Household heterogeneity, aggregation, and the distributional impacts of environmental taxes

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Abstract

This paper examines the incidence of an environmental tax using theoretical and numerical general equilibrium models that allow for heterogeneous households, fully general forms of preferences, differential spending and income patterns, differential factor intensities in production, and fully general forms of substitution among inputs of capital, labor, and pollution. First, we focus on the household aggregation problem and find that the incidence of an environmental tax can be qualitatively affected by the level of consistency with which household heterogeneity is integrated into the analysis. Distributional impacts of environmental taxes based on partial and general equilibrium analyses that fail to consistently integrate household heterogeneity are thus likely to be biased significantly. Second, we apply the heterogeneous household model to analyze the distributional effects of a U.S. carbon tax. We find strong evidence that such a tax would be regressive. While this result is robust with respect to varying households' and firms' characteristics, the regressivity is dampened considerably if labor is a good substitute for pollution relative to capital.

Keywords: Environmental taxes, General equilibrium, Heterogeneous households, Distributional effects, Sources side, Uses side, Non-homothetic preferences
JEL: H23, Q52

1. Introduction

The public acceptance for environmental taxes depends crucially on their distributional consequences. A plethora of applied research in public and environmental economics has investigated the incidence of environmental taxes in various policy settings. Not seldom, however, the empirical evidence whether a specific tax is regressive or not is mixed—even if the incidence of a given tax instrument is analyzed in a similar or identical policy context. Differences arise because the incidence analysis does not consider all relevant channels through which an environmental tax affects market outcomes (see, e.g., Atkinson & Stiglitz (1980) and Fullerton & Metcalf (2002) for a discussion of incidence impacts in the public finance literature). Environmental taxes often appear to be regressive on the "uses side of income" as they affect more heavily the welfare of the poorest households than of the richest ones, since poorer households spend a larger fraction of

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their income on polluting goods (e.g., energy or electricity). "Sources side of income" impacts can dampen or even offset the regressive incidence on the uses side to the extent that environmental tax policies affect the returns to factors of production that are disproportionately owned by richer households and used intensively in the production of dirty relative to clean industries (e.g., capital).[3] Partial equilibrium assessments, comprising Input-Output analysis as a widely used method, ignore general equilibrium effects, including sources side impacts.[4] Incidence analyses using a general equilibrium framework typically employ a single, representative household model to derive factor and output price changes which are used in an ex-post calculation to determine tax burdens across households. A problem with such an approach is that it neglects the impact of household heterogeneity on the equilibrium allocation, thus ignoring a potentially relevant channel for tax incidence. Despite the high policy relevance and academic interest for understanding the incidence of price-based pollution control policies, a theoretical analysis of the distributional impacts of environmental taxes in a general equilibrium setting with heterogeneous households is lacking.[5]

This paper develops a theoretical general equilibrium model in the spirit of Harberger (1962) of the incidence of an environmental tax that features heterogeneous households, fully general forms of preferences, differential spending and income patterns, differential factor intensities in production, and fully general forms of substitution among inputs of capital, labor, and pollution. Its purpose is two-fold. First, we investigate the implication of the household aggregation problem for the general equilibrium incidence of environmental taxes. In the absence of identical homothetic preferences for each individual or homothetic preferences and collinear initial endowment vectors (i.e., identical income shares), aggregated preferences depend on the distribution of income (Polemarchakis, 1983).[6] Thus acknowledging heterogeneity in tastes undercuts the representative consumer framework that is used to calculate the general equilibrium effects on output and factor prices (Kortum, 2010). How severe is the aggregation problem, i.e. to what extent are incidence results derived from a general equilibrium analysis which ignores household heterogeneity and the potential for non-homothetic preferences biased? Second, we apply the heterogeneous household model to assess the incidence of a tax on carbon dioxide (CO_2) emissions in the United States. Is a US carbon tax regressive? We assess the incidence on the sources and uses side, and explore how sensitive incidence results are with respect to key characteristics governing households' and firms' behavior.

Our paper builds on a small but growing literature that uses analytical general equilibrium models to study the incidence of environmental taxes. Our model builds on a series of influential papers by Fullerton and others (Fullerton & Heutel, 2007, 2010; Fullerton et al., 2012; Fullerton & Monti, 2013) that extend the Harberger (1962) model and previous theoretical work by Rapanos (1992, 1995) to develop a model

[3]The regressivity of many environmental taxes on the uses side, including carbon pricing in the context of climate policy, constitutes a serious concern for policymakers and has been investigated extensively in the literature (Poterba, 1991; Metcalf, 1999; Fullerton et al., 2012). Gasoline taxes are generally found to be progressive on the uses side (Sterner, 2012). More recently, work by Fullerton & Heutel (2007), Araar et al. (2011), and Rausch et al. (2011) has also scrutinized the sources side impacts of carbon taxation.

[4]Williams III & Goulder (2003) shows that under typical conditions the simple "excess-burden triangle" formula substantially underestimates the excess burden of commodity taxes, in some cases by a factor of 10 or more, because it ignores general equilibrium interactions—most important, interactions between the taxed commodity and the labor market.

[5]This paper focuses on the incidence of an environmental tax itself, i.e., we do not consider the question of how recycling the revenue from an environmental tax can shape distributional impacts across households. While a number of studies have shown that various revenue recycling options have the potential to offset unintended distributional consequences (for example, Metcalf, 2007; Parry & Williams III, 2010; Rausch et al., 2010b), the choice and design of revenue recycling constitute a separate and distinct policy issue which first requires an understanding of the incidence impacts *per se*.

[6]On a more fundamental conceptual level, and not related to the incidence of (environmental) taxation, the aggregation problem for heterogeneous consumers in general equilibrium models has been studied by Ackermann (2002) based on prior work by Rizvi (1994) and Martel (1996).

which represents pollution as an input along with capital and labor and that allows for general forms of substitution between inputs. We extend the model presented in Fullerton & Heutel (2007) which assumes a single consumer to include heterogeneous households. We additionally allow for preferences to be non-homothetic, therefore going beyond the standard assumption of homothetic preferences employed by most analyses that assess the incidence of environmental taxes. By fully integrating household heterogeneity in the model, our paper also differs from the contributions in Fullerton & Heutel (2010) and Fullerton et al. (2012) that use price impacts derived from the single-consumer model in Fullerton & Heutel (2007) to determine the burdens of a carbon tax on households using household survey data from the "Consumer Expenditure Survey". Fullerton & Monti (2013) do integrate two types of households into an analytical general equilibrium model and investigate the distributional impacts of a pollution tax swap where revenues are recycled through a (pre-existing) wage tax of low-income workers; they do not, however, focus on the impact of household heterogeneity on equilibrium outcomes.

The present paper is also related to the literature that uses computational methods to investigate the distributional impacts of environmental taxes. A widespread approach is to use Input-Output analysis to derive price changes for different consumers goods which are then applied to calculate tax burdens for households based on micro-household survey data. Examples include Robinson (1985) who studies the distributional burden of industrial abatement in the U.S. economy or, related to energy and CO_2 emissions taxes, Poterba (1991) who analyzes the incidence of gasoline taxes; Bull et al. (1994); Hassett & Metcalf (2009) compare a tax based on energy content and a tax based on carbon, and Metcalf (1999, 2009) analyze a revenue-neutral package of environmental taxes, including a carbon tax, an increase in motor fuel taxes, and taxes on various stationary source emissions. Dinan & Rogers (2002) assess the efficiency and distributional impacts of a U.S. cap-and-trade program for CO_2 emissions, and Mathur & Morris (2014) investigate the distributional effects of a carbon tax in broader U.S. fiscal reform. Other works study the incidence impacts of greenhouse gas emissions pricing policies across household income groups for different countries (e.g., Labandeira & Labeaga (1999) for Spain, Callan et al. (2009) for Ireland, and Jiang & Shao (2014) for China). Common to these studies is that they adopt a partial equilibrium perspective that does not consider behavioral changes, focuses on the uses sides of the incidence only, and employs welfare metrics that are not consistent with utility-maximizing micro-economic behavior.

A few papers use numerical general equilibrium models to derive price impacts on commodity and factor prices. Metcalf et al. (2008) carry out an analysis of carbon tax proposals and find that a carbon tax is highly regressive but that the regressivity is reduced due to sources side effects to the extent that resource and equity owners bear some fraction of the tax burden. Similarly, Araar et al. (2011); Dissou & Siddiqui (2014) use the prices effects generated to assess the distributional impact of a carbon tax on households. None of these studies, however, consider the impact of household heterogeneity on equilibrium outcomes as their analysis is based on a single, representative consumer general equilibrium model.

Lastly, some papers do consistently incorporate heterogeneous households into a numerical general equilibrium framework thus capturing the impact of household heterogeneity on equilibrium prices as well as the uses and sources side impacts. For example, Rausch et al. (2010a,b) investigate the incidence of a US carbon tax in a model with nine households representing different income classes and find that the overall impact is neutral to modestly progressive due to sources side effects (including inflation-indexed government transfers). Williams III et al. (2014, 2015) and Chiroleu-Assouline & Fodha (2014) employ calibrated overlapping generations models to assess the distributional incidence across generations but they do not model intra-cohort heterogeneity. A major weakness of analyses based on numerical simulation models is that they have to rely on specific functional forms with limited forms of substitution. In contrast, the present paper studies the incidence of an environmental tax in a theoretical model with general forms of substitution among inputs for both firms and households. Moreover, none of these studies investigates the impact of household heterogeneity on equilibrium outcomes and the household aggregation problem.

The key result of the present paper is that the household aggregation problem can have severe implications for analyzing the incidence of environmental taxes: basing the analysis on a single, representative household model can yield both qualitative and quantitative conclusions that differ from a general equilibrium model which consistently integrates household heterogeneity. The important implication for analyzing environmental tax policy is that both partial equilibrium methods, including Input-Output analysis, and general equilibrium analyses that fail to consistently integrate household heterogeneity should be used with care or best be avoided altogether.

We use theoretical and numerical analyses to derive and support our results. In the case of homothetic preferences we show that the impact of household heterogeneity on the equilibrium allocation can be characterized by two statistical quantities which reflect the degree of household heterogeneity in terms of expenditure and income shares and the second-order properties of households' preferences (i.e., the utility function). These metrics provide an intuitive way to express the deviation between the realistic case with heterogeneous households and a hypothetical case of identical households. We provide examples of conditions for households' and firms' characteristics—determining their respective equilibrium choices in response to an environmental tax—under which the aggregation bias does or does not matter. For example, for cases with limited substitutability between inputs of capital, labor, and pollution in production, factor and output price changes can be reversed in turn yielding qualitatively different incidence results across groups of poor and rich households. Moreover, we find that there exist for (almost) any benchmark economy, specified by production data and the distribution of preferences and endowments, plausible values for production elasticities such that factor price changes derived from a single, representative household model are of opposite sign relative to the model with heterogeneous households. We furthermore find that allowing preferences to be non-homothetic can qualitatively affect the burden of the environmental tax on factors of production, thus highlighting the potential importance of relaxing the assumption of homothetic preferences in order to fully capture the general equilibrium incidence of an environmental tax.

