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This paper is from the
GTAP Annual Conference on Global Economic Analysis
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A simple structure for CGE models

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Abstract

Although computable general equilibrium (CGE) models are now used widely in policy analysis, their growing complexity has become an entry barrier for potential users, and the equation systems are becoming lengthy, hard to understand and difficult to modify or debug.

The complexity is partly due to the lack of a clearly defined equation system that is used to solve the model. It is also the consequence of models being designed for multiple uses, with some variables and equations only required for some uses and many unnecessary for most uses.

In spite of their apparent complexity, the structure of CGE models is simple, based on standard national accounts data and microeconomic theory.

This paper proposes an approach to CGE model building that is based on a 'basic model'. This basic model is built around a simple and transparent structure that is easily adapted to meet the requirements of different applications. The paper explains how such a simple structure is constructed for a basic CGE model and how the basic model is modified and extended for different applications.

In section 2, a few simple models are used to explain the basic structure of equilibrium models and how the model solutions are reached. The insights drawn from these simple models are then applied in section 3 to build a large policy-oriented global CGE model. This example illustrates how the approach is used to produce a simple and transparent equation structure and an initial solution strategy, starting from a standardised data structure.

This basic model can be used as a platform to build extensions or modifications required for more sophisticated applications. An example is provided in section 4 to demonstrate that, by adding a few more variables and equations, the basic global model can be quickly modified to include more sophisticated features.

The paper concludes with some remarks on the implications of this approach for modelling in general.

Introduction

Computable general equilibrium (CGE) models have been used to analyse a wide range of policy issues, such as the economic implications of trade liberalisation, economic development and greenhouse gas abatement. To meet the growing demand for these types of analyses, more CGE models have been built and many existing models have been constantly updated and modified.[1] As more features are added, the structure becomes increasingly complex and unwieldy, making it difficult to interpret results and adapt the models for new applications.

The difficulty of interpreting model results is not new for CGE model users. Although the optimising behaviour of individual economic agents is easy to conceptualise, their interactions in a general equilibrium model can make it difficult to understand the drivers of results. For this reason, CGE model are viewed as a "black box" (Bouët, 2008). The problem of tracking and understanding results is exacerbated by the complex structure of the computer code used in many models.

Numerous surveys attempt to help model users understand the behaviour of CGE models. Some attempt to demystify CGE model structures (Mitra-Kahn, 2008) while others try to reconcile differences in the results from different models and highlight differences in assumptions (Piermartini and Teh, 2005; Bouët, 2008). Back-of-the-envelope calculations are also used to isolate the drivers of results in a more sophisticated CGE model (Dixon and Rimmer, 2002).

As a model grows in size and complexity, it becomes difficult to modify the code to adapt it to new applications. Adaptability is especially important for institutional users, where the same modelling platform is often used for many applications (van Meijl 2012). There is a temptation to maintain modifications from one use to the next, which contributes to growth and complexity in the code. Features that are not relevant to the application at hand have the potential to interact with drivers and with each other, making new adaptations more costly.

Some attempts have been made to address this problem. One recent example is a modular approach developed by the Agricultural Research Institute in the Netherlands (LEI) (van Meijl and Woltjer, 2012). This approach modifies slightly the standard GTAP code to produce a number of modules; users can choose which modules they want to use for their

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[1] For example, since the first publication in 1993, the MMRF model of Australia has grown from a simple multi-region model to a more sophisticated large model that incorporates many new features for modelling federal-state budgetary relationship or carbon policies (Meagher and Parmenter, 1993 and Adams, et al. 2010).

applications via a graphical interface. This approach lowers the entry barrier for less experienced model users, an important advantage in a research institution.

However, the problems of result traceability and model adaptability are due, fundamentally, to the complexity of the model code itself. Without simplifying the model code, it is not possible to resolve these problems.

The main reason for code complexity is simply that too many variables are included, obscuring the basic theory and cluttering the model structure. Examples of such variables include:

- general purpose variables that can be used to facilitate an array of different policy applications

- variables that are not used in the solution process, but for reporting or analysing results, or for checking model balances or errors

- variables required for various implementations of one model, such as historical or decomposition mode, comparative static or dynamic versions.

In this paper, we argue that CGE models can be constructed using a simple structure. This is because equilibrium models consist of two types of equations that describe the model's theory and its solution. Moreover, CGE models use data from standard national accounts, especially input-output tables and they define agents' behaviours using familiar microeconomic theory.[2] The equation system should reflect the simplicity of the data and theory.

While the approach was developed in response to difficulties associated with adapting models to new applications, there are many other benefits to a simple and transparent structure. A simple structure makes a model easy to understand, which is crucial for interpreting results. It also lowers entry barriers for new model users and reduces the reliance on expert modellers.

Application-specific models can be developed from the basic CGE structure and less experienced model users can be involved in the model building process. This may be particularly important for institutions where a group of researchers might work simultaneously on different applications, developing them from the same basic structure. Expertise can be spread widely, favouring collaboration among researchers in contrast to the reliance on a few specialist modellers.

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[2] The Australian tradition has favoured the use of input-output tables and concentrated on the production account. Other traditions have used a social accounting matrix approach, with a greater emphasis on institutional accounts.

The rest of this paper elaborates on how such a simple structure is built. In the next section, the basic structure of equilibrium models is laid out in its simplest form, along with a way of thinking about how to reach a model solution. The insights drawn from these simple models are then applied in the following section to building a larger, policy-oriented global CGE model to illustrate how the approach produces a simple and transparent equation structure for a large model.

The simple equation structure is designed to make it easy to build extensions or modify. An example is provided in the fourth section to demonstrate that the basic global model can be easily adapted to a more sophisticated version, which incorporates bilateral capital stocks and investment flows, necessary for modelling foreign investment and capital accumulation.

The paper concludes with general remarks on the implications of the approach for model building.

Equilibrium model structures: some examples

CGE models are structurally simple, consisting of two types of equations. One type defines a set of variables used to describe the model theory, while the other specifies a set of conditions under which an equilibrium solution can be found.

These structural features are common to all equilibrium models, including the simplest ones. A single-good partial equilibrium model is used to illustrate this point.

A single-good partial equilibrium model

Let the supply of a good, X, be defined as a positive function of the good price $\boldsymbol{P}^*$ and the demand for the good, Q, as a negative function of its price $\boldsymbol{P}^*$ and a positive function of an exogenously given income $\bar{\boldsymbol{Y}}$, as follows,

$$X = f^S(\boldsymbol{P}^*) \tag{1}$$
$$Q = f^D(\ \bar{\boldsymbol{Y}}, \boldsymbol{P}^*) \tag{2}$$

These two equations contain three endogenous variables, in of which X and Q are *defined* in eqs. 1 and 2 and $\boldsymbol{P}^*$ is *undefined*.[3] To find a solution for this system, a third equation

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[3] A bold letter with a bar ⁻ on top is used to denote an exogenously *defined* variable, while a bold letter with superscript * is used to denote an *undefined* variable.

must be introduced. A natural candidate for this is a *market clearing condition* (MCC),[4] linking the two defined variables X and Q as follows,

$$X = Q \tag{3}$$

With this third equation, the system is closed. The number of endogenous variables equals the number of equations, so that a unique solution for the equilibrium values of the three endogenous variables can be found.

Despite its simplicity, this model already contains the two basic components of all equilibrium models: a 'theoretical structure' (eqs. 1 and 2 that define the consumption and production behaviours as some functions of an unknown price) and a 'solution structure' (the MCC that links eqs. 1 and 2 to determine the undefined price).

No matter how large or how complex a model's theory, the entire theoretical structure is built on a set of undefined prices. The supply and demand equations can potentially be substituted for two sides of the MCC equations to determine those undefined prices.

There are two potential approaches to solving an equilibrium model. Traditionally, the three equations can be used to form a system of simultaneous equations with three endogenous variables (X, Q, P^*) and one exogenous variable ($\bar{Y}$),

$$F(X, Q, P^*, \bar{Y}) = 0 \tag{4}$$

In this approach, the only distinction in variables is between exogenous and endogenous. All endogenous variables are treated equally as unknown. The method does not recognise which variable is a price and which is a quantity, and, therefore, it cannot tell whether the solution is economically valid or not.

A more economically intuitive approach is an iterative one. This approach sees the solution as resulting from a process of market interactions, which allows the unknown price to converge towards an equilibrium. Therefore, the model solution is not an instantaneous one, but the result of a *search process* which results in all markets clearing. In this process, not all endogenous variables are seen as unknown. For a given P^*, variables X and Q can be calculated from eqs. 1 and 2. If the two quantities are not equal, an adjustment can be made in P^* to reduce the gap between X and Q. This process continues until the gap is closed by an equilibrium value of P^*.

This "trial-and-error" approach holds some economic intuition. It is related to Walras' idea of *tâtonnement* (a trial-and-error process). To implement this approach, however, model

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[4] In this paper, the term MCC refers only to a specific set of equations, which are related to the general equilibrium solution of *undefined* variables, to be discussed in detail below.

builders and users need to identify which endogenous variables are *defined* and which are *undefined*.

- *Defined* variables are those determined either by an equation as a function of other variables, or set equal to a constant, outside the model system. The values of these variables can be thought of as "known" to the modeller in the sense that they can be calculated from the values of the variables that are used to define them.

- *Undefined* variables are not determined by any equation in the model or as a constant outside the model and thus their values are 'unknown'. How to determine the values of these variables needs to be specified explicitly in the model equation system.

Like variables, model equations can also be seen as composed of two groups: one used to define endogenous variables, and the other used to link some defined variables with each other, like MCCs that equate demands and supplies, or with undefined variables.

