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Implications of Broader Gaussian Quadrature Sampling Strategy in the Contest of the Special Safeguard Mechanism

(Preliminary Draft)

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Abstract

This paper explores the sensitivity of model statistics to breadth of sampling used under the Sensitivity Analysis – a method very often used for General Equilibrium models and an inbuilt tool in General Equilibrium Modeling Package (GEMPACK). We force the Stroud and Liu Gaussian Quadratures available in Systematic Sensitivity Analysis (SSA) in GEMPACK to sample from beyond their usual $\sqrt{2}\sigma$ and 1σ limits. The breadth of sampling domain could be an important aspect for models that are highly non-linear and where results depend on thresholds and regime switches. In the Special Safeguards (SSM) example used here, we find that the broader sampling technique gives us very different results for second tier tariffs on wheat imports.

Key Words: Gaussian Quadrature, Sampling, Simulation Analysis

JEL Code: C15, C46, C63, C83

I.   Introduction

In recent years, Sensitivity Analysis has become more commonplace in the context of Computable General Equilibrium (CGE) analyses. It has long played a role in assessing the robustness of model results (e.g. Wigle 1991; Harrison et al., 1992). However, its domain of application has now extended to model validation (Valenzuela et al. 2007; Beckman et al. 2010), trade policy analysis (Hertel et al. 2010; Verma et al. 2011), and climate volatility (Ahmed et al. 2010). These new areas of application have raised important questions about SSA methodology.

One issue that has recently drawn attention from users is that Gaussian Quadratures (GQ) proposed by Stroud or Liu restrict the variation around the mean to no more than $\sqrt{2}\sigma$ and 1σ respectively. This restricted variation, might or might not take one as far from mean as is desirable for a problem. This may have implications for applications where the CGE model has different policy regimes which kick in at different threshold levels. In the context of productivity shocks to agriculture, for example, the existing mechanism definitely fails to simulate the impact of extreme weather events. Preckel et al. (2011) suggest that the presently restricted sampling (as in Original Stroud or Liu – OS or OL) may lead to distorted estimates of mean and standard deviations of endogenous variables in the sensitivity results and provide a method to push GQ to sample from extreme ends of a parameter's distribution.

This paper offers an application using the new quadrature proposed by Preckel et al., to sample more widely from the shock distributions than the OS or OL quadratures allow in their present implementations. We call the new implementation results – the Broad Sample Stroud or Liu (BS or BL). For technique of broader sampling used, see (Preckel et al 2011); the

implementation of the technique in the current SSA framework in GEMPACK is elaborated upon in the methodology section of this work.

For application we chose the one used by Hertel et al (2010) – analyzing the implications Special Safeguard Mechanism (SSM) which have been proposed as a method of shielding countries from import surges. The impacts of a specific special safeguard mechanism likely depend substantially on its design. The one under discussion is based broadly on the current special agricultural safeguard, which includes two triggers—one based on the price of imports and one on the quantity[1] of imports (GATT 1994). The quantity-based safeguard can be used when imports in a year exceed base imports – a three-year moving average of imports. The duty that can be applied increases as imports exceed this base. Imports of 110–115 percent of the base allow an additional duty of 25 percent of the current binding or 25 percentage points, imports of 115-135 percent of the base allow an additional duty of 40 percent of the binding or 40 percentage points, and imports of more than 135 percent of the base allow an additional duty of 50 percent of the binding or 50 percentage points. At present only the first two tiers of the policy are modeled in this this application.

Hertel et al. find that the quantity-based safeguard reduces imports, raises domestic prices, and boosts mean domestic production in the countries that implement it. However, rather than insulating developing countries in those regions from price volatility, the quantity-based safeguard increases domestic price volatility, largely by restricting imports when domestic output is low and prices are high. The authors estimate that the quantity based safeguard would shrink average wheat imports nearly 50 percent in some regions. However, all of these results are based on the Stroud sampling strategy, which does not sample beyond the $\sqrt{2}\sigma$ of the productivity

distributions. This may explain why their results hardly ever trigger the second level of the quantity-based SSM tariff regime.

In order to explore the sensitivity of these findings to the breadth of sampling used by Gaussian Quadrature, we rerun their model (a reduced dimensions version of it), under broader sampling strategy. *A priori* we expect that the number of times a specific regime kicks in under the two quadrature types – broader sampling versus standard sampling – to be different though we cannot say if it would significantly alter the means of the distributions of the endogenous variables. Comparisons are made for the results of the new quadrature with those of the old quadratures. By making such a comparison we hope to assess the usefulness of broader sampling techniques.