Using an empirically calibrated version of the theoretical model, based on historic data for the U.S. economy, we find strong evidence that a U.S. carbon tax would be regressive. This result is relatively robust to varying characteristics of both households and firms. Our analysis, however, points to the importance of including sources of income impacts for tax incidence analysis. As we find that sources side effects tend to be progressive, basing the incidence analysis on the uses side effects only may overestimate the regressivity of the environmental tax and can lead to qualitatively false conclusions. On the other hand, our finding that sources side impacts drive most of the variation in welfare impacts calls for a careful model specification of how both polluting and non-polluting firms respond to an environmental tax.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 derives closed-form expressions to assess the incidence of an environmental tax, and presents and interprets our theoretical results. Section 4 uses an empirically calibrated version of the theoretical model to derive additional results by means of numerical analysis. Section 5 concludes.

2. Model

We consider a static and closed economy with two sectors and two factors of production. A "clean" good is produced using capital and labor, and a "dirty" good is produced using capital, labor and pollution. Capital and labor are supplied inelastically and are mobile across sectors. The government taxes pollution, returning the revenue lump-sum to households. Our general equilibrium model follows closely Harberger (1962) and Fullerton & Heutel (2007) but differs in two important aspects. First, we introduce heterogeneous households that differ in terms of their preferences and income patterns derived from endowments of capital and labor. Second, we generalize the representation of household behavior by allowing for non-homothetic preferences. Using log-linearization, we analytically solve for first-order changes in equilibrium prices and quantities following an exogenous change in the pollution tax rate. Our model allows us to quantify the general equilibrium incidence of the environmental tax in the context of an economy with no a-priori restrictions placed on the number and characteristics of households.

The clean sector production function $X = X(K_X, L_X)$ and the dirty sector production function $Y = Y(K_Y, L_Y, Z)$ are assumed to exhibit constant returns to scale, where K_X, K_Y, L_X, and L_Y are the quantities of capital and labor used in each sector.[7] Let indexes n and m denote sectors and factors of production, respectively. The total amounts of factors of production in the economy are exogenously given and fixed: $K_X + K_Y = \bar{K}$ and $L_X + L_Y = \bar{L}$. Totally differentiating the resource constraints yields:

$$\hat{K}_X \frac{K_X}{\bar{K}} + \hat{K}_Y \frac{K_Y}{\bar{K}} = 0 \tag{1}$$

$$\hat{L}_X \frac{L_X}{\bar{L}} + \hat{L}_Y \frac{L_Y}{\bar{L}} = 0\,, \tag{2}$$

where a hat denotes a proportional change, e.g., $\hat{K}_X \equiv dK_X/K_X$. Pollution ($Z$) has no equivalent resource constraint and is a choice of the dirty sector. To ensure a finite use of pollution in equilibrium, we assume a pre-existing positive tax on pollution, τ_Z.

Firms in sector X can substitute between factors in response to changes in the wage rate (w) and capital rental rate (r) according to an elasticity of substitution in production, σ_X. Differentiating the definition for σ_X yields:

$$\hat{K}_X - \hat{L}_X = \sigma_X(\hat{w} - \hat{r})\,. \tag{3}$$

The production decision of firms in sector Y depends additionally on the pollution price they face, which is given by the pollution tax rate τ_Z. We model the choice between the three inputs of capital, labor and pollution by means of the Allen elasticities e_{ij} between inputs i and j (Allen, 1938). The 3×3 matrix of Allen elasticities is symmetric (i.e., $e_{ij} = e_{ji}$), its diagonal entries are less or equal to zero (i.e., $e_{ii} \leq 0$), and at most one of the three independent off-diagonal elements can be negative. Furthermore, e_{ij} is positive whenever inputs i and j are substitutes, and negative whenever they are complements. Totally differentiating input demand functions for sector Y, which describe the dirty sector's cost minimization problem, and dividing by the appropriate input level, yields:[8]

$$\hat{K}_Y - \hat{Z} = \theta_{YK}(e_{KK} - e_{ZK})\hat{r} + \theta_{YL}(e_{KL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{KZ} - e_{ZZ})\hat{\tau}_Z \tag{4}$$

$$\hat{L}_Y - \hat{Z} = \theta_{YK}(e_{LK} - e_{ZK})\hat{r} + \theta_{YL}(e_{LL} - e_{ZL})\hat{w} + \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z\,, \tag{5}$$

where θ_{mn} is the share of sector m's revenue paid to factor n, e.g. $\theta_{XK} = \frac{rK_X}{p_X X}$. Let p_X and p_Y denote output prices for X and Y, respectively. Under the assumption of perfect competition, the following expressions hold:

$$\hat{p}_X + \hat{X} = \theta_{XK}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X) \tag{6}$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK}(\hat{r} + \hat{K}_Y) + \theta_{YL}(\hat{w} + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Z + \hat{Z}) \tag{7}$$

$$\hat{X} = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{8}$$

$$\hat{Y} = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{9}$$

[7] Note that the production side of our model is the same as for the single-consumer model of Fullerton & Heutel (2007). In describing production we thus follow closely the model description in Fullerton & Heutel (2007, pp. 574-75).

[8] Appendix Appendix A in Fullerton & Heutel (2007) derives equations (4)-(9).

Households, indexed by $h = \{1, \ldots, H\}$, maximize utility by choosing optimal consumption of goods X and Y subject to an income constraint.[9] Each household inelastically supplies fixed factor endowments $\bar{K}^h$ and $\bar{L}^h$ which satisfy the following relations: $\sum_h \bar{K}^h = \bar{K}$ and $\sum_h \bar{L}^h = \bar{L}$. Income for household h is therefore given by $M^h = w\bar{L}^h + r\bar{K}^h + \xi^h \tau_Z Z$, where ξ^h is the share of the pollution tax revenue redistributed lump-sum to household h. Since the tax revenue is returned entirely to households, it follows that $\sum_h \xi^h = 1$.

Before proceeding, we emphasize that the focus of our analysis in this paper is on understanding the "pure" incidence of a pollution tax, i.e., without the impact resulting from how the tax revenue is redistributed. To isolate the pure incidence impacts, we therefore assume that the redistribution of tax revenues occurs in a way that does not influence the relative burdens across households.[10]

Following Hicks & Allen (1934), we parameterize non-homothetic consumer preferences for the two goods using the elasticity of substitution between goods X and Y in utility σ^h, and the income elasticities of demand for goods X and Y, denoted by $E^h_{X,M}$ and $E^h_{Y,M}$ respectively.[11] Appendix A derives the following expressions for changes in demand by household h in response to output and factor price changes:

$$\hat{X}^h - \hat{Y}^h = \sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h \hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h) \tag{10}$$

$$\hat{X}^h = -(\alpha^h E^h_{X,M} + (1-\alpha^h)\sigma^h)\hat{p}_X - [(1-\alpha^h)E^h_{X,M} - (1-\alpha^h)\sigma^h]\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \tag{11}$$

with $\hat{M}^h = \hat{w}\frac{w\bar{L}^h}{M^h} + \hat{r}\frac{r\bar{K}^h}{M^h} + \frac{\tau_Z Z}{p_X X + p_Y Y}(\hat{\tau}_Z + \hat{Z})$.

Finally, totally differentiating the market clearing conditions for the two consumption goods, $X = \sum_h X^h$ and $Y = \sum_h Y^h$, yields:

$$\hat{X} = \sum_h \frac{X^h}{X}\hat{X}^h \tag{12}$$

$$\hat{Y} = \sum_h \frac{Y^h}{Y}\hat{Y}^h\,. \tag{13}$$

Equations (1)–(13) are $11 + 2H$ equations in $11 + 2H$ unknowns ($\hat{K}_X$, $\hat{K}_Y$, $\hat{L}_X$, $\hat{L}_Y$, $\hat{w}$, $\hat{r}$, $\hat{p}_X$, $\hat{X}$, $\hat{p}_Y$, $\hat{Y}$, $\hat{Z}$, $H \times \hat{X}^h$, $H \times \hat{Y}^h$). Following Walras' Law, one of the equilibrium conditions is redundant, thus the effective number of equations is $10 + 2H$. We choose X as the numéraire good, which implies $\hat{p}_X = 0$.

The square system of model equations then endogenously determines all the above unknowns as functions of benchmark parameters (characterizing the equilibrium before the tax change), behavioral parameters (elasticities of production and consumption), and the exogenous positive change in the pollution tax ($\hat{\tau}_Z > 0$).

3. Analytical results and interpretations

When solving for the model unknowns as functions of the exogenous tax change, we are ultimately interested in the distributional incidence of the environmental tax. Let v^h denote the indirect utility function

[9] We assume that pollution, or environmental quality, is separable in utility, thus not influencing the optimal consumption choice. The incidence analysis carried out in this paper thus focuses on utility derived from market consumption only.

[10] The redistribution scheme which ensures that all households are impacted in the same proportion relative to their income is given by: $\frac{\xi^h}{M^h} = \frac{1}{\sum_h M^h} \equiv \frac{1}{p_X X + p_Y Y}$.

[11] Homothetic preferences are represented by the special case $E^h_{X,M} = E^h_{Y,M} = 1$. In this case the first-order behavior of households can be sufficiently described by σ^h, as for example in Fullerton & Heutel (2007).

of household h, and dv^h the change in utility caused by an increase in the pollution tax rate by $d\tau_Z$. To compare impacts across households, we express utility changes in monetary terms relative to income: $\frac{dv^h}{M^h \partial_{M^h} v^h}$ measures the amount of income which would cause a change in utility equal to dv^h at prices prior to the tax change, expressed relative to the income of household h. To isolate the distributional dimension from the economy-wide cost of the tax, we focus on the welfare impact of each household relative to the average welfare change. This ensures that results do not depend on the choice of numéraire.

We can then write the welfare impact of household h relative to the average economy-wide monetary loss per unit of income as:[12]

$$\Phi^h \equiv \frac{dv^h}{M^h \partial_{M^h} v^h} - \frac{1}{\sum_{h'} M^{h'}} \sum_{h'} \frac{dv^{h'}}{\partial_{M^{h'}} v^{h'}} = \underbrace{-(\gamma - \alpha^h)\hat{p}_Y}_{=\text{Uses of income impact}} + \underbrace{(\theta_L^h - \theta_L)\hat{w} + (\theta_K^h - \theta_K)\hat{r}}_{=\text{Sources of income impacts}}, \tag{14}$$

where $\theta_K^h \equiv \frac{r\bar{K}^h}{M^h}$ and $\theta_L^h \equiv \frac{w\bar{L}^h}{M^h}$ are the capital and labor income shares of household h, and $\theta_K \equiv \frac{r\bar{K}}{p_X X + p_Y Y}$, $\theta_L \equiv \frac{w\bar{L}}{p_X X + p_Y Y}$ and $\gamma \equiv \frac{p_X X}{p_X X + p_Y Y}$ are the value shares of capital, labor and the clean sector in the economy.

The welfare decomposition underlying equation (14) enables an intuitive economic interpretation of the various channels through which household characteristics determine incidence. On the one hand, for given changes in the prices of goods and factors, variation in impacts across households arises for two reasons. First, households differ in how they spend their income. For a given increase in the price of the dirty good ($\hat{p}_Y > 0$), consumers of the dirty good are more negatively impacted as compared to consumers of the clean good. This impact is referred to as the uses of income impact. Second, in a general equilibrium setting, a pollution tax also impacts factor prices. Households which rely heavily on income from the factor whose price falls relative to the other will be adversely impacted compared to the average household. These impacts are referred to as sources of income impacts.

