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MODELLING INVOLUNTARY UNEMPLOYMENT IN APPLIED GE MODELS

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Abstract

Involuntary unemployment is a regrettable feature of the world, but not of most general equilibrium models. Standard attempts to incorporate involuntary unemployment into such models have rested on mechanistic and arbitrary assumptions about wage rigidity. This paper recognises that the difficulty of modelling the reality of involuntary unemployment is that the currently used models lack a crucial piece of information: the role that the level of unemployment plays in labour market decisions.

This paper outlines a theory in which unemployment rates enter into the decision making process of individual agents which in turn yields the missing equation. It then draws on the empirical literature to parameterise it and incorporates this equation in the GTAP model to derive a dynamic GTAP model with endogenous unemployment (DGTAPU). The model is then simulated with a number of stylised scenarios and illustrative results are presented.

It is hoped that this paper will induce additional research in improving the specification of the 'new' equation and the precision of policy analysis.

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Introduction

With some exceptions (such as Annabi, 2003), it is a common practice in applied general equilibrium models to maintain that unemployment rate is exogenously determined and therefore take labour supply as exogenously given. Production sectors demand labour to minimise costs subject to a constant returns to scale production function. A flexible wage rate clears the labour market. All available labour is always employed. This modelling approach is commonly taken to mean that GE models maintain the assumption of full employment. Given the prevalence of unemployment across the world this perception alone is sufficient to cast doubt on the results projected by CGE models.

To address this, to some extent, some modellers employ alternative labour market closure rules. In a short run closure, the wage rate is taken as exogenous while labour supply is assumed to be available in unlimited quantities at this price. In a long run closure, however, the labour supply (and employment level) is treated as exogenous and the wage rate is determined endogenously to clear the labour market.[2] With the short run model closure rule, the deviation of the employment level from the given supply of labour determines the unemployment level provided that the given wage rate is higher than the market clearing wage rate. In the long run, however, a flexible wage rate clears the labour market. In this case, the unemployment rate is either zero (we have full employment) or fixed at the base year level.

CGE models that have an elaborate population module, such as GTEM, also take age and gender specific participation rates as given, which is then used to convert the working age population into the labour force. By assuming a given unemployment rate the labour force is then converted into labour supply which is matched with the aggregate labour demand by a market clearing wage rate (Pant, 2007). Even in these models the short run model closure rule is required to project short run changes in the unemployment rate. Along the long run equilibrium trajectory, the unemployment rate remains exogenous. Attempts have been made in some models to bring back any policy induced short run movements in the unemployment rates to their 'natural' rate by specifying a transition mechanism. (Dixon and Rimmer, 2003; Adam et al., 2015).

A fixed 'involuntary' unemployment rate–or the natural rate–is a limitation of most of the CGE models, such as the derivatives of the ORANI model of the Australian economy and the GTAP mode of the global economy; these models take labour supply as given (Dixon et al., 1982; Hertel, 1997).

In order to ascertain whether this limitation has introduced serious biases on the projections of the impacts of policy changes, such as trade liberalisation of some form or climate change mitigation policies, some modellers have experimented with alternative specifications of the labour market and concluded that the labour market closure rules do matter (Bontout and Jean, 1998).

Since the key reason for the failure to include the 'involuntary' unemployment rate as an endogenous variable in the long run model closure is that the labour market just does not have enough equations to do the job. The standard three equations of the labour market – the supply, demand and the market clearing condition– determine the supply, demand and the market clearing price (i.e., the wage rate) and therefore cannot determine the unemployment rate. If we wish to determine the unemployment rate endogenously, then one of the other three variables had to be

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[2] Some models apply nominal rigidity only when labour demand falls and allow wage flexibility when labour demand rises to improve the realism.

determined from outside. The model requires an additional equation for each of the labour market types and a model consistent theory that yields the equation.

This paper aims to accomplish this task by finding an equation that is theoretically and empirically defensible.

A so-called 'wage curve' representing an inverse relationship between real wage rates and unemployment rates, which is also consistent with the Phillips curve as well as many theories that establish unemployment in equilibrium, such as the efficiency wage hypothesis could provide the missing link. Because this is an empirical relationship, its integration with a CGE model could be made more acceptable if a theoretical structure that leads to the inverse relationship could be established.

This aim of this paper is to derive a 'wage curve' for a dynamic CGE model, such as the GTAP model and implement it. The wage curve augmented dynamic CGE model then can be used to examine the impacts of any policy change while allowing the unemployment rates to change endogenously.

By successfully doing so, this paper extends the capabilities of GTAP type recursively dynamic CGE models which can then be used to reassess the welfare impacts of policy changes and other relevant variables such as unemployment rates.

This paper is divided in five sections. Section 2 develops a new modelling framework for the labour market that yields the missing equation of the labour market. This equation also provides a theoretical foundation to the empirical 'wage curve' and perhaps the Phillips curve and the required interface to embed the wage curve into the conventional CGE models. Section 3 adds the missing equations to the GE model and calibrates their parameters using the base year data and the information provided by the 'wage curve'. Section 4 simulates the wage curve augmented Dynamic GTAP model for exogenously given labour productivity growth under alternative values for the elasticity of factor substitution parameter. Section 5 discusses the results and concludes the paper. By doing so this paper accomplishes two tasks: (1) it shows, without assuming that workers shirk, that involuntary unemployment exist in equilibrium even with flexible wage rates and competitive markets (work in progress); (2) it enhances the capability of CGE models by enabling them to project impacts of policy changes on the unemployment rate; and (3) it demonstrates, by example, that if labour productivity growth occurs in sectors operating on highly inelastic segments of the aggregate labour demand curve then this may cause the unemployment rate to rise.

