Global Trade Analysis Project
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This paper is from the
GTAP Annual Conference on Global Economic Analysis
https://www.gtap.agecon.purdue.edu/events/conferences/default.asp

Estimating Trade Elasticities and Asymmetric Trade Costs under Firm Heterogeneity [*]

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printdate: April 15, 2013

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1   Introduction

There is considerable interest in the literature for the estimation of the correct elasticity of trade flows with respect to trade costs, the trade elasticity (Head and Ries (2001), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), Romalis (2007), Baier and Bergstrand (2009), Feenstra, et al.   (2010), Simonovska and Waugh (2011), Simonovska and Waugh   (2011), Caliendo and Parro (2012)).[1] Getting a correct measure for the trade elasticity is important. It is a crucial parameter in the prediction of the effects of trade policy on trade flows and welfare. In many trade models the trade elasticity and domestic spending shares are sufficient statistics to determine the welfare gains from trade (Arkolakis, et al. (2012)).

The trade elasticity can be identified from variation in tariffs. Head and Ries (2001), Romalis (2007) and Caliendo and Parro (2012) are examples of studies following this approach. Eaton and Kortum (2002) employ price differences across countries as a proxy for trade costs to get an estimate of the trade elasticity. Simonovska and Waugh (2011) refine this approach using a simulated method of moments estimator. The current paper takes a different approach and identifies the trade elasticity using data on import shares, distance and gross output. In the conventional gravity equation distance cannot be used to identify the trade elasticity as the elasticity of distance with respect to trade flows consists of the elasticity of trade flows with respect to trade costs (the trade elasticity) and the elasticity of trade costs with respect to distance.

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[1] Hillberry and Hummels (2012) provide an overview of the literature.

Employing general equilibrium conditions following from input cost adjustment required to balance trade and a first order Taylor approximation around the free trade equilibrium, a second gravity equation is derived in the paper. The second gravity equation expresses the weighted sum of import shares of an exporter in its trading partners as a function of the weighted sum of distance with these export partners. Due to the general equilibrium conditions, the coefficient on the sum of distances in the second gravity equation is different from the coefficient of distance in the conventional gravity equation. Identification of the trade elasticity takes then place in two steps. In a first step, import shares are regressed on distance and importer and exporter fixed effects to get an estimate of the trade elasticity times the distance elasticity. In a second step the GE restricted gravity equation is estimated using the fitted values of distance from the first step. The coefficient on distance is different in the second stage, while still being a function of the trade elasticity. This enables the identification of the trade elasticity and using the coefficients from the first step also the distance elasticity.

The estimator is derived for the firm heterogeneity Melitz (2003) model. Therefore, full estimation of the trade elasticity requires solving the theoretical models and iterating between estimation of the trade elasticity and solving the model to gather free trade gross output shares.

Using the NBER-UN world trade data (Feenstra, et al. (2005)) for the largest 48 economies in the world, approximating free trade gross ouput shares with actual output shares and employing the estimator based upon the Melitz model leads to an estimated trade elasticity of about two. This estimate is low in comparison to other work in the literature.

The estimator proposed in this paper has some important advantages in comparison to the available estimators in the literature. An important advantage in comparison to the estimators employing tariff data is that the estimator based upon distance does not suffer from endogeneity and omitted variable bias. Tariff changes often go along with other types of reforms that might also affect trade. Moreover, tariffs might be driven by trade. In comparison to both the Eaton and Kortum (2002) and Simonovska and Waugh (2011) estimator and the estimators employing tariffs, another advantage is that the current estimator can take into account all trade, also trade in services. The other approaches are restricted to trade in goods where tariffs are available and trade in goods where price data are available. A possible disadvantage of the estimator proposed in the current paper is that one of the gravity equations relies on a taylor approximation around the free trade equilibrium. Also Baier and Bergstrand (2009) use a Taylor expansion to approximate the multilateral resistance terms in the Anderson and Van

Wincoop (2003) gravity equation.

