# FORWARD-LOOKING ENERGY ELASTICITY PARAMETERS FOR NESTED CES PRODUCTION FUNCTION

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## Introduction

Elasticities of substitution are the key parameters in CGE modeling. However an estimation and validation of the parameters is not straightforward. One of the ways is to use a historical data. Technological shifts between types of fuels and capital and fuels, observed in the past, potentially could be used to calibrate or econometrically estimate the elasticity coefficients. However, historical trends do not describe all the possible investment options available now. Moreover such estimates will be based on investment decisions made in particular economic condition, policies, and available technological/investment options in the time of decision. Application of the parameters for evaluation of future policy options involves undesirable (and unavoidable) assumption that future technological options are equal or similar to those in the past. A variety of new technological options will be disregarded from the analysis.

A more natural way to model technological options is Bottom-Up technological models. The reference energy system modes have an extensive representation of energy sector, and take into account currently available and expected technological options, but consider only part of an economy and lack connectivity with other sectors, f.i. do not provide a demand respond. Therefore their application is usually limited to the energy sector. There are several attempts to connect the top-down (CGE/AGE) and bottom-up models known as “soft” or “hard link” (see f.i. Böhringer and Rutherford 2006, 2008). However both methodologies require significant reduction of the models’ scale or some compromise in connectivity between the models. The methodology proposed in the paper might be considered as another way of hybrid modeling where bottom-up model is used to calibrate parameters for a top-down model. It is expected, the energy nest of a CGE model should provide results similar to the bottom-up model.

### Methodology

The methodology included two stages. On the first stage we generate a random sample of states of the world (SOW) and apply a multi-sector energy RU-TIMES models for electric power, and iron and steel sectors to simulate a sample of cost-efficient solutions for each of the randomly generated SOW. On the second stage we apply econometrics to approximate the simulated sample of cost-efficient solutions with a for-level nested CES production function. Comparing to historical data, where only one (supposedly optimal) solution is observed for particular economic conditions in the past, the randomly simulated sample represents a full set of optimal solutions for a number of possible combination of unknown variables (f.i. fuel prices). Therefore simulated data has an advantage over historical in two ways. First, it takes into account currently available and expected in the future technological options (in TIMES model). Second, simulated data accommodates a huge variety of economic conditions which are not observable in the real life, but theoretically possible.

## Specification of numerical experiment

To generate a sample of optimal fuel mix structure under given economic conditions, we apply a bottom-up model and solve it for various SOWs. Here are the main specifications of the experiment:

1. Each economic sector is modeled separately (one bottom-up model for each sector).
2. Time horizon is from 2010 to 2030.
3. Prices are randomly assigned for each commodity (coal, gas, oil) and each SOW, and are constant for the all period of consideration.
4. Unlimited supply of each energy source under fixed price.
5. Final demand is growing with constant rate and equal for all SOW.

The only difference between SOWs is a set of energy prices (gas, oil, coal). We performed 2000 model runs to generate a sample of the same size for CES estimates.

## Estimation of multi-level nested CES function

Estimation of CES function is not straightforward due to non-linearity. There are several methods exist for one level functions. However it is even more difficult to estimate multi-level, nested CES structure. In the paper we apply Bayesian econometrics to estimate CES functions. The applied method is similar to Tsurumi, Tsurumi (1976), but extended for multi-level cases. As a result, we estimate the system of following three equations for each level of CES function:

$$y = ad \cdot \left(d_1 \cdot x_1^{-\rho} + (1-d) \cdot \left(d_2 \cdot x_2^{-\rho_2} + \cdots\right)^{\rho_1/\rho_2}\right)^{-1/\rho_1} \cdot e^{\varepsilon}$$

$$\varepsilon \sim \mathrm{N}(0, \sigma^2)$$

where:

y — final product

$x_i$ - production factor for CES production function

$d_i$ — factor share parameter for CES production function

ad — productivity for CES production function

E — elasticity parameter for CES production function

ε — errors

ρ — elasticity parameter

$$E = \frac{1}{1+\rho}$$

The structure of multi-level CES function is presented on Figure 1. The resulting estimates for four Russian industries – iron and steel, electric power sector, transport sector and residential and commercial sector presented below.

