number of factories in the domestic processing industry. This is consequent to the price of the primary product being brought down more by the bigger market power of a monopsonistic industry.

Similarly, matrices 1b and 2b give us the number of processing factories $newD_t$ to be set up at a given year t, following the anticipations that the processing industry make based on the resulting state of the system at year $t-1$, for all the set of couple of values $(Q_{P,t-1}, n_{t-1})$. We can use this information to see when the number of factories will remain steady, even if some factories arrive at their obsolescence limit. Those situations correspond in particular to the states when: $newD(Q_{P,t-1}, n_{t-1} = 0)_t = n_{t-1}$. They are highlighted in green on Tables 1b and 2b.

We can notice that the steady number of factories in the processing industry is as ex-

Table 2: Reaction matrices of the primary agricultural production $Q_{P,t}$ and the number of new factories set-up $newD_t$ to a set of possible previous state of the system $(Q_{P,t-1}, n_{t-1})$ under the monopsonistic option

(a) Reaction of the primary agricultural production $Q_{P,t}$

n_{t-1} \ $Q_{p,t-1}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	20	20	20	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	20	20	20	14	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	20	20	20	20	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	20	20	20	20	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	20	20	20	20	13	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	20	20	20	20	15	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	20	20	20	20	17	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	20	20	20	20	19	9	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	20	20	20	20	20	10	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	20	20	20	20	20	12	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	20	20	20	20	20	13	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	20	20	20	20	20	14	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	20	20	20	20	20	15	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0
14	20	20	20	20	20	16	8	2	0	0	0	0	0	0	0	0	0	0	0	0	0
15	20	20	20	20	20	16	9	2	0	0	0	0	0	0	0	0	0	0	0	0	0

The red line correspond to the states where the primary agricultural production remains steady: $Q_{P,t} = Q_{P,t-1}$

(b) Reaction of the number of new factories set-up $newD_t$

n_{t-1} \ $Q_{p,t-1}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	0	1	1	1	1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	6
1	0	0	0	0	0	0	0	1	1	1	2	2	2	3	3	3	3	4	4	4	5
2	0	0	0	0	0	0	0	0	0	0	1	1	1	0	2	2	2	3	3	3	4
3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	2	2	2	3
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	2
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

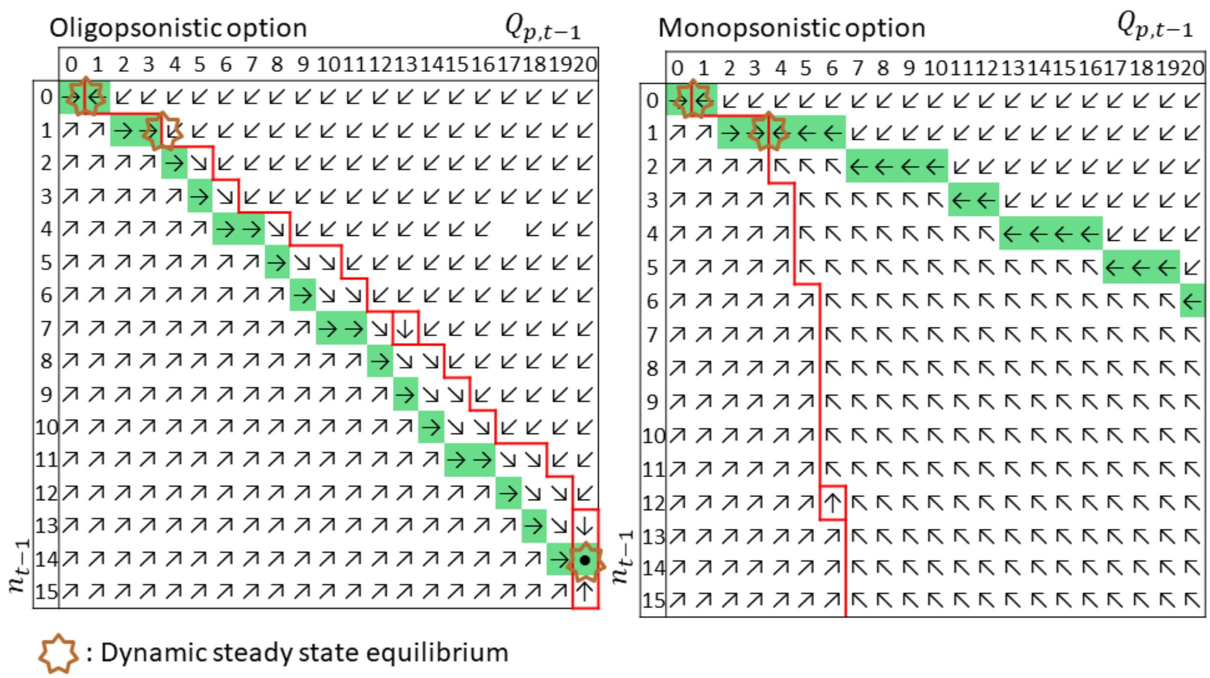
The squares highlighted in green correspond to the states of the system where the number of factory remains steady, even if some factories arrive at their obsolescence limit: $newD(Q_{P,t-1}, n_{t-1} = 0)_t = n_{t-1}$.

