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SAM elaboration as a multiobjective adjustment problem

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Abstract

When elaborating a SAM many data adjustments have to be made. In our study we assume we need to adjust the household data on income to the data on labor payments derived from the IO Table, assuming that the data on households is the one that needs to be adjusted. In this adjustment both the distribution of each labor income among the different household types and the distribution of the different labor incomes among each household type have to be simultaneously taken into account.

We propose multicriteria optimization techniques to solve this problem and to find a set of efficient solutions. We then show different ways to rank or select between these efficient solutions. We include various examples to show the effects of applying different decision criteria.

1. Introduction.

When elaborating a SAM many data adjustments have to be made. Just to take an example, in most of the cases the data sources used to distribute labor payments between labor categories (e.g. gender, training etc) are not the same as the one's needed to distribute labor income among different types of households. This data issue explains the differences in both schemes. In our study we elaborate an example that could be used later on to solve the need to adjust the household data on income to the data on labor payments derived from the IO Table, assuming that the data on households is the one that needs to be adjusted. In this adjustment both the distribution of each labor income among the different household types and the distribution of the different labor incomes among each household type have to be simultaneously taken into account.

We propose multicriteria optimization techniques to solve this problem. The objectives are expressed in terms of the column and row coefficients and we find a set of efficient solutions. We formulate this adjustment problem as a multicriteria optimization problem where the objectives are expressed in terms of the column and row coefficients, and we use multicriteria optimization techniques such as compromise programming to find a set of efficient solutions. We use the minimum sum of cross entropies as separation measure between the initial and final coefficients matrices.

We propose multicriteria optimization techniques to solve this problem and to find a set of efficient solutions. We then show different ways to rank or select between these efficient solutions. We use multiobjective programming combining both the weighting method with the constraint methods. We include various examples to show the effects of applying different decision criteria.

2. Adjustment criteria, multiobjective formulations and preliminary analysis.

Given a non-negative $m \times n$ matrix, $X^0 = \left(x_{ij}^0\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \in \mathrm{M}_{m\times n}\left(\mathbb{R}\right)$, and $\vec{r} = \begin{bmatrix} r_1 & \cdots & r_m \end{bmatrix}^t \in \mathbb{R}^m$, we want to determine an $m \times n$ matrix of positive elements, $\alpha = \left(\alpha_{ij}\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \in \mathrm{M}_{m\times n}\left(\mathbb{R}^+\right)$, which satisfies $\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i,\ i = 1,\dots,m,$ and optimizes certain objective. This objective is defined as a *separation measure between the initial and final matrices*, $X^0 = \left(x_{ij}^0\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$ and $X^t = \left(\alpha_{ij} x_{ij}^0\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$, or between the initial and final row and column coefficient matrices.

For a matrix of non-negative elements, $X = \left(x_{ij}\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \in \mathrm{M}_{m\times n}\left(\mathbb{R}^+\right)$, we denote $S_+ = \left\{(i,j) \in \{1,\dots,m\} \times \{1,\dots,n\} : x_{ij} > 0\right\}$. The matrix of row coefficients, $A = \left(a_{ij}\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \in \mathrm{M}_{m\times n}\left(\mathbb{R}^+\right)$, and that of column coefficients, $B = \left(b_{ij}\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \in \mathrm{M}_{m\times n}\left(\mathbb{R}^+\right)$, are defined as follows:

$$a_{ij} = \frac{x_{ij}}{\sum_{k=1}^{n} x_{ik}}, \quad i = 1,\dots,m,\ j = 1,\dots,n,$$
$$b_{ij} = \frac{x_{ij}}{\sum_{k=1}^{m} x_{kj}}, \quad i = 1,\dots,m,\ j = 1,\dots,n.$$

Let $A^0 = \left(a_{ij}^0\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$ and $B^0 = \left(b_{ij}^0\right)_{\substack{i=1,\dots,m \\ j=1,\dots,n}}$ be the initial row and column coefficient matrices.

If $x_{ij} = \alpha_{ij} x_{ij}^0$ with $\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i,\ i = 1,\dots,m,$ we have $a_{ij} = \dfrac{\alpha_{ij} x_{ij}^0}{\sum_{k=1}^{n} \alpha_{ik} x_{ik}^0} = \dfrac{\alpha_{ij} x_{ij}^0}{r_i}$ and $b_{ij} = \dfrac{\alpha_{ij} x_{ij}^0}{\sum_{k=1}^{m} \alpha_{kj} x_{kj}^0}$.

