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Time Use and the Care Economy in Latin America: A General Equilibrium Analysis

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1. Introduction

In Latin America – and elsewhere – (Maybud, 2015; Lopez Montaño, 2015; DANE, 2017), women of all ages are the principal providers of care for children, the elderly, the disabled, and for whole families and communities, at home and in private and public institutions. For instance, according to ILO, there are over 53 million domestic workers across the world and 83 percent of them are women. For instance, the care economy in Colombia as defined below represents between 19.1 and 20.4 percent of GDP depending on the valuation method (López Montaño et al., 2015). In addition, DANE (2017) shows that among women (men), between 2012 and 2015, 60 (18.8) percent worked on the care economy outside the System of National Accounts (SNA). In 2012-2013, women represented 79.4 percent of the unpaid workers in activities including those outside the SNA.

In this study, and following Alexander and Baden (2000) (p. 22), we broadly define the care economy as "the part of human activity, both material and social, that is concerned with the process of caring for the present and future labor force, and the human population as a whole, including the domestic provisioning of food, clothing and shelter". In addition, the authors add that "social reproduction is the provisioning of all such needs throughout the economy, regardless of whether it is of a paid or unpaid nature".[1]

In this paper, our interest is in assessing the impact of various policy measures on the care economy in Latin America. For instance, we quantify the – short- and long-run -- macro and meso impacts of an increase in the government provision of goods and services related to the care economy.

[1] ILO provides a narrower definition stating that the care work is broadly defined as looking after the physical, psychological, emotional and developmental needs of one or more other people (Maybud, 2015). It comprises paid or unpaid direct care of young children, frail elderly, Ill, and disabled. Besides, the care economy spans the public and private spheres, with a wide range of providers such as formal or informal domestic workers, childcare services, hospitals, convalescence homes, and hospices.

In terms of results, we expect to find relevant impacts from simulating changes in the government supply of services related to the care economy, such as childcare services. Furthermore, we plan to single out the key parameter governing the impact of such shocks. In fact, we expect that parameters such as the elasticity of substitution between male and female workers and the price-elasticity of domestic work will have a significant impact on our quantitative results. Thus, we will perform a sensitivity analysis regarding the value assigned to these particular parameters.

2. Method and Data

Model Structure

In terms of method, we extend GEM-Core, a recursive-dynamic CGE model documented in Cicowiez and Lofgren (2017), to make it gender-aware and single out aspects related to the care economy. In terms of literature, our model contains the elements that Fontana (2013) describes as those required to analyze gender issues in general and the care economy in particular.[2] Specifically, our model includes: (1) male and female labor categories, (2) formal and informal segments in the labor market[3], (3) unpaid domestic work, (4) unpaid labor in activities not related to the care economy such as family workers in agricultural activities, and (5) labor-leisure choice or labor supply decision for men and women separately. In Latin America, female employment – both paid and unpaid -- predominates in sectors related to the care economy; consequently, female labor supply is highly dependent on their development.

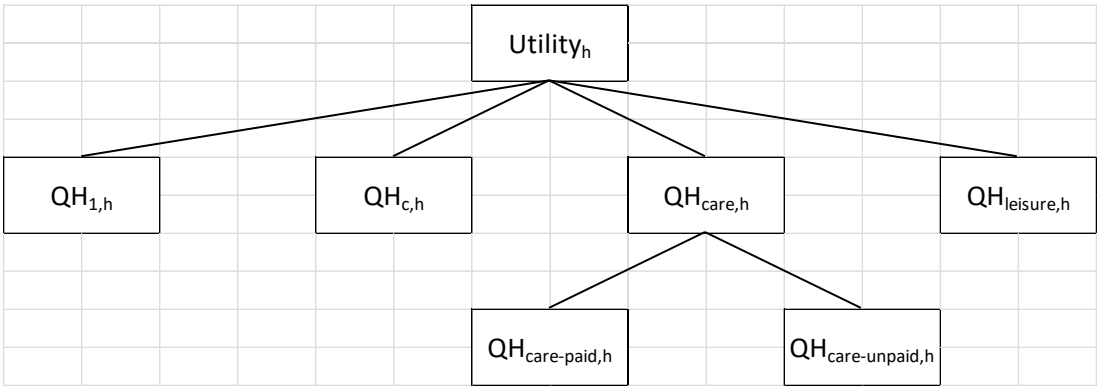
In the model, activities related to the unpaid care economy and "leisure production" are modeled as any other activity in the model; i.e., they are represented by a production function with their output (only) sold in the domestic market. However, unlike other activities, these activities only use labor -- mainly female in the first case -- as input.

[2] Also, our model has similarities with those of Fontana and Wood (2000), Fofana et al. (2005), and Mitik and Decaluwé (2009).

[3] Certainly, distinguishing between the formal and informal segments of the labor market is particularly relevant when discussing female work and the care economy (Maybud, 2015).

Alternatively, the demand of unpaid care services and leisure can be considered as home production for home consumption by the households. In the model, it is required that households demand unpaid care services and leisure produced using their own labor endowment. Technically, incorporating labor/leisure trade-off as described here implies that the consumption basket of a typical representative household comprises of goods, services, and leisure; i.e., household utility is defined at "full consumption" levels rather than only on commodities (see Figure 2.1).

Figure 2.1: utility function



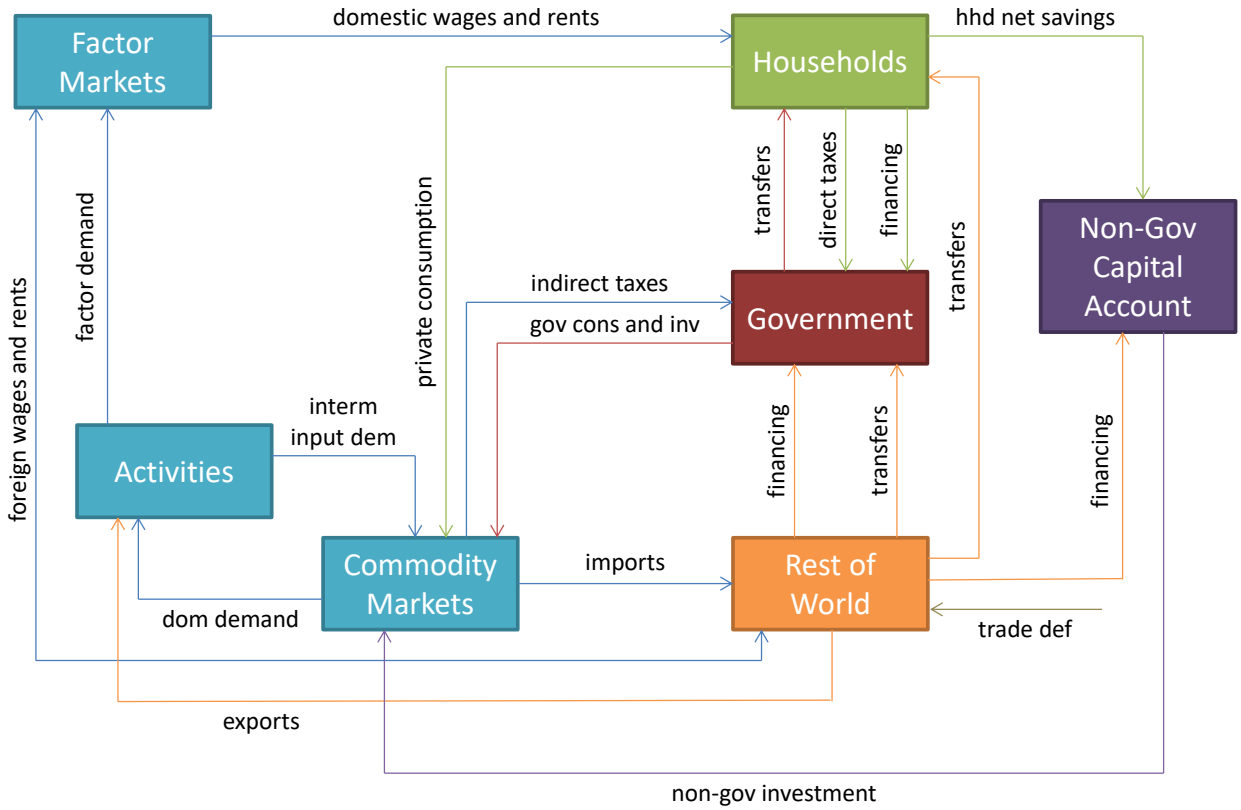
Source: Authors' elaboration.

