AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Global Trade Analysis Project
https://www.gtap.agecon.purdue.edu/

This paper is from the
GTAP Annual Conference on Global Economic Analysis
https://www.gtap.agecon.purdue.edu/events/conferences/default.asp

Modeling cross-border investment in CGE: some alternatives and mechanisms

Csilla Lakatos* Terrie Walmsley†

Preliminary draft. Do not quote.

April 16, 2009

1 Introduction

Trade policy analysis has been at the core of the classic CGE exercise, but with the growing importance of cross-border investment flows, the focus of applied general equilibrium models needs to shift towards alternatives and mechanisms of modeling international investment, in general, and FDI, in particular. Nevertheless, the lack of bilateral data on foreign assets and liabilities has often compelled CGE modelers to employ a somewhat artificial representation of foreign investment. As in other models - WorldScan (Lejour, Veenendaal, and Verweij 2006) - the dynamic GTAP model (GDyn) in its current version adopts the so-called global trust to collect savings and allocate investment on behalf of the representative household.

The objective of this paper is twofold.

First, we present a discussion of alternatives for modeling cross-border investment in applied general equilibrium.

Second, we aim to replace the global trust in the dynamic GTAP model and opt for the explicit modeling of bilateral investment flows. The data of bilateral FDI flows and stocks that serves as a base of our modeling exercise has been built by CEPII, France and it is entirely documented in (Boumellassa, Gouel, and Laborde 2007): contrary to other data sources, this database is fully consistent, balanced and suitable for use in a CGE work. The explicit

*GTAP/Purdue University. Email: clakatos@purdue.edu
†GTAP/Purdue Univeristy. Email: twalmsle@purdue.edu

representation of foreign investment in GDyn entails a preliminary treatment of the data for the construction of an initial regional level cross-ownership matrix using RAS techniques.

Few CGE models have clear theoretical foundations with regard to the allocation mechanism of investment across destinations. Modeling options can be divided into three categories: firstly, allocation based on investment demand; secondly, allocation based on investment supply; and thirdly, ad-hoc allocation mechanisms.

CGE models with allocation based on investment demand (first mechanism) distribute investment given the amount of savings. Applied models of investment demand could be built on strong theoretical foundations, however it has been shown that with this type of specification models exhibit extreme fluctuations with respect to changes in relative profitability (Bourguignon, Branson, and De Melo 1989).

Most known CGE models follow the pioneering work of (Petri 1997) in modeling bilateral FDI and adopt an investment supply type of allocation (second mechanism): total wealth is distributed across destinations (sectors and regions) as a function of relative rates of return characterized by a constant elasticity of transformation function (CET). The capital owner's goal is to maximize the net of his wealth subject to the diversification constraints imposed by the CET. We can thus argue that the investment decision is not taken in a totally ad-hoc manner, but in an optimization framework where investment is allocated to regions with the highest rates of return and at the same time taking into account investors' preferences for a given mix of diversification. Investment will depend on changing relative rates of return, but it will not be concentrated in the region with the highest rate of return. Once again, we can prove that this method is similar to a portfolio allocation rule, where preferences for diversification are reflected in the calibrated parameters of the CET. As a critique to this method, we can argue that rates of return, especially in real asset CGE models, cannot be interpreted as a "price", but more like the ratio of two prices that are driven by other mechanisms in the model.

The GDyn (Ianchovichina and McDougall 2000) due to its specific disequilibrium approach for modeling international capital mobility and the theory of adaptive expectations is an example of the third category. As in other recursive dynamic models, in each period the equilibrium level of savings determines the total amount of investment expenditure. The allocation of investment across destinations is determined by two mechanisms: elimination of errors in expectations and the equalization of rates of return in the long run.

2 Mobility of capital in CGE

CGE models offer a variety of possibilities for modeling capital mobility depending on the type and level of disaggregation of available data (capital stocks, income from equity, cross-ownership) and the objectives of the modeling exercise.

As a starting point, we need to differentiate between mobility of physical capital and mobility of capital referring to the movement of resources that through investment of the current period become capital in the next period.

