Trade costs and borders in the world of global value chains*

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Abstract

There is a growing body of statistical evidence of the importance of value chains for the global economy. The perception of longer value chains with more border crossings raised concerns about multiple trade barriers and associated costs. In the existing literature, however, the investigations of the accumulation of trade costs through the multi-stage production rarely extended beyond illustrative examples. The likely reasons are poor data and technical difficulties inherent in the newly developed accounting methods that focus on value added flows irrespective of border crossings. This paper proposes two new approaches to quantify the accumulation of trade costs along global value chains and a measure of the average number of border crossings in value chains. These approaches build on the inter-country input-output accounting frameworks that trace gross trade flows backward to their initial origin or forward to their ultimate destination. Data from the World Input-Output Database are supplemented with estimates derived from the UN Comtrade and UN TRAINS, allowing for an experimental computation of the accumulated import tariffs faced by exporters in 2001, 2005 and 2010. At the aggregate country and sector levels, the accumulation of import tariffs is found to be pervasive but moderate. The average number of border crossings exhibits a slow upward trend, but the accumulated tariffs decline quickly. Trade liberalization therefore neutralizes the risk of higher cumulative protection associated with the international fragmentation of production. The findings suggest that the input-output accounting frameworks may significantly extend the frontier of trade policy analysis in the world of global value chains.

1 Introduction

A value chain signifies that goods and services are produced in sequential stages. At each stage, enterprises purchase intermediate inputs, add value to them, and sell their outputs to other enterprises. These enterprises, in turn, produce their own outputs and the process continues. With the advent of the international fragmentation of production, value chains became global. According to a 2013 report by the OECD, WTO and UNCTAD for the G-20 Leaders Summit, "Value chains have become a dominant feature of the world economy" (OECD et al., 2013).

It is widely recognized that the growing fragmentation of production across borders may have important implications for trade and investment policies. When value chains are global,

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intermediate inputs cross national borders multiple times as their value is carried forward from one production stage to the next. The output is then a "bundle of many nations' inputs" (Timmer et al., 2013), but conventional gross trade statistics that inform trade policies attribute the origin only to the last known producing sector and exporting country. Policy measures that target these sectors or countries may not work well in the world of global value chains (OECD and WTO, 2012) because "what you see is not what you get!" (Maurer and Degain, 2010). It is therefore critical to understand where value is created and how it accumulates along the production chain.

It is also true that traded products bear a "bundle" of trade costs, as multiple border crossings entail multiple trade barriers and additional associated costs. The OECD has concluded, "The way in which tariffs and other protective measures at the border affect value chains needs to be taken into account in policy making and negotiations" (OECD, 2013, chap.3). There has been growing concern that, whereas nominal protection is now relatively low, cumulative protection can still be pervasive as the result of a magnification effect along the entire value chain. As an illustrative example, Ferrantino (2012) calculates that the uniform tariff of 10% is compounded exponentially along the value chain and is reported to reach 34% after five production stages and 75% after ten stages. The OECD (2013, chap.3) offers a similar rationale for the "tariff amplification effect:" the uniform tariff of 10% increases to 22% and 60% of the price of the final product after five and ten production stages, respectively.

Although the impact of global value chains on trade, the environment and jobs is now well established, there is only limited empirical evidence on the magnification of trade costs. The first authors to address this problem focused on explaining the cascading effect of tariff reduction. Hummels et al. (1999) suggest that "because the good-in-process crosses multiple borders, tariffs and transportation costs are incurred repeatedly", then "reductions in trade barriers yield a multiplied reduction in the cost of producing a good sequentially in several countries".

Investigating the magnification effect in more detail, Yi (2010) attributes it to two distinct causes. The first is the border effect: goods produced at various stages in different countries cross national borders during the production process and thus incur trade costs multiple times. The second is the vertical specialization effect: import tariffs apply to the customs value of gross exports as though imported goods were wholly produced in the exporting country, while they may actually carry values added in other countries earlier in the production process. Obviously, these two effects are not entirely separate: vertical specialization occurs when intermediate products cross multiple borders.

Theoretical trade models with embedded multi-stage production led to diverging conclusions. Yi (2003, 2010) identifies magnified and nonlinear trade responses to changes in import tariffs and other trade costs. In a similar exercise, Johnson and Moxnes (2013) find that fragmentation of production does not play an important role in inflating trade elasticity.

The measurement of trade costs in the global value chain environment is intimately connected with the renewed interest in the input-output framework first pioneered by Leontief (1936) and later adopted in numerous studies for the purpose of holistic value chain analysis. Tamamura (2010) and Koopman et al. (2010) are perhaps the first to provide numerical estimates of cumulative trade costs using inter-country input-output tables. Tamamura (2010) employs a form of the Leontief price model based on the 2000 Asian International Input-Output Table to examine the effect of import tariff reduction under China–Japan–ASEAN free trade agreements. He calls it "the repercussion effect" on production costs, resulting from the elimination of tariffs on all imports. Koopman et al. (2010) provide an illustrative calculation of magnified trade costs covering both bilateral transportation margins and import tariffs faced by exporting countries in 2004. In this exercise, they assemble a multi-regional input-output table from the GTAP database and compute transportation margins and tariffs applicable to value added flows rather than to gross exports.

Fally (2012) develops a formula to compute cumulative transport costs and shows that the result has a linear relationship with his index of "embodied production stages". Although not explicitly noted in his paper, Fally's measure of cumulative transport costs can be derived from the Leontief price model in the same way that Tamamura (2010) derives his tariff-to-output ratio.

Rouzet and Miroudot (2013) present an elaborate exposition of the concept of the cumulative tariff and the relevant computational method. They provide estimates of bilateral cumulative tariffs for various countries and industries that are based on the OECD inter-country input-output table and UNCTAD TRAINS data. Their version of cumulative tariffs can also be addressed in the Leontief price model.

This paper discusses three methods to quantify the accumulation of trade costs along global value chains. One of these methods builds on the Leontief price model and is conceptually equivalent to the earlier formulations in Tamamura (2010), Fally (2012) and Rouzet and Miroudot (2013). Two other methods build on accounting frameworks that trace gross trade flows through multi-stage production processes either backward to their initial origin or forward to their ultimate destination. A specific contribution of this paper is the development of a new measure of the incremental trade costs that arise at the border of one country (partner) with respect to both direct and indirect exports from another country (exporter) where indirect exports are "hidden" in third country exports. The derivation of this measure is possible because the underlying gross exports accounting framework discerns border crossings. Therefore, another contribution is a method to compute the average number of border crossings in global value chains.

The proposed measures are empirically tested using data from the World Input-Output Database (WIOD) and the UNCTAD TRAINS database for 2001, 2005 and 2010. At the aggregate country and sector levels, the accumulation effect of import tariffs is found to be moderate, though it may matter for certain bilateral linkages in the country-sector dimension. It is shown that longer value chains with more border crossings have not resulted in higher cumulative protection in external markets. Furthermore, cross-border value chains are effective channels for a "leakage" of preferences to non-members under free trade agreements.

The remainder of the paper proceeds as follows. Section 2 reviews the setup of the inter-country input-output system and discusses its utility in consistently modeling international trade costs. Methods of accounting for the accumulation of trade costs and multiple border crossings along global value chains are then briefly explained. Section 3 describes the data used for the experimental computations. The findings are discussed in Section 4. Finally, Section 5 provides a summary and recommendations for future research.

2 Accounting method

2.1 The input-output framework: notation and setup

Input-output tables are not the only analytical tool useful in exploring global value chains, but are perhaps the preferred choice for an economy-wide analysis. The existing alternatives – case studies of individual products (see an overview in Ali-Yrkkö and Rouvinen 2015) or analyses of trade in parts and components (e.g., Ng and Yeats 1999) – inevitably face the problem of value chain boundaries, i.e., the impossibility of capturing an entire production cycle that may consist of an infinite series of inter-industry interactions. Input-output tables provide an elegant solution to this problem, but at the expense of relatively high sector aggregation and a time lag in data availability.

Global value chain analysis requires a global input-output table where single-country tables are combined and linked via international trade matrices. Such inter-country or multi-regional input-output tables have been described by Isard (1951), Moses (1955), and Leontief and Strout (1963), among others, but have not been compiled at a global scale until late 2000s. The release of experimental global input-output datasets, including WIOD, Eora, Exiobase, OECD ICIO, GTAP-MRIO[1] and others,[2] has fuelled research into the implications of global value chains on trade, the economy and the environment.

Conceptually, an input-output table may be viewed as a comprehensive value chain representation of an economy. As such, it organizes data on the exchange of intermediate inputs among industries, the generation of value added by industries, and sales of final products to consumers. In an inter-country input-output table, the data are organized according to both country and industry classifications: each flow has the country and industry of origin (except value added) and country and industry of destination (except final products).

If there are K countries and N economic sectors in each country, the key elements of the inter-country input-output system may be described by block matrices and vectors. The KN×KN matrix of intermediate demand $\mathbf{Z}$ is therefore as follows:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1k} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{k1} & \mathbf{Z}_{k2} & \cdots & \mathbf{Z}_{kk} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{Z}_{rs} = \begin{bmatrix} z_{rs}^{11} & z_{rs}^{12} & \cdots & z_{rs}^{1n} \\ z_{rs}^{21} & z_{rs}^{22} & \cdots & z_{rs}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{rs}^{n1} & z_{rs}^{n2} & \cdots & z_{rs}^{nn} \end{bmatrix}$$

The lower index henceforth denotes a country with $r \in K$ corresponding to the exporting country and $s \in K$ to the partner country. The upper index denotes the sector. $\mathbf{Z}_{rs}$ is therefore an N×N matrix where each element z_{rs}^{ij} is the monetary value of the intermediate inputs supplied by the producing sector $i \in N$ in country r to the purchasing (using) sector $j \in N$ in country s.

Similarly, the KN×K matrix of final demand is:

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_{11} & \mathbf{f}_{12} & \cdots & \mathbf{f}_{1k} \\ \mathbf{f}_{21} & \mathbf{f}_{22} & \cdots & \mathbf{f}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f}_{k1} & \mathbf{f}_{k2} & \cdots & \mathbf{f}_{kk} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{f}_{rs} = \begin{bmatrix} f_{rs}^{1} \\ f_{rs}^{2} \\ \vdots \\ f_{rs}^{n} \end{bmatrix}$$

Each block $\mathbf{f}_{rs}$ is an N×1 vector with elements f_{rs}^{i} representing the value of the output of sector i in country r sold to final users in country s.

Total output of each sector is recorded in the KN×1 column vector $\mathbf{x}$:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_{1} \\ \mathbf{x}_{2} \\ \vdots \\ \mathbf{x}_{k} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{x}_{r} = \begin{bmatrix} x_{r}^{1} \\ x_{r}^{2} \\ \vdots \\ x_{r}^{n} \end{bmatrix}$$

And the value added by each sector is recorded in the 1×KN row vector $\mathbf{v}$:

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{1} & \mathbf{v}_{2} & \cdots & \mathbf{v}_{k} \end{bmatrix} \quad \text{where a block element} \quad \mathbf{v}_{s} = \begin{bmatrix} v_{s}^{1} & v_{s}^{2} & \cdots & v_{s}^{n} \end{bmatrix}$$

[1]Multi-regional versions of GTAP input-output tables were compiled on an *ad hoc* basis in various research projects and were not publicly released.

[2]See the special issue of *Economic Systems Research*, 2013, vol. 25, no. 1 for an overview.

$\mathbf{v}_s$ is a 1×N vector where each element v_s^j describes the value added generated by sector j in country s throughout the production process.

To better reflect the results of production, net of any taxes, subsidies or margins related to sales, the transactions in $\mathbf{Z}$ and $\mathbf{F}$ should be valued at basic prices. Meanwhile, from the producer's perspective, intermediate inputs should enter the calculation at purchasers' prices, inclusive of all costs associated with their purchase. Accordingly, the taxes or margins payable on intermediate inputs should also be accounted for as inputs to production. These are usually recorded as 1×KN row vectors below $\mathbf{Z}$:

$$\mathbf{m}(g)_{(\mathbf{Z})} = \begin{bmatrix}\mathbf{m}(g)_{(\mathbf{Z})1} & \mathbf{m}(g)_{(\mathbf{Z})2} & \cdots & \mathbf{m}(g)_{(\mathbf{Z})k}\end{bmatrix}$$
$$\text{where a block element} \quad \mathbf{m}(g)_{(\mathbf{Z})s} = \begin{bmatrix}m(g)^1_{(\mathbf{Z})s} & m(g)^2_{(\mathbf{Z})s} & \cdots & m(g)^n_{(\mathbf{Z})s}\end{bmatrix}$$

$\mathbf{m}(g)_{(\mathbf{Z})s}$ is a 1×N row vector of the g^{th} margin where each element $m(g)^j_{(\mathbf{Z})s}$ is the amount of tax paid, subsidy received or trade/transport margin on all intermediate inputs purchased by sector j in country s. $\mathbf{m}(g)_{(\mathbf{Z})}$ is in fact a condensed form of the valuation layer that conforms to the dimension of $\mathbf{Z}$:

$$\mathbf{M}(g)_{(\mathbf{Z})} = \begin{bmatrix}\mathbf{M}(g)_{(\mathbf{Z})11} & \mathbf{M}(g)_{(\mathbf{Z})12} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})1k}\\ \mathbf{M}(g)_{(\mathbf{Z})21} & \mathbf{M}(g)_{(\mathbf{Z})22} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})2k}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{M}(g)_{(\mathbf{Z})k1} & \mathbf{M}(g)_{(\mathbf{Z})k2} & \cdots & \mathbf{M}(g)_{(\mathbf{Z})kk}\end{bmatrix}$$
$$\text{where a block element} \quad \mathbf{M}(g)_{(\mathbf{Z})rs} = \begin{bmatrix}\mathbf{M}(g)^{11}_{(\mathbf{Z})rs} & \mathbf{M}(g)^{12}_{(\mathbf{Z})rs} & \cdots & \mathbf{M}(g)^{1n}_{(\mathbf{Z})rs}\\ \mathbf{M}(g)^{21}_{(\mathbf{Z})rs} & \mathbf{M}(g)^{22}_{(\mathbf{Z})rs} & \cdots & \mathbf{M}(g)^{2n}_{(\mathbf{Z})rs}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{M}(g)^{n1}_{(\mathbf{Z})rs} & \mathbf{M}(g)^{n2}_{(\mathbf{Z})rs} & \cdots & \mathbf{M}(g)^{nn}_{(\mathbf{Z})rs}\end{bmatrix}$$

In N×N matrices $\mathbf{M}(g)_{(\mathbf{Z})rs}$, each element $m(g)^{ij}_{(\mathbf{Z})rs}$ depicts the amount of g^{th} margin (tax paid, subsidy received or trade/transport cost) paid on intermediate inputs purchased by sector j in country s from sector i in country r. $\mathbf{M}(g)_{(\mathbf{Z})}$ is then a matrix of bilateral margins that changes the valuation of intermediate inputs. If the sector that produces the margins, e.g., domestic trade and transportation services, is modelled as endogenous to the inter-industry system (in other words, is inside $\mathbf{Z}$), the summation of $\mathbf{M}(g)_{(\mathbf{Z})}$ column-wise will result in a zero vector $\mathbf{m}(g)_{(\mathbf{Z})}$. Taxes and subsidies on products are usually recorded as exogenous to the system, so vector $\mathbf{m}(g)_{(\mathbf{Z})}$ contains non-zero values. International transport margins are also modelled as though they were provided from outside the system, which is the result of the "Panama assumption" (see Streicher and Stehrer 2015 for an extensive discussion).

For a complete account of trade costs later in this section, valuation terms should also be compiled with respect to final products – 1×K row vector $\mathbf{m}(g)_{(\mathbf{F})}$ and KN×K matrix $\mathbf{M}(g)_{(\mathbf{F})}$.

The fundamental accounting identities in the monetary input-output system imply that total sales for intermediate and final use equal total output, $\mathbf{Zi}+\mathbf{Fi} = \mathbf{x}$, and the purchases of intermediate and primary inputs at basic prices plus margins and net taxes on intermediate inputs equal total input (outlays) that must also be equal to total output, $\mathbf{i}'\mathbf{Z}+\sum_{g=1}^{G}\mathbf{m}(g)_{(\mathbf{Z})}+\mathbf{v} = \mathbf{x}'$, where $\mathbf{i}$ is an appropriately sized summation vector and G is the number of the valuation layers (margins).[3]

[3]We assume here that the inter-country input-output table does not contain purchases abroad by residents

Gross bilateral exports in the inter-country input-output system may be obtained by summing the international sales of outputs for intermediate and final use:

$$\mathbf{E}_{bil} = \begin{bmatrix} 0 & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\ \mathbf{e}_{21} & 0 & \cdots & \mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & 0 \end{bmatrix} \quad \text{where a block element} \quad \mathbf{e}_{rs} = \begin{bmatrix} e^1_{rs} \\ e^2_{rs} \\ \vdots \\ e^n_{rs} \end{bmatrix}$$

Block elements $\mathbf{e}_{rs}$ are N×1 vectors where each entry $e^i_{rs} = \sum_{j=1}^{N} z^{ij}_{rs} + f^i_{rs}$, $r \neq s$.