The separation measure chosen is the sum of cross entropies. We apply this separation measure for row and column coefficients leading to two different objectives:

$$f_H(X,X^0)=\sum_{(i,j)\in S_+} a_{ij}\ln\frac{a_{ij}}{a_{ij}^0}=\sum_{(i,j)\in S_+}\frac{x_{ij}^0}{r_i}\alpha_{ij}\ln\left(\frac{r_i^0}{r_i}\alpha_{ij}\right)$$

$$f_V(X,X^0)=\sum_{(i,j)\in S_+} b_{ij}\ln\frac{b_{ij}}{b_{ij}^0}=\sum_{(i,j)\in S_+} x_{ij}^0\frac{\alpha_{ij}}{c_j}\ln\left(c_j^0\frac{\alpha_{ij}}{c_j}\right)$$

where $c_j=\sum_{i=1}^{m}x_{ij}=\sum_{i=1}^{m}\alpha_{ij}x_{ij}^0,$ and $c_j^0=\sum_{i=1}^{m}x_{ij}^0,\ j=1,\dots,n.$

Particular cases.

Case 1: $\alpha_{ij}=\alpha_i,\ \forall i,j.$

In this case, from the total row conditions, it follows that $\alpha_i=\frac{r_i}{r_i^0},\ i=1,\dots,m,\ j=1,\dots,n.$ Then $A=A^0,\ f_H(X,X^0)=0$ and

$$f_V(X,X^0)=\sum_{(i,j)\in S_+}\frac{\frac{r_i}{r_i^0}x_{ij}^0}{\sum_{k=1}^{m}\frac{r_k}{r_k^0}x_{kj}^0}\ln\left(c_j^0\frac{\frac{r_i}{r_i^0}}{\sum_{k=1}^{m}\frac{r_k}{r_k^0}x_{kj}^0}\right).$$

Case 2: $\alpha_{ij}=\beta_j,\ \forall i,j.$

If a vector $\vec{\beta}=\left[\beta_1\ \cdots\ \beta_n\right]^t\in\mathbb{R}^n$ with $\beta_j>0,\ j=1,\dots,n,$ such that $X^0\vec{\beta}=\vec{r}$ exists, then for $X=\left(\beta_j x_{ij}^0\right)_{\substack{i=1,\dots,m\\ j=1,\dots,n}},\ B=B^0,\ f_V(X,X^0)=0$ and

$$f_H(X,X^0)=\sum_{(i,j)\in S_+}\frac{x_{ij}^0}{r_i}\beta_j\ln\left(\frac{r_i^0}{r_i}\beta_j\right).$$

In order to determine the smaller interval of variation of positive α_{ij} in columns which guarantees a solution to the system of linear equations $\sum_{j=1}^{n}\alpha_{ij}x_{ij}^{0}=r_i,\, i=1,\ldots,m,$ we solve the following optimization problem:

$$
\begin{aligned}
&\min z \\
&s.t. \\
&\sum_{j=1}^{n}\alpha_{ij}x_{ij}^{0}=r_i, \quad i=1,\ldots,m, \\
&l_j \le \alpha_{ij} \le u_j, \quad i=1,\ldots,m,\, j=1,\ldots,n, \\
&u_j - l_j \le z, \quad j=1,\ldots,n, \\
&\alpha_{ij} \ge \varepsilon, \quad i=1,\ldots,m,\, j=1,\ldots,n
\end{aligned}
\tag{1}
$$

where ε is a very small value which is introduced to avoid zero solutions. The optimum is $z^{*}=\max\left\{u_j-l_j,\, j=1,\ldots,n\right\},$ that is, we minimize the maximum range $u_j-l_j.$ In particular, if $z^{*}=0$ then a vector $\vec{\beta}=\left[\beta_1 \quad \cdots \quad \beta_n\right]^{t}\in\mathbb{R}^{n}$ with $\beta_j>0,\, j=1,\ldots,n,$ verifying $X^{0}\vec{\beta}=\vec{r}$ exists.