Physical capital mobility is not seen as actual movement of capital by a change in geographic location. Instead, the two alternative theories are: mobility of physical capital through trade[1] and mobility through depreciation.[2] Mobility of physical capital is not the main objective of my analysis and therefore by capital mobility I refer to the mobility of resources hereafter.

When considering the specification of capital mobility in a CGE model, there are three main types of issues that have to be considered:

- whether or nor capital should be mobile across sectors;

- whether or not capital should be mobile across borders;

- whether capital accumulation is described as a part of an intertemporal or recursive optimization[3].

If capital is specified as being sector specific, sectoral rates of return will have to be different and they cannot follow any predetermined patter. In this case, sectoral rates of return are endogenous and adjust in order to clear any differences between demand and supply of sectoral capital. If capital is chosen to be mobile perfectly mobile across sectors, there is an average rate of return that clears the aggregate market for capital. As argued by (Devarajan and Offerdal 1989) sectorally fixed capital reflects short run and perfect mobility could be associated with the long run.

Mobility of capital across borders does not necessarity refer to mobility across national state borders, but to whatever spacial unit is under consideration in the modeling excersize. If capital is assumed to be perfectly mobile across borders there is one aggregate international market for capital that is cleared by a unique international rate of return. In this approach, it is required that total supply of capital is know a-priori. On the other hand, if capital mobility is imperfect the specification will make use of exogenously defined elasticities that decribe the responsiveness of capital flows to rate of return differentials.

[1] Analogous with the theory that trade in factors is seen as a substitutes for trade in goods (Mundell 1957).

[2] Based on this approach, physical capital is fungible (putty) and it can take the concrete form necessary. The value of depreciated capital is used to finance capital formation.

[3] The same categories have been described in a comprehensive survey by (Islam 1999).

Finally, in the context of a multi-period dynamic optimization process capital mobility could be modeled using either a forward looking or recursive dynamic specification. In recursive dynamics agents are assumed to be myopic and the current period decisions are based entirely on current period variables. The forward looking specification assumes perfect foresight and relies heavily on (and it is sensitive) the projected path of state variables.

On the overall, there is no unique aproach that would perform optimally, but the choice of the specification of capital mobility should depend on the context and the objectives of the modeling exersize.

Few CGE models have clear theoretical foundations with regard to the allocation mechanism of investment across destinations. Modeling options can be divided into two main categories: investment demand models and non-investment (ad-hoc) allocation of investment .

CGE models with allocation based on investment demand distribute investment given the amount of savings. Applied models of investment demand could be built on strong theoretical foundations, however it has been shown that with this type of specification models exhibit extreme fluctuations with respect to changes in relative profitability (Bourguignon, Branson, and De Melo 1989).

In the second category there are models that determine investment based on an ad-hoc specification or more or less closely related to a theory of investment supply. Most known CGE models follow the pioneering work of (Petri 1997) in modeling bilateral FDI and adopt an investment supply type of allocation: total wealth is distributed across destinations (sectors and regions) as a function of relative rates of return characterized by a constant elasticity of transformation function (CET). The capital owner's goal is to maximize the net of his wealth subject to the diversification constraints imposed by the CET.

GDyn (Ianchovichina and McDougall 2000) due to its specific disequilibrium approach for modeling international capital mobility and the theory of adaptive expectations is an example of non-demand investment models. As in other recursive dynamic models, in each period the equilibrium level of savings determines the total amount of investment expenditure. The allocation of investment across destinations is determined by two mechanisms: elimination of errors in expectations and the equalization of rates of return in the long run. Cross-entropy optimization applied in preserving initial wealth allocation in GDyn could be considered as ad-hoc with no theoretical foundations, but we argue that it could be seen as a quasi-portfolio diversification rule given the equalization of rates of return in the long run.