In this paper, our focus is the interaction between the market and non-market provision of services related to the care economy. Consequently, we assume that households can substitute between unpaid care work – in LA, usually provided by mostly female family members -- and care services provided by the market and the government. Firstly, household consumption expenditure is distributed across (composite) commodities according to a Stone-Geary utility function, from which a linear expenditure system is derived. Secondly, households decide on the composition of the composite commodities through a Constant Elasticity (CES) function. Alternatively, the model allows for a different classification of consumer and producer goods and services. In fact, a "make matrix" is used

where each consumed good can be composed of one or more supplied goods, combined using a CES function.[4]

In any single year, GEM-Core has the structure summarized in Figure 2.2. As indicated by the figure, which serves as the reference point for this model overview, the major building blocks of the model are activities (the entities that carry out production), commodities (activity outputs and/or imports; linked to markets), factors (also linked to markets), and institutions (households, enterprises, the government, and the rest of the world). Given the relatively detailed treatment of the financing of private investment (compared to most other CGE models), the private (non-government) capital account also has its own box. In the following model presentation, we assume that the different blocks have the disaggregation presented in the above SAM (Table 2.1).

Figure 2.1. Overview of GEM-Core



[4] In fact, our treatment is more general, allowing for a household-specific "make" or transition matrix mapping consumed commodities to supplied commodities. In this application, and similarly to Ferri et al. (2005), we use CES sub-utility functions to combine consumption both goods and services both within and beyond the SNA boundary.

Source: Authors' elaboration.

Data

The database for GEM-Core consists of a SAM complemented by data related to factor employment, factor and population stocks, elasticities, and a GDP projection.

In 2010, Colombia was the first country in the Latin American region to pass a Law (Ley 1413) instructing the National Administrative Department of Statistics (DANE) to expand the National Accounts by developing a Satellite Account on the Economy of Care (Cuenta Satélite Economía del Cuidado) and the National Survey on Time Use (Encuesta Nacional de Uso del Tiempo; ENUT). Thus, in this paper we assume that Colombian data regarding the care economy is representative of LA. Naturally, the availability of official statistics regarding the care economy greatly facilitates the process of building a SAM for an archetype Latin American country extended beyond the boundaries of the SNA.

Thus, we first built a relatively standard 2015 Social Accounting Matrix (SAM) for using the supply and use tables and the integrated economic accounts for various Latin America countries. Besides, in order to expand the SAM beyond the SNA, we use the 2014 Satellite Account on the Economy of Care (Cuenta Satélite Economía del Cuidado) and the 2013-2014 National Survey on Time Use (Encuesta Nacional de Uso del Tiempo; ENUT). Specifically, we used these two sources of information to account for gender and the care economy in our 2015 extended SAM for Latin America.

Naturally, in order to study the care economy, the SAM (and model) disaggregation has to be relevant in terms of factors (i.e., by gender, skill level, and formality status), households (rural/urban, male/female household head), and industries (i.e., male and female dominated sectors). For each labor category, we use the ENUT to compute hours worked in activities within the SNA, hours spent working in the care economy outside the realm of the SNA, and hours spent in leisure activities (see Figure 2.3 and Figure 2.4). Interestingly, we do not use the number of workers to determine the labor input used by the activities. Also, our SAM differentiates between paid work and unpaid work such as (a) contributing to family

business; (b) activities for family consumption (e.g., fetching food, water, or fuel); and (c) meal preparation, shopping, laundry, and cleaning.

Figure 2.3: hours worked within and outside the NA

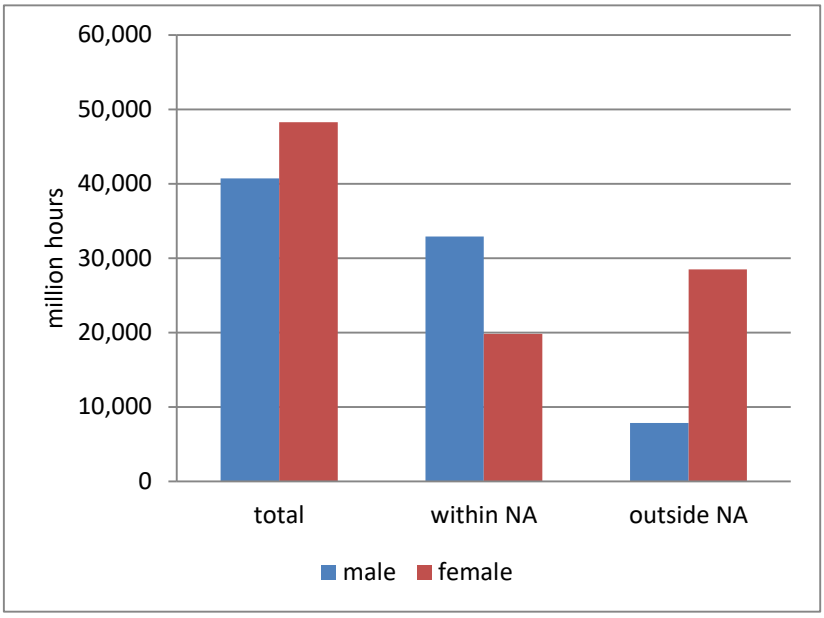
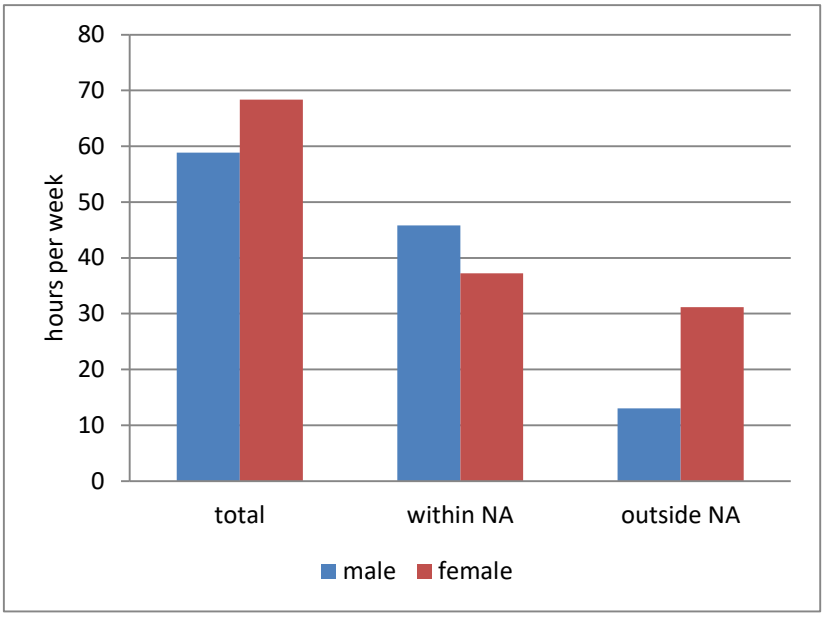


Figure 2.4: average hours per week worked within and outside the NA



3. Simulations

[TBC]

4. Conclusions

[TBC]

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Appendix A: Extended GEM-Core Mathematical Statement

This appendix presents a mathematical statement of GEM-Core, showing the relationships that, together with the database, determine the results of model simulations. A good understanding of the structure of the model and its database is needed to well understand the simulation results. The appendix is divided into two subsections, notation (A.1) and equations (A.2). Throughout, the presentation is organized around a set of tables.

A.1. Notation

Table A.1.1 explains notational principles, designed to make it easy to understand the statement. Tables A.1.2-A.1.5 define model sets, variables, Latin-letter parameters, Greek-letter parameters, respectively. In each of these tables, the items are arranged alphabetically. Given that this model is dynamic, a time index is part of the domains of all variables and the parameters that are most likely to change over time.

All model components are potentially active but whether they are used in any given application depends on the disaggregation of the database. In Table A.1.2 the right-most column shows the set definitions in the macro application of this paper. Parameters, variables, and equations for which the domains are empty sets are not part of the model and most of those in the model have very few set elements; as a result, the model that is solved is very small. In addition, the sets over which the model items are defined have very few items; as a result, the model that is solved is in fact very small.[5]

Table A.1.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	see the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	QG_c

[5] More concretely, as a consequence of the fact that the sets *FOTH, TAC, TACD, TACE,* and *TACM* are empty (Table A.1.2) and in the absence of an explicit subsidy account in the SAM, the model in this application does not include the variables *QT* and *SUBCT* (Table A.1.3), the parameters *icd*, *ice*, and *icm* (Table A.1.4), and the equations TRD13 (Table A.2.2) and INS13 (Table A.2.3). To exemplify the role of few members in most sets, note that, in each year *t*, the variable $QE_{c,t}$ is only defined over one *c*, the private commodity.