II. Methodology

i. *Concept*

The methodology section here basically outlines the implementation of the idea developed in Preckel et al. Let's begin by thinking about the inputs into any SSA exercise. SSA usually assesses the sensitivity of model results to changes in exogenous inputs. Once the user has identified the key parameters, outside information (from literature or data) is used for determining the "*amount of variation*" in the parameters. We usually get this outside information in terms of standard deviation for a given exogenous variable e.g. standard deviation in historic yield changes; or in terms of cut off points e.g. the upper and/or lower bound estimates in literature for elasticity of factor substitution. This section shows how under each scenario one can make the OS and OL, sample from given variation such that the cut off points (or any point

beyond the $\sqrt{2}\sigma$ and 1σ are also sampled by OS and OL respectively. We first highlight this in a generic framework before getting down to specifics. As the analysis here pertains to the SSA application in GEMPACK we restrict attention to symmetric triangular and uniform distributions.

a. *Case I: Known Mean and Standard Deviation*

For a symmetric triangular distribution (and therefore we deal here with only the right half of the distribution) with known mean (μ) and standard deviation (σ), one can determine the end point (E_a) corresponding to the probability density that one needs to cover:

$$P(\mu \leq x \leq E_a) = a/2$$

E_a can be written as:

$$E_a = \mu + s(a) \cdot \sigma \tag{1},$$

where the scaling factor $s(a)$ is a scalar and its value depends on the chosen a.[2]

For the extreme endpoint such that the entire domain is spanned

$$P(\mu \leq x \leq E_1) = 1/2 \qquad (so\ a = 1);$$

the relevant value of $s(a)$ of a triangular distribution (TD) equals $\sqrt{6}$ and the same for the uniform distribution (UD) is $\sqrt{3}$ therefore the relevant right tail endpoints are

$$E_1^t = \mu + \sqrt{6}\,\sigma \qquad \text{and} \qquad E_1^u = \mu + \sqrt{3}\,\sigma \qquad\qquad (2).$$

The end point E_a (for $a \in [0,1]$) is relevant as the input into (SSA).[3] The only thing that

changes in the RHS of equation (1) is the scaling factor $s(a)$. With some algebraic manipulation

it can be shown that for a symmetric TD, $s(a) = \left(1 - \sqrt{1-a}\right)\sqrt{6}$ and for a symmetric UD,

$s(a) = a\sqrt{3}$; and E_a can be thus be written as follows

$$E_a^t = \mu + \left(1 - \sqrt{1-a}\right)\sqrt{6}\,\sigma \qquad \text{and} \qquad E_a^u = \mu + a\sqrt{3}\,\sigma \qquad\qquad (3).$$

For example, if we want the shocks to be sampled from the TD with $\sigma = 1$ and $a = 0.90$

(so sampling from 90 percent of the total distribution area or equivalently leaving out the 5

percent of the distribution in each tail); the end point to be specified in SSA "*Amount of*

Variation" box should be $\left(1 - \sqrt{1-0.9}\right)\sqrt{6} = 1.67$. The same σ and a values for a symmetric

UD will yield the *Amount of Variation* = 1.56. [4]

Equation (3) can be used to figure out the *Amount of Variation* for any desired value of a,

for a symmetric triangular and uniform distributions. Let's for now concentrate on the TD

example and call [0, 1.67], the true[5] triangular distribution domain.

Giving this input of 1.67 as the *Amount of Variation* to SSA would however lead the OS

or OL quadrature to pick up shock values that are never more than $\sqrt{2}\sigma$ or σ respectively. For

our TD example this means that any values between 1.414 and 1.67 will never be chosen by OS

quadrature and those between 1 and 1.67 will not be sampled by OL. The effective distributions

that are sampled from, are thereby restricted to [0, 1.414] for Stroud and [0, 1] for Liu.

Equivalently one can say that samples shocks are drawn from only approximately 82 percent (for

Stroud's) and 65 percent (for Liu's) of the distribution's area; and this area is symmetrically concentrated around the mean of the distribution. The tails of the distribution are never sampled from; this can be a big mistake when one considers climate models (think extreme weather shocks) or regime switching models where low probability but high magnitude shocks can alter the results sometimes irreversibly so.