The key to the demand-driven input-output analysis is the Leontief inverse, which, in the case of the inter-country input-output table is defined as follows:

$$(\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} \mathbf{I}-\mathbf{A}_{11} & -\mathbf{A}_{12} & \cdots & -\mathbf{A}_{1k} \\ -\mathbf{A}_{21} & \mathbf{I}-\mathbf{A}_{22} & \cdots & -\mathbf{A}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A}_{k1} & -\mathbf{A}_{k2} & \cdots & \mathbf{I}-\mathbf{A}_{kk} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1k} \\ \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{k1} & \mathbf{L}_{k2} & \cdots & \mathbf{L}_{kk} \end{bmatrix} = \mathbf{L}$$

$\mathbf{A}_{rs}$ blocks are N×N technical coefficient matrices where an element $a^{ij}_{rs} = \frac{z^{ij}_{rs}}{x^j_s}$ describes the amount of input by sector i in country r required per unit of output of sector j in country s. In block matrix form, $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$. Leontief inverse $\mathbf{L}$ is a KN×KN multiplier matrix that allows total output to be expressed as a function of final demand: $\mathbf{x} = \mathbf{A}\mathbf{x}+\mathbf{F}\mathbf{i} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{F}\mathbf{i} = \mathbf{L}\mathbf{F}\mathbf{i}$.

2.2 Interpreting trade costs in the input-output framework

The input-output system described above captures all transactions within and between countries related to production, generation of income, final consumption and accumulation of capital. The compilation of input-output data follows national accounting conventions.

The System of National Accounts (SNA) and related input-output manuals do not explicitly discuss trade costs, but these can be identified as various inputs to production. Those trade costs that change the valuation of products from basic to producers' and purchasers' prices are represented as the valuation layers $\mathbf{M}(g)_{(\mathbf{Z})}$ and $\mathbf{M}(g)_{(\mathbf{F})}$ in the input-output tables and can be condensed to the respective $\mathbf{m}(g)_{(\mathbf{Z})}$ and $\mathbf{m}(g)_{(\mathbf{F})}$ vectors. These include trade and transport margins, and taxes less subsidies on products. Margins can be understood as purchases of services from the trade and transport sectors (SNA, 2009, para 6.67, 14.126-14.130) while taxes and subsidies are payments to/from the government (SNA, 2009, para 7.88-7.96). In the literature on trade costs (e.g., Anderson and van Wincoop 2004), margins are referred to as distribution costs; taxes on imports are parallel to tariff measures and partially parallel to non-tariff measures at the border.

Other trade costs that relate to non-tariff measures at and behind the border, e.g., expenditures on customs procedures, conformity assessments, etc., correspond to purchases of intermediate inputs from the relevant supplying sectors. It may not be feasible to quantify these expenses separately from production or distribution costs.

or domestic purchases by non-residents or any statistical discrepancies. The sum of intermediate purchases at basic prices, net taxes, margins on intermediate inputs and value added at basic prices is therefore equal to the sector output at basic prices.

The exportation or importation of certain goods and services may involve payments for permits or licenses from the government, and these are recorded in national accounts as other taxes on production (SNA, 2009, para 7.97), which are part of value added (primary inputs to production).[4] In input-output accounts, however, other taxes on production related to international trade are not distinguished from all other taxes less subsidies on production.

In sum, trade costs may be treated in the input-output framework as valuation layers, intermediate inputs or primary inputs to production. Given the internationally recognized standards for the compilation of input-output data and the underlying supply-use tables, the data on valuation layers are the most accessible for trade cost accounting. These data cover a significant share of trade costs, including distribution costs and taxes on traded products.

In an inter-country input-output table, the representation of valuation layers is somewhat more complex than in a national table because taxes and transport charges apply at both origin and destination. Accordingly, in between the basic price at origin and the purchasers' price at destination, there are FOB and CIF prices. FOB is the price of a good at the border of the exporting country, or the price of a service delivered to a non-resident, including transport charges and trade margins up to the point of the border, and including any taxes less subsidies on the goods exported. CIF is the price of a good delivered at the border of the importing country, or the price of a service delivered to a resident, before the payment of any import duties or other taxes on imports or trade and transport margins within the country (Eurostat, 2008, p.164).

Ideally, an inter-country input-output table requires at least six valuation layers, as Fig. 1 shows. Layers 1-4 in Fig. 1 apply to international trade transactions, or off-diagonal blocks of $\mathbf{Z}$ and $\mathbf{F}$ matrices, while layers 5 and 6 apply to both international trade and domestic transactions, or all blocks thereof. For an exhaustive trade cost analysis, it is important to separate taxes (subsidies) at destination that apply to imports only and to all products irrespective of their origin. As SNA (2009, para 7.91) explains, "imported goods on which all the required taxes on imports have been paid when they enter the economic territory may subsequently become subject to a further tax, or taxes, as they circulate within the economy". This is an important distinction between Fig. 1 in this paper and Fig. 1 in Streicher and Stehrer (2015), upon which it is based. Note also that the valuation layers in Fig. 1 may be disaggregated to provide more detail, e.g., the taxes less subsidies layer may be split into taxes and subsidies, and trade and transport margins may be split into trade margins and transport margins.

The sequence of production stages within the value chain can be approximated as a power series (see Miller and Blair, 2009):

$$\mathbf{Fi} + \mathbf{AFi} + \mathbf{AAFi} + \mathbf{AAAFi} + \ldots = \left(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots\right)\mathbf{Fi} = \mathbf{LFi}$$

where $\mathbf{Fi}$ is the column vector of output for final use (row sum of matrix $\mathbf{F}$). In this backward decomposition, the production of final output $\mathbf{Fi}$ involves the use of intermediate inputs at each production stage (tier) t, equal to $\mathbf{A}^t\mathbf{Fi}$.[5] Each term in this decomposition is at its basic price as recommended for the input-output analysis. The basic price reflects the purchase of intermediates at purchasers' prices and value added at basic prices (Eurostat, 2008, p.92). Then, at each tier t, the basic price of output absorbs the valuation terms from the previous tier and, recursively, from all tiers before that. All sequentially applied valuation terms become inseparable from the "bundle of inputs", and no power series exists

[4] A known issue in national accounts is distinguishing taxes from service fees payable to the government to ensure compliance with regulatory measures (see SNA, 2009, para 7.80). This affects the treatment of trade costs – either as intermediate or primary inputs – and may be particularly pronounced in the case of service suppliers.

[5] The first tier is $t = 0$.

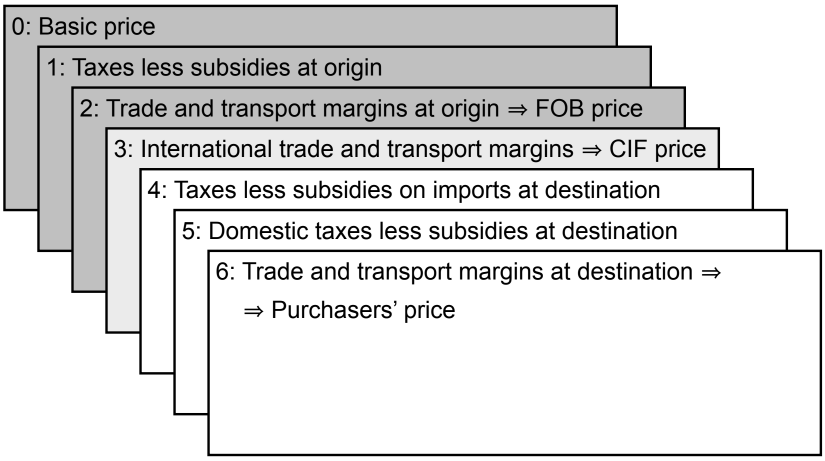

Figure 1: A minimum set of valuation layers in an inter-country input-output table
Author's adaptation of Fig. 1 from Streicher and Stehrer (2015)

for the valuation matrices. The input-output framework therefore does not support the logic of the exponential magnification of trade costs or margins as discussed by Ferrantino (2012). The input-output calculus of trade costs confirms the accumulation effect, but does so in different ways, which are reviewed in more detail in the following subsections.

2.3 Price model

The price model shows how the vector $\mathbf{m}(g)_{(\mathbf{Z})}$ propagates along the value chain. Let $\mathbf{p}$ be the column vector of index prices of industry output as in the standard Leontief price model (see Miller and Blair, 2009). The equilibrium condition requires that the price of industry output is entirely explained by the prices of intermediate and primary inputs:

$$\mathbf{p}'\hat{\mathbf{x}} = \mathbf{p}'\mathbf{Z} + \sum_{g=1}^{G} \mathbf{m}(g)_{(\mathbf{Z})} + \mathbf{v}$$

where $\mathbf{x}$ and $\mathbf{Z}$ should be interpreted in revised quantity terms (Miller and Blair, 2009). Post-multiplying by $\hat{\mathbf{x}}^{-1}$ leads to:

$$\mathbf{p}' = \mathbf{p}'\mathbf{A} + \sum_{g=1}^{G} \mathbf{m}(g)_{c(\mathbf{Z})} + \mathbf{v}_c$$

where $\mathbf{m}(g)_{c(\mathbf{Z})}$ is the 1×KN row vector of margin coefficients with the elements $m(g)^j_{c(\mathbf{Z}),s} = \dfrac{m(g)^j_{(\mathbf{Z})s}}{x^j_s}$, and $\mathbf{v}_c$ is the 1×KN row vector of value added coefficients with the elements $v^j_{c,s} = \dfrac{v^j_s}{x^j_s}$. Solving for $\mathbf{p}$ yields:

$$\mathbf{p}' = \sum_{g=1}^{G} \mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L} + \mathbf{v}_c\mathbf{L} \tag{1}$$

In the price model without an exogenous change of the primary input coefficients, the index price $\mathbf{p}$ will be equal to 1. Then, the $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ and $\mathbf{v}_c\mathbf{L}$ multipliers will give the shares of valuation (margins, net taxes) and value added in the equilibrium prices. In other words, each j, s^{th} element in the $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ vector corresponds to the part of the equilibrium price of the output of industry j in country s that accounts for the margins/taxes incurred directly by industry j in country s and indirectly by other industries along the downstream value chain. Note that, in line with the Leontief price model, $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ should be interpreted as the cost-push multipliers that translate an initial primary input coefficient or a change thereof into an index price of output or its change.

The $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ vector of multipliers can be recognized as a key term that Tamamura (2010) uses to study the effect of an import tariff reduction on production costs. It is also equivalent to the measure of the cumulative transport costs suggested by Fally (2012), though he uses a different notation and derives this measure from his recursive definition of the number of production stages. Rouzet and Miroudot (2013) combine the tariff-price multipliers $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ with the direct import tariffs to derive their measure of cumulative tariffs. To show this, let $\mathbf{T}_{(KN\times KN)}$ denote a KN×KN matrix of bilateral import tariff rates[6] where the elements τ_{rs}^{ij} do not differentiate across partner country sectors j, and let $\mathbf{m}(\tau)_{c(\mathbf{Z})}$ denote the row vector of import tariff coefficients with the elements $m(\tau)_{c(\mathbf{Z}),s}^{j} = \frac{\sum_{r=1}^{K}\sum_{i=1}^{N} z_{rs}^{ij}\tau_{rs}^{ij}}{x_s^j}$. Then, Rouzet and Miroudot's (2013) version of cumulative tariffs can simply be written as:[7]

$$\mathbf{T}_{(KN\times KN)cum} = \mathbf{T}_{(KN\times KN)} + \left[\mathbf{m}(\tau)_{c(\mathbf{Z})}\mathbf{L}\right]' \mathbf{i}' \tag{2}$$

$\mathbf{T}_{(KN\times KN)cum}$ above corresponds to the purchasers' price concept because it allocates direct tariff rates on top of the tariffs accumulated in the basic price of exports.

Either employed as a stand-alone multiplier vector, or in the matrix version of Rouzet and Miroudot (2013), $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ accounts for the cumulative impact of margins/taxes as an input to production in country r on the price of gross exports from country r to country s, but ignores the sectoral and national origin of the inputs that carried those margins/taxes. $\mathbf{m}(g)_{c(\mathbf{Z})}\mathbf{L}$ multipliers show how the price of the output would reduce if all import tariffs were set to zero.

2.4 Cumulative trade costs based on the value added accounting framework

A value added accounting framework traces the origin of gross exports to the sectors that initially contribute value added to those exports. This is a backward decomposition that reallocates all observed bilateral export flows into the unobserved value added flows between origins and destinations. The key element in a value added accounting framework is the "global" Leontief inverse $\mathbf{L}$. Koopman et al. (2012) and Stehrer (2013) are well known examples of such decomposition. Replacing the value added coefficients $\mathbf{v}_c$ with the margin or tax coefficients $\mathbf{m}(g)_{c(\mathbf{Z})}$, i.e., the amount of margin or tax payable per unit of output, enables the analyses of trade costs as embodied valuation terms.

For an illustrative purpose, split bilateral gross exports into exports of intermediate and final products:

[6] Tariff rates need to be expressed as decimals, or percentages divided by 100.

[7] The original formulation of Rouzet and Miroudot (2013), using the notation of this paper, is as follows:

$$\mathbf{T}_{(KN\times KN)cum} = \mathbf{T}_{(KN\times KN)} + \left[\sum_{t=0}^{\infty} \mathbf{i}'(\mathbf{A}\circ\mathbf{T}_{(KN\times KN)})\mathbf{A}^t\right]' \mathbf{i}'$$

where $\circ$ signifies the element-by-element multiplication. Given that $\mathbf{A}\circ\mathbf{T}_{(KN\times KN)} = \mathbf{M}(\tau)_{c(\mathbf{Z})}$, $\mathbf{i}'\mathbf{M}(\tau)_{c(\mathbf{Z})} = \mathbf{m}(\tau)_{c(\mathbf{Z})}$ and $\sum_{t=0}^{\infty}\mathbf{A}^t = \mathbf{L}$, this formula can be re-written in the form of equation (2).

$$\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$$

where the modified "check" operators extract off-diagonal block elements from block matrices but do not apply to the elements within those blocks. $\breve{\mathbf{Z}}_{(KN\times K)}$ is the matrix of intermediate demand condensed to the KN×K dimension (i.e., aggregated across partner country sectors) with the diagonal blocks set to zero:

$$\breve{\mathbf{Z}}_{(KN\times K)} = \begin{bmatrix} 0 & \mathbf{Z}_{12}\mathbf{i} & \cdots & \mathbf{Z}_{1k}\mathbf{i} \\ \mathbf{Z}_{21}\mathbf{i} & 0 & \cdots & \mathbf{Z}_{2k}\mathbf{i} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{k1}\mathbf{i} & \mathbf{Z}_{k2}\mathbf{i} & \cdots & 0 \end{bmatrix} \quad \text{where a block element} \quad \breve{\mathbf{Z}}_{(KN\times K)rs} = \begin{bmatrix} z_{rs}^{1\bullet} \\ z_{rs}^{2\bullet} \\ \vdots \\ z_{rs}^{n\bullet} \end{bmatrix}$$

In the formulation above, $\mathbf{i}$ is an N×1 summation vector and the upper index $n\bullet$ signifies that the intermediate inputs of the producing sector n are aggregated across purchasing sectors.

A respective direct bilateral g^{th} valuation layer is given by:

$$\underset{t=0}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})}$$

The above margins/taxes change the valuation of direct exports, or exports at tier 0.

Following the logic of sequential production stages, exports of intermediate and final products require intermediate inputs at the previous stage: $\mathbf{A}\breve{\mathbf{Z}}_{(KN\times K)} + \mathbf{A}\breve{\mathbf{F}}$. This involves the corresponding valuation at tier 1, counting tiers backwards:

$$\underset{t=1}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{F}}$$

The above changes the valuation of intermediate inputs involved in the production of direct exports $\breve{\mathbf{Z}}_{(KN\times K)}$ and $\breve{\mathbf{F}}$. To show this explicitly, we will zoom in a typical block element in $\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)}$:

$$\left[\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{Z}}_{(KN\times K)}\right]_{rs} = \sum_{t\neq s}^{K} \begin{bmatrix} \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{1u} z_{ts}^{u\bullet} \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{2u} z_{ts}^{u\bullet} \\ \vdots \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{nu} z_{ts}^{u\bullet} \end{bmatrix}$$

For a pair of exporter r and partner s, each element in the matrix above extracts the margin or tax incurred in the production of intermediate input z of sector u exported to country s at tier 0 and allocates that margin or tax to country r because it supplied the products subject to those margins or taxes at tier 1. Similarly, a typical block element in $\mathbf{M}(g)_{c(\mathbf{Z})}\breve{\mathbf{F}}$ is:

$$\left[\mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{F}}\right]_{rs} = \sum_{t\neq s}^{K} \begin{bmatrix} \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{1u} f_{ts}^{u} \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{2u} f_{ts}^{u} \\ \vdots \\ \sum_{u=1}^{N} m(g)_{c(\mathbf{Z})rt}^{nu} f_{ts}^{u} \end{bmatrix}$$

In fact, the matrix of margin coefficients $\mathbf{M}(g)_{c(\mathbf{Z})}$ applies here in the same way that the matrix of technical coefficients $\mathbf{A}$ does, but counts embodied primary, not intermediate inputs.

Intermediate inputs two tiers back are equal to: $\mathbf{A}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{A}\mathbf{A}\check{\mathbf{F}}$. And the corresponding valuation at tier 2 is:

$$\underset{t=2}{\mathbf{M}(g)_{(\mathbf{E})}} = \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{F}}$$

The above changes the valuation of embodied intermediate inputs two tiers back. Each element in either matrix counts the amount of g^{th} margin/tax payable on inputs supplied at tier 2.