Since output and factor price changes are not independent of households' characteristics, two additional and more indirect determinants of incidence emerge from the expression (14). First, in an economy with heterogeneous households, output and factor prices are not independent of the distribution of households' consumption profiles and factor endowments across the population; welfare changes for a given household type do not only depend on its own characteristics but also on those of other households in the economy. Second, even in an economy with identical households, the specifics of the household's behavioural response to price and income changes can affect equilibrium outcomes.

To analytically study incidence in our model, we now proceed by deriving expressions that characterize how output and factor prices changes depend on model parameters. Appendix B derives the following general solutions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ following a change in τ_Z:

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{15a}$$

$$\hat{w} = \frac{\theta_{XK}\theta_{YZ}}{D}\left[A(e_{ZZ} - e_{KZ}) - B(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z \tag{15b}$$

[12]Recall that p_X is the numéraire. Then $dv^h = \partial_{p_Y} v^h dp_Y + \partial_{M^h} v^h dM^h = \partial_{p_Y} v^h p_Y \hat{p}_Y + \partial_M^h v^h \left(\hat{w} w\bar{L}^h + \hat{r} r\bar{K}^h + \xi^h \tau_Z Z[\hat{\tau}_Z + \hat{Z}]\right)$, and use Roy's identity (i.e., $\partial_{p_Y} v^h = -Y^h \partial_{M^h} v^h$). Recall furthermore that the incidence-neutral tax re-distribution scheme is characterised by $\xi^h = \frac{M^h}{\sum_h M^h}$.

$$\hat{r} \quad = \quad -\frac{\theta_{XL}\theta_{YZ}}{D}\left[A(e_{ZZ}-e_{KZ})-B(e_{ZZ}-e_{LZ})+(\gamma_K-\gamma_L)(\delta-\sum_h \frac{\phi_Z^h}{\theta_{YZ}})\right]\hat{\tau}_Z\,, \qquad (15c)$$

where $\gamma_K \equiv \frac{K_Y}{K_X}$, $\gamma_L \equiv \frac{L_Y}{L_X}$, $\beta_L \equiv \theta_{XL}\gamma_L + \theta_{YL}$, $\beta_K \equiv \theta_{XK}\gamma_K + \theta_{YK}$, $A \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} - \sum_h \phi_Z^h)$, $B \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} - \sum_h \phi_Z^h)$, $C \equiv \beta_K + \beta_L + \theta_{YZ} - \sum_h \phi_Z^h$, $D \equiv C\sigma_X + A[\theta_{XK}\theta_{YL}(e_{KL}-e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})] - B[\theta_{XK}\theta_{YL}(e_{LL}-e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK}-e_{ZK})] - (\gamma_K-\gamma_L)(\theta_{XK}(\theta_{YL}\delta - \sum_h \phi_L^h) - \theta_{XL}(\theta_{YK}\delta - \sum_h \phi_K^h))$. The remaining expressions depend explicitly on household characteristics: $\phi_L^h \equiv (1-\frac{\alpha^h}{\gamma})E_{X,M}^h \frac{w\bar{L}^h}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{w\bar{L}^h}{M^h}$, $\phi_K^h \equiv (1-\frac{\alpha^h}{\gamma})E_{X,M}^h \frac{r\bar{K}^h}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{r\bar{K}^h}{M^h}$, $\phi_Z^h \equiv (1-\frac{\alpha^h}{\gamma})E_{X,M}^h \frac{\xi^h\tau_Z Z}{p_Y Y} + \frac{Y^h}{Y}(E_{Y,M}^h - E_{X,M}^h)\frac{\xi^h\tau_Z Z}{M^h}$ and $\delta \equiv \sum_h \frac{Y^h}{Y}\Big(\sigma^h + (\frac{\alpha^h}{\gamma}-1)(\sigma^h - E_{X,M}^h) + (E_{Y,M}^h - E_{X,M}^h)(1-\alpha^h)\Big)$.

Note that in general $\hat{w} = -\frac{\theta_{XK}}{\theta_{XL}}\hat{r}$. Thus, in order to understand the burden of the change in the pollution tax on the returns to factors, it is sufficient to study the change in the returns to capital, keeping in mind that—given our choice of the numéraire good—$\hat{w}$ always has the opposite sign as $\hat{r}$.

While the interpretation of the general solution is limited by its complexity, it is apparent from the analytical expressions above that going beyond a single consumer and introducing multiple, heterogeneous households with non-homothetic preferences into the model in general has a first-order impact on the market equilibrium, and thus on the incidence results. By considering expressions (15a)–(15c) one can identify the following two effects, which have also previously been identified in the context of the Harberger (1962) model. The $(\gamma_K - \gamma_L)(\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}})$ term in equations (15b) and (15c) represents the *output effect*: the tax on sector Y reduces output, and consequently depresses the returns to the factor used intensively in the dirty sector. The sign of the output effect follows this intuition only if the denominator D is positive, which in general is not the case, even for identical households (or equivalently a single household) and homothetic preferences (Fullerton & Heutel, 2007). Introducing multiple, heterogeneous households and non-homothetic preferences adds another layer of complexity to this indeterminacy, since $\delta - \sum_h \frac{\phi_Z^h}{\theta_{YZ}}$ cannot in general be signed, whereas this expression is positive for identical households with homothetic preferences.

The other terms in equations (15b) and (15c) embody the *substitution effects*, which reflect the reaction of firms to factor price changes. Again, while for the case with identical households and homothetic preferences the constants A and B can be signed as positive, this is not the case in our more general model. The substitution effect thus also bears a greater degree of indeterminacy as compared to the Fullerton & Heutel (2007) model.

To better understand the various effects at work, it is necessary to depart from the generality of the above expressions. We therefore consider a series of special cases in which we impose restrictions on household and production characteristics in order to seek definitive results for the changes in prices and returns to factors, and therefore better understand the implications for incidence. First, we present a special case for production under which household characteristics have no impact on price changes. Second, we consider cases which allow for full household heterogeneity in terms of preferences and income patterns but where preferences are assumed to be homothetic. Third, the role of non-homothetic preferences is investigated for cases with identical households. Comparing these special cases will be useful to understand the interaction of production and household characteristics in determining the changes in output and factor prices, and consequently incidence. Lastly, the most general case described by the combination of non-homothetic preferences and heterogeneous households is studied by means of numerical analysis.

3.1. Equal factor intensities in production

Consider first the case in which both industries have the same factor intensities, i.e., both are equally capital and labor intensive. Under this assumption, the price changes derived from a model with heterogeneous households are identical to those derived from a single household model.

Proposition 1. *Assume both sectors have the same factor intensities, i.e., $\gamma_K = \gamma_L$. Then, $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ are independent of household characteristics and depend only on production parameters.*

Proof. If $\gamma_K = \gamma_L$, then $A = B = \gamma_K C$. It then follows from (15a)–(15c) that all terms containing household characteristics in the expressions for $\hat{p}_Y$, $\hat{w}$ and $\hat{r}$ cancel out. □

Proposition 1 implies that in the case of equal factor intensities across industries, price changes derived from a single household model with homothetic preferences are sufficient to determine incidence of an environmental tax, even in an economy with different household types. Intuitively, as long as factor intensities are equal, changes in demands for X and Y do not affect *relative* demands for capital and labor thus implying that relative factor prices are unaffected. Factor price changes are thus determined by the "first-order" response of firms alone, as accounting for "first-order" household behavioral responses in combination with "first-order" firm responses would capture a second-order effect. The sign of factor price changes therefore depends only on production characteristics. Incidence remains in general undetermined, since it depends on how these price changes affect individual households, as determined by their income and expenditure shares.

3.2. Heterogeneous households with homothetic preferences

How does household heterogeneity affect equilibrium factor and output price changes following a change in the pollution tax? We focus on the conditions under which household heterogeneity can qualitatively reverse price changes as compared to a single-household model. These cases thus illustrate conditions which would give rise to significantly biased incidence results when using—as is often done in the literature—a simplified general equilibrium that abstracts from household heterogeneity. To provide a clear intuition, we first restrict our attention to the case with homothetic preferences.

For homothetic preferences, the heterogeneity of households can be described by the households' population distribution of the three following household characteristics: (i) expenditure shares α^h, (ii) income shares θ_L^h, and (iii) elasticities of substitution in utility σ^h.[13] Accordingly, we can summarize household heterogeneity by the following two quantities. First, we measure the degree in which expenditure and income patterns are correlated. To this end, we define the covariance between the expenditure share of the clean good and the labor income share as:

$$cov(\alpha^h, \theta_L^h) \equiv \sum_h (\alpha^h - \gamma) M^h (\theta_L^h - \theta_L) .$$

The covariance is, for example, positive if households who earn an above average share of their income from labor (i.e., $\theta_L^h > \theta_L$) spend an above average share of their income on the clean good (i.e., $\alpha^h > \gamma$).

Second, we quantify the interaction between expenditure shares α^h and substitution elasticities σ^h by defining the effective elasticity of substitution between clean and dirty goods in utility as:

$$\rho \equiv \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \left(\frac{\alpha^h}{\gamma} (\sigma^h - 1) + 1 \right) .$$

[13] Note that, for given ξ^h, a given θ_L^h uniquely determines θ_K^h.

ρ can be interpreted as a generalized weighted average of the σ^h's.[14]

Proposition 2 proves that the two quantities $cov(\alpha^h, \theta_L^h)$ and ρ are indeed sufficient to fully characterize the impact of household heterogeneity on equilibrium prices and the level of pollution. For homothetic preferences, the system of equations (15a)–(15c) characterizing price changes in the general case simplifies to the following expressions, where the expression for $\hat{w}$ has been omitted due to its simple relationship to $\hat{r}$ (see Appendix C.1 for the derivation):

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XK} - \theta_{YK}\theta_{XL})\theta_{YZ}}{D_H}\left[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho\right]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \quad (16a)$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_H}\left[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)\rho\right]\hat{\tau}_Z\,, \quad (16b)$$

where $A_H \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ})$, $B_H \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ})$, $C_H \equiv \beta_K + \beta_L + \theta_{YZ}$, $D_H \equiv C_H\sigma_X + A_H(\theta_{XK}\theta_{YL}(e_{KL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{KK} - e_{ZK})) - B_H(\theta_{XK}\theta_{YL}(e_{LL} - e_{ZL}) - \theta_{XL}\theta_{YK}(e_{LK} - e_{ZK})) - (\gamma_K - \gamma_L)\rho(\theta_{XK}\theta_{YL} - \theta_{XL}\theta_{YK}) - (\gamma_K - \gamma_L)\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}$. Proposition 2 then follows directly:

Proposition 2. *If preferences are homothetic, the impact of household heterogeneity on output and factor price changes in equilibrium only depends on two quantities describing individual households' characteristics: (i) the covariance between the expenditure share of the clean good and the labor income share, $cov(\alpha^h, \theta_L^h)$, and (ii) the effective elasticity of substitution between clean and dirty goods in utility, ρ.*

Proof. Equations (16a)–(16b). □

Using the quantities $cov(\alpha^h, \theta_L^h)$ and ρ, we are now in a position to investigate one key question of the paper: under what conditions are price and pollution changes from an economy populated by heterogeneous households with homothetic preferences identical to those derived from an economy with a single representative household? The next two propositions describe conditions in terms of household preferences and income patterns under which models with and without household heterogeneity yield identical equilibrium outcomes.