If $z^{*}=u_k^{*}-l_k^{*}>0,$ we can find positive values α_{ij} with $-z^{*}\le\alpha_{ij}-\alpha_{1j}\le z^{*},$ for any row *i* and column *j*, which satisfy $\sum_{j=1}^{n}\alpha_{ij}x_{ij}^{0}=r_i,\, i=1,\ldots,m.$ Using the optimum z^{*} we can determine more adjusted bounds for columns by solving the problem

$$
\begin{aligned}
&\min \sum_{j=1}^{n}\left(u_j-l_j\right) \\
&s.t. \\
&\sum_{j=1}^{n}\alpha_{ij}x_{ij}^{0}=r_i, \quad i=1,\ldots,m, \\
&l_j \le \alpha_{ij} \le u_j, \quad i=1,\ldots,m,\, j=1,\ldots,n, \\
&u_j - l_j \le z^{*}, \quad j=1,\ldots,n, \\
&\alpha_{ij} \ge \varepsilon, \quad i=1,\ldots,m,\, j=1,\ldots,n.
\end{aligned}
\tag{2}
$$

If the optimal solution to this problem is given by $l_j^{*}, u_j^{*},\, j=1,\ldots,n,$ we have positive coefficients α_{ij} with $l_j^{*}\le\alpha_{ij}\le u_j^{*}, \forall i, j,$ which satisfy $\sum_{j=1}^{n}\alpha_{ij}x_{ij}^{0}=r_i,\, i=1,\ldots,m.$

We are interested in matrices $\alpha = \left(\alpha_{ij}\right)_{\substack{i=1,\ldots,m \\ j=1,\ldots,n}}$ such that the differences between coefficients α_{ij} are small, both for each column and each row.

Example 1.

Consider the following scenarios:

a) $X_a^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \\ 5 & 9 & 20 & 18 \end{bmatrix}, \vec{r}_a = \begin{bmatrix} 49 \\ 35 \\ 65 \\ 55 \end{bmatrix} \quad \Rightarrow \quad z^* = 0.$

b) $X_b^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \\ 2 & 30 & 14 & 46 \end{bmatrix}, \vec{r}_b = \begin{bmatrix} 63 \\ 42 \\ 69 \\ 129 \end{bmatrix} \quad \Rightarrow \quad z^* = 0.032608696.$

c) $X_c^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \\ 2 & 30 & 14 & 46 \end{bmatrix}, \vec{r}_c = \begin{bmatrix} 89 \\ 42 \\ 69 \\ 178 \end{bmatrix} \quad \Rightarrow \quad z^* = 0.348442371.$

d) $X_d^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \end{bmatrix}, \vec{r}_d = \begin{bmatrix} 63 \\ 42 \\ 69 \end{bmatrix} \quad \Rightarrow \quad z^* = 0.$

e) $X_e^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \end{bmatrix}, \vec{r}_e = \begin{bmatrix} 89 \\ 42 \\ 69 \end{bmatrix} \quad \Rightarrow \quad z^* = 0.348442371.$

In scenarios a) and d) there is a vector $\vec{\beta} = [\beta_1 \ \cdots \ \beta_n]^t \in \mathbb{R}^n$ with $\beta_j > 0,\ j = 1, \ldots, n,$ such that $X^0 \vec{\beta} = \vec{r}$. In examples b), c) and e) such a vector $\vec{\beta}$ does not exist, in these cases we can find positive values α_{ij} with $\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i,\ i = 1, \ldots, m,$ verifying $|\alpha_{ij} - \alpha_{kj}| \le z^*, \forall i, k,$ for any column *j*. Observe that, from known algebraic results, the solution to the system $X^0 \vec{\beta} = \vec{r}$ in case a) is unique, in case d) we have a infinite set of solutions.

Multiobjective adjustment formulations.

We want to find a matrix $\alpha = (\alpha_{ij})_{\substack{i=1,\ldots,m \\ j=1,\ldots,n}} \in \mathrm{M}_{m \times n}(\mathbb{R}^+)$ which provides a "balanced" solution respect to the horizontal and vertical objectives, $f_H(X, X^0)$ and $f_V(X, X^0)$, with moderate differences between values α_{ij} for each column and row.