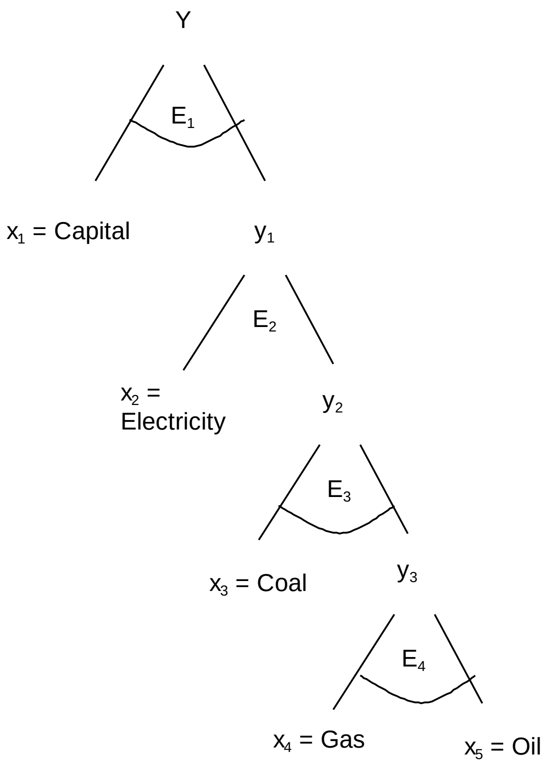


**Figure 1. Typical Nested CES production function structure.**

## Four-level elasticity parameters estimates for Russian industries

On the generated with RU-TIMES one-sector models data we estimate for level elasticity (E1-E4) and share (d1-d4) parameters, as well as scale or productivity factor (ad) for each year of the modeled horizon (2010-2030). The resulting estimates are presented in the Table 1.

Often, in general equilibrium models accepted standard industrial structure, without regard to the specific sector. For example, the use of capital and petroleum products in the iron and steel industry is directly proportional to output, as seen in the Figure 2. Putting oil on the lower level of the production function can reduce the quality of the evaluation and additional errors. In this paper choose optimal production structures, which are shown in Figure 8. The remaining fuel is aggregated by Leontief function.

As follows from the figure below, the elasticity parameters have a tendency to grow over time. This fact has an intuitive understanding that the possibility for technological/fuel switch is higher when we have longer time.

Also, we will have more possibility for switching between fuels if we expect higher economic growth, higher final demand for our products. To meet growing demand one should extend producing capacities. New investments can be done in any technology, based on coal or gas. Therefore the elasticity should be higher. A comparison of estimates for 1% and 3% growth is shown on Figure 2.