pected higher when the domestic primary production in primary product at time $t-1$ is bigger, corresponding to a bigger anticipation of the future supply on the primary product market. More interestingly, we notice that both in the monopsonistic and the oligopsonistic cases, the steady number of factories can be null if the domestic primary agricultural production is too small. This can be explained by the indivisibility of the capital in the processing industry. If there is not enough input to supply at a reasonable price, no factory can take advantage of increasing returns to scale to be profitable enough. We notice also that in the monopsonistic case, the dynamic equilibrium number of factories for a given domestic input supply is smaller than in the oligopsonistic case. This can be explained by the zero-profit-condition which operate under the oligopsonistic option. Contrary to the monopsonistic case, factories will keep being build, so new companies will keep entering until the anticipated profits are exhausted.

Finally, we can combine those information on the reaction of the primary agricultural production and the number of factories to analyze the direction of the system at every state resulting from the set of couple $(Q_{P,t-1}, n_{t-1})$. This is what is made on Figure 12 by constructing diagrams of the direction of the reaction of the system under the oligopsonistic option and the monopsonistic option. Those enable to highlight the steady states equilibrium of the system, when both the primary agricultural production and the number of factories remain steady.

We notice that both under the monopsonistic and the oligopsonistic options, there exists

Figure 12: Diagrams of the long-run dynamic reaction of the moder under oligopsonistic and monopsonistic option



multiple dynamic steady state equilibria where the number of factories in the processing industry and the domestic primary product supply stay constant. This proves the existence of possible hysteresis in this dynamic model of an agri-food sector. We can imagine that initial conditions or exogenous shocks can put the system at one equilibrium or another.

Among the several equilibria, a higher domestic primary agricultural production coincides always with a bigger number of factories in the domestic processing industry. This proves the existence of aggregation economies, with the synergy between farmers that gather and produce the same primary product in a region to supply enough for a domestic processing industry to flourish.

5 Dynamic simulations

In this section, we present some examples of results of dynamic simulations made with this model, calibrated on some hypothetical cases. Those simulations have been made under the oligopsonistic option of the model, however some similar results can be observed under the monopsonistic option. The parameters of the system for those simulations are presented in Appendix B