This decomposition can be continued backwards to an infinitely remote tier. Compiling the valuation of intermediate inputs at all tiers will result in:

$$\begin{aligned}\underset{t=1,2,\ldots,\infty}{\mathbf{M}(g)_{(\mathbf{Z},KN\times K)}} &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \\ &+ \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\mathbf{A}\check{\mathbf{Z}}_{(KN\times K)} + \cdots + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}^{t}\check{\mathbf{Z}}_{(KN\times K)} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\left(\mathbf{I} + \mathbf{A} + \mathbf{A}\mathbf{A} + \cdots + \mathbf{A}^{t}\right)\check{\mathbf{Z}}_{(KN\times K)} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{Z}}_{(KN\times K)}\end{aligned}$$

Similarly, the cumulative valuation of final products will yield:

$$\begin{aligned}\underset{t=1,2,\ldots,\infty}{\mathbf{M}(g)_{(\mathbf{F})}} &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\check{\mathbf{F}} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\check{\mathbf{F}} + \\ &+ \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}\mathbf{A}\check{\mathbf{F}} + \cdots + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{A}^{t}\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\left(\mathbf{I} + \mathbf{A} + \mathbf{A}\mathbf{A} + \cdots + \mathbf{A}^{t}\right)\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{F}}\end{aligned}$$

Combining the multi-tiered valuation of intermediate and final products allows for the cumulative accounting of trade costs corresponding to the g^{th} valuation layer:

$$\begin{aligned}\mathbf{M}(g)_{(\mathbf{E})cum} &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{Z}}_{(KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\check{\mathbf{F}} = \\ &= \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})} + \mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil}\end{aligned} \tag{3}$$

The $\mathbf{M}(g)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil}$ term involves the double-counting of embodied valuation in the same way that $\hat{\mathbf{v}}_c\mathbf{L}\mathbf{E}_{bil}$ involves the double-counting of value added. The core difference is that value added does not move internationally and $\hat{\mathbf{v}}_c$ is therefore a KN×KN diagonal matrix, unlike $\mathbf{M}(g)_{c(\mathbf{Z})}$.

If g corresponds to import tariffs τ, $\mathbf{M}(\tau)_{(\mathbf{Z},KN\times K)}$ can be written as $\breve{\mathbf{Z}}_{(KN\times K)} \circ \mathbf{T}$ and $\mathbf{M}(g)_{(\mathbf{F})}$ can be written as $\breve{\mathbf{F}} \circ \mathbf{T}$. The matrix of margin coefficients becomes equal to:

$$\mathbf{M}(\tau)_{c(\mathbf{Z})} = \mathbf{M}(\tau)_{(\mathbf{Z})}\hat{\mathbf{x}}^{-1} = \breve{\mathbf{Z}} \circ \mathbf{T}_{(KN\times KN)}\hat{\mathbf{x}}^{-1} = \mathbf{A} \circ \mathbf{T}_{(KN\times KN)}$$

where $\circ$ signifies the element-by-element multiplication. Then the cumulative import tariff is:

$$\begin{aligned}\mathbf{M}(\tau)_{(\mathbf{E})cum} &= \breve{\mathbf{Z}}_{(KN\times K)} \circ \mathbf{T} + \breve{\mathbf{F}} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{L}\mathbf{E}_{bil} = \\ &= \mathbf{E}_{bil} \circ \mathbf{T} + \left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{L}\mathbf{E}_{bil} \end{aligned} \quad (4)$$

where $\mathbf{M}(\tau)_{(\mathbf{E})cum}$ is the KN×K matrix of cumulative import tariffs in monetary terms and $\mathbf{T}$ is the matrix of bilateral import tariff rates in the country-sector by country (KN×K) dimension. Read this equation as follows: cumulative tariffs (in monetary terms) are equal to the direct tariffs on bilateral exports plus the tariffs embodied in bilateral exports throughout the entire value chain. An important distinction as compared to the formula of Rouzet and Miroudot (2013) is that the embodied valuation term $\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{L}\mathbf{E}_{bil} = \mathbf{M}(\tau)_{c(\mathbf{Z})}\mathbf{L}\mathbf{E}_{bil}$ is not uniform across producing countries. It accounts for tariffs as the embodied primary inputs payable on the products of sector i in country r regardless of whether r is a direct or t^{th} tier supplier. Thus, it traces cumulative tariffs backwards to the origin of the products subject to those tariffs. To put it more explicitly, it captures the tariffs payable on inputs at their origin and records these as embodied inputs at their destination. Therefore, one important drawback of this measure is that it cannot capture the indirect valuation of services.[8]

Finally, the element-by-element ratios of cumulative tariffs (or margins and net taxes, in general) to gross bilateral exports translate the estimates in monetary terms into percentages that are more convenient for trade policy analysis, e.g., in comparison with direct tariff rates:[9]

$$\mathbf{T}_{cum} = \mathbf{M}(\tau)_{(\mathbf{E})cum} \oslash \mathbf{E}_{bil} = \mathbf{T} + \left(\left(\mathbf{A} \circ \mathbf{T}_{(KN\times KN)}\right)\mathbf{L}\mathbf{E}_{bil}\right) \oslash \mathbf{E}_{bil} \quad (5)$$

where $\oslash$ is the element-by-element division. For brevity, $\mathbf{T}_{cum}$ will be referred to as "cumulative tariffs".

2.5 Incremental trade costs based on the gross exports accounting framework

A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain. Koopman et al. (2010) and Wang et al. (2013) propose the accounting frameworks that may be classified under this type.[10]

[8]Since equation (4) captures the tariffs at origin, and the direct tariffs on services are zero, the indirect (embodied) tariffs on services will also be zero. Meanwhile, in Rouzet and Miroudot's (2013) formula, the cumulative tariffs on services will be uniform across partner countries and will not show the variation of value chains in the bilateral country setting. This problem is addressed in the next subsection by a model that employs the gross exports accounting framework.

[9]It is impossible to obtain the tariff rate in percentage terms if the respective bilateral exports are zero. This also applies to the implicit tariff rates suggested in subsection 2.5.

[10]The delimitation between the gross exports accounting framework and the value added accounting framework is primarily intended for the reader's understanding of the underlying decomposition concept. In the existing literature, the elements of the backward and forward decompositions may be combined in a

An essential requirement for a gross exports accounting framework is the ability to account for sequential border crossings. The Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is not suitable because it is indifferent to the national origin of intermediate inputs. Another "global" inverse, described by Muradov (2015), addresses this issue:

$$\mathbf{H} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$$

where the modified "hat" and "check" operators extract, respectively, diagonal and off-diagonal block elements from block matrices but do not apply to the elements within those blocks. $\mathbf{H}$ is a KN×KN matrix of multipliers that is capable of sequentially identifying exports at tier t used to produce exports at the next tier $t+1$, or "exports embodied in exports" in a multi-country setting. Here, tiers denote production stages only when products cross national borders. An algebraic manipulation shows the relationship between the new "global" inverse and the standard Leontief inverse: $\mathbf{H} = \left(\mathbf{I} - \widehat{\mathbf{A}}\right)\mathbf{L}$. A detailed technical exposition may be found in the Appendix A.

The power series of $\mathbf{H}$ model the path of a "melting" portion of the initial exports until it is entirely consumed (used) at an infinitely remote t^{th} tier:

$$\mathbf{HE}_{bil} = \mathbf{E}_{bil} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil} + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{2}\mathbf{E}_{bil} + \cdots + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{bil}$$

Each term in this decomposition describes a portion of the initial exports that reaches partner after t tiers or border crossings. Replacing $\mathbf{E}_{bil}$ with a matrix of bilateral margins or taxes (subsidies) $\mathbf{M}(g)_{(\mathbf{E})}$ leads to the incremental valuation of those initial exports at the partner side:

$$\mathbf{HM}(g)_{(\mathbf{E})} = \mathbf{M}(g)_{(\mathbf{E})} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{M}(g)_{(\mathbf{E})} + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{2}\mathbf{M}(g)_{(\mathbf{E})} + \\ + \cdots + \left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{M}(g)_{(\mathbf{E})}$$

Obviously, $\mathbf{M}(g)_{(\mathbf{E})}$ is the margin or tax paid on direct exports. The second term $\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{M}(g)_{(\mathbf{E})}$ records the margin or tax paid on partner bilateral exports (2nd tier) which are in fact a part of the initial exports from the country of origin (1st tier). The remaining terms record margins or taxes in the same way at each successive tier, or after each border crossing. In other words, at t^{th} tier from the origin, the respective term in the power series above reallocates direct margins at destination in proportion to indirect exports at origin.

The summation of terms in this forward decomposition may therefore be treated as an incremental resistance term $\mathbf{M}(g)_{(\mathbf{E})inc}$ because trade costs arise incrementally in the exporter–partner relationship:

$$\mathbf{M}(g)_{(\mathbf{E})inc} = \mathbf{HM}(g)_{(\mathbf{E})} \tag{6}$$

where $\mathbf{M}(g)_{(\mathbf{E})} = \mathbf{M}(g)_{(\mathbf{Z},KN\times K)} + \mathbf{M}(g)_{(\mathbf{F})}$.

single formulation. For example, Wang et al. (2013) employ value added multipliers while tracing the use of direct exports. This helps in discerning the country of origin of added value contained therein, but not in discerning its sectoral origin.

For an intuitive interpretation of equation (6), consider the specific case of import tariffs:

$$\mathbf{M}(\tau)_{(\mathbf{E})inc} = \mathbf{H}(\mathbf{E}_{bil} \circ \mathbf{T}) \tag{7}$$

Each element in the KN×K matrix $\mathbf{M}(\tau)_{(\mathbf{E})inc}$ counts all tariffs (in monetary terms) payable on the products of sector i in country r at the border of country s regardless of whether s is a direct or t^{th} tier partner. Like the cumulative measure of tariffs $\mathbf{M}(\tau)_{(\mathbf{E})cum}$ derived from the value added accounting framework above, the $\mathbf{M}(\tau)_{(\mathbf{E})inc}$ term involves double counting of the import tariffs paid. However, it does so in a different way: it incrementally captures the tariffs payable at (the border of) destination and records these as exports at origin. Equation (7) is therefore capable of quantifying the indirect tariffs on services because it keeps track of services embodied in goods that are subject to tariffs.

The implicit tariff rates in this case are as follows:

$$\mathbf{T}_{inc} = \mathbf{M}(\tau)_{(\mathbf{E})inc} \oslash \mathbf{E}_{bil} = (\mathbf{H}(\mathbf{E}_{bil} \circ \mathbf{T})) \oslash \mathbf{E}_{bil} = \mathbf{T} + ((\mathbf{H} - \mathbf{I})(\mathbf{E}_{bil} \circ \mathbf{T})) \oslash \mathbf{E}_{bil} \tag{8}$$

where $\oslash$ is the element-by-element division. For brevity, $\mathbf{T}_{inc}$ will be referred to as "incremental tariffs".[11]

2.6 Cumulative and incremental trade cost accounting: an illustrative example

From equations (5, 8), it is apparent that both cumulative and incremental approaches count direct trade costs as these apply to cross-border transactions plus indirect costs that propagate through multi-stage production. A simplified example will show how the different accounting methods handle indirect trade costs. We assume that there are two countries, exporter (producer) and partner (user) that do not directly trade with each other. There are only two types of products, goods and services. Third countries A and B process intermediate goods and services purchased from the exporter and sell the processed goods to the partner as outlined in Fig. 2. From the perspective of value added or gross exports accounting, indirect flows exist and are effectively subject to indirect tariffs.

The cumulative method counts all tariffs that apply to the exporter's inputs at the border of third countries. These are the inputs that, after processing in countries A and B, will eventually reach the partner. In this way, the tariffs are recorded when the inputs leave the origin. In Fig. 2, the cumulative tariff is equal to a 10% tariff applied by third country A on the exporter's good worth 30 units, plus a 5% tariff applied by third country B on the exporter's good worth 20 units, which totals 4. Direct tariffs and, hence, cumulative tariffs do not apply to services.

The incremental method counts all tariffs that apply to the exporter's inputs at the border of the partner where they are hidden in third country exports. The tariffs are recorded when the embodied inputs reach the destination. In Fig. 2, the incremental tariff is equal to a 15% tariff applied on country B's goods where 40 units are sourced from the exporter through country A, including 10 units of services. The total incremental tariff is 6 units.

Both cumulative and incremental tariffs should not be understood as the amounts actually payable on traded products. Rather, they quantify the accumulated bilateral resistance or protection that a product faces as it moves along the entire value chain from exporter to partner.

[11] The terms "cumulative" and "incremental" are introduced here for easier reference to the two different accounting techniques.

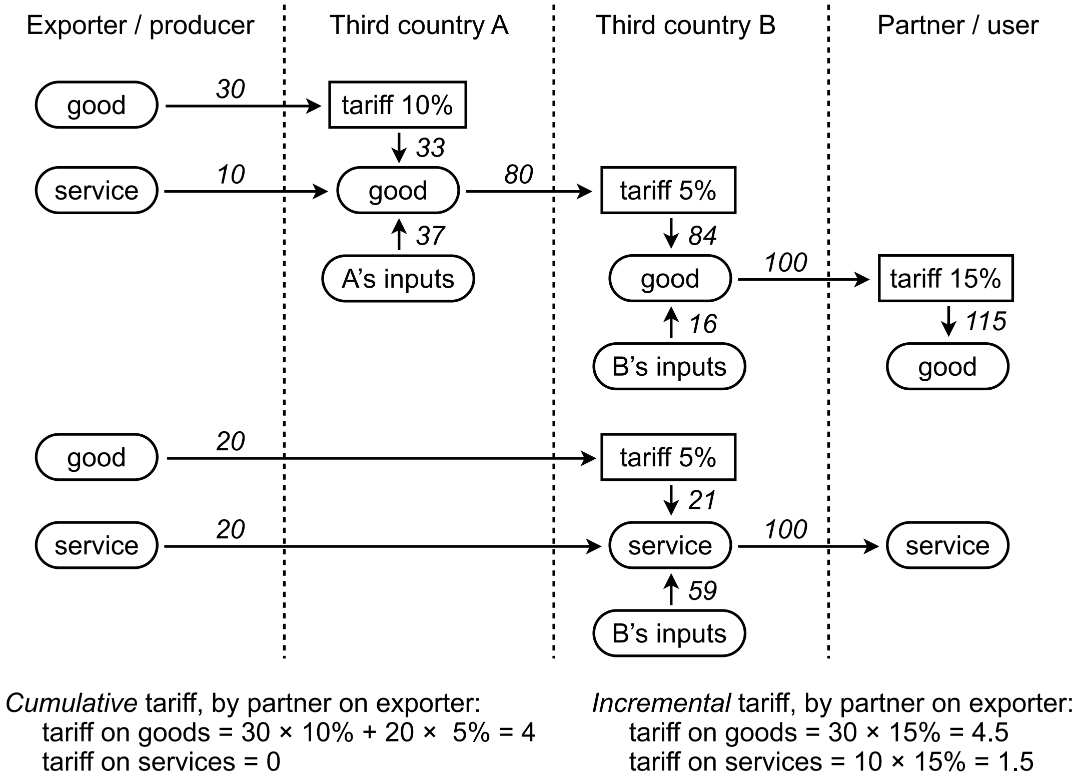

Figure 2: A simplified example of cumulative and incremental tariff accounting

For an exemplary calculation of the tariff multiplier described in subsection 2.3, we will treat country B in Fig. 2 as a producer and exporter. The tariff multiplier would sum up the tariffs paid by third country B ($80\times5\% = 4$ embodied in goods and $20\times5\% = 1$ embodied in services), third country A ($30\times10\% = 3$ embodied in goods) and would relate the result (7 in goods and 1 in services) to country B's output at basic prices; this is beyond the example in Fig. 2. If this output is assumed to equal country B's exports to its partner country (100 units of goods and 100 of services), the tariff multiplier will be 0.07 for goods and 0.01 for services. Adding the direct tariff by the partner country results in Rouzet and Miroudot's (2013) version of a cumulative tariff rate: $7\% + 15\% = 22\%$ for goods and $1\% + 0\% = 1\%$ for services. In this case, the origin of the inputs subject to tariffs or the countries that apply those tariffs will no longer be distinguished. Hence an analytical limitation of the tariff multiplier: it is impossible to measure the cumulative impact of tariffs along the downstream value chain on products of a particular exporting sector/country. Meanwhile, the cumulative and incremental methods enable the measurements of tariffs both applied by importing countries and faced by exporting countries/sectors.

Fig. 2 reveals that the tariff multiplier captures tariffs on services because goods are embodied in those services. Conversely, the incremental method counts tariffs on services because services are embodied in goods. The cumulative tariff counts tariffs that may eventually be embodied in services, but records those as tariffs on goods only.

The interpretation of trade cost accounting techniques may be more intricate in cases involving other valuation layers, e.g., taxes (subsidies) on exports or trade and transport margins at origin. In a general case, the cumulative and incremental methods allow the measurement of accumulated trade costs between the country of origin (exporter, producer) and the country of destination (partner, user), including direct and indirect costs. In the cumulative formulation, indirect trade costs are counted when valuation layers apply to the transactions between the exporter and the third countries. In the incremental formulation, these costs are counted when the same valuation layers apply to the transactions between the third countries and the partner. The incremental method captures trade costs further downstream on the value chain, and if direct tariffs are higher as the product approaches the final user, the incremental tariff will exceed the cumulative tariff, as in Fig. 2.

2.7 Number of border crossings

Previous studies, including Hummels et al. (1999) and Yi (2010), have identified multiple border crossings as a key factor behind the magnification of trade costs in global value chains. Measuring the number of border crossings per se is of significant analytical interest.

As noted in subsection 2.5, an essential property of the multiplier matrix $\mathbf{H}$ is the ability to trace a "melting" portion of the initial exports until it is entirely consumed (used) at an infinitely remote t^{th} tier. The sum of net exports that end up in partner final demand at each tier t yields cumulative exports.