The concept of sampling shocks outside the restricted range is what we call broader sampling. The theory is explained in Preckel et al. (2011). One can implement it using existing SSA tools. Basically it involves doing the SSA twice – once with the original distribution expanded and the second time with the original distribution contracted – and then using a probability weights to combine the results of the two. The details follow.

The first step is to choose the expansion parameter α. This would depend on how far along in the true distribution, do we want to go, to sample the shocks. If one wants to largest possible shock to be 1.67 then it translates into equating the effective and the true distributions. Now given that Stroud quadrature would never pick up a shock greater than $\sqrt{2}\sigma$, one would need to scale up the σ to use the existing mechanism but still pick a shock bigger than $\sqrt{2}\sigma$! One can rely on equation (3) to determine this expansion parameter to be $\left(1 - \sqrt{1-a}\right)\sqrt{3}$. When we want the entire domain of distribution to be spanned (and we have argued above that this means $a = 1$) then the expansion factor is $\sqrt{3}$. Table 1 below provides the expressions for the expansion factors in terms of a.

Table 1: Expansion parameter for Standard Deviation of Original Distribution

	Triangular Distribution	Uniform Distribution
Stroud's Quadrature	$(1 - \sqrt{1-a})\sqrt{3}$	$a\sqrt{3/2}$
Liu's Quadrature	$(1 - \sqrt{1-a})\sqrt{6}$	$a\sqrt{3}$

Source: Authors' Derivations

Once we have the expansion parameters figured out, one can solve the following system of equations given by Preckel et al. to get the contraction factor (β) and probability (p).

$$\alpha^2 p + \beta^2(1-p) = 1$$

$$\alpha^4 p + \beta^4(1-p) = \kappa$$

κ has been shown to equal the ratio of theoretic kurtosis for the symmetric distribution (2.4 for TD and 1.8 for UD) to the actual kurtosis of the original SSA sample inputs. This problem is programmed as a small GAMS utility and with inputs for κ and a from the user, will give the outputs needed for SSA implementation in GEMPACK.

b. *Case II: Known Mean and Cut-off points*

Sometimes however it could be that the information available for SSA are the mean and the extreme (true) end-point. In such a scenario we already know the variation V; where $V = E_1 - \mu$. But again to make sure that Stroud and Liu quadratures do not restrict this range of variation, we can use the following scaling factors for the variation.

Table 2: Expansion factor for Variation in Original Distribution

	Triangular Distribution	Uniform Distribution
Stroud's Quadrature	$\sqrt{3}/2$	$\sqrt{3}/2$
Liu's Quadrature	$\sqrt{6}$	$\sqrt{3}$

Source: Authors' Derivations

ii. *Broad Sample: SSA Inputs and Output*

With the values for α and β handy; all that is required, is scale the original σ by these factors. Let's call these σ_α and σ_β respectively. Equation (3) then for a chosen value of a and these standard deviations will give us the new extreme points. For e.g. E_a^{ETD} will be the extreme point for the expanded triangular distribution (ETD) and E_a^{CTD} is the same for the contracted triangular distribution (CTD). Similarly we have those for the UD.

$$E_a^{ETD} = \mu + \left(1 - \sqrt{1-a}\right)\sqrt{6}\,\sigma_\alpha \quad \text{and} \quad E_a^{EUD} = \mu + a\sqrt{3}\,\sigma_\alpha \tag{4}.$$

$$E_a^{CTD} = \mu + \left(1 - \sqrt{1-a}\right)\sqrt{6}\,\sigma_\beta \quad \text{and} \quad E_a^{CUD} = \mu + a\sqrt{3}\,\sigma_\beta \tag{5}.$$

So the amount of variation that one would specify for the SSA in its ETD version (the expanded triangular distribution version) would be given by $E_a^{ETD} - \mu = \left(1 - \sqrt{1-a}\right)\sqrt{6}\,\sigma_\alpha$. One can similarly use (4) and (5) to determine the same for other versions. Note that the only difference in comparison to the original triangular distribution SSA is that we use a scaled version of the σ. Also note that though (4) and (5) can be used to determine endpoints for both Stroud and Liu the values will differ because σ_α and σ_β will differ owing to the different expansion factors for the two quadratures (as in Table 1). With these new inputs for extreme end points we do the SSA

twice. The amount of variation for a Stroud TD for our application, obtained using the above procedure, are reported in Table 3.

