MAMS:* A framework for analyzing MDG and poverty reduction strategies

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*MAMS – MAquette for MDG Simulation; MDG = Millennium Development Goal

Introduction

This paper documents a dynamic Computable General Equilibrium (CGE) model that is designed to analyze strategies for achieving the Millennium Development Goals (MDGs) and, more broadly, strategies for growth and poverty reduction in developing countries. Our reference to the term "maquette" (French for model, sketch, or layout) reflects that the model is designed to capture the key processes for MDG achievement in a manner that is applicable to a wide range of countries. When applying the model to any given country, the key requirement is the development of a country-specific database that, among other things, summarizes the findings from sector studies on MDGs. In addition, it may often be necessary to adapt parts of the model structure to country-specific conditions.

An economywide approach is needed in MDG analysis given that the pursuit of key MDG-related policies typically has effects across the economy that feed back into the processes that determine MDG achievement. In its treatment of these processes, our approach considers the important roles that are played by the provision of MDG-related services (including health and education), as well as the social and economic status of the population (including per-capita household consumption and MDG achievements in related areas), incentives, and the state of infrastructure. In this process, external financing needs depend on economic performance in general, including growth in domestic government revenues.

The model may be solved in a "multi-pass" mode, one period at a time starting with the base year and moving forward, or in a "single-pass" mode, simultaneously for all time periods covered. It may be divided into three modules – "within-period" and "between-period" modules of a core CGE model and an MDG module – all of which are integrated into a simultaneous system of linear and non-linear equations.[1] The within-period module (in essence a static CGE model) captures the bulk of the production, consumption,

[1] This model is an extension of the static, standard CGE model in Lofgren et al. (2002) and the MDG model in Bourguignon et al. (2004). In addition to adding time and incorporating the MDG module, the model extends the earlier model by endogenizing factor productivity (which depends on economic openness and government capital stocks), and tracking the assets (liabilities) of the different institutions (factor endowments, domestic government debts, and foreign debts). The computer code is written in GAMS.

investment and trade decisions of the economy in any given time period. The between-period module provides linkages over time by updating selected parameters (including factor supplies, population, and factor productivity) on the basis of exogenous trends and past endogenous variables. The MDG module captures the processes that determine MDG achievement, most importantly the provision of services in the areas of education, health, water and sanitation, and other public infrastructure. The size and skill composition of the labor force is endogenized, in large measure depending on the evolution of education. The MDG module has feedback effects into the rest of the economy, most importantly via the labor market. In the model, growth depends on the accumulation of production factors (labor at different educational levels, private capital, and other factors such as land, if present) and changes in factor productivity, which is influenced by the accumulation of government capital stocks and openness to foreign trade.

For each time period, the model equations give a comprehensive account of decisions and related payments involving production (activities produce outputs using factors and intermediate inputs), consumption (by households and the government), investment (private, including FDI, and government), trade (both domestic and foreign), taxation, transfers between institutions (households, government, and the rest of the world), and links between factors and institutions. They also cover the market constraints, macro balances, and dynamic updating equations under which the agents operate. Given this comprehensive picture of the economy, for each year, a model solution defines a wide range of indicators of economic performance.

Poverty and inequality analysis, as in other CGE models, can be performed in several ways. The simplest but least desirable method uses an elasticity calculation for poverty given changes in per-capita household consumption. Representative-household or survey-based microsimulation approaches are preferable. The former assume fixed distributions of income or consumption within each household group, providing welfare estimations directly from the CGE model results. The latter can be either top-down, feeding CGE simulation results to a household model, or integrated, with the household model built directly into MAMS.

In essence, the current model structure is recursive: the decisions of economic agents depend on the past and the present, not the future, i.e., for the most part, forward-looking behavior is not assumed.[2] However, when the model is solved in single-pass mode, non-recursive features may be (and have been) incorporated; for example, the government may select fixed growth rates over time (or different degrees of front- or back-loading) for different government services so that MDG targets are achieved in 2015. Such decisions are forward-looking since they depend on future values outside of direct control of the government (such as household consumption per capita).[3] Potentially, when the model is solved in single-pass mode, selected aspects of private producer and consumer behavior could also be forward-looking, incorporating variable values from the future.

The disaggregation of the model is data-driven and flexible in most areas: subject to computer memory constraints, it can handle any number (one or larger) of primary factors, households, production activities, and commodities.[4] The applicability of the model to specific policy issues depends in part on the degree of disaggregation. For example, the analysis of issues related to poverty requires a relatively detailed disaggregation of household income sources (from factor endowments and the production activities in which they are employed) and consumption. If poverty and inequality is analyzed using a representative-household or integrated micro-simulation approach, then the household sector must also be disaggregated (highly disaggregated for the integrated micro-simulation case). For many analytical purposes, it is also preferable to disaggregate non-government production into multiple sectors.

In the rest of this document, we describe the different parts of our model, making frequent reference to the accompanying tables: Tables 1-2 (notation and equations for the

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[2] As a result of this absence of non-recursive features, the within-period module, including the subset of the MDG equations that only refer to the current period, can be solved recursively, one period at a time, with updating of selected parameters between each solution. This updating is described in the between-period module and in a second subset of the MDG equations with lagged relationships.

[3] Solving the model in the single-pass mode has the additional advantage of being computationally more efficient. On the other hand, especially during the developmental stage, it is often easier to solve and debug the model when it is run in the multi-pass mode.

[4] Mechanically, the model can accommodate one or more “enterprises”, i.e., non-consuming domestic non-government institutions that earn capital rents and allocate these rents to other institutions (domestic or foreign – the owners of the enterprises) after paying taxes and saving. However, this would require additional changes in the parts of the model that view other institutions as the direct owners of capital stocks that evolve over time as a function of depreciation and new investments by these institutions.

core CGE model) and 3-4 (notation and equations for the MDG Module) present the formal model; Table 5 summarizes the closure rules for the macro balances.

Throughout this paper, the following notational conventions apply: endogenous variables are written in upper case Latin letters; if the variable is fixed, a bar is placed above the name. Parameters have Greek or lower-case Latin letters. Subscripts refer to set indices. A "0" superscript is used to refer to base-year variable values. Otherwise, superscripts are exponents (i.e., not part of the name of the variable or parameter that is at the lower level). In the presence of the "0" superscript, the time subscript (*t*) has been suppressed.

Core CGE Model: Within-Period Module

In essence, the within-period module defines a one-period, static CGE model.[5] As shown in Table 2, it is divided into blocks covering prices, production and trade, domestic institutions, investments, and system constraints and macro variables. The price system of the model is extensive, especially for commodities. The *price block* (Equations 1-11) defines prices that can be expressed as functions of other endogenous variables (as opposed to being free variables that perform market-clearing functions). Among these prices, it is worth noting that transactions costs (the cost of moving the commodity between the border and the demanders or suppliers, or between domestic demanders and suppliers) are accounted for in the definitions of demander (domestic-currency) import prices, supplier (domestic-currency) export prices, and demander prices for domestic output sold domestically (Equations 1, 2, and 4). Whereas the transformation of output between exports and domestic sales typically is imperfect, the model allows for the special case of perfect transformability with zero exports as one possible outcome. For this case, export and domestic supplier prices are identical if exports are non-zero. If exports are zero, the domestic supplier prices may (and are likely to) exceed the export prices (Equation 3). Various aggregative prices – for composite supplies, for produced

[5] Apart from the fact that variables are time indexed, most of the "within-period" module is similar to the standard, static CGE model described in Lofgren *et al.* (2002). This document provides more detail on model features and references to the CGE modeling literature.

commodities, and value-added – are derived from relationships that define total revenue or costs as the sum of disaggregated receipts or payments (equations 5-7 and 9). The price of the aggregative intermediate commodity for any activity depends on its commodity composition and the prices of the commodities involved (Equation 8). The model is homogeneous of degree zero in prices, with the CPI serving as the model numéraire (Equation 10).[6] Alternative, the price index for non-tradables may serve as numéraire (Equation 11).

The *production and trade block* (Equations 12-29) includes the first-order conditions for profit-maximizing production and transformation decisions as well as cost-minimizing domestic demand decisions. Given available technology and market prices (taken as given in a perfectly competitive setting), producers maximize profits. The technology is defined by a nested, two-level structure. At the top, two alternatives are possible, either a Leontief aggregation of value-added and an aggregate intermediate or a CES aggregation of the same two variables (Equations 12-15). At the bottom, these are linked to a CES aggregation of primary factors (a value-added function) and a Leontief aggregation of intermediate inputs (Equations 16-18). Given that the national accounts rarely attribute value-added to government capital, the CES value-added function is limited to labor.[7] Each activity produces one or more outputs with fixed yield coefficients (Equation 19). Any commodity may be produced and marketed by more than one activity. A CES approach, assuming profit-maximizing producer behavior, is used to aggregate market sales of any commodity from different activities (Equations 20-21). Production is transformed into exports and domestic sales on the basis of a CET (Constant Elasticity of Transformation) function. The profit-maximizing, optimal ratio between the quantities of exports and domestic sales is positively related to the ratio between the corresponding supply prices (Equations 22-23). A less complex relationship applies to production without exports or without domestic sales (Equation 24). For any

[6] The computer code permits the user to choose either the CPI or the price index for non-tradables as numéraire. As long as the model is homogeneous of degree zero in prices, this choice has no impact on the equilibrium values of real variables. This homogeneity condition is not met under the macro closures where the savings or the domestic borrowing variables of the government are fixed. In these cases, it is implicitly assumed that the fixed variables are indexed to the numéraire.

[7] Nevertheless, the model accounts for the fact that government capital stocks indeed are needed in the production of government services: in the investment block, government investments are determined on the basis of capital demands (linked to the level of government service production) and capital stocks.

exported commodity, two alternatives are possible for export demand, either exogenous prices (in foreign currency) combined with an infinitely elastic demand or price-sensitive export demands (defined by constant-elasticity functions) with the foreign currency prices determined by domestic conditions and the exchange rate. (Equation 25 applies to the constant-elasticity case.)

Domestic demanders are assumed to minimize the cost of imperfectly substitutable imports and commodities from domestic production according to an Armington (CES aggregation) function (Equations 26-27). For commodities with only one supply source the supply from this source equals the composite supply (Equation 28). The transactions (trade and transport) demand for any service commodity is the sum of demands arising from domestic sales, exports, and imports, each of which is the product of the quantity traded and a fixed input coefficient (showing the quantity of the service commodity per unit of trade; Equation 29).

The *domestic institution block* (Equations 30-45) accounts for the receipts and expenditures of all domestic institutions, both government and non-government (households) as well as selected payment flows to and from the rest of the world. The equations are structured to accommodate databases with any number of households, one government, and one entity representing the rest of the world. The payments in this block are highly interrelated since institutions often are both at the receiving and paying ends. First, factor incomes are defined as a function of domestic wages (which may vary across activities) and employment levels, augmented by factor incomes from the rest of the world (Equation 30) and allocated across different institutions (domestic and foreign) in value shares that depend on factor endowment shares (Equations 31-32). Domestic non-government institutions: (i) earn net interest incomes, defined as the difference between net interest earnings from loans to the government and net interest payments to the rest of the world on borrowing from abroad (Equation 33); (ii) transfer fixed shares of their incomes (net of direct taxes and savings) to other institutions (domestic or foreign) (Equation 34); and (iii) earn total gross incomes defined as the sum of factor incomes, net interest incomes, and transfers from other institutions (with separate treatments depending on whether the sending institution is the government, the rest of the world, or another domestic non-government institution, and on whether the receiving institution is a household or non-household) (Equation 35); (iv) pay direct taxes according to rates that are fixed unless adjusted as part of the government closure rule; cf. discussion below (Equation 36); and (v) save out of incomes net of direct taxes according to marginal (and average) rates that are endogenous, depending on changes in per-capita incomes (if the elasticity of savings with respect to per-capita income is different from unity). Alternatively, for any given institution, the savings rate may be adjusted as part of the government closure (discussed below) (Equations 37-38).

For households, incomes net of direct taxes, savings, and transfers to other institutions (defined in Equation 39) are allocated across different commodities according to LES (Linear Expenditure System) demand functions, defined in per-capita form with separate equations for demands from the market and from own-production (Equations 40-41).

For the remaining domestic institution, the government, current incomes come from taxes (which are disaggregated into a wide range of categories), factor endowments (the government may own various non-labor factors), and transfers from other domestic institutions and the rest of the world (Equation 42). The (re)current expenditures of the government are divided into consumption, transfers to domestic institutions (CPI-indexed) and the rest of the world (fixed in foreign currency), and interest payments on domestic and foreign debt (Equations 43). For each period except the first, real government consumption, disaggregated by commodity, is defined as the level in the previous year times a growth factor that consists of multiple terms (Equation 44). For the base case, these terms are all exogenous; in simulations with other rules for determining government consumption (including simulations targeting MDGs), one of these may be (partly) endogenous.[8] Finally, government savings is simply the difference between current revenues and current expenditures (Equation 45).

The *investment block* (Equations 46-55) defines the transformation of savings into different types of investments, including adjustments for financial transactions (some of which involve the rest of the world) and foreign direct investment. The prices of new

[8] $RQGT_t$ is flexed if the absorption share of total government consumption is fixed. $RQGCT_{c,t}$ is flexed when some target influenced by this specific government service (c) is fixed for year t. For the most straightforward case, $rqgadj_{c,c',t}$ is 1 when c = c' and zero otherwise. However, if the analyst wants one or more kind of government consumption to grow in tandem with another, more than one c may have a value of 1 for any given c'. However, each c is linked to only one *c'*.

capital stocks depend on their composition and market prices (Equation 46). Government fixed investment spending is defined on the basis of these prices and real government fixed investment demand (the determination of which is defined below) (Equation 47). This fixed investment value is financed by some combination of government savings (net of spending on stock changes), new government bonds, and Central Bank borrowing, foreign borrowing, and foreign grants (which is a category separate from government transfers from the rest of the world) (Equation 48). According to one possible closure, government borrowing is flexible, assuring that this equality holds. (If so, the other terms would be exogenous or determined via other mechanisms.) Government bond borrowing and Central Bank borrowing is allocated across selected non-government institutions on the basis of their savings shares (Equations 49-50).[9] The fixed investment value of each domestic non-government institution is defined as its savings, net of spending on stock changes and lending to the government, and augmented by borrowing and grants from the rest of the world. For the rest of the world, the fixed investment value is simply the value of foreign direct investment (FDI; fixed in foreign currency) times the exchange rate; the FDI term is invariably fixed at zero for domestic institutions (Equation 51). For each non-government institution, investments in different capital stocks are determined by total fixed investment values, the prices of capital goods, and exogenous value shares of different capital stocks; the value share is unity if the database only specifies a single private capital type (Equation 52).[10] Government investment demand by capital stock is determined by the difference between the anticipated capital demand next year (drawing on production growth from last year) and the capital stock that would remain if no investments were made; if the demand is positive, then the equation will hold as an

[9] The savings shares are adjusted by a distortion term ($gbdist_i$) that reflects deviations between household shares for government borrowing and savings. Implicitly, the burden of Central Bank borrowing (which is tantamount to money printing) is felt by other agents since it extracts real purchasing power from them by reducing the value of the old money that they hold. In the absence of an explicit treatment of money in this model, this burden is here allocated across households on the basis of their savings shares. The government is assumed to pay no interest on is debt to the Central Bank (even if interest is paid, Central Bank profits tend to be recycled to the government). Given this, this debt stock is not monitored.