When $z^* = 0$ there is a vector $\vec{\beta} = [\beta_1 \ \cdots \ \beta_n]^t \in \mathbb{R}^n$ with $\beta_j > 0,\ j = 1, \ldots, n,$ such that $X^0 \vec{\beta} = \vec{r}$, and for $X^t = (\beta_j x_{ij}^0)_{\substack{i=1,\ldots,m \\ j=1,\ldots,n}}$ we have $f_V(X, X^0) = 0$. We can be interested in maintaining equal coefficients for each column ($\beta_j > 0,\ j = 1, \ldots, n$), and obtain those which minimize $f_H(X, X^0)$ and the difference between them, with this aim we consider a second objective defined as $\max_j \{\beta_j\} - \min_j \{\beta_j\}$ and formulate the following bi-objective program:

$$\begin{aligned}
&\min \left(f_H(X, X^0), u - l \right) \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \beta_j x_{ij}^0 = r_i, \quad i = 1, \ldots, m, \qquad\qquad (3) \\
&l \le \beta_j \le u, \quad j = 1, \ldots, n, \\
&\beta_j \ge \varepsilon, \quad j = 1, \ldots, n.
\end{aligned}$$

Example 2.

Consider the scenario d) in Example 1 where $X_d^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \end{bmatrix}, \vec{r}_d = \begin{bmatrix} 63 \\ 42 \\ 69 \end{bmatrix}$ and $z^* = 0.$ If we optimize the two objectives $f_H(X, X^0)$ and $u - l$ independently, we obtain the following objective values and solutions:

$\left(f_H(X, X^0), u - l \right) = (0.6374, 2.0945)$ with $\vec{\beta} = [0.6066 \quad 0.5801 \quad 0.4291 \quad 2.2038],$

$\left(f_H(X, X^0), u - l \right) = (0.6434, 1.6881)$ with $\vec{\beta} = [0.5662 \quad 0.5246 \quad 0.5246 \quad 2.2127].$

In order to obtain alternative solutions to Problem (3) we can apply multicriteria optimization techniques such as multiobjective programming, compromise programming, goal programming and others. In Example 2, using the constraint method for multiobjective programming, we obtain an alternative solution solving the problem:

$$\begin{aligned} & \min f_H(X, X^0) \\ & \text{s.t.} \\ & \sum_{j=1}^{n} \beta_j x_{ij}^0 = r_i, \quad i = 1, \ldots, m, \\ & l \le \beta_j \le u, \quad j = 1, \ldots, n, \\ & u - l \le \gamma \\ & \beta_j \ge \varepsilon, \quad j = 1, \ldots, n, \end{aligned} \tag{4}$$

where γ is a value comprised between 1.6881 and 2.0945; so, for γ=1.75 we obtain $\left(f_H(X, X^0), u - l \right) = (0.6379, 1.75)$ and $\vec{\beta} = [0.5951 \quad 0.5643 \quad 0.4563 \quad 2.2063].$

If $z^* > 0$ a vector $\vec{\beta} = [\beta_1 \ \cdots \ \beta_n]^t \in \mathbb{R}^n$ with $\beta_j > 0,\ j = 1, \ldots, n,$ such that $X^0 \vec{\beta} = \vec{r},$ does not exist and, for any column, the minimum range of variation of α_{ij} which provides a positive solution to the system $\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i, i = 1, \ldots, m,$ is $z^*.$ If we are interested in the objectives $f_H(X, X^0)$ and $f_V(X, X^0),$ and simultaneously we want to

keep the variation ranges fixed to $z^*,$ we solve the problem

$$
\begin{aligned}
&\min\left(f_H(X, X^0), f_V(X, X^0)\right) \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i, \quad i = 1,\dots,m, \\
&-z^* \le \alpha_{ij} - \alpha_{kj} \le z^*, \quad i,k = 2,\dots,m,\ j = 1,\dots,n, \\
&\alpha_{ij} \ge \varepsilon, \quad i = 1,\dots,m,\ j = 1,\dots,n.
\end{aligned}
\tag{5}
$$

In constraints $-z^* \le \alpha_{ij} - \alpha_{kj} \le z^*$ the bounds may be relaxed replacing $-z^*$ and z^* by $g\left(z^*\right), h\left(z^*\right)$ (also when $z^* = 0$).

Example 3.