2.1 Investment demand

Based on the economic theory of the firm's investment demand as formulated by (Nickell 1978) the firm maximizes the present value of its cash flows subject to the capital accumulation constraint:

$$\begin{aligned} \max V = \int_0^\infty e^{-rt} \left[p_t F(K_t, L_t) - w_t L_t - q_t I_t \right] dt \\ s.t. K_t = I_t - \delta K_t \\ K_0 = \overline{K_0} \end{aligned} \tag{1}$$

where r is a constant discount rate, L_t is the volume of labor employed at time t, K_t is the capital stock in place, I_t is the volume of investment, p_t is the price of output, w_t is the wage rate, q_t is the price of the investment good, δ is the depreciation rate and $F(K_t, L_t)$ is the production function. The solution to this optimization problem will lead to an investment demand function of the form:

$$I_t = \frac{1}{\gamma} \left(\frac{Q_t}{q_t} - 1 \right) \tag{2}$$

where Q_t is marginal cost of new capital and γ is a constant. Basically, we note that investment will be positive if the ratio $\frac{Q_t}{q_t}$ is bigger than 1, i.e. if the marginal cost of new capital is lower than the price of investment goods.

Applied CGE models of investment demand[4] are more or less closely related to the theoretical model of (Nickell 1978). Investment is determined based on the amount of available savings, while equilibrium is achieved either by the adjustment of an endogenous the interest rate or adjusting current account.

Despite the fact that investment demand models are built on strong theoretical foundations, (Lemelin and Decaluwe 2007) show that Equation 2 implies a demand elasticity of investment that is too high and leads to unstable models. In this sense, investment demand models exhibit high fluctuations with respect to changes of in relative profitability (Bourguignon, Branson, and De Melo 1989).

The main disadvantage of the investment demand specification lies in the high volatility of results and in the fact that it allows for negative investment (disinvestment) - a feature that could generate strange welfare results. Any shock that impacts the relative rate of return in combination with the reduced flexibility of the CES type specification of investment leads to disinvestment in the model. In addition, the model in its current form is calibrated on bilateral investment flows data: it has been argued in the literature that investment flows,

[4] For a review of these models see (Lemelin and Decaluwe 2007).

as opposed to stocks, are volatile and highly dependent on current economic conditions (Boumellassa, Gouel, and Laborde 2007).[5]

2.2 Investment supply

This specification is analogous with that developed by (Petri 1997) on modeling FDI in CGE.

Investment supply is driven by the total wealth to be allocated across destinations and acts as a portfolio diversification problem: the household's wealth (or in other words the value of its portfolio) is allocated at the beginning of each period subject to a diversification constraint. This specification could be compared with the consumer's choice among goods or the firm's choice among inputs subject to a budget/technology constraint, as it yields the same CES-type supply functions.

As defined by Equation 3, wealth in period t equals wealth in period $t-1$ in addition to savings of period t.

$$W_t = W_{t-1} + S_t \tag{3}$$

The allocation of wealth across regions is the following: wealth of region r is allocated in a two-stage budgeting problem across regions characterized by a separable constant elasticity of transformation (CET) function. The first stage of the portfolio-type optimization the household maximizes its total wealth subject to the diversification constraints:

$$\begin{aligned} \max W_r &= \sum_s RR_{rs} I_{rs} \\ s.t.\ \ I_r &= \sum_s \left[\alpha_{rs}^{-1/\sigma_s} I_{rs}^{\frac{\sigma_s+1}{\sigma_s}} \right]^{\frac{\sigma_s+1}{\sigma_s}} \end{aligned} \tag{4}$$

Relative investment is a function of relative rates of return and the due to the diversification constraint and the imperfect substitution investment does not crowed into the region with the highest rate of return. Investment decisions are differentiated by the country of origin and characterized by an imperfect substitution (σ_r) between preferences in investing in different countries. The variation in preferences of the calibrated parameters could be attributed either risk aversion or other features not captured by rates of return. Investment is driven by relative rates of return, and this specification does not require the

[5] Large take-overs, especially in relatively small countries could highly influence the yearly recorded investment flows data.

cross-entropy minimization theory for the diversification constraint as not all investment concentrate in the region with the highest profitability.