[10] Typically, the model will only have one private capital stock, i.e. the value of the share parameter is unity for this capital type. If the model has more than one private capital stock, the allocation between the different stocks may be endogenized, possibly deviating from the base-level allocation in response to changes in relative profit rates.

equality (Equations 53-54).[11] Finally, total investment demand by commodity source is defined given gross changes in capital stocks (both private and government) and the capital composition parameter (Equation 55).

The *system constraint and macro block* (Equations 56-62) explicitly captures the over-all resource constraints under which the economy operates as well as the determinants of TFP (total factor productivity) in the different production activities. For each market factor (all factors except government capital stocks), the supply is defined as the sum of institutional endowments (Equation 56). In the markets for the same set of factors, quantities demanded and supplied are set equal (Equation 57). Each market is cleared by the flexible, factor-specific wage variable (*WF*), which, at a more disaggregated level, can differ across activities by an activity-, factor-, and time-specific wage distortion (or differential) term (*WFDIST* ; cf. equations 17 and 30). In the special case of no wage differentials, this term is fixed at unity. Alternatively, in some cases it may be desirable to impose an exogenous time path for the employment of specific factors in selected activities (drawing on other pieces of information, for example data on the expected evolution of sectors based on the exploitation of natural resources). If so, the user only has to flex the wage distortion variable (*WFDIST*) and fix the employment variable (*QF*) for the relevant combination of factors, activities, and time periods. This can be done without changing the closure rule for other activities. Other alternatives for the factor market may be added, including closures with explicit unemployment.

For composite commodities, the supply is also set equal to the sum of demands (Equation 58). As noted above, the composite supplies stem from two sources, imports and domestic supplies to domestic markets, each of which has its own market clearing variable, the import quantity (*QM*) and the supplier price (*PDS*), respectively. The balance of payments constraint (Equation 59) imposes equality between foreign exchange uses (spending on imports, factor incomes and transfers to the rest of the world, and interest payments on foreign debts) and sources (export revenues, transfers, factor

[11] A mixed-complementary formulation is used with a lower limit of zero for the government investment variable (DKGOV) and government investment defined as being larger than or equal to zero. This treatment assures that, if DKGOV is positive, then investment will be equal to anticipated demand whereas, if DKGOV is zero, then it may be larger than a negative investment demand level (which would reflect the exceptional case of a rate of production decline in excess of the depreciation rate). Equation 54 transfers the value of DKGOV to DKINS for the government, a variable that is used elsewhere in the model across all capital stocks.

incomes, borrowing, grants, and FDI).[12] Finally, this block includes three macro variables, real GDP at market prices and the real trade-GDP ratio (both of which appear in other model equations), and real GDP at factor cost (which, as will be noted below, may be used as a fixed target in the calibration of TFP growth for the base run; Equations 60-62).

Core CGE Model: Between-Period Module

The equations in the between-period module update household populations and institutional stocks of assets and liabilities and TFP for selected activities. Invariably the equations in this module include lagged relationships. As shown in Table 2, the equations in this module do not apply to the first year, for which the values of the variables defined in this module are fixed.

As we will see in the MDG module, we assume that each household group is endowed with an unchanged share of the economywide total of every labor factor. The population of each household in any year is defined as its population in the preceding year times the growth factor of its labor force, multiplied by a population scaling factor (Equation 63). As a result, if the household has a relatively large (small) share of the labor types that grow most rapidly, then its population will also grow more (less) rapidly. As long as the model has more than one household, the resulting growth rates for each household group will deviate from its natural growth, indicating that, implicitly, parts of the population migrate between household groups. The population scaling factor assures that the sum of the representative household populations is equal to the exogenous population total (Equation 64). In effect this means that, if the aggregate labor force participation rate goes up (down), then this change will be imposed across all household groups.

For the different assets and liabilities held by households as well as for other aspects of their behavior (such as consumption patterns), the general assumption is that the

[12] Implicitly, an additional system constraint, the savings-investment balance, also holds: by channeling domestic savings and the terms that make up foreign savings to investment, the model equations assure that total savings and total investment are equal.

different household groups retain their original characteristics subject to adjustments needed to assure that the sums across the different household equal known totals. Accordingly, for the category of (non-labor) factors with exogenous total household stocks (land may be an example), the endowment of each household is defined as the product of an exogenous per-capita endowment, population, and a factor-specific scaling factor (Equation 65). The scaling factor is adjusted to assure that the total across all households equals an exogenous total (Equation 66).

For capital, the stock of each institution (households and others) is defined as the sum of its old, (potentially) redistributed capital stock, new investments, and exogenous adjustments (which may reflect the impact of natural disasters or institutional changes, removing parts of the capital stock from production) (Equation 67). For non-households, the old, redistributed stock is simply what remains of its non-depreciated stock of last year. For households, the non-depreciated stock is adjusted upwards for population growth and multiplied by a factor-specific scaling factor (Equation 68). The total redistributed capital stock across all household groups is equal to the total, non-depreciated stock (Equation 69). The scaling factor will adjust to assure that this constraint is satisfied. Except for the absence of depreciation, identical relationships hold for foreign debt (Equations 70-72) and government bonds (Equations 73-75). For foreign debt, the treatment is potentially more complex since the model allows for the possibility of non-paid interest (which is added to the debt) and debt relief.

Finally, the total factor productivity of each activity is defined as the product of a trend term, changes due to capital accumulation, and changes due to changes in economic openness (defined by the real trade-GDP ratio). The effects of capital accumulation and changes in openness depend on the values of exogenous, constant elasticities – if they are set at zero, the effect is zero and only the trend term will matter (Equation 76). The trend term is typically exogenous. However, as part of the model calibration, it may be endogenized in a setting where real GDP at factor cost is fixed in every time period (Equation 77).[13] The trade-GDP ratio, the most common indicator of economic openness

[13] When developing the model base run, the trend term may be exogenous. If so, the analyst needs to monitor overall GDP growth and, if needed adjust the exogenous growth term (CALTFPGT). The estimates of initial capital stocks and depreciation rates may also have to be revisited. Alternatively, the trend term may be endogenized while real GDP (at factor cost) is fixed in each year. If so, the analyst

(in terms of outcome, not policy stance) is defined in real terms (to avoid the impact of nominal changes, for example due to exchange rate depreciation). The impact of changes in this ratio is lagged in order to avoid large sudden changes. The fact that the elasticity parameters are disaggregated (by activity for trade and by activity and function for capital) make it possible to specify different channels and magnitudes for the productivity effects of trade and different types of government investments.

MDG Module

The MDG module (Equations 78-91) is a core component of the model. It specifies the mechanisms that determine the values for the indicators related to the different MDGs and educational behavior as well as the size and disaggregation (by educational achievement) of the labor force. The rest of the economy, which was presented in the preceding sections, influences the evolution of the MDGs and the educational sector through variables related to household consumption, the provision of different types of MDG-related services, labor wages, and capital stocks in infrastructure. In its turn, the MDG module influences the rest of the economy through its impact on the size and composition of the labor force. In addition, the evolution of one set of MDGs can influence other MDGs. The notation and the equations of the MDG module are presented in Tables 3 and 4.

The educational component, which is disaggregated by cycle (with three cycles as a typical level of disaggregation), consists of Equations 78-88. Within any cycle, the model endogenizes the following aspects of student behavior (or outcomes):

- the shares of the enrolled that graduate from their current grade, drop out, or repeat the grade next year (referred to as *grd*, *dropout*, and *rep*). Note that the term “graduate” throughout this paper and the model not only refers to students who successfully finish a given education cycle, but is also used to refer to students who successfully complete a grade (i.e., pass) and continue to

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should review the resulting overall growth rate for the trend term and, if needed, adjust the real GDP levels that are imposed. For non-base runs, trend TFP growth is typically fixed, permitting real GDP growth to be influenced by factor accumulation and endogenous TFP changes.

a higher grade within the cycle. The sum of these shares is unity – i.e., during the school year, a student must either graduate, drop out, or become a repeater (this applies to each grade and for each cycle as a whole);

- the shares among the graduates from their current grade (*grd*) who graduate from their current cycle (*grdcyc*) or continue to a higher grade within this cycle (*contcyc*). I.e., in terms of shares: *grdcyc* + *contcyc* = *grd*;
- the shares among cycle graduates who exit the school system (*grdexit*) or continue to next cycle (*grdcont*). The sum of these shares is also unity. For graduates from the last cycle, the share of those who exit is unity; and
- the share of the cohort of the 1^{st} year in primary school that enters school (*g1entry*).

Equations 78-84 define the share variables that identify different aspects of student behavior. For each cycle, a logistic function (Equation 78) defines the shares for 1^{st} year in-cohort entry, for graduates from the current grade, and for graduates who decide to continue to next cycle (i.e. *g1entry*, *grd*, and *grdcont*, the elements of the set *BLOG*). The logistic functional form was selected since it makes it possible to impose extreme (for education it is a maximum) values for the function and to incorporate extraneous information about elasticities and conditions under which target values are achieved. Another advantage is that it allows for segments of increasing and decreasing marginal returns to improvements in the determinants of educational behavior. The only endogenous variable in the logistic function (*SHREDINT*), is defined in a constant-elasticity formulation (Equation 79) as a function of: (i) educational quality; (ii) wage incentives, defined as relative wage gains from continued schooling (i.e., the relative wage gain that students can achieve if they complete a cycle that is sufficiently high to enable them to climb to the next higher level in the labor market); (iii) the under-five mortality rate (a proxy for the health status of the school population); (iv) the size of the infrastructure capital stock; and (v) household consumption per capita. The parameters in Equations 78-79 are selected as follows:

- the parameter $exted$ shows the extreme (maximum) value to which the behavior share should converge (here one) as the value of the intermediate variable approaches infinity;

- the parameter αed is calibrated so that, under base-year conditions, the behavioral share replicates the base-year value;
- the parameters βed and ϕed are calibrated so that the two equations: (i) replicate the base-year elasticities of the behavioral share (*SHRED*) with respect to the arguments of the constant-elasticity function; and (ii) achieve a behavioral target (e.g. a share very close to one for *g1entry*) under a set of values for the arguments of the constant-elasticity function that have been identified by other studies; and
- the value of the parameter γed determines how the base-year point on the logistic function is positioned relative to the inflection point (where the curve switches from increasing to decreasing marginal returns as the determinants of educational behavior improve).

Equation 79 is calibrated so that, in the base year (under base-year conditions), $SHRedint_{b,c,t} = SHRed^{0}_{b,c}$. (Note that these two terms enter the denominator of the second term in equation 78.)

Separate equations define two of the determinants of the CE function. Educational quality (*EDUQUAL*) is defined as the ratio between real services per student (total services divided by total enrollment) in the current year and in the base year; i.e., in the base-year, educational quality is indexed to one (Equation 80). Average real household consumption per capita (*QHPC*) is defined as total household consumption at base-year prices divided by total population (Equation 81).

Drawing on the shares defined in the preceding equations, the shares for repeaters, dropouts, and cycle graduates exiting from the school system (*rep*, *dropout*, and *grdexit*; elements in the set *BRES*) are defined residually (Equation 82). The formulation considers the fact that, as noted above, selected shares have to sum to unity. If more than one variable in *BRES* has to be adjusted in relation to one or more elements in *BLOG* (as is the case for the adjustment of shares for repeaters and dropouts in response to changes in the share of graduates), then all adjusted variables are scaled up or down by the same factor.[14] The share of graduates from a cycle (*grdcyc*) is defined as the share for the total number of graduates in the cycle (*grd*) divided by the number of years in the cycle

[14] The equation is formulated so that it works for cases with one or more than one term in any of the sums over related shares (defined by the mappings MBB and MBB2) in either of the sets BRES and BLOG.

whereas the residual share is assigned to graduates within a cycle (*contcyc*) (Equations 83-84). We use the net completion rate as our MDG 2 indicator. It is defined as the product of the relevant 1st-year primary school entry rate (*g1entry*) and the graduation rates (*grd*) over time for the cohort that graduate from primary school in the current year (Equation 85).[15]

Drawing on the above information, we can define the number of enrolled students by cycle and year. The number of "old" enrolled students in any cycle (i.e., those who were enrolled in the same cycle last year) is defined as the sum of those who: (i) continue within the cycle after successful completion of an earlier grade; and (ii) repeat the grade they were in last year (Equation 86). The number of "new" enrolled students is defined as the sum of: (i) graduates from the relevant earlier cycle last year who chose to continue; (ii) cohort entrants (only for the 1st primary cycle); and (iii) other, non-cohort entrants entering any cycle in the educational system (Equation 87).[16] The total number of enrolled students in a cycle is the sum of old and new students (Equation 88).

Drawing on these enrollment variables, institutional labor endowments are defined as the sum of the following components (Equation 89): (i) remaining labor from the preceding year; (ii) new labor force entrants among students who exited from the school system in the previous year (with separate terms for graduates and dropouts); (iii) new labor force entrants from the non-student population who reach the age at which they, to the extent that they seek work, become part of the labor force. Depending on their highest completed grade, the new labor force entrants are allocated to a specific labor category.

The treatment underlying MDGs 4, 5, 7a and 7b is similar but less complex. For these, a logistic function directly defines the MDG indicators as a function of an intermediate variable that is defined in a related CE function (Equations 90-91). The values for the parameters $extmdg$, αmdg , βmdg , and ϕmdg are defined following the same principles as the corresponding parameters in the logistic and CE functions for

[15] In other words, in order for 100% of the cohort to complete the primary cycle in time, it is necessary that all of them enter at the time of their first year and after that all manage to graduate in each year (i.e., successfully complete each grade) up to the final year of the cycle. Given that we do not generate separate graduation rates for students in the relevant cohort (as opposed to students outside this cohort), we assume that the rates for in-cohort students are identical to the over-all rates for students in the cycle.

[16] This category includes non-cohort entrants to the 1st primary year of primary school (at least during a transitional period of primary school expansion). It may also include immigrants from other countries.

education. The arguments of the CE function are similar except for that the relevant service supply is expressed in per-capita form (not per enrolled student).