Exogenous variables**	Upper-case Latin letters with a bar*	$\overline{QFS}_f$
Parameters**	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$; ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and factor wages (rents) start with *P*, *Q*, and *WF*, respectively.
**The distinction between exogenous variables and parameters is that the latter always have exogenous values whereas the former under alternative assumptions may be endogenous.

Table A.1.2. Sets

Name	**Description**	**Elements**
$a \in A$	activities (production sectors or industries)	act-prv, act-gov
$c \in C$	commodities (i.e., goods and services)	com-prv, com-gov
$c \in CD(\subset C)$	commodities with domestic sales of domestic output	com-prv, com-gov
$c \in CE(\subset C)$	exported commodities	com-prv
$c \in CM(\subset C)$	imported commodities	com-prv
$c \in CT(\subset C)$	transactions commodities (services paid under distribution margins)	
$d \in D$ $(INSD \cup FCAP \cup A \cup TAC)$	domestic demanders (or demand types): institutions (for consumption), investment by capital type, activities, transactions (distribution margins)	hhd, gov, f-cap, act-prv, act-gov
$f \in F$	factors	f-lab, f-cap, f-capg
$f \in FVA(\subset F)$	factors that earn value added (in SAM)	f-lab, f-cap
$f \in FCAP(\subset F)$	capital factors	f-cap
$f \in FCAPG(\subset FCAP, \not\subset FVA)$	gov't capital factors (do not earn value-added)	f-capg
$f \in FCAPNG(\subset FCAP, \subset FVA)$	non-gov't capital factors (earn value-added)	f-cap
$f \in FLAB(\subset FVA)$	labor factors (earn value-added)	f-lab
$f \in FOTH(\subset FVA,$ $\not\subset FLAB, \not\subset FCAP)$	other factors (earn value-added; not capital or labor)	
$f \in FUEND(\subset FVA)$	factors with endogenous unemployment rate	f-lab
$i \in INS$	institutions	hhd, gov, row

$i \in INSD \left(\subset INS \right)$	domestic institutions	hhd, gov
$i \in INSDNG \left(\subset INSD \right)$	domestic non-government institutions	hhd
$i \in INSNG \left(\subset INS \right)$	non-gov't institutions (rest of world and elements in *INSDNG*)	hhd, row
$h \in H \left(\subset INSDNG \right)$	households	hhd
$t \in T$	time periods (simulation years)	2015-2030
$t \in TMIN$	base period (first simulation year)	2015
$tac \in TAC$	transactions (distribution) types (domestic, import, export)	--
$tacd \in TACD \left(\subset TAC \right)$	transactions (distribution) for domestic sales	--
$tace \in TACE \left(\subset TAC \right)$	transactions (distribution) for exports	--
$tace \in TACM \left(\subset TAC \right)$	transactions (distribution) for imports	--

Table A.1.3. Variables

Name	**Description**
CPI_t	consumer price index
$DKA_{f,a,t}$	change in capital stock *f* allocated to activity *a*
$DKINS_{i,f,t}$	investment by institution *i* (in *INS*) in capital stock *f*
DPI_t	domestic producer price index (PDS-based)
EG_t	total current government expenditure
$EH_{h,t}$	consumption expenditure for household *h*
EXR_t	exchange rate (local currency per unit of foreign currency
$INV_{i,t}$	value of investment (including stock change) for institution *i* (in *INSNG*)
$INVG_t$	value of investment (including stock change) for government
$MPS_{i,t}$	marginal propensity to save for domestic non-government institution *i* (in *INSDNG*)
$MPSSCAL_t$	*MPS* scaling factor
$NFFG_t$	net foreign financing of government (FCU)
$NFF_{i,t}$	net foreign financing for non-government institution *i* (in *INSDNG*) (FCU)
$PA_{a,t}$	output price for activity *a*
$PDD_{c,t}$	demand price for commodity *c* (in *C*) produced and sold domestically
$PDS_{c,t}$	supply price for commodity *c* (in *C*) produced and sold domestically
$PE_{c,t}$	price for export of *c* (in *C*) (LCU) (net of export taxes and distribution margin)

$PK_{f,t}$	price (per unit of) of capital stock *f*
$PM_{c,t}$	price for import of *c* (in *C*) (LCU) (includes import tariffs and distribution margin)
$PQD_{c,d,t}$	composite commodity price for *c* (in *C*) for domestic demander (type) *d* (in *D*) [includes commodity subsidies, all taxes (including VAT and sales tax), and distribution margins]
$PQS_{c,,t}$	composite commodity price for *c* (includes import tariffs and distribution margins but not sales tax, commodity subsidies, or VAT)
$PX_{c,t}$	producer price for commodity *c*
$PVA_{a,t}$	value-added price for activity *a*
$QA_{a,t}$	level of activity *a*
$QD_{c,t}$	quantity sold domestically of domestic output *c*
$QE_{c,t}$	quantity of exports of commodity *c* (in *C*)
$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$QFINS_{i,f,t}$	endowment of institution *i* (in *INSD)* of factor *f*
$QG_{c,t}$	quantity of government consumption of commodity *c*
$QGSCAL_t$	government consumption scaling factor
$QH_{c,h,t}$	quantity consumed of commodity *c* by household *h*
$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*
$QINV_{c,t}$	quantity of investment demand for commodity *c* (investment by source)
$QINVSCAL_t$	investment scaling factor
$QM_{c,t}$	quantity of imports of commodity *c* (in *C*)
$QQ_{c,t}$	quantity of composite demand (and supply) of commodity *c* (in C)
$QT_{c,t}$	quantity of trade and transport services demand for commodity *c* (in C)
$QX_{c,t}$	quantity of domestic output of commodity *c* (in C)
$RGDPMP_t$	real GDP at market prices (at constant base-year prices)
$SAVF_t$	foreign savings (FCU)
$SAVG_t$	government savings
$SAV_{i,t}$	savings of domestic non-government institution *i* (in *INSDNG*)
$SHIF_{i,f,t}$	share for institution *i* (in *INSD*) in the income of factor *f*
$SUBCT_t$	government spending on commodity subsidies
$TFP_{a,t}$	total factor productivity for activity *a*
$TFPSCAL_t$	scaling of total factor productivity
$TRDGDP_t$	real foreign trade (exports+imports) and GDP ratio
$TRII_{i,i',t}$	transfers to institution i (in *INS*) from domestic non-government institution *i'* (in *INSDNG*)

$UERAT_{f,t}$	unemployment rate for factor *f*
$WALRAS_t$	variable check on Walras' law (which is satisfied if value is zero)
$WF_{f,t}$	economywide wage of factor *f*
$WFAVG_{f,t}$	average wage for factor *f* (in *FCAPNG*)
$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$YF_{f,t}$	income of factor *f*
YG_t	government current revenue
$YI_{i,t}$	income of (domestic non-government) institution *i* (in *INSDNG*)
$YIF_{i,f,t}$	income of institution *i* (in *INSD*) from factor *f*

Table A.1.4. Latin letter parameters

Name	**Description**
$capcomp_{c,f}$	quantity of commodity *c* per unit of new capital stock *f*
$cwts_{c,h}$	weight of commodity *c* in consumption basked of household *h*
$depr_{f,t}$	rate of depreciation for capital stock *f*
drf_t	change in foreign reserves (FCU)
$dwts_c$	weight of commodity *c* in the DPI (PDS-based producer price index)
$ica_{c,a}$	quantity of intermediate input *c* per unit of activity *a*
$icd_{c,c'}$	input of *c* for trade and transportation per unit of commodity *c*' produced and sold domestically
$ice_{c,c'}$	transactions input of *c* per unit of commodity *c*' export
$icm_{c,c'}$	transactions input of *c* per unit of commodity *c*' imports
$invshr_{f,i,t}$	share for capital stock *f* in investment spending of institution *i* (in *INSNG*)
$mpsb_{i,t}$	baseline marginal propensity to save for domestic non-gov't institution *i* (in *INSDNG*)
$ndfg_t$	net domestic financing to government (indexed to numéraire) (FCU)
$nff_{i,t}$	net foreign financing to institution *i* (in *INSD*) (FCU)
$pop_{ac,t}$	population of *ac* (household *h* in *H* or country total)
$pwe_{c,t}$	export price for commodity *c* (in foreign currency)
$pwm_{c,t}$	import price for commodity *c* (in foreign currency)
$qdstk_{c,i,t}$	change in stock (inventories) of *c* for institution *i* (in *INSD*)
$qfinsb_{i,f,t}$	endowment for institution *i* (in *INSD*) of factor *f* (in *FOTH*)
$qgb_{c,t}$	baseline quantity of government consumption of commodity *c*
$qg01_{c,t}$	0-1 parameter turning on-off potential scaling of gov consumption of *c*
$qinvb_c$	base-year quantity of investment (GFCF) demand for *c*