2. An alternative model of the labour market

2.1 Firm's equilibrium with standard labour market modelling

Let assume that the production function of a firm is given by

(1) $\qquad Y = F^j\left(L_j^*, K_j\right)$

Where Y is the units of output, F(.) is a production function with standard properties defined over the units of efforts L_j^* and units of the capital stock K_j employed by sector j.

We maintain that all firms take all prices as given and choose units of factor employment to maximize profits subject to their production technology (1). Their problem can be stated as:

(2) $\quad \underset{L_j^*, K_j}{MAX} \quad \Pi = P_j F^j\left(L_j^*, K_j\right) - W^* L_j^* - R K_j$

Where W^* represents the wage rate per unit of efforts, R is the rental rate of capital and P_j is the output price faced by sector j. Let efforts supplied by a unit of physical labour be an exogenous parameter, A, so that we have

(3) $\quad L_j^* = A\, L_j$ and $W^* = W/A$

The first order conditions for profit maximization for the firm are:

(4) $\quad P_j F_{L_j^*} - W^* = 0$ and

(5) $\quad P_j F_{K_j} - R = 0$

Where subscripts on F denote respective partial derivatives. Equations (4) and (5) state that firms should pay the factors their respective value of marginal products to maximise their profits. Since, $AF_{L^*} = F_L$, the condition (4) can also be written in a 'translated form' as:

(6) $\quad P_j\, F_{L_j} = P_j A F_{L_j^*} = A W^* = W.$

That means that paying value of marginal products for a unit of efforts is the same as paying value of its marginal products for the person hour. Solving equation (4) or (6) yields

(7) $\quad L_j = L_j(W, R, P_j; A)$

And similarly solving equation (5) yields the sectoral demand for capital services as

(8) $\quad K_j = K_j(W, R, P_j).$

Aggregating the demand for labour across all firms, the labour market clearing condition can be written as:

(9) $\quad \sum_j L_j = (1 - u)\, L$ and

Where L is the total labour force available for work at any wage rate and u is the unemployment rate, given exogenously.

Equation (9) states that the sum of labour demanded by all sectors must be equal to the labour available for employment, which is the unemployment rate adjusted labour force, L. A flexible wage rate will make the demand adjust to the available quantity of employable labour through equation (7).

Similarly, the equilibrium in the market for capital services can be written as:

(10) $\quad \sum_j K_j = K$

Where K is the aggregate stock of capital in the economy (with a constant utilisation rate). Equation (10) states that total demand for capital (services) is just equal to its supply. To maintain this equality, the rental rate R adjusts as required. Capital and labour are assumed to be perfectly mobile across sectors.

For given u, R, P and L, equation (7) and (9) describe the labour market equilibrium. In particular equation (9) determines the equilibrium wage rate, W, at which all available labour, adjusted by the unemployment rate, is employed. Similarly, for given W, P and K, equations (8) and (10) describe the market equilibrium for capital services and, in particular, equation (10) determines the market clearing rental rate, R.

For given unemployment rate, u, and output price vector P, equation (9) and (10) simultaneously determine W and R that establishes a 'general' equilibrium in a small open economy with given endowments by clearing the domestic factor markets. Commodity markets will be cleared by trade.

Thus, a standard description of the aggregate labour market consists of three equations – one describing the aggregate demand, another describing the supply of labour and one market clearing condition – determining the three variables, quantities of equilibrium demand, supply and the price, that is the wage rate, that clears the labour market. In the simplest model described above, we did not require a separate supply equation as it was exogenously given and stylizes, in bare bones, how the labour market is modelled in most computable general equilibrium models.

This type of labour market modelling, however, has two interrelated issues. First, although rising unemployment rates are a matter of concern to the society, no individual agent takes the unemployment rate into account while making their own choices. Therefore, there is no behavioural relationship that can connect the unemployment rate as a consequence of the choices made by the optimizing agents. Therefore, it has to remain inherently exogenous. Second, in mathematical terms, there is no equation that could be used to determine an additional variable, the unemployment rate.

In the next section we will modify this model by making the worker responsive to the labour market conditions. In particular, we will assume that the effort level, A, of a worker rises with the unemployment rate.

2.2   A modified model of the labour market

We start this section with the key assumption of this paper

Assumption 1: Let W_c be the competitive wage rate at which the labour market clears without unemployment, if the workers do not care about unemployment and W be any other market wage rate. Let us assume that workers supply efforts A according to the rule

(11) $A = A(u; \omega)$, where $\omega = \dfrac{W}{W_c}$.

Then, $A(u; \omega)$ can be viewed as an hours augmenting fear function of the work force which reflects the behavioural response of the workers to the unemployment rate, u, given ω.

Assumption 2: It is also assumed that

(12) $A(0,1) = 1$ and $A_u(u; \omega) > 0$ for $A < A^{max}$ and $A_u(u; \omega) < 0$ for $A > A^{max}$.