The structure of variables and equations in a CGE model can be illustrated in figure 1. The upper side of the line in the figure represents the number of variables, while the other side of the line represents the number of equations. To solve the model, the traditional approach divides all variables into exogenous and endogenous groups to make the number of endogenous variables equal to the number of equations.

Figure 1   **Structure of variables and equations**
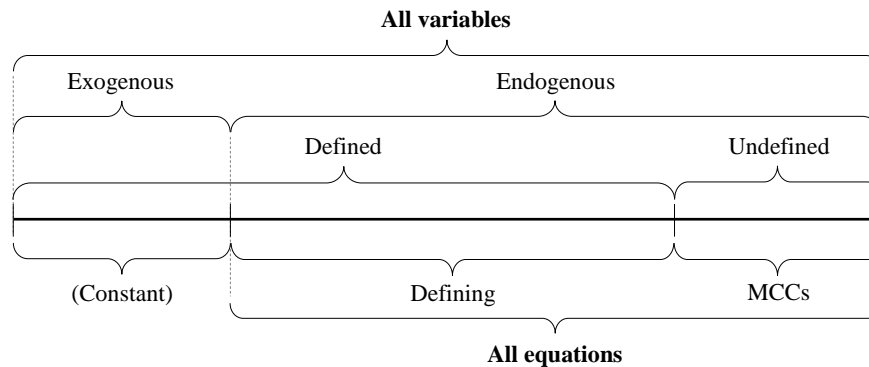
The iterative approach divides variables into defined and undefined. The equations can also be divided into two groups: one defining the endogenous group of defined variables and the other specifying MCCs. As the numbers of defined variables and the corresponding equations match, the only condition for the model to have a solution is to make sure each undefined variable has its corresponding MCC equation.

Applying this distinction to the model above, endogenous variables X, Q and exogenous variable $\bar{Y}$ are all defined and only P^* is undefined. The first two equations are used to

define the two endogenous variables X and Q, respectively, while the third equation is used to link X and Q to specify a MCC. As endogenous variables X and Q are already defined as functions, once the value of P^* is known, the values of X and Q can be recovered.

The entire system is fundamentally based on the undefined price P^*. By substituting out X and Q, the three-equation system can be reduced to the MCC equation with one undefined variable, P^*. This is also the case for all CGE models.

This simple example shows that not all variables and equations in a model are equally important for the model's solution. The solution lies fundamentally in the undefined variables that must be solved for with the corresponding MCCs. The defined variables, together with the defining equations, are also important because, only through them, can the supply and demand sides of the MCCs be defined as functions of the unknown prices.

Viewing variables and equations in this way is helpful to simplify model structure. Once the behavioural equations are written down and the undefined prices are identified, the model system can be closed with the corresponding MCCs.

The first closure for an equilibrium model is referred to as a "*natural*" closure. Alternative closures can be implemented easily by swapping exogenous and endogenous variables as is the practice in CGE modelling.

The complexity of a model and the computing time for a solution, depend, to a large extent, on the number of undefined variables. Large-scale CGE models with the consumption and production behaviours of multiple agents usually have a complex solution structure, composed of a large number of undefined price variables, not only for goods but also for factors, which are required to be solved jointly through their correspondent MCCs. In the following, two types of simple models with different production technologies are discussed.

A production-consumption model (DRTS)

Let the upward-sloping supply now be derived from the profit-maximising behaviour of a firm using capital (K) and labour (L) with a *diminishing return to scale* (*DRTS*) technology.[5] The supply of the good, X, is now defined as a function of two factor factors, R^* and W^*, as well as the price of the good, P^*,

$$X = f^S(P^*, R^*, W^*) \tag{5}$$

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[5] The *DRTS* production function is used here only for illustrating the implication of a conventional upward sloping supply curve for equilibrium models. The same supply curve can also be derived from other functional forms, such as *constant elasticity of transformation* (*CET*) functions.

This is derived from the firm's profit-maximising behaviour. Associated demands for the two factors are defined as functions of output and factor prices,

$$Q^K = f^K(X, \boldsymbol{R}^*, \boldsymbol{W}^*) \tag{6}$$
$$Q^L = f^L(X, \boldsymbol{R}^*, \boldsymbol{W}^*) \tag{7}$$

Income Y is now derived endogenously as the value of output,

$$Y = \boldsymbol{P}^* Q \tag{8}$$

The four new endogenous variables for factors, Q^K, Q^L, $\boldsymbol{R}^*$ and $\boldsymbol{W}^*$. Q^K and Q^L, are defined by eqs 6 and 7 while the two factor prices, $\boldsymbol{R}^*$ and $\boldsymbol{W}^*$, are undefined. Two more MCCs for factors must be introduced, in this case, with two exogenously given factor supplies,

$$Q^K = \bar{X}^K \tag{9}$$
$$Q^L = \bar{X}^L \tag{10}$$

Equations 2, 3 and 5 to 10 form a complete system for this production-consumption model. The solution to this system now lies in three undefined prices, $\boldsymbol{P}^*$, $\boldsymbol{R}^*$ and $\boldsymbol{W}^*$, to be solved simultaneously from three MCC, eqs 3, 9 and 10.

DRTS models are structurally more complex because they contain many undefined price variables. In fact, most CGE models assume *constant return to scale* (*CRTS*) production technology. The following section will show that, with *CRTS* technology, the number of undefined price variables can be reduced dramatically, so the solution structures of *CRTS* models is actually much smaller and simpler than other models.

A production-consumption model (CRTS)

Assume now that the firm uses *CRTS* technology. As a result, the price of the good is equal to unit cost, that is, a function of factor prices,

$$P = f^P(X, \boldsymbol{R}^*, \boldsymbol{W}^*) \tag{11}$$

This turns P from an undefined variable in the previous model into a defined variable.

With zero pure profit, the profit-maximising supply, X, can no longer be defined as a function of its own price P. Under *CRTS*, the supply curve is a horizontal line, determined by its price. Equation 3, $X = Q$, may still be included, but it is no longer a MCC for the good in the *CRTS* model. This equation actually defines X as a function of Q, because P is now defined by the sum of the costs and does not need a MCC to solve. This implies that

any quantity of *X*, defined by *Q*, is profit-maximising. Therefore, *X* can be substituted out by *Q* and, eq. 3 is deleted from the system.

Compared with the consumption model, the models with production are more complex (table 1). Under *DRTS*, the number of endogenous variables increases from three to eight and the undefined variables include both goods and factor prices. With *CRTS* technology, the supply function is replaced by a 'price function', due to "zero profit condition", and the solution structure includes only factor prices.

Table 1   **Comparing the structures of three simple models**

Consumption model			*DRTS model*			*CRTS model*		
Defined variables: endogenous								
Q	$Q = f^D(\bar{Y}, P^*)$	(2)	Q	$Q = f^D(Y, P^*)$	(2)	Q	$Q = f^D(Y, P)$	(2)
X	$X = f^S(P^*)$	(1)	X	$X = f^S(P^*, R^*, W^*)$	(5)	P	$P = f^P(R^*, W^*)$	(11)
			Q^K	$Q^K = f^K(X, R^*, W^*)$	(6)	Q^K	$Q^K = f^K(Q, R^*, W^*)$	(6)
			Q^L	$Q^L = f^L(X, R^*, W^*)$	(7)	Q^L	$Q^L = f^L(Q, R^*, W^*)$	(7)
			Y	$Y = P^* Q$	(8)	Y	$Y = R^* X^K + W^* X^L$	(8)
Defined variables: exogenous								
$\bar{Y}$			$\bar{X}^K, \bar{X}^L$			$\bar{X}^K, \bar{X}^L$		
Undefined variables and MCCs								
P^*	$X = Q$	(3)	P^*	$X = Q$	(3)			
			R^*	$Q^K = \bar{X}^K$	(9)	R^*	$Q^K = \bar{X}^K$	(9)
			W^*	$Q^L = \bar{X}^L$	(10)	W^*	$Q^L = \bar{X}^L$	(10)

In the *CRTS* model, the solution lies only in two factor prices, R^* and W^*, to be determined jointly from their MCCs. If one of these prices is chosen as the *numeraire*, one MCC equation becomes redundant and the other price may be calculated manually.

In most CGE models, factors are much fewer than commodities or industries. If production is characterised by *CRTS*, therefore, the undefined variables are confined to a small number of factor prices. No matter how many goods and industries a model may have, the number of undefined variables remains to be determined by the number of factors. This simplifies the solution structure significantly.

Table 1 also reveals another important feature of the *CRTS* models: the results of these models are driven only by demand-side activities. The output supplies are determined by their demands. With factor supplies fixed, the entire theoretical structure can be seen as defining factor demands as functions of their prices. The model can therefore be solved simply by adjusting the factor prices required for the demands to suit exogenous supplies.

In table 1, the equations are arranged in two sections: the defined variables are included in the first section, and the undefined variables are identified and linked with their underlying MCC equations in the second section. These two sections form a complete and closed equation system. In the following section, a large-sized global CGE model will be used as an example to demonstrate how this simple structure is applied in modelling practice.

A simple structure for a global CGE model

The model is built on a set of data drawn from the standard GTAP database. The database holds the key to the simplification of the equation system. In the following, the structure of the database is outlined first. This is followed by specifying the equation system of a basic global model. This simple structure makes the model easy to adapt to new applications.

To facilitate presentation, the basic model is developed in levels. Its computer implementation is not addressed. That said, some of the conventions used will be familiar to GEMPACK and GAMS users.

The sets used in the basic model are listed in table 2.