$shii_{i,i'}$	share of institution *i* (in *INS*) in the income (net of direct taxes and savings) of domestic non-gov't institution *i'* (in *INSDNG*)
$sub_{c,d,t}$	rate of subsidy on commodity *c* (in *C)* for demander *d* (in *D*)
$ta_{a,t}$	rate of tax on gross output value for activity *a*
$te_{c,t}$	rate of tax on commodity *c*
$tf_{f,t}$	rate of direct tax on factor *f*
$tfpb_{a,t}$	exogenous component of TFP for activity *a*
$tm_{c,t}$	rate of import tariff on commodity *c*
$tq_{c,t}$	rate of sales tax on commodity *c*
$trnsfr_{ac,i,t}$	transfers from institution i (gov't or rest of world) to *ac* [where *ac* is institution *i* (in *INS*) or factor *f* (in *F*)](LCU if from gov't; FCU if from rest of world)
$tva_{c,d,t}$	rate of value-added tax on commodity *c* (in *C*) for demander *d* (in *D*)
$ty_{i,t}$	rate of direct tax on domestic non-gov't institution *i* (in *INSDNG*)
$ueratb_{f,t}$	exogenous unemployment rate for factor *f* (not in *FUEND*)
wfb_f	exogenous economywide wage term for activity-specific factors
$wfdistb_{f,a}$	exogenous activity-specific wage term for mobile factors

Table A.1.5. Greek letter parameters

Name	**Description**
$\alpha_{i,t}^{sav}$	intercept in savings function for institution *i* (in *INSDNG*)
$\beta_{c,h}$	share parameter in LES function for household consumption of commodity *c*
$\gamma_{c,h}^{min}$	minimum quantity in LES function for household consumption of commodity *c*
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity *c* (top of nest)
δ_c^{ds}	share parameter for domestic sales in CET function for commodity *c* (top of nest)
δ_c^{e}	share parameter for exports in CET function for aggregated commodity *c* (in *C*) (top of nest)
δ_c^{m}	share parameter for imports in Armington function for commodity *c* (top of nest)
$\delta_{f,a}^{va}$	share parameter for factor *f* in CES VA function for activity *a*
$\eta_{a,f}^{tfp}$	elasticity of TFP in activity *a* with respect to gov't capital stock *f*
η_f^{wf}	elasticity of wage for factor *f* (in *FUEND*) with respect to unemployment rate
$\theta_{a,c}$	yield of output *c* per unit of activity *a*
κ_f	sensitivity of the allocation of new capital for *f* (in *FCAPNG*) across activities (in *A*) to current deviations of activity capital rents from the

	economywide average
ρ_c^q	exponent in Armington function for commodity *c*
ρ_a^{va}	exponent in CES VA function for activity *a*
ρ_c^x	exponent in CET function for commodity *c* (top of nest)
σ_c^q	elasticity of substitution between supplies of domestic output and imports in Armington function for *c* (top of nest)
σ_a^{va}	elasticity of substitution between factors in CES VA function of activity *a*
σ_c^x	elasticity of transformation between domestic sales and exports in CET function for *c* (top of nest)
φ_c^q	shift parameter in Armington function in which domestic sales and imports of commodity *c* (in *C*) are aggregated to composite supply *(*top of nest)
φ_a^{va}	shift parameter for CES VA function of activity *a*
φ_c^x	shift parameter in CET function for commodity *c (*top of nest)

A.2. Equations

The equations are split into four blocks:

1. Production and factors;
2. Domestic and aggregate foreign trade;
3. Current accounts of domestic institutions
4. Investment, system constraints, and numéraire.

Each section of the presentation covers one block and has its equations stated in one table. In model simulations, it is possible to choose among alternative assumptions for (i) payments linking the government, domestic non-government institutions, and the rest of the world; and (ii) the equilibrating mechanisms (the closures) for macro balances, factor markets, and markets for exports and imports. The assumptions used in this paper are presented in Section 3.[6] In this appendix, we apply the following set of relatively simple assumptions:

- Government budget: The government balance is cleared by adjustments in government investment in the context of rule-based or exogenous levels for other government payments (including exogenous values for tax rates, quantities of government consumption, and foreign and domestic financing).

[6] The User Guide that accompanies ISIM-GEM-Core provides full details on these and other model features.

- Savings-investment: The level of domestically financed private investment is determined by the level of financing from domestic non-government institutions, for which the marginal propensities to save are fixed. Government investment is financed as part of the government budget.
- Balance of payments: The balance is cleared by adjustments in the real exchange rate, which influence export and import quantities and values; other items in the balance of payments (including transfers, foreign investment, and net foreign financing) are exogenous or determined by other rules.
- Factor markets:
    - Private capital is activity-specific (not mobile across activities), with an activity-specific market-clearing wage.
    - Other factors (including labor) are mobile across activities; unemployment is endogenous for selected factors (typically labor).
- Foreign markets for exports and imports. Both world export and import prices are exogenous (i.e., the small country assumption).

A.2.1. Production and factors

The equations in this block are found in Table A.2.1. They cover the determination of production by sector, demands for factors and intermediates, TFP, factor wages (or rents), unemployment, and factor incomes.

The activity levels (*QA*), which drive the level of commodity production by each activity, is a CES function of factor employment, scaled to account for the contribution of intermediate inputs (PRD1). Factor demands (*QF*) are a function of the parameters of the production function, wages, and the price of value added (i.e., the payment to factors per unit of the activity), in a setting where the producers maximize profits while taking prices and wages as given (PRD2). *TFP* by activity is a function of an exogenous trend parameter, a scaling parameter (which typically is endogenous for the base simulation but otherwise exogenous), ratios between current and base-year government capital stocks, and openness to trade as the ratio between (a) the sum of real exports and imports; and (b) real GDP (PRD3). (The

latter ratio is defined in Table A.2.2.) In this equation, the impacts of government capital stocks and openness to trade are both captured by a constant-elasticity formulation.

Other variables related to production are determined by activity levels, other parameters, and prices. Intermediate demands (*QINT*) are a Leontief fixed-coefficient function of activity levels (PRD4). Likewise, commodity output levels (*QX*) are driven by activity levels multiplied by fixed yield coefficients (θ), summed over all relevant activities (PRD5). Depending on the values of the yield coefficients, any commodity may be produced by more than one activity and any activity may produce more than one commodity. The value-added price (*PVA*), which appeared above in the factor demand functions (PRD2), is defined as the price (or revenue) per unit of an activity (*PA*) net of activity taxes and the intermediate input cost per activity unit (PRD6). For any activity, *PA* is the product of yields and unit producer prices, summed over all outputs (PRD7).

The treatment of factor markets is rich, making it possible for the analyst to select among alternative assumptions with regard to mobility, unemployment, and supply growth. In this mathematical statement, we assume that (i) private capital his fully employed and activity-specific (with endogenous allocations of private capital created by new investment); and (ii) that other factors (labor and natural resources, if any) are mobile and may or may not have endogenous unemployment (depending on a set definition). Other configurations are possible.

Table A.2.1 shows the treatment of the markets for factors other than private capital, which is treated in Table A.2.4 given its links to investment and its special treatment of mobility. For non-labor factors, the unemployment (excess-capacity) rates are fixed (PRD8). For labor, wages are determined by a "wage curve", which is a function of the base-year wage and the ratios between current and base-year values for the CPI (the numéraire, which in practice does not change) and the unemployment rate, *UERAT*, which is endogenous and raised to a negative elasticity (PRD9). For all factors, the activity-specific wage term (*WFDIST*) is fixed (PRD10) and, irrespective of whether unemployment is endogenous or not, the factor market equilibrium conditions state that total employment equals total supplies adjusted for unemployment (or excess capacity) (PRD11).

Given the above-stated treatment, the factor market equilibrium conditions (PRD11) are cleared via adjustments in the economywide wage variable (*WF*). For factors not in the set

FUEND, the quantities supplied for employment (the RHS of PRD11) are fixed in any given time period; given this, the full adjustment burden falls on the LHS and the quantities demanded (defined in equation PRD2). For factors in *FUEND* (often labor), the adjustment is shared between the demand and supply sides. For example, for the case of excess demand (in PRD11, LHS is larger than RHS), an increase in *WF* would simultaneously (i) reduce *QF* and the LHS value of PRD11 (via PRD2); and (ii) reduce the unemployment rate (*UERAT* via PRD8), thereby raising the RHS value of PRD11.