One could argue that the investment decision is not taken in a totally ad-hoc manner, but in an optimization framework where investment is allocated to regions with the highest rates of return and at the same time taking into account investors' preferences for a given mix of diversification. Investment will depend on changing relative rates of return, but it will not be concentrated in the region with the highest rate of return.

The model is calibrated on bilateral/sectoral investment stocks data and it could be considered as less volatile than the investment demand specification calibrated on investment flows. As a critique to this method, we can argue that rates of return, especially in real asset CGE models, cannot be interpreted as a "price", but more like the ratio of two prices that are driven by other mechanisms in the model.

3 Bilateralizing investment in GDyn

The lack of bilateral data on foreign assets and liabilities has often compelled CGE modelers to employ a somewhat artificial representation of foreign investment. The dynamic GTAP model in its current version adopts a fictional entity called global trust to collect savings and allocate investment on behalf of the representative household.

This approach could lead to the distortion of foreign asset holdings. For instance, as results of a strong and differentiated change in rates of return across regions the representative household is expected to rebalance its portfolio of foreign asset holdings. The global trust will use an average rate of return, but one could argue that different regions might differ in their propensity to rebalance their portfolios.

The data of bilateral FDI flows and stocks that serves as a base of this modeling exercise has been built by CEPII, France and it is entirely documented in (Boumellassa, Gouel, and Laborde 2007): contrary to other data sources, this database is fully consistent, balanced and suitable for use in a CGE work. The development of this database now allows us to replace the global trust and calibrate the model on actual bilateral investment data.

Bilateralizing investment in GDyn at this stage is equivalent with the elimination of the global trust as the entity that collects savings and redistributes regional investment allowing households to invest directly abroad. As a first step, the three income vectors representing income of the household from for-

eign assets, income of the global trust on assets based in region r and income of the household from domestic assets have been replaced with a single bilateral income matrix $YQB(r, s)$ in the main the basedata of the model. (r is the region that owns assets and receives payments, while s owes assets and pays dividends).

3.1 Cross-entropy minimization: preserving wealth allocation in GDyn

The GDyn in its current version does not make use portfolio allocation theory in determining gross ownership positions, i.e. investors reactions are based only on (expected) rates of return. The cross-entropy minimization theory (Golan, Judge, and Robinson 1994) applied in preserving initial wealth-allocation thus acts as a quasi-portfolio diversification rule given the equalization of rates of return on the long run. The use of the cross-entropy theory employed in GDyn can be motivated with the following reasons:

- as in the short and medium run the model allows for differences in rates of return, there is no mechanism that would stop the concentration of investment in the region with the highest rate of return;

- the entropy allocation rule would preserve the so-called "home bias puzzle" of investment; according to this empirical regularity discovered by (French and Poterba 1991) countries tend to allocate a significant share of their portfolio to their domestic assets;

- in order to avoid negative values for both gross foreign assets and liabilities: no matter what the exogenous shock to income/wealth variables, cross-entropy minimization would keep initial positive share as positive during the simulation.

Cross-entropy minimization in its most general form can be defined as the minimization of the degree of divergence between two partitions of a given total value subject to different constraints.

$$\min CE = \sum_r \sum_s sh_{rs} \log \frac{sh_{rs}}{sh_0_{rs}} \tag{5}$$

In this specific case we can define:

$$sh_{rs} = \frac{w_{rs}}{\sum_r wh_r} \tag{6}$$

$$sh_0_{rs} = \frac{w_0_{rs}}{\sum_r wh_0_r} \tag{7}$$

where w_0_{rs} and w_{rs} are the bilateral cross-ownership matrices in period t and $t+1$,respectively and wh_r and wh_0_r are total wealth of household r in period t and $t+1$,respectively.