Within-Period Rules for Macro Balances

Like other economywide models, MAMS includes three macroeconomic balances: for the government, the rest of the world (the balance of payments), and savings-investments (loanable funds). In the computer code, the user chooses among a set of pre-programmed alternative closure rules for these balances. (The set can easily be expanded.) The choices made have no influence on the solution for the base year but will almost invariably influence the results for other years and across different simulations. The preprogrammed alternatives are summarized in Table 5. [17]

For the government balance (Equations 42, 43, 45, and 48), under the first closure, government savings (the difference between current government revenues and current government expenditures) is a flexible residual while all tax rates are fixed. Flexible domestic government borrowing covers the gap in the government capital account (between government investment costs and financing from other sources: government savings, foreign borrowing and foreign grants). The second closure (GOV-2) differs in that domestic government borrowing is fixed at some exogenous level (defined by year and simulation) while the direct tax rates of domestic non-government institutions are adjusted to assure that government savings suffice to cover the gap in the government capital account. According to the direct tax adjustment rule, the rates for selected institutions are adjusted endogenously by the same number of percentage points. Instead of relying on domestic sources of financing, the third government closure (GOV-3) assumes that foreign grants cover the gap in the government capital account in a setting where direct tax rates and domestic government borrowing are fixed. The fourth closure (GOV-4) combines elements of the second and third closure: both direct tax rates and foreign grants are flexible while government savings is fixed. This closure may be used

[17] For a discussion of macro closures in the context of the standard CGE model, see Lofgren et al. (2002, pp. 13-17). Table 5 includes the alternatives that are likely to be used most frequently. The GAMS code includes additional options, *inter alia*, for the closures where direct tax and savings rates are adjusted for selected institutions. These options involve scaling of rates that otherwise would have prevailed instead of uniform percentage point changes.

in a setting where direct tax rates are adjusted in response to an increase in government recurrent spending (perhaps with the objective of achieving some MDG target) at the same time as foreign grants respond to a related increase in government capital spending. The fifth government closure (GOV-5) is identical to the third except for that, instead of foreign grants, foreign borrowing covers the gap in the government capital account.

Under all of the above rules, it is assumed that government consumption is determined according to some other rule (i.e., adjustments in government consumption are not part of the mechanisms for achieving government balance). Three alternative rules for determining government consumption are specified (cf. equation 54):[18]

1. fixed real quantity by commodity and year; flexible absorption share; this is the rule that was assumed in the preceding model presentation;
2. fixed absorption share and flexible scaling of exogenous consumption quantities by a factor that is uniform across all commodities; i.e., fix the variable indicating the absorption share of government consumption and flex the variable $\overline{QCSHRABSADJ}_t$; and
3. for selected commodities and time periods, flexible growth rates combined with fixed targets for MDG indicators related to the government commodity (service) (i.e., flex the variable $\overline{RQGCT}_{c,t}$ for selected c and t); for remaining commodities, fixed real quantity by commodity and year and flexible absorption share (same as 1).

For the balance of payments (Equation 59), which is expressed in foreign currency, three closures are specified. The first closure (ROW-1) has a flexible real exchange rate whereas all items on the capital account, including foreign grants, are fixed. Given this, foreign savings (the current account deficit) is implicitly fixed. To exemplify how this works: if, *ceteris paribus*, the balance of payments is in deficit, then a depreciation of the real exchange rate eliminates this deficit by (i) reducing spending on imports (a fall in import quantities at fixed world prices); and (ii) increasing earnings from exports (an increase in export quantities at fixed world prices). The other two rest-of-world closures are designed to work with specific government closures. If, for the *government* balance,

[18] In addition to the equations in Tables 2 and 4, the complete computer model includes an equation that defines the absorption share of government consumption, a variable that is fixed under rule 2.

the third or fourth closure is used (GOV-3 or GOV-4), then the second rest-of-world closure (ROW-2) should be activated: it combines a flexible exchange rate with flexible foreign grants (where the latter in effect is determined to balance the government accounts); similarly, if for the government, the fifth closure is used (GOV-5), then the third rest-of-world closure (ROW-3), which assumes flexible foreign borrowing, should be used.

In the savings-investment balance (which is implicit in the model), government investment spending is determined by the demand for capital by the different government service activities whereas the determination of government savings depends on the closure rule adopted for the government balance. Hence, the alternative closures for the savings-investment balance focus on the non-government components of investment and savings. The closures are either investment-driven (the value of savings adjusts) or savings-driven (the value of investment adjusts). The first closure (SI-1) is investment driven: private investment is a fixed share of total absorption (which may vary over time). [19] In order to assure that total savings and investment values are equal, the savings rates of selected non-government institutions are adjusted by a uniform number of percentage points (the variable *DMPS* is flexed; Equation 37). The second closure (SI-2) makes the opposite assumptions: the variable for savings adjustment (*DMPS*) is fixed while the private investment absorption share is flexible. The last closure (SI-3) is identical to the first except for that the absorption share that is fixed defines *total* investment.

[19] The complete computer model includes equations that define the absorption shares of private and total investment (not included in Tables 2 or 4). These absorption variables are fixed or flexible depending on the macro closure.

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Table 1: Sets, parameters, and variables for core CGE modules of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	Activities	$f, f' \in F$	factors
$a \in ACES\ (\subset A)$	activities with CES function between Value Added and Intermediate inputs	$f \in FCAP (\subset F)$	capital factors
$a \in ALEO\ (\subset A)$	activities with Leontief fn between Value Added and Intermediate inputs	$f \in FCAPGOV (\subset FCAP)$	government capital factors
$c \in C$	Commodities	$f \in FEXOG (\subset F)$	factors with exogenous growth rates
$c \in CD (\subset C)$	commodities with domestic sales of domestic output	$f \in FLABN (\subset F)$	non-labor factors
$c \in CDN (\subset C)$	commodities not in *CD*	$h \in H (\subset INSDNG)$	households (incl. NGOs)
$c \in CE (\subset C)$	exported commodities	$i \in INS$	institutions (domestic and rest of world)
$c \in CEN (\subset C)$	commodities not in *CE*	$i \in INSD (\subset INS)$	domestic institutions
$c \in CECETN (\subset C)$	exported commodities without CET function	$i \in INSDNG (\subset INSD)$	domestic non-government institutions
$c \in CM (\subset C)$	imported commodities	$i \in INSNG (\subset INS)$	non-government institutions
$c \in CMN (\subset C)$	commodities not in *CM*	$(f, a) \in MFA$	mapping showing that disaggregated factor *f* is used in activity *a*
$c \in CT (\subset C)$	transaction service commodities	$t \in T$	time periods

PARAMETERS – LATIN LETTERS			
$capcomp_{c,f}$	quantity of commodity *c* per unit of new capital *f*	$pwse_{c,t}$	world price for export substitutes (FCU)
$cwts_c$	weight of commodity *c* in the CPI	$qdst_{c,i,t}$	quantity of stock change
$depr_f$	depreciation rate for factor *f*	$\overline{qe}_{c,t}$	export demand for *c* if *pwe* = *pwse* (world price for subs)
$dwts_c$	domestic sales price weights	$qfachhtot_{f,t}$	total household stock of exogenous, non-labor factors
$fdebtrelief_{i,t}$	foreign debt relief for domestic institution *i*	$qfacinsadj_{i,f,t}$	exogenous factor stock adjustment

$fdi_{i,t}$	foreign direct investment by institution *i* (rest of world) (FCU)	$qfpc_{i,f,t}$	per-capita quantity of exogenous-supply factor *f* by institution *i* and year *t*
$fintrat_{i,t}$	interest rate on foreign debt for domestic institution *i* (paid)	$rqgadj_{c,c',t}$	parameter linking government consumption growth across commodities
$fintratdue_{i,t}$	interest rate on foreign debt for domestic institution *i* (due)	$shii_{i,i'}$	share of net income of *i*' to *i* (*i*' $\in$ INSDNG)
$fprd_{f,a,t}$	productivity of factor *f* in activity *a*	$ta_{a,t}$	tax rate for activity *a*
$gbdist_{i}$	distortion factor for gov borrowing from institution *i*	$te_{c,t}$	export tax rate
$gfcfshr_{f,i,t}$	share of gross fixed capital formation for institution *i* in capital factor *f*	$tf_{f,t}$	direct tax rate for factor *f*
$gintrat_{i,t}$	interest rate on government bonds for domestic institution *i*	$tfp01_{a,t}$	0-1 parameter for activities with endogenous TFP growth
$ica_{c,a}$	quantity of *c* as intermediate input per unit of aggregate intermediate in activity *a*	$tfpelasqg_{a,f,t}$	elas'y of TFP for a w.r.t. to government capital cap stock *f*
$icd_{c,c',t}$	trade input of *c* per unit of commodity *c*' produced & sold domestically	$tfpelastrd_{a}$	elasticity of TFP for *a* w.r.t. to GDP trade share
$ice_{c,c',t}$	trade input of *c* per unit of commodity *c*' exported	$tfptrdwt_{t,t'}$	weight of period *t*' in tfp-trade link in *t*
$icm_{c,c',t}$	trade input of *c* per unit of commodity *c*' imported	$tins01_{i}$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$ifa_{f,a}$	quantity of capital f per unit of government activity *a*	$\overline{tins}_{i,t}$	exogenous component in direct tax rate for domestic institution *i*
$inta_{a}$	quantity of aggregate intermediate input per unit of activity *a*	$tm_{c,t}$	import tariff rate
iva_{a}	quantity of value-added per unit of activity *a*	$tq_{c,t}$	rate of sales tax
$mps01_{i}$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates	$trnsfr_{i,i',t}$	exogenous transfer from institution *i*' to institution *i*
$poptot_{t}$	total population by year	$trnsfr_{f,i',t}$	exogenous transfer from institution *i*' to factor *f*
$pwe_{c,t}$	export world price of *c* (FCU)	$trnsfrpc_{i,i',t}$	per-capita transfers from institution *i*' to hhd institution *i*
$pwm_{c,t}$	import world price of *c* (FCU)	$tva_{a,t}$	rate of value-added tax for activity *a*

PARAMETERS – GREEK LETTERS			
αa_a	shift parameter for top level CES function	$\delta va_{f,a}$	CES value-added function share parameter for factor *f* in activity *a*
αac_c	shift parameter for domestic commodity aggregation function	$\gamma h_{a,c,h}$	per capita household subsistence consumption of home commodity *c* from activity *a*
$\alpha vag_{a,t}$	exogenous component of efficiency (TFP) for activity *a*	$\gamma m_{c,h}$	per capita household subsistence cons of marketed commodity *c*
αq_c	Armington function shift parameter	ρa_a	CES top level function exponent
αt_c	CET function shift parameter	ρac_c	domestic commodity aggregation function exponent
$\beta h_{a,c,h}$	marginal share of household consumption on home commodity *c* from activity *a*	ρq_c	Armington function exponent
$\beta m_{c,h}$	marginal share of household consumption spending on marketed commodity *c*	ρsav_i	elasticity of savings rate with respect to per-capita income for institution (household) *h*
δa_a	share parameter for top level CES function	ρt_c	CET function exponent
δac_a	share parameter for domestic commodity aggregation function	ρva_a	CES value-added function exponent
δq_c	Armington function share parameter	$\theta_{a,c}$	yield of output *c* per unit of activity *a*
δt_c	CET function share parameter		

VARIABLES			
$\alpha va_{a,t}$	efficiency parameter in the CES value-added function	$PQ_{c,t}$	composite commodity price
$\alpha va2_{a,t}$	endogenous TFP trend term by *a*	$PVA_{a,t}$	value-added price (factor income per unit of activity)
$CALTFPGT_t$	Calibration factor for TFP growth	$PX_{c,t}$	aggregate producer price for commodity
$CBBOR_{i,t}$	government Central Bank borrowing (deficit monetization)	$PXAC_{a,c,t}$	price of commodity *c* from activity *a*
$CBBORTOT_t$	government Central Bank borrowing (deficit monetization)	$QA_{a,t}$	quantity (level) of activity
CPI_t	consumer price index	$QD_{c,t}$	quantity sold domestically of domestically produced *c*
$DGBOND_{i,t}$	change in holding of government bonds for domestic institution *i*	$QE_{c,t}$	quantity of exports of commodity *c*
$DGBONDTOT_t$	total change in holding of government bonds	$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$DKGOV_{f,t}$	gross government investment in *f*	$QFACINS_{i,f,t}$	real endowment of factor *f* for institution *i*
$DKINS_{i,f,t}$	gross change in capital stock (investment in) *f* for institution *i*	$QFCAPRED_{i,f,t}$	stock of redistributed capital by *ins* - *f* - *t*
$DMPS_t$	uniform point change in savings rate of selected domestic institutions	$QFS_{f,t}$	quantity supplied of factor *f*
DPI_t	producer price index for non-traded output	$QFSCAL_{f,t}$	scaling factor for constraint on total factor stock
$DTINS_t$	uniform point change in direct tax rate of selected domestic institutions	$QG_{c,t}$	quantity of government consumption of commodity *c*
EG_t	government expenditures	$QH_{c,h,t}$	quantity consumed by household *h* of marketed commodity *c*
$EH_{h,t}$	consumption spending for household	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from act *a* by hhd *h*
EXR_t	exchange rate (LCU per unit of FCU)	$QINTA_{a,t}$	quantity of aggregate intermediate input used by activity *a*
$FBOR_{i,t}$	foreign borrowing for domestic institution *i*	$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*
$FDEBT_{i,t}$	foreign debt for domestic inst *i*	$QINV_{c,t}$	quantity of investment demand for commodity *c*
$FDEBTRED_{i,t}$	redefined foreign debt stock for household *i*	$QM_{c,t}$	quantity of imports of commodity *c*
$FDEBTSCAL_t$	scaling factor for foreign debt stocks to meet aggregate constraint	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)

$FGRANT_{i,t}$	foreign grants to domestic institution *i* (FCU)	$QT_{c,t}$	quantity of trade and transport demand for commodity *c*
$GBOND_{i,t}$	endowment of government bonds for *i*	$QVA_{a,t}$	quantity of (aggregate) value-added
$GBONDRED_{i,t}$	redefined government bond holding for household *i*	$QX_{c,t}$	aggregated quantity of domestic output of commodity
$GBONDSCAL_{i,t}$	scaling factor for hhd holdings of gov bonds to meet aggregate constraint	$QXAC_{a,c,t}$	quantity of output of commodity *c* from activity *a*
$GDPREAL_t$	real GDP at market prices	$RQGT_t$	real government consumption growth for all *c* in *t* relative to *t-1*
$GDPREALFC_t$	real GDP at factor cost	$RQGCT_{c,t}$	real government consumption growth of *c* in *t* relative to *t-1*
$GSAV_t$	government savings	$SHIF_{i,f,t}$	share of institution *i* in income of factor *f*
$INSSAV_{i,t}$	savings of domestic non-government institution *i*	$TINS_{i,t}$	direct tax rate for domestic non-government institution *i*
$INVVAL_{i,t}$	investment value for institution *i*	$TINSADJ_t$	direct tax scaling factor
$MPS_{i,t}$	marginal propensity to save for domestic non-gov institution *i*	$TRDGDP_t$	foreign trade as share of GDP
$MPSADJ_t$	savings rate scaling factor	$TRII_{i,i',t}$	transfers from institution *i'* to *i* (both in the set INSDNG)
$PA_{a,t}$	activity price (unit gross revenue)	$WF_{f,t}$	average price of factor
$PDD_{c,t}$	demand price for commodity *c* produced & sold domestically	$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$PDS_{c,t}$	supply price for commodity *c* produced & sold domestically	$YF_{f,t}$	income of factor *f*
$PE_{c,t}$	export price (domestic currency)	YG_t	government revenue
$PINTA_{a,t}$	aggregate intermediate input price for activity *a*	$YI_{i,t}$	income of domestic non-government institution
$PK_{f,t}$	price of new capital stock *a*	$YIF_{i,f,t}$	income to domestic institution *i* from factor *f*
$PM_{c,t}$	import price (domestic currency)	$YIINT_{i,t}$	interest payment on government bonds to INS
$POP_{i,t}$	population by household		