Irrespective of the treatment of the markets, the total income for each factor (*YF*), also including private capital, is the product of the two wage terms and quantities employed, summed over all activities, plus net factor transfers (or income) from abroad, adjusted for the exchange rate (PRD12).

Table A.2.1. Equations for production and factors

PRD-1	$QA_{a,t} = TFP_{a,t} \cdot \varphi_a^{va} \left(\sum_f \delta_{f,a}^{va} \cdot QF_{f,a,t}^{-\rho_a^{va}} \right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A, t \in T$	Value added
PRD-2	$QF_{f,a,t} = \left(\frac{PVA_{a,t}}{WF_{f,t} \cdot WFDIST_{f,a,t}} \right)^{\sigma_a^{va}} \cdot \left(\delta_{f,a}^{va} \right)^{\sigma_a^{va}} \left(TFP_{a,t} \cdot \varphi_a^{va} \right)^{\sigma_a^{va}-1} QA_{a,t}$	$f \in FVA$ $a \in A$ $t \in T$	Factor demands
PRD-3	$TFP_{a,t} = tfpb_{a,t} \cdot \overline{TFPSCAL_t} \cdot \prod_{f \in FCAPG} \left(\frac{QFINS_{gov,f,t}}{QFINS_{gov,f}^{00}} \right)^{\eta_{a,f}^{tfp}} \cdot \left(\frac{TRDGDP_t}{TRDGDP^{00}} \right)^{\eta_{a,trdgdp}^{tfp}}$	$a \in A, t \in T$	Total factor productivity
PRD-4	$QINT_{c,a,t} = ica_{c,a} \cdot QA_{a,t}$	$c \in C, a \in A$ $t \in T$	Intermediate demands
PRD-5	$QX_{c,t} = \sum_{a \in A} \theta_{a,c} \cdot QA_{a,t}$	$c \in C, t \in T$	Output
PRD-6	$PVA_{a,t} = PA_{a,t}\left(1 - ta_{a,t}\right) - \sum_{c \in C} PQD_{c,a,t} \cdot ica_{c,a}$	$a \in A, t \in T$	Value-added price
PRD-7	$PA_{a,t} = \sum_{c \in C} \theta_{a,c} \cdot PX_{c,t}$	$a \in A, t \in T$	Activity price
PRD-8	$UERAT_{f,t} = ueratb_{f,t}$	$f \in FVA, f \notin FUEND$ $f \notin FCAPNG$	Exogenous unemploy-ment rates

PRD-9	$WF_{f,t} = WF_f^{00}\left(\frac{\overline{CPI_t}}{CPI^{00}}\right)\left(\frac{UERAT_{f,t}}{UERAT_f^{00}}\right)^{\eta_f^{wf}}$	$f \in FUEND$ $t \in T$	Wage curve
PRD-10	$WFDIST_{f,a,t} = wfdistb_{f,a}$	$f \in FVA$ $f \notin FCAPNG$	Exogenous activity-specific wage term for mobile factors
PRD-11	$\sum_{a \in A} QF_{f,a,t} = \left(1 - UERAT_{f,t}\right) \cdot \sum_{i \in INSD} QFINS_{i,f,t}$	$f \in FVA, t \in T$ $f \notin FCAPNG$	Factor markets
PRD-12	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$f \in FVA, t \in T$	Factor income

A.2.2. Domestic and aggregate foreign trade

Table A.2.2 covers the allocation of domestic commodity demands between imports and domestic output and the allocation of domestic output between exports and domestic sales. Equations TRD1-TRD3 are related to prices. In TRD1, the export price received by producers, *PE*, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient (*ice*) and the input price, summed over all inputs. In analogous fashion, equation TRD2 defines the domestic currency import price for demanders, *PM*, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price. In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (*pwe* and *pwm*) are exogenous. Equation TRD3 links the demander and supplier prices for domestic output sold domestically, *PDD* and *PDS*: the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output; as will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation INV3).

The commodity demand, *QQ*, is a CES aggregation of imports and domestic purchases, named the Armington function after its originator (TRD4); *QQ* is referred to as a "composite" demand given that it is met from different sources. Equation TRD5 defines the

composite demands for commodities that (as opposed to those covered by TRD4) do not have both imports and domestic purchases.

For commodities with both sources, domestic demanders are assumed to minimize the cost of any composite demand quantity subject to the Armington function and subject to the relative prices. The first-order conditions (FOCs) are made up of the Armington function itself (TRD4), and an equation that specifies the optimal demand ratio (*QM/QD*) as a function of the ratio between the prices of domestic output and imports (*PDD/PM*) (TRD6). The composite price *PQS* is implicitly defined by TRD7 given that the other variables in this equation are determined by other relationships. At the composite commodity level, a distinction is made between *PQS* and *PQD*. As shown by TRD8, the distinction is that *PQD* (the price paid by domestic demanders) is adjusted to account for sales taxes, value-added taxes, and subsidies; given that both value-added taxes and subsidies always or often have different rates for different demander categories, *PQD* is disaggregated along this additional dimension, captured by the index *d*.

Turning to the production side, a constant-elasticity-of-transformation (CET) function defines the frontier for allocations of domestic output (*QX*, defined in the preceding section) between exports and domestic sales (*QE* and *QD*, respectively) (TRD9) for outputs that, according to base data, have non-zero values for both destinations. Equation TRD10 defines the equivalent of this transformation for outputs with only domestic sales or only exports.

For outputs with both destinations, producers are assumed to maximize the revenue of any output quantity subject to the CET function and relative prices. The FOCs are made up of the CET function and an equation that specifies the optimal supply ratio (*QE/QD*) as a function of the ratio between the prices of exports and domestic sales (*PE/PDS*) (TRD11)**.** The average producer output price, *PX*, is defined as a weighted average of the prices received for domestic sales and exports (TRD12). (In section A.2.1, *PX* influences production decisions and revenues.) The demand for trade and transport services is a function of real domestic and foreign trade volumes, using a fixed-coefficient formulation (TRD-13). The final two equations in this block define the real trade-GDP ratio and real GDP, which is the denominator in this ratio (TRD-14 and TRD-15).

Table A.2.2. Equations for domestic and aggregate foreign trade

TRD-1	$PE_{c,t} = \left(1 - te_{c,t}\right) EXR_t \cdot pwe_{c,t} - \sum_{c' \in CT} \sum_{tace \in TACE} PQD_{c',tace,t} ice_{c',c}$	$c \in CE$ $t \in T$	Export price
TRD-2	$PM_{c,t} = \left(1 + tm_{c,t}\right) EXR_t \cdot pwm_{c,t} + \sum_{c' \in CT} \sum_{tacm \in TACM} PQD_{c,tacm,t} \cdot icm_{c',c}$	$c \in CM$ $t \in T$	Import price
TRD-3	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in CT} \sum_{tacd \in TACD} PQD_{c,tacd,t} icd_{c',c}$	$c \in C$ $t \in T$	Domestic demand price for domestic output
TRD-4	$QQ_{c,t} = \varphi_c^q \left(\delta_c^m \cdot QM_{c,t}^{-\rho_c^q} + \delta_c^{dd} \cdot QD_{c,t}^{-\rho_c^q} \right)^{-\frac{1}{\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Composite demand if use of imports and domestic output
TRD-5	$QQ_{c,t} = QM_{c,t} + QD_{c,t}$	$\left(c \in CM \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CM\right),$ $t \in T$	Composite demand if not use of both imports and domestic output
TRD-6	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \frac{\delta_c^m}{\delta_c^{dd}} \right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Import-domestic demand ratio
TRD-7	$PQS_{c,t} \cdot QQ_{c,t} = \left(PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t} \right)$	$c \in C$ $t \in T$	Composite demand price
TRD-8	$PQD_{c,d,t} = PQS_{c,t} \left(1 + tq_{c,t}\right) \left(1 - sub_{c,d,t}\right) \left(1 + tva_{c,d,t}\right)$	$c \in C, d \in D$ $t \in T$	Adjusted composite demand price
TRD-9	$QX_{c,t} = \varphi_c^x \left(\delta_c^e \cdot QE_{c,t}^{\rho_c^x} + \delta_c^{ds} \cdot QD_{c,t}^{\rho_c^x} \right)^{\frac{1}{\rho_c^x}}$	$c \in CE \cap CD$ $t \in T$	Output transfor-mation if both exports and domestic sales
TRD-10	$QX_{c,t} = QE_{c,t} + QD_{c,t}$	$\left(c \in CE \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CE\right),$ $t \in T$	Output transforma tion if not both exports and domestic

