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Calibrating the Modified An Implicitly Directly Additive Demand System (MAIDADS) for Applied Economic Modeling

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Abstract

The main objective of this paper is to make a good use of the recent study that modified the AIDADS (An Implicitly Directly Additive Demand System). The modified AIDADS (MAIDADS) is able to capture varying consumer behaviors across different expenditure levels and also allows minimum consumption quantities to change as utility varies. These features would attract attention from researchers conducting applied economic analysis which involves consumers at wide range of expenditure levels; consumers at developing and developed countries, households at different income levels, and evolution of expenditure patterns over the long-run simulated often under CGE models, e.g. for 50 or 100 years from the benchmark year. The parameters in the MAIDADS are estimated with the 2005 International Comparison Program dataset, and we obtained elasticities as well as the minimum consumption quantities for 147 countries and 12 expenditure categories. With these estimation results, the MAIDADS can be incorporated into applied economic models.

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1 Introduction

Recent developments in the area of study on consumer demand system have enabled researchers conducting applied economic analysis to construct practical and elaborated economic models. A good example can be found in the consumer demand system implemented in Computable General Equilibrium (CGE) models. A widely used specification of the Cobb-Douglas demand system has been extended to the Linear Expenditure System (LES) by incorporating subsistence levels of consumption goods.

Rimmer and Powell (1992) developed An Implicitly Directly Additive Demand system (AIDADS) which allows marginal budget shares to vary as expenditure changes. Their generalization of the LES overcomes an empirical weakness found in the LES, such as the constant marginal budget shares.

Demand equation for good i in the AIDADS (Rimmer and Powell, 1992) can be expressed as:

$$x_i = \theta_i + \frac{\alpha_i + \beta_i e^u}{(1+e^u)\, p_i} \left(c - \sum_j p_j \theta_j \right), \qquad \text{(AIDADS)}$$

where x_i and p_i are consumption quantity and price of good i, and total consumption expenditure is c, and level of utility is u. Note that a parameter of subsistence consumption quantity, θ_i, is constant as well as α_i and β_i. Restrictions on the parameters are $0 \leq \alpha_i, \beta_i \leq 1$, and $\sum_i \alpha_i = \sum_i \beta_i = 1$, and $0 \leq \theta_i < x_i$.

MAIDADS

Preckel, Cranfield, and Hertel (2010) extends the consumer demand system developed by Rimmer and Powell (1992) by allowing subsistence consumption quantities, θ_i, to vary along with utility, thereby referred to as Modified An Implicitly Directly Additive Demand System (or MAIDADS).

The constant parameter of subsistence consumption quantity, θ_i in AIDADS is defined as a function of utility, u, with nonnegative parameters of δ_i, τ_i, ω. Thus,

$$\theta_i(u) = \frac{\delta_i + \tau_i e^{\omega u}}{1 + e^{\omega u}}.$$

Note that the MAIDADS becomes AIDADS as δ_i is equal to τ_i as a special case.

2 Calibration of MAIDADS

A system of the MAIDADS demand equations is calibrated for each country, by solving the minimization problem specified as follows.

$$
\begin{aligned}
\min_{u,\omega,\alpha,\beta,\delta,\tau} \quad & \sum_{i}^{n} \left(\frac{s_i}{\hat{s}_i} - 1 \right)^2 \\
s.t. \quad & \\
& s_i = \frac{(\delta_i + \tau_i e^{\omega u})\,\hat{p}_i}{(1 + e^{\omega u})\,\hat{c}} + \frac{\alpha_i + \beta_i e^{u}}{(1 + e^{u})\,\hat{c}} \left(\hat{c} - \sum_{j}^{n} \hat{p}_j \frac{\delta_j + \tau_j e^{\omega u}}{1 + e^{\omega u}} \right) \\
& \sum_{i}^{n} \alpha_i = 1 \\
& \sum_{i}^{n} \beta_i = 1 \\
& \hat{x}_i > \frac{\delta_i + \tau_i e^{\omega u}}{1 + e^{\omega u}} \geq 0 \\
& \omega \geq 0 \\
& \alpha_i, \beta_i, \delta_i, \tau_i \geq 0 \quad \forall i.
\end{aligned}
$$