In other words, we also assume that $A(u; \omega)$ is concave in u with an interior maximum such that the effort level first rises with the unemployment rate, reaches its maximum and then declines with the rise in the unemployment rate. The function $A(u; \omega)$ is assumed to be increasing and convex in ω over a relevant range.

'Workers hold fear of unemployment' is a common assumption. Shapiro and Stiglitz (1984) assumed that all workers shirk and employers use unemployment to discipline the workers and showed that there is unemployment in equilibrium. Contrary to their assumption, we assume that all workers are conscientious. **They do not shirk**. They work diligently to fulfil their contracts. Not only that, if there is unemployment in the market then the workers would work even longer hours without being compensated for the extra work just to improve the competitiveness of their employer and retain their job. However, there is a limit to which they can work extra hours for free. If the unemployment rate rises above the threshold they will start reducing extra hours perhaps because they start looking for alternative ways of living.

For a firm, all prices and the unemployment rate are given. Therefore, the unemployment induced effort level, $A(u; \omega)$ of its workforce is also given. The firm utilises this information while maximising its profits, but it cannot choose the unemployment rate. A competitive firm is too small to do so.

Given (11), the defining equation (3) can be rewritten as

(3a) $L_j^* = A(u, \omega)L_j$ and $W^* = W/A(u, \omega)$.

The first order conditions (4) and (5) for profit maximization for the firm remain unchanged. However, acknowledging the dependence of the effort level on unemployment and the wage rates, the equation (6), pertaining to the firm's equilibrium employment of labour, can be written as:

(6a) $P_j F_L = P_j A(u, \omega)F_{L^*} = A(u, \omega)W^* = W$.

The remaining equations of the simplified model remain the same.

Equation (6a) can be solved for a firm's labour demand as

(7a) $L_j = L_j(W, R, P_j; u)$

We can now use (7a) and (9) to solve for W, given u. We still do not have enough equations to solve for u. It is, however, clear that for each u there is a profit maximizing allocation of resources and a set of prices that clear the markets. Given this the question naturally arises on whether there is a particular u that maximises the profits of all firms across all possible values of u? This question will be answered in the next subsection.

2.3 Joint maximisation by all firms

Suppose the firms collude to choose the optimal unemployment rate for them or persuade the government to impose an employment tax (payroll tax) so that the joint profit maximising unemployment rate emerges in equilibrium. What should be such unemployment rate? This question can be answered by solving the following problem:

(13) $\underset{L_j^*, K_j, u}{MAX}$ $\Pi(L_j^*, K_j, u) = \sum_j P_j F(L_j^*, K_j) - \sum_j W^* L_j^* - \sum_j R K_j$

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Subject to the resource constraint given by (8) and (9) and W^* and L_j^* are as defined in (3a).

By setting the first order derivatives of the joint profit function with respect to the choice variables equal to zero we get

(14) $P_j F_{L_j^*} - W^* = 0,$

(15) $P_j F_K - R = 0$ and

(16) $\sum_j \left(P_j F_{L_j^*} \frac{\partial L_j^*}{\partial u} \right) = 0$

Clearly, equation (14) and (15) are the same as equations (4) and (5) and therefore need no explanation. Using (3a), equation (16) can be rewritten as

(17) $W^* \frac{\partial A(u,\omega)}{\partial u} \cdot \sum_j L_j = 0$

And using the market clearing condition (9), equation (17) can be rewritten as

(18) $\frac{W}{A} (1-u) L \frac{\partial A(u,\omega)}{\partial u} = 0.$

Clearly, equation (18) is just another way of expressing the joint profit maximizing condition (16) and it can be understood as follows.

Since (1-u) cannot be zero for an interior solution and W, L and A are finite, positive numbers, satisfaction of the equation (18) requires that

(19) $\frac{\partial A(u,\omega)}{\partial u} = 0.$

Equation (19) clearly shows that, for each wage rate, the unemployment rate that maximises the worker's effort level would maximise the joint profit of the firms. This is so because the unemployment rate thus determined minimises the unit cost of effort at the given wage rate.

Solving the condition (19) one can obtain an implicit function

(20) $G(u, W; W_c) = 0$

Which describes a locus of pairs of the real wage rate and the unemployment rate at which the unemployment rate minimises the unit cost of efforts at the given hourly real wage rate. The joint profits would be maximised by the right combination of the unemployment rate and the hourly wage rate, together with the other conditions of equilibrium contained in the model. Hence, in the general equilibrium of an economy where workers fear unemployment, the labour market clears with an equilibrium unemployment rate.

2.4   An example of the effort supply function

In the absence of detailed study on the existence and nature of the effort supply function $A(u, \omega)$ let us assume that it is described by a quadratic function:

(21) $A(u, \omega) = \left[1 - \gamma + \gamma \omega^{2\theta} \right] + \alpha u - \beta \omega^{\theta} (u)^2$

Where α, β, γ and θ are positive parameters. Clearly, $A(0,1) = 1$. That means the standard hours supply, when there is no unemployment and workers are paid the market clearing wage rate, is 1 unit. Any increase in the wage rate and/or the unemployment rate will induce additional hours supplied for free.