The last two columns contain the values corresponding to the Pareto optimal solution to Problem (5) when we minimize $f_H\left(X, X^0\right) + f_V\left(X, X^0\right)$.

Scenario	X^0	$\vec{r}$	z^*	$g\left(z^*\right)$	$h\left(z^*\right)$	f_H	f_V
1	X_a^0	$\vec{r}_a$	0	$-z^*$	z^*	0.0771	0
2	X_c^0	$\vec{r}_c$	0.3484	$-z^*$	z^*	1.7129	2.7246
3	X_d^0	$\vec{r}_d$	0	$-z^*$	z^*	0.6374	0
4	X_e^0	$\vec{r}_e$	0.3484	$-z^*$	z^*	1.5139	4.3747
5	X_a^0	$\vec{r}_a$	0	$-(z^*+1)$	z^*+1	0.0001	0.0015
6	X_c^0	$\vec{r}_c$	0.3484	$-2z^*$	$2z^*$	0.0616	0.0985
7	X_d^0	$\vec{r}_d$	0	$-(z^*+1)$	z^*+1	0.0016	0.0189
8	X_e^0	$\vec{r}_e$	0.3484	$-2z^*$	$2z^*$	0.0564	0.1081

Table 1

In some of these scenarios, the solutions obtained are not useful because, due to the limitation imposed on the variation ranges for α_{ij}, the solution contains values close to zero (values equal to the lower bound imposed on α_{ij}). In Table 1, we can observe some scenarios where this limitation is in conflict with the horizontal and vertical objectives ($f_H\left(X, X^0\right)$ and $f_V\left(X, X^0\right).$).

In order to combine the minimization of objectives $f_H\left(X, X^0\right)$ and $f_V\left(X, X^0\right).$ with the minimization of the differences between coefficients α_{ij} in columns and rows, we define the following objectives:

$$\min f_H(X, X^0) \text{ and } \min f_V(X, X^0)$$

$$\min_j \left\{ \max_i \left\{ \alpha_{ij} \right\} - \min_i \left\{ \alpha_{ij} \right\} \right\}$$

$$\min \sum_i \left(\max_j \left\{ \alpha_{ij} \right\} - \min_j \left\{ \alpha_{ij} \right\} \right)$$

$$\min \sum_j \left(\max_i \left\{ \alpha_{ij} \right\} - \min_i \left\{ \alpha_{ij} \right\} \right)$$

which are included in the following formulation:

$$\begin{aligned}
&\min \left(f_H(X, X^0), f_V(X, X^0), \max_j \left\{ u_j^c - l_j^c \right\}, \sum_{j=1}^{n} \left(u_j^c - l_j^c \right), \sum_{i=1}^{m} \left(u_i^r - l_i^r \right) \right) \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i, \quad i = 1, \ldots, m, \\
&l_j^c \le \alpha_{ij} \le u_j^c, \quad i = 2, \ldots, m, \ j = 1, \ldots, n, \\
&l_i^r \le \alpha_{ij} \le u_i^r, \quad i = 2, \ldots, m, \ j = 1, \ldots, n, \\
&\alpha_{ij} \ge \varepsilon, \quad i = 1, \ldots, m, \ j = 1, \ldots, n.
\end{aligned} \tag{6}$$

To solve this problem we introduce three of the objectives in the set of constraints leading to the program:

$$
\begin{aligned}
&\min\left(f_H(X, X^0), f_V(X, X^0), \right)\\
&\text{s.t.}\\
&\sum_{j=1}^{n} \alpha_{ij} x_{ij}^0 = r_i, \quad i = 1, \ldots, m,\\
&l_j^c \le \alpha_{ij} \le u_j^c, \quad i = 2, \ldots, m,\ j = 1, \ldots, n,\\
&l_j^r \le \alpha_{ij} \le u_j^r, \quad i = 2, \ldots, m,\ j = 1, \ldots, n,\\
&u_j^c - l_j^c \le \gamma\left(z^*\right)\\
&\sum_{j=1}^{n} \left(u_j^c - l_j^c\right) \le \rho^c\\
&\sum_{i=1}^{m} \left(u_i^r - l_i^r\right) \le \rho^r\\
&\alpha_{ij} \ge \varepsilon, \quad i = 1, \ldots, m,\ j = 1, \ldots, n,
\end{aligned}
\tag{7}
$$

where $\gamma\left(z^*\right), \rho^c,$ and ρ^r, are fixed. We can obtain efficient solutions assigning different weights in the objective functions for different values of $\gamma\left(z^*\right), \rho^c,$ and ρ^r, this procedure is a mixture of weighting and constraint methods for multiobjective programming.