Table 2   **Sets used in the model and database**

Sets	*Definitions*
COM(1,…,*m*):	Commodities (indexed by *i* for commodity or *j* for industry)
REG(1,…,*n*):	Regions (indexed by *r* for home or *s* for host)
USR(COM,*hou*,*gov*,*inv*):	Users of commodities (indexed by *u*)
SRC(*dom*,*imp*):	Sources of commodities (indexed by *s*)
FAC(*lab*,*cap*,*land*):	Factors of production (indexed by *f*)
NCF(*lab*,*land*):	Non-capital factors (indexed by *l*)
MCOM(1,…,*h*):	Margin commodities (indexed by *m*)
NCOM(1,…,*k*):	Non-margin commodities (=COM–MCOM) (indexed by *i*)

Database structure

As CGE models are calibrated to a database, the structure of the equation system is, to a large extent, determined by the structure of the database. The value of each cell is explained by relevant variables. Therefore, the number of data items determines the number of variables to be introduced in the model system. To restrict the number of variables to be introduced, the number of data items is kept to a minimum. The following example explains how this can be achieved.

As in any other CGE model, at the centre of this global database is a set of regional input-output tables, valued at basic prices.[6] These tables are linked with each other through a set of trade matrixes, valued at various world prices.

A standard database structure for a representative region r is illustrated in Figure 2. The input-output table and the set of trade matrixes consist of six matrixes and two vectors, which represent economic activities, expressed in base year US dollars (millions). To link the database with the equation system, the values are expressed as a function of the relevant variables – in most cases, the product of a price and a quantity.[7]

1. Input-output table

The first part of the database is an extended input-output table, which can be divided into six matrixes and two vectors. The six matrixes are:

- Purchases of domestically produced and imported goods by domestic users valued at basic prices ($P_{(i,r,s)} Q_{(i,u,r,s)}$);
- Values of domestic indirect taxes/subsidies on these purchases ($VT_{(i,u,r,s)}^{dom}$);
- Exports of non-margin goods to each destination region, valued at basic prices ($P_{(i,r,"dom")} Q_{(i,r,s)}^{trd}$);[8]
- Values of taxes/subsidies on non-margin exports ($VT_{(i,r,s)}^{exp}$);
- Purchases of primary factors of production at the basic prices ($P_{(f,j,r)}^{fac} Q_{(f,j,r)}^{fac}$);
- Value of taxes on factor purchases ($VT_{(f,j,r)}^{fac}$).

The two vectors are:
- Exports of margin goods at basic prices ($V_{(m,r)}^{mexp} = P_{(m,r,"dom")} Q_{(m,r)}^{mexp}$);
- Value of production tax on industry outputs ($VT_{(j,r)}^{prd}$).

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[6] This example uses an input-output framework, which emphasises the production account. It would also be possible to use a SAM framework, which emphasises institutional distribution. Both frameworks emphasise income and other balances. For the purpose of this paper, the GTAP database was restructured to fit the standard input-output framework.

[7] The model is presented in levels to facilitate presentation. In this form, the model can be presented without having to define shares as would be required if the model was presented in linear percentage changes. Although the latter is convenient in many applications, the levels presentation is convenient for the purpose of the paper.

[8] The non-margin export matrix and the export tax matrix can also be seen as a part of the world trade data. They are included in the national input-output table to show the links between the regional data and the world trade data.

Figure 2 Database structure for a representative region (r)

1. Input-output table (r)

USER(u)

COM(j) *inv* *hou* *gov*   REG(s)

COM(i) $P_{(i,r,"dom")}\, Q_{(i,u,r,"dom")}$ $V_{(m,r)}^{mexp}$ $P_{(i,r,"dom")}\, Q_{(i,r,s)}^{trd}$

COM(i) $P_{(i,r,"imp")}\, Q_{(i,u,r,"imp")}$

COM(i) $VT_{(i,u,r,"dom")}^{dom}$ $VT_{(i,r,s)}^{exp}$

COM(i) $VT_{(i,u,r,"imp")}^{dom}$

FAC(f) $P_{(f,j,r)}^{fac}\, Q_{(f,j,r)}^{fac}$

FAC(f) $VT_{(f,j,r)}^{fac}$

$VT_{(j,r)}^{prd}$

$(= P_{(i,r,"dom")}\, Q_{(j,r)}^{dom})$

2. World trade matrixes (r)

REG(s)   REG(s)

$P_{(i,s,r)}^{imp}\, Q_{(i,s,r)}^{trd}$ $\Sigma_m\, V_{(m,i,s,r)}^{mimp}$
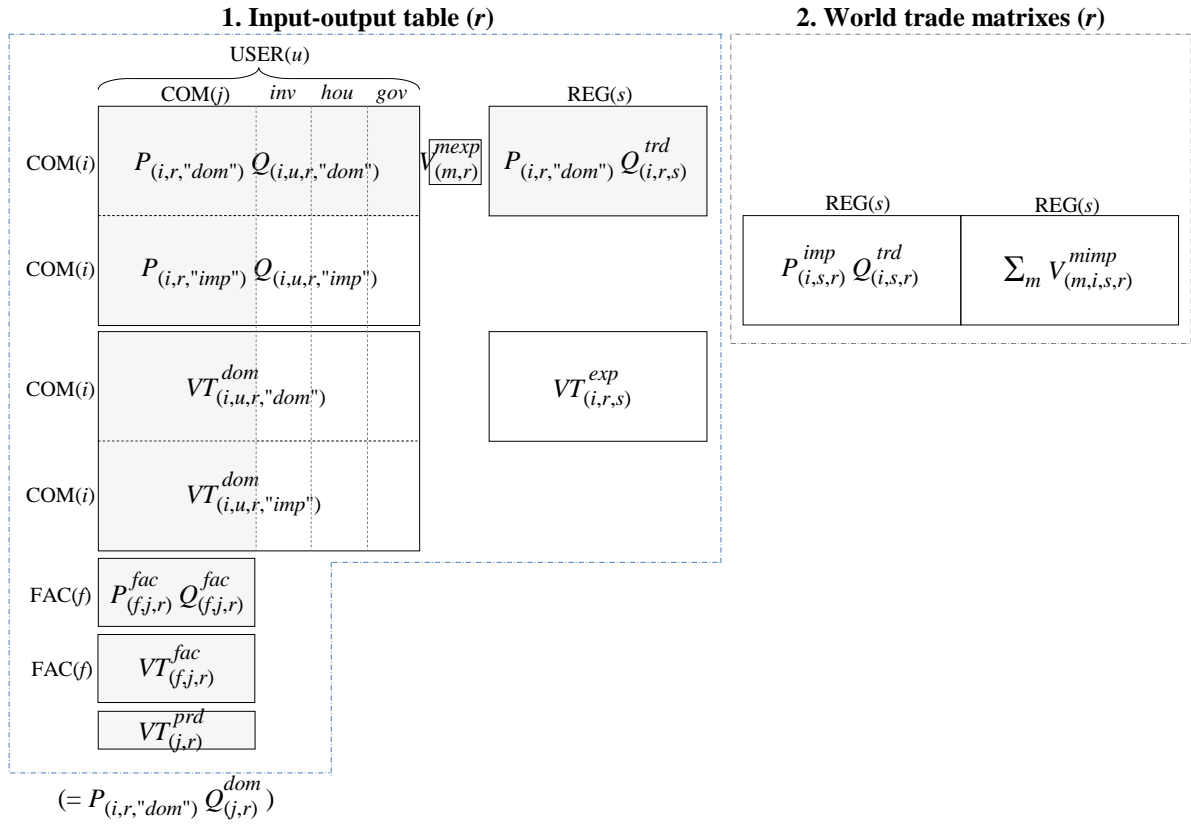
The format of this input-output table is convenient for verifying important accounting balances. For example, the total sales of products at basic prices (the row total $= P_{(i,r,"dom")}$ $(\Sigma_u\, Q_{(i,u,r,"dom")} + \Sigma_s\, Q_{(i,r,s)}^{trd} + (Q_{(m,r)}^{mexp})))$ should be equal to the basic values of their outputs produced by industries (the column total $= P_{(j,r,"dom")}\, Q_{(j,r)}^{dom}$). This balancing condition indicates an important relationship to be used later in the model system, that is, the supply of a product must be equal to the sum of its demands.

2. World trade matrixes

The second part of the database has only two matrixes: a domestic basic value import matrix $(P_{(i,r,s)}^{imp}\, Q_{(i,r,s)}^{trd})$ and an import margin matrix $(\Sigma_m\, V_{(m,i,r,s)}^{mimp})$. Other trade data can be derived from this database. For example,

- the *fob* values of exports: $(P_{(i,r,s)}^{fob}\, Q_{(i,r,s)}^{trd}) = P_{(i,r,"dom")}\, Q_{(i,r,s)}^{trd} + VT_{(i,r,s)}^{exp}$

- the *cif* values of imports: $(P_{(i,r,s)}^{cif}\, Q_{(i,r,s)}^{trd}) = (P_{(i,r,s)}^{fob}\, Q_{(i,r,s)}^{trd}) + \Sigma_m\, V_{(m,i,s,r)}^{mimp}$

- the values of import tariffs: $VT_{(i,s,r)}^{imp} = P_{(i,s,r)}^{imp}\, Q_{(i,s,r)}^{trd} - (P_{(i,r,s)}^{cif}\, Q_{(i,r,s)}^{trd})$

These "derived" data items need not be included. Similarly, data aggregations, such as regional incomes or savings, can be derived from the database and need not be added as separate items too. As a principle, a database should contain only "primary" data, valued at basic prices, so as to produce a parsimonious database.

Each region also has an aggregate capital stock, which is assumed to be owned and used by that region and tax revenues on factor incomes. Overall, the entire database contains only 12 data items,[9] which implies that few variables are required to describe the model.

In the following section, these variables are used to build the equation structure. The model theory consists of the basic components that are widely used in similar global models. The purpose of building it here is to demonstrate the simplicity of the required structure.

Equation structure

The general structure of CGE models can be illustrated in figure 3. As in simple models, it has two types of equations, one used to define the theory and the other used to specify solution conditions (figure 3). In the theory part, the behaviours of different users are defined separately. In the solution part, there are MCC equations. Additional equations may be needed to allocate savings to investment across industries or regions.