Taking the partial derivative of A with respect to u yields

(22) $\quad \frac{\partial A}{\partial u} = \alpha - 2\beta\omega^\theta u$

Hence use of equation (22) in the equation (19) yields the optimal unemployment rate as

(24) $\quad u = \frac{\alpha}{2\beta\omega^\theta}.$

Thus, there is an unemployment rate, given by the equation (24) for each ω, and hence for each real wage rate W, that maximizes the joint profits of the price taking firms. The equation (19), the first order condition of joint profit maximization, shows that at the unemployment rate given by equation (24), maximizes the effort level of each worker at the given wage rate.

Equation (24) does not, however, determine what unemployment rate or the wage rate that will prevail in equilibrium. It simply traces out all possible combinations of the pair (u, W) at which unit cost of efforts will be minimised at each W to all firms. For each value of W, the equilibrium market wage rate, equation (24) therefore yields the equilibrium unemployment rate at which the profits of all firms will be maximised.

At this point it is instructive to note that the equation (24) or the equation (20) is similar to the I-S curve or the L-M curve described in any textbook of macroeconomics. Each curve describes the locus of possible equilibrium points for just one market. General equilibrium requires that other markets also be in equilibrium at the same time.

As the market clearing wage rate is determined at the general equilibrium of the economy, the equilibrium unemployment rate is also determined at the general equilibrium of the economy. This unemployment rate produced by this model thus confirms with the concept of the 'natural' rate as coined by Friedman (1958).

2.5   The stability of unemployment equilibrium in a competitive economy

The beauty of the competitive labour market described in section 2.1 (or in any textbook) is that at the market clearing wage rate there is no one left who is looking for the job at the going wage rate and no one is looking for to hire or to fire any workers at the going wage rate. Therefore, there is no incentive for anyone to deviate from the market outcome. Any deviation would be corrected by the market forces and so the equilibrium remains stable (subject to the appropriate slope restrictions).

In the labour market equilibrium described above, in section 2.4, there are unemployed workers at the going wage rate. Would not these workers offer themselves to the employers at lower than the going wage rate to get employed is a natural question to ask. If they have incentives to offer their service at lower than the going wage rate and the employers have got the incentives to accept the

offer and employ them then eventually the unemployment rate in the competitive equilibrium will have to go down to zero (or to the frictional level).

There is no doubt that the unemployed workers would be offering their services at a wage rate that is lower than the equilibrium wage rate satisfying the

joint maximisation conditions. In deed we have assumed a vertical labour supply function when expressed in physical units. So the question of the stability of equilibrium boils down to whether an individual firm finds it profitable to hire its workers below the market clearing rate and increase its employment level.

To answer this question fully, suppose that (u^0, W^0) represents the general equilibrium solution of the labour market under joint maximisation.

Proposition 1: If $\frac{W}{A}\frac{\partial A(u,\omega)}{\partial W}\big|_{(u^0,W^0)} > 1$, then the equilibrium (u^0, W^0) is sustained in a competitive equilibrium. There will be no incentive for a firm to employ a worker offering to work at wages lower than W^0.

Proof: It follows from the condition that a percentage reduction in the wage rate from the equilibrium rate lowers the hours supplied by the worker by more than a percent.

Hence we will have

(25) $\quad \frac{W^1}{A(u^0,\ \omega^1)} > \frac{W^0}{A(u^0,\ \omega^0)}$, for all $W^1 < W^0$.

Condition (25) shows that the unit cost of effort will be higher to a firm paying hourly wage rates that are lower than the wage rate that emerges at the joint profit maximising equilibrium. Consequently, no firm would not accept any worker at lower wage rates and (u^0, W^0) represents a Nash equilibrium.

Whether the assumptions of proposition 1 will be satisfied is an empirical question. However, given the effort supply function (21) it is possible to derive some conditions under which the assumptions of proposition 1 can be satisfied.

Differentiating both sides of equation (21) with respect to ω, we get

(26) $\quad \frac{\omega}{A}\frac{\partial A}{\partial \omega} = \theta\left[1 + \frac{\gamma\omega^{2\theta}-\alpha u-1}{A}\right].$

Given the definition of ω in equation (11), we can obtain by differentiation,

(27) $\quad \frac{W}{\omega}\frac{d\omega}{dW} = 1$

Combining (26) and (27) we can derive

(28) $\quad \frac{W}{A}\frac{\partial A(u,\omega)}{\partial W}\big|_{(u^0,W^0)} = \theta\left[1 + \frac{\gamma\omega^{2\theta}-\alpha u-1}{A}\right].$

Using (24) and (27) we can see that

(29) $\frac{du}{u} = -\theta.\frac{dW}{W}$

Which is the wage curve studied empirically by Blanchflower and Oswald (1995). Drawing from their 'empirical law', which states that the unemployment elasticity of real wage rate equal to -0.1, we can write that

(30) $\theta = 10$.

Using (30) in (28) we get

(31) $\frac{W}{A}\frac{\partial A(u,\omega)}{\partial W}|_{(u^0,W^0)} = 10\left[1 + \frac{\gamma\omega^{2\theta} - \alpha u^0 - 1}{A(u^0,W^0)}\right]$.