<u>Example 4.</u>

Consider $X_e^0 = \begin{bmatrix} 1 & 15 & 7 & 23 \\ 10 & 2 & 4 & 15 \\ 12 & 17 & 13 & 21 \end{bmatrix}, \vec{r}_e = \begin{bmatrix} 89 \\ 42 \\ 69 \end{bmatrix}$. Table 2 shows the objective values for the solutions obtained when we minimize each objective disregarding the other ones. The diagonal of the table contains the ideal values.

Objective	$z^* = \max_j \{u_j^c - l_j^c\}$	$\sum_{j=1}^{n}(u_j^c - l_j^c)$	$\sum_{i=1}^{m}(u_j^r - l_j^r)$	f_H	f_V
$z^* = \max_j \{u_j^c - l_j^c\}$	**0.3484**	1.3938	8.1127	1.5140	4.3889
$\sum_{j=1}^{n}(u_j^c - l_j^c)$	0.6969	**0.6969**	8.8096	1.6909	0.0064
$\sum_{i=1}^{m}(u_j^r - l_j^r)$	0.8395	3.3582	**0**	0	0.1131
f_H	0.8395	3.3582	0	**0**	0.1131
f_V	0.8356	0.8584	8.0695	1.2676	**0.0094**

Table 2

The optimal matrices $\alpha = (\alpha_{ij})_{\substack{i=1,\ldots,m \\ j=1,\ldots,n}}$ corresponding to cases in Table 2 are shown in Table 3 where the last cell includes the solution obtained when we minimize the function $w_1 f_H(X, X^0) + w_2 f_V(X, X^0)$ subject to constraints of Problem (7) with $w_1 = w_2 = 1$ and $\gamma(z^*) = 0.70,\ \rho^c = 1.80,\ \text{and}\ \rho^r = 4.$ The objective values obtained are $(f_H, f_V, \gamma, \rho^c, \rho^r) = (0.3263, 0.0467, 0.70, 1.80, 4).$

$$z^* = \max_j \left\{ u_j^c - l_j^c \right\}$$

	1	2	3	4
1	0.3484	1.0667	0.3484	3.0527
2	1.00E-9	0.7183	1.00E-9	2.7042
3	1.00E-9	0.7183	1.00E-9	2.7042

$$\sum_{j=1}^{n} \left(u_j^c - l_j^c \right)$$

	1	2	3	4
1	1.00E-9	0.7183	1.00E-9	3.4011
2	1.00E-9	0.7183	1.00E-9	2.7042
3	1.00E-9	0.7183	1.00E-9	2.7042

$$\sum_{i=1}^{m} \left(u_j^r - l_j^r \right)$$

	1	2	3	4
1	1.9348	1.9348	1.9348	1.9348
2	1.3548	1.3548	1.3548	1.3548
3	1.0952	1.0952	1.0952	1.0952

$$f_H$$

	1	2	3	4
1	1.9348	1.9348	1.9348	1.9348
2	1.3548	1.3548	1.3548	1.3548
3	1.0952	1.0952	1.0952	1.0952

$$f_V$$

	1	2	3	4
1	0.1649	0.6304	0.1651	3.4010
2	0.1643	0.6089	0.1643	2.5655
3	0.1643	0.6098	0.1643	2.5965

$$f_H\left(X, X^0\right) + f_V\left(X, X^0\right)$$

	1	2	3	4
1	0.6817	2.1986	0.6817	2.1986
2	0.5523	2.0074	0.5877	2.0074
3	0.4706	1.4986	0.4928	1.4986

Table 3

Conclusions

These examples show how the multicriteria optimization techniques allow us to combine various adjustment criteria and to obtain more equilibrated solutions. In the last model, where we combined the five objectives, we can observe how we obtained more realistic solutions.

These results offer a tool to solve the SAM adjustment problems mentioned in the introduction of this work.

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