Prices, $\hat{p}$, and quantities, $\hat{x}$, are obtained from the ICP 2005 data (World Bank, 2008), and then consumption expenditure, $\hat{c} = \sum_i \hat{p}_i \hat{x}_i$, and initial shares, $\hat{s}_i = \hat{p}_i \hat{x}_i / \hat{c}$, are calculated. This minimization problem was solved numerically.

ICP 2005 data records 12 consumption commodities for 147 counties / regions, and Table 1 and Table 2 reports their listing respectively.

Table 1: Commodities from ICP 2005 Data

1	Food and non-alcoholic beverages	7	Transport
2	Alcoholic beverages and tobacco	8	Communication
3	Clothing and footwear	9	Recreation and culture
4	Housing, water, electricity, gas and other fuels	10	Education
5	Furnishings, household equipment and household maintenance	11	Restaurants and hotels
6	Health	12	Miscellaneous goods and services

Source: World Bank (2008)

Table 2: ICP 2005 Country / Region Listings

No.	Code	Name	No.	Code	Name	No.	Code	Name
1	ZAR	Congo, Dem. Rep.	50	CPV	Cape Verde	99	KAZ	Kazakhstan
2	GNB	Guinea-Bissau	51	PRY	Paraguay	100	BLR	Belarus
3	NER	Niger	52	GNQ	Equatorial Guinea	101	MEX	Mexico
4	LBR	Liberia	53	BOL	Bolivia	102	LBN	Lebanon
5	MOZ	Mozambique	54	SYR	Syrian Arab Republic	103	LVA	Latvia
6	CAF	Central African Republic	55	AZE	Azerbaijan	104	MAC	Macao, China
7	RWA	Rwanda	56	SWZ	Swaziland	105	HRV	Croatia
8	MWI	Malawi	57	NAM	Namibia	106	POL	Poland
9	AGO	Angola	58	BWA	Botswana	107	LTU	Lithuania
10	MLI	Mali	59	GEO	Georgia	108	SVK	Slovak Republic
11	GIN	Guinea	60	FJI	Fiji	109	EST	Estonia
12	BFA	Burkina Faso	61	GAB	Gabon	110	KOR	Korea, Rep.
13	TGO	Togo	62	MDV	Maldives	111	BHR	Bahrain
14	MDG	Madagascar	63	MDA	Moldova	112	HUN	Hungary
15	SLE	Sierra Leone	64	ARM	Armenia	113	CZE	Czech Republic