Although not necessary,

(32) $\frac{W}{A}\frac{\partial A(u,\omega)}{\partial W}|_{(u^0,W^0)} > 1$ if $\gamma > \frac{1+\alpha u^0}{\omega^{2\theta}}$.

Numerical exercises over a possible range of values for (u,ω) show that it is possible to find values for parameters α, β, γ and θ such that the assumption of proposition 1 is satisfied.

Given that the stability of the unemployment equilibrium so far rests on a specific form of the effort supply function and a particular type of parametric configuration can only be taken as a possibility.

However, even in situations when the stability of the unemployment equilibrium cannot be guaranteed, it can be argued that there is an incentive to all employers to collude and enforce the wage rate that emerges at the unemployment equilibrium that maximises their joint profits.

Conjecture 1: Employers may support trade unions demanding wage rate at W^0 which is higher that than the competitive wage rate with zero involuntary unemployment rate.

Conjecture 2: Employers may support minimum wage legislation to enforce the wage rate at W^0.

3   The DGTAPU model

The implementation of the labour market modelling outlined above in a general equilibrium model like GTAP (Hertel, 1997) to arrive at a dynamic GTAP model with endogenous determination of unemployment rates (DGTAPU) is quite straight forward.

We just need to add linearised versions of equation (21), (24) and the market clearing condition (9) into the model and specify the parameters of the model numerically.

For notational simplicity let us denote the percentage change of a variable X by p_ X, defined as p_X= $100 * \frac{dX}{X}$.

Differentiating equation (21) totally around the equilibrium point, we obtain after simplification

(33) $p\_A = \left(1 + \frac{\gamma\omega^{2\theta} - 1 - \alpha u}{A}\right).p\_u$.

The variable p_A is included as an additional labour productivity variable in the model. Changes in the unemployment rates thus affect the 'labour augmenting technical change' endogenously via the changes in the effort levels of the workers and then affect the sectoral labour demand in turn.

Similarly, the linearised equation (29) can be expressed as:

(34) $p_{\_u} = -\theta\, p\_W$, (empirical evidence indicates $\theta > 0.$)

135 empirical studies spanning 43 countries on the wage curve as listed in Blanchflower and Oswald (2005) including Blanchflower and Oswald (1990, 1994, 1995a, 1995b and 2005; Card, 1995; Blanchard and Katz, 1997 and 1999) have established that there exists an inverse relationship between the real wage rate and the unemployment rate. In the case of the US, the unemployment rate elasticity of the real wage rate has been found to be - 0.1, which means that in our case $\theta = 10$. A similar value has been found for many other countries (Blanchflower and Oswald, 2005) and this empirical law has passed some further tests (Card, 1995).

With this parameterization, (29) shows that unemployment rates fall with the rise in equilibrium real wage rates. Equation (33) indicates that a fall in unemployment rates endogenously reduces 'labour productivity' via reduced supply of free hours provided by the workers.

Other parameters can be calibrated as follows. As defined above, ω is the ratio of the current real wage to the real wage that would prevail when the economy attains full employment ($u = 0$). The model with exogenous unemployment rate can be simulated to find the wage rate that is required to reduce the unemployment level to zero over the entire simulation horizon and use this trajectory of the equilibrium wage rate as the data. To our surprise it requires less that 2 percent fall in the real wage rate to bring the full employment back.

The base year unemployment rate can be used in (24) to calibrate the value of β, taking α and γ as free parameters. Assuming that there exists a maximum level of effort a worker can exert relative to the work level standard (or a standard full time job =1), and denote it by A-max, equation (21) can be used to solve for α using the base year values of the wage rates, unemployment rates, calibrated values of other parameters given γ. Finally, γ remains to be specified by the modeller.

This completes the specification of the additional equation and calibration of the new parameters for a model like GTAP.

4   Illustrative Simulation Results

In this section we present the impacts on unemployment rates in regions experiencing labour augmenting technical progress by simulating the DGTAPU model. Since the impact of labour augmenting technical progress on the employment of labour depends on the size of the elasticity of factor substitution and the cost share of labour, we simulate the model under two sets of elasticity of factor substitution 1.5 and 0.3 for all sectors and in all countries. This uniformity on the parameter value has been forced for simplicity.

We have selected a uniform growth on all otherwise exogenous real factors to describe the business as usual case. This was done so not to portray that land supply globally, for example, would actually expand at 2 per cent per annum for the next 100 years, but to examine whether the modified model

also displays the properties of Solow-Swan growth models by relaxing all the real constraints in the model by a uniform 2 per cent per year for the next 100 years.

In the policy scenario, uniform labour augmenting productivity growth of 3 percent per annum has been assumed for the next 100 years for all countries. We expect unemployment rates to fall as a result of the shock when the elasticity of factor substitution is 1.5 and to rise when the elasticity of factor substitution is too low, say 0.3.

The impact on the unemployment rates by region under these scenarios are given from Figure1-6.