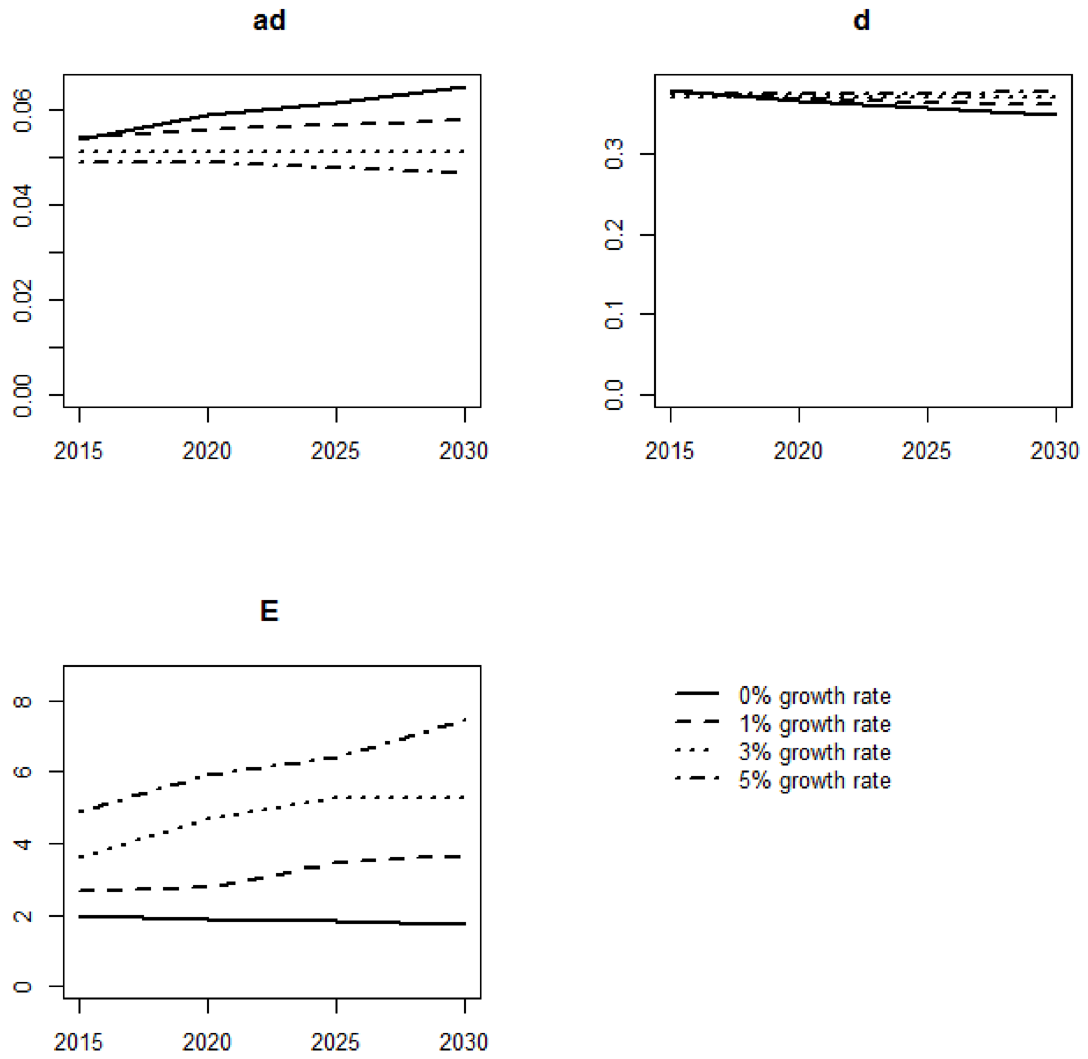


**Figure 2. Result estimation elasticities for Iron and Steel with different growth path.**

Another feature is that productivity grows less in the case of higher rates of economic growth, which at first glance counterintuitive. The more growth, the more introduced new, more efficient production capacity and therefore increasing productivity grows stronger.

This occurs for two reasons. Fuel consumption per unit of output is reduced if the share parameter increases so that more weight gets fuel use, which is much more. The second reason is specific to the steel industry. Energy efficiency is strongly dependent on the use of scrap, the volume of which is limited.

One possible solution is to fix a parameter when evaluating equity, which significantly reduces the quality of the assessment. The figure below shows residual sum of squares of estimation with fix and flexible share parameter d. Quality assessment is significantly reduced if we fix the share parameter for all periods.

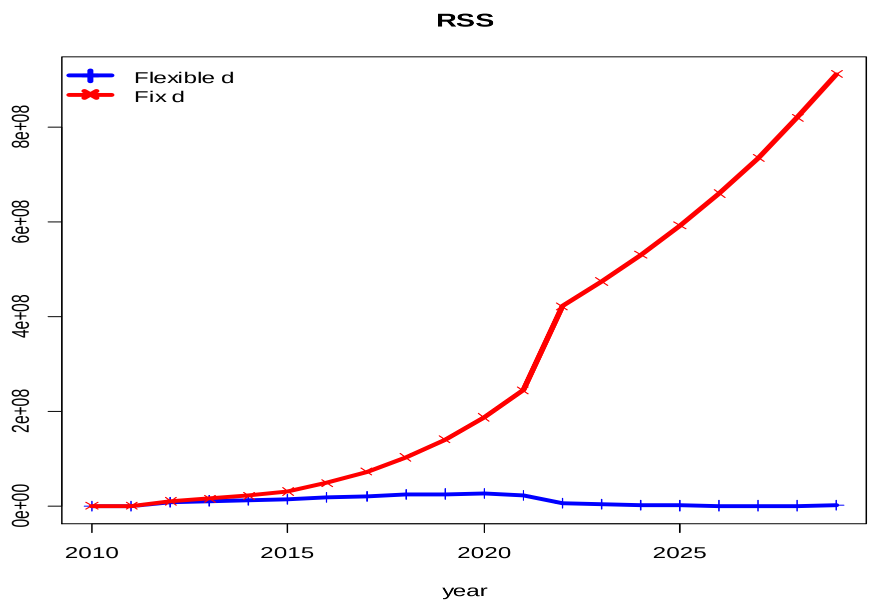


**Figure 3. Residual sum of squares of estimation with fix and flexible share parameter d**

## Some conclusions

The developed methodology applies bottom-up energy models to estimate nested CES elasticity parameters for tom-down (CGE) models. The resulting estimates have several advantages over historical elasticity parameters estimates. First, they take into account all available now and in the future technological options (based on bottom-up model specification). Second, they take into account all possible set of economic variables, instead of only one observed in the past. Therefore the methodology is much better in approximation of technological switching.