Table 2. Equations for the core CGE modules of MAMS Model:

CORE CGE MODEL: WITHIN-PERIOD MODULE

#	Equation	Domain	Description
Price Block			
(1)	$PM_{c,t} = pwm_{c,t} \cdot \left(1+tm_{c,t} \right) \cdot EXR_t + \sum_{c' \in C} \left(PQ_{c',t} \cdot icm_{c',c,t} \right)$ $\begin{bmatrix} \textit{import price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \textit{import price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \textit{tariff} \\ \textit{adjustment} \end{bmatrix} \cdot \begin{bmatrix} \textit{exchange rate} \\ (LCU \textit{ per } FCU) \end{bmatrix} + \begin{bmatrix} \textit{transaction} \\ \textit{costs} \end{bmatrix}$	$c \in CM$ $t \in T$	Import price
(2)	$PE_{c,t} = pwe_{c,t} \cdot \left(1-te_{c,t} \right) \cdot EXR_t - \sum_{c' \in C} \left(PQ_{c',t} \cdot ice_{c',c,t} \right)$ $\begin{bmatrix} \textit{export price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \textit{export price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \textit{tariff} \\ \textit{adjustment} \end{bmatrix} \cdot \begin{bmatrix} \textit{exchange rate} \\ (LCU \textit{ per } FCU) \end{bmatrix} - \begin{bmatrix} \textit{transaction} \\ \textit{costs} \end{bmatrix}$	$c \in CE$ $t \in T$	Export price
(3)	$PDS_{c,t} \geq PE_{c,t}$ $\begin{bmatrix} \textit{domestic supply} \\ \textit{price} \end{bmatrix} \geq \begin{bmatrix} \textit{export price} \\ (LCU) \end{bmatrix}$	$c \in$ $(CD \cap$ $CECETN)$ $t \in T$	Domestic floor price (= export price) for non-CET exportables
(4)	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in C} \left(PQ_{c',t} \cdot icd_{c',c,t} \right)$ $\begin{bmatrix} \textit{domestic demander} \\ \textit{price} \end{bmatrix} = \begin{bmatrix} \textit{domestic supplier} \\ \textit{price} \end{bmatrix} + \begin{bmatrix} \textit{transaction} \\ \textit{costs} \end{bmatrix}$	$c \in CD$ $t \in T$	Domestic demander price for domestic commodity
(5)	$PQ_{c,t} \cdot \left(1-tq_{c,t} \right) \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}$ $\begin{bmatrix} \textit{absorption} \\ (\textit{at demand prices} \\ \textit{net of sales tax}) \end{bmatrix} = \begin{bmatrix} \textit{domestic demander} \\ \textit{price times} \\ \textit{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \textit{import price} \\ \textit{times} \\ \textit{import quantity} \end{bmatrix}$	$c \in$ $(CD \cup CM)$ $t \in T$	Absorption
(6)	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$ $\begin{bmatrix} \textit{producer price} \\ \textit{times marketed} \\ \textit{output quantity} \end{bmatrix} = \begin{bmatrix} \textit{domestic supplier} \\ \textit{price times} \\ \textit{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \textit{export price} \\ \textit{times} \\ \textit{export quantity} \end{bmatrix}$	$c \in$ $(CD \cup CE)$ $t \in T$	Marketed output value
(7)	$PA_{a,t} = \sum_{c \in C} PXAC_{a,c,t} \cdot \theta_{a,c}$ $\begin{bmatrix} \textit{activity} \\ \textit{price} \end{bmatrix} = \begin{bmatrix} \textit{producer prices} \\ \textit{times yields} \end{bmatrix}$	$a \in A$ $t \in T$	Activity price
(8)	$PINTA_{a,t} = \sum_{c \in C} PQ_{c,t} \cdot ica_{c,a}$ $\begin{bmatrix} \textit{aggregate} \\ \textit{intermediate} \\ \textit{input price} \end{bmatrix} = \begin{bmatrix} \textit{intermediate input cost} \\ \textit{per unit of aggregate} \\ \textit{intermediate input} \end{bmatrix}$	$a \in A$ $t \in T$	Aggregate intermediate input price

(9)	$$PA_{a,t} \cdot (1 - ta_{a,t}) \cdot QA_{a,t} = PVA_{a,t} \cdot QVA_{a,t} + PINTA_{a,t} \cdot QINTA_{a,t}$$ $$\begin{bmatrix} \textit{activity price} \\ (\textit{net of taxes}) \\ \textit{times activity level} \end{bmatrix} = \begin{bmatrix} \textit{value-added} \\ \textit{price times} \\ \textit{quantity} \end{bmatrix} + \begin{bmatrix} \textit{aggregate} \\ \textit{intermediate} \\ \textit{input price times} \\ \textit{quantity} \end{bmatrix}$$	$a \in A$ $t \in T$	Activity revenue and costs
(10)	$$\overline{CPI}_t = \sum_{c \in C} PQ_{c,t} \cdot cwts_c$$ $$\begin{bmatrix} CPI \end{bmatrix} = \begin{bmatrix} \textit{prices times} \\ \textit{weights} \end{bmatrix}$$	$t \in T$	Consumer price index
(11)	$$DPI_t = \sum_{c \in CD} PDS_{c,t} \cdot dwts_c$$ $$\begin{bmatrix} \textit{price index for} \\ \textit{non-tradables} \end{bmatrix} = \begin{bmatrix} \textit{supplier price for output} \\ \textit{marketed domestically} \\ \textit{times weights} \end{bmatrix}$$	$t \in T$	Price index for non-tradables

Production and trade block

(12)	$$QA_{a,t} = \alpha a_a \cdot \left(\delta a_a \cdot QVA_{a,t}^{-\rho a_a} + (1 - \delta a_a) \cdot QINTA_{a,t}^{-\rho a_a} \right)^{-\frac{1}{\rho a_a}}$$ $$\begin{bmatrix} \textit{quantity of aggregate} \\ \textit{activity} \end{bmatrix} = CES \begin{bmatrix} \textit{demand for value - added} \\ \textit{and demand for aggregate} \\ \textit{intermediate input} \end{bmatrix}$$	$a \in ACES$ $t \in T$	CES technology: Aggregate activity production function
(13)	$$\frac{QVA_{a,t}}{QINTA_{a,t}} = \left(\frac{PINTA_{a,t}}{PVA_{a,t}} \cdot \frac{\delta a_a}{1 - \delta a_a} \right)^{\frac{1}{1+\rho a_a}}$$ $$\begin{bmatrix} \textit{value-added - intermediate} \\ \textit{input quantity ratio} \end{bmatrix} = f \begin{bmatrix} \textit{intermediate input -} \\ \textit{value-added price ratio} \end{bmatrix}$$	$a \in ACES$ $t \in T$	CES technology: Aggregate value added - intermediate input ratio
(14)	$$QVA_{a,t} = iva_a \cdot QA_{a,t}$$ $$\begin{bmatrix} \textit{demand for} \\ \textit{value-added} \end{bmatrix} = f \begin{bmatrix} \textit{activity} \\ \textit{level} \end{bmatrix}$$	$a \in ALEO$ $t \in T$	Leontief technology: Demand for aggregate value-added
(15)	$$QINTA_{a,t} = inta_a \cdot QA_{a,t}$$ $$\begin{bmatrix} \textit{demand for aggregate} \\ \textit{intermediate input} \end{bmatrix} = f \begin{bmatrix} \textit{activity} \\ \textit{level} \end{bmatrix}$$	$a \in ALEO$ $t \in T$	Leontief technology: Demand for aggregate intermediate input
(16)	$$QVA_{a,t} = \alpha va_{a,t} \cdot \left(\sum_{f \in F} \delta va_{f,a} \cdot \left(fprd_{f,a,t} \cdot QF_{f,a,t} \right)^{-\rho va_a} \right)^{-\frac{1}{\rho va_a}}$$ $$\begin{bmatrix} \textit{quantity of aggregate} \\ \textit{value-added} \end{bmatrix} = CES \begin{bmatrix} \textit{factor} \\ \textit{inputs} \end{bmatrix}$$	$a \in A$ $t \in T$	Value-added

(17)	$WF_{f,t}\cdot\overline{WFDIST}_{f,a,t} = PVA_{a,t}\cdot\left(1-tva_{a,t}\right)\cdot QVA_{a,t}$ $\cdot\left(\sum_{f'\in F}\delta va_{f',a}\cdot\left(fprd_{f',a,t}\cdot QF_{f',a,t}\right)^{-\rho va_a}\right)^{-1}\cdot\delta va_{f,a}\cdot fprd_{f,a,t}^{-\rho va_a}\cdot QF_{f,a,t}^{-\rho va_a-1}$ $\begin{bmatrix}\text{marginal cost of}\\ \text{factor } f \text{ in activity } a\end{bmatrix} = \begin{bmatrix}\text{marginal revenue product}\\ \text{of factor } f \text{ in activity } a\end{bmatrix}$	$a\in A$ $f\in F$ $t\in T$	Factor demand
(18)	$QINT_{c,a,t} = ica_{c,a}\cdot QINTA_{a,t}$ $\begin{bmatrix}\text{intermediate demand}\\ \text{for commodity } c\\ \text{from activity } a\end{bmatrix} = f\begin{bmatrix}\text{aggregate intermediate}\\ \text{input quantity}\\ \text{for activity } a\end{bmatrix}$	$c\in C$ $a\in A$ $t\in T$	Disaggregated intermediate input demand
(19)	$QXAC_{a,c,t}+\sum_{h\in H}QHA_{a,c,h,t} = \theta_{a,c}\cdot QA_{a,t}$ $\begin{bmatrix}\text{quantity of output}\\ \text{of commodity } c\\ \text{from activity } a\end{bmatrix}+\begin{bmatrix}\text{quantity consumed of}\\ \text{home commodity } c\\ \text{from activity } a \text{ in}\\ \text{all households}\end{bmatrix}=\begin{bmatrix}\text{activity-specific}\\ \text{marketed}\\ \text{production of}\\ \text{commodity } c\end{bmatrix}$	$a\in A$ $c\in C$ $t\in T$	Commodity production and allocation between market and home
(20)	$QX_{c,t} = \alpha ac_c\cdot\left(\sum_{a\in A}\delta ac_{a,c}\cdot QXAC_{a,c,t}^{-\rho ac_c}\right)^{-\frac{1}{\rho ac_c}}$ $\begin{bmatrix}\text{aggregate marketed}\\ \text{production of}\\ \text{commodity } c\end{bmatrix} = CES\begin{bmatrix}\text{output of commodity } c\\ \text{from activity } a\end{bmatrix}$	$c\in$ $(CE\cup CD)$ $t\in T$	Output aggregation function
(21)	$\frac{PXAC_{a,c,t}}{PX_{c,t}} = QX_{c,t}\cdot\sum_{a'\in A}\left(\delta ac_{a',c}\cdot QXAC_{a',c,t}^{-\rho ac_c}\right)^{-1}\cdot\delta ac_{a,c}\cdot QXAC_{a,c,t}^{-\rho ac_c-1}$ $\begin{bmatrix}\text{ratio of price of commodity } c\\ \text{from acitivty } a \text{ to}\\ \text{average output price}\end{bmatrix} = f\begin{bmatrix}\text{aggregate marketed commodity}\\ \text{output and output of commodity } c\\ \text{from activity } a\end{bmatrix}$	$a\in A$ $c\in C$ $t\in T$	Ratio of prices for output aggregation function
(22)	$QX_{c,t} = \alpha t_c\cdot\left(\delta t_c\cdot QE_{c,t}^{\rho t_c}+(1-\delta t_c)\cdot QD_{c,t}^{\rho t_c}\right)^{\frac{1}{\rho t_c}}$ $\begin{bmatrix}\text{aggregate marketed}\\ \text{domestic output}\end{bmatrix} = CET\begin{bmatrix}\text{export quantity, domestic}\\ \text{sales of domestic output}\end{bmatrix}$	$c\in(CD\cap$ $CECET)$ $t\in T$	Output transformation (CET) function
(23)	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}}\cdot\frac{1-\delta t_c}{\delta t_c}\right)^{\frac{1}{\rho t_c-1}}$ $\begin{bmatrix}\text{export-domestic}\\ \text{supply ratio}\end{bmatrix} = f\begin{bmatrix}\text{export-domestic}\\ \text{price ratio}\end{bmatrix}$	$c\in(CD\cap$ $CECET)$ $t\in T$	Export-domestic supply ratio