Figure 1: Impact of labour productivity growth in EU unemployment rates, Sig=1.5
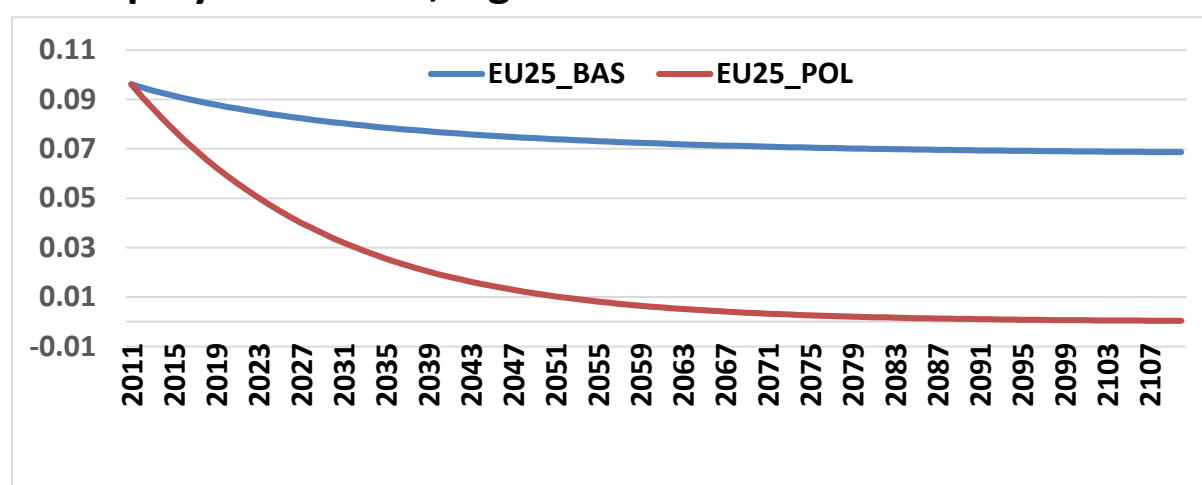

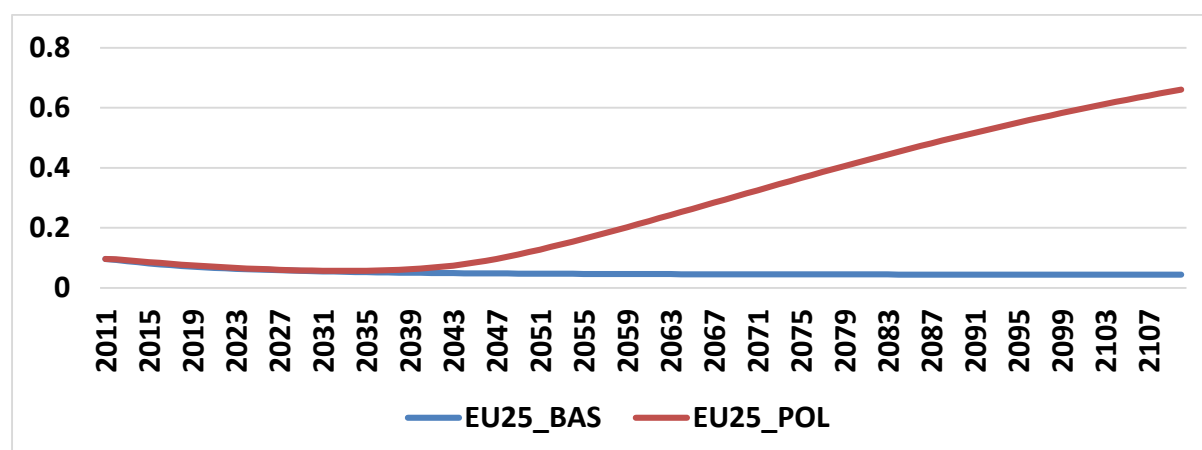## Figure 2: Impact of labour productivity growth in EU unemployment rates, Sig=0.3

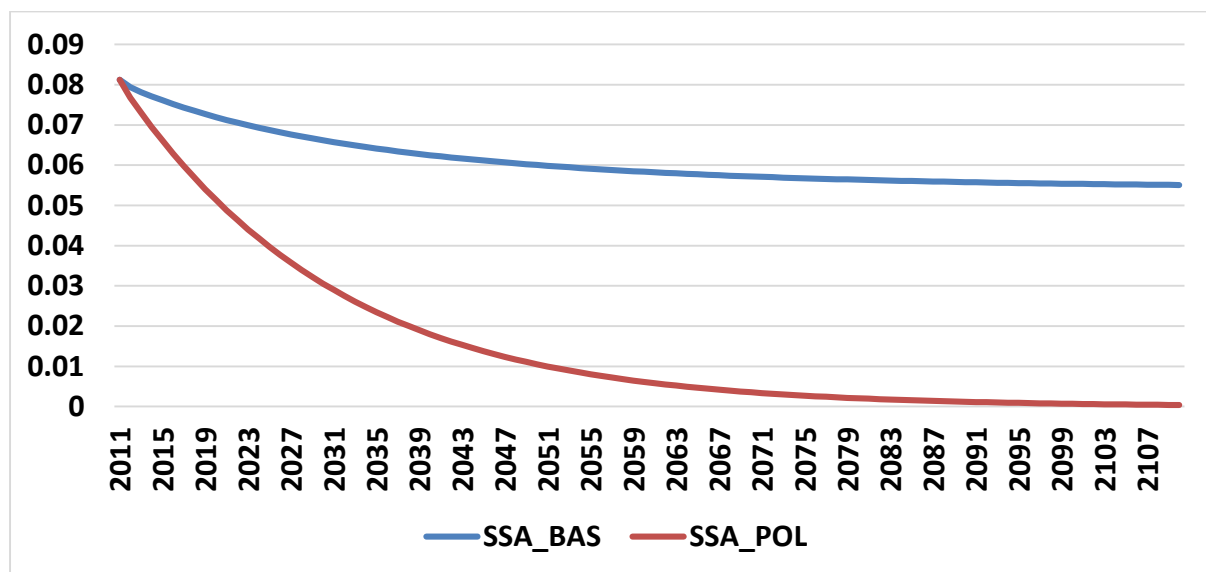**Figure 3: Impact of labour productivity growth in SSA unemployment rates, Sig=1.5**