There are several key observations should be mentioned regarding energy elasticity parameters for the CGE modeling:

- Elasticity parameters depend on horizon of planning (experiment). Longer horizon of planning usually lead to higher potential of switching between fuels and technologies, i.e. elasticity parameters are higher.
- Assumption of higher economic growth should result in higher elasticity of substitution. Currently existing capacities limit an opportunity for a technological maneuver in the short and medium run periods. However expansion of production assumes investments in new capacities.

- Technological shift and share parameters also depend on experiment horizon and should be considered for adjustment in CGE experiments.

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## Appendix: Details of estimation of nested CES function

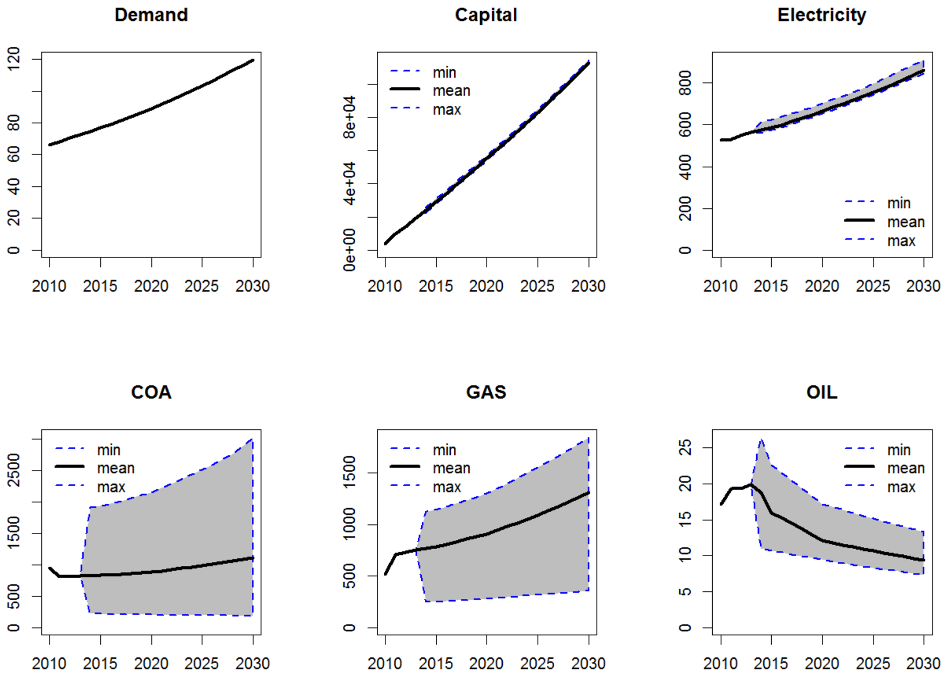


**Figure 4. Final demand and fuel consumption range after numerical experiment for Iron and Steel for 3% growth rate.**

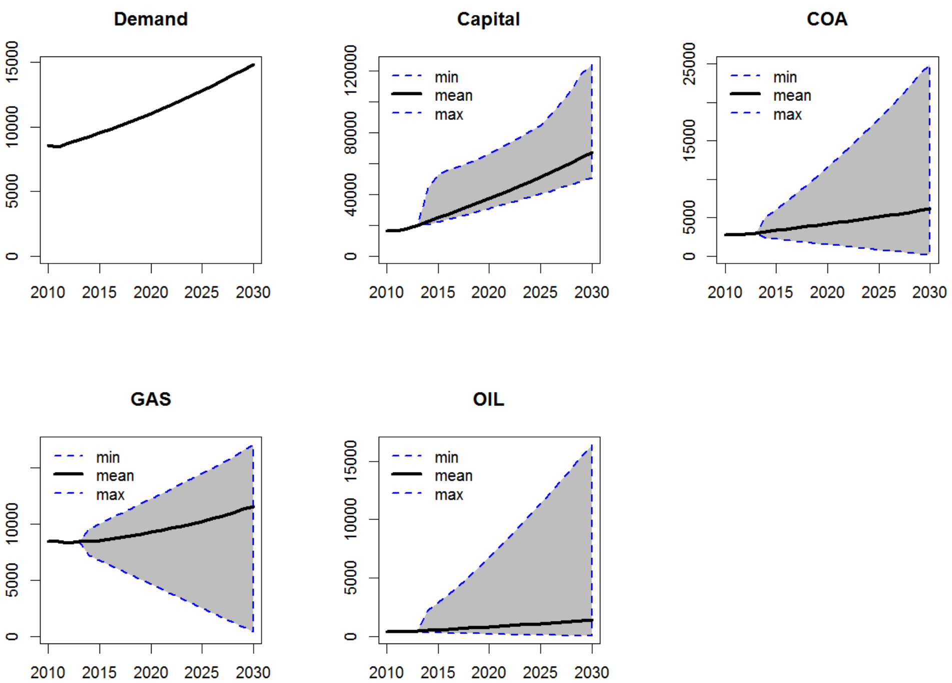


**Figure 5. Final demand and fuel consumption range after numerical experiment for Electricity for 3% growth rate.**

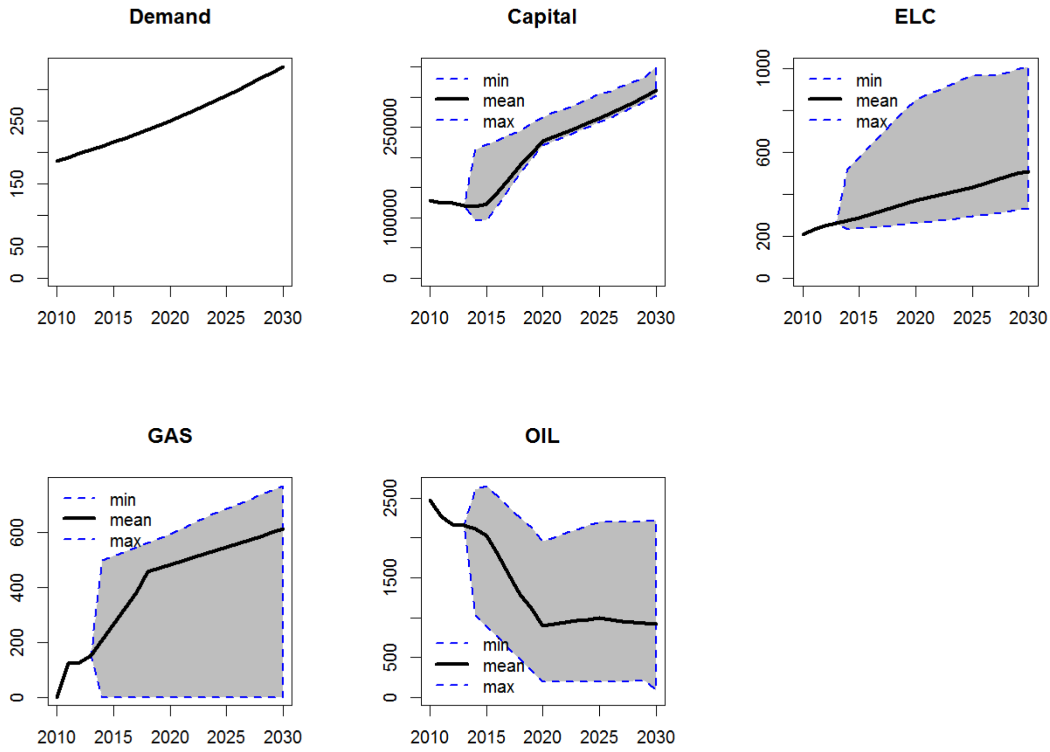


**Figure 6. Final demand and fuel consumption range after numerical experiment for Transport for 3% growth rate.**

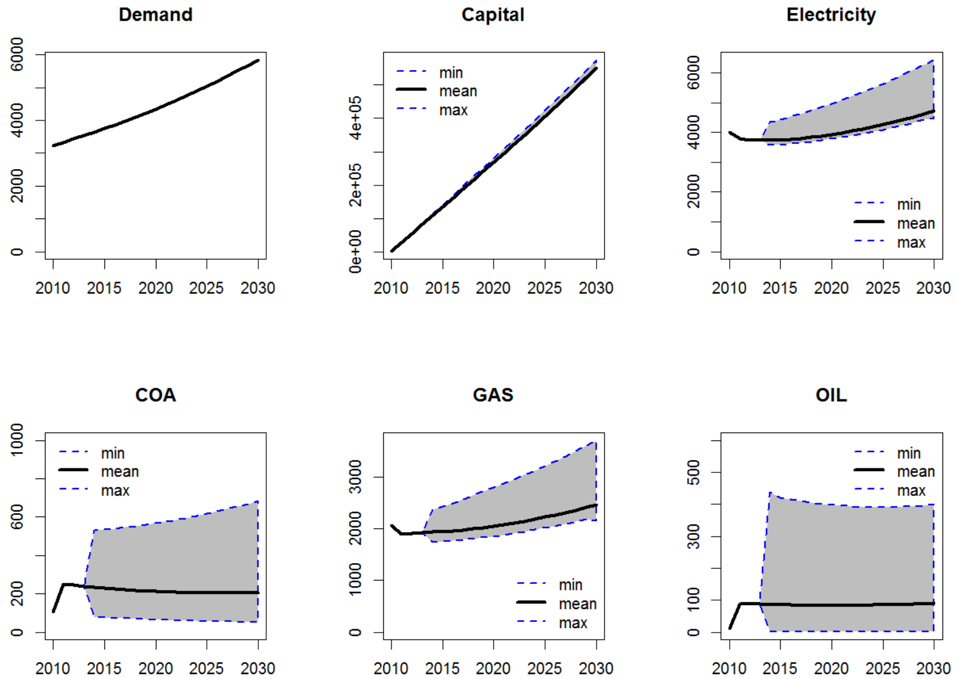


**Figure 7. Final demand and fuel consumption range after numerical experiment for Residential and Commercial for 3% growth rate.**