Proposition 3. *Assume homothetic preferences and (i) identical expenditure shares ($\alpha^h = \gamma$, $\forall h$) or (ii) identical income shares ($\theta_L^h = \theta_L$, $\forall h$). Then: output and factor price changes are identical to those for a single household characterised by homothetic preferences, clean good expenditure share γ, and elasticity of substitution between clean and dirty goods in utility equal to the effective elasticity ρ.*

Proof. Either of the above assumptions (i) and (ii) implies $cov(\alpha^h, \theta_L^h) = 0$. From equations (16a)–(16b) it is then easy to see that price changes are identical to those derived for an economy with a single consumer with homothetic preferences, clean good expenditure share γ, and elasticity of substitution in utility ρ. □

It follows that in the case with homothetic preferences and either identical expenditure shares or identical income shares (or both), households behave in the aggregate as a single representative household characterized by an elasticity of substitution in utility given by ρ. In the case with identical expenditure shares, the effective elasticity is equal to the weighted average of the individual households' substitution elasticities: $\rho = \frac{1}{\sum_h M^h}\sum_h M^h\sigma^h$. The resulting aggregate behavior is thus completely independent of patterns of income from capital and labor, and does not depend on the number of households. This result, however, breaks down if households have identical income shares but exhibit heterogeneity on the expenditure side. In the latter case, the value of ρ depends on the interaction between expenditure shares α^h and the substitution elasticities of individual households σ^h: if households with an above average expenditure

[14] To see this, consider the case with equal expenditure shares across households, i.e. $\alpha^h = \gamma$, $\forall h$. Then, $\rho = \sum_h M^h\sigma^h / \sum_h M^h$.

share on the dirty good have higher substitution elasticities, the single representative household responds in a more price-elastic manner as compared to a case with the same σ^h's but α^h's that are identical across households.

Proposition 3 motivates the definition of ρ as well as its interpretation as the "effective" elasticity of substitution between clean and dirty goods: when $cov(\alpha^h, \theta_L^h) = 0$—that is when either the households are identical on the expenditure or the income side (or both)—then in the aggregate, households *effectively* behave like a single household with substitution elasticity ρ. As will become clear below, this aggregation result does not hold in the more general case for $cov(\alpha^h, \theta_L^h) \neq 0$. While Proposition 3 describes the conditions for household heterogeneity which allow for consumer aggregation, it is clear that in reality consumers differ in ways which would violate these conditions. A central question for our purpose of incidence analysis is then to investigate to what extent household heterogeneity can reverse output and factor price changes, hence giving rise to qualitatively different outcomes on the uses and sources side of income for heterogeneous households.

Proposition 4. *Assume different factor intensities (i.e., $\gamma_K \neq \gamma_L$) and correlated income and consumption patterns (i.e., $cov(\alpha^h, \theta_L^h) \neq 0$). Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1, \forall h$). Then, for any observed consumption and production decisions before the tax change, there exist production elasticities (i.e., σ_X and e_{ij}) such that the relative burden on factors of production is opposite compared to the model with a single consumer, coupled to the same production side data.*

Proof. See Appendix C.2. □

Proposition 4 suggests that for a truthful portrayal of incidence impacts among heterogeneous households it is pivotal to consider the impact of household heterogeneity on equilibrium outcomes. It proves that, in the presence of heterogeneous households, the sources of income impacts from a pollution tax not only differ quantitatively but can yield qualitatively different predictions when relying on factor price changes derived from a single-household general equilibrium model. Importantly, the possibility of reversed factor price changes does not depend on a particular distribution of households' characteristics as long as the covariance between income and expenditure patterns is non-zero. It seems to be indisputable that $cov(\alpha^h, \theta_L^h) \neq 0$ is the most relevant case which describes reality.

To further illustrate the range of (differing) equilibrium outcomes which depend on the nature and degree of household heterogeneity, we provide an example for a special case of our simple economy. Consider unit-elastic preferences ($\sigma^h = 1$) and Leontief production ($\sigma_X = e_{ij} = 0$). Under these assumptions, equations (16a) and (16b) for price changes can be written as:

$$\hat{p}_Y = -\frac{cov(\alpha^h, \theta_L^h)}{D_{H,1}\gamma p_Y Y}(\gamma_K - \gamma_L)\theta_{YZ}\hat{\tau}_Z \tag{17a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,1}}(\gamma_K - \gamma_L)\hat{\tau}_Z\,, \tag{17b}$$

where $D_{H,1} \equiv (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{cov(\alpha^h,\theta_L^h)}{\gamma p_Y Y}$. Proposition 5 then follows directly:

Proposition 5. *Assume homothetic, unit-elastic preferences (i.e., $\sigma^h = 1$), Leontief technologies in clean and dirty good production (i.e., $\sigma_X = e_{ij} = 0$), and that the dirty sector is relatively capital-intensive (i.e., $\gamma_K > \gamma_L$). Then, the following holds:*[15]

[15]Note that for the case where the dirty sector is relatively labor-intensive (i.e., $\gamma_K < \gamma_L$), the sign of all the results in Proposition 5 is the opposite.

(i) if consumers are identical on the sources or uses side of income, or both: $\hat{p}_Y = 0$, $\hat{w} > 0$, *and* $\hat{r} < 0$.

(ii) If labor ownership and clean good consumption have a negative covariance, then $\hat{p}_Y > 0$, $\hat{w} > 0$ *and* $\hat{r} < 0$.

(iii) If labor ownership and clean good consumption have a positive covariance, then $\hat{p}_Y < 0$, $\hat{w} > 0$, $\hat{r} < 0$ *if the covariance is* low *(i.e.,* $D_{H,1} > 0$*), and* $\hat{p}_Y > 0$, $\hat{w} < 0$, $\hat{r} > 0$ *if the covariance is* high *(i.e.,* $D_{H,1} < 0$*).*

Proof. Equations (17a)–(17b). □

Proposition 5 illustrates that, following a change in the pollution tax, widely differing equilibrium outcomes are possible and depend on the type and degree of household heterogeneity. Depending on assumptions about heterogeneity of households' expenditure and income patterns, one can generate almost any combination of $\hat{p}_Y \gtreqless 0$, $\hat{w} \gtreqless 0$, $\hat{r} \gtreqless 0$. This suggests that even in a simple stylized general equilibrium model as ours, the incidence results of a pollution tax change can bring about quite different qualitative conclusions as far as uses and sources of income side impacts across heterogeneous consumers are concerned.

Note that the results in Proposition 5 hold under the assumption of unit-elastic preferences. One can easily show on the other hand that for a model with a single household and Leontief production, $\hat{p}_Y = 0$. Hence, Proposition 5 provides an example of a case in which patterns of output and factor price changes derived from an economy with heterogeneous households cannot be generated in an economy with the same production characteristics, coupled to any single representative consumer with homothetic preferences. This argument provides additional support for our hypothesis that it is pivotal to consistently integrate household heterogeneity in general equilibrium models when assessing the incidence of environmental taxes across different household groups.

3.3. Identical households with non-homothetic preferences

Our results have so far proven that household heterogeneity can have a qualitative result on the market equilibrium following an increase in a pollution tax, with implications for incidence. We now abstract from household heterogeneity in order to focus on the effect of non-homothetic preferences on the market equilibrium. To achieve this, we assume that households are identical.

As the following special case illustrates, accounting for non-homothetic preferences can also qualitatively effect the market equilibrium. Assume that all cross-price elasticities have the same positive value c: $\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$. Price changes are then of the following form:

$$\hat{p}_Y = -\frac{\theta_{XK}\theta_{XL}\gamma\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)^2(E_{Y,M} - E_{X,M})]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \tag{18a}$$

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{ID}}[(\gamma_K - \gamma_L)(E_{Y,M} - E_{X,M})(1-\gamma)]\hat{\tau}_Z\,, \tag{18b}$$

where $E^h_{X,M} \equiv E_{X,M}$ and $E^h_{Y,M} \equiv E_{Y,M}$ $\forall h$, $D_{ID} \equiv C_{ID} + A_{ID}\theta_{XL} + B_{ID}\theta_{XK} + (\gamma_K - \gamma_L)^2\theta_{XK}\theta_{XL}\frac{\gamma}{1-\gamma}$, $A_{ID} \equiv \gamma_L\beta_K + \gamma_K(\beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $B_{ID} \equiv \gamma_K\beta_L + \gamma_L(\beta_K + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y})$, $C_{ID} \equiv \beta_K + \beta_L + \theta_{YZ} + (E_{X,M} - E_{Y,M})\frac{\tau_Z Z}{p_X X + p_Y Y}$.

In order to determine the sign of the price changes, we define the following *Condition 1*: $D_{ID} > 0$. Condition 1 holds if the expenditure share on the clean good increase with income ($E_{X,M} > E_{Y,M}$). It also holds when the clean good expenditure share decreases with income ($E_{Y,M} > E_{X,M}$), but the difference between the income elasticities is not too large. We can then prove that a wide range of possible combinations of output and factor price changes are possible in this special case, depending on the parameters describing the non-homothetic preferences.

Proposition 6. *Assume identical households and equal cross-price elasticities ($\sigma^h = \sigma_X = e_{KL} = e_{KZ} = e_{LZ} \equiv c > 0$). Then, the following holds:*

(i) If preferences are homothetic, then $\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z$, and $\hat{w} = \hat{r} = 0$.

(ii) Assume that the dirty sector is relatively capital-intensive (i.e. $\gamma_K > \gamma_L$).[16]

(a) If Condition 1 holds, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$.

(b) If Condition 1 does not hold, then for $E_{Y,M} > E_{X,M}$: $\hat{p}_Y > \theta_{YZ}\hat{\tau}_Z$, $\hat{w} < 0$ and $\hat{r} > 0$, and for $E_{Y,M} < E_{X,M}$: $\hat{p}_Y < \theta_{YZ}\hat{\tau}_Z$, $\hat{w} > 0$ and $\hat{r} < 0$.

Proof. Equations (18a)–(18b). For (i): use $E_{Y,M} = E_{X,M}$. □

We have therefore illustrated that there exist cases where the relative burden on factors of production depends crucially on the interaction between production characteristics and the income elasticities of demand for the clean and the dirty goods. It follows that, by extending the Fullerton & Heutel (2007) model to incorporate household heterogeneity and non-homothetic preferences, we have added two dimensions that can both qualitatively alter the economy's reaction to an exogenous increase in the pollution tax. Both features are therefore in general significant for incidence.

4. Numerical analysis

We now assign plausible values to parameters to numerically examine the theoretical effects derived above. Given a version of our model which is calibrated to current data for the U.S. economy and available estimates from the literature, we seek to answer the following questions. First, how severe is the aggregation bias? In other words, how plausible is the assumption that the overall effect of a pollution tax on factor and output prices with heterogeneous consumers is the same as in the aggregate model with a single representative household? Employing the empirically calibrated model as a starting point, we explore how sensitive the answer is with respect to the degree of household heterogeneity, the structure of preferences, and the interplay with production parameters. In doing so, we also focus on the implications of the aggregation bias for incidence analysis, i.e., does using a simplified aggregate model produces qualitatively identical results in terms of a progressive or regressive incidence pattern. Second, we use the calibrated model to assess the incidence of a tax on carbon dioxide (CO_2) emissions in the U.S. Is a U.S. carbon tax regressive? Given the considerable uncertainty surrounding the parametrization of firm and household behaviour, we explore the robustness of the incidence result through sensitivity analysis by varying household and production characteristics and by identifying the relative importance of uses and sources effects of income.