The KN×K matrix of cumulative exports $\mathbf{E}_{cum}$ may be computed in two alternate ways yielding the same result (see Appendix A for a detailed derivation procedure).

First, cumulative exports may be computed as a function of final demand in partner countries:

$$\mathbf{E}_{cum} = \mathbf{H}\check{\mathbf{F}} + (\mathbf{H} - \mathbf{I})\widehat{\mathbf{F}} = \mathbf{HF} - \widehat{\mathbf{F}} \tag{9}$$

where the first term $\mathbf{H}\check{\mathbf{F}}$ accumulates direct and indirect exports of final products after all border crossings, and the second term $(\mathbf{H} - \mathbf{I})\widehat{\mathbf{F}}$ accumulates direct and indirect exports of intermediates eventually transformed into final products for partner use. This formulation is required for the derivation of the weighted average number of border crossings, while a rearrangement into $\mathbf{HF} - \widehat{\mathbf{F}}$ is useful for the implementation of equation (9).

Second, cumulative exports may be computed as a function of bilateral and total gross exports:

$$\mathbf{E}_{cum} = \mathbf{HE}_{bil} - (\mathbf{H} - \mathbf{I})\mathbf{E}_{tot} = \mathbf{H}(\mathbf{E}_{bil} - \mathbf{E}_{tot}) + \mathbf{E}_{tot} \tag{10}$$

In $\mathbf{E}_{cum}$, each element describes the amount of product of sector i in country r that is eventually used for final demand in country s, delivered as direct or indirect exports. Total cumulative exports to all destinations are equal to total direct gross exports:

$$\mathbf{E}_{cum}\mathbf{i} = \mathbf{E}_{bil}\mathbf{i}$$

The above is parallel to the summation of output embodied in final demand $\mathbf{LFi} = \mathbf{x}$.

Each t^{th} term in the power series of $\mathbf{H}$ therefore corresponds to a t^{th} border crossing.[12] The logic of the average propagation length (Dietzenbacher et al., 2005; Ye et al., 2015) suggests that the total number of border crossings $1 + 2 + 3 + \cdots + t$ be weighted by the share of direct and indirect exports at each successive tier in the cumulative exports at all tiers:

[12] The input-output model treats the border(s) between exporter and partner as a single border.

$$c = 1 \times \frac{\substack{\textit{direct exports} \\ \textit{of final products}} + \substack{\textit{direct exports} \\ \textit{of intermediates}}}{\substack{\textit{cumulative} \\ \textit{exports}}} + 2 \times \frac{\substack{\textit{indirect exports} \\ \textit{of final products} \\ \textit{to } 2^{nd} \textit{ tier partner}} + \substack{\textit{indirect exports} \\ \textit{of intermediates} \\ \textit{to } 2^{nd} \textit{ tier partner}}}{\substack{\textit{cumulative} \\ \textit{exports}}} +$$
$$+ \cdots + t \times \frac{\substack{\textit{indirect exports} \\ \textit{of final products} \\ \textit{to } t^{th} \textit{ tier partner}} + \substack{\textit{indirect exports} \\ \textit{of intermediates} \\ \textit{to } t^{th} \textit{ tier partner}}}{\substack{\textit{cumulative} \\ \textit{exports}}}$$

where c is the weighted average number of border crossings and intermediates are those transformed into final products without leaving the territory of the t^{th} tier partner. For the derivation of this measure in block-matrix form, we will first define weights separately for each of the two terms in equation (9). The count of the number of borders crossed by final products $\mathbf{H}\breve{\mathbf{F}}$ starts from 1:

$$1\breve{\mathbf{F}} \oslash \mathbf{E}_{cum} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 3\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

And the count of the number of borders crossed by intermediates for final use in partner countries $(\mathbf{H}-\mathbf{I})\widehat{\mathbf{F}}$ starts from 0 because the first domestic delivery of final products does not involve border crossings:

$$0\widehat{\mathbf{F}} \oslash \mathbf{E}_{cum} + 1\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 2\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

Adding up the two expressions above yields the bilateral weighted average number of border crossings:

$$\mathbf{C} = 1\left(\breve{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} +$$
$$+ \cdots + t\left(\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum}$$

We may easily verify that the sum of all weights implicitly applied to $\mathbf{F}$ is a KN×K matrix where all elements are equal to 1. Pre-multiplying the numerator (the expressions in brackets) by $\left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)$ and then by $\left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$ shows that:

$$1\mathbf{I} + 2\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1} + 3\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{2} + \cdots + t\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1} = \mathbf{H}^{2}$$

$$0\mathbf{I} + 1\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} + 2\left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{2} + \cdots + t\left(\breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{t} = \mathbf{H}(\mathbf{H} - \mathbf{I})$$

Then the equation of the weighted average number of border crossings can be simplified to:

$$\mathbf{C} = \left(\mathbf{H}^2\breve{\mathbf{F}} + (\mathbf{H} - \mathbf{I})\mathbf{H}\widehat{\mathbf{F}}\right) \oslash \mathbf{E}_{cum} = \left(\mathbf{H}^2\mathbf{F} - \mathbf{H}\widehat{\mathbf{F}}\right) \oslash \left(\mathbf{H}\mathbf{F} - \widehat{\mathbf{F}}\right) \tag{11}$$

The "hat" operator in equation (11) applies to the blocks of $\mathbf{F}$, not to the elements therein. $\mathbf{C}$ is a KN×K matrix where each element c^i_{rs} may be interpreted as the weighted average number of border crossings along the path of a product of sector i from country r to its final user in country s. The lowest value of the element c^i_{rs} is 1 when sector i in country r only exports final products. This is in line with the conventional wisdom confirming that exported products cross borders at least once.

3 Data

A number of global inter-country input-output databases have recently become available, building on various philosophies of construction and offering different types of coverage and content. WIOD, Eora, Exiobase, the OECD ICIO model and various multi-regional versions of GTAP datasets contain inter-country input-output tables that are compatible with the matrix setup in subsection 2.1. However, none of these contain the full sequence of valuation layers as shown in Fig. 1. At best, Eora discerns four valuation layers: subsidies on products, taxes on products, trade margins and transport margins, but does not separate those relevant to origin, destination, and international transit. WIOD records the information on valuation that is needed to change the national supply-use tables from purchasers' prices to basic prices, but does not utilize it to produce consistent valuation layers for the symmetric world table. It is worth noting that only Eora and Exiobase re-price imports from CIF prices recorded at destination into basic prices at origin (observed by Bouwmeester et al., 2014, p.520).

The reasonable balance between country and sector detail, the transparency of the compilation procedures and the availability of the underlying supply and use tables make the World Input-Output Database (WIOD) a convenient source of data for computing the proposed measures of trade cost accumulation in global value chains. The WIOD database is the outcome of a project funded by the European Commission and implemented by a consortium of 11 international partners. It contains a series of national and inter-country supply-use tables and input-output tables supplemented by sets of socio-economic and environmental indicators for 1995-2011. WIOD includes 27 European Union member states, 13 other major non-European economies, plus estimates for the rest of the world (RoW). The classification used in the WIOD discerns 35 industries and 59 products, based on NACE rev.1 (ISIC rev. 3) and CPA, respectively. The WIOD project is recognized for its benchmarking of inter-country input-output data against updated national account aggregates, ensuring accuracy in handling international merchandise and services trade statistics. It has been widely used for quantitative research into the various implications of global value chains (Timmer et al., 2015).[13]

An important drawback is that the international trade transactions in the WIOD remain at FOB prices, and, thus, include export taxes less subsidies, trade and transport margins paid at origin, on top of basic prices. This is because the data on international flows of

[13] The database and related information are available at `http://www.wiod.org`.

intermediates and final products in the WIOD are taken from national use tables for imports where the FOB price is treated as the basic price. Moreover, information from the valuation layers in national supply-use tables is not useful for re-pricing imports into the basic prices of the exporting country. This is because the WIOD compilers assumed that calculations of the margin and tax rates by product should not apply to exports (Dietzenbacher et al., 2013, p.80). Further complications arise because of the non-uniform price concepts used in national accounting practices. For example, the national supply and use tables for the USA in the WIOD contain tables of margins and net taxes where all elements are zero, and the use of products at basic prices is equal to their use at purchasers' prices.

A customization of the WIOD data, leading to the full sequence of valuation layers for the purpose of this paper, appears to be a complex procedure and will likely result in an arduous modification of the entire inter-country input-output table. Meanwhile, two valuation layers may be readily compiled, creating only minor inconsistencies with the original world input-output tables in the WIOD – the matrices of international trade and transport margins and the matrices of import taxes at destination (layer 3 and partially layer 4 from Fig. 1). These matrices were compiled for 2001, 2005 and 2010 in the product-by-industry format and were transformed into the symmetric industry-by-industry format. The underlying data were extracted from the UN Comtrade and UN TRAINS databases[14].

The compilation of the matrices of international trade and transport margins involved the following manipulations:

- using UN Comtrade data on total bilateral gross exports and imports among 40 WIOD countries to obtain a uniform aggregate CIF/FOB ratio;
- applying the uniform CIF/FOB ratio to the international trade blocks of the WIOD international use tables (goods only), following the approach of Lenzen et al. (2012) in the construction of Eora;
- running the standard RAS balancing procedure on the resulting matrix of margins, using the vectors of bilateral international trade and transport margins from the WIOD international use tables as constraints;
- transforming the rectangular matrix of international trade and transport margins (of dimension country-product × country-industry) into a square matrix (country-industry × country-industry) using the Eurostat model D (fixed product sales structure assumption); the columns for the rest of the world (RoW) are now missing because a use table for the RoW is not available;
- applying the uniform CIF/FOB ratio to the columns in the original world input-output table that correspond to the RoW as the importing country (including intra-RoW trade present in the "domestic" block of RoW); this yields an estimate of the international trade and transport margins payable on exports to the RoW.

The result is entirely consistent with the original world input-output table except the RoW as importer. In the WIOD world input-output tables, total international trade and transport margins on RoW imports are zero, while they are non-zero in the estimates obtained here. An immediate solution is to offset these non-zero margins by adding an appropriate row with the negative signs as a statistical discrepancy term.

The following is a brief description of the compilation of the bilateral import tariff matrices for the WIOD symmetric input-output tables:

[14] UN Comtrade and UN TRAINS were accessed via the World Integrated Trade Solution (WITS).

- extracting bilateral import tariff data from the UN TRAINS at ISIC Rev.3 two-digit level for 40 WIOD countries (MFN and preferential rates with the *ad valorem* equivalents of the non-*ad valorem* rates);
- computing the actual tariff rates as simple averages of the MFN and preferential rates, assuming that the preference utilization is 50%;
- applying bilateral import tariff rates to goods in WIOD international use tables (re-priced CIF using the international trade and transport margins); the tariff rates differentiate across partner countries but are uniform across purchasing industries in each partner country;
- transforming the rectangular matrix of import tariffs paid (in monetary terms, of dimension country-product × country-industry) into a square matrix (country-industry × country-industry) using the Eurostat model D; the columns for the RoW are missing at this stage;
- creating a "proxy" rest-of-world reporter in UN TRAINS, covering ∼60% of trade between the RoW and WIOD countries; extracting data on bilateral import tariff rates at ISIC Rev.3 two-digit level between WIOD countries and the "proxy" rest-of-world region and on intra-RoW international transactions (MFN and preferential rates with the *ad valorem* equivalents of the non-*ad valorem* rates, preference utilization assumed at 50%);
- aggregating bilateral import tariff rates from ISIC Rev.3 into the WIOD 35 industry classification using additional data on bilateral tariff line imports at ISIC Rev.3 two-digit level;
- applying the obtained tariff rates to the imports by the RoW from WIOD countries and intra-RoW transactions in the original world input-output table (re-priced CIF using the respective international trade and transport margins); this yields an estimate of the import tariffs payable on exports to the RoW.

The result includes the matrices of import tariffs in monetary terms on intermediate inputs $\mathbf{M}(\tau)_{(\mathbf{Z})}$ and final products $\mathbf{M}(\tau)_{(\mathbf{F})}$ that are used as an exemplary valuation layer to test the proposed accounting techniques. These matrices cannot be benchmarked on the WIOD data and are only partially consistent with the original world input-output table. For example, taxes less subsidies on products, including import taxes, are zero in the USA, while in the resulting valuation layer they are non-zero and are unlikely to be offset by net taxes on domestic products. Again, this is a problem inherent to sourcing the primary data from national accounts. Statistical discrepancy terms may be introduced where necessary (below the row of value added) to balance the output in the world input-output table.

4 Results and discussion

4.1 The accumulation effect of import tariffs is pervasive but moderate

The computation of bilateral cumulative or incremental tariff rates, as seen in equations (5, 8), yields matrices in the country-sector by country (KN×K) dimension, which in the case of the WIOD is 1435×41. Various aggregation options are available to reorganize these data

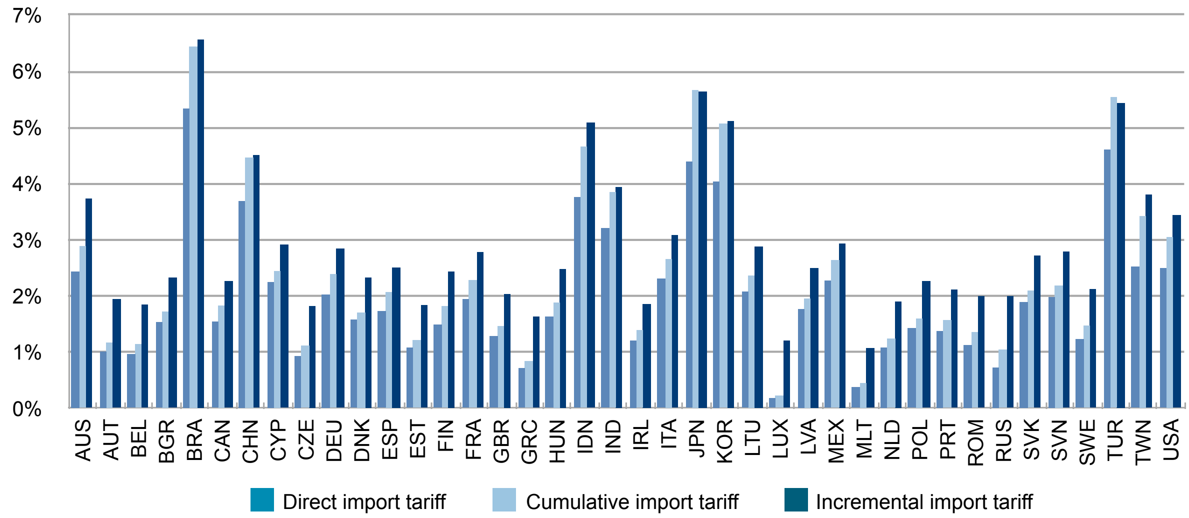

Figure 3: Direct and accumulated import tariffs faced by exporting country, 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of countries in the WIOD is in Table C.1, Appendix C.

by exporting country, exporting sector or importing country for a sensible visualization. The aggregated tariffs are necessarily trade-weighted.

From the perspective of market access for exporters, the average direct tariffs across all partners are generally low (see Fig. 3). Out of 40 countries in the WIOD (apart from the RoW), for only 7 countries did the average direct tariff exceed 3% in 2010, and for only one country was it higher than 5%. Brazil and Luxembourg faced, respectively, the highest (5.3%) and the lowest (0.2%) tariffs. The simple average import tariff for all 40 exporters declined from 3.2% in 2001 to 2.2% in 2005 and to 2.0% in 2010. The low average level of import tariffs is partly the result of accounting for bilateral and regional preferences arising from new free trade agreements. It also reflects the WIOD's focus on the European Union members that apply low MFN tariffs and zero tariffs with respect to their intra-regional imports.

Cumulative and incremental tariffs in Fig. 3 indicate that the average resistance to exports does not significantly increase when the multi-stage production is taken into account. For all 40 exporters, the simple average cumulative tariff went down from 3.9% in 2001 to 2.7% in 2005 and to 2.4% in 2010. The incremental method produces consistently higher estimates: 4.4% in 2001, 3.2% in 2005 and 2.9% in 2010.

By definition, cumulative and incremental tariffs may be split into direct tariffs on exports plus indirect tariffs on embodied inputs identified in two different ways. The indirect portion provides a good indication of the accumulated resistance effect. The largest indirect tariffs in 2010 are revealed by the incremental approach for Indonesia (3.76% direct tariff + 1.33% indirect tariff), Australia (2.44%+1.30%), and Taiwan (2.52%+1.28%), in addition to the cumulative approach for Japan (1.39%+1.28%). In none of these cases does the average tariff for exporters double as a result of value chain accounting. An indirect tariff in 2010 is higher than a direct tariff when counted by the incremental approach only for Luxembourg (0.18%+1.02%), Malta (0.38%+0.69%), Russia (0.73%+1.27%) and Greece (0.72%+0.92%). These are also the countries that face some of the lowest direct import tariffs.

While direct import tariffs tend to decline, the change in the relative importance of indirect tariff exhibits a complex pattern. In terms of cumulative tariff, the ratio of indirect tariff to direct import tariff across all export markets decreased both from 2001-2005 and from 2005-2010 for 14 countries in the WIOD. For 21 countries, this ratio first increased but later decreased, and it was lower in 2010 than in 2001 for 14 countries of those 21 (see Fig. 4). The cumulative accounting therefore indicates that the accumulated resistance effect has become somewhat less significant.