The remainder of this section shows how the simple structure in table 3 is used to build this global model. To keep the system simple and transparent, only those variables that are essential to the model's solution are introduced; any reporting or balance checking variables are not presented here.

The theory for this basic global model can be specified with 22 equation blocks,[10] organised in four sections:

1. Consumption: region and user demands for goods (table 3)

2. Production: industry outputs and demands for factors (table 4)

3. Income distribution: final user's income and expenditure (table 5)

4. Solution condition: market equilibrium for factors (table 6).

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9 To cover the same amount of information, a standard GTAP database uses 36 data items.

10 The variables and equations in the model structure are also referred to as 'blocks'. This is because most of them are multi-dimensional ones, which represent many individual variables and equations.

Figure 3   General structure of CGE model equations with CRTS technologies

|		
Theory	Consumption by user	Demands for goods
		Non-basic prices for goods
	Production by industry	Basic prices for goods (zero pure profit condition)
		Demands for factors
		Non-basic prices for factors
	Income/expenditure by final user	Income for final users
		Expenditure for final users
		Expected rates of return (EROR) to investment
Solution	Allocation of factors for production	MCCs for factors to determine *undefined* basic prices for factors
	Allocation of savings for investment	Equalisation of ERORs to determine allocation of savings

The first three sections are used to define the model theory: consumption and production behaviours and the price system in which these behaviours are linked. The last section specifies the general equilibrium conditions under which the model can be solved. The behaviours of economic agents are determined by goods and factors prices. The basic price is the foundation of this system, from which all other prices are derived (figure 4).

Most equations define an endogenous variable, which appears on the left hand of an equation. A defined variable is described by the equation title so that a separate list of variable names is not provided. Undefined variables and exogenous variables are identified and named wherever they are first introduced. The equations specifying behaviours are highlighted by a box, which is normally followed by equations that define the variables used only in the behavioural equations. The discussion focusses on how a given theory is specified with few variables and equations, rather than any specific functional forms.[11]

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[11] For example, the theory describing consumer demand, shown in eq.8 of table 3, might be specified as Cobb-Douglas, Extended Linear Expenditure System or as Constant Difference of Elasticity. Any of these specifications fits in the framework of this model.

Figure 4 **Price structure of the global model**
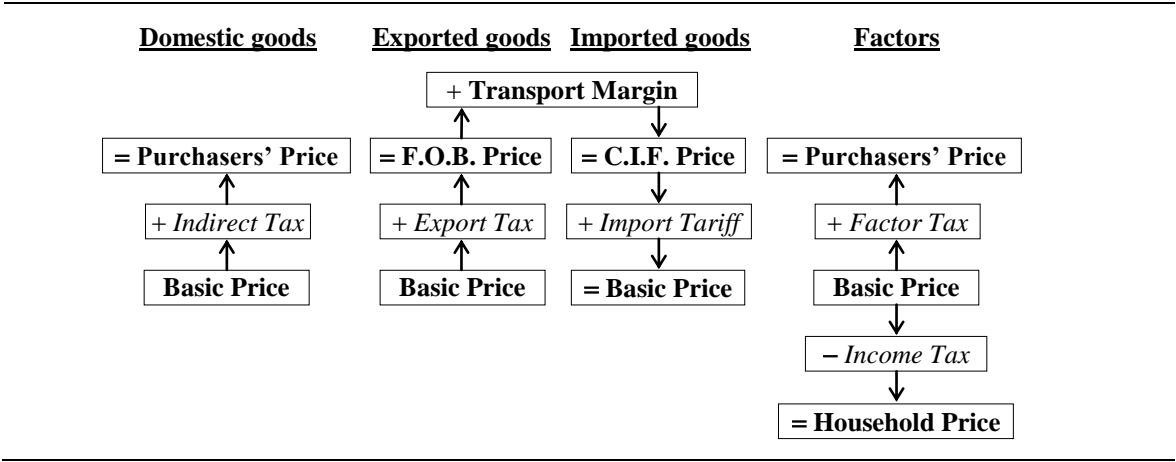
1. Demand for goods by user and region (eqs.1-9)

This section describes consumption behaviours in the model. Due to the simple database only nine equation blocks are needed to capture the consumption behaviours of all four users (table 3). This section is centred around three demand functions: two-tier Armington demands for imports (eq. 1) and for domestically produced goods and import composites (eq. 6), and top-level demands for composite goods (eq. 8).

The upper-tier CES demands represent users' demands for composite goods, composed of domestic goods and import composites. The functional form of composite demands depends on the nature of its user. Industries or investors use composite goods as intermediate inputs for goods production or capital formation; their demands are assumed to be of Leontief. The functional forms for household or government demands could be chosen from a range of options. The simple structure of the equation system allows for alternative functions to be implemented quickly.

Table 3   **Structure for a basic global CGE model (1): consumption**

1. Region and user's demands for goods (1-9)

(1) CES demand of region s for import i from region r

$$Q_{(i,r,s)}^{trd} = CES\,(P_{(i,r,s)}^{imp},\, P_{(i,s,"imp")},\, Q_{\_u(i,s,"imp")}) \qquad\qquad (i \in COM;\ r,s \in REG)$$

where $P_{(i,s,"imp")}$ is a CES price index for composite import i in region s, to be defined in eq. 16 in table 4.

(2) Domestic basic prices of import i from region r to region s

$$P_{(i,r,s)}^{imp} = P_{(i,r,s)}^{cif}\,(1 + \mathbf{\textcolor{red}{t_{(i,r,s)}^{imp}}}) \qquad\qquad (i \in COM;\ r,s \in REG)$$

where $\mathbf{\textcolor{red}{t_{(i,r,s)}^{imp}}}$ is the *ad valorem* rate of an import tariff.

(3) World *cif* price of import i from region r to region s

$$P_{(i,r,s)}^{cif} = P_{(i,r,s)}^{fob} + \frac{1}{Q_{(i,r,s)}^{trd}} \sum_m Q_{(m,i,r,s)}^{mrg}\, P_{(m)}^{mrg} \qquad\qquad (i \in COM;\ r,s \in REG)$$

(4) World *fob* price of export i from region r to region s

$$P_{(i,r,s)}^{fob} = P_{(i,r,"dom")}\,(1 + \mathbf{\textcolor{red}{t_{(i,r,s)}^{exp}}}) \qquad\qquad (i \in COM;\ r,s \in REG)$$

where $\mathbf{\textcolor{red}{t_{(i,r,s)}^{exp}}}$ is the *ad valorem* rate of an export tax.

(5) Regional total demand for good i from source s

$$Q_{\_u(i,r,s)} = \sum_u Q_{(i,u,r,s)} \qquad\qquad (i \in COM;\ r \in REG;\ s \in SRC)$$

(6) CES demand for good i from source s by user u in region r

$$Q_{(i,u,r,s)} = CES\,(P_{(i,u,r,s)}^{t},\, P_{\_s(i,u,r)}^{t},\, Q_{\_s(i,u,r)}) \qquad (i \in COM;\ u \in USR;\ r \in REG;\ s \in SRC)$$

where $P_{\_s(i,u,r)}^{t}$ is a CES price index for composite good i for user u in region r,

$$P_{\_s(i,u,r)}^{t} = CES\,(P_{(i,u,r,"dom")}^{t},\, P_{(i,u,r,"imp")}^{t}) \qquad\qquad (i \in COM;\ u \in USR;\ r \in REG)$$

(7) Purchasers' price of good i from source s for user u in region r

$$P_{(i,u,r,s)}^{t} = P_{(i,r,s)}\,(1 + \mathbf{\textcolor{red}{t_{(i,u,r,s)}^{dom}}}) \qquad\qquad (i \in COM;\ u \in USR;\ r \in REG;\ s \in SRC)$$

where $\mathbf{\textcolor{red}{t_{(i,u,r,s)}^{dom}}}$ is the *ad valorem* rate of an indirect tax.

(8) Demand for composite good i by user u in region r

$$Q_{\_s(i,u,r)} = \begin{cases} Leontief\,(Q_{(u,r)}^{dom}) & (i,\,u \in COM;\ r \in REG) \\[4pt] f\,(E_{(u,r)},\, P_{\_s(i_1,u,r)}^{t},\, \ldots,\, P_{\_s(i_n,u,r)}^{t}) & (i \in COM;\ u{=}hou;\ r \in REG) \\[4pt] f\,(E_{(u,r)},\, P_{\_s(i,u,r)}^{t}) & (i \in COM;\ u{=}gov;\ r \in REG) \\[4pt] Leontief\,(E_{(u,r)},\, P_{\_is(u,r)}^{t}) & (i \in COM;\ u{=}inv;\ r \in REG) \end{cases}$$

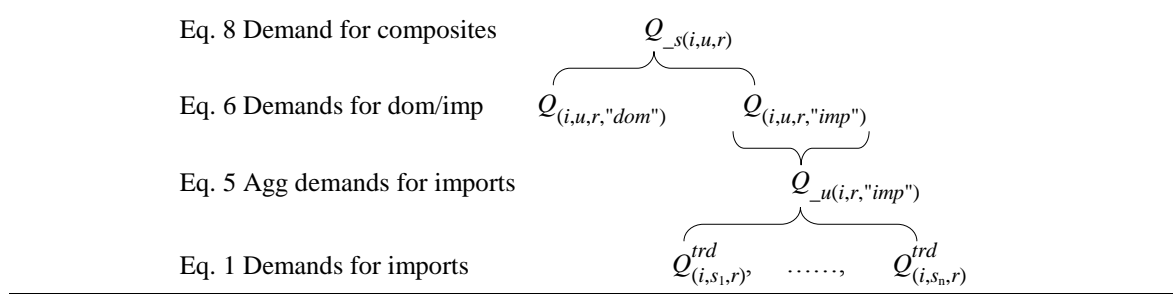(9) Purchases' price index for composite goods for user u in region r

$$P_{\_is(u,r)}^{t} = \frac{1}{Q_{\_is(u,r)}} \sum_i Q_{\_s(i,u,r)}\, P_{\_s(i,u,r)}^{t} \qquad\qquad (u \in USR;\ r \in REG)$$

where $Q_{\_is(u,r)}$ is total demand for composite goods by user u in region r,

$$Q_{\_is(u,r)} = \sum_i Q_{\_s(i,u,r)} \qquad\qquad (u \in USR;\ r \in REG)$$

The lower- and the upper-tier CES demand functions are linked with each other by a variable that aggregates the demands for composite imports over all users, $Q_{\_u(i,r,s)}$, defined in eq. 5. The nesting structure of the behavioural equations in this section is illustrated in figure 5.