Figure 4: Impact of labour productivity growth in SSA unemployment rates, Sig=0.3
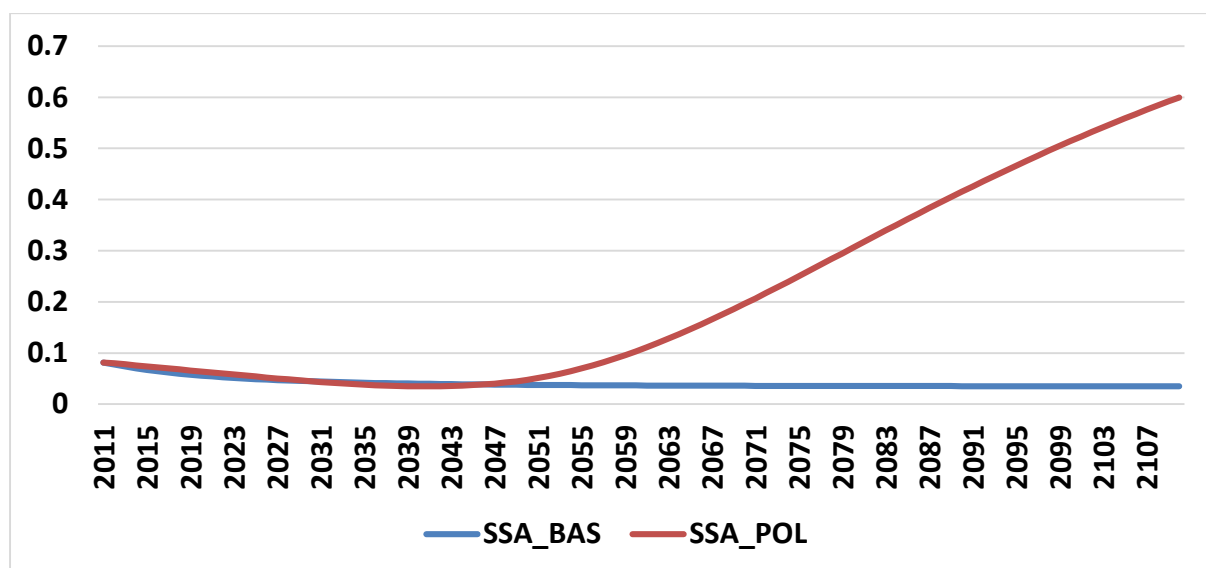
Figure 5: Impact of labour productivity growth in RoW unemployment rates, Sig=1.5
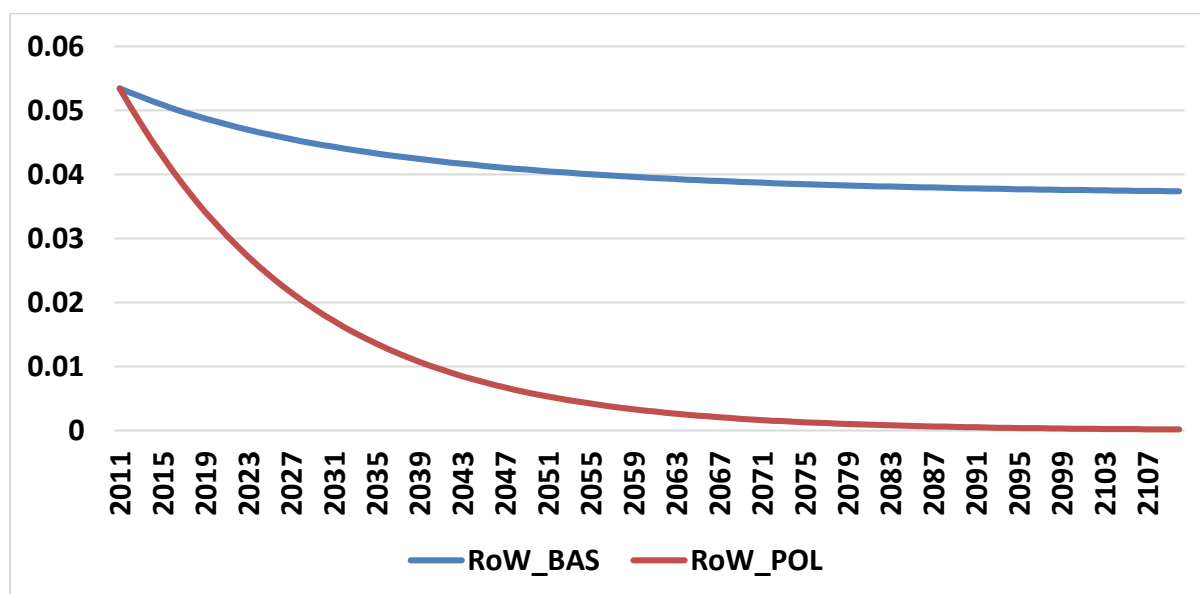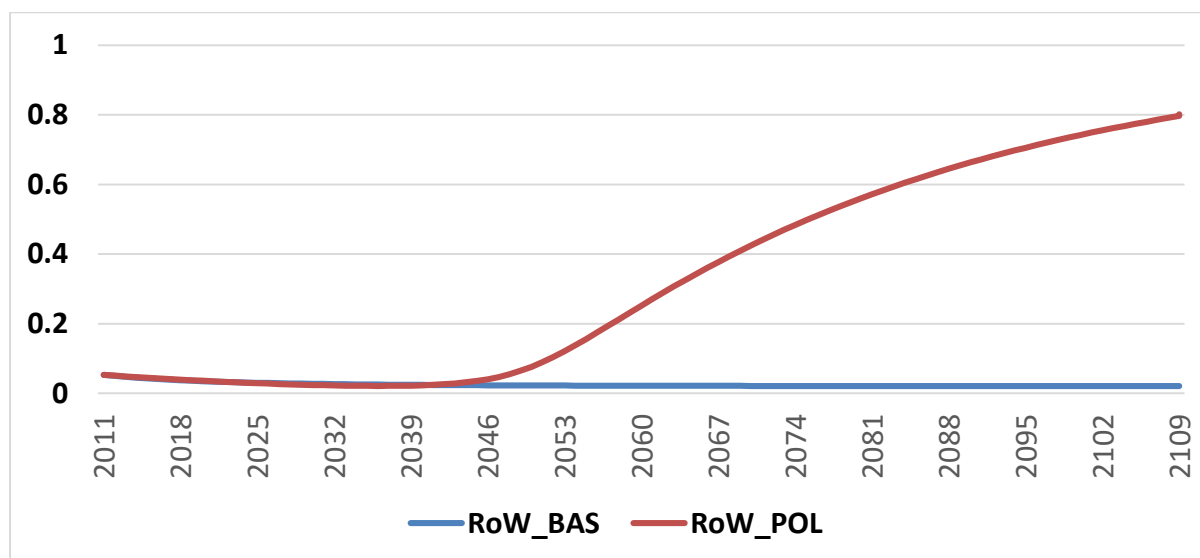