Following the approach in Novy (2012), the theoretical structure and the estimated trade elasticities are used in a second step to calculate trade cost measures as a function of import shares and gross output. The novelty of the approach in comparison to Novy (2012) is that asymmetric measures of trade costs are derived. The trade cost measures do not require any Taylor approximation and follow from the theoretical restrictions of the Melitz (2003) firm heterogeneity model. The trade cost measure is an aggregate of iceberg and fixed trade costs: with the minimum set of data it is not possible to distinguish between these two components of trade costs.

2   Theoretical Model

Consider an economy with J countries. A gravity equation will be derived for the Melitz model. Input bundles into production Z_{vi} in country i in period v consist of factor input bundles L_{vi} and intermediate input bundles I_{vi} according to a Cobb Douglas production function with a fraction β_{vi} spent on factor inputs. Intermediate input bundles are identical to final good bundles. The price of composite input bundles is indicated by a_{vi}, the price of factor input bundles by w_{vi} and the price of intermediate and final goods bundles by P_{vi}.

The model is characterized by CES preferences across a continuum of varieties and Melitz type firm heterogeneity. The substitution elasticity between varieties is σ and the distribution of firm productivities is Pareto with shape parameter θ. As is well known from the literature (Chaney (2008), Arkolakis, et al. (2012)) θ is also the trade elasticity in this model, i.e. the elasticity of trade flows with respect to trade costs. This subsection will present four steps to derive two theoretical gravity equations that can be used to derive an estimator of θ. The firs step is to write the import share of country i goods in country j in period v, s_{vij}, as follows (derivation in webappendix):

$$s_{vij} = \frac{Z_{vi}\left(\tau_{vij}a_{vi}\right)^{-\theta}f_{vij}^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\sum_{k}Z_{vk}\left(\tau_{vkj}a_{vk}\right)^{-\theta}f_{vkj}^{-\frac{\theta-\sigma+1}{\sigma-1}}} \qquad (1)$$

τ_{vij} and f_{vij} are respectively the iceberg and fixed trade costs for trade from country i to country j. Sunk entry costs f_e are set equal across countries and fixed trade costs are paid in bundles

of the destination country.[2]

Equation (1) can be rearranged to solve for a_{vi}:

$$a_{vi} = \left(\frac{1}{s_{vij}} \frac{Z_{vi} t_{vij}^{-\theta}}{\sum\limits_k Z_{vk} \left(t_{vkj} a_{vk} \right)^{-\theta}} \right)^{\frac{1}{\theta}} \tag{2}$$

In equation (2) iceberg trade costs and fixed trade costs are combined into one term:

$$t_{vij} = \tau_{vij} f_{vij}^{-\frac{\theta - \sigma + 1}{\theta(\sigma - 1)}} \tag{3}$$

The second step is to write the value of gross output in country i as a function of the imports in its trading partners:

$$a_{vi} Z_{vi} = \sum\limits_m a_{vm} Z_{vm} ES_{vm} s_{vim} \tag{4}$$

ES_{vm} is a measure of trade imbalances in country m and measures the ratio of spending E_{vm} relative to output $a_{vm} Z_{vm}$ (excess spending ES):

$$ES_{vm} = \frac{E_{vm}}{a_{vm} Z_{vm}} \tag{5}$$

Rearranging equation (4) and substituting the expression for a_{vi} in equation (2) gives:

$$1 = \sum\limits_m \left(\frac{s_{vij}}{s_{vmj}} \right)^{\frac{1}{\theta}} \frac{t_{vij}}{t_{vmj}} \left(\frac{Z_{vm}}{Z_{vi}} \right)^{\frac{\theta + 1}{\theta}} ES_{vm} s_{vim} \tag{6}$$

Solving equation (6) for the import share s_{vij} leads to the following implicit gravity equation:

$$s_{vij} = t_{vij}^{-\theta} Z_{vi}^{\theta + 1} \left(\sum\limits_m s_{vmj}^{-\frac{1}{\theta}} t_{vmj}^{-1} Z_{vm}^{\frac{\theta + 1}{\theta}} E_{vm} s_{vim} \right)^{-\theta} \tag{7}$$

The third step is to take a log Taylor approximation of equation (7) around the equilibrium with $s_{vij} = \frac{\overline{Z_{vi}}}{\sum\limits_s \overline{Z_{vs}}}$ and $t_{vij} = t_v$.[3] After a sequence of steps this leads to the following first

---

[2]Fixed trade costs paid in bundles in of the origin country would change the exposition, but not the final gravity equation.