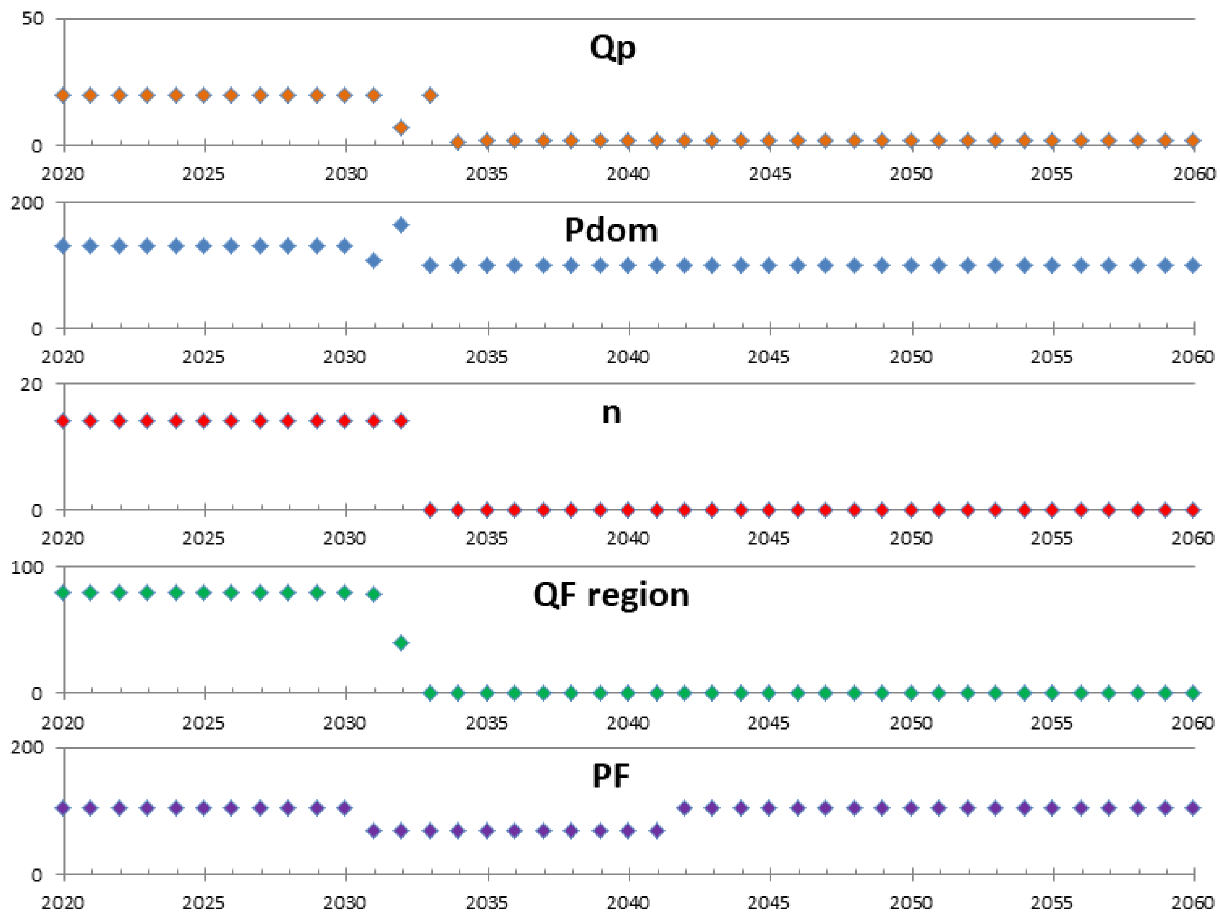
5.1 Hysteretic behavior

Figures 13 and 14 present the results for the domestic primary production $Q_{P,t}$, the domestic price of the primary product $P_{dom,t}$, the number of factories n_t, the total regional production of final processed product $Q_{F,t}^{region} = Q_{F,t} \cdot n_t$ and its price $P_{F,t}$. In both simulation, the system is first at a steady state for the first 30 years so all variables are stable from $t = 2020$ to $t = 2030$. We then shock the exogeneous price of the final processed product $P_{F,t}$ for ten years from 2031 to 2041 with a down-shock for the first simulation on Figure 13 and an up-shock on the second on Figure 14.

We can notice on Figure 13 that on the first two years of the shock, the number n_t of factories remains constant. This is because the durability of the factories does not give incentives to the industry to abandon factories before they reach obsolescence, as they have already been paid for. We notice that in these first two years of the shock, the price of the primary product first decrease in 2031 by transmission of the down-shock on the final product price, however this price spike again in 2032. This is because in 2032, the domestic primary production has reduced following the decrease of the price in 2031. However, the demand remains high in the short-run because some factories are still in place. So this imperfect adjustment in time gives temporarily an erratic movement to prices and supply.