			sales
TRD-11	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \frac{\delta_c^{ds}}{\delta_c^{e}} \right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cap CD$ $t \in T$	Export-domestic sales ratio
TRD-12	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$c \in C$ $t \in T$	Producer output price
TRD-13	$QT_{c,t} = \sum_{c' \in C} \left(icm_{c,c'} \cdot QM_{c',t} + ice_{c,c'} \cdot QE_{c',t} + icd_{c,c'} \cdot QD_{c',t} \right)$	$c \in CT$ $t \in T$	Trade and transport margin demands
TRD-14	$TRDGDP_t = \frac{\sum_{c \in C} EXR^{00} \cdot pwe_c^{00} \cdot QE_{c,t} + \sum_{c \in C} EXR^{00} \cdot pwm_c^{00} \cdot QM_{c,t}}{RGDPMP_t}$	$t \in T$	Real trade-GDP ratio
TRD-15	$RGDPMP_t = \sum_{\substack{c \in C \\ h \in H}} PQD_{c,h}^{00} \cdot QH_{c,h,t} + \sum_{\substack{c \in C \\ f \in FCAP}} PQD_{c,f}^{00} \cdot capcomp_{f,c} \cdot \sum_{i \in INS} DKINS_{i,f,t} + \sum_{c \in C} PQD_{c,dstk}^{00} \cdot \sum_{i \in INS} qdstk_{c,i,t} + \sum_{c \in C} PQD_{c,gov}^{00} \cdot QG_{c,t} + \sum_{c \in C} EXR^{00} \cdot pwe_c^{00} \cdot QE_{c,t} - \sum_{c \in C} EXR^{00} \cdot pwm_c^{00} \cdot QM_{c,t}$	$t \in T$	Real GDP at market prices

A.2.3. Current payments by domestic institutions

This equation block explains payments that are part of the current accounts of domestic institutions, i.e. current incomes and spending for households, the government and enterprises. In the model and its database, it is necessary to include at least one household, and, in practice, models applied to countries in this world invariably have a government. Enterprises are optional. Even though the model and the database can handle multiple representative households, this mathematical statement assumes for simplicity that there is only one household. The sets for institutions distinguish between *INSD* (all domestic institutions), *INSDNG* (all non-government domestic institutions, i.e. households and enterprises), and *H* (households, which may include "non-profit institutions in service of households"). Enterprises differ from households in that they do not consume.

On the income side, the shares of domestic institutions in factor incomes, *SHIF*, are defined on the basis of their stock (or endowment) shares (INS1); the stocks (*QFINS*) are defined

below in Section A.2.4. The factor incomes of domestic institutions, *YIF*, are a function of these shares, factor incomes (*YF*) net of direct taxes, and exogenous payments of factor incomes to the outside world (INS2). (*YF* was defined Section A.2.1.) Using this information, the total incomes of domestic non-government institutions, *YI*, are the sum of factor incomes, transfers from the government (indexed to the numéraire, in this case the CPI), transfers from abroad, and transfers from other institutions in *INSDNG* (INS3). (Government incomes are defined in a separate equation.)

The values for consumption and transfer spending by domestic non-government institutions are defined after deducting payments for direct taxes and savings. The mathematical statement treats direct tax rates as exogenous (policy-determined) but demonstrates alternative treatments for savings rates. The marginal propensity to save, *MPS*, is the product of an institution-specific rate (which may change over time) and a scaling parameter (INS4). If the latter is flexible, then total savings are adjusted endogenously in the context of restrictions on the total quantity or value of private investment financed by domestic non-government institutions. Here, the scaling parameter is fixed, meaning that investment spending must be flexible. (This is discussed in Section A.2.4.) Institution-specific savings values, *SAV*, are a linear function of *MPS* and income net of direct taxes, with an optional (non-zero) intercept, which is indexed to the numéraire (INS5). The presence of an intercept is essential when base-year data indicate that some household groups have negative savings – without a separate (and negative) intercept and the related assumption that marginal and average savings rates differ, higher incomes would in this setting reduce savings further below zero. Transfers from institutions in *INSDNG* to other institutions (in *INS*), *TRII*, are fixed shares of their incomes net of direct taxes and savings (INS6). For households, consumption spending, *EH*, is defined as income net of direct taxes, savings, and transfers to other institutions (INS7). Household consumption demands, *QH*, are a function of population, prices, and total spending (EH) (INS8); the demand functions are derived from the maximization of a Stone-Geary utility function subject to the total spending and prices. It is referred to as a linear expenditure system (LES) since spending on any commodity (the product of price and quantity) is a linear function of *EH* – this is evident if one multiplies both sides of INS8 by the price variable (*PQD*).

The remaining equations in this block define current government receipts and spending. Government receipts, *YG*, are the sum of tax revenues, domestic and foreign transfers, and factor incomes (INS9). Domestic transfers are exogenous and indexed to the numéraire; foreign transfers are exogenous in FCU. The taxes are made up of direct taxes on institutions and factors; domestic indirect taxes on sales, value-added, and activity revenues; export taxes; and import tariffs. To make the mathematical statement more easily digestible, value-added taxes (VATs) (*YTAXVAT*) are defined in a separate equation (INS10). Whether a given tax is part of an application depends on the database. As indicated, the VAT rates are disaggregated by commodity demanded, demander, and time period.

Current government spending, *EG*, is the sum of spending on consumption, domestic transfers, transfers abroad, and subsidies (INS11). The quantities of government consumption, *QG*, are defined on the basis of a trend term (*qgb*) that may be scaled selectively (by commodity and time period) (INS12); the impact of a given value for the scaling variable *QGSCAL* depends on the level of the parameter *qg01* – as indicated by its name, we propose that it be set at values between 0 and 1. In the current mathematical statement, *QGSCAL* is exogenous; if it were endogenous, it could be used to clear the government budget. Subsidy spending, *SUBCT*, is also defined in a separate equation (INS13); subsidy rates are similar to VAT rates in that they are disaggregated by commodity, demander, and time period.

Table A.2.3. Equations for current payments by domestic institutions

INS-1	$SHIF_{i,f,t} = \frac{QFINS_{i,f,t}}{\sum_{i' \in INSD} QFINS_{i',f,t}}$	$i \in INSD$ $f \in FVA$ $t \in T$	Shares of factor incomes to domestic institutions
INS-2	$YIF_{i,f,t} = SHIF_{i,f,t}\left(\left(1-tf_{f,t}\right)YF_{f,t} - EXR_t \cdot trnsfr_{row,f,t}\right)$	$i \in INSD$ $f \in FVA$ $t \in T$	Factor income to domestic institutions
INS-3	$YI_{i,t} = \sum_{f \in FVA} YIF_{i,f,t} + trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $+trnsfr_{i,row,t} \cdot EXR_t + \sum_{i' \in INSDNG} TRII_{i,i',t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution income
INS-4	$MPS_{i,t} = mpsb_{i,t} \cdot \overline{MPSSCAL}_t$	$i \in INSDNG$ $t \in T$	Marginal propensity to save