4.1. Data and Calibration

In order to situate our study in the context of the literature, we calibrate our model to data used previously for a two-sector general equilibrium environmental tax incidence analysis. For this purpose, we chose the production and consumption data of Fullerton & Heutel (2010). They aggregate a data set of the U.S. economy to a '"dirty" and a "clean" sector, where the dirty sector comprises the highly CO_2-intensive industries (electricity generation, transportation and petroleum refining). As in Fullerton & Heutel (2010)

[16]Note that for the case with $\gamma_K < \gamma_L$, the results for $\hat{w}$ and $\hat{r}$ are of opposite signs to the analogous expressions in Proposition 6 (ii). The results for $\hat{p}_Y$ remain unchanged, as long as factor intensities differ ($\gamma_K \neq \gamma_L$).

Table 1: Household expenditures on clean and dirty goods and household income by source for annual expenditure deciles (in % of total expenditure for a given household group)

Expenditure	Income sources		Expenditure by commodity	
decile	Labor	Capital	Clean	Dirty
1	42.8	13.5	85.5	14.5
2	74.5	13.8	84.8	15.2
3	86.3	16.2	85.4	14.6
4	103.5	18.0	86.1	13.9
5	108.8	20.4	86.8	13.2
6	114.4	29.4	87.7	12.3
7	118.8	31.2	88.5	11.5
8	120.0	38.4	89.2	10.8
9	124.6	45.1	90.7	9.3
10	93.4	54.7	94.1	5.9

Notes: Household data is based on "Consumer Expenditure Survey" (CEX) data as shown in Fullerton & Heutel (2010).

we assume an initial and pre-existing carbon tax of \$15 per metric ton of CO_2. Our comparative-static analysis considers a 100% increase in the carbon tax.

All prices in the benchmark are normalised to one, and quantities are normalised such that the total value of the economy is equal to one, i.e., $p_X X + p_Y Y = 1$. Calibrated values for outputs and inputs are as follows: $X = 0.929$, $L_X = 0.579$, $L_Y = 0.029$, $K_X = 0.350$, $K_Y = 0.037$, and $Z = 0.005$. Households are grouped by annual expenditure deciles,[17] and data for expenditures by clean and dirty goods as well as capital and labor income are shown in Table 1. Note that we abstract from government transfers.[18]

Incorporating heterogeneous households in a calibrated general equilibrium model of the U.S. economy requires that—at the aggregate level—data describing household consumption and income are consistent with the production data on output by sector and aggregate, economy-wide factor income. To reconcile data sources, we adjust the household data to be consistent with aggregate production data while preserving the relative characteristics of household expenditures across household groups (expenditure deciles). More specifically, data adjustments for each expenditure decile are as follows. First, we scale income to mach expenditure while keeping fixed the decile's capital-to-labor ratio. Second, we scale the capital ownership of all deciles by a common factor in order for aggregate household income by factor to match production side data, whilst preserving the relative capital ownership amongst deciles. Third, we perform an analogous scaling for consumption of the dirty good. This procedure yields a data with consistent household and production data which is used to calibrate the general equilibrium model.

[17] It is well-known in the literature on tax incidence that absent a fully dynamic framework, categorizing households by expenditure deciles is a better proxy for lifetime income as compared to a ranking based on annual income deciles (see, for example, Poterba1991; Fullerton & Heutel, 2010).

[18] Incorporating government transfers in the model would require including other taxes in our analysis in order to finance these transfer payments. As tax revenue would change following a change of the pollution tax, this would imply that other taxes also would have to adjusted simultaneously to ensure that the government budget is balanced. We deliberately refrain from including this aspect in our analysis as it would add significant complexity and push the paper more towards the issue of (green) tax reform. In abstracting from government transfers, one should bear in mind that they are to a large extent indexed to inflation, thus effectively protecting poorer households—which receive a higher proportion of transfers—from increases in prices for final consumption goods. Adding government transfers to the picture would thus probably make our incidence results look somewhat less regressive.

For our central case parametrization of production elasticities we follow Fullerton & Heutel (2010) assuming $\sigma_X = 1$, $e_{KL} = 0.1$, $e_{KZ} = 0.2$, and $e_{LZ} = -0.1$. This implies that capital is a better substitute for pollution than labor. For the single household model, Fullerton & Heutel (2010) assume that the elasticity of substitution between the clean and the dirty good in utility is unity, and that preferences are homothetic. Our central case is based on analogous assumptions for each household group, i.e., $\sigma^h = 1$ and $E^h_{X,M} = E^h_{Y,M} = 1$, $\forall h$. Note that while these parameter choices reflect central case assumptions, we perform extensive sensitivity analysis to check for the size of the aggregation bias and the incidence patterns from increases in the pollution tax.

4.2. Size of the Aggregation Bias and Implications for Incidence Analysis

We now measure the aggregation bias introduced by modeling an economy comprising heterogeneous households and potentially non-homothetic preferences as an economy with a single representative household and with preferences that are restricted to being homothetic.

From the theoretical analysis above we know that heterogeneous households and non-homothetic preferences can have a significant effect on price changes following an increase in the pollution tax. In this section we compute price changes for our model calibrated to the real world data from Table 1, where expenditure deciles are heterogeneous. By comparing these price changes with price changes derived from a model calibrated to the same aggregate data, but with a single representative household, we then deduce the magnitude of the aggregation bias on prices.

Biased price changes translate into biased welfare results. We therefore quantify the bias introduced by the simplified representative household approach with homothetic preferences on the distribution of welfare impacts across expenditure deciles. To perform this comparison we define the "Welfare Aggregation Bias", Γ, as follows:

$$\Gamma = \Omega^{-1} \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left| \Phi^h - \Phi^h_{\text{Homothetic, Aggregate Household Model}} \right| , \tag{19}$$

where $h \in \{Expenditure\ Deciles\}$, Φ^h is the household level welfare impact (determined by Equation 14), and $\Omega \equiv \sum_h \frac{M^h}{\sum_{h'} M^{h'}} \left|\Phi^h\right|$ represents the average welfare impact. $\Phi^h_{\text{Homothetic, Aggregate Household Model}}$ is also determined via Equation 14 for the same expenditure deciles, based on their expenditure and income shares, but price changes are now derived from a single household model calibrated to the same production data, but with a single household representing aggregate demand. This household is assumed to have homothetic preferences, and an elasticity of substitution in utility between clean and dirty consumption given by the expenditure-weighted average of the elasticities of the individual deciles.[19]

Γ yields a measure of the average difference in welfare impacts derived under the consistent approach and the generally biased representative household approach. If $\Gamma = 0$, then the welfare results derived under the two approaches are identical. If on the other hand Γ is large, then the bias on the household level welfare results is large compared to the average welfare impacts, i.e., the pattern of incidence is significantly affected by the aggregation bias.

Table 2 reports on the price changes for our model with heterogeneous expenditure deciles, as well as the welfare aggregation bias. Price changes for the model with a representative household with homothetic preferences are also reported below the table. We consider a range of alternative cases which are motivated by the considerable uncertainty surrounding both the household survey data as well as the specification of household parameters.

[19] i.e., $\sigma_{\text{Homothetic, Aggregate Household Model}} = \frac{1}{\sum_{h'} M^{h'}} \sum_h M^h \sigma^h$.

Table 2: Price changes and welfare aggregation biases (in %) for alternative parameter assumptions

	cov_{Low}						cov_{High}					
	ρ_{Low}			ρ_{High}			ρ_{Low}			ρ_{High}		
	$\hat{p}_Y$	$\hat{r}$	Γ	$\hat{p}_Y$	$\hat{r}$	Γ	$\hat{p}_Y$	$\hat{r}$	Γ	$\hat{p}_Y$	$\hat{r}$	Γ
Homothetic preferences												
$\sigma_X = 1.5$	7.21	-0.07	1.40	7.20	-0.09	1.43	7.22	-0.05	3.23	7.20	-0.11	3.39
$\sigma_X = 1$	7.20	-0.11	2.21	7.20	-0.13	2.27	7.21	-0.08	4.96	7.19	-0.16	5.53
$\sigma_X = 0.5$	7.17	-0.21	5.14	7.16	-0.26	5.45	7.19	-0.15	10.64	7.14	-0.32	12.38
$e_{K/LZ} = \pm 0.5$	7.26	0.13	1.61	7.26	0.10	1.69	7.27	0.15	3.88	7.25	0.07	4.27
$e_{K/LZ} = \mp 0.5$	7.07	-0.57	5.36	7.07	-0.60	5.07	7.08	-0.54	9.66	7.06	-0.62	10.19
Non-homothetic preferences												
$E_{Y,M^{Rich}} > E^{Poor}_{Y,M}$	7.20	-0.10	2.36	7.2	-0.13	2.11	7.21	-0.07	5.41	7.19	-0.16	4.81
$E_{Y,M^{Rich}} < E^{Poor}_{Y,M}$	7.20	-0.11	2.06	7.2	-0.13	2.42	7.21	-0.08	4.50	7.18	-0.16	5.85

Notes: Cases shown in rows above vary one parameter at a time while keeping all other parameters at their respective central case values. Price changes for the corresponding single-household model with homothetic preferences are the following ($\%\hat{p}_Y$, $\%\hat{r}$): $\sigma_X = 1.5$: (7.21, -0.08); $\sigma_X = 1$: (7.20, -0.12); $\sigma_X = 0.5$: (7.17, -0.23); $e_{K/LZ} = \pm 0.5$: (7.26, 0.11); $e_{K/LZ} = \mp 0.5$: (7.07, -0.58).

In addition to our central case assumptions which are based on observed data for the U.S. economy (see Section 4.1), we consider cases in which the distributions of expenditure shares by good and income shares by source are varied across households relative to the base case: cov_{low} and cov_{high} represent cases where the covariance measure is respectively halved and doubled relative to the central case, representing cases where there is respectively less and more heterogeneity across households. Different assumptions with respect to higher-order properties of households' utility functions are also considered, by introducing heterogeneity in the price and income elasticities of demand across households. In the case labeled ρ_{low} we assume that poorer households (i.e., lower expenditure deciles) are described by a lower elasticity of substitution between clean and dirty goods relative to the richer households, whilst for ρ_{high} we assume the opposite.[20] We also display two cases with non-homothetic preferences: the first ($E^{Rich}_{Y,M} > E^{Poor}_{Y,M}$) where poorer households are characterised by dirty good expenditure shares that increase more slowly with income compared to richer households, and the second ($E^{Rich}_{Y,M} < E^{Poor}_{Y,M}$) where the opposite holds.

To further explore the significance of the aggregation bias, we interact different cases regarding household characteristics with alternative assumptions about the production side, i.e., cases which differ with respect to the substitutability between capital and labor in the clean and between capital, labor, and pollution in the dirty sector.