Finally we need to combine the results (mean and standard deviations) of the expanded and the contracted SSAs using the probability p. And the formula for Broad sample mean (μ_{BS}) and broad sample standard deviation (σ_{BS}) can be shown to be as follows:

$$\mu_{BS} = p\mu_\alpha + (1-p)\mu_\beta \tag{6};$$

$$\sigma_{BS} = \left[p\sigma_\alpha^2 + (1-p)\sigma_\beta^2 + p(1-p)\left(\mu_\alpha - \mu_\beta\right)^2\right]^{\frac{1}{2}} \tag{7}.$$

These formulae apply to both, distribution of inputs (in our case sample shock values for wheat yields) as well as the distribution of endogenous model variables (additional SSM tariffs for our application).

III. Results

Using the formulae mentioned just above now we compare the SSA inputs and outputs for the broad sample SSA to that of the original SSA for the Stroud and Liu TD, for our SSM application. We chose to span the entire distribution domain, i.e. we chose the value of $a = 1$.

i. ***Moments of Yield Distribution: Mean and Variance (just to show that these should still be the same)***

We start with showing the extreme points and the min and max shocks to all the regions in our model. Appendix Table 3 provides the yield shocks for wheat in the regions. The original

extreme end points (given by $\sqrt{6}\sigma$ in column 1 of Table 3) are taken from Hertel et al. As one can see the maximum shock values sampled under the initial SSA setup never exceed $\sqrt{6}\sigma/\sqrt{3}$ or $\sqrt{2}\sigma$ whereas those under the broad sample SSA, span just about the whole distribution. The acronyms OSTD and BSSTD refer to Original Stroud Triangular Distribution and Broad Sample Stroud Triangular Distribution respectively.

The expansion and contraction of the original GQ is done subject to the conditions that the moments of distribution of the original sample shocks and the weighted moments of distribution of the two new distributions remain the same. This is what we check in Table 4. The mean and standard deviations of the original sample shocks can be calculated as

$$\mu_{OS} = \sum_i x_i/n \qquad \text{and} \qquad \sigma_{OS} = [\sum_i (x_i - \mu_{OS})^2/n]^{\frac{1}{2}} \quad \text{for} \quad i = 1 \dots n \qquad (8),$$

where x denotes the sample shock value chosen by the original Stroud GQ. These values can be accessed from reports saved by the SSA utility, which also provides the above two statistics for the shock values. We compare the results given by (8) to those given by (6) and (7).

Table 3: Stroud's End-Points (EP) and Extreme Shocks for Wheat Yields (percentage)

Region	Original EP for OSTD	Expanded EP for BSSTD	Contracted EP for BSSTD	Max shock (OSTD SSA)	Max shock (BSSTD SSA)
Australia	26.0	45.0	21.8	15.0	26.0
China	32.2	55.8	26.9	18.5	32.1
Japan	15.0	26.0	12.5	8.7	15.0
Other East Asia	21.5	37.2	18.0	12.4	21.4
South East Asia	24.5	42.4	20.5	14.1	24.5
Canada	16.2	28.1	13.6	9.3	16.1
USA	18.4	31.9	15.4	10.6	18.4
Mexico	24.5	42.4	20.5	14.1	24.4
Argentina	34.7	60.1	29.0	20.0	34.7
Brazil	86.1	149.1	72.0	49.5	85.7
Rest of Latin America	22.1	38.3	18.5	12.8	22.1
EU15	15.0	26.0	12.5	8.6	14.9
Other EUR	38.1	66.0	31.9	22.0	38.1
Russia	55.4	96.0	46.4	31.8	55.2
MENA	21.1	36.5	17.7	12.2	21.1
Sub-Saharan Africa	23.7	41.0	19.8	13.6	23.6
Rest of the World	64.5	111.7	54.0	26.3	45.6

Source: Hertel et al. and Model SSA Simulations

BSSTD: Broad Sample Stroud Triangular Distribution; OSTD: Original Stroud Triangular Distribution

Table 4: Comparing Mean and Standard Deviation of Sample Shocks to Wheat Yields in Original versus the Broad Sample