Figure 5   **Structure of consumption equations**

Eq. 8 Demand for composites	$Q\_s(i,u,r)$
Eq. 6 Demands for dom/imp	$Q_{(i,u,r,"dom")}$ $Q_{(i,u,r,"imp")}$
Eq. 5 Agg demands for imports	$Q\_u(i,r,"imp")$
Eq. 1 Demands for imports	$Q^{trd}_{(i,s_1,r)}, \quad \ldots\ldots, \quad Q^{trd}_{(i,s_n,r)}$

Eqs. 2-5 are used to define variables used in the lower-tier CES demands for imported goods in eq. 1. Eq. 7 defines a price variable, $P^{t}_{(i,u,r,s)}$, used in the upper-tier CES demand in eq. 6. Eq. 9 defines another price variable, $P^{t}_{\_is(u,r)}$, used in the top-level demand function in eq. 8.

The other endogenous variables in this section are defined in the following sections.

2. Industry outputs and demands for factors (eqs.10-15)[12]

The structure of production is very similar to that of the simple *CRTS* model. It contains six equation blocks (table 4), three of which are in the simple *CRTS* model:

- eq. 10 defines $Q^{dom}_{(j,r)}$, the aggregate demands for regional outputs

- eq. 12 defines $Q^{fac}_{(f,j,r)}$, the CES demands for factors

- eq. 15 defines $P_{(j,r,s)}$, the basic prices of regional products

These equations are common in all *CRTS* models. Eqs. 10 and 15 are implied by the input-output balancing conditions.

Eq. 13 defines the purchaser prices of factors, $P^{tfac}_{(f,j,r)}$), a variable used to define factor demands. Eq. 11 defines the demand for trade margin, which is required by the transportation data in the database.

---

[12] Demands for intermediate inputs were addressed in the previous section with the demands from all users, including industry users.

Table 4   **Structure of a basic global CGE model (2): production**

2. Industry outputs, demands for factors (10-15)

(10) Total demand for output from industry j in region r

$$Q_{(j,r)}^{dom} = \begin{cases} Q_{\_u(j,r,"dom")} + \sum_s Q_{(j,r,s)}^{trd} & (j \in NCOM; r \in REG) \\[2mm] Q_{\_u(j,r,"dom")} + \sum_s Q_{(j,r,s)}^{trd} + Q_{(j,r)}^{mexp} & (j \in MCOM; r \in REG) \end{cases}$$

(11) CES demand for margin good m exported from region r

$$Q_{(m,r)}^{mexp} = CES\,(P_{(m,r,"dom")},\, P_{(m)}^{mrg},\, Q_{\_irs(m)}^{mrg}) \qquad (m \in MCOM; r \in REG)$$

where $P_{(m)}^{mrg}$ is a CES price index for composite margin good m,

$$P_{(m)}^{mrg} = CES\,(P_{(m,r_1,"dom")},\, \dots,\, P_{(m,r_n,"dom")}) \qquad (m \in MCOM)$$

and $Q_{\_irs(m)}^{mrg}$ is the world demand for composite margin good m, a global sum of demands of trade in non-margin good i for composite margin good m, defined as a Leontief function,

$$Q_{(m,i,r,s)}^{mrg} = Leontief\,(Q_{(i,r,s)}^{trd}) \qquad (m \in MCOM; i \in COM; r,s \in REG)$$

(12) CES demand for factor f used by industry j in region r

$$Q_{(f,j,r)}^{fac} = CES\,(P_{(f,j,r)}^{tfac},\, P_{\_f(j,r)}^{tfac},\, Q_{(j,r)}^{dom}) \qquad (f \in FAC; j \in COM; r \in REG)$$

where $P_{\_f(j,r)}^{tfac}$ is a CES price index for composite factor in industry j in region r,

$$P_{\_f(j,r)}^{tfac} = CES\,(P_{("land",j,r)}^{tfac},\, P_{("cap",j,r)}^{tfac},\, P_{("lab",j,r)}^{tfac}) \qquad (j \in COM; r \in REG)$$

(13) Purchaser price for factor f in industry j of region r

$$P_{(f,j,r)}^{tfac} = P_{(f,j,r)}^{fac}\,(1 + \boldsymbol{t_{(f,j,r)}^{fac}}) \qquad (f \in FAC; j \in COM; r \in REG)$$

where $\boldsymbol{t_{(f,j,r)}^{fac}}$ is the *ad valorem* rate of a tax on factor f used in industry j of region r.

(14) CET supply of land in industry j in region r (industry **MCC for land**)

$$X_{(j,r)}^{lnd} = CET\,(\boldsymbol{P^{*\,fac}_{("land",j,r)}},\, P_{\_j("land",r)}^{fac},\, \boldsymbol{\bar{X}_{("land",r)}^{fac}}) \qquad (j \in COM; r \in REG)$$

where $\boldsymbol{\bar{X}_{("land",r)}^{fac}}$ is the exogenous supply of land in region r, $\boldsymbol{P^{*\,fac}_{("land",j,r)}}$ is the *undefined* basic price of land and $P_{\_j("land",r)}^{fac}$ is a CET price index for composite land in region r,

$$P_{\_j("land",r)}^{fac} = CET\,(\boldsymbol{P^{*\,fac}_{("land",j_1,r)}},\, \dots,\, \boldsymbol{P^{*\,fac}_{("land",j_m,r)}}) \qquad (r \in REG)$$

(15) Basic price for good j from source s in region r

$$P_{(j,r,s)} = \begin{cases} \dfrac{1}{Q_{(j,r)}^{dom}}\left(\sum_i Q_{\_s(i,j,r)}\, P_{\_s(i,j,r)}^{t} + \sum_f Q_{(f,j,r)}^{fac}\, P_{(f,j,r)}^{tfac}\right)(1 + \boldsymbol{t_{(j,r)}^{prd}}) & (j \in COM; r \in REG; s=dom) \\[3mm] CES\,(P_{(j,k_1,r)}^{imp},\, \dots,\, P_{(j,k_n,r)}^{imp}) & (j \in COM; r \in REG; s=imp) \end{cases}$$

where $\boldsymbol{t_{(j,r)}^{prd}}$ is the rate of a production tax.

Eq. 14 is a CET supply of industry-specific land. This equation can be regarded as optional because it is based on a particular assumption on the supply of land. According to this assumption, land is imperfectly transformable between its uses in certain industries. [13]

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[13] Alternatively, the industry-specific supply of land, $X_{(j,r)}^{lnd}$, may be set as exogenous so that eq. 14 is removed and replaced by an equation to define the exogenous variable $\bar{X}_{("land",r)}^{fac}$ as the sum of $X_{(j,r)}^{lnd}$ over all industries.

3. Final users' income and expenditure (eqs.16-19)

Compared with the simple model, this income section is more complex because this model has three final users (households, governments and investors) and a complex income distribution among them.

Four equation blocks (table 5) define incomes and expenditures for three final users. These are all straightforward accounting relationships.

- Eq. 16 defines incomes for three final users, $Y_{(u,r)}$: household disposable income is equal to the sum of factor returns, net of income tax; government income is tax revenue; investor's income is derived from household and government savings. Eq. 17 defines price $P_{(f,r)}^{tinc}$, which is required to define household income in eq. 16.

- Eq. 18 defines expenditures for three final users, $E_{(u,r)}$: household and government incomes, net of savings, equal their expenditures on final consumptions, while regional investment is funded by domestic savings and *net foreign investment* (NFI), $Y_{(r)}^{NFI}$.

Table 5   **Structure of a basic global CGE model (3): income and expenditure**

3. Final user's income and expenditure (16-19)

(16) Household disposable income, government income and regional savings

$$
Y_{(u,r)} = \begin{cases} \sum_{f\in\text{FAC}} P_{(f,r)}^{tinc}\ \bar{X}_{(f,r)}^{fac} & (u=hou;\ r\in\text{REG}) \\ (Total\ Tax\ Revenue) & (u=gov;\ r\in\text{REG}) \\ \sum_{i=hou,gov} Y_{(i,r)}\ s_{(i,r)} & (u=inv;\ r\in\text{REG}) \end{cases}
$$

where $s_{(i,r)}$ is the saving rate for final user i (=*hou*, *gov*).

(17) Post-income tax price for factor f in region r

$$P_{(f,r)}^{tinc} = P^{*\,fac}_{\_j\,(f,r)}\,(1-t_{(f,r)}^{inc}) \qquad (f\in\text{FAC};\ r\in\text{REG})$$

where $t_{(f,r)}^{inc}$ is the *ad valorem* rate of a tax on the income of factor f.