In 2033, the number of factories starts to decrease to reach 0 in 2034. Then, all the endogenous variables of the model are brought down, in particular the price of the primary product and subsequently the domestic primary production. We observe that after the down-shock on the price of the final product is over in 2041, no factories are set-up again, the price of the primary product and the domestic production remain low. This is then an example of hysteretical behavior. The system has reached and remain in a low production steady-state equilibrium. As a consequence of agglomeration economies, no company has incentives to enter the market as the domestic production of input appear too low and reciprocally, the price of the primary product is too low to make more farms switch to the production of primary product again.

Figure 13: Dynamic evolution of the local agri-food system in the case of a down-shock on the final product price



On Figure 14, where we simulate a symmetrical up-shock on the price of the final processed product, the results are quite different. The number of factories increases dramatically as well as the price of the input and the production of final product as soon as 2042, the year following the shock. This is because the processing industry build new factories to take advantage of this high price as soon as it have perceived it. However, the domestic production of primary product remains stable as it has reached its maximum regional production. Therefore, the processing industry imports most of its input. Once, the shock is over, the number of factories remains high for 5 years as some have been built in the last year of the shock. This keeps the demand for primary product high in the short-run and therefore its price. However, once these factories reach their obsolescence, the number of factories gets back to its pre-shock level and the system reach back to its original steady stade equilibrium. Therefore, the hysteresis in this case is limited in time.

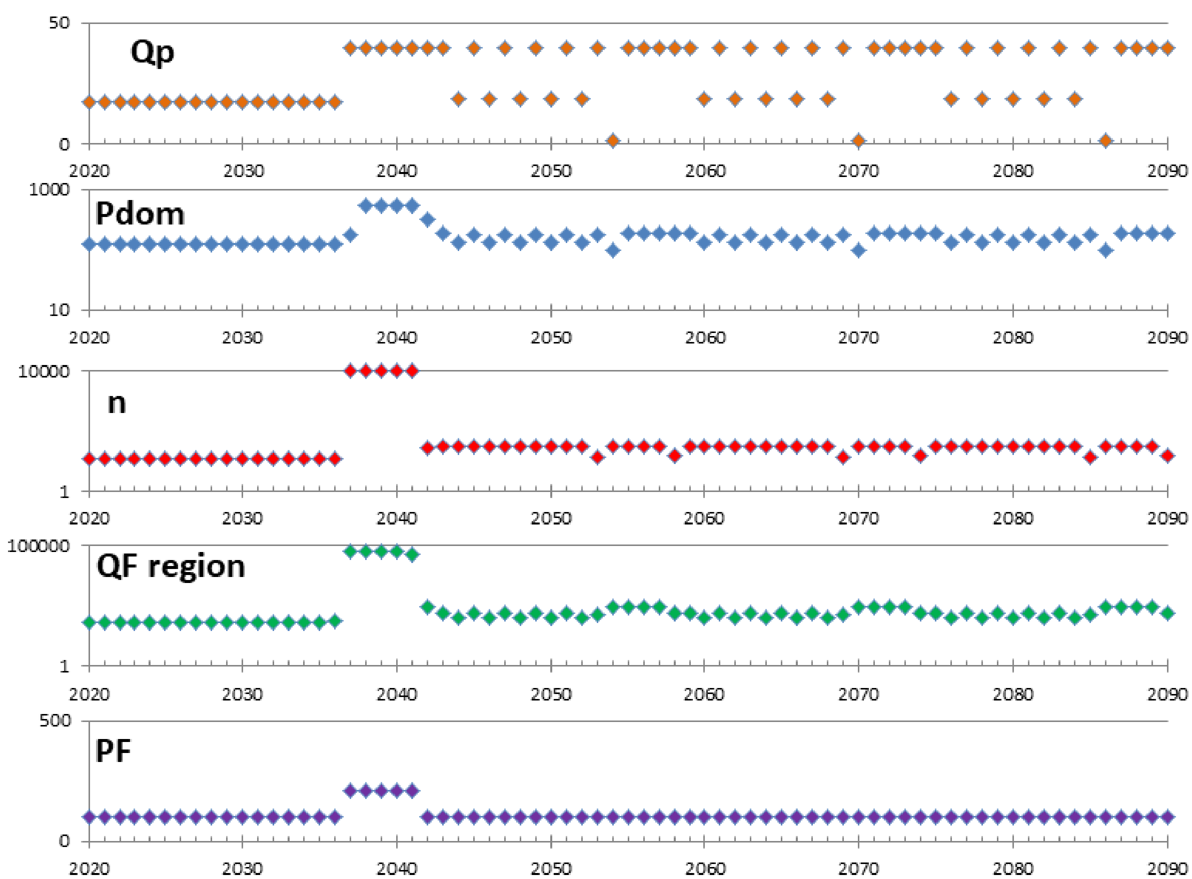
Figure 14: Dynamic evolution of the local agri-food system in the case of a up-shock on the final product price