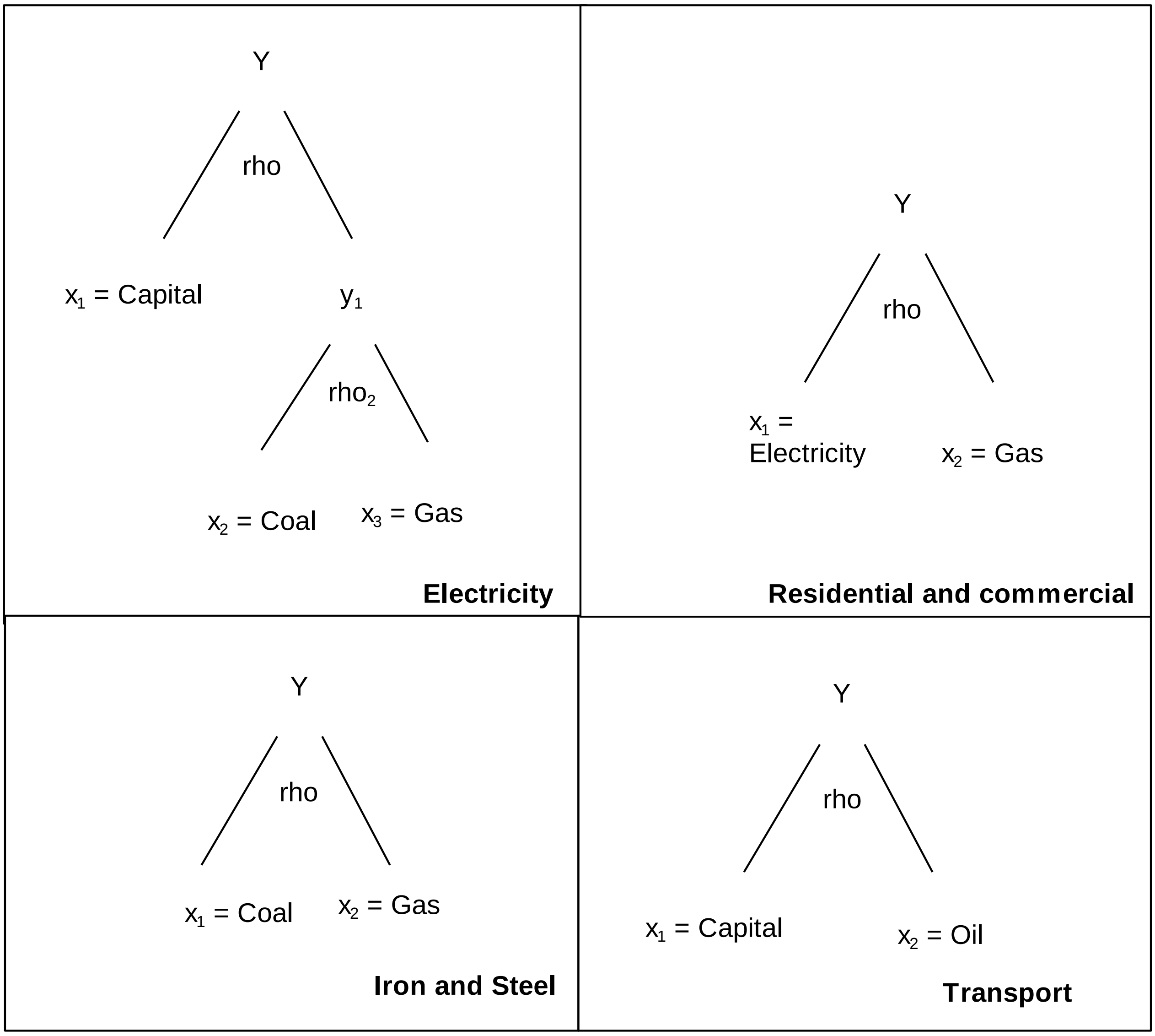


**Figure 8. Optimal nested CES production function for different sector for Russia.**

**Table 1. Estimation results of four-level CES functions for Iron and Steel.**

| Parameter | Growth | Value | 2015 | 2020 | 2025 | 2030 |
|---|---|---|---|---|---|---|
| ad | 3% | mean | 0.12 | 0.13 | 0.13 | 0.19 |
| | | sd | 0.00 | 0.00 | 0.01 | 0.07 |
| E | | mean | 0.22 | 0.23 | 0.22 | 0.22 |
| | | sd | 0.53 | 0.47 | 0.49 | 0.51 |
| d | | mean | 0.21 | 0.10 | 0.11 | 0.14 |
| | | sd | 0.07 | 0.04 | 0.07 | 0.07 |
| E2 | | mean | 0.17 | 0.22 | 0.22 | 0.56 |
| | | sd | 0.80 | 0.51 | 0.44 | 0.30 |
| d2 | | mean | 0.83 | 0.90 | 0.88 | 0.72 |
| | | sd | 0.10 | 0.05 | 0.07 | 0.19 |
| E3 | | mean | 0.26 | 0.31 | 0.45 | 0.46 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | sd | 0.43 | 0.42 | 0.53 | 0.40 |
| d3 | | mean | 0.46 | 0.31 | 0.31 | 0.20 |
| | | sd | 0.09 | 0.10 | 0.08 | 0.08 |
| E4 | | mean | 3.93 | 2.97 | 3.16 | 1.01 |
| | | sd | 0.85 | 0.81 | 0.82 | 0.39 |
| d4 | | mean | 0.82 | 0.77 | 0.71 | 0.36 |
| | | sd | 0.11 | 0.19 | 0.22 | 0.37 |

**Table 2. Estimation results of CES functions for Iron and Steel.**

| Parameter | Growth | Value | 2015 | 2020 | 2025 | 2030 |