(24)	$$QX_{c,t} = QD_{c,t} + QE_{c,t}$$ $$\begin{bmatrix} \textit{aggregate} \\ \textit{marketed} \\ \textit{domestic output} \end{bmatrix} = \begin{bmatrix} \textit{domestic market} \\ \textit{sales of domestic} \\ \textit{output [for} \\ c \in (CD \cap CEN)] \end{bmatrix} + \begin{bmatrix} \textit{exports [for} \\ c \in (CE \cap CDN)] \end{bmatrix}$$	$c \in$ $(CD \cap CEN) \cup$ $(CE \cap CDN) \cup$ $(CD \cap CECETN)$, $t \in T$	Output transformation for outputs without exports, exports without domestic sales, and non-CET exports with domestic sales
(25)	$$QE_{c,t} = \overline{qe}_{c,t} \cdot \left(\frac{PWE_{c,t}}{pwse_{c,t}} \right)^{\rho e_c}$$ $$\begin{bmatrix} \textit{export} \\ \textit{demand} \end{bmatrix} = f \begin{bmatrix} \textit{trend export quantity, world price} \\ \textit{for exports relative to world} \\ \textit{price for export substitutes} \end{bmatrix}$$	$c \in CED$ $t \in T$	Export demand with constant-elasticity demand function
(26)	$$QQ_{c,t} = \alpha q_c \cdot \left(\delta q_c \cdot QM_{c,t}^{-\rho q_c} + (1 - \delta q_c) \cdot QD_{c,t}^{-\rho q_c} \right)^{-\frac{1}{\rho q_c}}$$ $$\begin{bmatrix} \textit{composite} \\ \textit{supply} \end{bmatrix} = f \begin{bmatrix} \textit{import quantity, domestic} \\ \textit{use of domestic output} \end{bmatrix}$$	$c \in$ $(CM \cap CD)$ $t \in T$	Composite supply (Armington) function
(27)	$$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \cdot \frac{\delta q_c}{1 - \delta q_c} \right)^{\frac{1}{1+\rho q_c}}$$ $$\begin{bmatrix} \textit{import-domestic} \\ \textit{demand ratio} \end{bmatrix} = f \begin{bmatrix} \textit{domestic-import} \\ \textit{price ratio} \end{bmatrix}$$	$c \in$ $(CM \cap CD)$ $t \in T$	Import-domestic demand ratio
(28)	$$QQ_{c,t} = QD_{c,t} + QM_{c,t}$$ $$\begin{bmatrix} \textit{composite} \\ \textit{supply} \end{bmatrix} = \begin{bmatrix} \textit{domestic use of} \\ \textit{marketed domestic} \\ \textit{output [for} \\ c \in (CD \cap CMN)] \end{bmatrix} + \begin{bmatrix} \textit{imports [for} \\ c \in (CM \cap CDN)] \end{bmatrix}$$	$c \in$ $(CD \cap CMN)$ $\cup$ $(CM \cap CDN)$, $t \in T$	Composite supply for non-imported outputs and non-produced imports
(29)	$$QT_{c,t} = \sum_{c' \in C'} \left(icm_{c,c',t} \cdot QM_{c',t} + ice_{c,c',t} \cdot QE_{c',t} + icd_{c,c',t} \cdot QD_{c',t} \right)$$ $$\begin{bmatrix} \textit{trade and transport} \\ \textit{demand for commodity c} \end{bmatrix} = \begin{bmatrix} \textit{from imports} \end{bmatrix} + \begin{bmatrix} \textit{from exports} \end{bmatrix} + \begin{bmatrix} \textit{from marketed} \\ \textit{domestic output} \end{bmatrix}$$	$c \in CT$ $t \in T$	Demand for transaction services

Domestic institution block

(30)	$$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot \overline{WFDIST}_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$$ $$\begin{bmatrix} \textit{income of} \\ \textit{factor f} \end{bmatrix} = \begin{bmatrix} \textit{sum of activity payments} \\ \textit{(activity-specific wages} \\ \textit{times employment levels)} \end{bmatrix} + \begin{bmatrix} \textit{income to factor f} \\ \textit{from Rest of World} \end{bmatrix}$$	$f \in F$ $t \in T$	Factor income

(31)	$SHIF_{i,f,t} = \dfrac{QFACINS_{i,f,t}}{\sum_{i' \in INS} QFACINS_{i',f,t}}$ $\begin{bmatrix} \textit{share of institution i in} \\ \textit{the income of factor f} \end{bmatrix} = \begin{bmatrix} \textit{endowment of institution i of factor f} \\ \textit{divided by total endowment of factor f} \end{bmatrix}$	$i \in INS$ $f \in F$ $t \in T$	Institutional shares in factor incomes
(32)	$YIF_{i,f,t} = SHIF_{i,f,t} \cdot \left[\left(1 - tf_{f,t}\right) \cdot YF_{f,t} \right]$ $\begin{bmatrix} \textit{income of} \\ \textit{institution i} \\ \textit{from factor f} \end{bmatrix} = \begin{bmatrix} \textit{share of income} \\ \textit{of factor f to} \\ \textit{institution i} \end{bmatrix} \cdot \begin{bmatrix} \textit{income of factor f} \\ \textit{(net of tax)} \end{bmatrix}$	$i \in INS$ $f \in F$ $t \in T$	Institutional factor incomes
(33)	$YIINT_{i,t} = gintrat_{i,t} \cdot GBOND_{i,t} - fintrat_{i,t} \cdot FDEBT_{i,t} \cdot EXR_t$ $\begin{bmatrix} \textit{net interest} \\ \textit{income of} \\ \textit{institution i} \end{bmatrix} = \begin{bmatrix} \textit{interest earnings} \\ \textit{on government} \\ \textit{bonds} \end{bmatrix} - \begin{bmatrix} \textit{interest} \\ \textit{payments} \\ \textit{on foreign debt} \end{bmatrix}$	$i \in INSDNG$ $t \in T$	Institutional net interest income
(34)	$TRII_{i,i',t} = shii_{i,i'} \cdot (1 - MPS_{i',t}) \cdot (1 - TINS_{i',t}) \cdot YI_{i',t}$ $\begin{bmatrix} \textit{transfer from} \\ \textit{institution i' to i} \end{bmatrix} = \begin{bmatrix} \textit{share of net income} \\ \textit{of institution i'} \\ \textit{transfered to i} \end{bmatrix} \cdot \begin{bmatrix} \textit{income of institution} \\ \textit{i', net of savings and} \\ \textit{direct taxes} \end{bmatrix}$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Intra-institutional transfers
(35)	$YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i' \in INSDNG'} TRII_{i,i',t} + YIINT_{i,t}$ $\begin{bmatrix} \textit{income of} \\ \textit{institution i} \end{bmatrix} = \begin{bmatrix} \textit{factor} \\ \textit{income} \end{bmatrix} + \begin{bmatrix} \textit{transfers from other} \\ \textit{domestic non-government} \\ \textit{institutions} \end{bmatrix} + \begin{bmatrix} \textit{net} \\ \textit{interest} \\ \textit{income} \end{bmatrix}$ $+ trnsfr_{i,gov,t} \cdot \overline{CPI}_t + trnsfrpc_{i,gov,t} \cdot POP_{i,t} \cdot \overline{CPI}_t$ $+ \begin{bmatrix} \textit{transfers from government} \\ \textit{to non-household institutions} \end{bmatrix} + \begin{bmatrix} \textit{transfers from} \\ \textit{government to households} \end{bmatrix}$ $+ trnsfr_{i,row,t} \cdot EXR_t + trnsfrpc_{i,row,t} \cdot POP_{i,t} \cdot EXR_t$ $+ \begin{bmatrix} \textit{transfers from Rest of World} \\ \textit{to non-household institutions} \end{bmatrix} + \begin{bmatrix} \textit{transfers from} \\ \textit{Rest of World to households} \end{bmatrix}$	$i \in INSDNG$ $t \in T$	Income of domestic, non-government institutions
(36)	$TINS_{i,t} = \overline{tins}_{i,t} \cdot \left(1 + \overline{TINSADJ}_t \cdot tins01_i\right) + \overline{DTINS}_t \cdot tins01_i$ $\begin{bmatrix} \textit{direct tax} \\ \textit{rate for} \\ \textit{institution i} \end{bmatrix} = \begin{bmatrix} \textit{base rate adjusted} \\ \textit{for scaling for} \\ \textit{selected institutions} \end{bmatrix} + \begin{bmatrix} \textit{point change} \\ \textit{for selected} \\ \textit{institutions} \end{bmatrix}$	$i \in INSDNG$ $t \in T$	Direct tax rates for domestic non-government institutions

(37)	$$MPS_{i,t} = \overline{MPS}_{i,t} \cdot \left(\frac{\left(1 - TINS_{i,t}\right) \cdot YI_{i,t}}{POP_{i,t}} \right)^{\rho sav_i - 1} \cdot \left(1 + \overline{MPSADJ}_t \cdot mps01_i\right)$$ $$\begin{bmatrix} marginal \\ propensity \\ to\ save \end{bmatrix} = \begin{bmatrix} exogenous \\ term \end{bmatrix} \times \begin{bmatrix} adjustment\ for \\ per\text{-}capita \\ post\text{-}tax\ income \end{bmatrix} \times \begin{bmatrix} scaling\ adjustment \\ for\ selected \\ institutions \end{bmatrix}$$ $$+ \overline{DMPS}_t \cdot mps01_i$$ $$+ \begin{bmatrix} point\text{-}change \\ adjustment\ for \\ selected\ institutions \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Savings rates for domestic non-government institutions
(38)	$$INSSAV_{i,t} = MPS_{i,t} \cdot \left(1 - TINS_{i,t}\right) \cdot YI_{i,t}$$ $$\begin{bmatrix} savings\ for \\ institution\ i \end{bmatrix} = \begin{bmatrix} savings \\ rate\ for \\ institution\ i \end{bmatrix} \cdot \begin{bmatrix} income\ of \\ institution\ i \\ (net\ of\ direct\ taxes) \end{bmatrix}$$	$i \in INSDNG$	Savings for domestic non-government institutions
(39)	$$EH_{h,t} = \left(1 - \sum_{i \in INSDNG} shii_{i,h}\right) \cdot \left(1 - MPS_{h,t}\right) \cdot (1 - TINS_{h,t}) \cdot YI_{h,t}$$ $$\begin{bmatrix} household\ income \\ disposable\ for \\ consumption \end{bmatrix} = \begin{bmatrix} household\ income,\ net\ of\ direct \\ taxes,\ savings,\ and\ transfers\ to \\ other\ non\text{-}government\ institutions \end{bmatrix}$$	$h \in H$ $t \in T$	Household consumption expenditure
(40)	$$QH_{c,h,t} = POP_{h,t} \cdot \left(\gamma m_{c,h} + \frac{\beta m_{c,h} \cdot \left(\left[\frac{EH_{h,t}}{POP_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma h_{a,c',h} \right)}{PQ_{c,t}} \right)$$ $$\begin{bmatrix} quantity\ of \\ household\ demand \\ for\ commodity\ c \end{bmatrix} = f \begin{bmatrix} household \\ consumption \\ spending,\ prices \end{bmatrix}$$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand for commodities from market
(41)	$$QHA_{a,c,h,t} = POP_{h,t} \cdot \left(\gamma h_{a,c,h} + \frac{\beta h_{a,c,h} \cdot \left(\left[\frac{EH_{h,t}}{POP_{h,t}} \right] - \sum_{c' \in C} PQ_{c',t} \cdot \gamma m_{c',h} - \sum_{a' \in A} \sum_{c' \in C} PXAC_{a',c',t} \cdot \gamma h_{a',c',h} \right)}{PXAC_{a,c,t}} \right)$$ $$\begin{bmatrix} quantity\ of\ household\ demand \\ for\ commodity\ c\ from\ activity\ a \end{bmatrix} = f \begin{bmatrix} household\ consumption \\ spending,\ prices \end{bmatrix}$$	$a \in A$ $c \in C$ $h \in H$ $t \in T$	Household consumption demand for own production

(42)	$YG_t = \sum_{i \in INSDNG} TINS_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t} + \sum_{a \in A} ta_{a,t} \cdot PA_{a\,t} \cdot QA_{a,t}$ $\begin{bmatrix} \textit{government} \\ \textit{revenue} \end{bmatrix} = \begin{bmatrix} \textit{direct taxes} \\ \textit{from institutions} \end{bmatrix} + \begin{bmatrix} \textit{direct taxes} \\ \textit{from factors} \end{bmatrix} + \begin{bmatrix} \textit{activity tax} \end{bmatrix}$ $+ \sum_{a \in A} tva_{a,t} \cdot PVA_{a,t} \cdot QVA_{a,t} + \sum_{c \in CM} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t} \cdot EXR_t$ $+ \begin{bmatrix} \textit{value-added tax} \end{bmatrix} + \begin{bmatrix} \textit{import tariffs} \end{bmatrix}$ $+ \sum_{c \in CE} te_{c,t} \cdot pwe_{c,t} \cdot QE_{c,t} \cdot EXR_t + \sum_{c \in C} tq_{c,t} \cdot PQ_{c,t} \cdot QQ_{c,t}$ $+ \begin{bmatrix} \textit{export taxes} \end{bmatrix} + \begin{bmatrix} \textit{sales tax} \end{bmatrix}$ $+ \sum_{f \in F} YIF_{gov,f,t} + \sum_{i \in INSDNG} TRII_{gov,i,t} + trnsfr_{gov,row,t} \cdot EXR_t$ $+ \begin{bmatrix} \textit{factor income} \end{bmatrix} + \begin{bmatrix} \textit{transfers from} \\ \textit{domestic institutions} \end{bmatrix} + \begin{bmatrix} \textit{transfers from RoW} \end{bmatrix}$	$t \in T$	Government recurrent revenue
(43)	$EG_t = \sum_{c \in C} PQ_{c,t} \cdot QG_{c,t} + \sum_{i \in INSDNH} trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $\begin{bmatrix} \textit{government} \\ \textit{spending} \end{bmatrix} = \begin{bmatrix} \textit{government} \\ \textit{consumption} \end{bmatrix} + \begin{bmatrix} \textit{transfers to domestic} \\ \textit{non-household institutions} \end{bmatrix}$ $+ \sum_{h \in H} trnsfrpc_{h,gov,t} \cdot POP_{h,t} \cdot \overline{CPI}_t + trnsfr_{row,gov,t} \cdot EXR_t$ $+ \begin{bmatrix} \textit{transfers to domestic} \\ \textit{households} \end{bmatrix} + \begin{bmatrix} \textit{transfers to} \\ \textit{Rest of World} \end{bmatrix}$ $+ \sum_{i \in INS} gintrat_{i,t} \cdot GBOND_{i,t} + fintrat_{gov,t} \cdot FDEBT_{gov,t} \cdot EXR_t$ $+ \begin{bmatrix} \textit{interest payment} \\ \textit{on domestic debt} \end{bmatrix} + \begin{bmatrix} \textit{interest payment} \\ \textit{on foreign debt} \end{bmatrix}$	$t \in T$	Government recurrent expenditures
(44)	$QG_{c,t} = QG_{c,t-1}$ $\cdot \left(1 + \overline{RQGT}_t + \sum_{c' \in C'} rqgadj_{c,c',t} \cdot \overline{RQGCT}_{c',t} \right)$ $\begin{bmatrix} \textit{real government} \\ \textit{consumption} \\ \textit{of c in t} \end{bmatrix} = \begin{bmatrix} \textit{real government} \\ \textit{consumption} \\ \textit{of c in t - 1} \end{bmatrix} \cdot \begin{bmatrix} 1 + \begin{bmatrix} \textit{growth} \\ \textit{rate} \end{bmatrix} \end{bmatrix}$	$c \in C$ $t \in T$ $t > 1$	Real government consumption
(45)	$GSAV_t = YG_t - EG_t$ $\begin{bmatrix} \textit{government} \\ \textit{savings} \end{bmatrix} = \begin{bmatrix} \textit{government} \\ \textit{recurrent revenue} \end{bmatrix} - \begin{bmatrix} \textit{government} \\ \textit{recurrent expenditures} \end{bmatrix}$	$t \in T$	Government savings