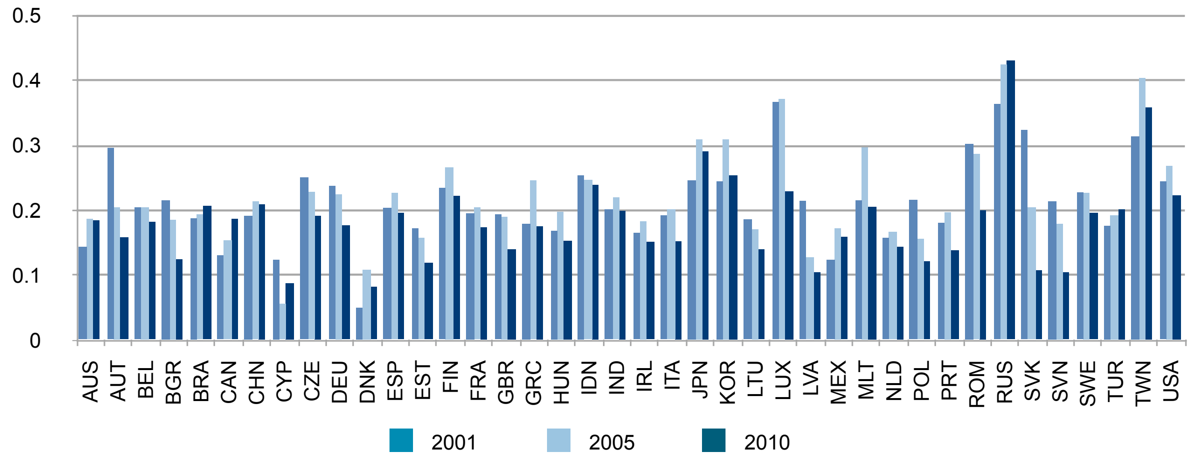

Figure 4: Ratio of indirect tariff to direct import tariff faced by exporting country: the cumulative approach

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

Note: the full list of countries in the WIOD is in Table C.1, Appendix C.

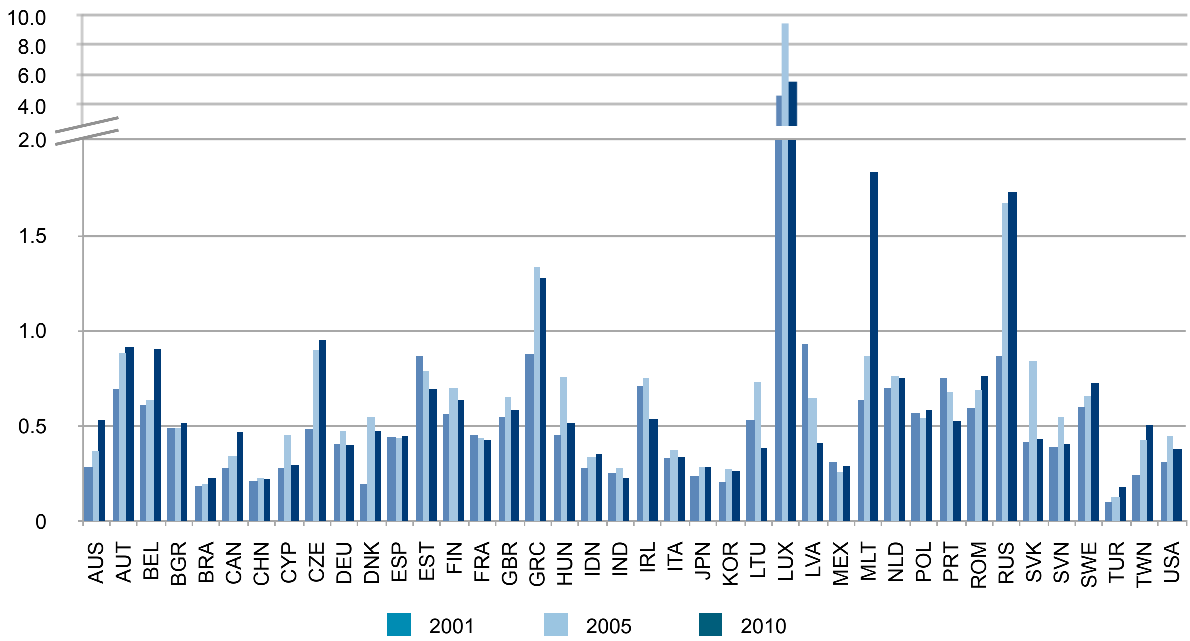

Figure 5: Ratio of indirect tariff to direct import tariff faced by exporting country: the incremental approach

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

Note: the full list of countries in the WIOD is in Table C.1, Appendix C.

In terms of incremental tariff, the ratio of indirect tariff to direct import tariff that exporters face in foreign markets increased both from 2001-2005 and from 2005-2010 for 14 countries. This ratio first increased but then decreased for 18 countries, and only for 4 countries in the WIOD did it decrease in both periods (see Fig. 5). For example, Indonesia faced indirect tariff because third countries levied tariffs on its intermediate exports (i.e.

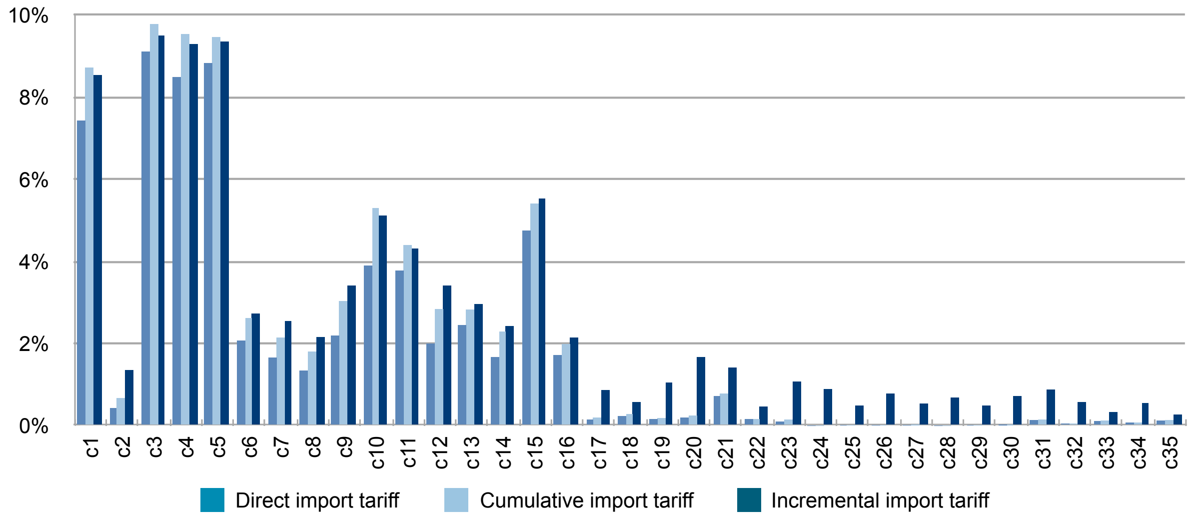

Figure 6: Direct and accumulated import tariffs faced by exporting sector, 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of sectors in the WIOD is in Table C.2, Appendix C.

cumulative indirect tariff) equal to 0.25 of the direct tariff faced in 2001 and 0.24 of the direct tariff in 2010. However, Indonesia faced indirect tariff because partners levied tariffs on third country exports (i.e. incremental indirect tariff) equal to 0.28 of the direct tariff faced in 2001 and 0.35 of the direct tariff faced in 2010. The accumulation effect of protection becomes more pronounced further downstream value chain, which will be addressed in more detail in subsection 4.3.

The computation of the trade-weighted cumulative tariff rates based on Rouzet and Miroudot's (2013) formula (not shown in Fig. 3) requires the same data as the cumulative tariff from equation (5) but a different weighting and aggregation scheme. The result is therefore very close to the cumulative tariff in Fig. 3, and only for China, India, Korea and Mexico is it slightly higher than both cumulative and incremental tariffs. See Appendix B for an illustration.

The aggregation of tariffs faced by exporting sectors reveals no significant accumulation effect of resistance along the value chain (see Fig. 6). In 2010, the sectors subject to the highest indirect tariffs were wholesale trade (0.19% direct tariff + 1.47% indirect tariff) and basic metals (2.00%+1.41%) in the incremental valuation, and rubber and plastics (3.90%+1.41%) and agriculture (7.44%+1.28%) in the cumulative valuation. Incremental tariffs tend to be higher than cumulative tariffs among sectors that face low direct tariffs, especially services, while the opposite is generally true for those sectors facing high tariffs in direct export markets.

Only for one goods-producing sector – mining and quarrying – does the accumulated resistance raise the indirect tariff at the partner border (that is, incremental valuation) by a magnitude above the direct tariff (0.42%+0.92%). For the service sectors, as expected, the direct and cumulative tariffs are close to zero[15] while the incremental tariff ranges from 0.26% on private households with employed persons to 1.66% on wholesale trade.

The accumulation effect, measured by the ratio of indirect tariff to direct import tariff

[15]In the supply-use framework, the output of service sectors may include goods. If the rectangular supply and use tables are transformed into square input-output tables with the Eurostat Model D (the default in the WIOD), the output of the service sectors will still contain goods and will therefore incur transport margins and tariffs. This is the reason for non-zero direct tariffs on some service sectors (c17 – c35) in Fig. 6.

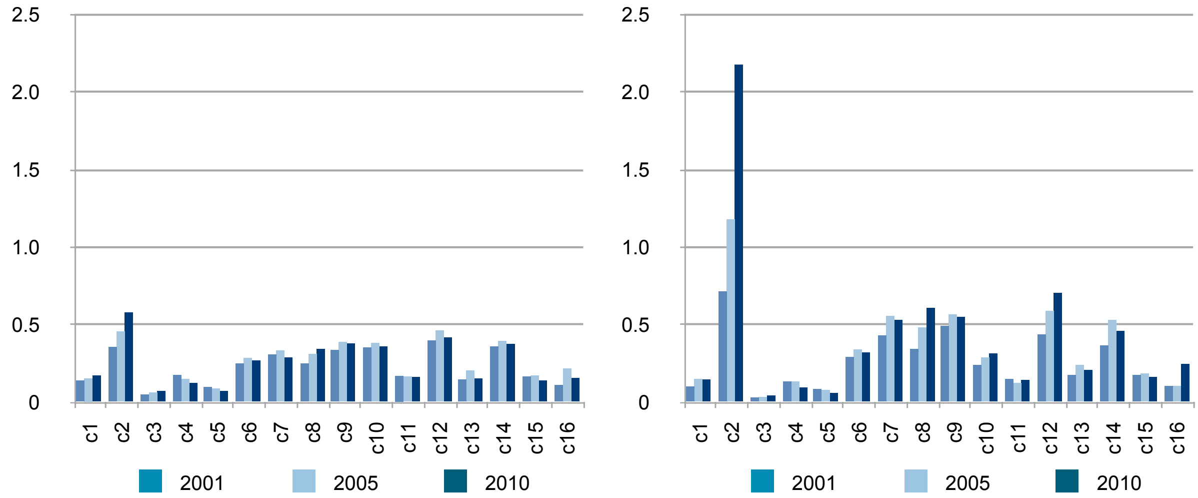

Figure 7: Ratio of indirect tariff to direct import tariff faced by exporting sector (goods only): the cumulative approach (left) and incremental approach (right), 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of sectors in the WIOD is in Table C.2, Appendix C.

faced in Fig. 7,[16] remained relatively stable in the cumulative valuation with the exception of the products of mining and quarrying sector for which the ratio increased from 0.36 in 2001 to 0.46 in 2005 and 0.58 in 2010. The incremental valuation reveals a more significant accumulation of resistance to exports. For the mining and quarrying sector, it increased from 0.71 in 2001 to 1.18 in 2005 and 2.17 in 2010. Indirect protection also accumulates at the partner border with respect to other sectors that produce inputs such as coke, petroleum products, basic metals and fabricated metal products. But the accumulation effect is less significant for sectors that export primarily final products: food, textiles, leather and footwear.

The aggregate figures of course disguise the variation in tariffs faced in individual markets and the related tariff accumulation effect. Yet the latter is modest for the largest exported items. For example, the direct tariff levied in Italy on China's exports of textile products was 9.9% in 2010. The total tariff that those products faced on the way to Italy is estimated at 12.0% and the total tariff paid directly and indirectly at Italy's (EU) border is 10.6%. In some cases, the accumulation may be more pronounced: the direct tariff in Russia on basic metals and fabricated metal supplied from India was 5.9% in 2010, while the total tariff paid along the production chain and at Russia's border were, respectively, 11.3% and 17.9%. However, the relative importance of such products for bilateral trade is usually low.

In the bilateral country-sector setting, both cumulative and incremental tariff rates may suffer from division by the marginal values of direct exports. For example, the cumulative tariff accruing to the exports of petroleum products from Greece to Finland is 91% in 2010 (while the direct tariff is 0%), and the incremental tariff on the exports of post and telecommunication services from Canada to China is 168% (direct tariff is 0%). These results should be interpreted with care. In the most extreme case, when direct exports are zero, the implicit cumulative or incremental tariff rate cannot be defined.

[16]Service sectors are not shown in Fig. 7 because the ratio involves division by direct tariffs that are close to zero

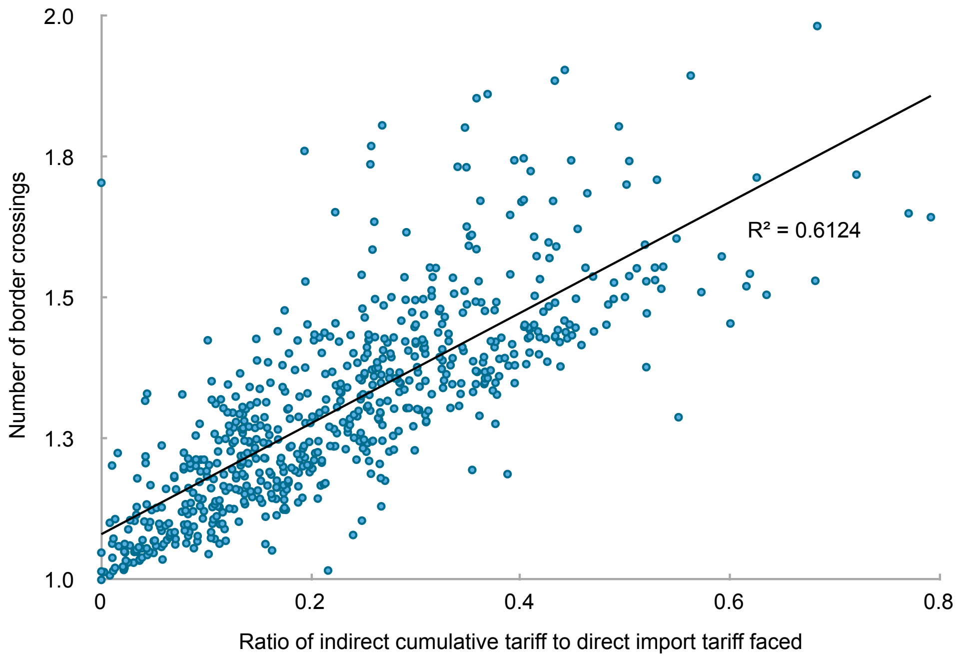

Figure 8: Relationship between indirect tariffs and number of border crossings, by exporting country-sector (goods only), 2010

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

4.2 Number of border crossings increases while cumulative tariffs decline

As noted in previous sections, the growing number of borders that intermediate inputs now have to cross because of the international fragmentation of production is thought to be the main force behind the accumulation of trade costs. Fig. 8 exposes this statement to an empirical test: for each exporting country-sector, the tariff accumulation effect across all partners (horizontal axis) is related to the average number of border crossings across all destinations (vertical axis). The accumulation effect is defined as the ratio of indirect cumulative tariff between an exporting country-sector and all its partners to direct import tariff.[17] The incremental tariff is less relevant for this exercise because of its excessive focus on the tariffs applicable at the partner border. The scatter plot only shows the results for goods-producing sectors (c1 – c16 in the WIOD) in 2010, as the results for service sectors may be biased because they face zero or minimal direct tariffs.

Fig. 8 confirms that, by and large, a higher accumulation effect is associated with more border crossings. However, the growing number of border crossings in a particular period of time does not bring about an increase in cumulative tariffs. Moreover, the change in the average number of border crossings across all partners has not been uniform (see Fig. 9). For 26 exporting countries in WIOD, this number increased in 2001-2005 but descended in 2005-2010. For 12 countries, it increased both in 2001-2005 and 2005-2010. 2 countries experienced a decline of this measure in both periods. The simple average number of border crossings for all exporters rose from 1.30 in 2001 to 1.35 in 2005 and stood at 1.34 in 2010.

Meanwhile, the cumulative tariff faced by total exports declined from 2001-2005 and from 2005-2010 for 29 countries. It first decreased but later increased for 10 countries and rose in both periods for only one country. The simple average cumulative tariff faced by all exporters went down from 3.86% in 2001 to 2.66% in 2005 and to 2.36% in 2010.

[17] This measure is derived from equation 5: $\mathbf{T}_{cum} \oslash \mathbf{T} - 1$.