Region	μ_{OS}	σ_{OS}	μ_{BS}	σ_{BS}	$\mu_{OS} - \mu_{BS}$	$\sigma_{OS} - \sigma_{BS}$
Australia	-3.51E-08	1.06E+01	-3.48E-08	1.06E+01	-3.00E-10	0.00E+00
China	1.32E-07	1.31E+01	8.83E-08	1.31E+01	4.37E-08	0.00E+00
Japan	-2.79E-07	6.12E+00	-2.86E-07	6.12E+00	7.00E-09	0.00E+00
Other East Asia	-3.56E-07	8.78E+00	-3.70E-07	8.78E+00	1.40E-08	0.00E+00
South East Asia	-7.05E-07	1.00E+01	-6.59E-07	1.00E+01	-4.60E-08	0.00E+00
Canada	-2.90E-07	6.61E+00	-2.19E-07	6.61E+00	-7.10E-08	0.00E+00
USA	7.19E-07	7.51E+00	6.84E-07	7.51E+00	3.50E-08	0.00E+00
Mexico	-6.83E-07	1.00E+01	-6.06E-07	1.00E+01	-7.70E-08	0.00E+00
Argentina	8.48E-07	1.42E+01	9.23E-07	1.42E+01	-7.50E-08	0.00E+00
Brazil	-1.28E-05	3.52E+01	-1.21E-05	3.52E+01	-7.00E-07	0.00E+00
Rest of Latin America	1.11E-06	9.02E+00	1.02E-06	9.02E+00	9.00E-08	0.00E+00
EU15	-1.36E-06	6.12E+00	-1.32E-06	6.12E+00	-4.00E-08	0.00E+00
Other EUR	3.28E-06	1.56E+01	3.15E-06	1.56E+01	1.30E-07	0.00E+00
Russia	-1.12E-06	2.26E+01	-1.13E-06	2.26E+01	1.00E-08	0.00E+00
MENA	-2.32E-06	8.61E+00	-2.21E-06	8.61E+00	-1.10E-07	0.00E+00
Sub-Saharan Africa	-4.10E-06	9.68E+00	-4.01E-06	9.68E+00	-9.00E-08	0.00E+00
Rest of the World	0.00E+00	2.63E+01	0.00E+00	2.63E+01	0.00E+00	0.00E+00

Source: SSA Input Report Files and Authors' Calculations

ii.   *Moments of Additional Tariffs: Mean and Variance*

Having satisfied ourselves that the shocks were in fact generated from a broader sample with the same means and standard deviations (Table 4) and also that the new shocks do span the entire original sample domain (Table 3) we next turn to variables of our interest, which in this application are the additional SSM tariffs. We report the same measures (mean and standard deviations: original and broad sample) for SSM additional tariffs in Tables 5 and 6 as those for wheat yields reported in Table 4. Note that as only 9 of the 17 regions resort to SSM as a policy measure, the additional tariffs are reported for only those 9.

As one can see from the Tables below, for yield distributions characterized by the same μ and σ (Table 4) we get slight differences in the distributions of the results. These differences depend on the non-linearity of the model results in the variables shocked and also the inter-dependency of results in General Equilibrium set up.

For our application we see the Tier 1 mean tariffs to be lower for the Broad Sample SSA in comparison to original SSA, for both Stroud and Liu quadratures but the same cannot be said about the Tier 2 mean tariffs. Standard deviations too present mixed results across the two Tiers and Quadrature types.

One result that does stand out however and needs further look is that under Broad Sample SSA Tier 2 mean tariffs are positive for more countries than under the original SSA. It's true for both Stroud and Liu quadratures. While only 3 (for Stroud) and 2 (for Liu) countries see positive Tier 2 tariffs with original SSA, the number jumps to 7 under the broad sample SSA.

Table 5: Mean and Standard Deviation for Additional SSM Tariffs under the Original and Broad Sample Stroud SSA