Investment block

(46)	$PK_{f,t} = \sum_{c \in C} capcomp_{c,f} \cdot PQ_{c,t}$ $\begin{bmatrix} \textit{price of new} \\ \textit{capital stock} \end{bmatrix} = \begin{bmatrix} \textit{total value of commodities c} \\ \textit{per unit of new capital} \end{bmatrix}$	$f \in FCAP$ $t \in T$	Price of new capital stock

(47)	$$INVVAL_{gov,t} = \sum_{f \in FCAPGOV} PK_{f,t} \cdot DKINS_{gov,f,t}$$ $$\begin{bmatrix} \textit{government fixed} \\ \textit{investment value} \end{bmatrix} = \begin{bmatrix} \textit{government spending} \\ \textit{on capital goods} \end{bmatrix}$$	$t \in T$	Government fixed investment value
(48)	$$INVVAL_{gov,t} = GSAV_t - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,gov,t} + DGBONDTOT_t$$ $$\begin{bmatrix} \textit{government fixed} \\ \textit{investment value} \end{bmatrix} = \begin{bmatrix} \textit{government} \\ \textit{savings} \end{bmatrix} - \begin{bmatrix} \textit{spending on} \\ \textit{stock changes} \end{bmatrix} + \begin{bmatrix} \textit{total change in holdings} \\ \textit{of government bonds} \end{bmatrix}$$ $$+CBBORTOT_t + \left(\overline{FBOR}_{gov,t} + \overline{FGRANT}_{gov,t}\right) \cdot EXR_t$$ $$+\begin{bmatrix} \textit{Government Central Bank} \\ \textit{borrowing (deficit monetization)} \end{bmatrix} + \begin{bmatrix} \textit{foreign borrowing and} \\ \textit{foreign grants (in LCU)} \end{bmatrix}$$	$t \in T$	Government investment financing
(49)	$$DGBOND_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_i \cdot INSSAV_{i',t}} \cdot DGBONDTOT_t$$ $$\begin{bmatrix} \textit{change in holdings of} \\ \textit{government bonds} \\ \textit{by institution i} \end{bmatrix} = \frac{\begin{bmatrix} \textit{savings by} \\ \textit{by institution i} \end{bmatrix}}{\begin{bmatrix} \textit{total institution} \\ \textit{savings value} \end{bmatrix}} \cdot \begin{bmatrix} \textit{(scaled) total change} \\ \textit{in holdings of} \\ \textit{government bonds} \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Allocation of government bond borrowing across domestic non-government institutions
(50)	$$CBBOR_{i,t} = \frac{gbdist_i \cdot INSSAV_{i,t}}{\sum_{i' \in INSDNG'} gbdist_i \cdot INSSAV_{i',t}} \cdot CBBORTOT_t$$ $$\begin{bmatrix} \textit{Government Central Bank} \\ \textit{borrowing by institution i} \end{bmatrix} = \frac{\begin{bmatrix} \textit{savings by} \\ \textit{by institution i} \end{bmatrix}}{\begin{bmatrix} \textit{total institution} \\ \textit{savings value} \end{bmatrix}} \cdot \begin{bmatrix} \textit{(scaled) total Government} \\ \textit{Central Bank borrowing} \end{bmatrix}$$	$i \in INSDNG$ $t \in T$	Allocation of the burden of Central Bank borrowing across domestic non-government institutions
(51)	$$INVVAL_{i,t} = INSSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,i,t} - DGBOND_{i,t}$$ $$\begin{bmatrix} \textit{non-government fixed} \\ \textit{investment value} \end{bmatrix} = \begin{bmatrix} \textit{savings} \end{bmatrix} - \begin{bmatrix} \textit{stock} \\ \textit{changes} \end{bmatrix} - \begin{bmatrix} \textit{change in holdings of} \\ \textit{government bonds} \end{bmatrix}$$ $$-CBBOR_{i,t} + \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} + fdi_{i,t}\right) \cdot EXR_t$$ $$-\begin{bmatrix} \textit{Government Central} \\ \textit{Bank borrowing} \end{bmatrix} + \begin{bmatrix} \textit{foreign borrowing, grants,} \\ \textit{and direct investment (in LCU)} \end{bmatrix}$$	$i \in INSNG$ $t \in T$	Investment financing for non-government institutions
(52)	$$PK_{f,t} \cdot DKINS_{i,f,t} = gfcfshr_{f,i,t} \cdot INVVAL_{i,t}$$ $$\begin{bmatrix} \textit{non-government spending} \\ \textit{on capital stock f} \end{bmatrix} = \begin{bmatrix} \textit{total fixed investment value} \\ \textit{times share for capital stock f} \end{bmatrix}$$	$i \in INSNG$ $f \in FCAP$ $t \in T$	Non-government investment by capital stock
(53)	$$DKGOV_{f,t} \geq \sum_{\substack{a \in A \\ \mid (f,a) \in MFA}} \left(ifa_{f,a} \cdot QA_{a,t} \cdot \frac{QA_{a,t}}{QA_{a,t-1}} \right) - QFACINS_{gov,f,t} \cdot (1 - depr_f)$$ $$\begin{bmatrix} \textit{gross government} \\ \textit{investment demand} \\ \textit{for capital} \end{bmatrix} = \begin{bmatrix} \textit{anticipated demand for capital} \\ \textit{next year (based on current} \\ \textit{production and its growth)} \end{bmatrix} - \begin{bmatrix} \textit{remaining capital stock} \\ \textit{(after depreciation) next} \\ \textit{year if no investment} \end{bmatrix}$$	$f \in FCAPGOV$ $t \in T$ $t > 1$	Real government demand for investment in capital stock f

(54)	$$DKINS_{gov,f,t} = DKGOV_{f,t}$$ $$\left[\begin{matrix} \textit{gross investment in } f \textit{ of} \\ \textit{institution ins (here "ins" = gov)} \end{matrix}\right] = \left[\begin{matrix} \textit{gross government investment} \\ \textit{demand for capital} \end{matrix}\right]$$	$f \in FCAPGOV$ $t \in T$ $t > 1$	Real government investment in capital stock f
(55)	$$QINV_{c,t} = \sum_{f \in FCAP} \left(capcomp_{c,f} \cdot \sum_{i \in INS} DKINS_{i,f,t} \right)$$ $$\left[\begin{matrix} \textit{real investment demand} \\ \textit{for commodity } c \end{matrix}\right] = \left[\begin{matrix} \textit{demand for c for each type of capital,} \\ \textit{summed over all institutions and capital types} \end{matrix}\right]$$	$c \in C$ $t \in T$	Total real investment demand by commodity

System constraint and macro block

(56)	$$QFS_{f,t} = \sum_{i \in INS} QFACINS_{i,f,t}$$ $$\left[\begin{matrix} \textit{supply of} \\ \textit{factor } f \end{matrix}\right] = \left[\begin{matrix} \textit{sum of all} \\ \textit{institutional endowments} \end{matrix}\right]$$	$f \in F$ $t \in T$	Factor supplies
(57)	$$\sum_{a \in A} QF_{f,a,t} = QFS_{f,t}$$ $$\left[\begin{matrix} \textit{demand for} \\ \textit{market factor } f \end{matrix}\right] = \left[\begin{matrix} \textit{supply of} \\ \textit{market factor } f \end{matrix}\right]$$	$f \in F$ $t \in T$	Factor markets
(58)	$$QQ_{c\,t} = \sum_{a \in A} QINT_{c,a,t} + \sum_{h \in H} QH_{c,h,t} + QG_{c,t}$$ $$\left[\begin{matrix} \textit{composite} \\ \textit{supply} \end{matrix}\right] = \left[\begin{matrix} \textit{intermediate} \\ \textit{use} \end{matrix}\right] + \left[\begin{matrix} \textit{household} \\ \textit{consumption} \end{matrix}\right] + \left[\begin{matrix} \textit{government} \\ \textit{consumption} \end{matrix}\right]$$ $$+QINV_{c,t} + \sum_{i \in INS} qdst_{c,i,t} + QT_{c,t}$$ $$+\left[\begin{matrix} \textit{fixed} \\ \textit{investment} \end{matrix}\right] + \left[\begin{matrix} \textit{stock} \\ \textit{change} \end{matrix}\right] + \left[\begin{matrix} \textit{trade and} \\ \textit{transport} \end{matrix}\right]$$	$c \in C$ $t \in T$	Composite commodity markets equilibrium
(59)	$$\sum_{c \in CM} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{f \in F} YIF_{row,f,t}}{EXR_t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{EXR_t}$$ $$\left[\begin{matrix} \textit{import} \\ \textit{spending} \end{matrix}\right] + \left[\begin{matrix} \textit{factor income} \\ \textit{to Rest of World} \end{matrix}\right] + \left[\begin{matrix} \textit{transfers from domestic} \\ \textit{non-gov institutions to RoW} \end{matrix}\right]$$ $$+trnsfr_{row,gov,t} + \sum_{i \in INSD} fintrat_{i,t} \cdot FDEBT_{i,t}$$ $$+\left[\begin{matrix} \textit{transfers from} \\ \textit{government to RoW} \end{matrix}\right] + \left[\begin{matrix} \textit{interest payment} \\ \textit{on foreign debt} \end{matrix}\right]$$ $$= \sum_{c \in CE} pwe_{c,t} \cdot QE_{c,t} + \sum_{i \in INSDNH} trnsfr_{i,row,t} + \sum_{h \in H} trnsfrpc_{h,row,t} \cdot POP_{h,t}$$ $$= \left[\begin{matrix} \textit{export} \\ \textit{revenue} \end{matrix}\right] + \left[\begin{matrix} \textit{transfers from RoW to domestic} \\ \textit{non-household institutions} \end{matrix}\right] + \left[\begin{matrix} \textit{transfers from RoW to} \\ \textit{domestic households} \end{matrix}\right]$$ $$+\sum_{f \in F} trnsfr_{f,row,t} + \sum_{i \in INSD} \left(\overline{FBOR}_{i,t} + \overline{FGRANT}_{i,t} \right) + fdi_{row,t}$$ $$+\left[\begin{matrix} \textit{factor income} \\ \textit{from RoW} \end{matrix}\right] + \left[\begin{matrix} \textit{borrowing} \\ \textit{from RoW} \end{matrix}\right] + \left[\begin{matrix} \textit{grants} \\ \textit{from RoW} \end{matrix}\right] + \left[\begin{matrix} \textit{foreign direct} \\ \textit{investment} \end{matrix}\right]$$	$t \in T$	Balance of payments (in foreign currency)

(60)	$$GDPREAL_t = \sum_{c \in C} \sum_{h \in H} PQ^0_c \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC^0_{a,c} \cdot QHA_{a,c,h,t}$$ $$\left[real\ GDP \right] = \begin{bmatrix} household\ market \\ consumption \end{bmatrix} + \begin{bmatrix} household\ own \\ production\ consumption \end{bmatrix}$$ $$+ \sum_{c \in C} PQ^0_c \cdot QG_{c,t} + \sum_{c \in C} PQ^0_c \cdot QINV_{c,t} + \sum_{c \in C} \sum_{i \in INS} PQ^0_c \cdot qdst_{c,i,t}$$ $$+ \begin{bmatrix} government \\ consumption \end{bmatrix} + \begin{bmatrix} fixed \\ investment \end{bmatrix} + \begin{bmatrix} stock \\ change \end{bmatrix}$$ $$+ \sum_{c \in CE} EXR^0 \cdot pwe^0_c \cdot QE_{c,t} - \sum_{c \in CM} EXR^0 \cdot pwm^0_c \cdot QM_{c,t}$$ $$+ \left[exports \right] - \left[imports \right]$$	$t \in T$	Real GDP at market prices
(61)	$$TRDGDP_t = \frac{\sum_{c \in CE} EXR^0 \cdot pwe^0_c \cdot QE_{c,t} + \sum_{c \in CM} EXR^0 \cdot pwm^0_c \cdot QM_{c,t}}{GDPREAL_t}$$ $$\begin{bmatrix} ratio\ of \\ trade\ to\ GDP \end{bmatrix} = \frac{\left[real\ trade \right]}{\left[real\ GDP \right]}$$	$t \in T$	Real Trade-GDP ratio
(62)	$$GDPREALFC_t = \sum_{a \in A} pva^0_a \cdot \left(1 - tva^0_{a,t} \right) \cdot QVA_{a,t}$$ $$\begin{bmatrix} real\ GDP \\ at\ factor\ cost \end{bmatrix} = \begin{bmatrix} value\text{-}added \\ net\ of\ taxes \end{bmatrix}$$	$t \in T$	Real GDP at factor cost
CORE CGE MODEL: BETWEEN-PERIOD MODULE			
Asset stock updating block			
(63)	$$POP_{h,t} = POPSCAL_t \cdot POP_{h,t-1} \cdot \frac{\sum_{f \in FLAB} QFACINS_{h,f,t}}{\sum_{f \in FLAB} QFACINS_{h,f,t-1}}$$ $$\begin{bmatrix} population\ of \\ household\ h \\ in\ year\ t \end{bmatrix} = \begin{bmatrix} population \\ scaling \\ factor\ for\ t \end{bmatrix} \cdot \begin{bmatrix} population\ of \\ household\ h \\ in\ year\ t\text{-}1 \end{bmatrix} \cdot \begin{bmatrix} factor\ for\ labor \\ force\ growth \\ for\ household\ h \end{bmatrix}$$	$h \in H$ $t \in T$ $t > 1$	Population by household
(64)	$$poptot_t = \sum_{h \in H} POP_{h,t}$$ $$\begin{bmatrix} total\ population \\ (exogenous) \end{bmatrix} = \begin{bmatrix} sum\ of\ population \\ for\ households\ h \end{bmatrix}$$	$t \in T$ $t > 1$	Population by household
(65)	$$QFACINS_{h,f,t} = QFSCAL_{f,t} \cdot POP_{h,t} \cdot qfpc_{h,f,t}$$ $$\begin{bmatrix} stock\ of\ non\text{-}labor \\ factor\ f\ by\ household \end{bmatrix} = \begin{bmatrix} scaling \\ factor \end{bmatrix} \cdot \begin{bmatrix} population\ for \\ household\ h \end{bmatrix} \cdot \begin{bmatrix} per\text{-}capita\ stock \\ for\ household\ h \end{bmatrix}$$	$h \in H$ $f \in (FEXOG \cap FLABN)$ $t \in T,\ t > 1$	Non-labor factors with exogenous total stocks
(66)	$$qfachhtot_{f,t} = \sum_{h \in H} QFACINS_{h,f,t}$$ $$\begin{bmatrix} total\ household\ stock\ of \\ exogenous,\ non\text{-}labor\ factors \end{bmatrix} = \begin{bmatrix} sum\ of\ disaggregated \\ household\ stocks \end{bmatrix}$$	$f \in (FEXOG \cap FLABN)$ $t \in T$	Constraint of total household stocks of exogenous, non-labor factors