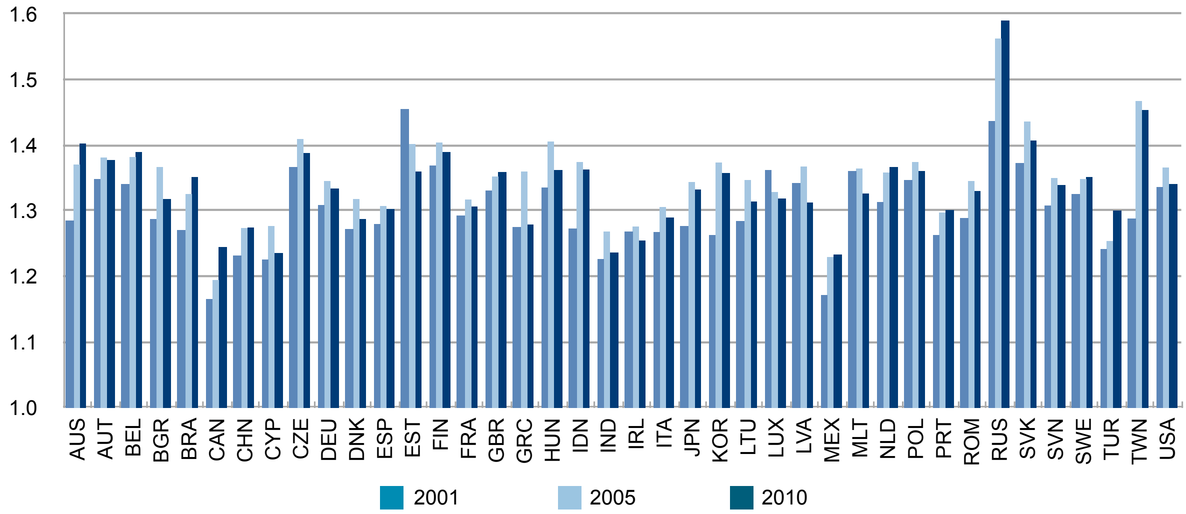

Figure 9: Weighted average number of border crossings, by exporting country
Source: WIOD database, author's calculations
Note: the full list of countries in WIOD is in Table C.1, Appendix C.

In sum, the number of border crossings rose slowly over 2001-2005-2010 while cumulative tariffs declined quickly. The continuous reduction in direct import tariffs neutralized the indirect tariff accumulation effect. As revealed in subsection 4.1, the direct tariff is still the largest component of the cumulative tariff.

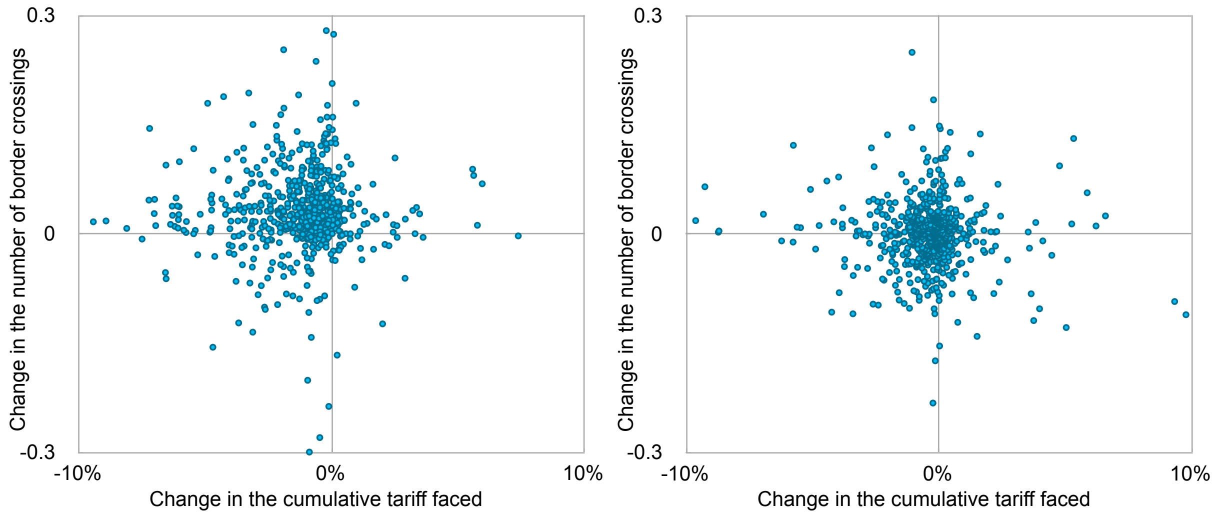

Figure 10: Relationship between the change in the number of border crossings and the change in the cumulative tariff faced, by exporting country-sector (goods only)
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

At the country-sector level, there is no clear unidirectional link between the change in the cumulative tariff (in percentage points) and the respective change in the number of border crossings (in dimensionless units). In Fig. 10, these changes are contrasted and differentiated between two periods. It is clear that from 2001-2005, a reduction in the cumulative tariff among goods-producing sectors was, in the vast majority of cases, associated with an increase in the number of borders to be crossed. In 2005-2010, such a pattern is barely discernable. We may observe that from 2001-2005, the international fragmentation of production increased the average number of borders a product was required to cross before consumption, but trade liberalization ensured that exporters benefited from this and did not face greater protection along the downstream value chain. Over the next 5 years, both fragmentation of production and liberalization of trade slowed down with a mixed but mostly neutral effect on exporters. The global economic and trade collapse of the late 2000s might at least partially explain this result.

4.3 Indirect protection is higher downstream in the value chain

The incremental tariff is usually higher than the respective cumulative tariff, as seen in Fig. 3 and Fig. 6. The simplified example in Fig. 2 indicates that both accounting approaches measure indirect tariffs on the same intermediate inputs travelling along the value chain. The cumulative approach counts indirect tariffs closer to the country of origin while the incremental approach counts those tariffs closer to the country of destination. If the tariffs on direct exports at destination are higher, the incremental tariff exceeds the cumulative tariff, as is the case in Fig. 2. The opposite is true if the tariffs at destination are lower.

In 2010, the higher incremental tariff rate in comparison with the cumulative tariff rate is relatively significant for the products of such sectors as mining and basic metals (see Fig. 6). These are also the sectors with the longest cross-border value chains leading to their eventual users. Food and beverages (c3), textile (c4) and leather products (c5) face incremental tariffs that are lower than cumulative tariffs, and their respective downstream cross-border value chains are among the shortest. This may also be observed in 2001 and 2005, see Fig. 11.

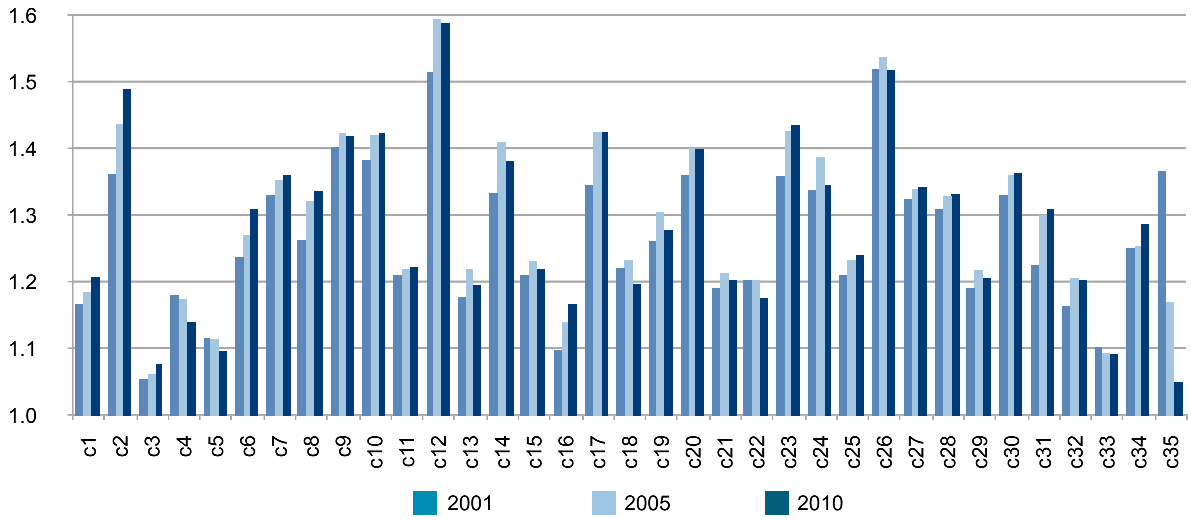

Figure 11: Weighted average number of border crossings, by exporting sector
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of sectors in the WIOD is in Table C.2, Appendix C.

Fig. 12 tests at the disaggregate country-sector level whether protection at destination increases with the number of border crossings in 2010. The visualization helps in discerning this relationship, though it is not very strong: the longer the cross-border value chain, the higher the protection that the embodied inputs face at the market of destination.

4.4 Preferential tariff reduction enhances indirect market access

Currently, it is common for most countries to enter into free trade agreements that mutually enhance market access for their goods and services. Rules of origin usually require

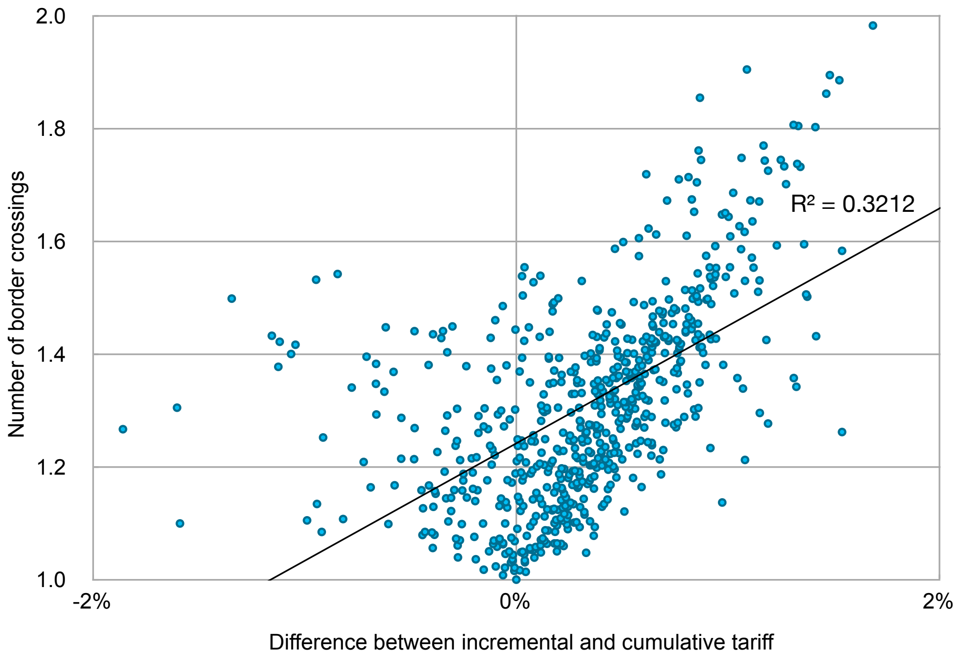

Figure 12: Relationship between the number of border crossings and the difference between the incremental tariff and the cumulative tariff faced, by exporting country-sector (goods only), 2010

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

that goods be wholly obtained within the member countries or that third country inputs therein be substantially transformed to qualify for preferential market access. These rules, however, are not designed to consistently account for foreign inputs (particularly service inputs) through multiple production tiers. The goods that qualify for preferential access under free trade agreements may therefore embody sizable amounts of third country inputs which will effectively also benefit from enhanced market access. Similarly, the intermediate inputs that move along the production chain between members and then to third countries will face lower total protection because transactions between the members are subject to lower tariffs. A simple simulation of cumulative and incremental tariffs under the Trans-Pacific Partnership (TPP) explicitly captures these effects.

The WIOD covers five of the twelve TPP members: Australia, Canada, Japan, Mexico and the USA. As a simulation exercise, direct tariff rates on imports from members were first set to zero, then the payable TPP tariffs were modelled as the simple average between zero and the effectively applied tariff rates of 2010 to account for incomplete preference utilization. This led to the modification of the matrices of import tariffs in monetary terms for both the intermediate inputs $\mathbf{M}(\tau)_{(\mathbf{Z})}$ and the final products $\mathbf{M}(\tau)_{(\mathbf{F})}$, which were inserted again into equations (5, 8). The input-output structure of domestic and cross-border transactions was held constant. Table 1 shows the percentage changes in cumulative and incremental tariffs among the five TPP countries and selected non-members.

Table 1 indicates that value chains blur the effects of bilateral or regional preferences. While the simulated direct tariff rates were set to half of their actual level, the reduction in the accumulated tariff rates does not reach 50%. For example, total direct and indirect tariffs on Japan's exports to Canada decreased by no more than 34% and those on Australia's exports to Mexico fell by 23% at best.

The most interesting finding from Table 1 is perhaps the indirect effect of the free trade

Table 1: Simulation of the tariff reduction effect under the Trans-Pacific Partnership, based on 2010 data

Percentage change in bilateral *cumulative* tariffs											
	Partner										
Exporter	AUS	BRA	CAN	CHN	DEU	IDN	JPN	KOR	MEX	RUS	USA
AUS	–	0.3	27.9	0.2	0.5	0.1	45.6	0.1	23.1	0.2	10.9
BRA	0.0	–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CAN	27.0	0.8	–	0.7	2.2	0.7	45.7	0.6	47.9	0.4	6.7
CHN	0.0	0.0	0.0	–	0.0	0.0	0.0	0.0	0.0	0.0	0.0
DEU	0.0	0.0	0.0	0.0	–	0.0	0.0	0.0	0.0	0.0	0.0
IDN	0.0	0.0	0.0	0.0	0.0	–	0.0	0.0	0.0	0.0	0.0
JPN	45.7	0.5	33.7	0.2	0.9	0.1	–	0.2	39.3	0.5	28.4
KOR	0.0	0.0	0.0	0.0	0.0	0.0	0.0	–	0.0	0.0	0.0
MEX	39.2	0.0	2.7	0.3	0.1	0.8	31.1	0.5	–	0.2	0.3
RUS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	–	0.0
USA	15.0	0.2	20.4	0.3	0.2	0.3	37.4	0.0	0.9	0.1	–
Percentage change in bilateral *incremental* tariffs											
	Partner										
Exporter	AUS	BRA	CAN	CHN	DEU	IDN	JPN	KOR	MEX	RUS	USA
AUS	–	0.0	22.4	0.0	0.0	0.0	46.1	0.0	16.1	0.0	9.6
BRA	4.3	–	1.8	0.0	0.0	0.0	2.0	0.0	0.9	0.0	0.3
CAN	23.4	0.0	–	0.0	0.0	0.0	46.8	0.0	42.4	0.0	4.3
CHN	3.1	0.0	1.0	–	0.0	0.0	0.7	0.0	1.7	0.0	0.3
DEU	1.0	0.0	1.3	0.0	–	0.0	2.6	0.0	0.9	0.0	0.3
IDN	6.3	0.0	3.0	0.0	0.0	–	1.7	0.0	3.2	0.0	0.8
JPN	45.6	0.0	31.9	0.0	0.0	0.0	–	0.0	39.8	0.0	27.2
KOR	1.8	0.0	1.2	0.0	0.0	0.0	0.8	–	1.4	0.0	1.0
MEX	37.8	0.0	4.8	0.0	0.0	0.0	40.3	0.0	–	0.0	0.4
RUS	8.0	0.0	4.2	0.0	0.0	0.0	2.2	0.0	4.3	–	1.3
USA	15.9	0.0	20.4	0.0	0.0	0.0	42.0	0.0	2.2	0.0	–

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

agreement on selected third countries. It reveals that TPP members face lower protection in non-member markets because the cumulative approach counts reduced TPP tariffs as embodied inputs in exports bound for the non-TPP partners. Most notably, the cumulative tariff rate on Canada's products entering Germany decreases by 2.2%. Here the protection applied to non-members in the TPP markets remains unchanged because TPP preferences do not apply to inputs from non-members.

As expected, the incremental accounting approach produces different results. Non-members are shown to benefit from the TPP because their inputs enter the member markets but are treated as products originating from the TPP partners. For example, the incremental tariff rate on Russia's exports to Australia is 8.0% lower. However, exports from TPP members to Russia are subject to the same tariff rates because incremental tariffs are counted at destination. The incremental method is perhaps more relevant for such calculations because it helps quantify the benefits – although marginal – of free trade agreements to the non-participating parties.

Table 2 reports the results of a similar simulation of tariff changes under the TPP with

Table 2: Simulation of the tariff reduction effect under the Trans-Pacific Partnership with China and the Republic of Korea, based on 2010 data

Percentage change in bilateral *cumulative* tariffs

	Partner										
Exporter	AUS	BRA	CAN	CHN	DEU	IDN	JPN	KOR	MEX	RUS	USA
AUS	–	5.1	35.7	46.0	10.2	1.9	47.7	48.6	33.7	8.5	21.5
BRA	0.0	–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CAN	30.7	1.6	–	44.2	5.7	3.5	46.9	44.8	48.2	2.3	7.5
CHN	43.6	0.6	41.2	–	1.7	1.6	40.3	45.9	45.4	1.0	42.5
DEU	0.0	0.0	0.0	0.0	–	0.0	0.0	0.0	0.0	0.0	0.0
IDN	0.0	0.0	0.0	0.0	0.0	–	0.0	0.0	0.0	0.0	0.0
JPN	48.1	5.5	44.6	48.1	14.1	2.7	–	48.1	46.9	12.9	43.0
KOR	46.3	1.7	44.5	45.7	11.8	4.8	42.3	–	46.7	3.1	39.6
MEX	39.9	0.1	2.8	37.3	1.1	3.0	32.4	31.6	–	3.3	0.5
RUS	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	–	0.0
USA	20.3	1.1	21.1	44.5	4.2	6.3	44.6	49.3	1.4	4.8	–

Percentage change in bilateral *incremental* tariffs

	Partner										
Exporter	AUS	BRA	CAN	CHN	DEU	IDN	JPN	KOR	MEX	RUS	USA
AUS	–	0.0	33.6	41.7	0.0	0.0	47.9	46.4	32.2	0.0	24.0
BRA	11.1	–	4.5	3.9	0.0	0.0	6.8	2.1	2.3	0.0	3.5
CAN	28.2	0.0	–	40.3	0.0	0.0	47.8	43.3	42.8	0.0	5.1
CHN	43.1	0.0	39.3	–	0.0	0.0	42.5	45.3	41.5	0.0	43.0
DEU	2.3	0.0	3.2	1.5	–	0.0	6.3	3.9	2.3	0.0	2.9
IDN	11.8	0.0	8.1	8.6	0.0	–	6.5	6.9	8.3	0.0	3.9
JPN	47.3	0.0	35.9	47.1	0.0	0.0	–	46.5	42.9	0.0	30.6
KOR	45.7	0.0	41.6	44.2	0.0	0.0	45.8	–	43.9	0.0	36.4
MEX	39.8	0.0	5.1	39.5	0.0	0.0	41.6	41.3	–	0.0	0.6
RUS	18.4	0.0	11.6	8.8	0.0	0.0	8.0	6.1	12.7	–	7.9
USA	20.5	0.0	20.9	39.2	0.0	0.0	44.2	47.9	2.7	0.0	–

Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations

two additional members included – China and Korea. Now the "leakage" of preferences is more apparent. The cumulative approach reveals that TPP members obtain enhanced indirect access to non-member markets: tariff facing Japan in Germany falls by 14.1%, in Russia by 12.9% and in Brazil by 5.5% (compare to, respectively, 0.9%, 0.5% and 0.5% in Table 1). Non-members benefit from enhanced indirect access to member markets as change in incremental tariff shows: tariff in Australia with respect to Brazil is lower by 11.1%, in Japan by 6.8% and the USA by 3.5% (4.3%, 2.0% and 0.3% in Table 1).