	China	Other East Asia	South Asia	Mexico	Argentina	Brazil	Rest of Latin America	MENA	Sub-Saharan Africa
					TIER 1				
μ_{OS}	9.5	0.0	4.3	3.3	6.2	10.2	2.9	2.1	2.5
μ_{BS}	9.1	0.0	3.4	2.8	5.5	9.9	2.6	1.9	2.2
$\mu_{OS} - \mu_{BS}$	**0.3**	**0.0**	**0.9**	**0.5**	**0.7**	**0.3**	**0.3**	**0.2**	**0.3**
σ_{OS}	11.9	0.0	7.6	5.3	9.6	11.9	5.1	3.9	4.8
σ_{BS}	11.4	0.0	6.5	5.4	8.4	11.7	5.2	4.3	4.9
$\sigma_{OS} - \sigma_{BS}$	**0.4**	**0.0**	**1.2**	**-0.1**	**1.2**	**0.3**	**-0.1**	**-0.4**	**-0.1**
					TIER 2				
μ_{OS}	1.2	0.0	0.0	0.0	0.1	3.6	0.0	0.0	0.0
μ_{BS}	1.1	0.0	0.3	0.1	0.4	3.3	0.1	0.0	0.1
$\mu_{OS} - \mu_{BS}$	**0.1**	**0.0**	**-0.3**	**-0.1**	**-0.3**	**0.3**	**-0.1**	**0.0**	**-0.1**
σ_{OS}	1.8	0.0	0.0	0.0	0.7	5.5	0.0	0.0	0.0
σ_{BS}	1.8	0.0	1.8	0.9	2.2	5.2	0.7	0.3	0.8
$\sigma_{OS} - \sigma_{BS}$	**0.1**	**0.0**	**-1.8**	**-0.9**	**-1.5**	**0.3**	**-0.7**	**-0.3**	**-0.8**

Source: Model SSA Results and Authors' Calculations

Table 6: Mean and Standard Deviation for Additional SSM Tariffs under the Original and Broad Sample Liu SSA

	China	Other East Asia	South Asia	Mexico	Argentina	Brazil	Rest of Latin America	MENA	Sub-Saharan Africa
TIER 1									
μ_{OS}	12.5	0.0	4.4	3.2	6.4	12.5	2.9	2.0	2.4
μ_{BS}	12.4	0.0	3.1	2.5	5.3	12.3	2.3	1.6	2.0
$\mu_{OS} - \mu_{BS}$	**0.1**	**0.0**	**1.2**	**0.7**	**1.1**	**0.2**	**0.6**	**0.4**	**0.4**
σ_{OS}	12.6	0.0	5.7	4.8	8.8	12.6	4.1	2.9	3.7
σ_{BS}	12.4	0.0	5.2	4.6	7.5	12.3	4.4	3.8	4.1
$\sigma_{OS} - \sigma_{BS}$	**0.2**	**0.0**	**0.6**	**0.1**	**1.3**	**0.3**	**-0.3**	**-0.9**	**-0.5**
TIER 2									
μ_{OS}	1.8	0.0	0.0	0.0	0.0	4.3	0.0	0.0	0.0
μ_{BS}	1.4	0.0	0.3	0.2	0.3	3.2	0.1	0.0	0.1
$\mu_{OS} - \mu_{BS}$	**0.4**	**0.0**	**-0.3**	**-0.2**	**-0.3**	**1.1**	**-0.1**	**0.0**	**-0.1**
σ_{OS}	2.0	0.0	0.0	0.0	0.0	5.2	0.0	0.0	0.0
σ_{BS}	1.9	0.0	1.7	1.3	1.8	5.3	1.2	0.4	0.7
$\sigma_{OS} - \sigma_{BS}$	**0.1**	**0.0**	**-1.7**	**-1.3**	**-1.8**	**-0.1**	**-1.2**	**-0.4**	**-0.7**

Source: Model SSA Results and Authors' Calculations

iii.   *Frequency of Policy Trigger*

The presence of positive tariffs for a greater number of countries, under the broad sample SSA, motivates one to look at the frequency of Tier 2 SSM tariffs being triggered under the two different types of SSAs. Table 7 below reports this information.

Instead of actual numbers of times that a Tier 2 tariff gets triggered for a given SSA exercise, we report the percentage of the simulations in which the tariff gets triggered; the reason being that the broader sample involves twice as many number of simulations as the original standard SSA and therefore the actual trigger numbers are not strictly comparable.

Having a total of 17 regions translates into the number of SSA simulations for OSTD, BSSTD, OLTD (Original Liu Triangular Distribution) and BSLTD (Broad Sample Liu Triangular Distribution) are 34, 68, 64 and 128 respectively. For broad sample SSA the simulations with both the contracted and expanded distributions are pooled together, but unlike for μ and σ, the frequency of trigger being a count data they are not weighted.

The point that stands out is that Tier 2 SSM tariffs are triggered more often under the broad sample simulations. and Liu (needs more explanation). The result here is expected because the samples being drawn now cover a wider domain of original distribution (recall that by construction, the μ and σ of input distribution remains the same even under Broader sample SSA).