(67)	$QFACINS_{i,f,t} = QFCAPRED_{i,f,t-1} + DKINS_{i,f,t-1} + qfacinsadj_{i,f,t-1}$ $\begin{bmatrix} \textit{stock of capital} \\ \textit{type f held} \\ \textit{by institution i} \end{bmatrix} = \begin{bmatrix} \textit{redistributed old} \\ \textit{capital stock} \end{bmatrix} + \begin{bmatrix} \textit{gross stock} \\ \textit{change in t-1} \end{bmatrix} + \begin{bmatrix} \textit{exogenous adjustment} \\ \textit{in capital stock} \end{bmatrix}$	$i \in INS$ $f \in FCAP$ $t \in T,\ t>1$	Capital stocks by institution
(68)	$QFCAPRED_{i,f,t} = (1-depr_f)\cdot QFACINS_{i,f,t} \quad if\ \ i \notin H$ $= (1-depr_f)\cdot QFACINS_{i,f,t}\cdot\left(QFSCAL_{f,t}\cdot\dfrac{POP_{i,t}}{POP_{i,t-1}}\right) \quad if\ \ i \in H$ $\begin{bmatrix} \textit{redistributed old capital} \\ \textit{stock by institution i} \end{bmatrix} = \begin{bmatrix} \textit{depreciated} \\ \textit{capital stock} \end{bmatrix}\cdot\begin{bmatrix} \textit{(if institution i is a household, then the depreciated} \\ \textit{capital stock is scaled for population changes)} \end{bmatrix}$	$i \in INS$ $f \in FCAP$ $t \in T,\ t>1$	Redistribution of old capital stocks
(69)	$\sum_{h\in H} QFCAPRED_{h,f,t} = \left(1-depr_f\right)\cdot\sum_{h\in H} QFACINS_{h,f,t}$ $\begin{bmatrix} \textit{total redistributed (and} \\ \textit{depreciated) household stock} \end{bmatrix} = \begin{bmatrix} \textit{sum of disaggregated (and depreciated)} \\ \textit{household stocks before redistribution} \end{bmatrix}$	$f \in FCAP$ $t \in T,\ t>1$	Constraint on total household capital stocks after redistribution
(70)	$FDEBT_{i,t} = FDEBTRED_{i,t-1} + FBOR_{i,t-1}$ $+\left(fintratdue_{i,t-1} - fintrat_{i,t-1}\right)\cdot FDEBT_{i,t-1} - fdebtrelief_{i,t-1}$ $\begin{bmatrix} \textit{foreign} \\ \textit{debt in t} \end{bmatrix} = \begin{bmatrix} \textit{redistributed} \\ \textit{foreign debt in t-1} \end{bmatrix} + \begin{bmatrix} \textit{foreign bor-} \\ \textit{rowing in t-1} \end{bmatrix} + \begin{bmatrix} \textit{unpaid interest on} \\ \textit{foreign debt in t-1} \end{bmatrix} - \begin{bmatrix} \textit{foreign debt} \\ \textit{relief in t-1} \end{bmatrix}$	$i \in INSD$ $t \in T$ $t > 1$	Foreign debt of domestic institutions
(71)	$FDEBTRED_{i,t} = FDEBT_{i,t} \quad if\ \ i \notin H$ $= FDEBT_{i,t}\cdot\left(FDEBTSCAL_t\cdot\dfrac{POP_{i,t}}{POP_{i,t-1}}\right) \quad if\ \ i \in H$ $\begin{bmatrix} \textit{redistributed old foreign} \\ \textit{debt by institution i} \end{bmatrix} = \begin{bmatrix} \textit{original} \\ \textit{debt} \end{bmatrix}\cdot\begin{bmatrix} \textit{(if institution i is a household, then the foreign} \\ \textit{debt is scaled for population changes)} \end{bmatrix}$	$i \in INSD$ $t \in T$ $t > 1$	Redistribution of old foreign debt
(72)	$\sum_{h\in H} FDEBTRED_{h,t} = \sum_{h\in H} FDEBT_{h,t}$ $\begin{bmatrix} \textit{total redistributed} \\ \textit{household debt} \end{bmatrix} = \begin{bmatrix} \textit{sum of household debts} \\ \textit{before redistribution} \end{bmatrix}$	$t \in T$ $t > 1$	Constraint on total household foreign debt after redistribution
(73)	$GBOND_{i,t} = GBONDRED_{i,t-1} + DGBOND_{i,t-1}$ $\begin{bmatrix} \textit{stock of government} \\ \textit{bond held by} \\ \textit{institution i} \end{bmatrix} = \begin{bmatrix} \textit{redistributed holdings of} \\ \textit{stock of government bond} \\ \textit{held by institution i in t-1} \end{bmatrix} + \begin{bmatrix} \textit{government} \\ \textit{borrowing} \\ \textit{from i in t-1} \end{bmatrix}$	$i \in INSDNG$ $t \in T$ $t > 1$	Government bond holdings of domestic institutions
(74)	$GBONDRED_{i,t} = GBOND_{i,t} \quad if\ \ i \notin H$ $= GBOND_{i,t}\cdot\left(GBONDSCAL_t\cdot\dfrac{POP_{i,t}}{POP_{i,t-1}}\right) \quad if\ \ i \in H$ $\begin{bmatrix} \textit{redistributed old bond} \\ \textit{holdings by institution i} \end{bmatrix} = \begin{bmatrix} \textit{original} \\ \textit{bond holdings} \end{bmatrix}\cdot\begin{bmatrix} \textit{(if institution i is a household, then the bond} \\ \textit{holdings is scaled for population changes)} \end{bmatrix}$	$i \in INSDNG$ $t \in T$ $t > 1$	Redistribution of holdings of old government bonds
(75)	$\sum_{h\in H} GBONDRED_{h,t} = \sum_{h\in H} GBOND_{h,t}$ $\begin{bmatrix} \textit{total redistributed} \\ \textit{household bond holding} \end{bmatrix} = \begin{bmatrix} \textit{sum of household bond holdings} \\ \textit{before redistribution} \end{bmatrix}$	$t \in T$ $t > 1$	Constraint on total household holdings of government bonds after redistribution

(76)	$$\alpha va_{a,t} = \alpha va2_{a,t} \cdot \prod_{f \in FCAP} \left[\frac{\sum_{i \in INS} QFACINS_{i,f,t}}{\sum_{i \in INS} QFACINS^{0}_{i,f}} \right]^{tfpelasqg_{a,f,t}} \cdot \left(\frac{\sum_{t' \in T} tfptrdwt_{t,t'} \cdot TRDGDP_{t'}}{TRDGDP^{o}} \right)^{tfpelastrd_{a}}$$ $$\begin{bmatrix} efficiency \\ term\ for \\ activity\ a \end{bmatrix} = \begin{bmatrix} endogenous \\ trend \\ term\ for \\ activity\ a \end{bmatrix} \cdot \begin{bmatrix} product\ of: ratio\ of\ all \\ current\ real\ capital \\ endowment\ f\ to\ inital \\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix} \cdot \begin{bmatrix} weighted\ avg.\ (over\ time) \\ of\ ratios\ of\ openness \\ to\ initial\ value,\ raised \\ to\ the\ relevant\ elasticity \end{bmatrix}$$	$a \in A$ $t \in T$	Efficiency (TFP) by activity
(77)	$$\alpha va2_{a,t} = \alpha va2_{a,t-1} \cdot \left(1 + \overline{CALTFPGT}_{t} \cdot tfp01_{a,t} + \overline{\alpha vag}_{a,t} \right)$$ $$\begin{bmatrix} endogenous \\ trend\ term\ for \\ activity\ a \end{bmatrix} = \begin{bmatrix} endogenous \\ trend\ term\ for \\ activity\ a\ in\ t-1 \end{bmatrix} \cdot \begin{bmatrix} growth \\ adjustment \\ factor \end{bmatrix}$$	$a \in A$ $t \in T$ $t > 1$	TFP trend term by activity

Table 3. Notation for MDG module of MAMS model

SETS			
Symbol	Explanation	Symbol	Explanation
$a \in A$	activities	$h \in H$	households (excl. NGOs) ={*h* = the single household}
$b \in B$	student behavioral characteristics ={*rep* = repeater; *dropout* = dropout; *grd* = graduate; *grdcont* = continuing graduate; *grdexit* = exiting graduate; *g1entry* = entrant to grade 1; *grdcyc*= grad from last cycle-year; *contcyc* = grad within cycle}	$b,b' \in MBB$	mapping between *b* (in *BRES*) and *b'* (in *BLOG*): ={(*rep*, *dropout*).*grd*, *grdexit.grdcont*}
$b \in BLOG$ $(\subset B)$	student behavior determined by logistic function ={*grd*, *grdcont*, *g1entry*}	$b,b' \in MBB2$	mapping between *b* (in *BRES*) and all elements *b'* (also in *BRES)* that are related to the same element(s) in *BLOG*: ={*rep*.(*rep*, *dropout*), *dropout*.(*rep*, *dropout*), *grdexit.grdexit*}
$b \in BRES$ $(\subset B)$	student behavior determined by residual scaling ={*rep* = repeater; *dropout* = dropout; *grdexit* = exiting graduate}	$c,c' \in MCE$	mapping private and public education into 1 education commodity, by cycle = {*c-edup1*.(*c-edup1*, *c-edup1ng*)} where *c-edup1ng* is private 1st cycle primary; similarly for *c-edup2*, *c-edus*, and *c-edut*.
$c \in C$	Commodities	$c,c' \in MCM$	mapping between aggregate (CMDG) and disaggregated MDG service commodities (CHLTH and CWTSN) = {*c-hlt*.(*c-hlt1g*, *c-hlt2g*, *c-hlt3g*, *c-hlt1ng*, *c-hlt2ng*, *c-hlt3ng*} and {*c-wtsn*.(*c-wtsn*)}
$c \in CEDU$ $(\subset C)$	education services ={*c-edup1* = 1st cycle primary; *c-edup2* = 2nd cycle primary; *c-edus* = secondary; *c-edut* = tertiary}; can include both private and public education	$mdg \in MDG$	selected MDG indicators ={*mdg2*, *mdg4*, *mdg5*, *mdg7a*, *mdg7b*}
$c \in CELA$	educational cycle that corresponds to the age at which non-students would enter the labor force	$mmdg2(t,b,t')$	MDG2 in t is defined as the product over selected combinations of b and t' (where $t' \in T11$)
$c \in CHLTH$ $(\subset C)$	health services (public) ={*c-hlt1g* = low-tech; *c-hlt2g* = medium-tech; *c-hlt3g* = high-tech}; corresponding private health services labeled with "*ng*"	$mdg \in MDGSTD$	MDG indicators ={*mdg4* = under-5 mortality rate; *mdg5* = maternal mortality rate; *mdg7a* = access to safe water; *mdg7b* = access to basic sanitation}
$cmdg \in CMDG$	aggregate MDG (non-education) service commodities = {*c-hlt* = aggregate health in MDG functions, not in C; *c-wtsn* = water-sanitation services}	$f,c \in MFC$	mapping between labor types and cycles of education ={*flab-n*.(*c-edup1*, *c-edup2*); *flab-s*.(*c-edus*); *flab-t*.(*c-edut*)}

$c \in CWTSN$ $(\subset C)$	water-sanitation service commodities {*c-wtsn* = water-sanitation services}	$mdgarg \in MDGARG$	arguments in CE function for MDGs ={*cmdg* = agg commodities; *mdg* = different MDGs; *fcapinf* = infrastructure capital stocks; *hhdconspc* = per-capita hhd consumption }
$edarg \in EDARG$	arguments in CE function for educational behavior ={*edu-qual* = qnty of services per student; *wage-prem* = skilled-unskilled wage ratio; *wage-prem2* = superskilled-skilled wage ratio; *mdg4* = under-five mortality rate; *fcapinf* = infrastructure capital stocks; *hhdconspc* = per-capita hhd consumption}	$t \in T$	time periods
$f \in FLAB$	labor factors {*f-labn* = less than 12 yrs of education; *f-labs* = 12-14 yrs of education (secondary education or 2 years of tertiary); *f-labt* = more than 14 yrs of education (at least 3 years of tertiary)	$t \in T11$	time periods including preceding years for MDG2 calculation

PARAMETERS			
$\alpha ed_{b,c}$	constant in logistic function for educational behavior	$grdcont01_{c,c'}$	0-1 constant showing that for *c'* next cycle is *c*
αlog_{mdg}	constant in logistic function for MDG achievement	ord_t	ordinal position of *t* in the set *T*
αm_{mdg}	constant in constant-elasticity function for intermediate MDG variable	$popageg1_t$	population in age cohort entering grade 1
$\beta ed_{b,c}$	constant in logistic function for educational behavior	$popagelab_t$	population in age cohort entering labor force (age at end of a model education cycle)
βlog_{mdg}	constant in logistic function for MDG achievement	$poptot_t$	total population in *t*
$\varphi ed_{b,c,edarg}$	elasticity of behavior *b* in cycle *c* w.r.t. argument *edarg* in educational constant-elasticity function	$qg1entncoh_{c,t}$	number of non-cohort (non-1st-year-primary) entrants to first cycle
$\varphi m_{mdg,mdgarg}$	elasticity of *mdg* w.r.t. argument *mdgarg* in constant-elasticity function for MDG	$shif^{0}_{i,f,t}$	share of domestic institution i in income of factor *f*
$depr_f$	depreciation rate for factor *f*	$shrdemot01_{c,c'}$	0-1 parameter showing that for dropouts from *c'* the highest cycle is *c*
$discrat$	discount rate	$shrlabent_{c,t}$	share of drop-outs and leavers in cycle *c* that enter the labor force
$exted_{b,c}$	maximum share for educational behavior *b* in cycle *c*	$shrlabent2_f$	share of labor type *f* of labor force entrants without education
$extm_{mdg}$	maximum value for MDG 7a and 7b; minimum value for MDG 4 and 5	$yrlev_c$	years in school cycle for each education cycle *c*

VARIABLES			
$EDUQUAL_{c,t}$	educational quality in cycle *c* in year *t*	$QFACINS_{i,f,t}$	endowment of labor type *f* for institution *i* in *t*
EG_t	government expenditures	$QH_{c,h,t}$	consumption of commodity *c* in *t* by household *h*
$INVVAL_{i,t}$	investment value for institution *i*	$QHA_{a,c,h,t}$	quantity consumed of home commodity *c* from activity *a* by household *h*
$MDGVAL_{mdg,t}$	value for MDG indicator *mdg* in *t*	$QHPC_t$	Per-capita household consumption in *t*
$MDGINT_{mdg,t}$	value for intermediate MDG indicator *mdg* in period *t* (entering logistic function)	$QQ_{c,t}$	quantity of goods supplied to domestic market (composite supply)
$PQ_{c,t}$	price of commodity *c* in *t*	$QXHLTH_{mdg,t}$	government and NGO provision of aggregated health services related to health MDG
$PXAC_{a,c,t}$	price of commodity *c* from activity *a*	$SHRed_{b,c,t}$	share of students in cycle *c* with behavior *b* in *t*
$QENR_{c,t}$	total number of students enrolled in cycle *c* in year *t*	$SHRedint_{b,c,t}$	value for intermediate indicator of educational behavior *b* in cycle *c* and time *t* (entering logistic function)
$QENROLD_{c,t}$	number of old students enrolled in cycle *c* in year *t*	$WF_{f,t}$	economywide wage for factor *f* in *t*
$QENRNEW_{c,t}$	number of new students enrolled in cycle *c* in year *t*		