Figure 6: Impact of labour productivity growth in RoW unemployment rates, Sig=0.3

When elasticity of factor substitution is sufficiently large, results presented in figures 1,3, and 5 show that unemployment rates fall to zero within few years of policy scenario. as the increased supply of 'efforts' makes labour cheaper relative to other factors, labour can substitute the scarce factors under the high elasticity of factor substitution case. Therefore, Labour productivity growth raises real wage and reduces the unemployment rate.

When the elasticity of substitution is rather small, 0.3 in this case, the results presented in figures 2,4 and 6 show that increased labour productivity causes unemployment to rise. It is so because the increased supply of 'efforts' can not substitute for other scarce factors easily and so the real wage rate has to fall and the unemployment rate rises.

So far the performance of the DGTAPU model has remained in line with what as been predicted by the theory. The ability to project the impacts on unemployment rates endogenously of any policy change makes the modified model more precise in the study of public policies than the models that take unemployment rates as given.

5   Summary and Conclusion.

In this paper we presented a model of the labour market based on the fundamental assumption that workers are conscientious but fear unemployment. Therefore, they tend to supply additional hours for free when there is higher unemployment in the market; and it is a common knowledge. This assumption is quite the opposite to the assumption of shirking models of Shapiro and Stiglitz (1984).

Given this behaviour on the part of the workers it was shown that the joint profit of firms is maximised at a positive unemployment rate, which maximises the effort level of every worker. It was also shown that there is an inverse relationship between the profit maximizing unemployment rate and the market clearing real wage rate. This relationship was shown to be consistent with the empirical wage curve found in the literature which did not have a simple explanation of what exactly did the curve represent. We have now shown that the wage curve indeed represents the cost minimising supply of efforts at all possible hourly wage rates when translated into the wage-employment plane and represents cost minimising labour supply curve as viewed by the employers.

Given that the wage curve is just one of the first order conditions of joint maximisation, it just represents a locus of the possible combination of (u,W) at which labour market could clear and therefore is very similar to the I-S curve in its nature. There is no indication of the causal relationship in the wage curve between the unemployment rate and the real wage rate as claimed in the literature.

The wage curve alone can not establish an equilibrium in the labour market. The general equilibrium would be established once the aggregate labour demand was brought into the picture with all the information regarding the equilibrium in all other markets.

However, the 'wage curve' turned out to be the missing equation in labour market modelling supported by the effort supply function.

The 'wage curve' augmented dynamic GTAP model projected changes in the unemployment rates as predicted by the theory and hence it is theoretically robust. It enables the modellers to conduct better policy analysis than the models with exogenous unemployment rates. Further study of the effort supply function may improve the precision of the labour market modelling.

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