[3]It can be easily seen that these values for s_{vij} and t_{vij} satisfy the gravity equation (6). Also, import shares add up to 1.

theoretical gravity expression:

$$
\begin{aligned}
\ln s_{vij} = & -\theta \ln t_{vij} + (\theta + 1) \ln Z_{vi} + \sum_m \omega_{vm} \ln s_{vmj} - (\theta + 1) \sum_m \omega_{vm} \ln Z_{vm} \\
& + \theta \sum_m \omega_{vm} \ln t_{vmj} - \theta \sum_m \omega_{vm} \ln E_{vm} - \theta \sum_m \omega_{vm} \ln s_{vim} \\
& - \theta \ln \sum_l \overline{E_{vl} Z_{vl}} + \theta \sum_m \omega_{vm} \left(\ln \overline{Z_{vm}} - \ln \overline{E_{vm}} \right)
\end{aligned}
\tag{8}
$$

With ω_{vm} defined as:

$$
\omega_{vm} = \frac{\overline{Z_{vm} E_{vm}}}{\sum_l \overline{Z_{vl} E_{vl}}}
\tag{9}
$$

The fourth step is to rewrite equation (8) summing the LHS and RHS over j using ω_{vj} as weights. This leads after a sequence of steps to a second theoretical gravity equation:

$$
\begin{aligned}
\sum_j \omega_{vj} \ln s_{vij} = & -\frac{\theta}{\theta + 1} \sum_j \omega_{vj} \ln t_{vij} + \ln Z_{vi} + \frac{1}{\theta + 1} \sum_j \omega_{vj} \sum_m \omega_{vm} \ln s_{vmj} \\
& - \sum_m \omega_{vm} \ln Z_{vm} + \frac{\theta}{\theta + 1} \sum_j \omega_{vj} \sum_m \omega_{vm} \ln t_{vmj} - \frac{\theta}{\theta + 1} \sum_m \omega_{vm} \ln E_{vm} \\
& - \frac{\theta}{\theta + 1} \ln \sum_l \overline{E_{vl} Z_{vl}} + \frac{\theta}{\theta + 1} \sum_m \omega_{vm} \left(\ln \overline{Z_{vm}} - \ln \overline{E_{vm}} \right)
\end{aligned}
\tag{10}
$$

Equation (6) can be used to get an expression for the ratio of trade costs $\frac{t_{vij}}{t_{vmj}}$:

$$
\frac{t_{vij}}{t_{vmj}} = \left(\frac{s_{vij}}{s_{vmj}} \right)^{-\frac{1}{\theta}} \left(\frac{Z_{vi}}{Z_{vm}} \right)^{\frac{\theta + 1}{\theta}} \frac{a_{vm} Z_{vm}}{a_{vi} Z_{vi}}
\tag{11}
$$

It is not possible to find expressions for t_{vij}, only the ratio of trade costs can be determined. With equation (11) the trade costs for importing from country i in country m can be expressed relative to intracountry trade costs in country m as:

$$
\frac{t_{vim}}{t_{vmm}} = \left(\frac{s_{vim}}{s_{vmm}} \right)^{-\frac{1}{\theta}} \left(\frac{Z_{vi}}{Z_{vm}} \right)^{\frac{\theta + 1}{\theta}} \frac{a_{vm} Z_{vm}}{a_{vi} Z_{vi}}
\tag{12}
$$

Equation (12) is similar to the measure for trade costs in Novy (2012). The difference is that the measure in equation (12) is asymmetric. It measures trade resistance of an importer with respect to a trading partner relative to trade with itself. The measure of trade costs in Novy (2012) is defined as:

$$
t_{vim}^{novy} = \frac{t_{vim} t_{vmi}}{t_{vmm} t_{vii}} = \left(\frac{X_{vim} X_{vmi}}{X_{vmm} X_{vii}} \right)^{-\frac{1}{\theta}}
\tag{13}
$$

This can be rewritten in terms of import shares as follows:

$$t_{vim}^{novy} = \left(\frac{s_{vim}s_{vmi}}{s_{vmm}s_{vii}} \right)^{-\frac{1}{\theta}} \tag{14}$$