16	CIV	Cote d'Ivoire	65	EG2	Egypt, Arab Rep. (AFR)	114	KWT	Kuwait
17	GHA	Ghana	66	EG1	Egypt, Arab Rep. (WAS)	115	BRN	Brunei Darussalam
18	BEN	Benin	67	PER	Peru	116	PRT	Portugal
19	BGD	Bangladesh	68	TUN	Tunisia	117	SVN	Slovenia
20	NPL	Nepal	69	JOR	Jordan	118	QAT	Qatar
21	MRT	Mauritania	70	ECU	Ecuador	119	SGP	Singapore
22	TCD	Chad	71	WAS	WESTERN ASIA	120	MLT	Malta
23	SEN	Senegal	72	COL	Colombia	121	ISR	Israel
24	NGA	Nigeria	73	ALB	Albania	122	NZL	New Zealand
25	CMR	Cameroon	74	THA	Thailand	123	GRC	Greece
26	COG	Congo, Rep.	75	VEN	Venezuela, RB	124	OEE	OECD-EUROSTAT
27	STP	Sao Tome and Principe	76	SAM	SOUTH AMERICA	125	ITA	Italy
28	KEN	Kenya	77	BRA	Brazil	126	ESP	Spain
29	DJI	Djibouti	78	MNE	Montenegro	127	FIN	Finland
30	LAO	Lao PDR	79	TUR	Turkey	128	CYP	Cyprus
31	SDN	Sudan	80	MYS	Malaysia	129	TWN	Taiwan, China
32	AFR	AFRICA	81	ZAF	South Africa	130	JPN	Japan
33	YEM	Yemen, Rep.	82	UKR	Ukraine	131	IRL	Ireland
34	KHM	Cambodia	83	WLD	WORLD	132	HKG	Hong Kong, China
35	IND	India	84	MKD	Macedonia, FYR	133	DEU	Germany
36	VNM	Vietnam	85	BIH	Bosnia and Herzegovina	134	BEL	Belgium
37	CHN	China	86	URY	Uruguay	135	DNK	Denmark
38	LSO	Lesotho	87	CIS	CIS	136	AUS	Australia
39	MAR	Morocco	88	CHL	Chile	137	FRA	France
40	PAK	Pakistan	89	SAU	Saudi Arabia	138	SWE	Sweden
41	IDN	Indonesia	90	SRB	Serbia	139	NLD	Netherlands
42	PHL	Philippines	91	ARG	Argentina	140	CAN	Canada
43	ASP	ASIA AND THE PACIFIC	92	OMN	Oman	141	CHE	Switzerland
44	TJK	Tajikistan	93	ROM	Romania	142	AUT	Austria
45	MNG	Mongolia	94	IRN	Iran, Islamic Rep.	143	GBR	United Kingdom
46	IRQ	Iraq	95	BGR	Bulgaria	144	NOR	Norway
47	LKA	Sri Lanka	96	RU1	Russian Federation (OEE)	145	ISL	Iceland
48	KGZ	Kyrgyz Republic	97	RU2	Russian Federation (CIS)	146	LUX	Luxembourg
49	BTN	Bhutan	98	MUS	Mauritius	147	USA	United States