5.2 Cyclical behavior

The simulation presented on Figure 15 is rather similar to the one on Figure 14. The system is at the origin at a productive steady state equilibrium and we impose an up-shock on the price of the final processed product, here for 5 years from 2036 to 2041. However, in this case the steady state does not correspond to the maximum regional production of primary product.

We then observe similar results as in the simulation on Figure 14, however this time, the system does not transition back to its original steady state equilibrium. Imperfect adjustment in time due to coordination problems among the agents and the lag between long-run investment decision and the short-run market consequences will make the system enter a cyclical behavior. Never the local production of primary product and the number of factories will reach simultaneously there steady-state level, therefore making the system impossible to stabilize.

Figure 15: Dynamic evolution of the local agri-food system in the case of a up-shock on the final product price



This is a consequence of the durability of capital in the model and lags between investment decisions and production. They create those cycles due to imperfect coordination leading to over- and under-investment periods. This feature is particularly interesting as we can find such business cycles in real agri-food sectors (Rieu, 1998; Aadland, 2004; Gouel, 2012). However, on a modeling perspective, this makes the model potentially unstable with an erratic behavior due to the intersection of multi-equilibria and cyclical behavior. If this feature can be representative of a reality, it makes the results complicated to analyze and very dependent to the original calibration.

6 Conclusion

The model presented in this paper is a first attempt at representing the agglomeration economies that can arise in agri-food sectors. We show that the specification of indivisible sunk-costs in the processing industry and transport costs for trade between a domestic region and another can generate agglomeration economies across the value-chains. We show that such external economies of scale can generate multiple steady-state dynamic equilibria and therefore be the source of an hysteretic behavior of the local agri-food system.

Krugman (1991) has shown the importance of the expectation specification to determine the equilibrium toward which an economy with multiple potential equilibria will evolve. We therefore underline the importance of the agents'myopic anticipations assumption in our model to generate the hysteresic behavior of the system. Indeed, we can imagin that a better coordination among the farmers as well as with processing industry would enable the model to always reach the most productive equilibrium. This could be the object of a future option to describe an integrated structure of the local agri-food sector.

This model has been build as a first step toward the integration of aggregation economies and hysteresis in applied model. The duality of the model specification between a short-run partial equilibrium model and a recursive long-run dynamic could enable the model to be plugged to a bigger general-equilibrium model in the future. This could allow the description of the hysteretic consequences of trade shocks transmission from a global system to one region's agri-food sector.

However, numerous modeling challenges remain before reaching this goal. In particular, the description of the processing industry's market power would generate substantial complications to keep integrating the anticipation of the rest of the whole market's reaction to its input demand in its short-run optimization program. Some further assumption on short-run anticipations could enable to tackle this issue. Applying the model to empirical cases would also represent a major calibration challenges. It could be particularly difficult to get relevant informations regarding the capital's durability and state of obsolescence. One would also have to consider the consequences of the heterogeneities across farms and factories : their consequences for the model and which data would enable their description. We may also want to represent the rest of the domestic region's economy as some potential linkages with labor and land markets may be relevant.