It should be noted that in Table 2 we do not report results for the base case for income and expenditure shares (cov_{Base}) since, as one would expect, the results simply represent intermediate outcomes between the cov_{Low} and cov_{High} cases. We furthermore do not report the results for our central case assumptions for substitution elasticities, i.e. for $\sigma^h = 1, \forall h$ (which would correspond to ρ_{Base}), since in this case we find that the aggregation bias is virtually zero. There is an intuitive reason for this result: when substitution elasticities are identical across households, for a given increase in the price of the dirty good, households

[20]For both ρ_{low} and ρ_{high}, we chose the σ^h's such that the elasticity of substitution between the clean and dirty consumption goods in the utility function of the single, representative consumer equals unity.

Table 3: Selected cases for which welfare aggregation bias is "large", i.e. incidence results across household groups differ qualitatively due to the aggregation bias

Expenditure	Case1		Case 2		Case 3	
decile	$\Phi^h_{Heterogeneous}$	$\Phi^h_{Aggregate}$	$\Phi^h_{Heterogeneous}$	$\Phi^h_{Aggregate}$	$\Phi^h_{Heterogeneous}$	$\Phi^h_{Aggregate}$
1	-0.15	-0.21	0.16	0.48	0.56	-0.67
2	0.21	-0.36	3.06	5.95	5.35	-6.03
3	0.23	-0.32	3.01	5.83	5.23	-5.89
4	0.31	-0.30	3.37	6.48	5.77	-6.49
5	0.29	-0.25	3.05	5.84	5.19	-5.83
6	0.12	-0.15	1.45	2.79	2.50	-2.81
7	0.14	-0.10	1.34	2.56	2.26	-2.54
8	0.01	-0.02	0.16	0.30	0.27	-0.31
9	-0.03	0.09	-0.63	-1.23	-1.11	1.26
10	-0.36	0.40	-4.16	-8.02	-7.15	8.04

Notes: Cases are defined as follows. Case 1: $\sigma_X = 0$, $\sigma^h = 2$, for $h = 1, \ldots, 5$, $\sigma^h = 0$, for $h = 6, \ldots, 10$, $e_{KL} = 0.1$, $e_{KZ} = 0.5$, and $e_{LZ} = 0.4$. Case 2: Leontief production, σ^h as for ρ_{low}, $E^h_Y = 2$, for $h = 1, \ldots, 7$, and $E^h_Y = 0$, for $h = 8, \ldots, 10$. Case 3 corresponds to the case in Proposition 4: $\sigma_X = 0$, $\sigma^h = 1$, $e_{KL} = -0.145$, $e_{KZ} = e_{LZ} = 0$, and $E^h_Y = 1$.

all substitute the same percentage of dirty good consumption with clean consumption. Abstracting from changes in income, it then follows that the aggregate change in consumption is the same as for a representative household with the same substitution elasticity. The numerical results show that in this case other effects that may depend on household heterogeneity are not of particular significance.

An analysis of the results reported in Table 2 delivers the following insights. First of all, the aggregation bias on the returns to capital is larger than on the price of the dirty good. The reason is that a given bias on the returns to capital is necessarily coupled to a bias on the returns to labor of the opposite sign. By the zero-profit condition both contribute to the bias on the price of the dirty good, which therefore assumes an intermediate value. As a percentage of the price changes, the bias on returns to capital is much larger than that on the price of the dirty good, since returns to factors are much less affected by the pollution tax compared to the price of the dirty good.

Secondly, we find that, for a given covariance between income and expenditure patterns, both the price of the dirty good and the returns to capital are decreasing in the effective elasticity ρ. Intuitively, the reaction of aggregate demand to an increase of the price of the dirty good is disproportionately affected by the households that consume the dirty good more intensively. Under the case ρ_{High}, these households' demand is more price elastic than the average demand, hence aggregate demand will react more elastically to an increase in the price of the dirty good as compared to the single, representative consumer. This in turn depresses demand for the dirty good more, leading to both a decrease in its price and the returns to the factor which is used intensively in the dirty industry, i.e. capital. An analogous explanation holds true for the ρ_{Low} case.

Third, the changes in the price of the dirty good and the return to capital are increasing in the absolute value of the covariance for ρ_{Low}, and decreasing in the absolute value of the covariance for ρ_{High}. Intuitively, a higher covariance means that households consuming an above-average share of the dirty good consume even more. This in turn magnifies the impact of the value of the effective elasticity (ρ_{Low} or ρ_{High}) on the determination of equilibrium price changes.

Fourth, Table 2 suggests a relatively minor impact of non-homothetic preferences on the aggregation bias: comparison with the case $\sigma_X = 1$ (which differs only in that it assumes homothetic preferences), shows that the bias on price changes and therefore on welfare for the cases reported is mainly driven by household heterogeneity rather than non-homothetic preferences.

In summary, we find that the aggregation bias for the empirically motivated cases shown in Table 2 is non-negligible. The magnitude of the welfare aggregation bias relative to the average welfare impact can be as large as 10%. The price aggregation bias in terms of factor returns, however, can be much larger. Heterogeneity in the elasticities of substitution in utility amongst expenditure deciles is necessary in order to observe a significant aggregation bias, and a higher degree of heterogeneity in income and expenditure patterns magnifies this bias. The non-homotheticity of household preferences, on the other hand, does not have a significant impact on incidence.

Lastly, Table 3 presents selected cases for which the aggregation bias is large enough to cause incidence patterns to be qualitatively different to the point that, for some households, the sign of the welfare impact is even reversed. The wide variation in welfare impacts across deciles in these cases emphasizes the fact that within the range of possible values of household parameters there exist equilibria in which the economy is particularly sensitive to an increase in the pollution tax. Although these cases are relatively "distant" to our central case assumptions, they illustrate the pitfalls in assessing distributional impacts of an environmental tax in an overly simplistic model with a single, representative consumer.

4.3. Applying the Heterogeneous Household Model: Is a U.S. Carbon Tax Regressive?

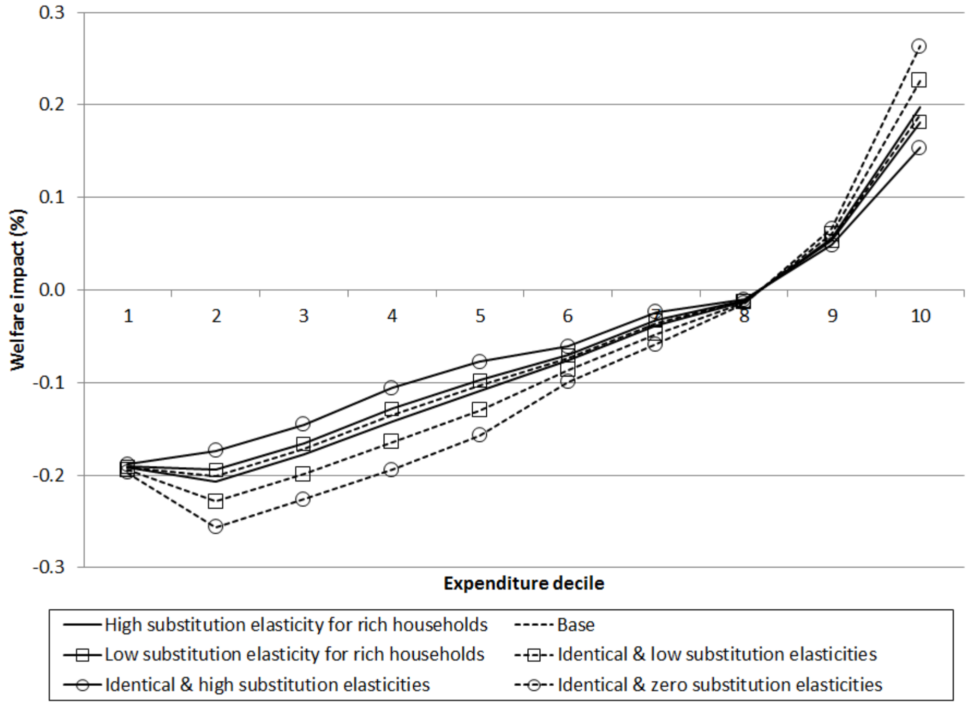
We now use our calibrated model to assess the incidence of a U.S. carbon tax. We explore the robustness of the incidence result through "piecemeal" sensitivity analysis by varying household and production elasticities. For each case, we identifying the relative importance of uses and sources effects of income. Figure 1a displays welfare impacts for a range of cases which vary household characteristics around the base case. We assume different values for σ^h, the elasticity of substitution in utility between clean and dirty goods. For "low" and "high" substitution cases, we set σ^h for different household groups as in ρ_{Low} and ρ_{High}, respectively. For cases with identical "zero", "low", and "high" substitution elasticities the following values are assumed, respectively: $\sigma^h = 0$, $\sigma^h = 0.5$, and $\sigma^h = 1.5$, $\forall h$. In all cases, household expenditure and income shares are left unchanged.

From Figure 1a it is evident that a carbon tax is regressive in the base case, and that this result is robust to varying household characteristics. Even if households are more able to substitute away from the taxed dirty good, as reflected by high σ^h's, the carbon tax puts disproportionately large burdens on households in lower expenditure deciles. The incidence is slightly more regressive for low values of σ^h as compared to cases with high values for σ^h. This is driven by the fact that for relatively low σ^h's, the burden from higher prices for the dirty good is borne to a larger extent by consumers, hence falling more heavily on those household groups that spend a relatively large fraction of their income on the dirty good. At the same time, as consumers are less able to substitute away from the dirty good, the reduction in the dirty sector output, Y, is relatively smaller, hence the return to capital, the factor used intensively in the production of Y, decreases by less. This explains why richer households with relatively high capital income shares (i.e., deciles 9 and 10) experience relative welfare gains from the carbon tax.

Figure 1b displays welfare impacts for a range of cases which vary production characteristics around base case assumptions. Cases shown vary either the elasticity of substitution between capital and labor in clean production, σ_X (halving and doubling the value from the base case), the substitutability between capital and labor vis-à-vis pollution, or a combination of the two. The case "K better substitute for Z" assumes $e_{KZ} = 0.5$ $e_{LZ} = -0.5$, and the case "L better substitute for Z" assumes $e_{KZ} = -0.5$ $e_{LZ} = 0.5$.