Source: World Bank (2008)

3 Elasticities in MAIDADS

Consumption expenditure elasticities, η_i, and uncompensated price elasticities, ε_{ij}, can be obtained from the MAIDADS demand equation and an implicit function defining the relationship between total consumption expenditure and utility:

$$
\begin{aligned}
f(u,c) &= \sum_i \mu_i(u) \ln\left(\frac{\mu_i(u)}{p_i}\right) + \ln c^*(u,c) - \ln A - u \equiv 1, \qquad (1)\\
\mu_i(u) &= \frac{\alpha_i + \beta_i e^u}{(1+e^u)},\\
c^*(u,c) &= c - \sum_i p_i \theta_i(u),\\
\theta_i(u) &= \frac{\delta_i + \tau_i e^{\omega u}}{1+e^{\omega u}},
\end{aligned}
$$

where A is a constant.

Following the derivation steps in Rimmer and Powell (1992) and van der Mensbrugghe (2001), firstly the marginal budget share, ρ_i, is calculated from the MAIDADS demand equation.

$$
\begin{aligned}
x_i &= \theta_i(u) + \frac{\mu_i(u)}{p_i} c^*(u,c) \qquad \text{(MAIDADS)}\\
\rho_i &= p_i \frac{\partial x_i}{\partial c}\\
&= p_i \left[\frac{\mu_i}{p_i}\frac{\partial c^*}{\partial c} + \frac{c^*}{p_i}\frac{\partial \mu_i}{\partial u}\frac{\partial u}{\partial c} + \frac{\partial \theta_i}{\partial u}\frac{\partial u}{\partial c} - \frac{\mu_i}{p_i}\left(\sum_j p_j \frac{\partial \theta_j}{\partial u}\frac{\partial u}{\partial c}\right)\right]\\
&= \mu_i + \frac{\partial u}{\partial c}\left[c^*\frac{\partial \mu_i}{\partial u} + p_i\frac{\partial \theta_i}{\partial u} - \mu_i\left(\sum_j p_j \frac{\partial \theta_j}{\partial u}\right)\right]\\
&= \mu_i + \frac{\partial u}{\partial c}\left[c^*\frac{e^u(\beta_i-\alpha_i)}{(1+e^u)^2} + p_i\frac{\omega e^{\omega u}(\tau_i-\delta_i)}{(1+e^{\omega u})^2} - \mu_i\left(\sum_j p_j \frac{\omega e^{\omega u}(\tau_j-\delta_j)}{(1+e^{\omega u})^2}\right)\right]\\
&= \mu_i + c^*\frac{\partial u}{\partial c}\left[\frac{e^u(\beta_i-\alpha_i)}{(1+e^u)^2} + \frac{\omega e^{\omega u}}{c^*(1+e^{\omega u})^2}\left(p_i(\tau_i-\delta_i) - \mu_i\sum_j p_j(\tau_j-\delta_j)\right)\right]\\
&= \mu_i + c^*\frac{\partial u}{\partial c}\Psi_i. \qquad (2)
\end{aligned}
$$

Total consumption expenditure and utility are combined in an implicit form, and thus the term, $\partial u/\partial c$, in the equation (2) is calculated by applying the implicit function

theorem to the MAIDADS identity defined in (1).

$$
\begin{aligned}
\frac{\partial u}{\partial c} &= -\frac{\partial f}{\partial c}\left[\frac{\partial f}{\partial u}\right]^{-1} \\
&= -\frac{1}{c^*}\left[\frac{\partial f}{\partial u}\right]^{-1} \\
&= -\frac{1}{c^*}\left[-1+\frac{e^u}{(1+e^u)^2}\sum_i(\beta_i-\alpha_i)\left(\ln\left(\frac{\mu_i}{p_i}\right)+1\right)-\frac{\omega e^{\omega u}}{c^*(1+e^{\omega u})^2}\sum_i p_i(\tau_i-\delta_i)\right]^{-1} \\
&= -\frac{1}{c^*}\left[-1+\frac{e^u}{(1+e^u)^2}\sum_i(\beta_i-\alpha_i)\ln(x_i-\theta_i)-\frac{\omega e^{\omega u}}{c^*(1+e^{\omega u})^2}\sum_i p_i(\tau_i-\delta_i)\right]^{-1} \\
&= -\frac{1}{c^*}\Omega
\end{aligned}
$$

Therefore, we can obtain the expression for the marginal budget share and the consumption expenditure elasticity as:

$$
\begin{aligned}
\rho_i &= \mu_i - \Psi_i\Omega, \\
\eta_i &= \frac{\partial x_i}{\partial c}\frac{c}{x_i} \\
&= \frac{\partial x_i}{\partial c}\frac{c}{x_i}\frac{p_i}{p_i} \\
&= \frac{\rho_i}{s_i} \\
&= \frac{1}{s_i}(\mu_i - \Psi_i\Omega).
\end{aligned} \tag{3}
$$

Uncompensated price elasticities, ε_{ij}, are also derived from the MAIDADS demand equation.