INS-5	$SAV_{i,t} = \alpha_{i,t}^{sav} \cdot \overline{CPI} + MPS_{i,t}\left(1-ty_{i,t}\right)YI_{i,t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution savings
INS-6	$TRII_{i,i'} = shii_{i,i'}\left(\left(1-ty_{i',t}\right)YI_{i',t} - SAV_{i',t}\right)$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Institutional transfers
INS-7	$EH_{h,t} = \left(1-ty_{h,t}\right)YI_{h,t} - SAV_{h,t} - \sum_{i \in INS} TRII_{i,h,t}$	$h \in H$ $t \in T$	Household consumption expenditure
INS-8	$QH_{c,h,t} = pop_{h,t}\left(\gamma_{c,h,t}^{min} + \frac{\beta_{c,h}\left(\frac{EH_{h,t}}{pop_{h,t}} - \sum_{c' \in C} PQD_{c',h,t} \cdot \gamma_{c',h,t}^{min}\right)}{PQD_{c,h,t}}\right)$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand
INS-9	$YG_t = \sum_{i \in INSDNG} ty_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t}$ $+\sum_{c \in C} tq_{c,t} \cdot PQS_{c,t} \cdot QQ_{c,t} + YTAXVAT_t$ $+\sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$ $+\sum_{c \in C} te_{c,t} \cdot pwe_{c,t} \cdot QE_{c,t} \cdot EXR_t$ $+\sum_{c \in C} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t} \cdot EXR_t$ $+trnsfr_{gov,row,t} \cdot EXR_t$ $+\sum_{i \in INSDNG} TRII_{gov,i,t} + \sum_{f \in F} YIF_{gov,f,t}$	$t \in T$	Government current receipts
INS-10	$YTAXVAT_t =$ $\sum_{c \in C}\sum_{a \in A}\left(1-sub_{c,a,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,a,t} \cdot QINT_{c,a,t}$ $+\sum_{c \in C}\sum_{h \in H}\left(1-sub_{c,h,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,a,t} \cdot QH_{c,h,t}$ $+\sum_{c \in C}\left(1-sub_{c,gov,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,gov,t} \cdot QG_{c,t}$ $+\sum_{c \in C}\sum_{f \in FCAP}\left(\left(1-sub_{c,f,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,f,t} \cdot capcomp_{c,f} \sum_{i \in INS} DKINS_{i,f,t}\right)$ $+\sum_{c \in C}\sum_{i \in INS}\left(1-sub_{c,dstk,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,dstk,t} \cdot qdstk_{c,i,t}$ $+\sum_{c \in C}\sum_{c' \in CDIS}\left(1-sub_{c,tacm,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tacm,t} \cdot icm_{c,c',r} \cdot QMR_{c',r,t}$ $+\sum_{c \in C}\sum_{c' \in CDIS}\left(1-sub_{c,tace,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tace,t} \cdot ice_{c,c',r} \cdot QER_{c',r,t}$ $+\sum_{c \in C}\sum_{c' \in C}\left(1-sub_{c,tacd,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tacd,t} \cdot icd_{c,c'} \cdot QD_{c',r,t}$	$t \in T$	VAT revenue
INS-11	$EG_t = \sum_{c \in C} PQD_{c,gov,t} \cdot QG_{c,t} + \sum_{i \in INSDNG} trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $+trnsfr_{row,gov,t} \cdot EXR_t + SUBCT_t$	$t \in T$	Government expenditure

INS-12	$QG_{c,t} = qgb_{c,t}\left(1+qg01_{c,t}\cdot\overline{QGSCAL}_t\right)$	$c\in C$ $t\in T$	Government consumption
INS-13	$SUBCT_t =$ $\sum_{c\in C}\sum_{a\in A} sub_{c,a,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QINT_{c,a,t}$ $+\sum_{c\in C}\sum_{h\in H} sub_{c,h,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QH_{c,h,t}$ $+\sum_{c\in C} sub_{c,gov,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QG_{c,t}$ $+\sum_{c\in C}\sum_{f\in FCAP} sub_{c,f,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot capcomp_{c,f}\sum_{i\in INS} DKINS_{i,f,t}$ $+\sum_{c\in C}\sum_{i\in INS} sub_{c,dstk,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot qdstk_{c,i,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} sub_{c,tacm,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icm_{c,c',r}\cdot QMR_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} sub_{c,tace,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot ice_{c,c',r}\cdot QER_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in C} sub_{c,tacd,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icd_{c,c'}\cdot QD_{c',r,t}$	$t\in T$	Commodity subsidy

A.2.4. Investment, system constraints, and numéraire

This block covers investment spending by different institutions, how it is financed, and how the new capital that is generated feeds into the economy. The specification of investment and its financing makes it possible to specify the remaining system constraints, the markets for private capital factors, commodity balances, and the balance of payments.[7] (In Sections A.2.1 and A.2.3, we covered the markets for other factors and the government balance, respectively.) In addition, we here specify the numéraire, needed for CGE models like GEM-Core.

For the government, investment spending (or gross capital formation), *INVG*, is defined as the sum of government savings (the difference between current receipts and spending), domestic net financing (indexed to the numéraire), and foreign net financing (exogenous in FCU) (INV1). The sum of the two financing terms is the government primary deficit. These two terms are referred to as net financing items since they represent the difference between new borrowing and interest payments; the latter do not appear explicitly in the

[7] The term system constraints refers to constraints that are not perceived by individual actors (like household budget constraints) but which the economic system nevertheless most respect (like a labor market constraint which says that the quantity employed equals the labor force net of unemployment).

model.[8] For domestic non-government institutions, the corresponding variable, *INV*, is the sum of own savings and net financing from abroad minus claims on investment funding to finance the government and add to foreign reserves (INV2). Before translating investment spending into quantities of new capital, it is necessary to specify prices; in equation INV3, the unit prices of new capital stocks, *PK*, are defined as the product of the price of commodity *c* and the matrix of capital composition coefficients (which shows the quantities of commodities *c* used as inputs per unit of any new capital stock *f*) summed over all *c*.

The next three equations define investment quantities by destination (by type of capital stock) by government and non-government institutions and investment quantities by source (the use of commodity inputs in the production of new capital). For the government, the quantity of new capital stock *f*, $DKINS_{gov,f,t}$, is defined as investment spending net of spending on new inventory (gross fixed capital formation or GFCF), multiplied by the spending share for *f*, and divided by *PK* to transform into stock quantities (INV4). In the corresponding equation for non-government investment (INV5), GFCF is the sum of (a) investment net of stock change spending for institutions in INSDNG; and (b) foreign investment (the value of which is exogenous in FCU). GFCF is allocated across different capital stocks (if more than one) in fixed spending shares and transformed into quantities of new capital by dividing by *PK*.[9] Final investment demands (i.e. investment quantities defined by the source of inputs into the construction of new capital), *QINV*, are defined as the product of the capital composition matrix and investment by capital stock, summed over all capital stocks (INV6).

For any capital stock, the endowments held by domestic institutions (government and non-government), *QFINS*, are defined as the sum of (a) the stock held in the previous year net of depreciation; and (b) new investment in the previous year (INV7). For the stocks of other factors, endowments are defined exogenously (INV8). The values for endowments were used in Section A.2.1 to define the supply sides of markets for non-capital factors and, in Section A.2.3, to define the distributional shares for factor incomes.

[8] Given simulation results for government and private net financing (domestic and foreign) and with the help of additional data on initial debt stocks, and real interest rates by time period, it is straightforward to compute debt stocks, which may be expressed in relation to other simulation results (like GDP or export values).

[9] From our experience, in most applications of models of this, it is preferable to have one type of private capital with constrained mobility. However, the option of multiple stocks may be useful if, for example, the model should mimic the allocation of foreign investment to specific sectors like mining.

The allocation of private capital stocks across activities responds to relative capital rents.[10] As an input to the formulation used, the average wage of private capital stock *f*, *WFAVG*, is defined as total rent to *f* divided by total employment of *f* (INV9). In equation INV10, the allocation of new private capital stock *f* to an activity *a*, *DKA*, is defined as the product of (a) an allocation based on current activity shares (i.e. total new investment in *f* times the current share of *a* in the use of *f*); and (b) an adjustment term that is above (below) unity if the wage of capital stock *f* in *a* is above (below) the economywide average, assuming a positive value for the parameter *κ* (Greek kappa; $\kappa \geq 0$). *κ* plays a crucial role in this formulation: the higher its value, the stronger the sensitivity of the allocation of new capital to differences in capital rents; if it is zero, the allocation of stock *f* does not change over time and if it is too high, capital rents may oscillate.[11]

Total employment of capital stock *f* in activity *a* in period *t*, $QF_{f,a,t}$, is defined as the stock installed in *t-1*, $QF_{f,a,t-1}$, net of depreciation, plus the quantity of new investment in stock *f* in *t-1* allocated to *a* , $DKA_{f,a,t-1}$ (INV11). This last equation may be seen as defining a set of activity-specific markets for capital stock *f* in which the quantity supplied (the right-hand side) is fixed within any period *t* (determined by past decisions) while the quantity demanded (the left-hand side) is determined by profit-maximization (cf. Section A.2.1). A wage variable defined over *f* and *a* is needed to clear this market. Accordingly, among the two wage variables that apply to any factor ($WF_{f,t}$ *and* $WFDIST_{f,a,t}$), equation INV12 fixes the economywide variable *WF* while leaving the activity-specific variable *WFDIST* flexible. The

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[10] This approach that is presented may have been first developed in Dervis et al. (1982, pp. 175-178). Our treatment deviates in one respect: for simplicity, we use capital rents by activity instead of profit rates.