(18) Expenditure of final user u in region r

$$
E_{(u,r)} = \begin{cases} Y_{(u,r)}\,(1-s_{(u,r)}) & (u=hou,\ gov;\ r\in\text{REG}) \\ Y_{(u,r)} + Y_{(r)}^{NFI} & (u=inv;\ r\in\text{REG}) \end{cases}
$$

(19) Net foreign investment in region r

$$Y_{(r)}^{NFI} = P_{(r)}^{NFI}\ \bar{Q}_{(r)}^{NFI} \qquad (r\in\text{REG})$$

where $P_{(r)}^{NFI}$ is the price of net foreign investment, defined as

$$P_{(r)}^{NFI} = \frac{1}{\bar{Q}_{(r)}^{NFI}}\,(\sum_{u\in\text{FUSR}} Q_{\_is(u,r)}\,P_{\_is(u,r)}^{t} - Q_{(r)}^{gdp}\,P_{(r)}^{gdp}) \qquad (r\in\text{REG})$$

where $Q_{(r)}^{gdp}$ and $P_{(r)}^{gdp}$ are real GDP and GDP deflator in region r, respectively.

If NFI flows were zero, the variable $Y_{(r)}^{NFI}$ would be set as endogenous and the theoretical system would be complete. However, NFI is not zero in the database and, therefore, must be determined by the model theory. Several options are possible. Eq. 19 provides a simple option.[14]

Eq. 19 defines regional NFI as the product of its price $P_{(r)}^{NFI}$ and quantity, $\bar{Q}_{(r)}^{NFI}$. The latter can be set as exogenous to fix NFI in real terms. As NFI inflows reflect the values of net imports, eq. 19 effectively implies that NFI inflows are determined by regional trade balances.

In this section, income and expenditure variables are aggregated from the variables that have already been introduced in the system. There is no need to introduce new variables for aggregation purpose, for example, defining each type of factor incomes or tax revenue separately and then aggregating them together. These aggregated variables may be useful for reporting modelling results, but lengthen the equation system and can slow solution time. More importantly, the inclusion of nonessential variables will make the model theory less transparent and more difficult to understand.

4. Market equilibrium conditions for factors (eqs.20-22)

The last section specifies the MCCs for factors (table 6). In this model, the supplies of three factors, $\bar{X}_{(f,r)}^{fac}$, are all set as exogenous at the regional level. Land is assumed to be imperfectly mobile across industries; it is supplied according to a CET supply function, as defined in eq. 14. Therefore, the MCC for land is specified at the industry level (eq. 20). This MCC is required to determine the general equilibrium values of the basic prices of industry-specific land, $P^{*fac}_{("land",j,r)}$, which is an undefined variable.

Table 6   Structure of a basic global CGE model (4): solution

4. Market equilibrium conditions for factors (20-22)

(20) Industry's MCC for land

$$X_{(j,r)}^{lnd} = Q_{("land",j,r)}^{fac}$$ ($j\in$COM; $r\in$REG)

(21) Regional MCC for labour and capital

$$\bar{X}_{(f,r)}^{fac} = \sum_j Q_{(f,j,r)}^{fac}$$ (f=lab, cap; $r\in$REG)

where $\bar{X}_{(f,r)}^{fac}$ is the exogenous supply of factor f (lab, cap) in region r.

(22) Basic prices for factor f (lab, cap) (**PEC for labour and capital**)

$$P_{(f,j,r)}^{fac} = P^{*fac}_{\_j\,(f,r)}$$ (f=lab, cap; $j\in$COM; $r\in$REG)

where $P^{*fac}_{\_j\,(f,r)}$ is the undefined price of factor f (lab, cap) in region r.

---

[14] An alternatively option is shown below.

Capital and labour are assumed to be homogenous and perfectly mobile across industries. As a result, the basic prices of industry-specific capital or labour are expected to be equal across all industries in equilibrium. This *price equalisation condition* (PEC) is given in eq. 22, in which the variable for regional basic prices of capital and labour, $P^{*~fac}_{\_j~(f,r)}$ ($f = cap$, lab), is undefined. To determine the general equilibrium values of these two prices, two MCC equations are set up for capital and labour, respectively (eq. 21).[15]

Eqs. 1 to 22 complete the specification of this global CGE model:[16]

- The 19 equations in tables 3 to 5 define 19 endogenous variable blocks that describe the model's theory

- the three equations in table 6 specify the MCCs for the three factors, which are used to find a solution for three undefined basic factor prices, and the associated PECs for mobile factors.

There is no need to count the number of endogenous variables and equations because every *defined* endogenous variable is defined by one equation in the theoretical part: their numbers are always identical. To solve the model, attention only needs to be paid to the *undefined* variables. It is important to clearly identify which variables are not defined and what MCCs are needed to determine them. In the solution part, therefore, the numbers of undefined variables and their associated MCC/PEC equations must be equal too.

The undefined variables in *CRTS* models are limited to basic factor prices. In this global model, the basic prices of three factors, $P^{*~fac}_{("land",j,r)}$, $P^{*~fac}_{\_j~("lab",r)}$ and $P^{*~fac}_{\_j~("cap",r)}$, are not defined. Therefore, three MCCs are needed to determine them. These variables and equations comprise the solution structure (table 7).

With factor supplies exogenously given, the general equilibrium values of these factor prices could be solved simultaneously from their MCC equations. This is because the demands for factors can be expressed, fundamentally, as a function of their own prices. All other endogenous variables could be substituted out, leaving only a small number of factor MCC equations in the equation system for solving the factor prices.

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[15] If, alternatively, capital supply is assumed to be fixed at the industry level, the basic prices of industry-specific capital stocks are not equal and become undefined. The MCCs for capital are then set at the industry level.

[16] According to Walras' law, in a general equilibrium model, one price variable can be chosen as a *numeraire* and, therefore, one equation becomes redundant and can be removed from the system.

Table 7   **Undefined variables and MCC equations in the basic model**

	Undefined variables		MCC equations	
$P^{*fac}_{("land",j,r)}$	Basic price of land	Eq.20	MCC for sectoral land	$Q^{fac}_{("land",j,r)}$
$P^{*fac}_{\_j("lab",r)}$	Basic price of labour	Eq.21	MCC for regional labour	$\bar{X}^{fac}_{("lab",r)}$
$P^{*fac}_{\_j("cap",r)}$	Basic price of capital	Eq.21	MCC for regional capital	$\bar{X}^{fac}_{("cap",r)}$

With this 'natural' closure, an initial general equilibrium solution can be computed. Alternative closures may be implemented by swapping selected exogenous and endogenous variables without changing the basic structure of the model.

Once the model's equation system is completed, other nonessential variables could be introduced to process modelling results for reporting and diagnostic purposes. Some examples of these variables may include

- Aggregations of quantities or values

- Weighted averages of various prices

- Decomposition of changes in outputs or real incomes

- Welfare indicators or decomposition

Adding these variables do not alter the model's solution, because they are not part of the equation system that produces the model's solution. Therefore, it is important to define these variables separately to maintain the simplicity and integrity of the model's essential equation system.

5. An alternative structure for allocating regional savings (eqs. 23-26)

In the basic model above, the NFI in a region was determined by its given trade pattern. Alternatively, NFI could be determined by investment behaviour.[17] This section shows how this alternative structure is implemented in the basic model.

Assuming that regional savings can be invested internationally seeking higher returns, this requires eq. 19 (table 5), which ties NFIs with trade patterns, to be replaced by four equations that specify investor's behaviour (table 8).

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[17] This assumption is adopted in the GTAP model (Hertel, 1997).

Table 8

An alternative structure for allocating regional savings to determine trade balance

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5. Global investment of regional savings (23-26)

(23) Expected rate of return to investment in region r

$$R^e_{(r)} = f(P^{tinc}_{("cap",r)}, P^t_{\_is("inv",r)}, X^{cap}_{t+1\,(r)}, \bar{X}^{fac}_{("cap",r)}) \qquad (r \in REG)$$

(24) Capital stock to be used in the next period

$$X^{cap}_{t+1\,(r)} = \bar{X}^{fac}_{("cap",r)}(1 - r^{dep}_{(r)}) + Q_{\_is("inv",r)} \qquad (r \in REG)$$

where $r^{dep}_{(r)}$ is the rate of capital depreciation.

(25) Equalisation of regional expected rates of return (**PEC for foreign investment**)

$$R^e_{(r)} = R^{*e}_{\_r} \qquad (r \in REG)$$

where $R^{*e}_{\_r}$ is the general equilibrium rate of return for the world as a whole.

(26) Global saving and investment equality (**MCC for global savings**)

$$\sum_r Y_{("inv",r)} = \sum_r E_{("inv",r)}$$

In this table, eqs. 23 and 24 define regional expected rates of return, $R^e_{(r)}$, as a function of the current return to capital, and the beginning- and end-of-period capital stocks.[18]

Eq. 25 specifies that expected rates of return are equalised globally in equilibrium. This PEC is associated with a MCC. This is required to determine the value of the NFI inflows, $Y^{*NFI}_{(r)}$, which becomes a undefined variable when eq. 19 is removed from the system.

Eq. 25 also introduces a new undefined variable, $R^{*e}_{\_r}$, the equilibrium rate of return to investment for the world as a whole. This variable ensures that the global savings-investment market clears. This global MCC for savings is specified in eq. 26.

The new model has 25 equation blocks, 21 of which are from the basic model specified above. The undefined variables and corresponding equations for MCCs/PECs are listed in table 9. The new savings and investment behaviour requires two more undefined variables than in the basic system.

Again, if one of the price variables is chosen as the *numeraire*, a corresponding equation becomes redundant and can be removed. A natural candidate for the redundant equation is eq. 26, global savings and investment equality.[19] With this equation removed, the number of equation blocks in the basic model is reduced to 24.

---

[18] This is an assumption used in many CGE models. Other specifications are possible. For example, Pant (2012) proposes a generic approach to investment allocation in recursive dynamic CGE models.