Figure 1: Welfare impacts ($\Phi^h_{Heterogeneous}$) of increased pollution tax across annual expenditure deciles

(a) Alternative assumptions about household characteristics



(b) Alternative assumptions about production characteristics

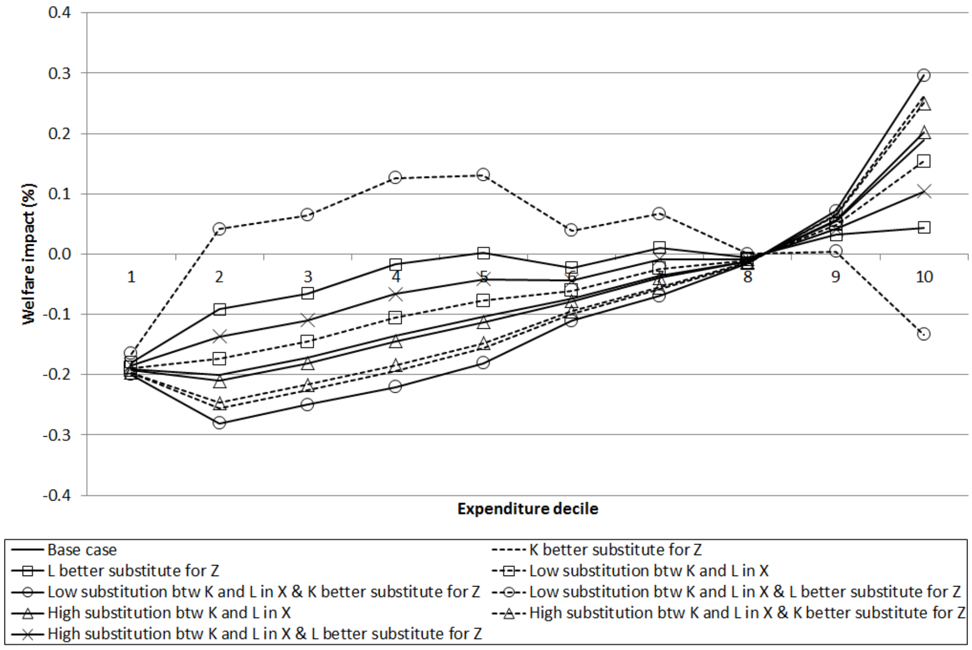


Table 4: Household welfare impacts by expenditure decile (in %) by uses and sources side of income for alternative household characteristics

Expenditure	Uses side	Sources side					
Decile	*all cases*[a]	Central case ($\sigma^h = 1$)	ρ_{low}	ρ_{high}	$\sigma^h = 1.5$	$\sigma^h = .5$	$\sigma^h = 0$
1	-0.19	0.00	0.00	0.00	0.00	0.00	0.00
2	-0.23	0.03	0.02	0.03	0.00	0.05	-0.03
3	-0.20	0.03	0.02	0.03	0.00	0.05	-0.03
4	-0.16	0.03	0.02	0.04	0.00	0.06	-0.03
5	-0.13	0.03	0.02	0.03	0.00	0.05	-0.03
6	-0.09	0.01	0.01	0.02	0.00	0.02	-0.01
7	-0.05	0.01	0.01	0.01	0.00	0.02	-0.01
8	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
9	0.06	-0.01	0.00	-0.01	0.00	-0.01	0.01
10	0.23	-0.04	-0.03	-0.04	0.00	-0.07	0.04

Notes: Cases shown in columns are identical to cases in Figure 1a. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown.

Table 5: Household welfare impacts by expenditure decile (in %) by uses and sources side of income for alternative production characteristics

Expenditure	Uses side	Sources side								
Decile	*all cases*[a]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	-0.19	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.01
2	-0.23	0.03	-0.03	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
3	-0.20	0.03	-0.02	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
4	-0.16	0.03	-0.03	0.14	0.06	-0.05	0.28	0.02	-0.02	0.10
5	-0.13	0.03	-0.03	0.13	0.05	-0.05	0.26	0.02	-0.02	0.09
6	-0.09	0.01	-0.01	0.06	0.02	-0.02	0.12	0.01	-0.01	0.04
7	-0.05	0.01	-0.01	0.06	0.02	-0.02	0.11	0.01	-0.01	0.04
8	-0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00
9	0.06	-0.01	0.01	-0.03	-0.01	0.01	-0.05	0.00	0.00	-0.02
10	0.23	-0.04	0.03	-0.18	-0.07	0.07	-0.35	-0.02	0.02	-0.12

Notes: Cases shown in columns are identical to cases in Figure 1b. [a]Uses side impacts are virtually identical for all the cases, hence only one column is shown. Columns are defined as follows: (1)=central case, (2)=K better substitute for Z ($e_{KZ} = 0.5$ and $e_{LZ} = -0.5$), (3)=L better substitute for Z ($e_{KZ} = -0.5$ and $e_{LZ} = 0.5$), (4)=Low substitutability between K and L in sector X ($\sigma_X = 0.5$), (5)=Low substitutability between K and L in sector X and K better substitute for Z, (6)=Low substitutability between K and L in sector X and L better substitute for Z (7)=High substitutability between K and L in sector X ($\sigma_X = 1.5$), (8)=X more price elastic and K better substitute for Z, (9)=High substitutability between K and L in sector X and L better substitute for Z.

The following insights emerge from Figure 1b. First, while for the majority of cases the carbon tax is shown to be regressive, there is considerable variation in welfare impacts depending on production parameters. Second, the pattern of distributional impacts depends largely on the substitutability of inputs in the production of the dirty good. If capital is a better substitute for pollution than labor, then the carbon tax is regressive, due to the regressivity of both the uses and the sources of income incidence. On the sources of income side, as the burden on factor prices falls on labor rather than capital, poorer households with high labor income shares experience large welfare losses, while richer households with high capital income shares experience larger relative gains. In contrast, the carbon tax is less regressive and can even in some cases be inversely U-shaped if labor is a relatively good substitute for pollution vis-à-vis capital, due to the progressivity of the sources of income incidence when the burden falls on capital rather than on labor. Third, higher values of σ_X imply flatter incidence curves, since this dampens burden on the returns to the factors of production.

For the cases shown in Figure 1, Tables 4 and 5 decompose welfare impacts into uses and sources side of income impacts. For the range of household or production characteristics that we consider, we find that uses side of income effects are markedly regressive and that–for a given household group–there is relatively little variation in the size of uses side impacts. The sources side impacts on the other hand tend to be mostly neutral or progressive, driven by the fact that burdens mostly fall on capital, and are much more sensitive to behavioural parameters as compared to the uses side impacts.[21]

To summarize, we find strong evidence that a U.S. carbon tax would be regressive. Our analysis, however, points to the importance of including sources of income impacts for tax incidence analysis. As we find that sources side effects tend to be progressive, basing incidence analysis on uses of income effects only may overestimate the regressivity of the distributional outcomes and may even lead in some cases to qualitatively false conclusions. On the other hand, our finding that sources side impacts drive most of the variation in welfare impacts also calls for a careful assessment—and model specification—of how both polluting and non-polluting firms respond to a pollution tax and ensuing factor price changes.

5. Conclusions

This paper has studied the general equilibrium incidence of an environmental tax in a simple theoretical model that allows for heterogeneous households, fully general forms of preferences, differential spending and income patterns, differential factor intensities in production, and fully general forms of substitution among inputs of capital, labor and pollution. Given the high interest of policymakers in the distributional impacts of environmental taxation and a large body of academic research on this issue, we have investigated the question to what extent incidence analyses are biased if household heterogeneity is not consistently integrated in the economic model. In other words, how important is the household aggregation bias when the purpose of the economic analysis is precisely to assess the distributional impacts of an environmental tax across groups of households that exhibit considerable heterogeneity?

Our analysis indeed shows that ignoring the household aggregation problem can have severe implications for analyzing the incidence of environmental taxes. In the theoretical part of the paper, we first provide an intuitive way to characterize the degree of household heterogeneity and the impact of heterogeneity on equilibrium outcomes (following a change in the environmental tax). We then provide conditions under which the household aggregation bias is large and incidence results vary substantially and can be fully reversed depending on the distribution of households' expenditure and income shares. We also characterize

[21] Note that the small variation in impacts for the 1^{st} and 8^{th} expenditure deciles reflects that these households have a capital-labor ratio which is similar to the sample's average. Hence, the sources side impacts relative to the average are small for these two deciles.

conditions for which the household aggregation problem is muted. We then assign plausible parameter values to numerically examine the theoretical model. Given a plausibly calibrated version of the model which is based on historic data for the US economy, we investigate the likely size of the aggregation bias in an empirical context. We find that the magnitude of the aggregation bias in terms of welfare is non-negligible and that there exist cases in which the incidence patterns for household income groups differ qualitatively, i.e., the sign of welfare impacts for particular household groups is reversed.

Our analysis and findings suggests that the household aggregation problem can have potentially very important consequences for assessing the incidence of environmental policy. It also suggests that incidence results from previous studies that were based on partial or general equilibrium approaches ignoring the impact of household heterogeneity on equilibrium outcomes may only have captured some but not all of the relevant channels through which the tax burden is determined. Future research on the distributional effects of environmental taxes should be based on sound general equilibrium analyses which consistently incorporate household heterogeneity.

We applied our model with full household heterogeneity to analyze the distributional impacts of a US carbon tax. We find strong support that such a tax would be regressive. This results is robust with respect to significant variations in parameters of firms and households that govern their equilibrium behavior in responses to a change in the carbon tax rate. We find, however, for plausible parameter assumptions cases in which the regressivity is significantly dampened and can even follow an inverse U-shaped pattern if labor is a relatively good substitute for pollution vis-à-vis capital, due to the progressivity of the sources side impacts when the burden falls on capital rather than on labor. In line with previous analyses, we find clear evidence that the uses side impacts are regressive but that the sources side effects tend to be progressive, and can even overturn the regressive uses side impacts.

While this paper provides the first analysis of the household aggregation problem in the context of the incidence and distributional effects of environmental tax policy, the underlying theoretical model is deliberately simple. As has been done in the rich literature that followed the original Harberger (1962) article, the model could be extended in many ways allowing, for example, for imperfect factor mobility, imperfect competition, non-constant returns to scale, international trade in goods and factor, and tax evasion. While this may in some cases change the specific results for distributional outcomes, the household aggregation problem and its implications for the general equilibrium incidence of environmental tax policy are, however, likely to exist even if some of the simplifying assumptions which underly our analysis would be relaxed.

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Appendix A. Derivation of equations (10) and (11)

Consider the household demand functions $X = X(p_X, p_Y, M)$ and $Y = Y(p_X, p_Y, M)$, where the household index h is omitted for simplicity. Define the income elasticities of demand of good X and Y as $E_{X,M} = \frac{M}{X}\frac{\partial X}{\partial M}$ and $E_{Y,M} = \frac{M}{Y}\frac{\partial Y}{\partial M}$, respectively. Let $E_{X,p_X} = -\frac{p_X}{X}\frac{\partial X}{\partial p_X}$ and $E_{Y,p_X} = -\frac{p_X}{Y}\frac{\partial Y}{\partial p_X}$ denote the respective own price elasticities of demand. As shown in Hicks & Allen (1934), at the equilibrium solution the following conditions hold: $E_{X,p_X} = \alpha E_{X,M} + (1-\alpha)\sigma$, $E_{Y,p_X} = \alpha E_{Y,M} - \alpha\sigma$, $E_{X,p_Y} = (1-\alpha)E_{X,M} - (1-\alpha)\sigma$, $E_{Y,p_Y} = (1-\alpha)E_{Y,M} + \alpha\sigma$, where σ is the elasticity of substitution between clean and dirty consumption in utility.