Substituting the shares from above:

$$CE = \sum_r \sum_s \left[\frac{w_{rs}}{\sum_r wh_r} \log \left(\frac{w_{rs} \sum_r wh_0_r}{w_0_{rs} \sum_r wh_r} \right) \right] \tag{8}$$

Multiply by $\sum_r wh_r$ and expand the above expression:

$$CE \sum_r wh_r = \sum_r \sum_s \left[w_{rs} \log \frac{w_{rs}}{w_0_{rs}} \right] - \sum_r \sum_s \left[w_{rs} \log \frac{\sum_r wh_r}{\sum_r wh_0_r} \right]$$

$$CE \sum_r wh_r = \sum_r \sum_s \left[w_0_{rs} \log \frac{w_{rs}}{w_0_{rs}} \right] - \left[\sum_r wh_r \log \frac{\sum_r wh_r}{\sum_r wh_0_r} \right]$$

Since $\sum_r wh_r$ and $\sum_r wh_0_r$ are given, rewrite the above equation:

$$FHHLD = CE + \sum_r wh_r \log \frac{\sum_r wh_r}{\sum_r wh_0_r}$$

$$FHHLD \sum_r wh_r = \sum_r \sum_s \left[w_{rs} \log \frac{w_{rs}}{w_0_{rs}} \right]$$

Thus the cross-entropy minimization could be summed up with:

$$\min F = FHHLD \sum_r wh_r = \sum_r \sum_s \left[w_{rs} \log \frac{w_{rs}}{w_0_{rs}} \right]$$

$$s.t. \sum_s w_{rs} = wh_r$$

$$\sum_r w_{rs} = wf_s$$

The Lagrangean could be written as the following:

$$\mathcal{L}\left[w_{rs}, \lambda_r, \mu_s\right] = \sum_r \sum_s \left[w_{rs} \log \frac{w_{rs}}{w_0_{rs}}\right] + \lambda_r \left[wh_r - \sum_s w_{rs}\right] + \mu_s \left[wf_s - \sum_r w_{rs}\right]$$

The first order conditions are the following:

$$\frac{\partial \mathcal{L}}{\partial w_{rs}} = \log \frac{w_{rs}}{w_0_{rs}} + 1 - \lambda_r - \mu_s = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\lambda_r} = wh_r - \sum_s w_{rs} = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\mu_s} = wf_s - \sum_r w_{rs} = 0 \tag{11}$$

Solving the F.O.Cs we get:

$$\lambda_r + \mu_s = \log \frac{w_{rs}}{w_0_{rs}} + 1 \tag{12}$$

$$\sum_s w_{rs} = wh_r \tag{13}$$

$$\sum_r w_{rs} = wf_s \tag{14}$$

Thus, Equation 12 is the one determining the dynamic bilateral cross-ownership matrix in GDyn, while Lagrangian multipliers λ_r and μ_s are determined by 13 and 14.

Please note that considering the fact that cross-entropy minimization entails an optimization based on shares, the system of equations will become overdetermined and singular if solved for all n shares (solve for $n-1$ shares).

This method of modeling investment in GDyn offers the advantage that it requires minimum adjustment to the present version of the model and the associated database: the adaptive expectations theory is preserved, while the database needs small adjustment with the inclusion of the bilateral income data. On the other hand, the disadvantage of the model lies in the fact that the cross-entropy type optimization holds the structure of the cross-ownership matrix close to that of the initial no matter what the shock imposed.

While the cross-entropy based investment allocation method requires minimal modifications to the existing version of GDyn, the investment demand and supply based methods require the elimination of the theory of adaptive expectations as in these cases relative investment is determined by relative rates of

return.

3.2 The composition of wealth: CET versus cross-entropy

The cross-entropy theory used in the construction of GDyn could mostly be motivated with its suitability in reproducing empirical findings such as the "home-bias" of investment of (French and Poterba 1991), portfolio diversification and for its properties in keeping original positive shares positive along the simulations. Nevertheless, some could dispute the lack of theoretical foundations in employing such a tool in modeling investment in CGE and argue in favor of a CET type allocation as in many recently developed CGE models.

Cross-entropy initially applied in matrix balancing[6] problems in economics as means of minimizing the degree of divergence between two partitions keeps the composition of the cross-ownership matrix in GDyn close to its base year structure. The underlying assumption could be motivated by the fact that the initial composition of the wealth matrix is the optimal one in the diversification and risk dispersion attempts of the investing agent.