[19] A variable calculating the difference between global net saving and net investment in the GTAP model is called "Walras slack". It is used to check model's overall balance.

Table 9	**Undefined variables and MCC/PEC equations in the alternative basic model**			
	Undefined variables		*MCC/PEC equations*	
$P^{*fac}_{("land",j,r)}$	Basic price of industrial land	Eq.20	MCC for sectoral land	$Q^{fac}_{("land",j,r)}$
$P^{*fac}_{j("lab",r)}$	Basic price of regional labour	Eq.21	MCC for regional labour	$\bar{X}^{fac}_{("lab",r)}$
$P^{*fac}_{j("cap",r)}$	Basic price of regional capital	Eq.21	MCC for regional capital	$\bar{X}^{fac}_{("cap",r)}$
$Y^{*NFI}_{(r)}$	Net foreign investment inflow	Eq.25	PEC for rates of return	$R^{e}_{(r)}$
$R^{*e}_{\_r}$	World expected rate of return	Eq.26	MCC for global savings	$\sum_r Y_{("inv",r)}$

In this model, regional net trade is determined by the equalisation of the expected rates of return to investments, rather than by the given trade pattern.

An extension of the basic model

The simple structure of the basic model can be used as a platform for modifications and extensions. Some examples include

- Splitting a commodity to allow for multiple goods to be produced by a single industry

- Splitting an industry to allow for alternative technologies to be used to produce the same output

- Introducing multiple households with different consumption patterns and income sources

- Introducing CET supply of exports

- Incorporating carbon emission data and carbon policy instruments

This section demonstrates how easily the basic global model can be transformed into a more sophisticated version to analyse policies that affect foreign direct investment (FDI) and services trade via commercial presence.

In the basic model above, there is an inconsistency between capital stocks and investment. On the one hand, the capital stock is fully owned and used by each region, which implies no foreign capital ownership. On the other hand, regional savings are allowed to be invested across regions, which implies the existence of foreign investment in the absence of foreign capital stocks. One way of overcoming this inconsistency is to introduce both bilateral foreign capital stock ownership and bilateral foreign investment flows.

Modification of the theory

In the extended global model, each region is assumed to source its capital stock from all foreign as well as domestic regions. To be consistent with the bilateral capital stock structure, regional savings are assumed to have been invested across all regions.

As with the basic model, the theory is designed on the basis of the database, which is extended in these two areas: capital stocks and investments. Once the database is extended, the equation structure is designed to explain how the new items are affected by the simulation.

Database extension

In the basic model database, capital stocks are included as a vector: each region is endowed with an aggregate capital stock. In the extension, this vector is replaced by a matrix whose elements show the value of the capital stock owned by home region *r* and used in host region *s* (part 3 in figure 6). It is expressed as the product of the price of capital goods and the supply of capital stock, $P^t_{\_is("inv",s)}\ \bar{X}^{cap}_{(r,s)}$, where $\bar{X}^{cap}_{(r,s)}$ is set as exogenous. Note that

- the sum of a row in this matrix gives the value of total capital stocks owned by a home region

- the sum of a column gives the value of total capital stocks used by a host region.

Figure 6   **Extension of the basic model database: bilateral capital stock and saving-investment matrixes**
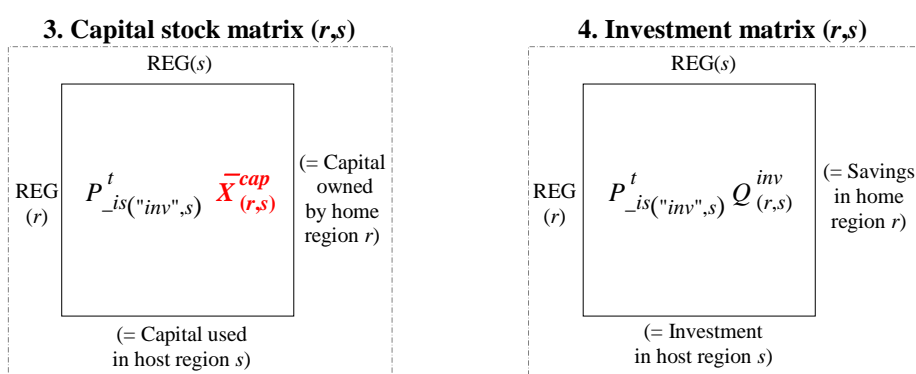
The latter is calibrated to the value of the capital stocks in the original database: the capital stock $P^t_{\_is("inv",r)}\ \bar{X}^{fac}_{("cap",r)}$ in the basic model is now calculated as $\Sigma_r\ P^t_{\_is("inv",r)}\ \bar{X}^{cap}_{(r,s)}$.

The other matrix to be added to the database is a bilateral saving-investment matrix (part 4 in figure 2). An element of this matrix shows the saving of home region r invested in host region s. It can be expressed as $E^{*\,inv}_{(r,s)} = P^{t}_{\_is("inv",s)}\, Q^{inv}_{(r,s)}$. Note also that

- the sum of a row is the total savings generated from a home region
- the sum of a column gives the total investment received by a host region.

The latter is calibrated to the value of total investment in the regional input-output table shown in figure 2, while the former is equal to the regional savings in the basic model. The differences between the column and the row sums are regional NFI inflows ($Y^{NFI}_{(r)}$).

Equation structure extension

As the extension of the database relates only to capital stocks and investment flows, only a small number of equation blocks that relate to capital and investment behaviour need to be modified.

1. Introducing demands for bilateral capital stocks

Regional capital stocks are assumed to be imperfectly substitutable in production; this is captured by a CES demand function for capital originating from each region. The required changes affect some equations in sections 2 through 4 (table 10).

The main change is a CES demand function for bilateral capital stocks defined in eq. 27 to be included in section 2 ("Industry outputs and demands for factors"). This demand is nested under the regional demand for composite capital, $Q^{fac}_{\_j("cap",s)}$, which is defined in eq. 28 as the sum of capital stocks used in all industries.

Two more new variables are introduced in eq. 27: the supply of bilateral capital stock, $\bar{X}^{cap}_{(r,s)}$, and the basic rental price of capital, $P^{*\,cap}_{(r,s)}$. As capital supply $\bar{X}^{cap}_{(r,s)}$ is defined exogenously, eq. 27 is implicitly a MCC for bilateral capital stock.

The basic rental price of capital, $P^{*\,cap}_{(r,s)}$, is an undefined variable in the extended model; it is determined by the MCC for capital. A previously undefined variable, $P^{fac}_{\_j("cap",s)}$, the regional price of capital stock, is now redefined as a CES price index for the host region's *composite* capital stock, composed of capital supplied by all regions.

Table 10   Extension of the basic model (1): introducing bilateral capital stocks

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2. Industry's outputs and demands for factors (modified)

(27) CES demand of region s for capital from region r (**MCC for bilateral capita stock**)

$$\bar{X}^{cap}_{(r,s)} = CES\ (Q^{fac}_{\_j\,("cap",s)},\ P^{fac}_{\_j\,("cap",s)}\ \boldsymbol{P}^{*cap}_{(r,s)}) \qquad\qquad (r,s\in REG)$$

where $\bar{X}^{cap}_{(r,s)}$ is the supply of capital from region r to s, and $\boldsymbol{P}^{*cap}_{(r,s)}$ is the basic rental price of capital, and $P^{fac}_{\_j\,("cap",s)}$ is linked to a CES price index,

$$P^{fac}_{\_j\,("cap",s)} = CES\ (\boldsymbol{P}^{*cap}_{("r_1",s)},\ \ldots,\ \boldsymbol{P}^{*cap}_{("r_m",s)}) \qquad\qquad (s\in REG)$$

(28) Total demand for factor f used in host region s

$$Q^{fac}_{\_j\,(f,s)} = \sum_j Q^{fac}_{(f,j,s)} \qquad\qquad (f\in FAC;\ s\in REG)$$

3. Final users' income and expenditure (modified)

(16r) Household disposable income in region r

$$Y_{(u,r)} = \begin{cases} \displaystyle\sum_f P^{tinc}_{(f,r)} X^{fac}_{(f,r)} + \sum_s (P^{tcap}_{(r,s)}\ \bar{X}^{cap}_{(r,s)} - P^{tcap}_{(s,r)}\ \bar{X}^{cap}_{(s,r)}) & (u=hou;\ r\in REG) \\[2mm] (Total\ Tax\ Revenue) & (u=gov;\ r\in REG) \\[2mm] \displaystyle\sum_{i=hou,gov} Y_{(i,r)}\ \boldsymbol{s_{(i,r)}} & (u=inv;\ r\in REG) \end{cases}$$

(17r) Post-income tax price for factor f in host region s

$$P^{tinc}_{(f,s)} = \begin{cases} \boldsymbol{P}^{*fac}_{\_j\,(f,s)}\ (1 - \boldsymbol{t}^{inc}_{(f,s)}) & (f=land,\ lab;\ s\in REG) \\[2mm] \dfrac{1}{Q^{fac}_{\_j\,(f,s)}} \displaystyle\sum_r \bar{X}^{cap}_{(r,s)}\ P^{tcap}_{(r,s)} & (f=cap;\ s\in REG) \end{cases}$$

where $\boldsymbol{t}^{inc}_{(f,s)}$ is the rate of a tax on the income of factor f in host region s.

(29) Post-income tax rental price for capital from home region r used in host region s

$$P^{tcap}_{(r,s)} = (1 - \boldsymbol{t}^{inc}_{("cap",s)})\ \boldsymbol{P}^{*cap}_{(r,s)} \qquad\qquad (r,s\in REG)$$

4. Regional supplies of factors (modified)

(21r) Supply of factor f (lab, cap) in region r (MCC for labour)

$$X^{fac}_{(f,r)} = \begin{cases} Q^{fac}_{\_j\,(f,r)} & (f=lab;\ r\in REG) \\[2mm] \displaystyle\sum_s \bar{X}^{cap}_{(r,s)} & (f=cap;\ r\in REG) \end{cases}$$

The two new variables, $\bar{X}^{cap}_{(r,s)}$, and $\boldsymbol{P}^{*cap}_{(r,s)}$, are linked with the basic model through two existing variables $Q^{fac}_{\_j\,("cap",s)}$ and $P^{fac}_{\_j\,("cap",s)}$.