5 Conclusion

There has been a growing body of statistical evidence and case studies supporting the importance of value chains in the global economy. The perception of longer value chains with more border crossings has raised concerns about higher indirect trade costs.

This paper has discussed the application of input-output analysis to measuring the number of border crossings and trade costs that accumulate along global value chains. The proposed indicator counts the weighted average number of borders that a product of a particular sector has to cross between the exporter and the partner country until it is entirely consumed in the latter. Meanwhile, three different measurements of accumulated trade costs are identified. Two of these may be considered contributions of this paper because, in contrast to one measure discussed in the literature, they are capable of discerning the origin and destination of products subject to trade costs along the global value chain. For the reader's convenience, these measures are labelled "cumulative" and "incremental" trade costs. In an exporter–partner relationship, "cumulative" costs indirectly apply to the transactions between the exporter and third countries while "incremental" costs indirectly apply to transactions between third countries and the partner country.

The application of the proposed accounting techniques to the inter-country input-output tables taken from the WIOD database yields several noteworthy findings. The experimental calculations only covered one type of trade cost – import tariffs – and the data were sourced from the UNCTAD TRAINS database. First, at the aggregate country or product level, direct import tariffs (as seen in 2010) are still the largest component of the cumulative or incremental tariff. Indirect tariff protection is unlikely to significantly hinder the flow of embodied inputs downstream along the value chain. Second, the indirect cumulative tariff rises with the average number of borders crossed. However, the continuous reduction of direct import tariffs neutralized the effect of the greater number of border crossings in value chains from 2001-2010. Third, the more borders crossed, the more costly is the indirect tariff protection further downstream the value chain, which is why the incremental tariff measurement is usually higher than the cumulative one. Fourth, trade cost propagation through global value chains erodes preferences under free trade agreements and effectively extends these preferences to non-participating countries. The more members join a free trade area, the more significant is the "leakage" of preferences to third countries, including developing economies. It should be stressed that the incremental measurement of trade costs is better suited to this type of analysis. It is compatible with the notion of indirect market access and is capable of accounting for indirect tariffs on services.

The findings suggest that input-output accounting frameworks may significantly extend the frontier of trade policy analysis in the world of global value chains. The critical issue for further research is the availability of data on the trade costs that change the price of products on their way from producer to purchaser. While international trade and transport margins and import tariffs can be directly accessed or easily estimated for most country pairs, there remain huge data gaps with respect to export taxes and subsidies, domestic trade and transport costs. The experience of writing this article indicates that existing and upcoming inter-country input-output datasets could be enhanced in several ways that would support an exhaustive trade cost analysis. First, re-price international trade flows into the basic prices of the exporting countries. Second, provide access to the underlying supply and use tables with valuation layers so that users may derive symmetric input-output table in alternative product-by-product formats, which is thought to be more convenient for trade cost analysis. Third, compile the full set of at least six valuation layers as shown in Fig. 1. Some layers (1-4 in Fig. 1) apply because goods are sent (services are supplied) from one country to another. Other layers (5 and 6) apply because goods and services are delivered to users within a single country without leaving it. Finally, increase sector resolution.

References

Ali-Yrkkö, J. and Rouvinen, P. (2015). Slicing up global value chains: a micro view. Journal of Industry, Competition and Trade *15*, 69–85.

Anderson, J. E. and van Wincoop, E. (2004). Trade Costs. Working Paper 10480 National Bureau of Economic Research Cambridge.

Bouwmeester, M. C., Oosterhaven, J. and Rueda-Cantuche, J. M. (2014). A new SUT consolidation method tested by a decomposition of value added and CO2 embodied in EU27 exports. Economic Systems Research *26*, 511–541.

Dietzenbacher, E., Los, B., Stehrer, R., Timmer, M. and De Vries, G. (2013). The construction of world input–output tables in the WIOD project. Economic Systems Research *25*, 71–98.

Dietzenbacher, E., Luna, I. R. and Bosma, N. S. (2005). Using average propagation lengths to identify production chains in the Andalusian economy. Estudios de economía aplicada *23*, 405–422.

Eurostat (2008). Eurostat Manual of Supply, Use and Input-Output Tables. Eurostat Luxembourg.

Fally, T. (2012). Production staging: measurement and facts. Unpublished manuscript University of Colorado–Boulder.

Ferrantino, M. J. (2012). Using Supply Chain Analysis to Examine the Costs of Non-Tariff Measures (NTMs) and the Benefits of Trade Facilitation. Staff Working Paper ERSD-2012-02 World Trade Organization Geneva.

Hummels, D., Ishii, J. and Yi, K.-M. (1999). The Nature and Growth of Vertical Specialisation in World Trade. Staff Reports No.72 Federal Reserve Bank of New York New York.

Isard, W. (1951). Interregional and regional input-output analysis: a model of a space-economy. The Review of Economics and Statistics *33*, 318–328.

Johnson, R. C. and Moxnes, A. (2013). Technology, trade costs, and the pattern of trade with multi-stage production. Unpublished paper Dartmouth College.

Koopman, R., Powers, W., Wang, Z. and Wei, S.-J. (2010). Give Credit Where Credit Is Due: Tracing Value Added in Global Production Chains. Working Paper 16426 National Bureau of Economic Research Cambridge.

Koopman, R., Wang, Z. and Wei, S.-J. (2012). Tracing Value-added and Double Counting in Gross Exports. Working Paper 18579 National Bureau of Economic Research Cambridge.

Lenzen, M., Kanemoto, K., Moran, D. and Geschke, A. (2012). Mapping the Structure of the World Economy. Environmental Science & Technology *46*, 8374–8381.

Leontief, W. (1936). Quantitative Input and Output Relations in the Economic Systems of the United States. The Review of Economics and Statistics *18*, 105–125.

Leontief, W. and Strout, A. (1963). Multiregional Input-Output Analysis. In Structural Interdependence and Economic Development, (Barna, T., ed.), pp. 119–149. Macmillan (St. Martin's Press) London.

Maurer, A. and Degain, C. (2010). Globalization and Trade Flows: What You See is Not What You Get! Economic Research and Statistics Division, Staff Working Paper ERSD-2010-12 World Trade Organization Geneva.

Miller, R. E. and Blair, P. D. (2009). Input-Output Analysis: Foundations and Extensions. Second edition, Cambridge University Press, New York.

Moses, L. N. (1955). The stability of interregional trading patterns and input-output analysis. The American Economic Review *45*, 803–826.

Muradov, K. (2015). Input-output calculus of international trade. Paper presented at the 23rd International Input-Output Conference, Mexico City.

Ng, F. and Yeats, A. (1999). Production Sharing in East Asia: Who Does What for Whom and Why? Policy Reseach Working Paper 2197 The World Bank Washington, DC.

OECD (2013). Interconnected Economies: Benefiting from Global Value Chains. OECD Publishing, Paris.

OECD and WTO (2012). Trade in Value-Added: Concepts, Methodologies And Challenges. Joint oecd-wto note.

OECD, WTO and UNCTAD (2013). Implications of Global Value Chains for Trade, Investment, Development and Jobs. Report prepared for the G-20 Leaders Summit, Saint Petersburg (Russian Federation).

Rouzet, D. and Miroudot, S. (2013). The cumulative impact of trade barriers along the value chain: An empirical assessment using the oecd inter-country input-output model. Paper presented at the 16th Annual Conference on Global Economic Analysis.

SNA (2009). System of National Accounts 2008. United Nations, European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, and the World Bank New York.

Stehrer, R. (2013). Accounting relations in bilateral value added trade. WIOD Working Paper Series No.14.

Streicher, G. and Stehrer, R. (2015). Whither Panama? Constructing a consistent and balanced world SUT system including international trade and transport margins. Economic Systems Research *27*, 213–237.

Tamamura, C. (2010). Cost Reduction Effects of ”pseudo FTAs” in Asia - Application of a price model based on a multilateral I/O table. Discussion Paper 226 Institute of Developing Economies, JETRO.

Timmer, M., Los, B., Stehrer, R. and de Vries, G. (2013). Fragmentation, Incomes and Jobs. An analysis of European competitiveness. GGDC Research Memorandum 130 Groningen Growth and Development Centre, Faculty of Economics and Business, University of Groningen.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R. and de Vries, G. J. (2015). An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. Review of International Economics *23*, 575–605.

Wang, Z., Wei, S.-J. and Zhu, K. (2013). Quantifying International Production Sharing at the Bilateral and Sector Levels. Working Paper No.19677 National Bureau of Economic Research Cambridge.

Ye, M., Meng, B. and Wei, S.-j. (2015). Measuring smile curves in global value chains. Discussion Paper 530 Institute of Developing Economies, JETRO.

Yi, K.-M. (2003). Can vertical specialization explain the growth of world trade? Journal of Political Economy *111*, 52–102.

Yi, K.-M. (2010). Can multistage production explain the home bias in trade? American Economic Review *100*, 364–393.

A Gross exports accounting framework and derivation of the new "global" inverse

A.1 The new "global" inverse

A gross exports accounting framework traces the destination of direct exports to their eventual users. This is a forward decomposition where the observed bilateral export flows are reallocated into the unobserved flows of embodied products as those pass through the downstream value chain.

By definition, bilateral gross exports comprise cross-border flows of intermediate and final products:

$$\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$$

Exports of intermediates can be expressed as a function of the partner country total output:

$$\breve{\mathbf{Z}}_{(KN\times K)} = \breve{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$$

where $\hat{\mathbf{x}}_{(KN\times K)}$ is the block-diagonalized vector of total output:

$$\hat{\mathbf{x}}_{(KN\times K)} = \begin{bmatrix} \mathbf{x}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{x}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{x}_k \end{bmatrix}$$

Total output $\hat{\mathbf{x}}_{(KN\times K)}$ is the sum of intermediates for domestic use, final products for domestic use and total exports, which in the KN×K block-diagonalized form can be written as:

$$\hat{\mathbf{x}}_{(KN\times K)} = \widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}$$

$\mathbf{E}_{tot}$ is the block-diagonalized matrix of total gross exports:

$$\mathbf{E}_{tot} = \begin{bmatrix} \mathbf{e}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{e}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{e}_k \end{bmatrix} \quad \text{where a block element} \quad \mathbf{e}_r = \begin{bmatrix} e_r^1 \\ e_r^2 \\ \vdots \\ e_r^n \end{bmatrix}$$

Block elements $\mathbf{e}_r$ are N×1 vectors where each entry $e_r^i = \sum_{s\neq r}^{K} \left(\sum_{j=1}^{N} z_{rs}^{ij} + f_{rs}^{i} \right)$.

Insert the decomposed $\hat{\mathbf{x}}_{(KN\times K)}$ into $\breve{\mathbf{Z}}_{(KN\times K)} = \breve{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)}$ and then into $\mathbf{E}_{bil} = \breve{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{F}}$ to obtain:

$$\mathbf{E}_{bil} = \breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}}$$

Now, gross bilateral exports are a sum of (a) direct exports of intermediates for domestic intermediate use by partner, (b) direct exports of intermediates for domestic final use by partner, (c) direct exports of intermediates for exports by partner and (d) direct exports of final products. The eventual use of exported intermediates described by the first term $\breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)}$ remains undetermined, i.e., these can either be embodied in domestic final use by partner or in partner exports. Accordingly, subsequent manipulations decompose this term until it is completely allocated between domestic final use and exports.

Using that $\widehat{\mathbf{Z}}_{(KN\times K)} = \widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} = \widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right)$ leads to an infinite series of inter-industry interactions:

$$\begin{aligned}
\mathbf{E}_{bil} &= \breve{\mathbf{A}}\widehat{\mathbf{Z}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\left(\widehat{\mathbf{Z}}_{(KN\times K)} + \widehat{\mathbf{F}} + \mathbf{E}_{tot}\right) + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\widehat{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{A}}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\mathbf{E}_{tot} + \breve{\mathbf{F}} = \dots
\end{aligned}$$

Compiling and rearranging all terms after $t \to \infty$ rounds of interactions results in:

$$\begin{aligned}
\underset{t\to\infty}{\mathbf{E}_{bil}} &= \breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)} + \left(\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t + \dots + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\right)\widehat{\mathbf{F}} + \\
&+ \left(\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t + \dots + \breve{\mathbf{A}}\widehat{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\widehat{\mathbf{A}} + \breve{\mathbf{A}}\right)\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)} + \breve{\mathbf{A}}\left(\left[\widehat{\mathbf{A}}\right]^t + \dots + \widehat{\mathbf{A}}\widehat{\mathbf{A}} + \widehat{\mathbf{A}} + \mathbf{I}\right)\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\left[\widehat{\mathbf{A}}\right]^t + \dots + \widehat{\mathbf{A}}\widehat{\mathbf{A}} + \widehat{\mathbf{A}} + \mathbf{I}\right)\mathbf{E}_{tot} + \breve{\mathbf{F}} = \\
&= 0 + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}
\end{aligned}$$

The elements in $\breve{\mathbf{A}}\left[\widehat{\mathbf{A}}\right]^t \hat{\mathbf{x}}_{(KN\times K)}$ are approaching zero with $t \to \infty$ because the column sums of $\mathbf{A}$ and $\widehat{\mathbf{A}}$ are less then 1 in a monetary IO table.

It is worth noting that, due to the known property of the block-diagonal matrices, $\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}$ is equal to a block-diagonal matrix of local Leontief inverses:

$$\begin{aligned}
(\mathbf{I} - \mathbf{A})^{-1} &= \begin{bmatrix} \mathbf{I} - \mathbf{A}_{11} & 0 & \cdots & 0 \\ 0 & \mathbf{I} - \mathbf{A}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{I} - \mathbf{A}_{kk} \end{bmatrix}^{-1} = \\
&= \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{11})^{-1} & 0 & \cdots & 0 \\ 0 & (\mathbf{I} - \mathbf{A}_{22})^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\mathbf{I} - \mathbf{A}_{kk})^{-1} \end{bmatrix}
\end{aligned}$$

The equation obtained above reallocates direct exports of sector i from the exporting country r according to their eventual use by the direct partner s:

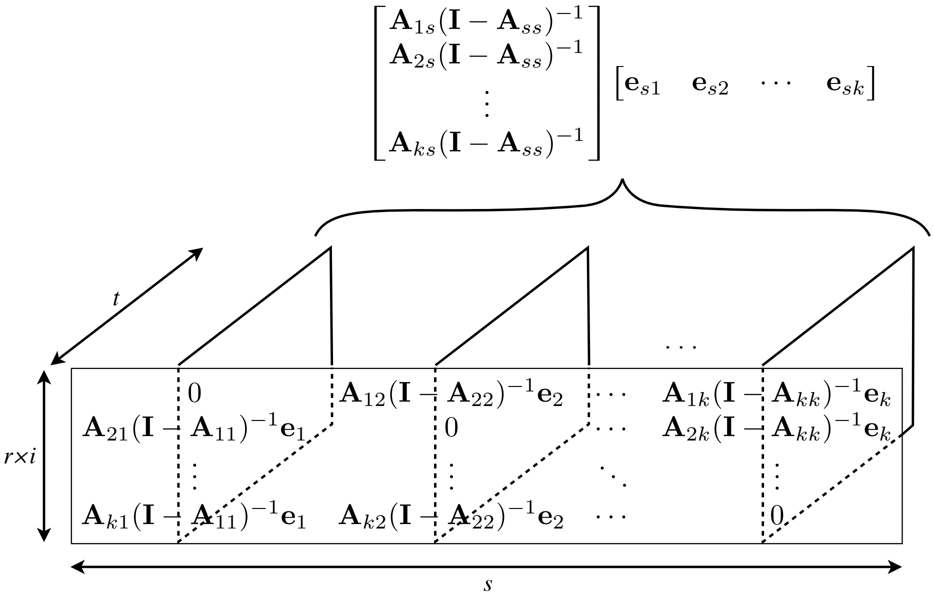

Figure A.1: Transformation of the $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ matrix into a 3rd-order tensor

$$\underbrace{\mathbf{E}_{bil}}_{\substack{\text{bilateral}\\ \text{gross}\\ \text{exports}}} = \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\substack{\text{intermediates}\\ \text{eventually transformed}\\ \text{by partner into final products}\\ \text{for domestic use}}} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}}_{\substack{\text{intermediates}\\ \text{eventually used}\\ \text{by partner for exports}}} + \underbrace{\breve{\mathbf{F}}}_{\substack{\text{final products}\\ \text{directly exported}\\ \text{to partner}\\ \text{for domestic use}}} \tag{A.1}$$

Note that exports in this type of decomposition embody value added from all sectors and all countries of origin. The component matrices represent flows of products (not value added) and are necessarily confined to direct gross exports. In other words, value chains are confined to the national borders. Each component flow can be expressed as a share of direct gross exports and will not exceed 100%. This decomposition is conceptually close to those in Koopman et al. (2010) and Wang et al. (2013), though differs in the way of identifying the eventual use of direct exports.