In the meantime, we argue that a CET-type allocation of investment will exhibit the relative inflexibility of the cross-entropy allocation with respect to the composition of the cross-ownership matrix. This argument could be explained with the so-called small shares problem of the CES-type specification (Kuiper and van Tongeren 2006).

A CES-type demand function, or more particularly an Armington style import demand or export supply will tend to underestimate trade creation no matter how significant reduction in barriers to trade are simulated: if there are little or no trade flows in the initial data the impact of trade liberalization on these flows will be insignificant. The small share problem of the Armington specification arises due to the calibration of parameters on initial database that will not adjust during the simulations.

In its most general form, a CES demand can be derived from a cost minimization problem subject to CES preferences and a budget constraint and will result in a demand function of the type:

$$X_i = \alpha_i \left(\frac{P_i}{P}\right)^{-\sigma} X \tag{15}$$

where X_i is input demand, X is the quantity of supplied output, α_i is a share parameter and σ is the elasticity of substitution between inputs. For the specific

[6] For instance, in the construction or updating of Social Accounting Matrices.

case of CES investment supply function X_i represents sectoral or regional investment supply, X is total wealth and the corresponding prices represent rates of return to investment. The CES price index is of the following form:

$$P = \left(\sum_i \alpha_i P_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{16}$$

Parameter α_i is calibrated on investment stocks of the base period and could be interpreted as the relative importance of investment to sector i or region r with respect to total wealth. If initially this share is zero or very small, post-simulation changes will not be significant.[7]

To sum up, the use of cross-entropy theory in the balancing of the cross-ownership matrix in GDyn could be motivated by many empirical regularities of investment behavior, while this specification exhibits a few of the inflexibilities of a CET-type demand.

References

Boumellassa, H. and Gouel, C. and Laborde, D. (2007). A Multisector, Multi-country FDI database for GTAP. 10th Annual GTAP Conference, Purdue University, USA.

Bourguignon, F. and Branson, W.H. and De Melo, J. (1989). Macroeconomic Adjustment and Income Distribution: A Macro Micro Simulation Model. OECD Development Centre.

Devarajan, S. and Offerdal, E. (1989). Capital Markets and Computable General Equilibrium Models: Comparative Statics without Apology. *Journal of Policy Modeling 11 (2).*

French, K.R. and Poterba, J.M. (1991). Investor Diversification and International Equity Markets. NBER working paper.

Golan, A. and Judge, G. and Robinson, S. (1994). Recovering Information in the Case of Underdetermined Problems and Incomplete Data. *Review of Economics and Statistics 76*, 541–9.

Ianchovichina, E. and McDougall, R. (2000). Theoretical Structure of Dynamic GTAP. *GTAP Technical Paper 17.*

Islam, N. (1999). Capital Mobility in CGE Models: a Survey.

[7] There could be significant changes if there are large changes in P_i or the elasticity of substitution is high.

Kuiper, M. and van Tongeren, F. (2006). Using Gravity to Move Armington: An Empirical Approach to the Small Initial Trade Share Problem in General Equilibrium Models. 9th Annual GTAP conference, Addis Ababa, Ethiopia.

Lejour, A. and Veenendaal, P. and Verweij, G.and van Leeuwen, N. (2006). WorldScan: A Model for International Economic Policy Analysis. *CPB Document*.

Lemelin, A. and Decaluwe, B. (2007). Issues in Recursive Dynamic CGE Modeling: Investment by Destination, Savings and Public Debt A Survey . *CIRPEE working paper*.

Mundell, R.A. (1957). International Trade and Factor Mobility. *The American Economic Review*, 321–335.

Nickell, SJ. (1978). *The Investment Decisions of Firms*. James Nisbet & Co Ltd.

Petri, P.A. (1997). Foreign Direct Investment in a Computable General Equilibrium Framework. Conference, Making APEC Work: Economic Challenges and Policy Alternatives.