Table 7: SSM Policy Trigger Frequency (percent of sample simulations)

	China	Other East Asia	South Asia	Mexico	Argentina	Brazil	Rest of Latin America	MENA	Sub-Saharan Africa
TIER 1									
OSTD	47%	0%	32%	38%	38%	44%	35%	26%	29%
BSSTD	46%	0%	40%	38%	47%	46%	35%	32%	32%
OLTD	50%	0%	47%	38%	47%	50%	42%	42%	42%
BSLTD	50%	0%	46%	43%	53%	50%	42%	41%	42%
TIER 2									
OSTD	29%	0%	0%	0%	3%	35%	0%	0%	0%
BSSTD	32%	0%	9%	6%	13%	35%	6%	3%	3%
OLTD	47%	0%	0%	0%	0%	47%	0%	0%	0%
BSLTD	44%	0%	23%	18%	23%	40%	16%	8%	11%

Source: Model SSA Results and Authors' Calculations

iv.   *Other Comparisons*

Across the two strategies for SSA, it turns out that the mean domestic prices are higher and more volatile when broader sample strategy is used. They are as much as 10 percent higher and 65 percent more volatile in Brazil, which is the country relying on SSM the most. However it is puzzling that the mean wheat imports under the broader sample strategy fall less or even rise somewhat. As expected both import quantities and output become more volatile.

IV.   Conclusions

The broader sample SSA strategy developed by Preckel et al. does indeed help one to cover domain of input distributions that were not plausible to sample from with the existing SSA mechanism. It does so by splitting the input distribution into two – an expanded and contracted – distributions, while still imposing the μ and σ of the original distribution on the new composite distribution.

As we have seen it does make a difference in the frequency of an extreme regime (Tier 2) coming into play and also its intensity. Whether or not this new approach to doing SSA is important depends on application at hand; but it might be of particular interest for climate change models.

References

Ahmed, S.A., T.W. Hertel and W. Martin. 2010. Climate volatility and trade policy in Tanzania. Selected Paper for *13th Annual Conference on Global Economic Analysis*, 9–11 June 2010, Penang, Malaysia.

Beckman, J., T.W. Hertel and W. Tyner. 2011. Validating Energy-Oriented CGE Models. *Energy Economics* (forthcoming).

Harrison, G.W., and H.D. Vinod. 1992. The Sensitivity Analysis of General Equilibrium Models: Completely Randomized Factorial Sampling Designs. *Review of Economics and Statistics* (74): 357-362.

Hertel, T.W., W. Martin and A. Leister. 2010. Potential Implications of a Special Safeguard Mechanism in the World Trade Organization: the Case of Wheat. *World Bank Economic Review*, Volume 24(2), p330-359.

Preckel, P.V., M. Verma, T.W. Hertel and W. Martin. 2011. Systematic Sensitivity Analysis for GTAP – Where We've Been and Where We're Going. Selected paper for *14th Annual Conference on Global Economic Analysis*, 16-18 June 2011, Venice, Italy.

Valenzuela, E., T. W. Hertel, R. Keeney and J. Reimer. 2007. Assessing Global Computable General Equilibrium Model Validity Using Agricultural Price Volatility, *American Journal of Agricultural Economics*; Vol. 89(2), p383-397.

Verma, M., T.W. Hertel and E. Valenzuela. (forthcoming). Are The Poverty Effects of Trade Policies Invisible? *World Bank Economic Review*.

Wigle, R. 1991. The Pagan-Shannon Approximation: Unconditional Systematic Sensitivity in Minutes, *Empirical Economics* 16:35-49.

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[1] Our work here focuses only on quantity triggers, for the ease of detailed comparison.

[2] The value for a is chosen by the user and depends on his/her belief about what portion of the triangular distribution is the true domain for the SSA sample shocks. We for now recommend to use the values of $a \in [0.83,1]$ for Stroud TD, $[0.65,1]$ for Liu TD, $[0.82,1]$ for Stroud UD and $[0.58,1]$ for Liu UD.

[3] Note that the way SSA operates in GEMPACK, the input is only the difference between the end point and the mean, which is only $\sqrt{6}\sigma$ (for TD) or $\sqrt{3}\sigma$ (for UD) when $a = 1$.

[4] The exact values for TD and UD are 1.67489307354169 and 1.55884572681199 respectively.

[5] By true we mean that we want the shocks to be drawn from the entire domain not have only part of it covered.