To accommodate the changes in the demands for capital, in section 3 ("final users' income and expenditure"), household disposable income $Y_{("hou",r)}$ is redefined in eq. 16r [20] to account for net foreign income (the second term in eq. 16r). The post-tax capital prices $P^{tinc}_{("cap",s)}$ are redefined accordingly in eq. 17r. A fourth new variable, $P^{tcap}_{(r,s)}$, the post-tax

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[20] The letter r is used here to denote a modified equation from the basic model.

rental price of capital, is introduced in eqs. 16r and 17r; it is defined in eq. 29 as a function of the basic price of capital $P^{*\,cap}_{(r,s)}$ and its income tax rate.

With the introduction of bilateral capital stocks $\bar{X}^{cap}_{(r,s)}$, the existing capital supply, $X^{fac}_{("cap",r)}$ defined in eq. 21 in section 4 ("Regional supplies of factors"), is redefined as the endogenous capital stock *owned* by a region in eq. 21r.

There are four new variables ($\bar{X}^{cap}_{(r,s)}$, $Q^{fac}_{\_j\,(f,s)}$, $P^{*\,cap}_{(r,s)}$ and $P^{tcap}_{(r,s)}$) and three new equations (eqs. 27-29) in table 10. The capital supply, $X^{cap}_{(r,s)}$, is an exogenous variable; the basic price, $P^{*\,cap}_{(r,s)}$, is an undefined variable and the other two variables are defined by eqs. 28 and 29. The undefined capital prices, $P^{*\,cap}_{(r,s)}$, are determined by the capital MCCs (eq. 27). Following this methodical process produces a valid closure.

2. Introducing bilateral investment of a region's savings

To extend foreign investment from net flows to bilateral gross flows, all variables and equations in section 5 are redefined by adding one more dimension. The modified variables are defined in eqs. 23r-26r in table 11. Eq. 26r links the sum of regional savings in the basic model with the sum of bilateral investments introduced in the extended model.

Two new variables are introduced, $E^{*\,inv}_{(r,s)}$ and $Q^{inv}_{(r,s)}$, the value and quantity indexes of investment by home region *r* into host region *s*. Two new equations are introduced:

- Eq. 30 defines $Q^{inv}_{(r,s)}$ as investment expenditure $E^{*\,inv}_{(r,s)}$ divided by the relevant capital good price index $P^{t}_{\_is\,("inv",s)}$.

- Eq. 31 links the sum of real investment $\sum_r Q^{inv}_{(r,s)}$ from all sources to real investment in the host region *s*, which is the sum of capital goods produced in the host region. This is the MCC for real investment in the host region, and determines the equilibrium value for undefined variable $Y^{*\,NFI}_{(s)}$.

Again, the numbers of new variables and new equations are equal: the closure of the extended model remains valid.

Table 11 Extension of the basic model (2): introducing bilateral investment flows

5. Global investment of regional savings (modified)

(23r) Expected rates of return to investment from home region r to host region s

$$R^e_{(r,s)} = f(P^{tcap}_{(r,s)}, P^t_{\_is\,("inv",s)}, X^{cap}_{t+1\,(r,s)}, X^{cap}_{(r,s)}) \qquad (r,s \in REG)$$

where $\gamma_{(s)}$ is a parameter that controls the sensitivity of capital growth to change in $R^e_{(r,s)}$.

(24r) Capital stock at the end of period t

$$X^{cap}_{t+1\,(r,s)} = X^{cap}_{(r,s)}\,(1 - r^{dep}_{(r,s)}) + Q^{inv}_{(r,s)} \qquad (r,s \in REG)$$

where $r^{dep}_{(r,s)}$ is the rate of capital depreciation.

(25r) Equalisation of expected rates of return (**PEC for rates of return**)

$$R^e_{(r,s)} = R^{*e}_{\_r} \qquad (r,s \in REG)$$

where $R^{*e}_{\_s\,(r)}$ is the general equilibrium rate of return to investment from home region r.

(26r) **MCC for global savings**

$$\sum_r Y_{("inv",r)} = \sum_r \sum_s E^{*\,inv}_{(r,s)}$$

where $E^{*\,inv}_{(r,s)}$ is the investment of savings from home region r to host region s.

(30) Demand for real investment from home region r to host region s

$$Q^{inv}_{(r,s)} = \frac{E^{*\,inv}_{(r,s)}}{P^t_{\_is("inv",s)}} \qquad (r,s \in REG)$$

(31) Total real investment in host region s (**MCC for real investment**)

$$Q_{\_is\,("inv",s)} = \sum_r Q^{inv}_{(r,s)} \qquad (s \in REG)$$

Overall, there are 30 equation blocks in this extended global CGE model, five of which are newly introduced (eqs. 27-31). Among the 25 original equation blocks in the basic model, only six equation blocks require modification (eqs. 16r, 17r, 23r-26r). Other equations and variables in the basic model remain unchanged.

The undefined variables in the extended model are listed in table 12. The general equilibrium values of these variables can be solved simultaneously from a system of MCC or PEC equations. This provides a 'natural' closure for the extended model. Again, eq. 26r will become redundant if one price variable is chosen as a numeraire.

Table 12 **Undefined variables and correspondent MCC/PEC equations in the extended model**

	Undefined variables		MCC/PEC equations	
$P^{*\,fac}_{("land",j,r)}$	Basic price of industrial land	Eq.20	MCC for industrial land	$Q^{fac}_{("land",j,r)}$
$P^{*\,fac}_{\_j\,("lab",r)}$	Basic price of region labour	Eq.21r	MCC for regional labour	$X^{fac}_{("lab",r)}$
$P^{*\,cap}_{(r,s)}$	Basic price of bilateral capital	Eq.27	MCC for bilateral capital	$X^{cap}_{(r,s)}$
$E^{*\,inv}_{(r,s)}$	Value of investment	Eq.25r	PEC for regional rates of return	$R^{e}_{(r,s)}$
$R^{*e}_{\_r}$	World expected rate of return	Eq.26r	MCC for global savings	$\Sigma_r\,Y_{("inv",r)}$
$Y^{*\,NFI}_{(s)}$	Net foreign investment inflow	Eq.31	MCC for regional investment	$Q_{\_is\,("inv",s)}$

A similar specification has been used recently to build the ANZEA model (APC and NZPC, 2012) to analyse the impacts of various aspects of economic integration between the two countries. The model incorporates bilateral capital stocks at the sectoral level, which is required for analysing the effects of sector-specific reforms between the two countries and between them and the rest of the world.[21]

With bilateral capital stocks and investment flows built in the structure, this model is also suitable for recursive dynamic simulations. This is because bilateral foreign investment and capital accumulations as well as foreign capital income flows are now be properly accounted for.

Concluding remarks

It is argued in this paper that CGE models are structurally simple and their equation systems can be presented in a simple and transparent framework. An alternative approach to CGE modelling is introduced and applied in the building of a simple and transparent basic global model, which can be adapted to meet the requirements of various applications. This approach can also be applied to build other CGE models, such as national models or multi-region models.

It should be emphasised that the purpose of simplifying a model's equation system is to make the model theory and structure clear and easy to understand. Therefore, the simplicity of the model's equation system should be judged against the clarity of the model theory. To maintain the model's theoretical clarity, all the basic components of the model theory must be retained, even if this means more variables are needed under certain

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[21] The equation system of the ANZEA model can be found in Supplementary Paper E of the report (APC and NZPC, 2012).

circumstances.[22] For example, additional variables may be needed to define supply and demand variables separately, to identify the model's solution structure explicitly.[23]

Because of the relative simplicity of the basic model, different application can easily be developed at relatively low cost to suit the needs of various projects. This modelling strategy relies on a set of application-specific models developed from a single basic CGE infrastructure.

Application-specific models are structurally simpler than general purpose models because they do not carry the 'excess baggage' of variables and equations that are not required for the particular application. This strategy reduces entry costs for many model users, and can encourage the use of CGE models in a wider area. It also reduces reliance on experts and increases users' control over the models they develop. Moreover, it facilitates the transfer of knowledge and of models to new users, and facilitates communication with non-modellers, who may wish to contribute to the model building process.

For institutions that use CGE models intensively for a wide-range of applications, this modelling strategy may be particularly helpful because it facilitates at an institutional level:

- maintenance and error checking, as the model code is transparent, structured and uncluttered, making it easy to identify potential errors

- the development of application-specific models

- the interpretation and comparison of modelling results

- the dissemination and communication within and outside the institution

- the training of staff new to CGE modelling by lowering entry costs

From a computing point of view, running times can be shortened significantly. This means that initial tests can be run more quickly to produce better simulation design. Sensitivity analysis on parameters, data and closure can also be facilitated by faster solution times as they involve running multiple simulations. More importantly, saving computing time gives researchers more time to check results and the verify model theory and data.

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[22] Variable substitution, which can make a model more compact, can be done by some software (for example, in GEMPACK). A slightly longer code can be clearer than one that is more elegant, but less transparent.

[23] An example is eq. 21 in the model system, "MCC for industry-specific land". Variable $X^{lnd}_{(j,r)}$ could have been substituted out so that eq. 21 were deleted and the MEC for land were implicitly incorporated in eq. 20. The reason for keeping this variable and equation in the system is to explicitly specify a MCC for land, distinguishable from the CET supply of land, defined in eq. 20.

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