Using these four conditions, changes in household h's demand for good X and Y given changes in the prices of goods and factor prices can be expressed, respectively, as:

$$\begin{aligned}\hat{X}^h &= \frac{1}{X^h}(p_X\partial_{p_X}X^h\hat{p}_X + P_Y\partial_{p_Y}X^h\hat{p}_Y + M^h\partial_{M^h}X^h\hat{M}^h)\\ &= -E^h_{X,p_X}\hat{p}_X - E^h_{X,p_Y}\hat{p}_Y + E^h_{X,M}\hat{M}^h\\ &= -(\alpha E^h_{X,M} + (1-\alpha)\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{X,M} - (1-\alpha)\sigma^h)\hat{p}_Y + E^h_{X,M}\hat{M}^h\,, \end{aligned} \tag{A.1}$$

and

$$\begin{aligned}\hat{Y}^h &= \frac{1}{Y^h}(p_X\partial_{p_X}\hat{p}_X + p_Y\partial_{p_Y}\hat{p}_Y + M^h\partial_{M^h}\hat{M}^h)\\ &= -E^h_{Y,p_X}\hat{p}_X - E^h_{Y,p_Y}\hat{p}_Y + E^h_{Y,M}\hat{M}^h\\ &= -(\alpha E^h_{Y,M} - \alpha\sigma^h)\hat{p}_X - ((1-\alpha)E^h_{Y,M} + \alpha\sigma^h)\hat{p}_Y + E^h_{Y,M}\hat{M}^h\,. \end{aligned} \tag{A.2}$$

Appendix B. Derivation of price and pollution changes in general solution (equations (15a)–(15c))

Subtract (8) from (6) and (9) from (7), to obtain:

$$\hat{p}_X = \theta_{XK}\hat{r} + \theta_{XL}\hat{w} \tag{B.1}$$

$$\hat{p}_Y = \theta_{YK}\hat{r} + \theta_{YL}\hat{w} + \theta_{YZ}\hat{\tau}_Z\,. \tag{B.2}$$

Substitute (12) and (13) into (8) and (9):

$$\sum_h \frac{X^h}{X}\hat{X}^h = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X \tag{B.3}$$

$$\sum_h \frac{Y^h}{Y}\hat{Y}^h = \theta_{YK}\hat{K}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}\,. \tag{B.4}$$

Solve (10) for $\hat{Y}^h$ and insert the result into (B.4). Rearrange to obtain:

$$\begin{aligned}&\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y - \hat{p}_X) + (E^h_{Y,M} - E^h_{X,M})(\alpha^h\hat{p}_X + (1-\alpha^h)\hat{p}_Y - \hat{M}^h)) =\\ &\qquad \sum_h \frac{Y^h}{Y}\hat{X}^h - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \end{aligned} \tag{B.5}$$

From (B.3), insert the following on the right-hand side of the equality: $+0 = \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \sum_h \frac{X^h}{X}\hat{X}^h$ and use the fact that X is chosen to be the numeraire, thus yielding:

$$\begin{aligned}&\frac{1}{Y}\sum_h Y^h(\sigma^h(\hat{p}_Y) + (E^h_{Y,M} - E^h_{X,M})((1-\alpha^h)\hat{p}_Y - \hat{M}^h)) =\\ &\sum_h \frac{M^h}{p_Y Y}(1 - \frac{\alpha^h}{\gamma})\hat{X}^h + \theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y - \theta_{YZ}\hat{Z}\,. \end{aligned} \tag{B.6}$$

Eliminate $\hat{X}^h$ from equation (B.6) by using equation (11), then insert the explicit expression for the budget change $\hat{M}^h$:

$$\begin{aligned}&\hat{p}_Y\delta = \sum_h \phi^h_L\hat{w} + \sum_h \phi^h_K\hat{r} + \sum_h \phi^h_Z\hat{\tau}_Z\\ &+\theta_{XK}\hat{K}_X + \theta_{XL}\hat{L}_X - \theta_{YK}\hat{K}_Y - \theta_{YL}\hat{L}_Y + (\sum_h \phi^h_Z - \theta_{YZ})\hat{Z}\,. \end{aligned} \tag{B.7}$$

Next, solve equations (1) and (2) for $\hat{K}_X$ and $\hat{L}_X$, and insert them into (B.7). Furthermore, insert equation (B.2) to eliminate $\hat{p}_Y$, thus obtaining:

$$\begin{aligned}(\sum_h \phi_Z^h - \theta_{YZ})\hat{Z} = (\delta\theta_{YK} - \sum_h \phi_K^h)\hat{r} + (\delta\theta_{YL} - \sum_h \phi_L^h)\hat{w} + (\delta\theta_{YZ} - \sum_h \phi_Z^h)\hat{\tau}_Z \\ +\hat{K}_Y(\theta_{XK}\gamma_K + \theta_{YK}) + \hat{L}_Y(\theta_{XL}\gamma_L + \theta_{YL})\,.\end{aligned} \tag{B.8}$$

Solve equations (4) and (5) for $\hat{K}_Y$ and $\hat{L}_Y$, and insert them into equation (B.8). This yields:

$$\begin{aligned}-C\hat{Z} = (-\sum_h \phi_K^h + \theta_{YK}(\delta + \beta_K(e_{KK} - e_{ZK}) + \beta_L(e_{LK} - e_{ZK})))\hat{r} \\ +(-\sum_h \phi_L^h + \theta_{YL}(\delta + \beta_K(e_{KL} - e_{ZL}) + \beta_L(e_{LL} - e_{ZL})))\hat{w} \\ +(-\sum_h \phi_Z^h + \theta_{YZ}(\delta + \beta_K(e_{KZ} - e_{ZZ}) + \beta_L(e_{LZ} - e_{ZZ})))\hat{\tau}_Z\,.\end{aligned} \tag{B.9}$$

Next eliminate $\hat{Z}$. To achieve this, substitute equations (1) and (2) into equation (3), obtaining:

$$-\gamma_K\hat{K}_Y + \gamma_L\hat{L}_Y = \sigma_X(\hat{w} - \hat{r})\,. \tag{B.10}$$

Substituting equations (4) and (5) into (4) and (B.10) yields:

$$\begin{aligned}\sigma_X(\hat{w} - \hat{r}) = (\gamma_L - \gamma_K)\hat{Z} + \theta_{YK}(\gamma_L(e_{LK} - e_{ZK})\hat{r} - \gamma_K(e_{KK} - e_{ZK}))\hat{r} \\ \theta_{YL}(\gamma_L(e_{LL} - e_{ZL})\hat{w} - \gamma_K(e_{KL} - e_{ZL}))\hat{w} + \\ \theta_{YZ}(\gamma_L(e_{LZ} - e_{ZZ})\hat{\tau}_Z - \gamma_K(e_{KZ} - e_{ZZ}))\hat{\tau}_Z\,.\end{aligned} \tag{B.11}$$

Now solve equation (B.11) for $\hat{Z}$ and equate to equation (B.9):

$$\begin{aligned}\Big((\gamma_K - \gamma_L)(-\sum_h \phi_K^h + \theta_{YK}\delta) + C\sigma_X + \theta_{YK}[-A(e_{KK} - e_{ZK}) + B(e_{LK} - e_{ZK})]\Big)\hat{r} \\ +\Big((\gamma_K - \gamma_L)(-\sum_h \phi_L^h + \theta_{YL}\delta) - C\sigma_X + \theta_{YL}[-A(e_{KL} - e_{ZL}) + B(e_{LL} - e_{ZL})]\Big)\hat{w} \\ = \Big((\gamma_L - \gamma_K)(-\sum_h \phi_Z^h + \theta_{YZ}\delta) + \theta_{YZ}[-A(e_{ZZ} - e_{KZ}) + B(e_{ZZ} - e_{LZ})]\Big)\hat{\tau}_Z\end{aligned} \tag{B.12}$$

Equations (B.1) and (B.12) are two equations in two unknowns, $\hat{r}$ and $\hat{w}$. Solve (B.1) for $\hat{w}$ and substitute into (B.12), solving for $\hat{r}$. Inserting $\hat{r}$ into (B.1) and (B.2) then delivers $\hat{w}$ and $\hat{p}_Y$, respectively. These expressions correspond to (15a)–(15c).

Appendix C. Special cases and proofs

Appendix C.1. Derivation of equations (16a)–(16b)

In the case of homothetic preference, $E^h_{X,M} = E^h_{Y,M} = 1$. We can therefore simplify some of the terms that reflect the heterogeneity of preferences in (15a)–(15c) as follows:

$$\sum_h \phi_Z^h = \sum_h (1 - \frac{\alpha^h}{\gamma})\xi^h \frac{\tau_Z Z}{p_Y Y} = \frac{\tau_Z Z}{p_X X p_Y Y} \sum_h (\gamma - \alpha^h) M^h = 0\,,$$

$$\sum_h \phi_L^h = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{w\bar{L}^h}{p_Y Y} = \frac{1}{\gamma p_Y Y} \sum_h (\gamma - \alpha^h) M^h \theta_L^h = -\frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}\,,$$

$$\sum_h \phi_K^h = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{r\bar{K}^h}{p_Y Y} = \sum_h (1 - \frac{\alpha^h}{\gamma}) \frac{M^h - w\bar{L}^h - \xi^h \tau_Z Z}{p_Y Y} = \frac{cov(\alpha^h, \theta_L^h)}{\gamma p_Y Y}\,,$$

$$\delta \equiv \rho := \frac{1}{p_Y Y} \sum_h (1 - \alpha^h) M^h \Big(\frac{\alpha^h}{\gamma}(\sigma^h - 1) + 1\Big) \geq \frac{1}{\gamma p_Y Y} \sum_h (1 - \alpha^h) M^h (\gamma - \alpha^h) = \frac{1}{\gamma p_Y Y} \sum_h M^h (\gamma - \alpha^h)^2 \geq 0\,.$$

Inserting these simplified expressions into the system of equations (15a)–(15c) delivers (16a)–(16b).

Appendix C.2. Proof of Proposition 4

If preferences are homothetic and unit-elastic, the change in returns to capital is given by:

$$\hat{r} = -\frac{\theta_{XL}\theta_{YZ}}{D_{H,2}}[A_H(e_{ZZ} - e_{KZ}) - B_H(e_{ZZ} - e_{LZ}) + (\gamma_K - \gamma_L)]\hat{\tau}_Z \tag{C.2}$$

where $D_{H,2} = C_H\sigma_X + e_{KL}[A_H\theta_{YL} + B_H\theta_{YK}] + e_{LZ}[B_H\theta_{XK}(\theta_{YZ} + \theta_{YL}) - A_H\theta_{XK}\theta_{YL}] + e_{KZ}[A_H\theta_{XL}(\theta_{YZ} + \theta_{YK}) - B_H\theta_{XL}\theta_{YK}] + (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma}cov(\alpha^h, \theta_L^h)$.

Since income and expenditure patterns are assumed to be correlated, the last term in $D_{H,2}$—which is the only term depending on household characteristics other than the aggregate ones—is non-zero. Note that on the other hand, for a single consumer, this term equals zero. It thus follows that one can choose Allen elasticities such that the sign is reversed when setting the last term to zero, i.e., when considering the model with a single consumer. An example of such a choice would be $\sigma_X = e_{KZ} = e_{LZ} = 0$ and $-[A_H\theta_{YL} + B_H\theta_{YK}]e_{KL} \in \left(min\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma}cov(\alpha^h, \theta_L^h), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}, max\{(\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL}) - (\gamma_K - \gamma_L)\frac{1}{p_Y Y_\gamma}cov(\alpha^h, \theta_L^h), (\gamma_K - \gamma_L)(\theta_{XL}\theta_{YK} - \theta_{XK}\theta_{YL})\}\right)$. As the numerator in (C.2) depends only on aggregate household characteristics, its value will be identical in both the heterogeneous and the single consumer case. It thus follows—for the given choice of Allen elasticities—that the signs of $\hat{w}$ and of $\hat{r}$ are reversed as compared to the model with a single household with homothetic preferences.