Multiplying the two asymmetric measures for trade costs in equation (12) leads to t^{novy}:

$$\begin{aligned}
\frac{t_{vim}t_{vmi}}{t_{vmm}t_{vii}} &= \left(\frac{s_{vim}}{s_{vmm}} \right)^{-\frac{1}{\theta}} \left(\frac{Z_{vi}}{Z_{vm}} \right)^{-\frac{\theta+1}{\theta}} \frac{a_{vm}Z_{vm}}{a_{vi}Z_{vi}} \left(\frac{s_{vmi}}{s_{vii}} \right)^{-\frac{1}{\theta}} \left(\frac{Z_{vm}}{Z_{vi}} \right)^{-\frac{\theta+1}{\theta}} \frac{a_{vi}Z_{vi}}{a_{vm}Z_{vm}} \\
&= \left(\frac{s_{vim}s_{vmi}}{s_{vmm}s_{vii}} \right)^{-\frac{1}{\theta}}
\end{aligned}$$

Hence, in the asymmetric index of the current paper market size plays a role, whereas in the Novy (2012) index it drops out.

3   Estimation Strategy

In this section the estimation strategy will be presented based on the theoretical gravity equations in the previous section. Trade costs t_{vij} (capturing both iceberg and fixed trade costs) is written as a function of distance d_{vij}:

$$\ln t_{vij} = \beta \ln d_{vij} \tag{15}$$

Other variables explaining trade costs or a different specification for the effect of distance on trade costs (including quadratic terms or using stepfunctions) can be easily added.

The estimation strategy consists of three steps. The first step is to estimate the first theoretical gravity equation (8) with data on import shares and distance. Adding importer time and exporter time fixed effects, respectively η_{vi} and ζ_{vj}, and using equation (15) for trade costs as a function of distance, equation (8) can be written as the first empirical gravity equation:

$$\ln s_{vij} = -\theta\beta \ln t_{vij} + \eta_{vi} + \zeta_{vj} + \varepsilon_{vij} \tag{16}$$

The second step is to estimate the second theoretical gravity equation (10) with data on import shares, distance and gross output shares. Using the fitted value for $\theta\beta$ from the first gravity equation (16), $\widetilde{\theta\beta}_M$, and adding a time fixed effect ν_v, equation (8) can be written as the second

empirical gravity equation:

$$\sum_j \omega_{vj} \ln s_{vij} = -\frac{1}{\theta+1} \sum_j \omega_{vj} \widetilde{\theta} \beta_M \ln d_{vij} + \ln Z_{vi} + \nu_v + \upsilon_{vij} \qquad (17)$$

Equation (17) enables the estimation of θ from the coefficient on the first term on the RHS. Z_{vi} is taken directly from the data. $\overline{Z_{vj}}$ corresponds with the zero gravity level of Z_{vj}. As a first approximation the actual values of Z_{vi} can be used.

In a third step the model can be solved with estimates for θ. Setting trade costs at the zero gravity level gives proper values for $\overline{Z_{vj}}$. These can be used to update the estimates for θ from the two estimating equations. Iterating between estimation and solving of the model will generate the final estimate for θ.

4   Data

Data are required on import shares, distance and gross output. The import shares s_{vij} are calculated based upon the NBER-UN trade flow data collected by Feenstra, et al. (2005) and data on GDP in current dollars from the IMF International Financial Statistics (IFS). Gross output bundles Z_{vi} are based upon GDP data in PPP terms from Conference Board (2012). Gross output value $a_{vi} Z_{vi}$ is also calculated from the GDP data in PPP terms from Conference Board (2012) employing the price index variable from Heston, et al. (2011) to get GDP in current dollars. Data on distance d_{ij} are taken from Clair et al (2004).[4] The distance data are calculated following the great circle formula, which uses latitudes and longitudes of the relevant capital cities. The distance for trade within a country was set equal to the internal distance values provided as well by Clair et al (2004). As a result of availability of the trade flow data, the sample ranges from 1960 to 2000. Following Di Giovanni and Levchenko (2012) the number of countries is limited to the largest 48 countries with the remainder of the trade flows attributed to a rest of the world (ROW).