|---|---|---|---|---|---|---|
| ad | 0% | mean | 0.05 | 0.06 | 0.06 | 0.07 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |
| E | | mean | 2.36 | 1.89 | 1.84 | 1.72 |
| | | sd | 0.98 | 0.97 | 0.96 | 0.95 |
| d | | mean | 0.38 | 0.37 | 0.36 | 0.35 |
| | | sd | 0.00 | 0.00 | 0.01 | 0.01 |
| ad | 1% | mean | 0.05 | 0.06 | 0.06 | 0.06 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |
| E | | mean | 2.68 | 2.79 | 3.45 | 3.66 |
| | | sd | 0.98 | 0.97 | 0.96 | 0.95 |
| d | | mean | 0.38 | 0.37 | 0.36 | 0.36 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.01 |
| ad | 3% | mean | 0.05 | 0.05 | 0.05 | 0.05 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |
| E | | mean | 3.30 | 4.70 | 5.82 | 5.26 |
| | | sd | 0.98 | 0.97 | 0.96 | 0.95 |
| d | | mean | 0.37 | 0.37 | 0.37 | 0.37 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.01 |
| ad | 5% | mean | 0.05 | 0.05 | 0.05 | 0.05 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |
| E | | mean | 3.95 | 5.90 | 6.41 | 8.64 |
| | | sd | 0.98 | 0.96 | 0.96 | 0.96 |
| d | | mean | 0.37 | 0.37 | 0.37 | 0.38 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |

**Table 3. Estimation results of two-level CES functions for Electricity.**

| Parameter | Growth | Value | 2015 | 2020 | 2025 | 2030 |
|---|---|---|---|---|---|---|
| ad | 3% | mean | 0.53 | 0.78 | 0.68 | 0.64 |
| | | sd | 0.04 | 0.04 | 0.02 | 0.03 |
| E | | mean | 0.32 | 6.28 | 11.57 | 13.33 |
| | | sd | 0.73 | 0.91 | 0.97 | 0.97 |
| d | | mean | 0.92 | 0.25 | 0.25 | 0.25 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | sd | 0.06 | 0.04 | 0.02 | 0.02 |
| E2 | | mean | 0.49 | 0.82 | 4.57 | 7.42 |
| | | sd | 0.79 | 0.76 | 0.89 | 0.94 |
| d2 | | mean | 0.05 | 0.11 | 0.14 | 0.14 |
| | | sd | 0.01 | 0.02 | 0.02 | 0.02 |
| ad | 5% | mean | 0.51 | 0.75 | 0.65 | 0.62 |
| | | sd | 0.03 | 0.04 | 0.02 | 0.03 |
| E | | mean | 0.38 | 9.21 | 11.92 | 13.74 |
| | | sd | 0.73 | 0.95 | 0.97 | 0.98 |
| d | | mean | 0.89 | 0.24 | 0.25 | 0.24 |
| | | sd | 0.05 | 0.02 | 0.02 | 0.02 |
| E2 | | mean | 0.52 | 1.02 | 5.88 | 6.85 |
| | | sd | 0.71 | 0.79 | 0.91 | 0.93 |
| d2 | | mean | 0.06 | 0.12 | 0.13 | 0.14 |
| | | sd | 0.01 | 0.02 | 0.02 | 0.02 |

**Table 4. Estimation results of CES functions for Transport.**

| Parameter | Growth | Value | 2015 | 2020 | 2025 | 2030 |
|---|---|---|---|---|---|---|
| ad | 3% | mean | 0.03 | 0.01 | 0.01 | 0.01 |
| | | sd | 0.00 | 0.00 | 0.00 | 0.00 |
| E | | mean | 1.83 | 3.20 | 10.55 | 13.59 |
| | | sd | 0.93 | 0.87 | 0.96 | 0.98 |
| d | | mean | 0.15 | 0.38 | 0.14 | 0.10 |
| | | sd | 0.04 | 0.18 | 0.03 | 0.01 |
| ad | 5% | mean | 0.03 | 0.01 | 0.01 | 0.01 |
| | | sd | 0.01 | 0.00 | 0.00 | 0.00 |
| E | | mean | 1.91 | 4.03 | 7.17 | 13.60 |
| | | sd | 0.91 | 0.86 | 0.96 | 0.98 |
| d | | mean | 0.15 | 0.33 | 0.18 | 0.10 |
| | | sd | 0.05 | 0.20 | 0.03 | 0.02 |

**Table 5. Estimation results of two-level CES functions for Residential and Commercial.**

| Parameter | Growth | Value | 2015 | 2020 | 2025 | 2030 |
|---|---|---|---|---|---|---|
| ad | 3% | mean | 1.07 | 1.16 | 1.24 | 1.31 |
| | | sd | 0.02 | 0.02 | 0.02 | 0.02 |
| E | | mean | 1.10 | 1.47 | 1.43 | 1.55 |
| | | sd | 0.60 | 0.66 | 0.64 | 0.67 |
| d | | mean | 0.91 | 0.92 | 0.92 | 0.92 |
| | | sd | 0.03 | 0.03 | 0.03 | 0.03 |
| ad | 5% | mean | 1.09 | 1.20 | 1.28 | 1.34 |
| | | sd | 0.02 | 0.02 | 0.02 | 0.03 |
| E | | mean | 1.20 | 1.49 | 1.50 | 1.62 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | sd | 0.62 | 0.64 | 0.66 | 0.67 |
| d | | mean | 0.91 | 0.92 | 0.92 | 0.92 |
| | | sd | 0.03 | 0.03 | 0.03 | 0.03 |