[11] With reference to INV10, by definition, $\sum_{a \in A} DKA_{f,a,t} = \sum_{i \in INSNG} DKINS_{f,i,t}$ for $f \in FCAPNG$ and $t \in T$. This can be shown as follows: For simplicity, replacing $\sum_{i \in INSNG} DKINS_{f,i,t}$ by DKI and $WF_{f,t} \cdot WFDIST_{f,a,t}$ by WFA_a, suppressing remaining *f* and *t* subscripts, and noting the definition of the average wage,

$$DKA_a = DKI \cdot \frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right); \sum_{a \in A} DKA_a = DKI \cdot \sum_{a \in A}\left(\frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right)\right)$$

$$= DKI\left(\frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}} + \kappa \cdot \sum_{a \in A}\left(\frac{WFA_a}{\overline{WF}} \frac{QF_a}{\sum_{a' \in A} QF_{a'}}\right) - \frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}}\right) = DKI\left(1 + \kappa\left(\frac{\overline{WF}}{\overline{WF}} - 1\right)\right) = DKI$$

simulated values for the product of the two variables show the scarcity value of private capital stocks by activity. In sum, for private capital, it is assumed that installed stocks cannot be reallocated while the analyst controls the extent to which the allocation of new capital will shift toward sectors with relatively high capital rents.

For each domestic commodity, the demand side is now complete. The equation INV13 defines total composite demand for any commodity, *QQ*, as the sum of consumption, investment (fixed capital formation and stock changes), intermediate demands, and demands for trade and transportation services (due to domestic and foreign trade). As specified in Section A.2.2, these demands generate demands for domestic output and/or imports. The markets for domestic output sold domestically are cleared by the linked variables *PDD* and *PDS*; to exemplify, for the case of excess demand, increases in both price variables would simultaneously reduce domestic demands for domestic outputs and increase the quantities of output sold domestically (raising the total output level by raising profitability and raising the share of output sold domestically).

The statement of investment financing completes the flows in the balance of payments, which is expressed in FCU. Equations INV14 and INV15 state the current and capital accounts, respectively, with foreign savings, *SAVF*, as the linking variable. In the current account balance, inflows are due to exports, and transfers from abroad while outflows are caused by imports, transfers from domestic non-government institutions, and factor incomes. The variable *SAVF* measures the current-account deficit; if outflows (the right-hand side) are larger (smaller) than inflows (the left-hand side), foreign savings are positive (negative).[12] In the capital account balance, the current account deficit is financed by net foreign financing to government and non-government institutions and foreign investment, net of increases of foreign reserves. By influencing export and import quantities in opposite directions, raising or reducing the trade balance in FCU, adjustments in the exchange rate, *EXR*, clear the balance of payments, making sure that the level of foreign savings matches the level that is financed on the right-hand side of the capital account.

[12] The variable *SAVF* deviates from the definition of foreign savings given that interest payments are an implicit part of the capital account instead of being part of the current account. The variable *SAVF* could more accurately be referred to as the primary deficit of the nation.

As a manifestation of Walras' law, in a CGE model like the one presented above, one equation should be removed to assure equality between the number of variables and independent equations; it is possible to check that the omitted equation holds in a post-calculation. Here we opt for the alternative of instead inserting one variable, named WALRAS, into one equation. Hence, the presence of *WALRAS* in the capital account of the balance of payments. In the absence of errors, the solution value for *WALRAS* should be (very close to) zero.

Finally, a well-specified CGE model like GEM-Core is homogeneous of degree zero in prices, meaning that only relative prices matter and that, if one set of relative prices solves the model, then any multiple of this set of prices would also solve the model (scaling all domestic prices and payments) without any influence on quantities. To anchor the price level, a price or price index, referred to as the numéraire, needs to be fixed, with the consequence that all other prices are measured relative to this numéraire. In this mathematical statement, the consumer price index, *CPI*, is the numéraire. Here, equation INV16 defines the CPI, which is fixed, on the basis of the base-year weights of household consumption payments by commodity and household type in total household consumption.[13]

Table A.2.4. Equations for investment, system constraints, and numéraire

INV-1	$INVG_t = \left(YG_t - EG_t\right) + ndfg_t \cdot \overline{CPI}_t + nff_{gov,t} \cdot EXR_t$	$t \in T$	Gov't primary deficit, investment value, and financing
INV-2	$INV_{i,t} = SAV_{i,t} + nff_{i,t} \cdot EXR_t - \left(\frac{SAV_{i,t}}{\sum_{i' \in INSDNG} SAV_{i',t}} \right) \left(ndfg_t \cdot \overline{CPI}_t + drf_t \cdot EXR_t \right)$	$i \in INSDNG$ $t \in T$	Non-gov't investment value and its financing
INV-3	$PK_{f,t} = \sum_{c \in C} PQD_{c,f,t} \cdot capcomp_{c,f}$	$f \in FCAP$ $t \in T$	Price of new capital

[13] As an alternative to CPI, the domestic producer price index (DPI) may serve as numéraire. In addition, it is often used as the denominator in the definition of the price-level-deflated (PLD) real exchange rate (REXR). Algebraically, with time subscripts omitted, $DPI = \sum_{c \in C} PDS_c \cdot dwts_c$ and $REXR = EXR / DPI$.

INV-4	$$DKINS_{gov,f,t} = \frac{invshr_{f,gov,t} \cdot \left(INVG_t - \sum_{c \in C} PQD_{c,gov,t} \cdot qdstk_{c,gov,t} \right)}{PK_{f,t}}$$	$f \in FCAPG$ $t \in T$	Gov't investment by government capital stock
INV-5	$$DKINS_{i,f,t} = \frac{invshr_{f,i,t}}{PK_{f,t}} \cdot \left(\left(INV_{i,t} - \sum_{c \in C} PQD_{c,i,t} \cdot qdstk_{c,i,t} \right)_{\mid i \in INSDNG} + \left(invf_{i,t} \cdot EXR_t \right)_{\mid i \in INSROW} \right)$$	$f \in FCAPNG$ $i \in INSNG$ $t \in T$	Non-gov't investment by private capital stock
INV-6	$$QINV_{c,t} = \sum_{i \in INS} \sum_{f \in FCAP} capcomp_{c,f} \cdot DKINS_{i,f,t}$$	$c \in C$ $t \in T$	Real investment demand (by source)
INV-7	$$QFINS_{i,f,t} = QFINS_{i,f,t-1} \left(1 - depr_{f,t-1}\right) + DKINS_{i,f,t-1}$$	$i \in INSD$ $f \in FCAP$ $t \in T$ $t \notin TMIN$	Accumulation of capital by domestic institutions
INV-8	$$QFINS_{i,f,t} = qfinsb_{i,f,t}$$	$i \in INSD$ $f \in FOTH$ $t \in T$	Exogenous institutional endowments for other factors
INV-9	$$WFAVG_{f,t} = \frac{\sum_{a \in A} WF_{f,a,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t}}{\sum_{a \in A} QF_{f,a,t}}$$	$f \in FCAPNG$ $t \in T$	Average wage (rent) by private capital stock
INV-10	$$DKA_{f,a,t} = \left(\sum_{i \in INSNG} DKINS_{f,i,t} \right) \left(\frac{QF_{f,a,t}}{\sum_{a' \in A} QF_{f,a',t}} \right) \cdot \left(1 + \kappa_f \left(\frac{WF_{f,t} \cdot WFDIST_{f,a,t}}{WFAVG_{f,t}} - 1 \right) \right)$$	$f \in FCAPNG$ $a \in A$ $t \in T$	Allocation of new private capital by activity
INV-11	$$QF_{f,a,t} = QF_{f,a,t-1} \left(1 - depr_{f,t-1}\right) + DKA_{f,t-1}$$	$f \in FCAPNG$ $a \in A, t \in T$ $t \notin TMIN$	Accumulation of private capital by activity
INV-12	$$WF_{f,t} = wfb_f$$	$f \in FCAPNG$	Exogenous economy-wide wage term for private capital
INV-13	$$QQ_{c,t} = \sum_{h \in H} QH_{c,h,t} + QG_{c,t} + QINV_{c,t} + \sum_{i \in INSD} qdstk_{c,i,t} + \sum_{a \in A} QINT_{c,a,t} + QT_{c,t}$$	$c \in C$ $t \in T$	Commodity balance

INV-14	$\sum_{c \in C} pwe_{c,t} \cdot QE_{c,t} + \sum_{ac \in INSD \cup F} trnsfr_{ac,row,t} + SAVF_t = \sum_{c \in C} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{EXR_t} + \sum_{f \in F} trnsfr_{row,f,t}$	$t \in T$	Current account of balance of payments
INV-15	$SAVF_t = \sum_{i \in INSD} nff_{i,t} + invf_t - drf_t + WALRAS_t$	$t \in T$	Capital account of balance of payments
INV-16	$\sum_{c \in C} \sum_{h \in H} PQD_{c,h,t} \cdot cwts_{c,h} = \overline{CPI_t}$	$t \in T$	Consumer price index