Table 4. Equations for MDG module of MAMS model

#	Equation	Domain	Description
(78)	$SHRed_{b,c,t} = exted_{b,c} + \dfrac{\alpha ed_{b,c}}{1+\gamma ed_{b,c} \cdot EXP\left(\beta ed_{b,c} \cdot \left(SHRedint_{b,c,t} - SHRed^{0}_{b,c}\right)\right)}$ $\begin{bmatrix} \text{student share with} \\ \text{behavior } b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{logistic function of intermediate} \\ \text{behavior variable } (SHRedint_{b,c,t}) \end{bmatrix}$	$b \in BLOG$ $c \in CEDU$ $t \in T$	Student behavior (logistic function)[20]
(79)	$SHRedint_{b,c,t} = SHRed^{0}_{b,c} \cdot \left(EDUQUAL_{c,t}\right)^{\varphi ed_{b,c,\text{"edu-qual"}}}$ $\cdot \left(\dfrac{WF_{\text{"f-labs"},t}}{WF_{\text{"f-labn"},t}}\right)^{\varphi ed_{b,c,\text{"wage-prem"}}} \cdot \left(\dfrac{WF_{\text{"f-labt"},t}}{WF_{\text{"f-labs"},t}}\right)^{\varphi ed_{b,c,\text{"wage-prem2"}}}$ $\cdot MDGVAL_{\text{"mdg4"},t}^{\varphi ed_{b,c,\text{"mdg4"}}} \cdot \left(\sum_{i \in INS} QFACINS_{i,\text{"fcapinf"},t}\right)^{\varphi ed_{b,c,\text{"fcapinf"}}} \cdot QHPC_{t}^{\varphi ed_{b,c,\text{"hhdconspc"}}}$ $\begin{bmatrix} \text{intermediate variable for student} \\ \text{share with behavior } b \text{ in cycle } c \end{bmatrix} = \begin{bmatrix} \text{exogenous} \\ \text{trend value} \end{bmatrix}$ $\cdot \begin{bmatrix} \text{influence of: education quality (service per student);} \\ \text{wage premia (for } c \le \text{ secondary and } c \ge \text{ tertiary, resp.);} \\ \text{student health (proxied by MDG4);} \\ \text{level of infrastructure; and per-capita household consumption} \end{bmatrix}$	$b \in BLOG$ $c \in C$ $t \in T$	Student behavior (constant-elasticity function defining intermediate variable)[21]
(80)	$EDUQUAL_{c,t} = \dfrac{\sum_{\substack{c' \in C \\ \lvert (c,c') \in MCE}} QQ_{c',t}}{QENR_{c,t}} \Bigg/ \dfrac{\sum_{\substack{c' \in C \\ \lvert (c,c') \in MCE}} QQ^{0}_{c'}}{QENR^{0}_{c}}$ $\begin{bmatrix} \text{educational quality} \\ \text{in cycle } c \text{ in year } t \end{bmatrix} = \begin{bmatrix} \textit{real} \text{ services per student} \\ \text{in } \textit{cycle } c \text{ in t} \end{bmatrix} \div \begin{bmatrix} \textit{real} \text{ services per student} \\ \text{in } \textit{cycle } c \text{ in base-year} \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Educational quality

[20] The α and β parameters in the logistic functions (Equations 83 and 90) have been calibrated so that (i) under base-year conditions, the left-hand side variables (showing student behavior shares or MDG values) will replicate base-year values; and (ii) under conditions derived from supporting studies of health and education, the left-hand side variables will take on values indicative of or compatible with MDG achievement.

[21] In the computer program, equations 84 and 91 (constant-elasticity functions defining intermediate variables for educational behavior or MDG achievement) are more complex in two respects. First, the terms that are raised to exponents, which represent elasticities, are all divided by base-year values. This formulation was preferred given our desire to simulate scenarios with changes in elasticities but without any changes in simulated base-year values for left-hand-side variables. Second, for the element *grdcont* $\in$ BLOG, the decision to continue to the next education cycle depends on the values for the right-hand side variables that correspond to the next cycle.

(81)	$$QHPC_t = \frac{\sum_{c\in C}\sum_{h\in H} PQ_c^0 \cdot QH_{c,h,t} + \sum_{a\in A}\sum_{c\in C}\sum_{h\in H} PXAC_{a,c}^0 \cdot QHA_{a,c,h,t}}{poptot_t}$$ $$\begin{bmatrix} \textit{real household cons -} \\ \textit{umption per capita} \end{bmatrix} = \begin{bmatrix} \textit{total household consumption at base -} \\ \textit{year prices divided by total population} \end{bmatrix}$$	$t \in T$	Real household consumption per capita
(82)	$$SHRed_{b,c,t} = \left(1 - \sum_{\substack{b'\in BLOG \\ \mid(b,b')\in MBB}} SHRed_{b',c,t}\right) \frac{SHRed^{0}_{b,c,t}}{\sum_{\substack{b'\in BRES \\ \mid(b,b')\in MBB2}} SHRed^{0}_{b',c,t}}$$ $$\begin{bmatrix} \textit{student share} \\ \textit{with behavior} \\ \textit{b in cycle c} \end{bmatrix} = \begin{bmatrix} \textit{residual value (1 less sum} \\ \textit{of shares for related} \\ \textit{elements in BLOG)} \end{bmatrix} \cdot \begin{bmatrix} \textit{initial share of b in} \\ \textit{total shares for related} \\ \textit{residual elements} \end{bmatrix}$$	$b \in BRES$ $c \in CEDU$ $t \in T$	Student behavior (defined residually, given left-hand side of the logistic function for education).
(83)	$$SHRed_{"grdcyc",c,t} = \frac{SHRed_{"grd",c,t}}{yrlev_c}$$ $$\begin{bmatrix} \textit{student share that} \\ \textit{graduates from} \\ \textit{cycle c in year t} \end{bmatrix} = \begin{bmatrix} \textit{student share that graduates} \\ \textit{in each grade within cycle c} \end{bmatrix} \div \begin{bmatrix} \textit{number of years} \\ \textit{in cycle c} \end{bmatrix}$$	$c \in CEDU$ $t \in T$	graduation rate by cycle (ratio cycle graduates over enrollment)
(84)	$$SHRed_{"contcyc",c,t} = SHRed_{"grd",c,t} - SHRed_{"grdcyc",c,t}$$ $$\begin{bmatrix} \textit{student share that} \\ \textit{continues in cycle} \\ \textit{c in year t} \end{bmatrix} = \begin{bmatrix} \textit{student share that} \\ \textit{graduates in each} \\ \textit{grade within cycle c} \end{bmatrix} - \begin{bmatrix} \textit{student share that} \\ \textit{graduates from} \\ \textit{cycle c in year t} \end{bmatrix}$$	$c \in CEDU$ $t \in T$	continuation rate by cycle
(85)	$$MDGVAL_{"mdg2",t} = \prod_{\substack{b\in B,t'\in T11 \\ \mid mmdg2(t,b,t')}} SHRed_{b,"c\text{-}edup1",t'}$$ $$\begin{bmatrix} \textit{first cycle primary school} \\ \textit{net completion rate} \end{bmatrix} = \begin{bmatrix} \textit{product of student shares} \\ \textit{for first cycle primary} \end{bmatrix}$$	$t \in T$	MDG 2
(86)	$$QENROLD_{c,t} = SHRed_{"contcyc",c,t\text{-}1} \cdot QENR_{c,t-1} + SHRed_{"rep",c,t\text{-}1} \cdot QENR_{c,t-1}$$ $$\begin{bmatrix} \textit{number old students} \\ \textit{enrolled in cycle c in t} \end{bmatrix} = \begin{bmatrix} \textit{enrolled in cycle c in t - 1} \\ \textit{who continue in c} \end{bmatrix} + \begin{bmatrix} \textit{enrolled in c in} \\ \textit{t - 1 who repeated c} \end{bmatrix}$$	$c \in CEDU$ $t \in T$ $t > 1$	Enrollment old students
(87)	$$QENRNEW_{c,t} = \sum_{c'\in C} grdcont01_{c,c'} \cdot SHRed_{"grdcont",c',t\text{-}1} \cdot SHRed_{"grdcyc",c',t\text{-}1} \cdot QENR_{c',t-1} + SHRed_{"g1entry",c',t\text{-}1} \cdot popageg1_t + qg1entncoh_{c,t}$$ $$\begin{bmatrix} \textit{number new students} \\ \textit{enrolled in cycle c in t} \end{bmatrix} = \begin{bmatrix} \textit{enrolled in preceding cycle c' in} \\ \textit{t - 1 who graduated and entered c} \end{bmatrix} + \begin{bmatrix} \textit{(cohort) students entering} \\ \textit{cycle c (c = primary)} \end{bmatrix} + \begin{bmatrix} \textit{(non - cohort) students entering} \\ \textit{c from outside school system} \end{bmatrix}$$	$c \in CEDU$ $t \in T$ $t > 1$	Enrollment new students

(88)	$QENR_{c,t} = QENROLD_{c,t} + QENRNEW_{c,t}$ $\begin{bmatrix} total\ number\ enrolled \\ in\ cycle\ c\ in\ t \end{bmatrix} = \begin{bmatrix} enrolled\ old\ students \\ in\ cycle\ c\ in\ t \end{bmatrix} + \begin{bmatrix} enrolled\ new\ students \\ in\ cycle\ c\ in\ t \end{bmatrix}$	$c \in CEDU$ $t \in T$ $t > 1$	Total Enrollment
(89)	$QFACINS_{i,f,t} = shif^{0}_{i,f,t} \cdot (1 - depr_f) \cdot \sum_{i' \in INS} QFACINS_{i',f,t-1}$ $\begin{bmatrix} endowment\ of\ labor\ type\ f \\ for\ institution\ i\ in\ t \end{bmatrix} = \begin{bmatrix} share\ of\ i\ in \\ labor\ type\ f \end{bmatrix} \cdot \begin{bmatrix} non\text{-}retired\ labor \\ from\ previous\ year \end{bmatrix}$ $+ \sum_{\substack{c \in C \\ \mid (f,c) \in MFC}} \left(shrlabent_{c,t-1} \cdot SHRed_{"grdexit",c,t-1} \cdot SHRed_{"grdcyc",c,t-1} \cdot QENR_{c,t-1} \right.$ $+ [enrolled\ in\ school\ in\ t-1,\ who\ graduate, exit\ the\ school\ system, and\ enter\ the\ labor\ force\ in\ t]$ $+ \sum_{c' \in C} \left(shrdemot01_{c,c'} \cdot shrlabent_{c',t-1} \cdot SHRed_{"dropout",c',t-1} \cdot QENR_{c',t-1} \right) \Big)$ $+ [enrolled\ in\ school\ in\ t-1,\ who\ dropout, and\ enter\ the\ labor\ force\ in\ t\ at\ the\ next\ lower\ level\ c]$ $+ shrlabent2_f \cdot \left(popagelab_t - \sum_{c \in CELA} QENRNEW_{c,t} \right)$ $[entrants\ to\ labor\ force\ from\ outside\ educational\ system\ who\ are\ of\ labor\text{-}force\text{-}age]$	$i \in INS$ $f \in FLAB$ $t \in T$ $t > 1$	Labor supply
(90)	$MDGVAL_{mdg,t} = extmdg_{mdg} + \dfrac{\alpha mdg_{mdg}}{1 + \gamma mdg_{mdg} \cdot EXP\left(\beta mdg_{mdg} \cdot \left(MDGINT_{mdg,t} - MDGVAL^{0}_{mdg}\right)\right)}$ $\begin{bmatrix} MDG \\ value \end{bmatrix} = \begin{bmatrix} logistic\ function\ of\ intermediate \\ MDG\ value\left(MDGINT_{mdg,t}\right) \end{bmatrix}$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (logistic function)
(91)	$MDGINT_{mdg,t} = \alpha m_{mdg} \cdot \left(\prod_{cmdg \in CMDG} \left(\sum_{\substack{c \in C \\ \mid (cmdg,c) \in MCM}} \frac{QQ_{c,t}}{poptot_t} \right)^{\varphi m_{mdg,cmdg}} \right)$ $\cdot \left(\sum_{i \in INS} QFACINS_{i,"fcapinf",t} \right)^{\varphi m_{mdg,"fcapinf"}} \cdot$ $\cdot \left(\prod_{mdg' \in MDGSTD} MDGVAL_{mdg',t}^{\varphi m_{mdg,mdg'}} \right) \cdot QHPC_t^{\varphi m_{mdg,"hhdconspc"}}$ $\begin{bmatrix} intermediate\ variable \\ for\ MDGs\ 4\ and\ 5 \end{bmatrix} = \begin{bmatrix} exogenous \\ parameter \end{bmatrix} \cdot \begin{bmatrix} influence\ of: real\ value\ for\ services\ per\ capita; \\ level\ of\ infrastructure;\ water\ and\ sanitation\ MDGs; \\ household\ consumption\ per\ capita; \end{bmatrix}$	$mdg \in MDGSTD$ $t \in T$	MDGs 4, 5, 7a, and 7b (constant-elasticity function defining intermediate variable)

Table 5. Alternative closure rules for macro balances

Government	**GOV-1**	**GOV-2**	**GOV-3**	**GOV-4**	**GOV-5**
Direct tax rates	fixed	flexible*	fixed	flexible*	fixed
Government savings	flexible	flexible	flexible	fixed	flexible
Government bond borrowing	flexible	fixed	fixed	fixed	fixed
Foreign borrowing	fixed	fixed	fixed	fixed	flexible
Foreign grants	fixed	fixed	flexible	flexible	fixed
Rest of World	**ROW-1**	**ROW-2**	**ROW-3**		
Exchange rate	flexible	flexible	flexible		
Foreign grants	fixed	flexible	fixed		
Foreign borrowing	fixed	fixed	flexible		
Savings-investment	**SI-1**	**SI-2**	**SI-3**		
Private investment absorption share	fixed	flexible	flexible		
Total investment absorption share	flexible	flexible	fixed		
Private savings rate	flexible	fixed	flexible		

Notes: * uniform point change for selected domestic non-government institutions