In the decomposition above, it is still unknown where the re-exported term $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is destined for. The next exercise will trace this flow to the next tiers of the value chain and allocate it according to its eventual use. A tier henceforth will correspond to cross-border flows of intermediate products.

The term $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ needs disaggregating according to the next country of destination, or second-tier partner. Given that $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ is a KN×K matrix that shows the flows among the exporting countries r and the first-tier partners s, our exercise requires extending the matrix to the third dimension KN×K×K. Then it will show the flows from the exporter r through the first-tier partner s to the second-tier partner t. This is visualized in Fig. **??**.

The result is a thee-dimensional matrix, or a 3rd-order tensor where the third dimension is constructed by computing the outer product of the s^{th} column in $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}$ and s^{th} row in $\mathbf{E}_{bil}$:

$$\begin{bmatrix} \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\ \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\ \vdots \\ \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{e}_{s1} & \mathbf{e}_{s2} & \cdots & \mathbf{e}_{sk} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \end{bmatrix}$$

These KN×K matrices are perpendicular to $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ and their row sums are equal to the s^{th} column of $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$. So the tensor contraction along the third dimension results in reverting to the KN×K matrix $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$.

In principle, the s^{th} row in $\mathbf{E}_{bil}$ may be replaced with the sum of the rows in the component matrices from $\mathbf{E}_{bil} = \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}$. Then the re-exported term may be disaggregated again into the fourth dimension (KN×K×K×K) and so on, which may lead to a series of high-dimensional tensors.

In order to keep data in a manageable form for the decomposition to the next tiers, we opt for the tensor contraction along the second dimension, that is first-tier partners s:

$$\sum_{s=1}^{K} \begin{bmatrix} \mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\ \mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \\ \vdots \\ \mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{e}_{s1} & \mathbf{e}_{s2} & \cdots & \mathbf{e}_{sk} \end{bmatrix} =$$

$$= \begin{bmatrix} \sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{1s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{2s}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s1} & \sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{s2} & \cdots & \sum_{s=1}^{K}\mathbf{A}_{ks}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{e}_{sk} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & \mathbf{A}_{12}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & \mathbf{A}_{1k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\ \mathbf{A}_{21}(\mathbf{I}-\mathbf{A}_{11})^{-1} & 0 & \cdots & \mathbf{A}_{2k}(\mathbf{I}-\mathbf{A}_{kk})^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{k1}(\mathbf{I}-\mathbf{A}_{11})^{-1} & \mathbf{A}_{k2}(\mathbf{I}-\mathbf{A}_{22})^{-1} & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\ \mathbf{e}_{21} & 0 & \cdots & \mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & 0 \end{bmatrix} =$$

$$= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$$

This operation results in a KN×K matrix $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ where the country of origin is still r while the country of destination is t, or the second-tier partner. Replace $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ in equation (A.1) with $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} = \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\text{2nd tier from } r \text{ to } t=s} + \underbrace{\breve{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} \tag{A.2}$$

The second term on the right side now captures intermediate exports from sector i of country r that are embodied in all exports to country s (which also appears as t at the next tier) via third countries. As a result, we disaggregate the second-tier partners at the expense of aggregating the first-tier partners. Importantly, the term on the left side in (A.2) no longer represents direct bilateral exports. Instead, it accounts for cumulative exports to the first- and second-tier partners.

Insert equation (A.1) into equation (A.2) to decompose bilateral exports to the second-tier partners:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} &= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil} + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{F}}\right) + \breve{\mathbf{F}} = \\
&= \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}} + \\
&+ \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}} + \breve{\mathbf{F}}
\end{aligned}$$

Replace again $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{tot}$ with $\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}$ and allocate the second-tier total exports to the third-tier bilateral exports:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd + 3d tier}} &= \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\widehat{\mathbf{F}}}_{\text{2nd tier from } r \text{ to } s} + \\
&+ \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\mathbf{E}_{bil}}_{\text{3rd tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\breve{\mathbf{F}}}_{\text{2nd tier from } r \text{ to } s} + \underbrace{\breve{\mathbf{F}}}_{\text{1st tier from } r \text{ to } s}
\end{aligned}$$

In this way, further decomposing and reallocating exports along the value chain to the t^{th} tier results in:

$$\begin{aligned}
\underbrace{\mathbf{E}_{bil}}_{\text{1st + \ldots + } t\text{th tier}} &= \sum_{1}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{tot} + \\
&+ \sum_{1}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t-1}\breve{\mathbf{F}} = \\
&= \sum_{0}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\widehat{\mathbf{F}} - \widehat{\mathbf{F}} + \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\mathbf{E}_{tot} + \\
&+ \sum_{0}^{t}\left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\breve{\mathbf{F}} - \left(\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{t}\breve{\mathbf{F}}
\end{aligned}$$

As the decomposition proceeds to an infinitely remote $t^{\text{th}} \to \infty$ tier, the re-exported term approaches zero and is eventually reallocated between intermediates and final products for domestic use:

$$\underset{\text{all tiers}}{\mathbf{E}_{bil}} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \widehat{\mathbf{F}} - \widehat{\mathbf{F}} + 0 + \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}} - 0 =$$
$$= \left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \widehat{\mathbf{F}} + \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}}$$

This is a way to trace bilateral exports throughout the whole value chain to the ultimate destination where they end up in partner final demand. The term on the left side can be treated as cumulative bilateral exports $\mathbf{E}_{cum}$ where the elements are smaller or larger than direct bilateral exports, subject to the mode of partner integration into the value chain:

$$\underbrace{\mathbf{E}_{cum}}_{\substack{\text{cumulative}\\ \text{exports}}} = \underbrace{\left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \widehat{\mathbf{F}}}_{\substack{\text{direct and indirect exports of intermediates}\\ \text{eventually transformed into final products}\\ \text{for domestic use}}} + \underbrace{\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \breve{\mathbf{F}}}_{\substack{\text{direct and indirect exports}\\ \text{of final products}}} =$$
$$= \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{F} - \widehat{\mathbf{F}} \tag{A.3}$$

Equation (A.3) is not a decomposition of actual trade flows. Rather, it should be understood as a way to compute cumulative bilateral exports $\mathbf{E}_{cum}$ where each element describes the amount of product by sector i of country r that is eventually used for final demand in country s, delivered as direct or indirect exports.

$\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1}$ is a new "global" multiplier matrix that will be denoted by $\mathbf{H}$ for brevity.

The derivation of the equation of cumulative bilateral exports is also possible with the use of an alternative transformation at each tier:

$$\underbrace{\mathbf{E}_{bil}}_{\text{1st + 2nd tier}} = \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil} + \breve{\mathbf{F}} =$$
$$= \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \widehat{\mathbf{F}} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{F}} -$$
$$- \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil} =$$
$$= \underbrace{\mathbf{E}_{bil}}_{\text{1st tier}} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{tot} + \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1} \mathbf{E}_{bil}$$

The continuous substitution of $\mathbf{E}_{bil}$ to an infinitely remote $t^{\text{th}} \to \infty$ tier will yield:

$$\mathbf{E}_{cum} = \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{bil} - \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{tot} + \mathbf{E}_{tot} =$$
$$= \left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} \mathbf{E}_{bil} - \left(\left(\mathbf{I} - \breve{\mathbf{A}}\left(\mathbf{I} - \widehat{\mathbf{A}}\right)^{-1}\right)^{-1} - \mathbf{I}\right) \mathbf{E}_{tot} =$$
$$= \mathbf{H}\mathbf{E}_{bil} - (\mathbf{H} - \mathbf{I})\mathbf{E}_{tot} \tag{A.4}$$

Cumulative bilateral exports can therefore be expressed as a function of either final demand or bilateral and total gross exports.

A.2 The relationship of new "global" inverse to the standard Leontief "global" inverse

The following manipulations show the relationship of **H** to the standard Leontief "global" inverse **L**:

$$\begin{aligned}
\mathbf{L} &= (\mathbf{I}-\mathbf{A})^{-1} \\
\mathbf{L}(\mathbf{I}-\mathbf{A}) &= (\mathbf{I}-\mathbf{A})^{-1}(\mathbf{I}-\mathbf{A}) \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}}-\breve{\mathbf{A}}) &= \mathbf{I} \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}})-\mathbf{L}\breve{\mathbf{A}} &= \mathbf{I} \\
\mathbf{L}(\mathbf{I}-\widehat{\mathbf{A}})(\mathbf{I}-\widehat{\mathbf{A}})^{-1}-\mathbf{L}\breve{\mathbf{A}}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} &= \mathbf{I}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
\mathbf{L}-\mathbf{L}\breve{\mathbf{A}}(\mathbf{I}-\widehat{\mathbf{A}})^{-1} &= (\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
\mathbf{L}\left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right) &= (\mathbf{I}-\widehat{\mathbf{A}})^{-1} \\
(\mathbf{I}-\widehat{\mathbf{A}})\mathbf{L} &= \left(\mathbf{I}-\breve{\mathbf{A}}\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}\right)^{-1} = \mathbf{H}
\end{aligned}$$

The above also shows that **H** exists as long as does **L**.

A.3 The equivalence between total cumulative exports and total direct gross exports

An important property is that total cumulative exports to all destinations are equal to total direct gross exports:

$$\mathbf{E}_{cum}\mathbf{i} = (\mathbf{H}\mathbf{E}_{bil}-(\mathbf{H}-\mathbf{I})\mathbf{E}_{tot})\,\mathbf{i} = \mathbf{H}\mathbf{E}_{bil}\mathbf{i}-\mathbf{H}\mathbf{E}_{tot}\mathbf{i}+\mathbf{E}_{tot}\mathbf{i} = \mathbf{E}_{bil}\mathbf{i}$$

The formulation above utilizes that, by definition, the sum of bilateral exports across all partners equals total exports.

B Comparison of cumulative import tariff measurements in this and previous papers

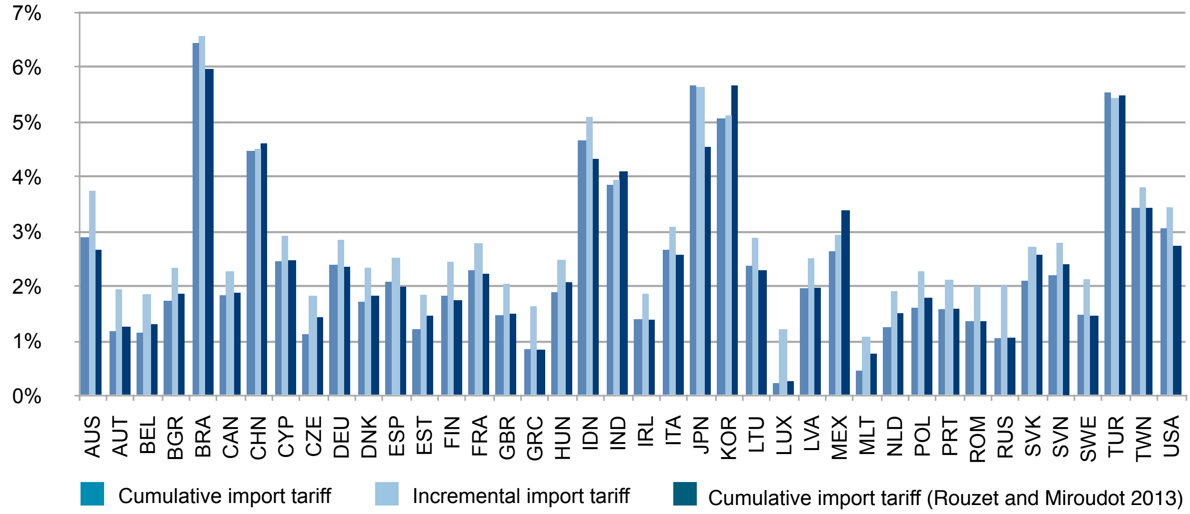

Figure B.1: Comparison of cumulative, incremental tariffs in this paper and Rouzet and Miroudot's (2013) version of cumulative tariff, faced by exporting country in 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of countries in the WIOD is in Table C.1, Appendix C.

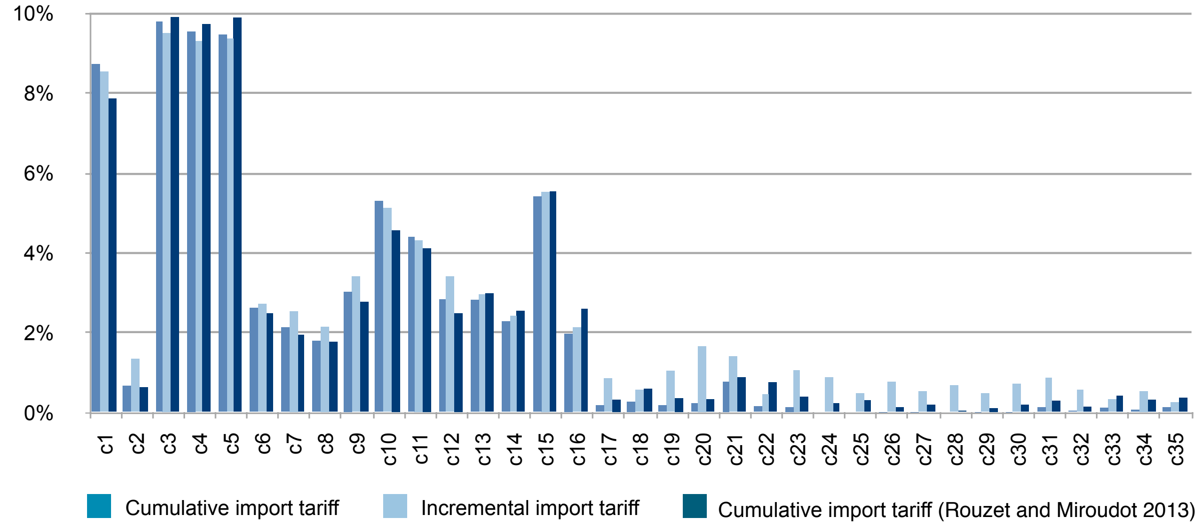

Figure B.2: Comparison of cumulative, incremental tariffs in this paper and Rouzet and Miroudot's (2013) version of cumulative tariff, faced by exporting sector in 2010
Source: WIOD, UN Comtrade and UN TRAINS databases, author's calculations
Note: the full list of sectors in the WIOD is in Table C.2, Appendix C.

C Countries and industries in the WIOD database

Table C.1: List of countries in the WIOD database

Country code	Country	Country code	Country
AUS	Australia	IRL	Ireland
AUT	Austria	ITA	Italy
BEL	Belgium	JPN	Japan
BGR	Bulgaria	KOR	Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CHN	China	LVA	Latvia
CYP	Cyprus	MEX	Mexico
CZE	Czech Republic	MLT	Malta
DEU	Germany	NLD	Netherlands
DNK	Denmark	POL	Poland
ESP	Spain	PRT	Portugal
EST	Estonia	ROM	Romania
FIN	Finland	RUS	Russian Federation
FRA	France	SVK	Slovak Republic
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HUN	Hungary	TUR	Turkey
IDN	Indonesia	TWN	Chinese Taipei
IND	India	USA	United States
		RoW	Rest of the World

Source: Dietzenbacher et al., 2013; `http://www.wiod.org`

Table C.2: List of industries in the WIOD database

WIOD code	NACE Rev.1/ ISIC Rev.3	Industry
c1	A – B	Agriculture, Hunting, Forestry and Fishing
c2	C	Mining and Quarrying
c3	15 – 16	Food, Beverages and Tobacco
c4	17 – 18	Textiles and Textile Products
c5	19	Leather, Leather and Footwear
c6	20	Wood and Products of Wood and Cork
c7	21 – 22	Pulp, Paper, Paper , Printing and Publishing
c8	23	Coke, Refined Petroleum and Nuclear Fuel
c9	24	Chemicals and Chemical Products
c10	25	Rubber and Plastics
c11	26	Other Non-Metallic Mineral
c12	27 – 28	Basic Metals and Fabricated Metal
c13	29	Machinery, Nec
c14	30 – 33	Electrical and Optical Equipment
c15	34 – 35	Transport Equipment
c16	36 – 37	Manufacturing, Nec; Recycling
c17	E	Electricity, Gas and Water Supply
c18	F	Construction
c19	50	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
c20	51	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
c21	52	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
c22	H	Hotels and Restaurants
c23	60	Inland Transport
c24	61	Water Transport
c25	62	Air Transport
c26	63	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
c27	64	Post and Telecommunications
c28	J	Financial Intermediation
c29	70	Real Estate Activities
c30	71 – 74	Renting of M and Eq and Other Business Activities
c31	L	Public Admin and Defence; Compulsory Social Security
c32	M	Education
c33	N	Health and Social Work
c34	O	Other Community, Social and Personal Services
c35	P	Private Households with Employed Persons

Source: Dietzenbacher et al., 2013; `http://www.wiod.org`