$$
\begin{aligned}
\varepsilon_{ij} &= \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i} \\
&= \frac{p_j}{x_i}\left[\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij}-\frac{\mu_i\theta_j}{p_i}\right)+\frac{c^*}{p_i}\frac{\partial\mu_i}{\partial u}\frac{\partial u}{\partial p_j}+1\frac{\partial\theta_i}{\partial u}\frac{\partial u}{\partial p_j}-\frac{\mu_i}{p_i}\sum_k p_k\frac{\partial\theta_k}{\partial u}\frac{\partial u}{\partial p_j}\right] \\
&= \frac{p_j}{x_i}\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij}-\frac{\mu_i\theta_j}{p_i}\right)+\frac{p_j}{x_i}\frac{\partial u}{\partial p_j}\left(\frac{c^*}{p_i}\frac{\partial\mu_i}{\partial u}+1\frac{\partial\theta_i}{\partial u}-\frac{\mu_i}{p_i}\sum_k p_k\frac{\partial\theta_k}{\partial u}\right) \\
&= \frac{p_j}{x_i}\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij}-\frac{\mu_i\theta_j}{p_i}\right)+\frac{p_j}{x_i}\frac{\partial u}{\partial p_j}\left(\frac{c^*}{p_i}\frac{e^u(\beta_i-\alpha_i)}{(1+e^u)^2}+\frac{\omega e^{\omega u}(\tau_i-\delta_i)}{(1+e^{\omega u})^2}-\frac{\mu_i}{p_i}\sum_k p_k\frac{\omega e^{\omega u}(\tau_k-\delta_k)}{(1+e^{\omega u})^2}\right)
\end{aligned}
$$

$$= \frac{p_j}{x_i}\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij} - \frac{\mu_i \theta_j}{p_i}\right) + \frac{c^* p_j}{p_i x_i}\frac{\partial u}{\partial p_j}\left[\frac{e^u\left(\beta_i - \alpha_i\right)}{\left(1+e^u\right)^2} + \frac{\omega e^{\omega u}}{c^*\left(1+e^{\omega u}\right)^2}\left(p_i\left(\tau_i - \delta_i\right) - \mu_i \sum_k p_k\left(\tau_k - \delta_k\right)\right)\right]$$

$$= \frac{p_j}{x_i}\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij} - \frac{\mu_i \theta_j}{p_i}\right) + \frac{c^* p_j}{p_i x_i}\frac{\partial u}{\partial p_j}\Psi_i \tag{4}$$

where

$$\phi_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$$

Applying the implicit function theorem on $\partial u / \partial p_j$, we will have the expression such as;

$$\begin{aligned} \frac{\partial u}{\partial p_j} &= -\frac{\partial f}{\partial p_j}\left[\frac{\partial f}{\partial u}\right]^{-1} \\ &= \left(\frac{\mu_j}{p_j} + \frac{\theta_j}{c^*}\right)\left[\frac{\partial f}{\partial u}\right]^{-1} \\ &= \frac{s_j c}{p_j c^*}\Omega. \end{aligned} \tag{5}$$

Therefore, the uncompensated price elasticities are

$$\begin{aligned} \varepsilon_{ij} &= \frac{p_j}{x_i}\left(-\frac{\mu_i c^*}{p_i^2}\phi_{ij} - \frac{\mu_i \theta_j}{p_i}\right) + \frac{c^* p_j}{p_i x_i}\frac{s_j c}{p_j c^*}\Psi_i \Omega \\ &= -\frac{\mu_i}{s_i c}\left(p_j \theta_j + \phi_{ij} c^*\right) + \frac{s_j}{s_i}\Psi_i \Omega \\ &= -\frac{\mu_i c^*}{s_i c}\left(\mu_j - \phi_{ij}\right) - s_j \eta_i \end{aligned} \tag{6}$$

4 Calibration Results

Consumption expenditure shares of 12 commodities for 147 countries / regions are calculated from the ICP 2005 data, and the Figure 1 reports the results. The panel (a) plots the points for food and non-alcoholic beverages, alcoholic beverages and tobacco, and clothing and footwear. It is remarkable that the share of food expenditure rapidly declines as total consumption expenditure increases along the horizontal axis. The smoothed lines in the panels are drawn to trace their statistical trend.

Once numerically solved the minimization problem for each country / region, all the parameters as well as the level of utility are obtained to compute the consumption expenditure elasticities. Figure 2 report the results. Note that there are 7 countries

excluded from the figure because of their elasticities becoming negative (Guinea-Bissau, Madagascar, Bangladesh, Sudan, Pakistan, Bhutan, and Syrian Arab Republic). In the panel (a), expenditure elasticities of food indicate a increasing trend at the higher expenditure levels.