To calculate import shares various approaches are followed. A first dinstinction is made between scaling the trade flows from Feenstra, et al. (2005) with total imports from the IFS dataset in either goods or goods and services or not scaling the trade flows with IFS import data. A second distinction is made between calculating import shares of ROW using the trade flows of the ROW countries (direct approach) or calculating the trade flows of the ROW countries

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[4]http://www.cepii.fr/anglaisgraph/bdd/distances.htm

Import shares calculation	$\theta\beta$	t-value	$\frac{\theta}{\theta+1}\beta$	t-value	Implied θ
direct no scaling	-1.4261	-215.64	-0.4878	-9.81	1.92
indirect no scaling	-1.4302	-215.81	-0.4402	-8.92	2.25
direct scaling goods	-1.4157	-214.80	-0.5444	-11.00	1.60
indirect scaling goods	-1.4202	-215.02	-0.4620	-9.40	2.07
direct scaling goodsservices	-1.3808	-209.49	-0.5586	-11.39	1.47
indirect scaling goodsservices	-1.3863	-209.82	-0.4772	-9.79	1.91

Table 1: Coefficients and t statistics of fixed effects log import shares regression and summed log import shares regression on distance and the implied trade elasticity

by substracting trade of the largest 48 countries from trade flows with all countries (indirect approach). Distance between the first 48 countries and ROW is calculated as a weighted average of distance between each country and all ROW countries with as weights GDP in PPP terms from Conference Board (2012).

5   Estimation Results

Table 1 displays the regression coefficients on distance and their standard errors of the first and second empirical gravity equation in the Melitz economy, equations (16) and (17). The first and third columns present the coefficients of the conventional gravity equation (16), $\theta\beta$, and of the new gravity equation (17) in summed shares, $\frac{\theta}{\theta+1}\beta$. The implied trade elasticity θ is displayed in the fifth column.

The estimation results of the conventional gravity equation do not vary much with the way the import shares were calculated. The new gravity equation in summed shares displays somewhat more variation, but coefficients are highly significant. The implied trade elasticity also varies somewhat. Most variation in the estimates is caused by the distinction between direct and indirect. The indirect approach is preferred, as it generates values for ROW import shares consistent with other countries' import shares.

The overall picture from table 1 is clear. The estimates suggest a trade elasticity of around 2, much lower than in the existing literature. Eaton and Kortum (2002) come to a trade elasticity of around 8. Simonovska and Waugh (2012) argue that the correct trade elasticity in the Melitz model based upon the methodology of Eaton and Kortum (2002) should be around 4.

6 Calculation of Trade Costs

To be completed

7 Concluding Remarks

This paper proposes a novel way to estimate the trade elasticity from a minimum set of data. Import shares, gross output bundles and distance are sufficient to identify the trade elasticity imposing general equilibrium restrictions in a Melitz economy. Estimates based upon the largest 48 countries in the UN-NBER dataset imply a rather low value for the trade elasticity of about 2. In ongoing work the current estimator is extended to Armington and Eaton and Kortum economies. Also, the corrected standard error will be calculated.

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Appendix A   Derivations of Equations

Equation (12)

Substituting equation (11) into equation (6) makes clear that it satisfies this equation:

$$
\begin{aligned}
1 &= \sum_m \left(\frac{s_{vij}}{s_{vmj}}\right)^{\frac{1}{\theta}} \left(\frac{s_{vij}}{s_{vmj}}\right)^{-\frac{1}{\theta}} \left(\frac{Z_{vi}}{Z_{vm}}\right)^{\frac{\theta+1}{\theta}} \frac{a_{vm}Z_{vm}}{a_{vi}Z_{vi}} \left(\frac{Z_{vm}}{Z_{vi}}\right)^{\frac{\theta+1}{\theta}} ES_{vm}s_{vim} \\
1 &= \sum_m \frac{a_{vm}Z_{vm}}{a_{vi}Z_{vi}} \frac{E_{vm}}{a_{vm}Z_{vm}} s_{vim} \\
a_{vi}Z_{vi} &= \sum_m E_{vm}s_{vim}
\end{aligned}
$$