Finally, if the potentially cyclical behavior of the model is an interesting feature, giving a representation of the imperfect coordination of the agents investments across time. It could lead to an erratic behavior of the model, at the intersection of cyclical and multi-equilibria behaviors. This makes the results potentially very dependent to initial situation and calibration and overall complexify their analysis. However, this highlights the role of capital durability and imperfect anticipations, leading to business cycles. Looking at the resilience of the system, this also show the potential role of the obsolescence of the capital as we can imagine that the system will not have the same resilience at different points of business cycles.

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Appendices

A Computation of the demand for primary product by the processing industry under perfect competition

Under perfect competition, the processing industry optimize its profit π on its demand for primary product, considering it is price taker on this market, so that P_{dom} is exogenous. It then maximize:

$$\begin{aligned}\pi &= P_F \cdot Q_F(D_F) - D_F \cdot P_{dom} \\ &= P_F \cdot \alpha \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1}{\theta}} - \frac{D_F}{n} \cdot P_{dom}\end{aligned} \tag{55}$$

Over D_F. The first order condition is then:

$$\frac{\partial \pi}{\partial} = 0 \tag{56}$$

It leads to:

$$
\begin{aligned}
P_F \cdot \alpha \cdot (1-\beta) \cdot \frac{(D_F)^{\theta-1}}{n^\theta} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1-\theta}{\theta}} - \frac{P_{dom}}{n} = 0 \\
\Leftrightarrow P_{dom} = P_F \cdot \alpha^\theta \cdot (1-\beta) \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot QF^{1-\theta} \\
\Leftrightarrow P_{dom} = P_F \cdot \alpha \cdot (1-\beta) \cdot \left(\frac{D_F}{n}\right)^{\theta-1} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta\right)^{\frac{1-\theta}{\theta}} \\
\Leftrightarrow (P_{dom})^{\frac{\theta}{1-\theta}} \cdot \left(\frac{D_F}{n}\right)^\theta = (P_F \cdot \alpha \cdot (1-\beta))^{\frac{\theta}{1-\theta}} \cdot \left(\beta K^\theta + (1-\beta) \cdot \left(\frac{D_F}{n}\right)^\theta\right) \\
\Leftrightarrow \left(\frac{D_F}{n}\right)^\theta \cdot \left[(P_{dom})^{\frac{\theta}{1-\theta}} - (1-\beta) \cdot (P_F \cdot \alpha \cdot (1-\beta))^{\frac{\theta}{1-\theta}}\right] = (P_F \cdot \alpha \cdot (1-\beta))^{\frac{\theta}{1-\theta}} \cdot \beta K^\theta \\
\Leftrightarrow D_F = n \cdot \left[\frac{(P_F \cdot \alpha \cdot (1-\beta))^{\frac{\theta}{1-\theta}} \cdot \beta K^\theta}{(P_{dom})^{\frac{\theta}{1-\theta}} - (1-\beta) \cdot (P_F \cdot \alpha \cdot (1-\beta))^{\frac{\theta}{1-\theta}}}\right]^{\frac{1}{\theta}}
\end{aligned}
\tag{57}
$$

B Parameters of the system in the numerical computations presented

Table 3: Parameters of the system for computation in sections 4 and 5.1

a^{dom}	1
P_{dom}^{ref}	100
σ_{dom}	-2
a^{ext}	15 000
P_{ext}^{ref}	150
σ_{ext}	-1.6
Q_0	15 000
τ_M	45
τ_X	50
α	1
β	0.5
K	40
θ	-0.2

δ	0.1
KM	30
PK	45
T_{farm}	1
Q_P^{farm}	20
nF	1000
Q_{farm}^{alt}	20
r	0.05
KF	0.04325
T_F	5
PF	105
P_{alt}	12

b /

Table 4: Parameters of the system for computation in section 5.2

a^{dom}	1
P^{ref}_{dom}	100
σ_{dom}	-2
a^{ext}	15 000
P^{ref}_{ext}	150
σ_{ext}	-1.6
Q_0	15 000
τ_M	45
τ_X	50
α	1
β	0.5
K	40
θ	-0.2

δ	0.1
KM	30
PK	45
T_{farm}	1
Q^{farm}_P	40
nF	1000
Q^{alt}_{farm}	20
r	0.05
KF	0.087
T_F	5
PF	105
P_{alt}	75