Figure 3 reveals that the ratio of the minimum consumption quantity to the level of consumption becomes lower for most of the commodities, as expenditure increases. However, there are exceptions for housing, health, miscellaneous goods.

Having calibrated the MAIDADS and computed elasticities, it is possible to project the expenditure shares. To make simple projections, we set the path of the total consumption expenditure as:

$$c_{t+1} = (1 + r/100)c_t,$$

for $t = 1, ..., 99, r = 3$. Figure 4 shows the results for selected counties; Vietnam, China, and Japan.

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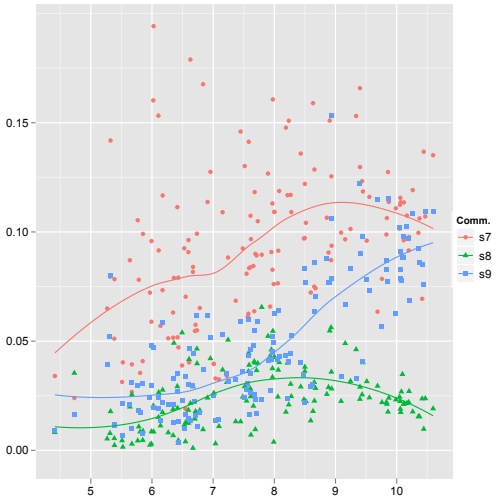
(a) 1: Food, 2: Alcoholic bev., 3: Clothing

(b) 4: Housing, 5: Furnishings, 6: Health



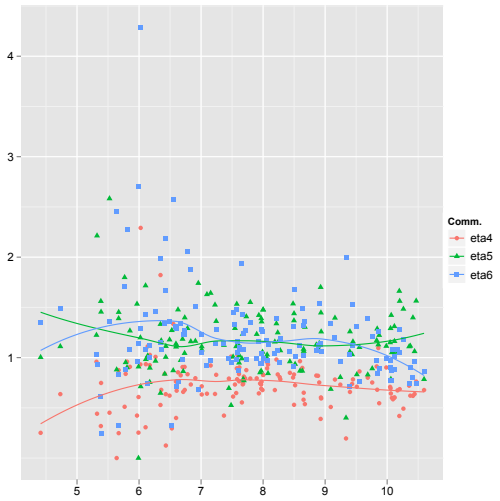
(c) 7: Trans., 8: Communication, 9: Recreation

(d) 10: Education, 11: Restaurants, 12: Misc.

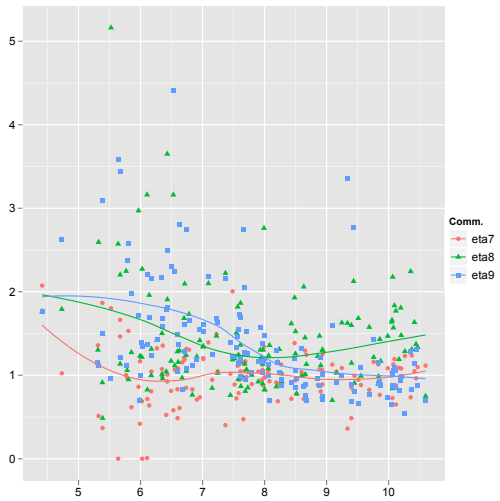
Source: World Bank (2008) and authors' calculations

Figure 1: Observed shares and expenditures (points), and smoothed (lines)

(a) 1: Food, 2: Alcoholic bev., 3: Clothing



(b) 4: Housing, 5: Furnishings, 6: Health

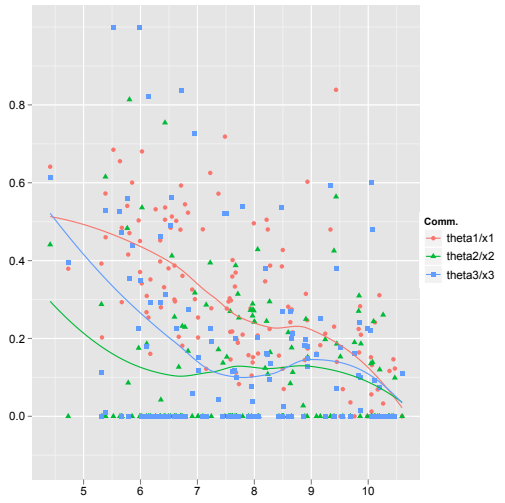


(c) 7: Trans., 8: Communication, 9: Recreation

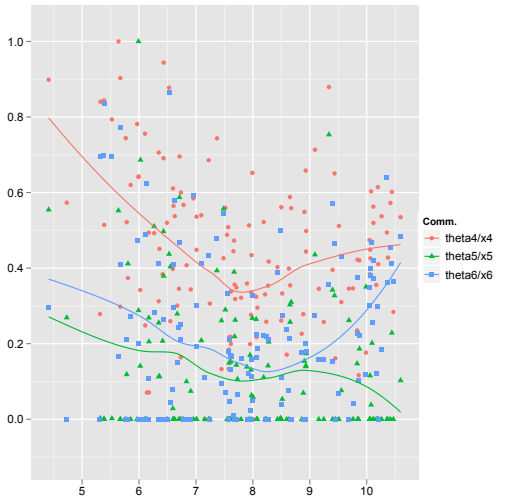
(d) 10: Education, 11: Restaurants, 12: Misc.

Source: Calibration results

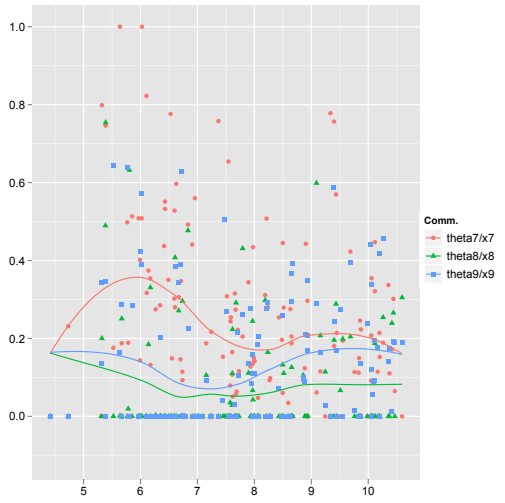
Figure 2: Expenditure elasticities (points), and smoothed (lines)



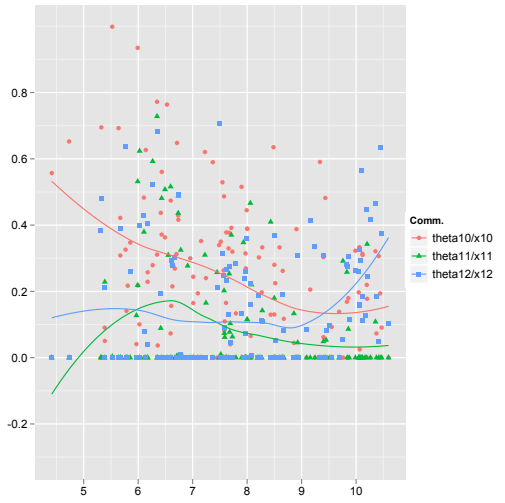
(a) 1: Food, 2: Alcoholic bev., 3: Clothing



(b) 4: Housing, 5: Furnishings, 6: Health



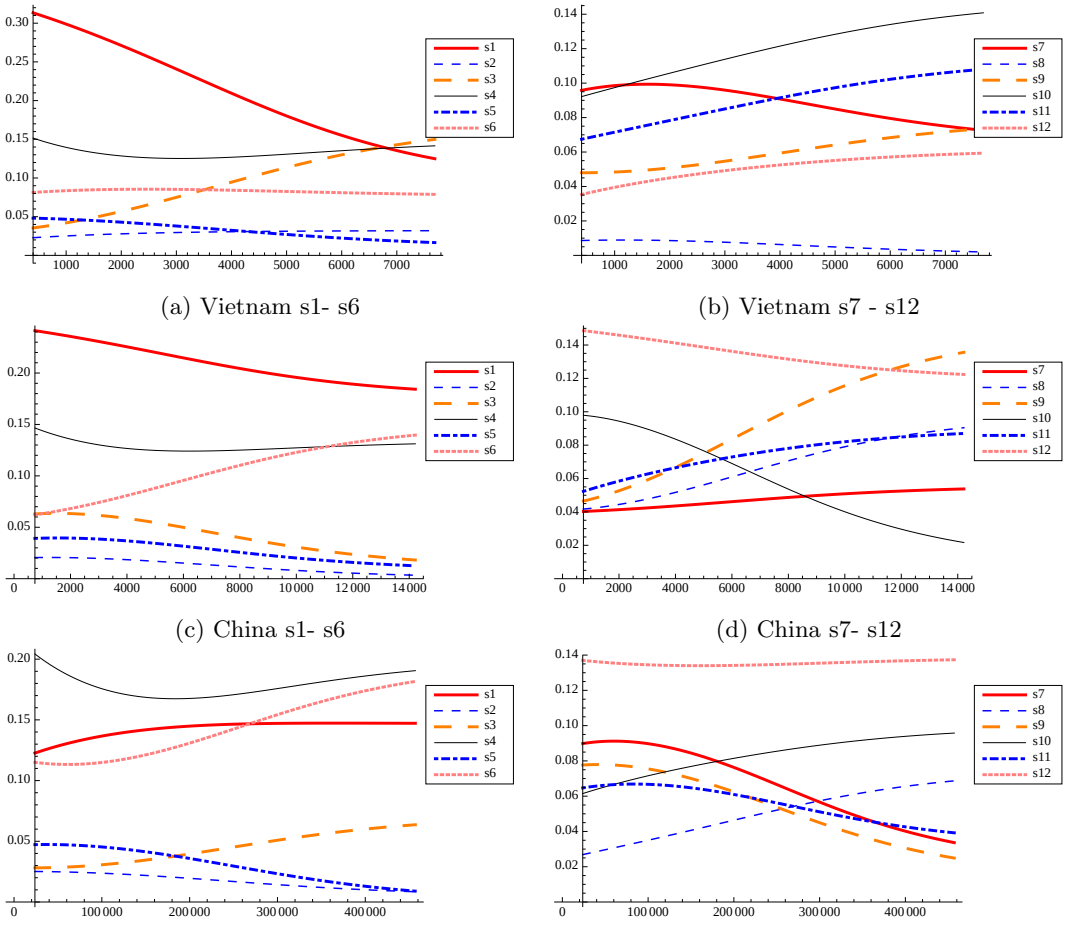
(c) 7: Trans., 8: Communication, 9: Recreation



(d) 10: Education, 11: Restaurants, 12: Misc.

Source: Calibration results

Figure 3: $\theta_i(u)/x_i$ and expenditures (points) and smoothed (lines)



(a) Vietnam s1- s6

(b) Vietnam s7 - s12

(c) China s1- s6

(d) China s7- s12

(e) Japan s1- s6

(f) Japan s7- s12

Note: s1: food, s2: alcoholic bev., s3: clothing, 4: housing, 5: furnishings, 6: health, 7: trans., 8: communication, 9: recreation, 10: education, 11: restaurants, 12: misc.

Source: Authors' calculations

Figure 4: Projected shares and expenditures for selected counties